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Ensuring efficiency and reliability of equipment with optimization of integrated tests

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ABSTRACT

The paper suggests a method for optimization of the process of integrated tests for complex technical equipment of automotive, aviation, and rocket systems based on an analysis of efficiency dynamics models. The method makes it possible to determine scopes of ground tests for complex technical equipment, which are minimally required to start field tests under the necessity to combine ground and field development. Exponential models of complex systems' development are used as efficiency dynamics models for the whole testing process as well as efficiency dynamics models at various levels of the hierarchy of tests. The paper addresses an optimization task when the structure of tests at each level of the hierarchy is specified, i.e. efficiency dynamics models are determined for each level. The authors determine optimal points of transition from one level of tests to another, considering the random nature of efficiency dynamics parameters. A method for optimization using determination of the optimal scope of field tests is given.

1. Introduction

Further improvement of methodological support for test planning is an important issue in experimental development of complex technical systems including integrated launch vehicles (ILVs). Thus far, scientists developed methodological support for scope planning of ground tests for flight vehicle (FV) systems and FV flight tests where the "efficiency – cost – time" criterion is fundamental [1]. Due to that support it is possible to determine the required testing scope based on minimum costs needed to achieve the specified levels of technical characteristics' estimates and reliability subject to testing time constraints, and plan the scope of FV systems' ground development prior to the start of flight tests. The procedure for experimental development regulated by codes and specifications provides for transition to the stage of flight tests after ground development is completed. As practice of testing various types of latest-generation FVs shows, due to various reasons (slow delivery, irregular financing, increased duration of tests due to upgrades, etc.), the actual duration of FV ground development exceeded the estimated duration, dictating the need to combine ground development with flight tests.

As for ILVs, a failure to complete ground development of ILV systems by the start of flight tests can lead to accidents at launch and, therefore, to larger costs. That is why it is very important to determine optimal (in terms of minimum total costs) scopes of ground development for ILV systems under the necessity to combine different stages of experimental development. In turn, total costs for experimental ILV development shall include costs for ground development, flight tests and possible damage in monetary terms, incurred in case of an accident at launch during flight tests that would include, among other things, costs for the lost spacecraft. Minimization of total costs for experimental ILV development cannot be achieved without optimal allocation of costs for ground development of individual systems, which is conditioned by the dependence of possible damage in monetary terms in case of accidents at launch on the level of systems' development. Therefore, it becomes possible to relate the damage in monetary terms to the scopes of ILV systems' ground tests and costs for such tests. Then, the task of optimizing allocation of costs for ground development and identifying the optimal scope of ILV systems' ground tests reduces to finding such number of tests that would ensure

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maximum reduction of total costs. The task can be defined as follows: to develop an optimal plan for experimental ground development of ILV systems under the necessity to combine ground and flight tests, it is required to determine such scopes of ground tests, implementation of which (prior to flight tests) will make it possible to minimize total costs that account for possible damage in monetary terms in case of accidents at launch during flight tests caused by a failure to complete ground development of ILV systems.

Numerous papers address methods of planning integrated tests for complex technical objects.

Antonov et al. [2] considered issues of planning the testing scope for high-reliability objects. During development and manufacturing of new specimens of systems, a task of identifying their reliability arises. Field tests performed according to a certain plan represent the most objective way to determine reliability characteristics. One of the most widely used test plans is [N,U,T] plan where N non-reparable specimens are tested within a time interval from 0 to T. It is assumed that, during tests, k items fail, while N-k items pass them successfully. Thus, after an experiment, we will have a mixed sample including k failures and N-k right censored observations. However, if the item tested is highly reliable, it is quite possible that in some time interval [0, T] there will be no failures, i.e. k will equal to 0 because the probability of failure in this interval is extremely small, and the number of items tested is limited.

Strukov and Senachin [3] considered issues of experiment planning in testing of internal combustion engines. However, the method of planning experimental researches does not make it possible to determine scopes of integrated tests to ensure the required specifications of items.

Shevchenko [4] suggested a method to plan scopes of ground development for integrated launch vehicles, minimizing total costs including costs for ground development, flight tests, and possible damage in monetary terms, incurred in case of accidents at launch during flight tests. The method makes it possible to determine scopes of ILV ground tests, which are minimally required to start field tests under the necessity to combine ground and field development.

Lukin et al. [5] suggested a method of planning the period of control tests for technical systems of long-term regular use, failure-free operation of which is characterized by gamma distribution, to verify requirements for mean time between failures and probability of achieving the target.

Lukin and Sukhoruchenkov [6] provided a rationale for methods of planning testing scopes for technical systems to confirm that parameters regarding normal distribution of random scalar characteristics of technical systems' performance comply with the specified requirements.

Ermakova [7] addressed tasks of planning and integrating tests for spacecrafts and their on-board systems at different stages of ground development and flight tests in order to meet the requirements for reliability and flying life of automatic spacecrafts (AS), reduction of resource consumption and time for their development as well as time for trial-run inspection.

In their another paper, Antonov et al. [8] considered issues of planning the testing scope for high-reliability objects as well. During development and manufacturing of new specimens of systems, a task of identifying their reliability arises. This is due to the fact that there are requirements concerning the need to present those indicators in passports and specifications of products supplied to the market. Field tests represent the most objective way to determine reliability characteristics. However, when manufacturing complex expensive items, it is impossible to test a large batch of finished products. Thus, it is required to determine the duration of field tests and the scope of items to be tested, provided that the requirements for the accuracy of the resultant estimate of reliability characteristics are specified. Scope planning is based on the manufacturer's requirements for the need to confirm the lower bound of probability of failure-free operation with the set confidence coefficient.

Vasilevsky et al. [9] also addressed tasks of planning and integrating tests for spacecrafts and their on-board systems at different stages of ground development and flight tests in order to meet the requirements for reliability and flying life of spacecrafts, provided a rationale for reduction of resource consumption and time for their development as well as time for trial-run inspection.

According to Pandian et al. [10], avionics (aeronautics and aerospace) industries must rely on components and systems of demonstrated high reliability. The paper discusses the issues that arise with the use of handbook-based methods in commercial and military avionics applications.

In this article [11], a high-order spacecraft test language, China aerospace test and operation language (CATOL), is given associated with the current test requirements; meanwhile, the structure of the language is presented.

XU et al. [12] conducted a study on a new airworthiness compliance verification method based on pilot aircraft-environment complex system simulation.

In their papers, Huang and Wang [13], Bayley et al. [14] addressed optimization of individual aspects of integrated testing.

According to Wang et al. [15], testability plays an important role in improving the readiness and decreasing the lifecycle cost of equipment. Aiming at the problems with a small sample of testability demonstration test data (TDTD) such as low evaluation confidence and inaccurate result, a testability evaluation method was proposed based on the prior information of multiple sources and Bayes theory.

According to Wang et al. [16], built-in-test (BIT) is responsible for equipment fault detection, so the test data correctness directly influences diagnosis results. The paper focuses on test results monitor and BIT equipment (BITE) failure judge, and a series of improved approaches is proposed.

2. Testing process optimization

During development and creation of new specimens of complex technical objects (automotive, aviation, space, etc.), various tests shall be conducted. These tests can be divided into two categories:

- Stage 1: stationary (bench) tests,
- Stage 2: field tests.

The cost of field tests is usually way more than the cost of bench tests (especially in design of aviation and rocket systems' equipment). In this case, it becomes important to determine the required scopes of tests at both stages in terms of their maximum efficiency and minimization of the cost of the whole testing scope. In this situation, the most difficult is to identify the moment of transition from the first test stage to the second one. Moreover, we shall remember that, during field tests, the environment is contaminated significantly with fuel and fuel combustion products, which means environmental problems and, therefore, commitment to reduce the scope of field tests.

When developing an integrated test program for complex technical objects (automotive, aviation, space, etc.), it is needed to solve the task of testing process optimization, i.e. the task of determining the optimal scope and content of all types of tests conducted during the design of objects.

Tests conducted at all levels of design development are interrelated, and they cannot be planned in isolation from the testing process. It is necessary to determine the scope and content of individual tests so that the testing process would have optimal qualities, i.e. would ensure the specified efficiency and reliability of the system at minimum financial and time expenditures.

To optimize the testing process, it is convenient to use a method based on the analysis of efficiency dynamics models. This method makes it possible to determine the optimal requirements for the efficiency of each level of tests (under the specified laws of efficiency dynamics for each level of the hierarchy and general required efficiency and reliability of the system). Various models of complex systems' development, such as exponential model, logistic model, etc., can be used as efficiency dynamics models during optimization.

Without limiting the generality of reasoning, to provide better clarity and simplicity of data, we adopt the exponential model as an efficiency dynamics model. It can be used both to describe the efficiency dynamics of the whole testing process and efficiency dynamics at various levels of the hierarchy. In the general case, the laws of efficiency dynamics at various levels of the hierarchy can vary, but that does not limit the possibility of using this method.

An analysis of the hierarchical structure of tests makes it possible to present the efficiency dynamics models in terms of time and cost at each i-th level of tests as follows:

$$W_i(\tau_i) = a_i - (a_i - W_{0i})\exp\{-\theta_i\tau_i\} \tag{1}$$

$$W_i(C_i) = b_i - (b_i - W_{0i})\exp\{-K_iC_i\} \tag{2}$$

where:

$t_{0i} \le \tau_i \le t_{0i+1}; C_{0i} \le C_i \le C_{0i+1};$

a_i, b_i — limit values of efficiency for the i-th level of tests;

W_{0i} — initial value of efficiency at the i-th level of tests;

θ_i, K_i — indicators of efficiency growth in terms of time and cost, respectively;

t_{0i}, C_{0i} — time and cost by the start of the i-th level of tests, respectively.

According to the specifics of various levels of tests, the following conditions are met:

$$\left.\begin{array}{l} a_i > a_{i-1}; \theta_i < \theta_{i-1}; \\ b_i > b_{i-1}; K_i < K_{i-1}. \end{array}\right\} \tag{3}$$

Let us accept the following generalized efficiency criterion as an optimality criterion:

$$E = \frac{Q}{S} \tag{4}$$

The maximum efficiency increase ensured by the implementation of an integrated testing program is considered as output Q:

$$Q = W_c - W_0 \tag{5}$$

where W_0 — efficiency by the start of the tests, W_c.— efficiency of completed tests.

Both the average time $\overline{T}$ necessary for the implementation of an integrated testing program and the average cost for the program $\overline{C}$ can be considered as costs S:

$$S\left\{\begin{array}{l}\overline{T}\\ \overline{C}\end{array}\right. \tag{6}$$

Since the value of output $W_c - W_0$ in this task is specified, the maximum of the efficiency criterion (4) is achieved at minimum costs S. Thus, the task of testing process optimization reduces to the task of developing such testing program that would require minimum time and cost expenditures to achieve the specified efficiency.

Let us analyze such setting of the optimization task when the structure of tests at each level of the hierarchy is specified, i.e. efficiency dynamics models are determined for each level.

With such an approach, the task is set in the following way.

Let us assume that by the start of the tests, i.e. at $t_0 = c_0 = 0$, the system is characterized by some initial efficiency W_0. After the tests, due to identification and elimination of design defects, it is necessary to improve the system efficiency up to some specified value W_c. It is known that transition from state W_0 into state W_c takes N stages corresponding to N levels of the hierarchy of tests. At each i-th stage of the tests, the system efficiency is improved from the initial value W_{0i} up to some value $W_i = W_{0i+1}$, which, in turn, is the initial value of efficiency for the next stage.

During the tests, the current system efficiency is improved according to the efficiency dynamics model typical for that stage.

Let us assume that the estimates of efficiency dynamics model parameters $\overline{a}_i, \overline{\theta}_i, \overline{b}_i, \overline{K}_i$ and their dispersions for each stage are known. Then, the time and cost necessary for transition of the system from state W_0 into state W_c at the specified parameters of the efficiency dynamics model can be determined only by the location of the points of transition from one level of tests to another, i.e. the initial values of efficiency W_{0i} at $i = 2, \ldots, n$. Therefore, to ensure optimal testing process, we shall find such transition points that would ensure the minimum total time and cost of the tests necessary for transition of the system from state W_0 into state W_c.

Let us consider at first the task of determining transition points minimizing the total time of testing at non-random values of efficiency dynamics parameters.

The total time and cost of testing represent a combination of time and costs at particular levels of the hierarchy of tests. In other words, the time and cost are additive criteria of optimality.

Moreover, based on expressions (1) and (2), it can be seen that the state of the system at the i-th stage W_i depends only on the state at the i-1 stage W_{0i} and does not depend on the way the system fell into the state W_{0i}.

Thus, the conditions for the application of the dynamic programming method to solve the optimization task are met.

In accordance with the dynamic programming method, we can start testing process optimization from the end of the tests, i.e. from the n-th stage, defining the value of the optimality criterion at the last, n-th stage as Φ_n.

Typically, the value of the particular criterion at this stage is adopted as this value, i.e.

$$\Phi_n = \tau_n \tag{7}$$

the value of the criterion $\Phi_{n,\ldots,i}$ at the last i-th stages:

$$\Phi_{n,\ldots,i} = \tau_n + \ldots + \tau_i \tag{8}$$

The value Φ_n at the specified parameters a_n, θ_n and the specified value W_c depends only on W_{0n}, which in this case represents "control". It is clear that the minimum value $\Phi_n^* = \tau_n^* = 0$ is achieved when: $W_{0n}^* = W_c$.

Moving on to minimizing the value $\Phi_{n,n-1}$ at the found optimal W_{0n}^*, we can determine the optimal value of $W_{0\,n-1}$, which will also equal to W_c.

Continuing with stage-by-stage optimization, we can derive a solution for any i-th step:

$$\Phi_{n,n-1\ldots,i} = 0 \tag{9}$$

where $W_{0i}^* = W_c$, $i = 2, \ldots, n$.

Thus, as a result of described optimization, we can derive a solution, according to which the minimum total time of tests (equal to zero) can be obtained when the system (after the tests at the first level of the hierarchy) improves its efficiency from W_0 to W_c. However, based on the given analysis of the hierarchical structure of the testing process, it is clear that, at the initial efficiency W_0, the system cannot be driven to state W_c at a single step.

In the case considered, no restrictions were placed on the acceptable region. However, such restriction exists: W_{0i} shall be found on the curve of efficiency dynamics at the i-1 level.

This restriction can be taken into account if we adopt the following as a function:

$$\Phi_n = \tau_n + \tau_{n-1} = \frac{1}{\theta_n}\ln\frac{a_n - W_{0n}}{a_n - W_c} + \frac{1}{\theta_{n-1}}\ln\frac{a_{n-1} - W_{0n-1}}{a_{n-1} - W_{0n}}. \tag{10}$$

At the specified parameters $\theta_n, \theta_{n-1}, a_n, a_{n-1}, W_c$, function Φ_n depends only on the values W_{0n}, W_{0n-1}, i.e.:

$$\Phi_n = \Phi_n(W_{0n}, W_{0n-1}). \tag{11}$$

According to the dynamic programming method, we can perform conditional optimization of Φ_n, assuming that the W_{0n-1} state is known.

Differentiating expression (10) with respect to W_{0n} and equating the derivative to zero, we can obtain a condition for optimal transition from the n-1 level to the n-th level:

$$\theta_n\left(a_n - W_{0n}^{*t}\right) = \theta_{n-1}\left(a_{n-1} - W_{0n}^{*t}\right) \tag{12}$$

Let us analyze this condition. The left side of the equation is derivative $\left.\frac{dW_n}{d\tau_n}\right|_{\tau_n=0}$, and the right side is derivative $\left.\frac{dW_{n-1}}{d\tau_{n-1}}\right|_{\tau_{n-1}}$.

Thus, the optimal transition point is the point where efficiency growth rates at the n-1 level at the transition point and at the n-th level at the initial point are equal.

Let us analyze ratio (12) to conclude that the extreme point is the minimum point. At $W_{0n} < W_{0n}^{*t}$, the following inequality is true:

$$\theta_n(a_n - W_{0n}) < \theta_{n-1}(a_{n-1} - W_{0n}).$$

In this case, the value of the derivative $\frac{\partial \Phi_n}{\partial W_{0n}} < 0$.

At $W_{0n} > W_{0n}^{*t}$:

$$\theta_n(a_n - W_{0n}) > \theta_{n-1}(a_{n-1} - W_{0n})$$

and the value of the derivative $\frac{\partial \Phi_n}{\partial W_{0n}} > 0$.

Thus, the extreme point matches the minimum of the value Φ_n.

Let us proceed with conditional optimization of $\Phi_{n,n-1}$ at the found optimal value:

$$W_{0n}^{*t} = \frac{\theta_n a_n - \theta_{n-1} a_{n-1}}{\theta_n - \theta_{n-1}} \tag{13}$$

and assuming that W_{0n-2} is known:

$$\Phi_{n,n-1} = \frac{1}{\theta_n}\ln\frac{a_n - W_{0n}^{*t}}{a_n - W_3} + \frac{1}{\theta_n - 1}\ln \times \frac{a_{n-1} - W_{0n-1}}{a_{n-1} - W_{0n}^{*t}} + \frac{1}{\theta_{n-2}}\ln\frac{a_{n-2} - W_{0n-2}}{a_{n-2} - W_{0n-1}} \tag{14}$$

Differentiating $\Phi_{n,n-1}$ with respect to W_{0n-1} and equating the derivative to zero, we can find a condition for optimal transition from the n-2 level to the n-1 level, similar to the previous condition (12):

$$\theta_{n-1}\left(a_{n-1} - W_{0n-1}^{*t}\right) = \theta_{n-2}\left(a_{n-2} - W_{0n-1}^{*t}\right) \tag{15}$$

In a similar way, we can find a condition for optimal transition from any i-1 level to the i-th level at $i = 2, \dots, n$:

$$\theta_i\left(a_i - W_{0i}^{*t}\right) = \theta_{i-1}\left(a_{i-1} - W_{0i}^{*t}\right) \tag{16}$$

whence it follows that:

$$W_{0i}^{*t} = \frac{\theta_i a_i - \theta_{i-1} a_{i-1}}{\theta_i - \theta_{i-1}} \tag{17}$$

The ratios derived have a clear physical sense. In fact, the specified segment of the trajectory $(W_c - W_0)$ can be passed in minimum time if the growth rate W is maximum. It is condition (16) that ensures the maximum speed of movement (Figure 1). If movement starts at some point $W_{0i} < W_{0i}^{*t}$ where the speed of movement along the i-1 curve of efficiency dynamics is higher than the speed of movement along the i-th curve, then time losses will be observed at the trajectory segment $(W_{0i}^{*t} - W_{0i}^{1})$. The situation is similar when $W_{0i} > W_{0i}^{*t}$.

The value of optimal time T can be determined as follows:

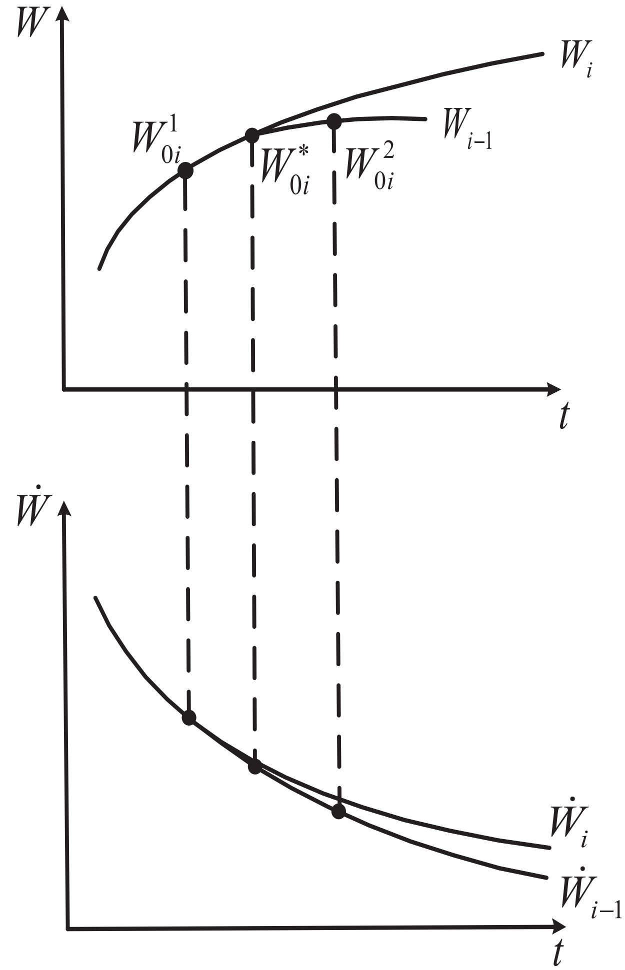

Figure 1. Choosing the optimal point of transition to the highest test level.

$$T^* = \sum_i \tau_i^* = \sum_i \frac{1}{\theta_i}\ln\frac{a_i - W_{0i-1}^{*t}}{a_i - W_{0i}^{*t}} \tag{18}$$

The condition of optimal transition (expressed as the equality of the derivatives of the current efficiency at the preceding and subsequent levels) can be obtained when all levels are described by logistic models of efficiency growth, as well as when some levels of the hierarchy of tests are described by exponential models, while the rest of them — by logistic ones.

Let us present the condition of optimal transition from the i-1 level to the i-th level of tests for the case when both levels are described by logistic models. In this case, the condition of equality of the derivatives at the transition point can be written as follows:

$$\frac{\theta_i}{a_i}\left(a_i - W_{0i}^{*t}\right) = \frac{\theta_{i-1}}{a_{i-1}}\left(a_{i-1} - W_{0i}^{*t}\right) \tag{19}$$

whence it follows that:

$$W_{0i}^{*t} = \frac{\theta_i - \theta_{i-1}}{\theta_i / a_i - \theta_{i-1}/a_{i-1}} \tag{20}$$

3. Determining optimal points of transition from stationary to field tests considering the random nature of efficiency dynamics parameters

Let us replace the random efficiency dynamics model with an averaged non-random model and solve the task as a deterministic problem of dynamic programming. The average value of test time is taken as an optimality criterion.

We will consider the following function as the particular average criterion $\overline{\Phi}_n$:

$$\overline{\Phi}_n = \overline{\tau}_n + \overline{\tau}_{n-1} = \frac{1}{\overline{\theta}_n}\ln\frac{\overline{a}_n - \overline{W}_{0n}}{\overline{a}_n - \overline{W}_c} + \frac{1}{\overline{\theta}_{n-1}}\ln\frac{\overline{a}_{n-1} - \overline{W}_{0n-1}}{\overline{a}_{n-1} - \overline{W}_{0n}} + \frac{1}{2}\left\{\frac{1}{\overline{\theta}_n}\left[\frac{1}{(\overline{a}_n - \overline{W}_c)^2} - \frac{1}{(\overline{a}_n - \overline{W}_{0n})^2}\right]\sigma^2(a_n) + \frac{1}{\overline{\theta}_{n-1}}\left[\frac{1}{(\overline{a}_{n-1} - \overline{W}_{0n})^2} - \frac{1}{(\overline{a}_{n-1} - \overline{W}_{0n-1})^2}\right]\sigma^2(a_{n-1}) + \frac{2}{\overline{\theta}_n^3}\ln\frac{\overline{a}_n - \overline{W}_{0n}}{\overline{a}_n - \overline{W}_c}\sigma^2(\theta_n) + \frac{2}{\overline{\theta}_{n-1}^3}\ln\frac{\overline{a}_{n-1} - \overline{W}_{0n-1}}{\overline{a}_{n-1} - \overline{W}_{0n}}\sigma^2(\theta_{n-1}) + \left[\frac{1}{\overline{\theta}_{n-1}}\frac{1}{(\overline{a}_{n-1} - \overline{W}_{0n})^2} - \frac{1}{\overline{\theta}_n}\frac{1}{(\overline{a}_n - \overline{W}_{0n})^2}\right]\sigma^2(W_{0n}) - \frac{1}{\overline{\theta}_{n-1}}\frac{1}{(\overline{a}_{n-1} - \overline{W}_{0n-1})^2}\sigma^2(W_{0n-1}) + \frac{1}{\overline{\theta}_n}\frac{1}{(\overline{a}_n - \overline{W}_c)} \times \sigma^2(W_c)\right\} \quad (21)$$

Let us find the conditional minimum of the function $\overline{\Phi}_n$, assuming that the values $\overline{a}_n, \overline{a}_{n-1}$, $\overline{\theta}_n, \overline{\theta}_{n-1}, \overline{W}_{0n-1}\overline{W}_c$, as well as $\sigma^2(a_n), \sigma^2(a_{n-1}), \sigma^2(\theta_n), \sigma^2(\theta_{n-1})\sigma^2(W_{0n-1})\sigma^2(W_{0n}), \sigma^2(W_c)$ are known. In this case, the function $\overline{\Phi}_n$ will depend only on the average value of $\overline{W}_{0n}$.

Differentiating expression (21) with respect to $\overline{W}_{0\,n}$ and equating the derivative to zero, we can find a condition for optimal transition from the n-1 level to the n-th level considering random characteristics of efficiency dynamics parameters. This condition can be written as follows:

$$\overline{\theta}_n\left(\overline{a}_n - \overline{W}_{0n}^{*t}\right)\Delta_{n-1}^t = \overline{\theta}_{n-1}\left(\overline{a}_{n-1} - \overline{W}_{0n}^{*t}\right)\Delta_n^t \quad (22)$$

where:

$$\Delta_n^t = 1 + \frac{\sigma^2\left(a_n - W_{0n}^{*t}\right)}{\left(\overline{a}_n - \overline{W}_{0n}^{*t}\right)^2} + \frac{\sigma^2(\theta_n)}{\overline{\theta}_n^2};$$

$$\Delta_{n-1}^t = 1 + \frac{\sigma^2\left(a_{n-1} - W_{0n}^{*t}\right)}{\left(\overline{a}_{n-1} - \overline{W}_{0n}^{*t}\right)^2} + \frac{\sigma^2(\theta_{n-1})}{\overline{\theta}_{n-1}^2}.$$

In the general case, the dispersion $\sigma^2(W_{0n})$ is unknown and depends on the choice of point W_{0n}. However, expressions Δ_n^t and Δ_{n-1}^t include relative values of the dispersions, i.e. ratios of the dispersions to the square of the estimated values. These ratios can be specified easily based on the average accuracy of the estimate of efficiency dynamics parameters, which is usually 10–300%.

Performing stage-by-stage optimization in accordance with the dynamic programming method, we can obtain a condition for optimal transition from the i-1 level to the i-th level of the hierarchy of tests in the following form:

$$\overline{\theta}_i\left(\overline{a}_i - \overline{W}_{0i}^{*t}\right)\Delta_{i-1}^t = \overline{\theta}_{i-1}\left(\overline{a}_{i-1} - \overline{W}_{0i}^{*t}\right)\Delta_i^t \quad (23)$$

where:

$$\Delta_i^t = 1 + \frac{\sigma^2\left(a_i - W_{0i}^{*t}\right)}{\left(\overline{a}_i - \overline{W}_{0i}^{*t}\right)^2} + \frac{\sigma^2(\theta_i)}{\overline{\theta}_i^2},$$

$$\Delta_{i-1}^t = 1 + \frac{\sigma^2\left(a_{i-1} - W_{0i}^{*t}\right)}{\left(\overline{a}_{i-1} - \overline{W}_{0i}^{*t}\right)^2} + \frac{\sigma^2(\theta_{i-1})}{\overline{\theta}_{i-1}^2},$$

whence it follows that:

$$W_{0i}^{*t} = \frac{\overline{\theta}\overline{a}_i\Delta_{i-1}^t - \overline{\theta}_{i-1}\overline{a}_{i-1}\Delta_i^t}{\overline{\theta}_i\Delta_{i-1}^t - \overline{\theta}_{i-1}\Delta_i^t}.$$

Applying the dynamic programming method for optimization of the average accuracy of tests, we can derive similar ratios that determine the optimal point of transition from the i-1 level to the i-th level of tests:

$$\overline{K}_i\left(\overline{b}_i - \overline{W}_{0i}^{*C}\right)\Delta_{i-1}^C = \overline{K}_{i-1}\left(\overline{b}_{i-1} - \overline{W}_{0i}^{*C}\right)\Delta_i^C \quad (24)$$

where:

$$\Delta_i^C = 1 + \frac{\sigma^2\left(b_i - W_{0i}^{*C}\right)}{\left(\overline{b}_i - \overline{W}_{0i}^{*C}\right)^2} + \frac{\sigma^2(K_i)}{\overline{K}_i^2},$$

$$\Delta_{i-1}^C = 1 + \frac{\sigma^2\left(b_{i-1} - W_{0i}^{*C}\right)}{\left(\overline{b}_{i-1} - \overline{W}_{0i}^{*C}\right)^2} + \frac{\sigma^2(K_{i-1})}{\overline{K}_{i-1}},$$

whence it follows that:

$$W_{0i}^{*C} = \frac{\overline{K}_i\overline{b}_i\Delta_{i-1}^C - \overline{K}_{i-1}\overline{b}_{i-1}\Delta_i^C}{\overline{K}_i\Delta_{i-1}^C - \overline{K}_{i-1}\Delta_i^C}.$$

It can be shown that the condition for optimal transition with regard to the cost (24) generally differs from the condition for optimal transition with regard to the time (22). For that purpose, let us consider several individual cases which are of our main interest. For the sake of simplicity and clarity, we assume that, at each level, the value of efficiency, ultimate with regard to the cost, equals to the value of efficiency, ultimate with regard to the time:

$$a_i = b_i; a_{i-1} = b_{i-1} \quad (25)$$

This means that, as for the current equipment development, the most advanced test facilities are used during tests. Let us also assume that the accuracy of the estimate of efficiency dynamics parameters is the same at all levels. Theт, the cost of tests at each level of the hierarchy can be associated with the time of tests using a proportional dependence:

$$C_i = \omega_i\tau_i \quad (26)$$

where ω_i — the proportionality factor.

Indeed, the time of tests can be determined as follows:

$$\tau_i = \tau'_i \cdot n \quad (27)$$

where

τ'_i — the time spent on one test;

n — the number of tests.

The cost is also proportional to the number of tests:

$$C_i = C'_i \cdot n \quad (28)$$

where

C'_i — the cost of one test.

The time spent on one test is composed of the time spent on preparation of the test, the time of the test itself and the time of analysis of the test results.

The cost of one test is composed of the costs for depreciation of test equipment during one test, depreciation of the tested specimen, as well as the cost of work of the operating personnel, etc.

Based on expressions (27) and (28), ratio (26) can be easily derived, where $\omega_i = \frac{C'_i}{\tau'_i}$ — the specific cost per unit of time for one test.

Applying condition (25) and ratios (27) and (28) to efficiency expressions (1) and (2), we can obtain a dependence between the efficiency growth indicators θ_i and K_i:

$$\theta_i = \omega_i K_i \tag{29}$$

Let us assume that the proportionality factor ω_i is the same for all levels, i.e. the specific cost is the same for the entire integrated testing program. In this case, as it follows from (23), (24), the choice of W_{0i}^{*t}, which ensures the minimum average time of tests, ensures the minimum cost of tests as well.

Let us consider a case when the specific cost at the *i*-1 level of tests is higher than that at the *i*-th level.

In this case, the optimum condition with regard to the cost requires a decrease in the value W_{0i}^{*C} as compared to the value W_{0i}^{*t} and, thus, dictates the need to increase the time of development at the *i*-th level of tests.

If the specific cost at the *i*-1 level is lower than that at the *i*-th level of tests, then to ensure the minimum cost, it is necessary to increase the time of development at the *i*-1 level.

Finally, let us consider a case when the accuracy of the parameters' estimate at the *i*-1 and the *i*-th levels is not the same. In this case, the time of development is re-allocated depending on the accuracy of the parameters' estimate: if the accuracy at the *i*-1 level is lower than that at the *i*-th level, the preference should be given to the *i*-th level, and vice versa.

4. Method for optimization of integrated tests using determination of the optimal scope of field tests as an example

For purposes of clear illustration of the optimization method, let us consider a specific example of determining the optimal scope of field tests. To solve the task, we will divide the whole hierarchy of tests into two categories: stationary tests and field development tests. Let us assume that changes in the efficiency at each level follow the exponential law. Let us also assume that the efficiency dynamics laws are completely specified for each level of tests, i.e. average values and dispersions of parameters that determine the efficiency dynamics at each level of the hierarchy are specified.

An approximate view of those curves is given in Figure 2.

Curve 1 corresponds to the efficiency growth in stationary tests, while curve 2 — in field tests.

As follows from the figure, if the entire development of the system up to the specified value of efficiency W_c was conducted only in a field test, then the time T would be needed. In case of stationary development tests, due to their specifics, the rate of efficiency growth is higher than that in a field test, but the ultimate value of efficiency a_s is lower than the specified value W_c. In this regard, to reduce the total time and cost of tests, development of the system up to the specified efficiency value W_{0f} that corresponds to point A should be conducted in stationary tests, while the final development up to the specified efficiency value W_c should be conducted in a field test. Let us determine the transition point from stationary to field tests that corresponds to the minimum of the average time or the average cost of tests.

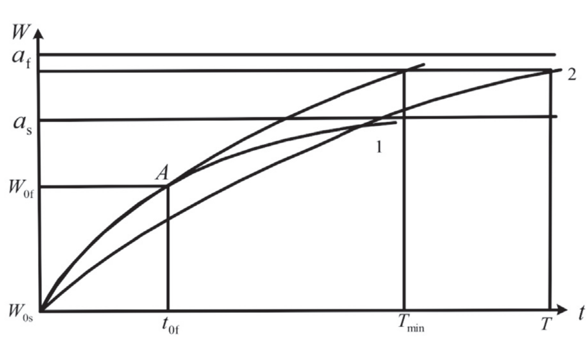

Figure 2. Optimal division into stationary and field development tests.

In accordance with Eqs. (23) and (24), we can derive a condition for optimal transition with regard to the time:

$$\bar{\theta}_f\left(\bar{a}_f - \bar{W}_{0f}^{*t}\right)\Delta_s^t = \bar{\theta}_s\left(\bar{a}_s - \bar{W}_{0f}^{*t}\right)\Delta_f^t,$$

where:

$$\Delta_f^t = 1 + \frac{\sigma^2\left(a_f - W_{0f}^{*t}\right)}{\left(\bar{a}_f - \bar{W}_{0f}^{*t}\right)^2} + \frac{\sigma(\theta_f)}{\bar{\theta}_f^2},$$

$$\Delta_s^t = 1 + \frac{\sigma^2\left(a_s - W_{0f}^{*t}\right)}{\left(\bar{a}_s - \bar{W}_{0f}^{*t}\right)^2} + \frac{\sigma^2(\theta_s)}{\bar{\theta}_s^2},$$

whence it follows that:

$$W_{0f}^{*t} = \frac{\bar{\theta}_f\bar{a}_f\Delta_s^t - \bar{\theta}_s\bar{a}_s\Delta_f^t}{\bar{\theta}_f\Delta_s^t - \bar{\theta}_s\Delta_f^t} \tag{30}$$

and a condition of optimal transition with regard to the cost:

$$\bar{K}_f\left(\bar{b}_f - \bar{W}_{0f}^{*C}\right)\Delta_s^C = \bar{K}_s\left(\bar{b}_s - \bar{W}_{0f}^{*C}\right)\Delta_f^C,$$

where:

$$\Delta_f^C = 1 + \frac{\sigma^2\left(b_f - W_{0f}^{*C}\right)}{\left(\bar{b}_f - \bar{W}_{0f}^{*C}\right)^2} + \frac{\sigma(K_f)}{\bar{K}_f^2},$$

$$\Delta_s^C = 1 + \frac{\sigma^2\left(b_s - W_{0f}^{*C}\right)}{\left(\bar{b}_s - \bar{W}_{0f}^{*C}\right)^2} + \frac{\sigma^2(K_s)}{\bar{K}_s^2},$$

whence it follows that:

$$W_{0f}^{*C} = \frac{\bar{K}_f\bar{b}_f\Delta_s^C - \bar{K}_s\bar{b}_s\Delta_f^C}{\bar{K}_f\Delta_s^C - K_s\Delta_f^C} \tag{31}$$

Conditions (30) and (31) coincide when:

$$a_f = b_f, a_s = b_s,$$

$$\omega_f = \omega_s,$$

$$\Delta_s^C = \Delta_f^C = \Delta_s^t = \Delta_f^t.$$

At $\omega_f < \omega_s$, it is cost-efficient to increase the scope of field development, but in this case the total time of tests slightly increases.

This is the case, for example, in development of relatively inexpensive one-time items, when the cost of stationary test facility operation is higher than the cost of field tests for such items.

In tests of expensive items, for example, spacecrafts, ratio $\omega_f > \omega_s$ is valid, and stationary tests represent the main type of development.

Thus, values of efficiency dynamics characteristics allow for rational allocation of time and costs between stationary and field tests.

5. Summary

The method for optimization of integrated tests of complex technical equipment of automotive, aviation, and rocket systems based on the analysis of efficiency dynamics models, suggested and considered in this paper, makes it possible to determine the scope of stationary tests for complex technical equipment that are minimally required to start field tests under the necessity to combine ground and field development. The exponential models of development of complex systems were used as efficiency dynamics models for the whole testing process as well as efficiency dynamics models at various levels of the hierarchy of tests. The optimization task considered in the paper, when the structure of tests at each level of the hierarchy is specified, i.e. when efficiency dynamics models are determined for each level, showed its efficiency in solving the tasks set in the paper. Determination of optimal points of transition from one level of tests to another, considering the random nature of efficiency dynamics parameters, was confirmed by the presented optimization method using determination of the optimal scope of field tests as an example.

6. Conclusion

Further improvement of methodological support for test planning is an important issue in experimental development of complex technical systems including integrated launch vehicles (ILVs). As for ILVs and other complex technical systems, a failure to complete ground development by the time of field tests can lead to accidents at launch and, therefore, to larger costs. The presented method for determination of points of transition from ground tests to field tests makes it possible to solve the problem efficiently.

Declarations

Author contribution statement

A.A. Boryaev & Zhu Yuqing: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

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Additional information

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