Integrating prospect theory with variable reference point into the conversion-based framework for linear ordinal ranking aggregation

Nana Liu[1] · Zeshui Xu[2] · Hangyao Wu[3] · Peijia Ren[4]

Abstract

Considering that converting linear ordinal ranking (LOR) information into interval utility values can not only improve the computability of LOR information but also explore the degree of preference for different alternatives for decision-makers hidden behind LOR information, this paper proposes a conversion-based LOR aggregation method to aggregate LOR information under risk. Given that the behaviours of decision-makers are influenced by risk, this paper adopts prospect theory to depict the decision-makers' behaviours under risk in the conversion-based aggregation process. To achieve this, the information energy for LOR is constructed firstly, and its features are analysed, which makes a basis for the conversion process. After that, the details about how to integrate the prospect theory with variable reference points into the conversion-based aggregation framework are presented. Finally, an example (exploring the financial product preferences of a group of respondents) evidences the practicality of the proposed method. Further, some analyses and discussions are conducted to verify the rationality and stability of the method.

Keywords Linear ordinal ranking aggregation · Information energy · Prospect theory

✉ Zeshui Xu
  xuzeshui@263.net

  Nana Liu
  liunana_lala@qq.com

  Hangyao Wu
  wwwhy1012@163.com

  Peijia Ren
  renpeijia@outlook.com

1 School of Business Administration, Chongqing Technology and Business University, Chongqing 400067, China

2 Business School, Sichuan University, Chengdu 610064, Sichuan, China

3 School of Management Science and Engineering, Chongqing Technology and Business University, Chongqing 400067, China

4 School of Business Administration, South China University of Technology, Guangzhou 510641, China

1 Introduction

Nowadays, individuals, enterprises and government departments are faced with various problems that require judgment and choice. Thus, how to obtain effective decision-making results has a huge and profound impact on people's production and life as well as social development. With the improvement of technology, the information we receive becomes more and more complex, making the decision-making process more challenging. Moreover, when solving realistic decision-making problems, due to incomplete access to information, it is always difficult for individuals to make appropriate decisions efficiently (Carneiro et al. 2021). To deal with the problem, group decision making, considering opinions from different decision-makers (DMs) comprehensively, is developed to eliminate the negative influence (Gou et al. 2019). Generally, the evaluation information provided by DMs can be segmented to two categories, i.e. the cardinal information and the ordinal information (Gonzalez-Arteaga et al. 2016). Cardinal information is mainly described by some crisp numbers and presented by some "absolute amounts". For

example, in some evaluation problems, DMs are required to provide precise scores, or specific values of an alternative under an indicator. On the contrary, ordinal information is commonly presented in a "relative amount" style (Chen et al. 2013), i.e. generating the evaluation information by comparing different alternatives, such as fuzzy preference relations (Georgescu 2007), linguistic preference relations (Liu et al. 2020b) and weak orders (Encheva 2012). In fact, for most of complex decision-making problems, DMs are hard to make objective judgments and provide precise evaluations directly. Therefore, ordinal information, as a more flexible expression way, exhibits its superiority in describing the comparative judgments of alternatives and possesses the application prospect in the complex decision-making environment. Linear ordinal ranking (LOR), in which all the alternatives are ranked from best to worst or inversely, comes out to be one of the most frequently used ordinal information and has been applied to many scenarios, such as product recommendation (Liu et al. 2020a), market segmentation (Liu et al. 2019), strategic alliance selection (Xu 2013) and the design of smart city (Dopazo and Martinezcespedes 2015), etc.

As the information aggregation is one of the most important issues in group decision-making (Vanicek et al. 2009), the LOR information aggregation has received much attention. The existing studies on LORs aggregation can be mainly divided into five categories, i.e. the position-based aggregation methods (Borda 1781; Cook and Kress 1985; Ali et al. 1986; Sese and Morishita 2001; Contreras 2010; Brandenburg et al. 2013; Hou 2015), the machine learning-based aggregation methods (Klementiev et al. 2008; Chen et al. 2011), the relationship-based aggregation methods (Yager 2001; Franceschini et al. 2015, 2016; Liang et al. 2018), the conversion-based aggregation methods (Chiclana et al. 1998; Wang et al. 2005; Dopazo and Martinez-Cespedes 2017; Liu et al. 2021) and other methods (Cook and Kress 1990; Dwork et al. 2001; Aledo et al. 2013). Considering that the conversion-based method can improve the computability of LOR information and dig the alternatives' utilities or DMs' preferences hidden behind the LOR information (Liu et al. 2021), this paper mainly aggregates the LOR information referring to the ideas in conversion-based methods.

In the conversion-based methods, Chiclana et al. (Chiclana et al. 1998) discussed the method to convert LOR into fuzzy preference relation in the fuzzy multi-objective problem and established a model that considered the consistency of preference information. Then, the converted information was aggregated by the average weighted operator, which provided a new idea for the LOR transformation. Through the establishment of a mathematical model, Wang et al. (2005) obtained the possible maximum and minimum values of the utility value of each alternative,

and the LORs were expressed in the form of interval utility values (IUVs), and then the calculation rules of IUVs were used to derive the aggregation results. The basic information aggregation mechanism was "LOR-IUV-LOR". In this process, the IUV could reflect the degree of preference of DMs more clearly. In addition, based on Perron's theorem, Hou et al. (2008) defined the ranking vector for LOR in the group decision-making environment through the eigenvector corresponding to the largest eigenvalue of the evaluation information matrix and then derived the aggregated ranking. Similarly, Dopazo et al. (2017) also aggregated the LORs by the priority weights. They proposed a two-stage method. Firstly, the priority weights of the alternatives were derived according to the LOR information, and then the final ranking was obtained based on the priority weight. In the first step, learning how to merge the preference matrix and collect group preferences from uncertain and possibly conflicting information was the main task. In the second step, combining the fuzzy preference relations and the properties of graph theory, the priority vectors were derived from the aggregated preference matrix. Recently, Yang et al. (2019) approximated the overall distribution of users' rankings by collecting a set of regional distributions that were selected from small-region independent models and then generated an aggregated ranking data set from the distributions. By working in a smaller domain instead of a larger domain, this method can significantly reduce the magnitude of added noise. Besides, combining with the concept of granular computing, Liu et al. (2021) proposed two models to convert LORs to IUVs to obtain the aggregated LOR.

With the above literature review, we can find that: (1) converting LOR to IUV instead of single-valued information can not only reduce information loss, but also avoid redundant information by controlling the length of IUV; (2) there are few studies that pay attention to the impact of the risks in the decision-making process on DMs, which may derive imprecise results; and (3) considering the DMs' decision-making behaviours under risks, some aggregation methods combine the prospect theory with preference-approval structures, in which the reference points are all fixed. However, in LORs, the reference points are undetermined, and DMs may have different attitudes towards risks. It is more reasonable to set a variable reference point. Therefore, in this paper, to reflect the different decision-making behaviours of DMs under risks, we introduce a new parameter to make the reference point variable in the aggregation process with the prospect theory.

The rest of the paper is organized as follows: Sect. 2 introduces some basic knowledge. Section 3 details the establishment of the proposed aggregation method and gives the framework of the method. Section 4 applies the proposed method to a numerical example and provides

some comparative analysis, sensitive analysis and simulation test to show the rationality and accuracy of the method. Finally, the contributions of the paper and some conclusions are presented in Sect. 5.

2 Preliminaries

In this section, we mainly present the basic knowledge that is used in this paper, i.e. the concepts of LOR, EPM, and prospect theory.

2.1 LOR and EPM

LOR is an efficient tool for DMs to express their preferences and have received much attention in recent years. Based on Refs. (Cook et al. 1986; Emond and Mason 2002), Liu et al. (2020a) gave the definition of LOR, in which all the alternatives in the finite set $\{A_1, A_2, \ldots A_n\}$ are ranked from the best to the worst or from the worst to the best linking by the symbols " $\sim$ ", " $\succ$ " and " $\prec$ ". Meanwhile, $A_i \succ A_j$ denotes that the alternative A_i is superior to the alternative A_j, $A_i \prec A_j$ denotes that the alternative A_i is inferior to the alternative A_j, and $A_i \sim A_j$ denotes that the preferences between A_i and A_j are the same.

Example 1 Assume that there are five alternatives in $\{A_1, A_2, A_3, A_4, A_5\}$, $o_i : A_1 \succ A_2 \succ A_3 \sim A_4 \succ A_5$ is one of the LORs.

In order to depict the information in LOR, the extended preference map (EPM) is introduced by Liu et al. (2020a) referring to the existing research of Gonzalez-Arteaga et al. (2016) and Hou and Triantaphyllou (2019), which is shown as follows:

Definition 1 Liu et al. (2020a) $P_j = [p_{i,j}]_{1 \times n}$ is called the EPM of a LOR o_j, where $p_{i,j}$ denotes the extended preference map element (EPME) of A_i in the LOR o_j. P_j must satisfy: (1) $\forall i, p_{i,j} \in I, I = \{1, 2, \ldots, i, \ldots, n\}$; (2) the cardinal of $p_{i,j}$ is 1, i.e. $|p_{i,j}| = 1$; (3) the elements in $P_j = [p_{i,j}]_{1 \times n}$ represent the positions of alternatives in o_j, and $p_{i,j} = 1 + n_{i,j}^b - (n_{i,j}^t - c_{i,j}^t)$, where $n_{i,j}^b$ denotes the number of alternatives that are better than A_i in o_j, $n_{i,j}^t$ denotes the total number of the alternatives that have the same preference degree but are better than A_i in o_j ($n_{i,j}^t \geq 2$ or $n_{i,j}^t = 0$), $c_{i,j}^t$ denotes the number of the group of the alternatives that have the same preference degree, but are better than A_i in o_j (in one group, there are at least two alternatives that have the same preference degree).

Remark 1 If the alternatives are all in the same preference level, then their corresponding EPMEs are all equal to 1.

Example 2 The corresponding EPM for o_1 : $A_1 \succ A_2 \succ A_3 \sim A_4 \succ A_5$ is $P_1 = [1, 2, 3, 3, 4]$. For the alternative A_1, $n_{1,1}^b = 0, n_{1,1}^t = 0, c_{1,1}^t = 0$, then $p_{1,1} = 1$. For the alternative A_2, $n_{2,1}^b = 1, n_{2,1}^t = 0, c_{2,1}^t = 0$, then $p_{2,1} = 2$. For the alternatives A_3 and A_4, $n_{3,1}^b = n_{4,1}^b = 2, n_{3,1}^t = n_{4,1}^t = 0, c_{3,1}^t = c_{4,1}^t = 0$, then $p_{3,1} = p_{4,1} = 3$. For the alternative A_5, $n_{5,1}^b = 4, n_{5,1}^t = 2, c_{5,1}^t = 1$, then, $p_{5,1} = 4$.

Besides, in order to capture some quantitative features of the EPM, Liu et al. (2021) gave some formulas to calculate the statistical values for the EPM $P_j = [p_{i,j}]_{1 \times n}$.

(1) The arithmetic mean: $M(P_j) = \frac{\sum_{i=1}^{n} p_{i,j}}{n}$;

(2) The variance: $V(P_j) = \frac{\sum_{i=1}^{n} (p_{i,j} - M(P_j))^2}{n}$;

(3) The standard deviation: $S(P_j) = \sqrt{\frac{\sum_{i=1}^{n} (p_{i,j} - M(P_j))^2}{n}}$;

(4) The coefficient of variation: $CV(P_j) = \frac{S(P_j)}{M(P_j)}$ (If $M(P_j) = 0$, then we hold the view that $CV(P_j) = 0$), where $M(P_j)$, $V(P_j)$, $S(P_j)$ and $CV(P_j)$ denote the arithmetic mean, the variance, the standard deviation and the coefficient of variation, respectively, $p_{i,j}$ is the EPME of the alternative A_i in o_j, n is the number of the alternatives.

2.2 Prospect theory

Prospect theory, a classic bounded rationality theory, is usually applied to depict the DMs' decision-making behaviours under risk, which distinguishes the choice process to two main phases, i.e. the editing phase and the evaluation process (Kahneman and Tversky 1979). There are several main operations in the editing phase, i.e. combination, segregation and cancellation, which transform the outcomes and probabilities associated with the offered prospects. In the evaluation phase, the edited prospects are assessed and the prospects with the highest value will be chosen (Kahneman and Tversky 1979). The theory points out that DMs usually have different attitudes towards gains and losses. Most of DMs are risk-averse to gains and are risk seeking to losses. Besides, DMs are more sensitive to losses than gains (Kahneman and Tversky 1979). Based on the features, the value function and weighting function in prospect theory are presented (Tversky and Kahneman 1992).

$$v(y) = \begin{cases} (y - y_0)^\alpha, & y \geq y_0 \\ -\lambda(y_0 - y)^\beta, & y < y_0 \end{cases} \quad (1)$$

$$w(p) = \begin{cases} \dfrac{p^\gamma}{[p^\gamma + (1-p)^\gamma]^{1/\gamma}}, & y \geq y_0 \\[3mm] \dfrac{p^\delta}{[p^\delta + (1-p)^\delta]^{1/\delta}}, & y < y_0 \end{cases} \quad (2)$$

where y_0 denotes the reference points, α and β are both positive real numbers that measure the curvature of the value function for gains and losses, respectively, λ denotes the coefficient of loss aversion and γ and δ are the risk attitude coefficients to gains and losses, respectively.

Then, the chart of the value function is shown in Fig. 1a, and the chart of the weighting function is shown in Fig. 1b.

3 Main method

In this section, the problem specification is firstly shown, which clarifies the foci of the proposed method. Then, the definition of information energy for LOR is given, and its properties are analysed. Finally, the whole framework of the conversion-based method combined with prospect theory is presented.

3.1 Problem specification

Suppose that there are n alternatives ($n \geq 2$) (denoted as $\{A_1, A_2, \ldots, A_i, \ldots, A_n\}$), m DMs (denoted as $\{D_1, D_2, \ldots, D_j, \ldots, D_m\}$), and each of them provides one LOR (denoted as $\{o_1, o_2, \ldots, o_j, \ldots, o_m\}$) to represent their preferences. Then, the corresponding EPMs for the LORs are obtained (denoted as $\{P_1, P_2, \ldots, P_j, \ldots, P_m\}$).

The conversion-based aggregation method proposed by Liu et al. (2021) assumes that the preferences of DMs are uniform (see Fig. 2a), which breaches reality due to the influence of risks on DMs' decision-making behaviours. To fit the realistic situation, this paper attempts to combine the prospect theory with the conversion-based LOR aggregation method.
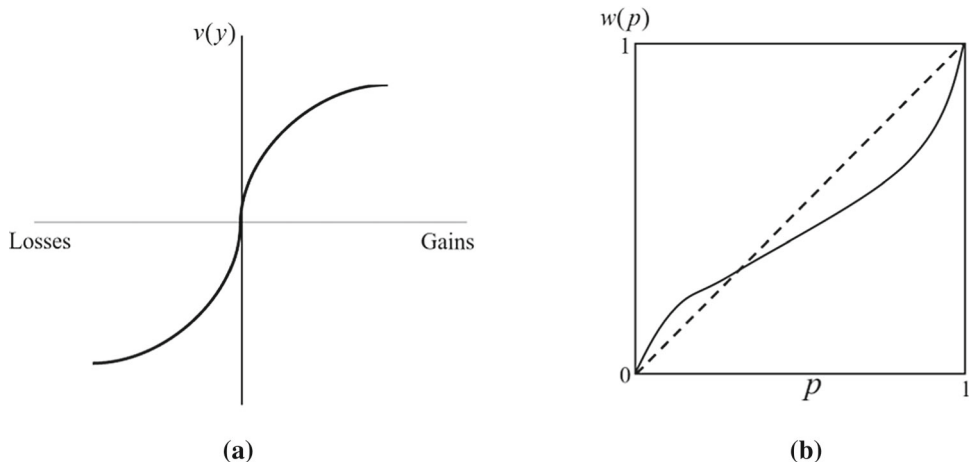
The basic idea of the proposed method is clearly illustrated by Fig. 2b, in which if the alternative A_i is recognized as gain and ranked at the kth position in the LOR o_j, the corresponding upper bound of its interval utility value is $U_{i_k j}^G$ ($k = 1, 2, \ldots, n$), the corresponding interval index is ρ_j^G; if the alternative A_i is recognized as loss and ranked at the kth position in the LOR o_j, the corresponding lower bound of its utility value is $U_{i_k j}^L$ ($k = 1, 2, \ldots, n$), the corresponding interval index is ρ_j^L, and $L_{i_k i_{k+1}}$ ($k \leq n-1$) denotes the maximum interval length of the IUV for the alternative A_i that is ranked at the kth position. It is worth noting that (1) the upper bounds of the IUVs for the alternatives in one LOR are different through the processing of the functions in the prospect theory; (2) the maximum lengths of the IUVs for the alternatives in one LOR are also different; and (3) the interval index for the alternatives in one LOR is variable. This method focuses on how to determine the upper bounds of the IUVs and the interval index for the alternatives in one LOR. Taking the advantage in reflecting the effectiveness of the LOR, in this paper, the information energy is used to determine the primary interval index. The primary values of the upper bounds follow the idea used by Liu et al. (2021). With these values, the functions in the prospect theory are utilized to determine the lower bounds of the IUVs.

3.2 Information energy for LOR

To measure the degree of fuzziness for fuzzy set, the concept of entropy was put forward by De Luca and Termini (1972). Correspondingly, the notion of the quantity measure for information, i.e. information energy, was proposed (Onicescu 1966). The information energy of a fuzzy set ie: $[0, 1] \rightarrow [0, 1]$ should follow the rules: (1) it is strictly decreasing in $[0, \frac{1}{2}]$ and strictly increasing in $[\frac{1}{2}, 1]$; (2) ie(0) = ie(1) = 1; and (3) ie$(\frac{1}{2}) = 0$ (Taheri and Azizi

Fig. 1 The charts for the value function and weighting function in the prospect theory
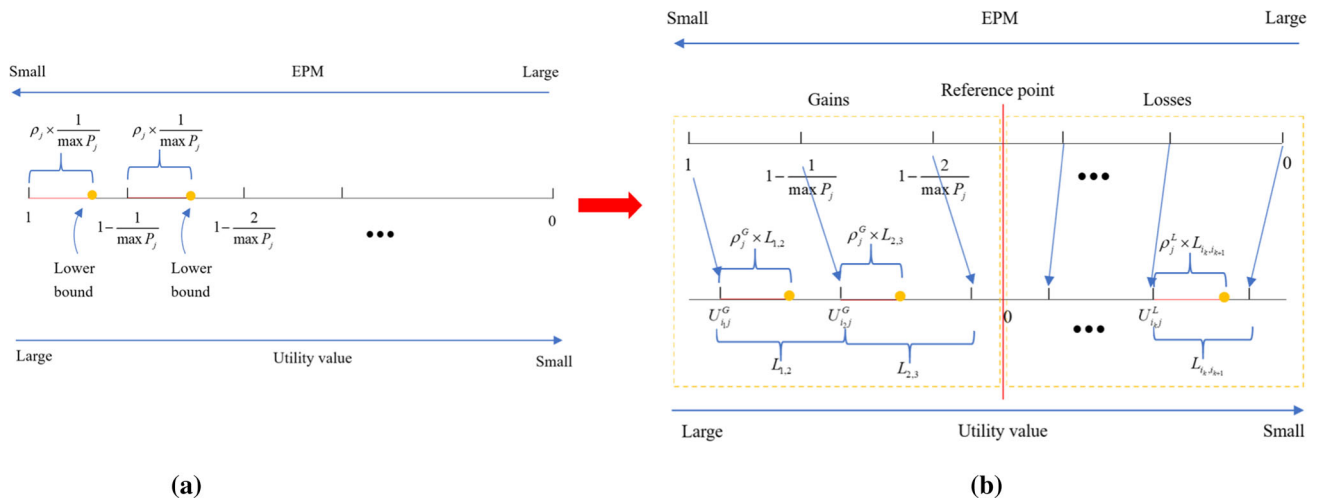

(a)

(b)

Fig. 2 The comparison of the two conversion-based methods

2007). The formula of the information energy for a fuzzy set (Dumitrescu 1977) is shown in Eq. (3).

$$\text{ie}(A(x)) = 2\left(A(x)^2 + A^c(x)^2\right) - 1 \tag{3}$$

where $\text{ie}(A(x))$ denotes the information energy for the fuzzy set A, $A(x)$ denotes the degree that the element x belongs to the fuzzy set, $A^c(x)$ denotes the degree that the element x not belongs to the fuzzy set and $A^c(x) = 1 - A(x)$.

Following the opinions in Ref. (Onicescu 1966), we firstly give the definition of the information energy for LOR.

Definition 2 The information energy for LOR is a measure of the effectiveness of the LOR. It should satisfy the rules below:

(1) If it is easy to find the superior alternatives and distinguish the DMs' preferences among the alternatives in a LOR, then the LOR should have the relatively higher information energy;

(2) If a LOR cannot provide effective information to distinguish the DMs' preferences among the alternatives, then it should have lower information energy.

Generally, in a LOR, if the more the equivalence symbols and the more forward the positions of equivalence symbols, then the more difficult to select the qualified alternatives, the information energy should be smaller. Then two extreme situations can be determined: (1) if there is no equivalence symbol in a LOR, it would be the most convenient for DMs to identify the pros and cons of the alternatives, and the corresponding LOR is considered the most effective, the corresponding information energy should be 1; (2) if the alternatives in a LOR are all linked by the symbol " $\sim$ ", it would be the most difficult to decide which alternative is the best, and the corresponding

information energy should be 0. Therefore, the number and the position of the equivalence symbol are the two basic factors that influence the value of information energy for a LOR. Meanwhile, as Fig. 3 shows, the change of the number and position of the equivalence symbol influence the sum of the elements in the corresponding EPM directly. The larger the number of the equivalence symbol and the more forward the position of the equivalence symbol, the lower the sum of the EPMEs in the corresponding EPM. Hence, we take the sum of the elements in the corresponding EPM as the numerical expressions of the two basic factors. In this case, the more difficult to choose the good alternatives and the lower the value of information energy. Therefore, we take "how easy is it to choose a good alternative" and "the corresponding value of information energy" as the consequences of the change of the two basic factors.

In this paper, when the position of the equivalence symbol keeps unchangeable, the sum of the EPMEs in the corresponding EPM increase or decrease monotonically as the number of the equivalence symbol decreases or increases. Similarly, when the number of the equivalence symbol keeps unchangeable, the sum of the EPMEs in the corresponding EPM increases or decreases monotonically as the position of the equivalence symbol goes backward or forward. Hence, in this paper, we take the sum of the EPMEs in the corresponding EPM as the main component during the establishment of information energy. In the original formula of information energy (Taheri and Azizi 2007), $ie(0) = ie(1) = 1$ and $ie(\frac{1}{2}) = 0$, which is not monotonically increasing or decreasing. Hence, we just use the half part of the formula in the conventional method, i.e. we let $ie(1) = 1$ and $ie(\frac{1}{2}) = 0$. Correspondingly, for a LOR that contains n alternatives and the relationships between each alternative are all confirmed, i.e. if all the
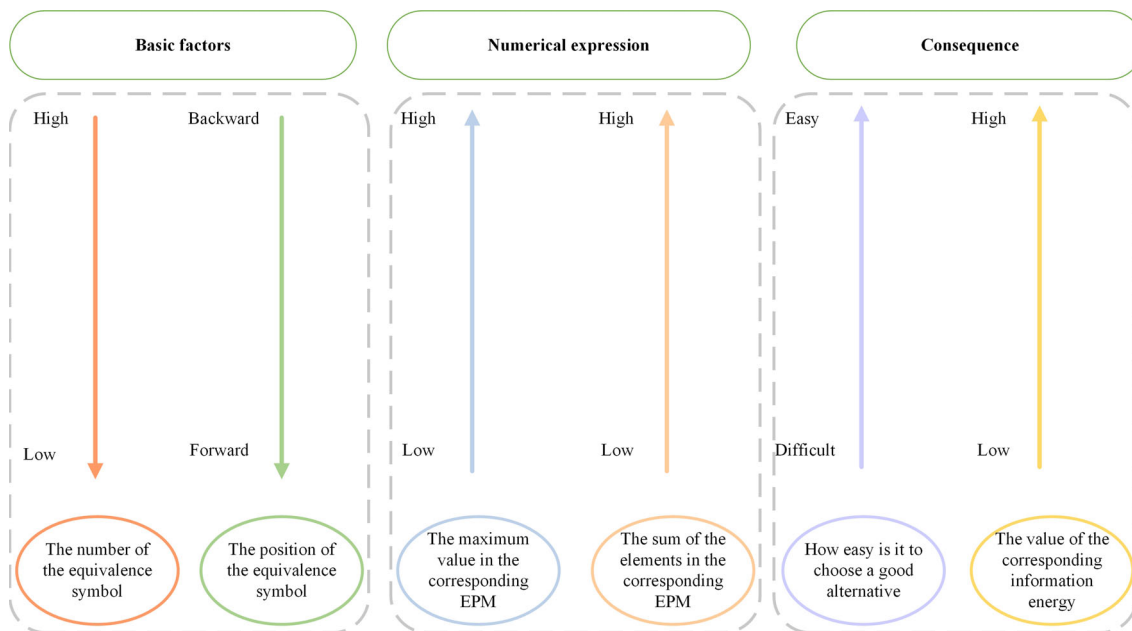
Fig. 3 The qualitative relationship among the five variates

alternatives are linked by the symbol "$\succ$", its corresponding information energy is 1; if the alternatives in a LOR are all linked by the symbol "$\sim$", the corresponding information energy is 0.

To guarantee the above assumptions, the linear mapping is utilized to derive the information energy of the LOR. For n alternatives, the two extreme situations should satisfy the following conditions:

(1) if all the n alternatives are linked by "$\succ$", then $\text{sum}(o_j) = \sum_{i=1}^{n} p_{i,j} = \frac{n(n+1)}{2} \rightarrow 1$;

(2) if all the n alternatives are linked by "$\sim$", then $\text{sum}(o_j) = \sum_{i=1}^{n} p_{i,j} = n \rightarrow 0.5$;

Then we can obtain the following formula:

$$
\begin{aligned}
IE_j &= 2\left(\left(\frac{\text{sum}(o_j) - n}{n(n-1)} + \frac{1}{2} \right)^2 + \left(\frac{1}{2} - \frac{\text{sum}(o_j) - n}{n(n-1)} \right)^2 \right) \\
&\quad - 1 \\
&= 4\left(\frac{\text{sum}(o_j) - n}{n(n-1)} \right)^2
\end{aligned}
$$

(4)

where IE_j denotes the information energy for LOR o_j, $\text{sum}(o_j)$ denotes the sum of the EPMEs in the corresponding EPM, $p_{i,j}$ denotes the EPMEs of the alternative A_i in LOR o_j and n denotes the number of the alternatives.

The information energy of LOR has the following properties:

(1) $0 \leq IE_j \leq 1$;

(2) If all the alternatives in a LOR are linked by "$\succ$", then $IE_j = 1$;

(3) If all the alternatives in a LOR are linked by "$\sim$", then $IE_j = 0$;

(4) In a LOR, if the number of alternatives is fixed, then, as the number of the equivalence symbols increases and the positions of them go forward, the value of the information energy tends to decrease generally.

Proof

(1) From Eq. (4), when $\frac{\text{sum}(o_j) - n}{n(n-1)} \geq 0$, the value of information energy increases with the value of $\frac{\text{sum}(o_j) - n}{n(n-1)}$ increases. Based on the quantitative features of the EPM, it is easy to know that the biggest value for $\frac{\text{sum}(o_j) - n}{n(n-1)}$ is 0.5 and the smallest value is 0. Hence, we can find that $0 \leq IE_j \leq 1$.

(2) If all the alternatives in a LOR are linked by "$\succ$", we can get that $\text{sum}(o_j) = \left(\frac{n(n+1)}{2} \right)$, and then $IE_j = 1$.

(3) If all the alternatives in a LOR are linked by "$\sim$", we can get that $\text{sum}(o_j) = n$, and then $IE_j = 0$.

(4) With the quantitative features of EPM, it is easy to find that when the number of alternatives in a LOR is fixed, as the number of the equivalence symbols increases and the positions of them go forward, the maximum value of the corresponding EPM decreases; meanwhile, the sum of the elements in the corresponding EPM decreases. Then, the value of information energy tends to decrease generally.

From Eq. (4), we can obtain Fig. 4, in which (a)-(g) correspond to the values of information energy of the LORs containing 3–9 alternatives, respectively. In each chart, the vertical axis denotes the value of information energy, and the horizontal axis denotes the serial number of the LORs. Generally, the greater the serial number, the more the equivalence symbols. The points that are in the same colours correspond to the LORs that have the same number of the symbol " $\sim$ ", in which the greater the serial number, the more forward the positions of them. In the figures, it presents a trend that when the number of the alternatives in a LOR is fixed, as the number of equivalence symbols increases and their positions become more forward, the value of information energy decreases with fluctuations. The fluctuations are mainly resulted from the change of the position of the equivalence symbol. For example, in Fig. 4d (the number of alternatives is 6), the corresponding LORs for the four points A, B, C and D are: $A_1 \sim A_2 \succ A_3 \succ A_4 \succ A_5 \sim A_6, A_1 \succ A_2 \succ A_3 \sim A_4 \sim A_5 \succ A_6$ and $A_1 \sim A_2 \sim A_3 \succ A_4 \succ A_5 \sim A_6, A_1 \succ A_2 \succ A_3 \sim A_4 \sim A_5 \sim A_6$. For A and B, the number of the equivalence symbols is the same, but the different equivalence symbols' positions result in that $\text{sum}(o_B) > \text{sum}(o_A)$, and then $\text{IE}_B > \text{IE}_A$. For C and D, although the number of equivalence symbols in C is less than those in D, the equivalence symbols in D are all located in the positions in the back and the equivalence symbols in C are in the front relatively. Therefore, $\text{IE}_D > \text{IE}_C$.

3.3 The conversion method combining with prospect theory

After calculating the information energy of LOR, the primary interval index is determined. Then, to explore the decision-making behaviours of the DMs under risks, the conversion-based aggregation process combined with prospect theory can be conducted.

Firstly, the primary upper bound can be derived by Eq. (5):

$$u_{i,j} = 1 - \frac{p_{i,j} - 1}{\max P_j} \tag{5}$$

where $u_{i,j}$ denotes the primary upper bound for the alternative A_i based on the LOR provided by the jth DM, $p_{i,j}$ denotes the EPME in the EPM P_j and $\max P_j$ is the maximum value in the EPM P_j.

Then, the primary values of the upper bounds and the interval index should be processed by the functions in the prospect theory. Before this step, the reference point should be determined, so that the gains and losses can be distinguished. In many existing studies, the reference point is usually fixed (Peng and Yang 2017; Zhang et al. 2017; Liu and Zhang 2021). In this paper, the value of the reference

point is not specified but determined by DMs themselves, which is shown in Eq. (6).

$$RP_j = 1 - \frac{(\eta \times \max P_j + (1 - \eta) \times \min P_j) - 1}{\max P_j} \tag{6}$$

where RP_j denotes the reference point of the jth LOR, $\max P_j$ is the maximum value in the EPM P_j, $\min P_j$ is the minimum value in the EPM P_j, η is the reference point index and $0 \le \eta \le 1$.

Then, the upper bounds of the IUVs for all the alternatives can be derived as follows:

$$U_{i,j} = \begin{cases} U_{i,j}^G = (u_{i,j} - RP_j)^\alpha, & u_{i,j} \ge RP_j \\ U_{i,j}^L = -\lambda(RP_j - u_{i,j})^\beta, & u_{i,j} < RP_j \end{cases} \tag{7}$$

where $U_{i,j}$ is the upper bound for the IUV of the alternative A_i in the LOR o_j, $U_{i,j}^G$ denotes the upper bound of the IUV of the alternative A_i that belongs to the gains in the LOR o_j and $U_{i,j}^L$ denotes the upper bound of the IUV of the alternative A_i that belongs to the losses in the LOR o_j.

Afterwards, the interval index can be derived through the weight function in the prospect theory.

$$\rho_{i,j} = \begin{cases} \dfrac{IE_j^\gamma}{[IE_j^\gamma + (1 - IE_j)^\gamma]^{1/\gamma}}, & u_{i,j} \ge RP_j \\ \dfrac{IE_j^\delta}{[IE_j^\delta + (1 - IE_j)^\delta]^{1/\delta}}, & u_{i,j} < RP_j \end{cases} \tag{8}$$

where $\rho_{i,j}$ denotes the interval index for the IUV of the alternative A_i in the LOR o_j and IE_j denotes the information energy of the LOR o_j.

Then, the lower bound can be determined as shown in Eqs. (9)-(10):

$$L_{i,j} = \begin{cases} U_{i,j} - \rho_{i,j} \times d_{i,j}, & u_{i,j} \ne RP_j \\ 0, & u_{i,j} = RP_j \end{cases} \tag{9}$$

$$d_{i,j} = \begin{cases} \dfrac{1}{\max P_j}, & p_{i,j} = \max P_j \\ U_{i,j}, & p_{i,j} = [(\eta \times \max P_j + (1 - \eta) \times \min P_j)] - 1 \\ 0, & p_{i,j} = (\eta \times \max P_j + (1 - \eta) \times \min P_j) \\ U_{i,j} - U_{k,j}, & \text{otherwise} \end{cases} \tag{10}$$

where $L_{i,j}$ is the lower bound of the IUV for the alternative A_i in the LOR o_j, $d_{i,j}$ denotes the distance between the alternative A_i and the alternative that ranks behind it, $[(\eta \times \max P_j + (1 - \eta) \times \min P_j)]$ is the minimum integer that is larger or equal to $(\eta \times \max P_j + (1 - \eta) \times \min P_j)$ and $U_{k,j}$ is the upper bound of the IUV for the alternative whose EPME satisfies that $p_{k,j} = p_{i,j} + 1$.

After deriving the upper bounds and lower bounds of the IUVs for all the alternatives in each LOR, we can follow
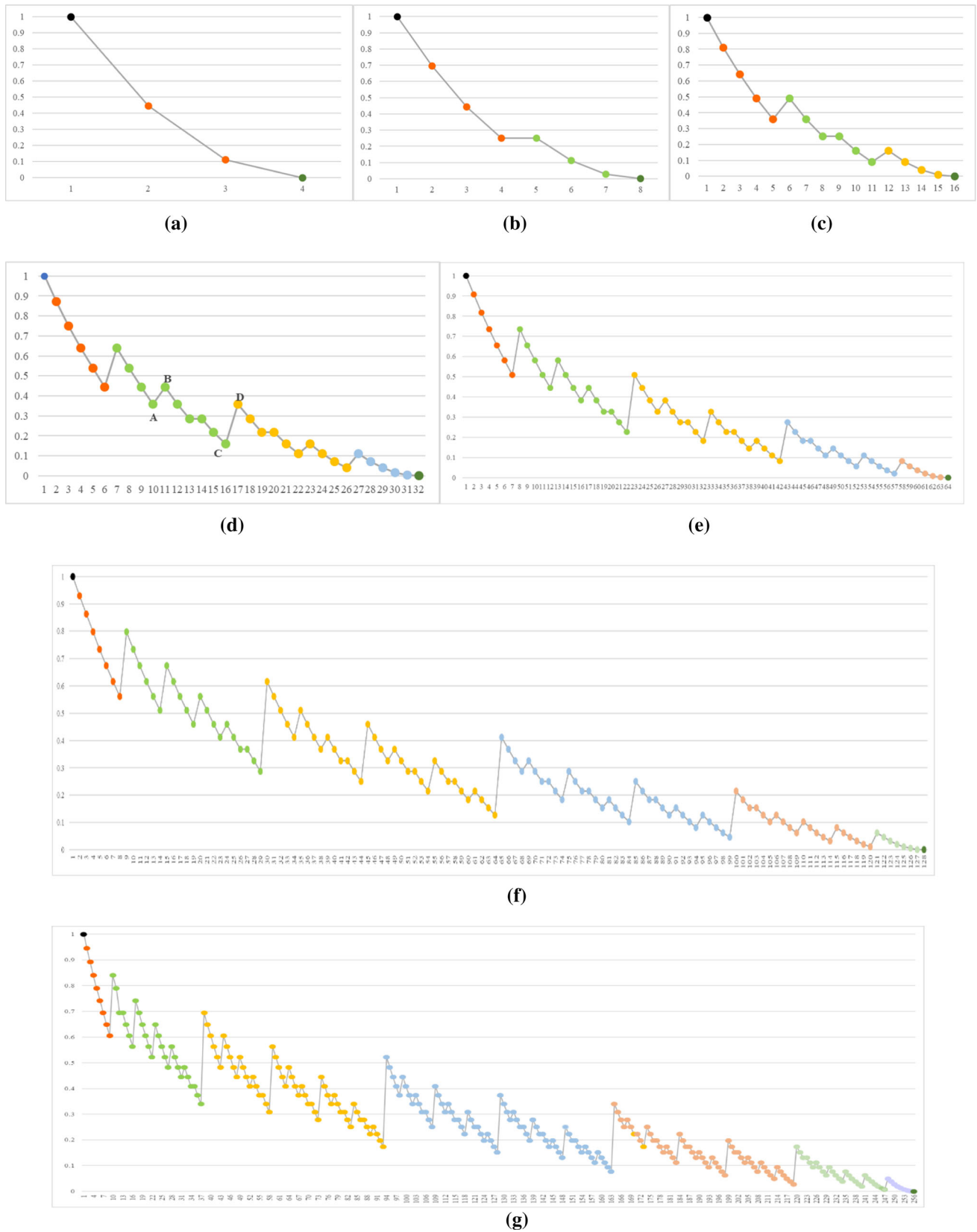
Fig. 4 The charts of information energy for the LORs with 3–9 alternatives

the classic aggregation operator for interval values to derive the collective IUVs for each alternative.

$$\begin{cases} U_i^C = \sum_{j=1}^m \omega_j \times U_{i,j} \\ L_i^C = \sum_{j=1}^m \omega_j \times L_{i,j} \end{cases} \qquad (11)$$

where U_i^C denotes the upper bound of the collective IUV for the alternative A_i, ω_j is the weight of the DM E_j and L_i^C denotes the lower bound of the collective IUV for the alternative A_i.

Then, the preference degree of each alternative can be calculated as follows:

$$\begin{aligned} \mathrm{PD}(A_i \succ A_k) &= \frac{\max\left(0, U_i^C - L_k^C\right) - \max\left(0, L_i^C - U_k^C\right)}{\left(U_i^C - L_i^C\right) + \left(U_k^C - L_k^C\right)}, \\ k &= 1, 2, \ldots, n \end{aligned}$$
$$(12)$$

where $\mathrm{PD}(A_i \succ A_k)$ denotes the degree of the alternative A_i better than the alternative A_k.

Then, the score of each alternative can be calculated and the final aggregated rank can be obtained:

$$S_i = \sum_{k=1}^n \mathrm{PD}(A_i \succ A_k) \qquad (13)$$

where S_i is the score of the alternative A_i, and the larger the score, the better the alternative A_i.

Figure 5 outlines the framework of the proposed method, which is divided into five parts:

(1) The information collection process: we collect the LOR information from different DMs;

(2) The primary transformation process: the LORs will be transformed into EPMs to support the further calculation, so that the value of information energy

for each LOR and the primary values of upper bounds for all the alternatives in each LOR can be derived based on Eqs. (4)-(5);

(3) The prospect theory-combined conversion process: the value function and the weighting function in the prospect theory will be utilized to convert EPMs into IUVs based on Eqs. (6)-(11);

(4) The scores derivation process: calculating the scores of alternatives based on Eq. (12);

(5) The aggregation LOR derivation process: deriving the final ranking based on the scores.

4 Illustrations

The outbreak of the COVID-19 definitely brings huge disaster to human society (Karabag 2020). The economic damages caused by COVID-19 have spread widely; primary sectors (including agriculture, petroleum and oil), second sectors (including manufacturing industry) and tertiary sector (including education, finance, health care and the pharmaceutical industry, hospitality, tourism and aviation, real estate and housing sector, information technology, media, research and development, food sector) are all influenced by it terribly (Nicola et al. 2020). As the pandemic continues to spread, the negative effects caused by it will last and exacerbate (Gupta et al. 2020). Many countries are trying to eliminate the economic impacts of COVID-19 as much as possible through different monetary or fiscal policies (Nicola et al. 2020). Under such situation, financial market is facing huge challenge. In the illustration part, we put the new method in the problem of detecting consumers' preferences among the common financial
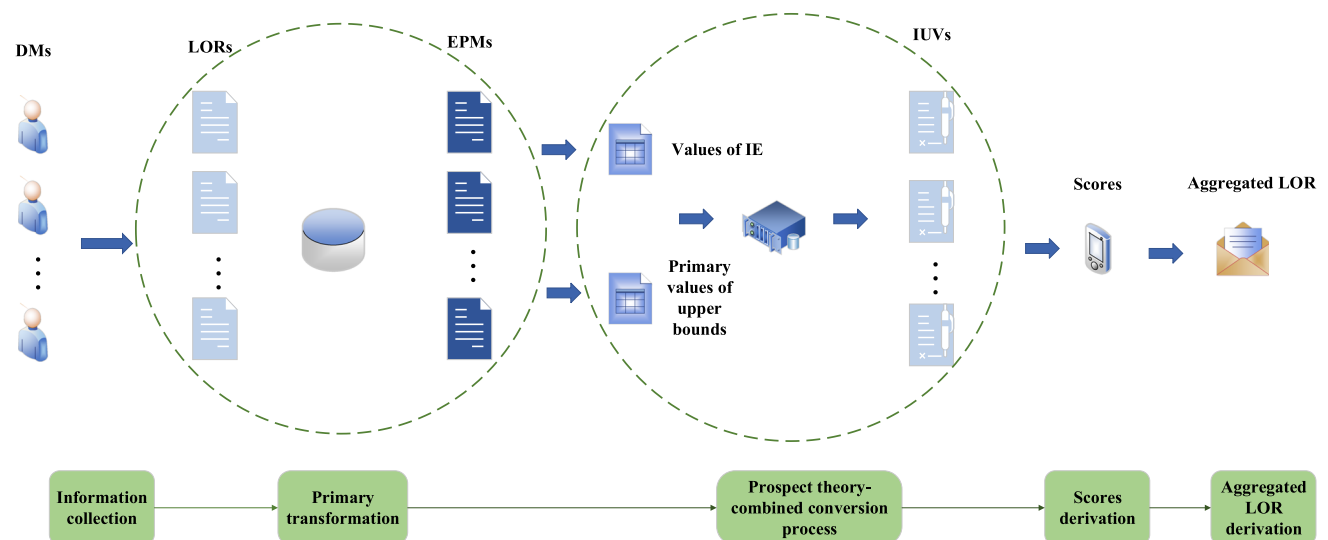

Fig. 5 The framework of the new method

products, i.e. deposit, gold, fund, stock, national debt, bond, foreign exchange, insurance and peer-to-peer lending (P2P), to present the usage of the method. Then, the discussions about the method are presented from two perspectives, i.e. comparative analysis and sensitive analysis. Besides, in order to further present the features of the method, simulation experiments are also conducted.

4.1 Numerical example

In this part, we collect the LOR information that reflects the preferences among the nine common financial products from 15 DMs. Their basic information, including gender composition, age composition and job composition, is shown in Fig. 6.

The LOR information they provided is shown in Table 1; meanwhile, the corresponding EPMs and information energy are calculated.

Then, referring to Eq. (5), the primary upper bounds for all the alternatives in each LOR can be calculated, which are shown in Table 2.

Then, in order to proceed to the value function and the weighting function in the conversion process, we need to determine the reference point for each LOR. In the numerical example, we let the reference point index for each LOR be equal to 0.5. Hence, the upper bounds processed by the value function can be derived (see Table 3), in which $\alpha = \beta = 0.88$ (Tversky and Kahneman 1992).

Similarly, the corresponding interval index can be determined, and the results are shown in Table 4 (we let $\gamma = 0.61$, $\delta = 0.69$ (Tversky and Kahneman 1992)).

After obtaining the upper bounds and the interval index, the lower bounds of the alternatives can be calculated through Eqs. (9)-(10) (see Table 5).

Based on the results in Tables 3 and 5, we can obtain the collective upper bounds and the collective lower bounds for all the alternatives (considering that the weights of DMs

have influences on the final results but we mainly focus on the prospect theory combination process in this paper, we let the DMs have the same weight). The collective IUV of each alternative is: $IUV_1 = [0.2113, 0.2945]$, $IUV_2 = [-0.0794, 0.0233]$, $IUV_3 = [0.1151, 0.2227]$, $IUV_4 = [-0.4391, -0.3150]$, $IUV_5 = [-0.0400, 0.0597]$, $IUV_6 = [-0.2712, -0.1682]$, $IUV_7 = [-0.4836, -0.3424]$, $IUV_8 = [-0.3312, -0.2035]$, $IUV_9 = [-0.9816, -0.9014]$.

Then, we can obtain the score of each alternative: $S_1 = 7.9403$, $S_2 = 5.3127$, $S_3 = 7.0597$, $S_4 = 1.6998$, $S_5 = 5.6873$, $S_6 = 3.7065$, $S_7 = 1.3645$, $S_8 = 3.2291$, $S_9 = 0$.

Finally, the aggregated result of the 15 LORs is: $A_1 \succ A_3 \succ A_5 \succ A_2 \succ A_6 \succ A_8 \succ A_4 \succ A_7 \succ A_9$. That is to say, the deposit is the most preferred financial product among these 15 financial products, even if its interest is relatively low. The second most preferred financial product is bond, followed by national debt. The stock, foreign exchange and P2P are less preferred, especially the P2P. From the aggregated result, we can see that the DMs prefer the financial products with high stability and relatively low risk under the impact of COVID-19, although it means the interest is relatively low. To the 15 participants, the phenomenon that foreign exchange is not popular can be understood under the impact of the epidemic. Besides, P2P has been controversial for a long time in China, and there are lots of issues on P2P. Therefore, during the pandemic, it is also difficult for consumers to invest in P2P. In this case, for the financial institutions, how to adapt to the new conditions, help consumers make profits, and stimulate the energy of market are worthy of in-depth thinking.

4.2 Discussions

In this subsection, we firstly compare the proposed method with some existing methods that have been proven to be effective in aggregating LOR information. In this part, in
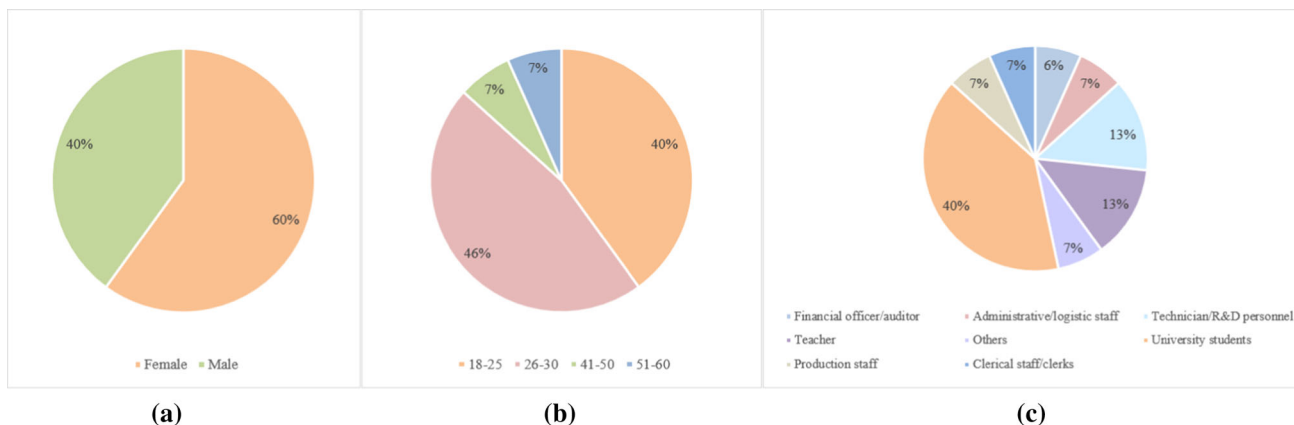

Fig. 6 The basic information of the DMs

Table 1 The LORs provided by DMs and their corresponding EPMs and information energy

DMs	LOR	EPM	IE
E_1	$A_3 \succ A_1 \sim A_8 \succ A_2 \sim A_4 \succ A_6 \succ A_5 \sim A_7 \sim A_9$	$[2, 3, 1, 3, 5, 4, 5, 2, 5]$	0.3403
E_2	$A_1 \succ A_8 \succ A_2 \sim A_3 \succ A_5 \succ A_6 \succ A_7 \succ A_4 \succ A_9$	$[1, 3, 3, 7, 4, 5, 6, 2, 8]$	0.6944
E_3	$A_1 \succ A_8 \succ A_5 \succ A_3 \succ A_4 \succ A_6 \succ A_2 \succ A_7 \succ A_9$	$[1, 7, 4, 5, 3, 6, 8, 2, 9]$	1.0000
E_4	$A_1 \succ A_2 \succ A_5 \succ A_4 \sim A_8 \succ A_3 \succ A_7 \succ A_6 \succ A_9$	$[1, 2, 5, 4, 3, 7, 6, 4, 8]$	0.7415
E_5	$A_1 \succ A_2 \succ A_3 \succ A_5 \succ A_6 \succ A_8 \succ A_7 \succ A_4 \succ A_9$	$[1, 2, 3, 8, 4, 5, 7, 6, 9]$	1.0000
E_6	$A_3 \succ A_7 \succ A_8 \succ A_2 \succ A_6 \succ A_5 \succ A_4 \succ A_1 \succ A_9$	$[8, 4, 1, 7, 6, 5, 2, 3, 9]$	1.0000
E_7	$A_1 \sim A_5 \succ A_2 \succ A_3 \succ A_4 \succ A_7 \succ A_6 \succ A_8 \succ A_9$	$[1, 2, 3, 4, 1, 6, 5, 7, 8]$	0.6049
E_8	$A_1 \succ A_3 \succ A_8 \succ A_2 \succ A_9 \succ A_6 \succ A_5 \succ A_7 \succ A_4$	$[1, 4, 2, 9, 7, 6, 8, 3, 5]$	1.0000
E_9	$A_1 \succ A_3 \succ A_2 \sim A_4 \succ A_5 \sim A_6 \sim A_7 \succ A_9 \succ A_8$	$[1, 3, 2, 3, 3, 3, 3, 5, 4]$	0.2500
E_{10}	$A_1 \sim A_3 \sim A_4 \succ A_2 \succ A_5 \succ A_6 \succ A_7 \sim A_8 \succ A_9$	$[1, 2, 1, 1, 3, 4, 5, 5, 6]$	0.2785
E_{11}	$A_3 \succ A_1 \succ A_5 \succ A_7 \succ A_6 \succ A_4 \succ A_2 \succ A_8 \succ A_9$	$[2, 7, 1, 6, 3, 5, 4, 8, 9]$	1.0000
E_{12}	$A_5 \succ A_1 \sim A_3 \succ A_6 \succ A_2 \sim A_7 \succ A_8 \succ A_4 \succ A_9$	$[2, 4, 2, 6, 1, 3, 4, 5, 7]$	0.4823
E_{13}	$A_2 \succ A_5 \succ A_6 \succ A_7 \succ A_1 \succ A_3 \succ A_4 \succ A_8 \succ A_9$	$[5, 1, 6, 7, 2, 3, 4, 8, 9]$	1.0000
E_{14}	$A_1 \succ A_5 \succ A_8 \succ A_3 \succ A_6 \succ A_4 \succ A_7 \succ A_2 \succ A_9$	$[1, 8, 4, 6, 2, 5, 7, 3, 9]$	1.0000
E_{15}	$A_1 \sim A_5 \sim A_8 \sim A_3 \sim A_4 \sim A_6 \sim A_7 \sim A_9 \sim A_2$	$[1, 1, 1, 1, 1, 1, 1, 1, 1]$	0

Table 2 The primary upper bounds for all the alternatives in each LOR

LOR	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
o_1	0.8000	0.6000	1.0000	0.6000	0.2000	0.4000	0.2000	0.8000	0.2000
o_2	1.0000	0.7500	0.7500	0.2500	0.6250	0.5000	0.3750	0.8750	0.1250
o_3	1.0000	0.3333	0.6667	0.5556	0.7778	0.4444	0.2222	0.8889	0.1111
o_4	1.0000	0.8750	0.5000	0.6250	0.7500	0.2500	0.3750	0.6250	0.1250
o_5	1.0000	0.8889	0.7778	0.2222	0.6667	0.5556	0.3333	0.4444	0.1111
o_6	0.2222	0.6667	1.0000	0.3333	0.4444	0.5556	0.8889	0.7778	0.1111
o_7	1.0000	0.8750	0.7500	0.6250	1.0000	0.3750	0.5000	0.2500	0.1250
o_8	1.0000	0.6667	0.8889	0.1111	0.3333	0.4444	0.2222	0.7778	0.5556
o_9	1.0000	0.6000	0.8000	0.6000	0.6000	0.6000	0.6000	0.2000	0.4000
o_{10}	1.0000	0.83333	1.0000	1.0000	0.6667	0.5000	0.3333	0.3333	0.1667
o_{11}	0.8889	0.3333	1.0000	0.4444	0.7778	0.5556	0.6667	0.2222	0.1111
o_{12}	0.8571	0.5714	0.8571	0.2857	1.0000	0.7143	0.5714	0.4286	0.1429
o_{13}	0.5556	1.0000	0.4444	0.3333	0.8889	0.7778	0.6667	0.2222	0.1111
o_{14}	1.0000	0.2222	0.6667	0.4444	0.8889	0.5556	0.3333	0.7778	0.1111
o_{15}	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

order to control variables, DMs are assigned the same weight. Then, based on the data in the numerical example, we test the influence of the coefficients in the value function and the weight function, respectively.

4.2.1 Comparative analysis

For the numerical example in 4.1, the comparisons are mainly carried out through the following aspects:

(1) The Borda method (Borda 1781): the score of each alternative can be calculated by Eq. (14):

$$S_i^B = \sum_{j=1}^{n} p_{i,j} \tag{14}$$

where S_i^B denotes the score of the alternative A_i and $p_{i,j}$ is the EPME of the alternative A_i in LOR o_j. In this method, the less the score, the better the alternative.

(2) The method proposed by Chiclana et al. (1998): this method transforms rankings to utility values of alternatives by the formula

$$U_{i,j}^{\text{Chiclana}} = \frac{\max P_j - p_{ij}}{\max P_j - 1} \tag{15}$$

where $U_{i,j}^{\text{Chiclana}}$ denotes the utility value of the alternative A_i in the LOR o_j.

Then, the collective utility values of the alternatives can be obtained through Eq. (16).

Table 3 The upper bounds processed by the value function

LOR	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
o_1	0.2426	0.0000	0.4465	0.0000	− 1.0046	− 0.5459	− 1.0046	0.2426	− 1.0046
o_2	0.4831	0.2292	0.2292	− 0.8084	0.0872	− 0.1961	− 0.5157	0.3593	− 1.0870
o_3	0.4899	− 0.5989	0.1446	0.0000	0.2662	− 0.3254	− 0.8557	0.3803	− 1.1022
o_4	0.4831	0.3593	− 0.1961	0.0872	0.2292	− 0.8084	− 0.5157	0.0872	− 1.0870
o_5	0.4899	0.3803	0.2662	− 0.8557	0.1446	0.0000	− 0.5989	− 0.3254	− 1.1022
o_6	− 0.8557	0.1446	0.4899	− 0.5989	− 0.3254	0.0000	0.3803	0.2662	− 1.1022
o_7	0.4831	0.3593	0.2292	0.0872	0.4831	− 0.5157	− 0.1961	− 0.8084	− 1.0870
o_8	0.4899	0.1446	0.3803	− 1.1022	− 0.5989	− 0.3254	− 0.8557	0.2662	0.0000
o_9	0.4465	0.0000	0.2426	0.0000	0.0000	0.0000	0.0000	− 1.0046	− 0.5459
o_{10}	0.4628	0.2952	0.4628	0.4628	0.1123	− 0.2526	− 0.6643	− 0.6643	− 1.0413
o_{11}	0.3803	− 0.5989	0.4899	− 0.3254	0.2662	0.0000	0.1446	− 0.8557	− 1.1022
o_{12}	0.3321	0.0000	0.3321	− 0.7471	0.4744	0.1804	0.0000	− 0.4060	− 1.0675
o_{13}	0.0000	0.4899	− 0.3254	− 0.5989	0.3803	0.2662	0.1446	− 0.8557	− 1.1022
o_{14}	0.4899	− 0.8557	0.1446	− 0.3254	0.3803	0.0000	− 0.5989	0.2662	− 1.1022
o_{15}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 4 The interval index of all the alternatives for each LOR

LOR	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
o_1	0.3396	0.3396	0.3396	0.3396	0.3538	0.3538	0.3538	0.3396	0.3538
o_2	0.5302	0.5302	0.5302	0.5837	0.5302	0.5837	0.5837	0.5302	0.5837
o_3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
o_4	0.5621	0.5621	0.6196	0.5621	0.5621	0.6196	0.6196	0.5621	0.6196
o_5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
o_6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
o_7	0.4766	0.4766	0.4766	0.4766	0.4766	0.5214	0.5214	0.5214	0.5214
o_8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
o_9	0.2907	0.2907	0.2907	0.2907	0.2907	0.2907	0.2907	0.2935	0.2935
o_{10}	0.3067	0.3067	0.3067	0.3067	0.3067	0.3132	0.3132	0.3132	0.3132
o_{11}	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
o_{12}	0.4116	0.4116	0.4116	0.4429	0.4116	0.4116	0.4116	0.4429	0.4429
o_{13}	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
o_{14}	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
o_{15}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

$$U_{i,c}^{\text{Chiclana}} = \sum_{j=1}^{m} \omega_j U_{i,j}^{\text{Chiclana}} \qquad (16)$$

where $U_{i,c}^{\text{Chiclana}}$ denotes the collective utility value of the alternative A_i, ω_j denotes the weight of the expert

E_j, and $\sum_{j=1}^{m} \omega_j = 1$. The larger the collective utility value, the better the alternative.

(3) The Wang et al.'s method (Wang et al. 2005): this method transforms rankings into IUVs for each alternative by the core model as follows:

$$(M1) \quad \min / \max z_1 = u_{i,j}$$
$$\text{s.t.} \quad u_{\sigma(k),j} - u_{\sigma(k+1),j} \geq \zeta, \quad \text{if} \quad u_{\sigma(k),j} \succ u_{\sigma(k+1),j} \quad k \in [1, n],$$
$$u_{\sigma(k),j} - u_{\sigma(k+1),j} = \zeta, \quad \text{if} \quad u_{\sigma(k),j} \sim u_{\sigma(k+1),j} \quad k \in [1, n]$$
$$\sum_{i=1}^{n} u_{i,j} = 1,$$
$$u_{i,j} \geq 0 \quad i \in [1, n]$$

where $u_{i,j}$ denotes the utility values of the alternatives, k denotes the sequence positions of the LOR from the best to worst, $\sigma(k)$ corresponds to the subscript of the alternative that is ranked at the kth position in the sequence, obviously $u_{\sigma(k),j}$ and $u_{\sigma(k+1),j}$ are adjacent in the sequence and ζ is the threshold that satisfies $0 \leq \zeta \leq \frac{2}{n(n-1)}$ (in this paper, we let $\zeta = \frac{2}{n(n-1)}$). The minimum and maximum values derived from the model will become the lower bounds and the upper bounds of the IUVs for the alternatives, respectively.

(4) The conversion-based algorithms proposed by Liu et al. (2021): the basic structure of the first algorithm in their study is similar to the method in this paper. The core of the first conversion algorithm is the model shown as follows:

$$(M2) \quad \min z_2 = \sum_{j=1}^{m} \sum_{i=1}^{n} \left[\left(U_{i,j}^{\text{Liu}} - U_{i,c}^{\text{Liu}} \right)^2 + \left(L_{i,j}^{\text{Liu}} - L_{i,c}^{\text{Liu}} \right)^2 \right]$$
$$\text{s.t.}$$
$$0 \leq \rho_j^{\text{Liu}} \leq 1$$

$$\varphi_{ik} = \begin{cases} 1 & \text{if the relationship between } A_i \text{ and } A_k \text{ derived from the two methods are the same} \\ 0 & \text{otherwise} \end{cases}.$$

where $U_{i,j}^{\text{Liu}}$ is the upper bounds of the IUV for the alternative A_i in the LOR o_j, $U_{i,c}^{\text{Liu}}$ denotes the upper bound of the collective IUV for the alternative A_i, $L_{i,j}^{\text{Liu}} \left(L_{i,j}^{\text{Liu}} = U_{i,j}^{\text{Liu}} - \rho_j^{\text{Liu}} \times \frac{1}{\max P_j} \right)$ denotes the lower

bounds of the IUV for the alternative A_i in the LOR o_j and $L_{i,c}^{\text{Liu}}$ denotes the lower bound of the collective IUV for the alternative A_i. Besides, the ways to calculate $U_{i,c}^{\text{Liu}}$ and $L_{i,c}^{\text{Liu}}$ in Ref. (Liu et al. 2021) are the same with the ways in this paper. After deriving $U_{i,c}^{\text{Liu}}$ and $L_{i,c}^{\text{Liu}}$, the scores for the alternatives can be calculated through Eq. 2. (12)-(13).

The aggregated results of these five different methods are shown in Table 6, in which the similarity rate between the results derived from the above five methods and the result obtained by the proposed method is calculated through Eq. (17).

$$\text{Similarity rate} = \frac{2 \sum_{i=1, i<k}^{n} \varphi_{ik}}{n(n-1)} \quad (17)$$

where n is the number of the alternative and φ_{ik} is a 0–1 variation and it satisfies:

From Table 6, we can conclude that: (1) all of these methods choose A_1, i.e. the deposit, as the best alternative, and in the most results, the top four choices are the same. From this perspective, we can see that the proposed method in this paper is relatively reasonable and reliable. (2) The

Table 5 The lower bounds of the alternatives

LOR	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
o_1	0.1602	0.0000	0.3773	0.0000	− 1.0754	− 0.7082	− 1.0754	0.1602	− 1.0754
o_2	0.4175	0.1539	0.1539	− 0.9673	0.0410	− 0.3826	− 0.6865	0.2903	− 1.1537
o_3	0.3803	− 0.8557	0.0000	0.0000	0.1446	− 0.5989	− 1.1022	0.2662	− 1.2133
o_4	0.4135	0.2862	− 0.3924	0.0382	0.1446	− 0.9810	− 0.6971	0.0382	− 1.1645
o_5	0.3803	0.2662	0.1446	− 1.1022	0.0000	0.0000	− 0.8557	− 0.5989	− 1.2133
o_6	− 1.1022	0.0000	0.3803	− 0.8557	− 0.5989	0.0000	0.2662	0.1446	− 1.2133
o_7	0.4241	0.2973	0.1615	0.0456	0.4241	− 0.6683	− 0.3627	− 0.9504	− 1.1459
o_8	0.3803	0.0000	0.2662	− 1.2133	− 0.8557	− 0.5989	− 1.1022	0.1446	0.0000
o_9	0.3872	0.0000	0.1721	0.0000	0.0000	0.0000	0.0000	− 1.0633	− 0.6805
o_{10}	0.4114	0.2391	0.4114	0.4114	0.0779	− 0.3815	− 0.7824	− 0.7824	− 1.0936
o_{11}	0.2662	− 0.8557	0.3803	− 0.5989	0.1446	0.0000	0.0000	− 1.1022	− 1.2133
o_{12}	0.2697	0.0000	0.2697	− 0.8890	0.4158	0.1062	0.0000	− 0.5571	− 1.1308
o_{13}	0.0000	0.3803	− 0.5989	− 0.8557	0.2662	0.1446	0.0000	− 1.1022	− 1.2133
o_{14}	0.3803	− 1.1022	0.0000	− 0.5989	0.2662	0.0000	− 0.8557	0.1446	− 1.2133
o_{15}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 6 The aggregated results derived from the above four methods and the numerical example

Method no	Aggregated LOR	Similarity rate (%)
(1)	$A_1 \succ A_3 \succ A_5 \succ A_2 \succ A_8 \succ A_6 \succ A_7 \succ A_4 \succ A_9$	94.44
(2)	$A_1 \succ A_3 \succ A_5 \succ A_2 \succ A_8 \succ A_6 \succ A_4 \succ A_7 \succ A_9$	97.22
(3)	$A_1 \succ A_5 \succ A_2 \succ A_8 \succ A_6 \succ A_3 \succ A_7 \succ A_4 \succ A_9$	83.33
(4)	$A_1 \succ A_3 \succ A_5 \succ A_2 \succ A_8 \succ A_6 \succ A_4 \succ A_7 \succ A_9$	97.22
The new method	$A_1 \succ A_3 \succ A_5 \succ A_2 \succ A_6 \succ A_8 \succ A_4 \succ A_7 \succ A_9$	/

similarity rates of the results are derived from the method proposed by Chiclana et al. (1998), and the first method proposed by Liu et al. (2021) is up to 97.22%, and the only one difference exists on the relationship between A_6 and A_8. The aggregated LOR derived from Borda method also has relatively high similarity rate (94.44%) with the aggregated result by the proposed method, and the main differences are reflected in the relationships between A_6 and A_8, and A_4 and A_7. The result derived from Wang et al.'s method (Wang et al. 2005) has a relatively low similarity rate (83.33%.) with the result by the proposed method. By the proposed method, A_3 is better than A_5, A_2, A_6 and A_8, while in Wang et al.'s method (Wang et al. 2005), the result is the opposite. From M-1, we can see that the model derives the upper bounds and the lower bounds through calculating the maximum and minimum values of the variables in the model, respectively. The dispersion degrees of the lengths of the IUVs derived from the Wang et al.'s method are larger than those derived from the proposed method (see Table 7, which shows the standard deviation of the interval lengths of the IUVs derived from the two methods). That is to say, the IUVs derived from the Wang et al.'s method may contain some redundant information to influence the accuracy of the result.

Then, we measure the consistency ratio between the individuals' LORs and the five aggregated LORs by:

$$\mathrm{CR}(A_i, A_k) = \frac{\sum_{j=1}^{15} \phi_{ik,j}}{15} \tag{18}$$

where $\mathrm{CR}(A_i, A_k)$ denotes the consistency ratio on the pair of alternative A_i and A_k and $\phi_{ik,j}$ is also a 0–1 variable, which satisfies the condition:

$$\phi_{ik,j} = \begin{cases} 1 & \text{if the relationship between } A_i \text{ and } A_k \text{ in } o_j \\ & \text{is the same to that in the aggregated LOR .} \\ 0 & \text{otherwise} \end{cases}$$

The detailed situations are shown in Fig. 7, in which the lines 1–5 denote the Borda method (Borda 1781), the method proposed by Chiclana et al. (1998), Wang et al.'s method (2005), the first method proposed by Liu et al. (2021), and the proposed method in this paper, respectively. From the figure, we can see that the consistency ratio of Wang et al.'s method has relatively more fluctuations, especially on the relationships between A_2 and A_3, A_3 and A_5, A_3 and A_6, A_3 and A_8. The consistency ratios between the individuals' LORs and the aggregated LORs in the other five methods are similar to each other, which shows that although the different methods have their own features, their results are relatively consistent. From this point, we can see that the proposed method has good rationality and reliability.

Further, the collective IUVs or utility values of the alternatives derived from the four methods, i.e. the method proposed by Chiclana et al. (1998) (signs in magenta), Wang et al.'s method (Wang et al. 2005) (signs in green), the method proposed by Liu et al. (2021) (signs in black) and the proposed method in this paper (signs in red), are shown in Fig. 8. In the figure, the vertical axis denotes the values of the IUVs or utility values and the horizontal axis denotes the different alternatives, where the interval from 1 to 2 corresponds to A_1, the interval from 2 to 3 corresponds to A_2 and the rest can be done in the same manner.

We can firstly see that the trend of the collective IUVs or utility values derived from these methods is similar. In addition, only the collective IUVs derived from the proposed method have values that less than 0. Investigating the features of the prospect theory that DMs are usually more sensitive to losses than gains, the IUV of the best alternative is closer to 0 than the IUV of the worst alternative. From this perspective, the proposed method not only has rationality and accuracy, but also reflects the features of the DMs' decision-making behaviours.
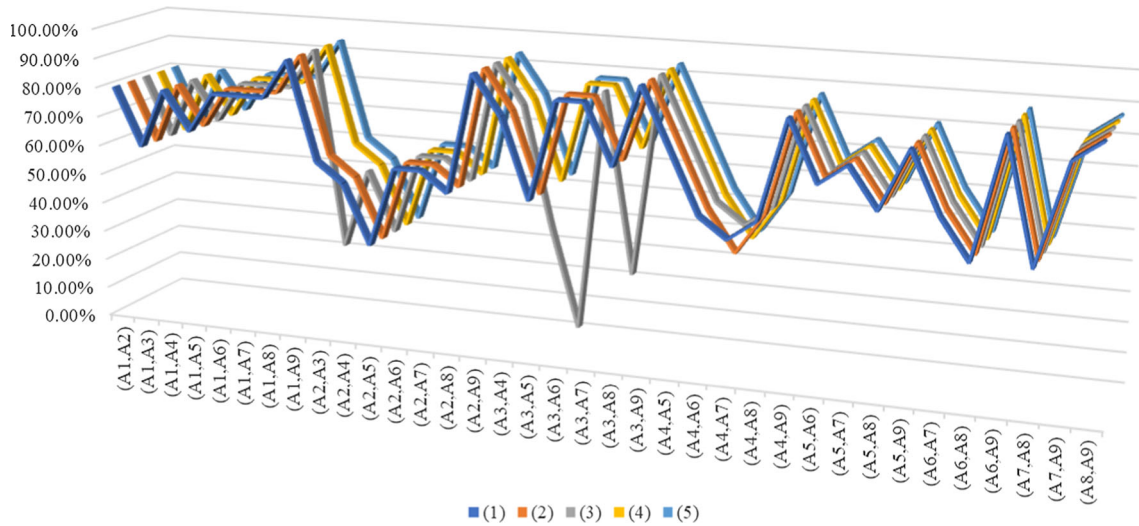
Table 7 The coefficient of variation of the interval length of the IUVs for the alternatives

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
Wang et al.'s method	1.8187	1.4137	2.1815	1.6282	1.6080	1.5353	1.4934	1.9216	1.5830
The proposed method	0.5749	0.8697	0.5945	0.8728	0.7765	0.9423	0.6285	0.5838	0.4883

Fig. 7 The consistency ratio between the individuals' LORs and the five aggregated LORs

**Fig. 8** The IUVs of the alternatives derived from the five different methods
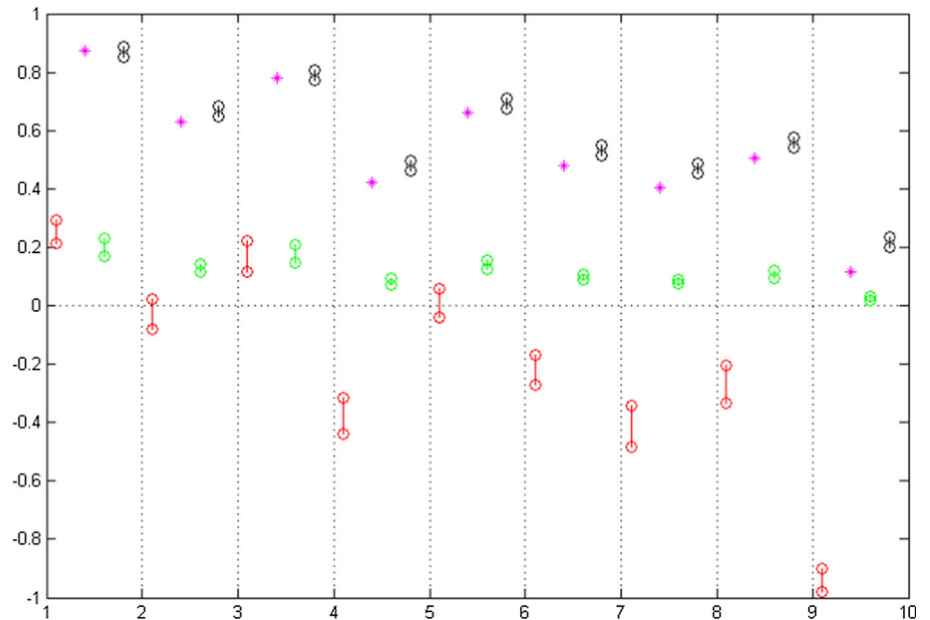
4.2.2 Sensitive analysis

Since there are some uncertain coefficients in the proposed method, i.e. the reference point index η, and α, β, λ, γ and δ in the value function and weighting function, we conduct the sensitive analysis to explore how the change of the coefficients influences the final scores of the alternatives. In Fig. 9, the figures a–f denote the analysis results of η, α, β, λ, γ and δ, respectively. The horizontal axis denotes the range of the coefficients, where η changes from 0 to 1, α changes from 0.68 to 1.21 (Tian 2019), β changes from 0.68 to 1.02(Tian 2019), λ changes from 1.25 to 2.25 (Tian 2019), γ changes from 0.55 to 0.721 (Tian 2019), δ changes from 0.36 to 0.84 (Tian 2019), and they all change with the

step width 0.001. The vertical axis denotes the scores of the nine alternatives.

In Fig. 9a, we can see that the change of η does not have obvious impact on the scores of A_1, A_2, A_3, A_5 and A_9. Although η changes, their scores just fluctuate in a very small range, and the final ranking of them remains unchanged. However, as the index η changes, especially η increases from 0 to around 0.4 and increases from around 0.7 to 0.8, and the gap between the scores of A_4 and A_7 becomes close. When η changes from around 0.4 to 0.75, their difference becomes more obvious. In addition, the change of η has some remarkable effects on the scores of A_6 and A_8. When the value of η is around 0.5 and 0.85, the relationship between A_6 and A_8 changes.

(a)

(b)

(c)

(d)

(e)

(f)
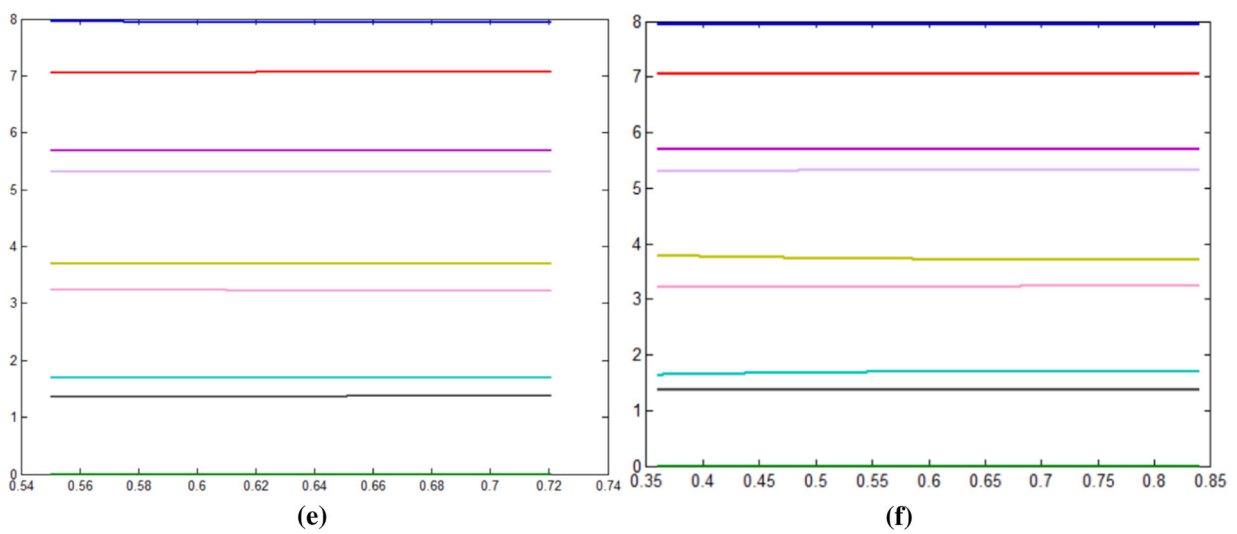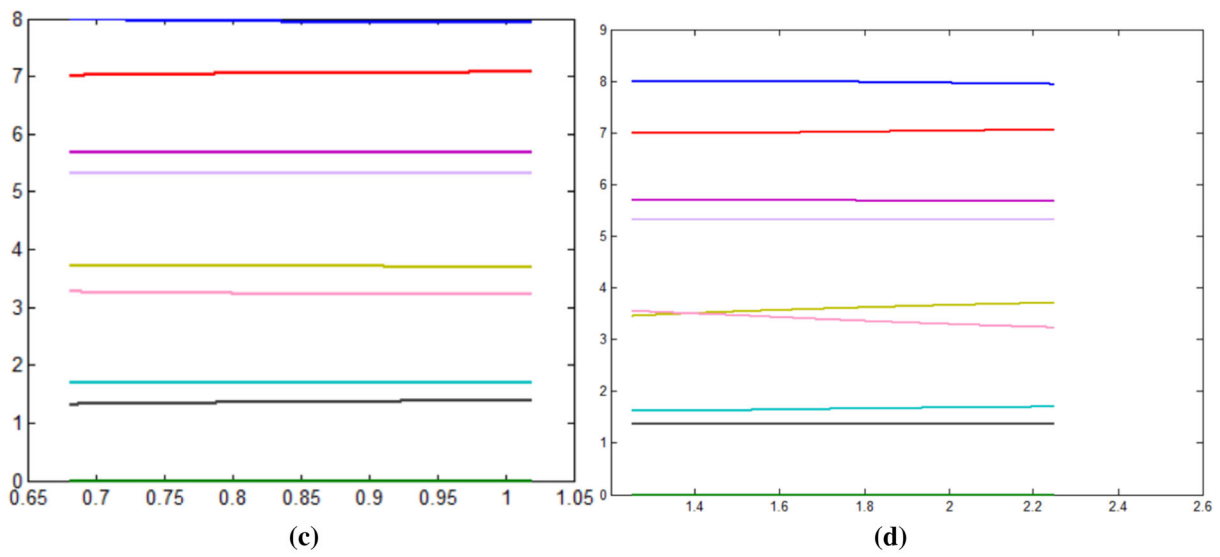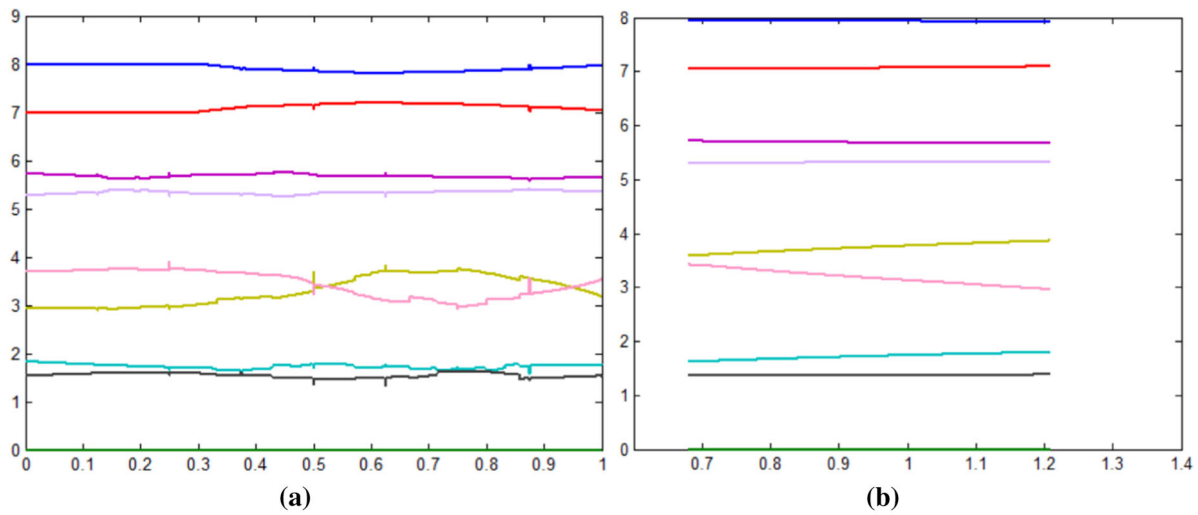
A1   A2   A3   A4   A5   A6   A7   A8   A9

◀ **Fig. 9** The sensitivity analysis of the coefficients

From Fig. 9b, c, we can see that the changes of α and β have little effect on the scores of the alternatives, but the aggregated LOR remain unchanged. As the value of β changes, only the difference between the scores of A_4 and A_7 shows a certain decreasing trend. When α changes, the difference between the scores of A_4 and A_7 increases. In addition, as α changes from 0.68 to 1.21, the difference of the scores between A_6 and A_8 has a significant increasing trend. From this perspective, the changes of α and β mainly have effects on the scores of A_4, A_7, A_6 and A_8.

When the value of λ changes within the interval, most of the scores of the alternatives are not influenced, and the relationship between A_6 and A_8 is affected obviously. From Fig. 9d, when λ is around 1.4, the ranking changes, and as the value of λ increases more, the difference of the scores between A_6 and A_8 tends to be more obvious. From Fig. 9e, f, it can be seen that the scores of the alternatives change much merely as the change of γ and δ.

Overall, the change of γ and δ influences the final aggregated results least. The change of α and β has a certain influence on the scores of alternatives, but they do not affect the final aggregated LOR. The change of λ also has certain impact on the scores of the alternatives, espe-cially A_6 and A_8. The change of η has the most significant impact on the scores of the alternatives. The score of each alternative is affected to a certain extent, and the ranking relationship between A_6 and A_8 is influenced the most obviously. We can find that A_6 and A_8 are ranked at the opposite parts of gains and losses in most of the evaluations from the DMs, and A_8 is ranked at the last position and the second from the bottom in some evaluations. Therefore, we infer that it is the reason that the ranking relationship between A_6 and A_8 is influenced the most obviously. From this perspective, we can see how significant that the change of reference point will affect the final result, and how important to choose the reference point.

4.3 Simulation experiment

Based on the sensitive analysis, we find that the change of reference point has relatively obvious impact on the scores of the alternatives. Therefore, we firstly design a simulation test to investigate the impact of the reference point index on the scores of alternatives. Then, considering that one of the prominent features of the proposed method is the combination with prospect theory, we also design a simulation experiment to investigate the difference between the method that removes the prospect theory and the proposed method in this paper.
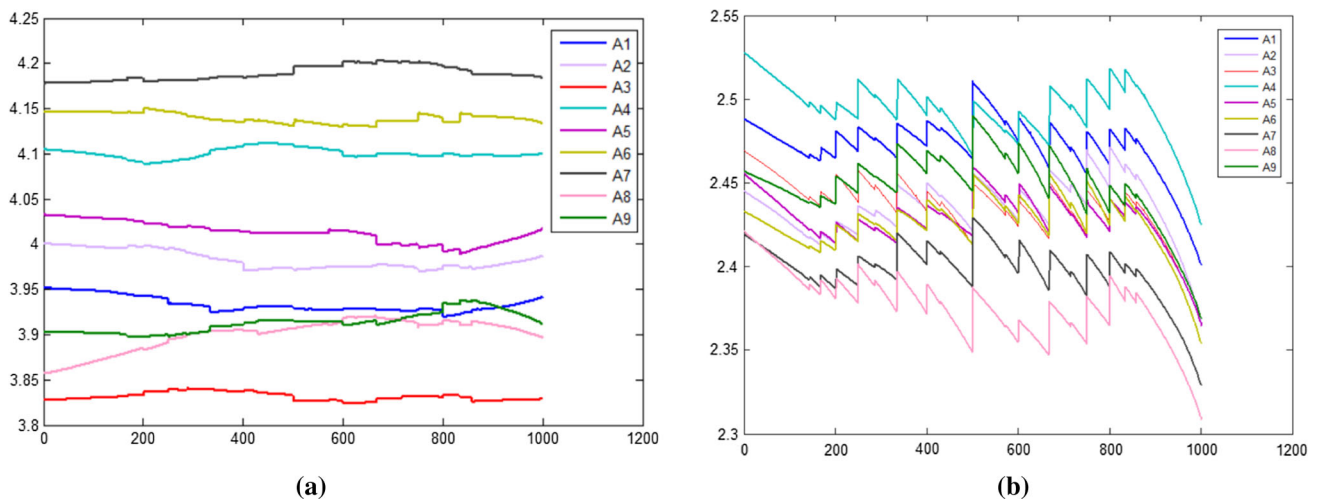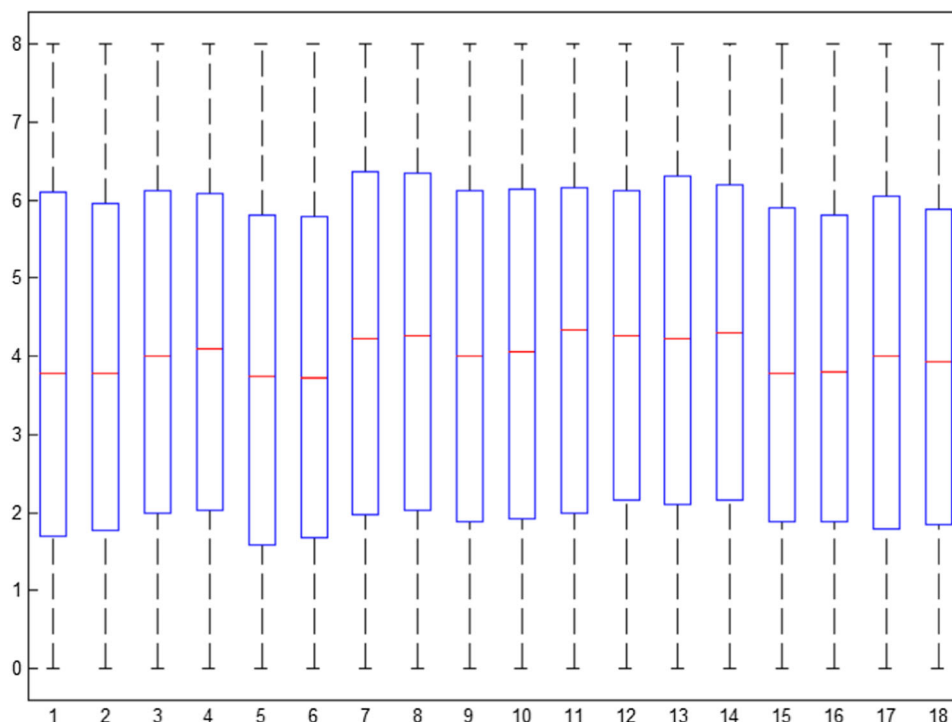

(a)   (b)

Fig. 10 The statistic features of the simulation test

Table 8 The average abstract differences between the scores derived from these two methods

$|S_{11} - S_{21}|$	$|S_{12} - S_{22}|$	$|S_{13} - S_{23}|$	$|S_{14} - S_{24}|$	$|S_{15} - S_{25}|$	$|S_{16} - S_{26}|$	$|S_{17} - S_{27}|$	$|S_{18} - S_{28}|$	$|S_{19} - S_{29}|$
0.3963	0.3985	0.4079	0.3867	0.3946	0.4058	0.4019	0.4183	0.4125

4.3.1 The simulation about reference point

In this simulation test, we let the number of the alternative be 9, and the EPMs of the LORs can be confirmed. Then, we use the software Matlab to generate all the possible EPMs to be the basis for the further calculation. In each round, we pick 15 EPMs randomly, and let the reference point index change from 0 to 1 with the step width 0.001. That is to say, in each round, the 15 EPMs selected randomly will be utilized 1001 times. In order to make the further charting process more convenient, we set the number of the total round to be 1001. That is to say, we will generate the 15 EPMs 1001 times, and in each time, the corresponding 15 EPMs will be used 1001 times. The horizontal axis denotes the change of η, i.e. η changes from 0 to 1 with the step width 0.001. The vertical axis in Fig. 10a denotes the average value of the scores for the nine alternatives in the 1001 random extraction test, and the vertical axis in Fig. 10b denotes the standard variation of the scores for the nine alternatives in the 1001 random extraction test.

Figure 10 shows the statistic features of the results in the simulation test. In order to investigate the influence of the reference point, we firstly let the value of reference point index remain unchanged and calculate the average score and the standard deviation of each alternative based on the results of the 1001 rounds. Figure 10a shows the situations of the average score, in which the horizontal axis denotes the 1001 times that the reference point index η changes

from 0 to 1 with the step width 0.001, and the vertical axis denotes the average score of each alternative. Figure 10b shows the situations of the standard deviation, in which the horizontal axis denotes the 1001 times that the reference point index η changes from 0 to 1 with the step width 0.001, and the vertical axis denotes the standard deviation of the score of each alternative.

From the figures, we can see that as the change of η, and the average scores mainly distribute within the interval [4.3, 4.75]. The average scores of the alternatives have tiny fluctuations, in which the average scores of most alternatives are relatively stable. Only the average score of A_8 presents the increasing trend. In addition, from the variation of standard deviations of the alternatives, the fluctuations have the similar trends, and as the value of η changes from 0 to 1, the standard deviations also have decreasing trend; especially when η is going to 0, the standard deviations decrease precipitously. The values of the standard deviations all distribute in the interval [2.3, 2.55]. From this perspective, the change of the reference point definitely has certain impact on the scores of the alternatives, but its impact on the scores of the alternatives seems have the similar effectiveness, so that do not change the final aggregated ranking to a large extent.

4.3.2 The simulation about the prospect theory process

Similarly, in the simulation test of this part, we also set the number of the alternative to be 9 and use the software

Matlab to generate the basic database, i.e., all the possible EPMs. Then, we let the experiment run 1000 times; in each time, the final scores of the alternatives will be calculated through the proposed method in this paper and the method removing the prospect theory process, respectively.

The data in Table 8 show the average abstract differences between the scores derived from these two methods, from which we can also see that the differences of the results derived from these two methods are not large.

In order to investigate the quantitative features of the results, the boxplots of the scores are shown in Fig. 11, in which the boxplots odd-numbered denote the scores of the alternative A_1 to the alternative A_9 based on the proposed method, and the boxplots even-numbered denote the scores of the alternative A_1 to the alternative A_9 based on the method without prospect theory.

From Fig. 11, we can see that the minimum values, medians and maximum values of the results derived from these two methods are close. Besides, from the positions of the different quantiles, the fluctuation degree of the results derived from the proposed method is little larger than the results derived from the method without the prospect theory. Considering the features of the prospect theory, we infer that it is because the alternatives are divided into two parts, i.e. gains and losses, and it holds the view that DMs are more sensitive to the losses than gains. From this perspective, although the method without the prospect theory shows the advantage to some extent on the less fluctuation, the proposed method in this paper can depict the DMs' risk behaviours more vividly with acceptable fluctuation.

5 Conclusion

This paper proposes a new conversion-based LOR aggregation method combined with prospect theory with changeable reference point. The contributions of the paper are shown as follows: (1) the information energy for LOR is proposed, which can reflect the effectiveness of the LOR and also enrich the measurement theory for LOR information; (2) through combining with the prospect theory, the new aggregation process can depict the decision-making behaviours under risks, which make the aggregation process reflect the preferences of DMs more vividly; (3) in this paper, the reference point is not fixed, and we let experts determine its value by giving reference point index, which is closer to the real decision-making environment and improve the aggregation method.

In this paper, based on the content in the illustration part, we can find that the coefficients only have little influence on the results, and it can reflect DMs' preferences under risks vividly through combining with the prospect theory. The results derived from the proposed method are

rational and stable. During the aggregation process, it can be used to mine the preferences of DMs hidden behind the LOR information, which is meaningful to the improvement of the conversion-based LOR aggregation method.

There are also some points that we can discuss in the future. For example, this paper mainly deals with the complete LOR information, which contains all the alternatives. In reality, sometimes DMs provide incomplete information. Under such situation, it is meaningful to consider the consistency and consensus issues, when aggregate this kind of information.

Author contributions NL contributed to conceptualization, methodology, validation, formal analysis, investigation and writing–original draft. ZX contributed to validation, writing–review and editing and supervision. HW contributed to conceptualization and writing–review and editing. PR contributed to writing–review and editing.

Declarations

References

Aledo JA, Gamez JA, Molina D (2013) Tackling the rank aggregation problem with evolutionary algorithms. Appl Math Comput 222:632–644

Ali I, Cook WD, Kress M (1986) Note—ordinal ranking and intensity of preference: a linear programming approach. Manag Sci 32:1642–1647

Borda JC (1781) Memoire sur les elections au scrutiny

Brandenburg FJ, Gleibner A, Hofmeier A (2013) Comparing and aggregating partial orders with Kendall tau distances. Discret Math Algorithms Appl 5:1360003

Carneiro J, Alves P, Marreiros G, Novais P (2021) Group decision support systems for current times: overcoming the challenges of dispersed group decision-making. Neurocomputing 423:735–746

Chen SC, Wang F, Song YQ, Zhang CS (2011) Semi-supervised ranking aggregation. Inf Process Manag an Int J 47:415–425

Chen SW, Liu J, Wang H, Augusto JC (2013) Ordering based decision making—a survey. Inf Fusion 14:521–531

Chiclana F, Herrera F, Herrera-Viedma E (1998) Integrating three representation models in fuzzy multipurpose decision making based on fuzzy preference relations. Fuzzy Sets Syst 97:33–48

Contreras I (2010) A distance-based consensus model with flexible choice of rank-position weights. Gr Decis Negot 19:441–456

Cook WD, Kress M (1985) Ordinal ranking with intensity of preference. Manag Sci 31:26–32

Cook WD, Kress M (1990) A data envelopment model for aggregating preference rankings. Manag Sci 36:1302–1310

Cook WD, Kress M, Seiford LM (1986) An axiomatic approach to distance on partial orderings. RAIRO Oper Res 20:115–122

Dopazo E, Martinezcespedes ML (2015) Rank aggregation methods dealing with incomplete information applied to smart cities. In: IEEE international conference on fuzzy systems, pp 1–7

Dopazo E, Martinez-Cespedes ML (2017) Rank aggregation methods dealing with ordinal uncertain preferences. Expert Syst Appl 78:103–109

Dumitrescu D (1977) A definition of an informational energy in fuzzy sets theory. Stud Univ Babes-Bolyai Math 22:57–59

Dwork C, Kumar R, Naor M, Sivakumar D (2001) Rank aggregation methods for the Web. In: The web conference, pp 613–622

Emond EJ, Mason DW (2002) A new rank correlation coefficient with application to the consensus ranking problem J. Multi-Criteria Decis Anal 11:17–28

Encheva S (2012) Automated decision making based on weak orderings. Int J Intell Inf Database Syst 6:30–44

Franceschini F, Maisano DA, Mastrogiacomo L (2015) A novel algorithm for fusing preference orderings by rank-ordered agents. Fuzzy Sets Syst 266:84–100

Franceschini F, Maisano D, Mastrogiacomo L (2016) Fusion of multi-agent preference orderings in an ordinal semi-democratic decision-making framework. Meas J Int Meas Confed 91:699–702

Georgescu I (2007) Fuzzy preference relations. In: Fuzzy choice functions.studies in fuzziness and soft computing, Springer, Berlin, pp 49–74

Gonzalez-Arteaga T, Alcantud JCR, de Andres CR (2016) A new consensus ranking approach for correlated ordinal information based on Mahalanobis distance. Inf Sci 372:546–564

Gou XJ, Xu ZS, Liao HC (2019) Group decision making with compatibility measures of hesitant fuzzy linguistic preference relations. Soft Comput 23:1511–1527

Gupta M, Abdelmaksoud A, Jafferany M et al (2020) COVID-19 and economy. Dermatol Ther 33:e13329

Hou FJ (2015) A consensus gap indicator and its application to group decision making. Gr Decis Negot 24:415–428

Hou FJ, Triantaphyllou E (2019) An iterative approach for achieving consensus when ranking a finite set of alternatives by a group of experts. Eur J Oper Res 5:570–579

Hou FJ, Liao AH, Wu QZ (2008) AlternatiVe ranking in prefIerence ordering-based social choice: eigenvector approach. J Nanjing Univ Sci Technol Sci Ed 32:80–83

Kahneman D, Tversky A (1979) Prospect theory: an analysis of decision under risk. Econometrica 47:263–292

Karabag SF (2020) An unprecedented global crisis! The global, regional, national, political, economic and commercial impact of the coronavirus pandemic. J Appl Econ Bus Res 10:1–6

Klementiev A, Roth D, Small K (2008) Unsupervised rank aggregation with distance-based models. In: 2008 International conference on machine learning, pp 472–479

Liang HM, Xiong W, Dong YC (2018) A prospect theory-based method for fusing the individual preference-approval structures in group decision making. Comput Ind Eng 117:237–248

Liu PD, Zhang P (2021) A normal wiggly hesitant fuzzy MABAC method based on CCSD and prospect theory for multiple attribute decision making. Int J Intell Syst 36:447–477

Liu JP, Liao XW, Huang W, Liao XZ (2019) Market segmentation: a multiple criteria approach combining preference analysis and segmentation decision. Omega 83:1–13

Liu NN, Xu ZS, Zeng X-J, Ren PJ (2020a) An agglomerative hierarchical clustering algorithm for linear ordinal rankings. Inf Sci 557:170–193

Liu PS, Diao HY, Zou L, Deng AS (2020b) Uncertain multi-attribute group decision making based on linguistic-valued intuitionistic fuzzy preference relations. Inf Sci 508:293–308

Liu NN, Xu ZS, Wu HY, Ren PJ (2021) Conversion-based aggregation algorithms for linear ordinal rankings combined with granular computing. Knowl Based Syst 219:106880

Luca AD, Termini S (1972) A definition of nonprobabilistic entropy in the setting of fuzzy sets theory. Inf Control 20:301–312

Nicola M, Alsafib Z, Sohrabic C et al (2020) The socio-economic implications of the coronavirus pandemic (COVID-19): a review. Int J Surg 78:185–193

Onicescu O (1966) Energie informationelle. Comptes Rendus L'académie Des Sci 263:841–842

Peng XD, Yang Y (2017) Algorithms for interval-valued fuzzy soft sets in stochastic multi-criteria decision making based on regret theory and prospect theory with combined weight. Appl Soft Comput 54:415–430

Sese J, Morishita S (2001) Rank aggregation method for biological databases. Genome Inform 12:506–507

Taheri SM, Azizi R (2007) On information energy of the image fuzzy sets. Inf Sci 177:3871–3881

Tian XL (2019) Fuzzy decision-making methods based on prospect theory and its application in venture capital. Sichuan University

Tversky A, Kahneman D (1992) Advances in prospect theory: cumulative representation of uncertainty. J Risk Uncertain 5:297–323

Vanicek J, Vrana I, Aly S (2009) Fuzzy aggregation and averaging for group decision making: a generalization and survey. Knowl Based Syst 22:79–84

Wang YM, Yang J-B, Xu D-L (2005) A preference aggregation method through the estimation of utility intervals. Comput Oper Res 32:2027–2049

Xu ZS (2013) Group decision making model and approach based on interval preference orderings. Comput Ind Eng 64:797–803

Yager RR (2001) Fusion of multi-agent preference orderings. Fuzzy Sets Syst 117:1–12

Yang JY, Cheng X, Su S et al (2019) Collecting preference rankings under local differential privacy. In: IEEE 35th international conference on data engineering, pp 1598–1601

Zhang ZX, Wang L, Rodriguez RM et al (2017) A hesitant group emergency decision making method based on prospect theory. Complex Intell Syst 3:177–187