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Fuzzy multicomponent stress-strength reliability in presence of partially accelerated life testing under generalized progressive hybrid censoring scheme subject to inverse Weibull model

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A R T I C L E I N F O

Method name:
Partially accelerated life testing under generalized progressive hybrid censoring scheme

Keywords:
Stress-strength reliability
Partially accelerated life testing
Generalized progressive hybrid censoring scheme
Inverse Weibull distribution
Fuzzy
Maximum likelihood estimation
Akaike information criterion
Kolmogorov–Smirnov test

A B S T R A C T

It typically takes a lot of time to monitor life-testing experiments on a product or material. Units can be tested under harsher conditions than usual, known as accelerated life tests to shorten the testing period. This study's goal is to investigate the issue of partially accelerated life testing that use generalized progressive hybrid censored samples to estimate the stress-strength reliability in the multicomponent case. Also, the fuzziness of the model is considered that gives more sensitive and accurate analyses about the underlying system. Maximum likelihood estimation method under the inverse Weibull distribution and using the generalized progressively hybrid censoring scheme is introduced to obtain an estimator for the fuzzy multicomponent stress-strength reliability. Also, an asymptotic confidence interval is deduced to examine the reliability of the fuzzy multicomponent stress-strength. Simulation study is conducted using maximum likelihood estimates and confidence intervals for the fuzzy multicomponent stress-strength reliability for different values of the parameters and different schemes. A real data application representing the data for the failure times for a certain software model is introduced to obtain the fuzzy multicomponent stress-strength reliability for different schemes.

- The fuzzy multicomponent stress-strength reliability is investigated under partially accelerated life testing and the generalized progressively hybrid censored scheme.
- An algorithm is introduced to simulate data for the censoring scheme.
- A real data application is presented to obtain the fuzzy multicomponent stress-strength reliability at different schemes.

Specifications table

Subject area:	Statistics
More specific subject area:	Stress-strength reliability
Method name:	Partially accelerated life testing under generalized progressive hybrid censoring scheme
Name and reference of original method:	A.A. Ismail, On designing constant-stress partially accelerated life tests under time-censoring, Strength of Materials 46 (2014) 132–139, https://doi.org/10.1007/s11223-014-9524-z
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Background

There are variety techniques of accelerated life testing. One of these techniques is the constant stress accelerated life testing where the stress applied to the product units in the test remains constant over time. Another technique is the progressive stress accelerated life testing in which the stress is increased over time. And the step stress accelerated life testing in which the test condition is altered for a predetermined amount of time or failures.

Zarrin et al. [1] introduced an estimation in constant stress partially accelerated life tests for Rayleigh distribution under type-I censoring. Kamal [2] presented the constant stress partially accelerated life test design for inverted Weibull distribution with type-I censoring. Ismail [3] considered the constant stress partially accelerated life tests with type-I censoring under Weibull distribution. Hyun and Lee [4] studied constant-stress partially accelerated life testing for log-logistic distribution with censored data. Almarashi [5] introduced an estimation for constant-stress partially accelerated life tests of generalized half-logistic distribution based on progressive type-II censoring. Ahmadini et al. [6] presented an estimation of constant stress partially accelerated life test for Fréchet distribution with type-I censoring. Dey et al. [7] introduced an inference on Nadarajah–Haghighi distribution with constant stress partially accelerated life tests under progressive type-II censoring. Yousef [8] presented a statistical inference for a constant-stress partially accelerated life tests based on progressively hybrid censored samples from inverted Kumaraswamy distribution. El-Sagheer et al. [9] introduced inferences for stress-strength reliability model in the presence of partially accelerated life test to its strength variable. Eliwa and Ahmed [10] introduced a reliability analysis of constant partially accelerated life tests under progressive first failure type-II censored data from Lomax model. Nassar and Elshahhat [11] introduced a statistical analysis of inverse Weibull constant-stress partially accelerated life tests with adaptive progressively type I censored data. Sarhan and Tolba [12] introduced an analysis for the stress-strength reliability under partially accelerated life testing using Weibull model.

The goal of fuzzy reliability is to give researchers the tools they need to sensitively and precisely analyze the underlying systems of life dependability. There are only two possible outcomes in the probability theory which is based on perception. To deal with the idea of partial truth, fuzzy theory is extended from its linguistic information foundation. While fuzzy reliability requires more information to produce the fuzzy value comparison of traditional reliability, fuzzy values are determined between true and false. Eryilmaz and Tütüncü [13] presented the stress strength reliability in the presence of fuzziness. Sabry et al. [14] discussed an inference of fuzzy reliability model for inverse Rayleigh distribution. Yazgan et al. [15] introduced a study for the fuzzy stress-strength reliability for weighted exponential distribution.

In accelerated life testing, the fuzziness is increased by the models used to transform life times under high stress levels in order to estimate life time distribution under usual stress. For results that are realistic, data fuzziness must be quantitatively described.

The models that are used in accelerated life testing to convert life times under high stress into life time distributions under typical stress raise the degree of fuzziness. For results that are realistic, data fuzziness must be quantitatively described.

Censoring is a useful technique that is frequently applied in lifetime tests. When there are not enough test units or when it is not possible to collect data for all test units due to a lack of time or resources, censoring is crucial in practical experiments. The mathematical ease of Type-I and Type-II censoring draws a lot of interest. In Type-I censoring, the test is terminated when a predetermined time has passed; in Type-II censoring, the test is terminated when a predetermined number of units fail. However, if the experimenter needs to remove units on a sporadic basis, both censoring strategies might not be appropriate. As a result, progressive Type-II censoring is thought to be preferable and has become popular recently. The intermittent removal of units is permitted in this censoring. Additionally, it helps you save some money and some time.

On the basis of various censoring methodologies, it is difficult to derive effective statistical procedures under various life testing experiments for the unknown interesting quantities. Hybrid censoring is the combination of Type-I and Type-II censoring schemes. In hybrid censoring scheme, the experiment ends after a certain amount of time has passed and a certain number of failures. Yousef et al. [16] used the generalized progressive hybrid censoring design to discuss the inference of stress-strength model based on the exponentiated exponential distribution. Nagy et al. [17] discussed the generalized Type-II progressive hybrid censoring sample from the Burr Type-XII distribution. Wang et al. [18] discussed the inference of Kumaraswamy distribution under generalized progressive hybrid censoring.

Numerous censoring schemes that can be used in reliability analysis can be found in the literature. Kohansal [19] introduced an estimation of reliability in a multicomponent stress-strength model for a Kumaraswamy distribution based on progressively censored sample. Hassan et al. presented [20] an estimation of multicomponent stress-strength reliability following Weibull distribution based on upper record values. Mahto et al. [21] introduced an estimation of reliability in a multicomponent stress–strength model for a general class of inverted exponentiated distributions under progressive censoring.

Jha [22] introduced multicomponent stress-strength reliability estimation based on unit generalized Rayleigh distribution. Saini et al. [23] presented the reliability estimation of multicomponent stress strength model for Burr XII distribution using progressively first-failure censored samples. Hu and Gui [24] introduced reliability inference of multicomponent stress–strength system based on Chen distribution using progressively censored data.

The Weibull distribution does not offer a satisfactory parametric fit if the data show a non-monotone and unimodal hazard rate function, so the inverse Weibull distribution is a better fit model than the Weibull distribution. Depending on the value of the shape parameter, the hazard rate function of the inverse Weibull distribution can be either decreasing or increasing. The inverse Weibull is used to model several types of data, including the time it takes for an insulating fluid to break down when subjected to the action of Diesel engine mechanical parts like pistons and crankshafts are under constant tension and deterioration.

Keller and Kamath [25] established the inverse Weibull distribution with two parameters. It has been used to simulate a variety of real-world scenarios, including the deterioration of mechanical parts like hammers and diesel drive shafts. The inverse Weibull distribution is used in analyzing data from reliability engineering and life testing experiments. Keller et al. [26] showed that the inverse Weibull distribution is the best in fitting the dataset of dynamic engine parts when compared to the other distributions. If X is a continuous random variable that follows the inverse Weibull distribution with shape and scale parameters β and γ, respectively, then the probability density function and its corresponding cumulative distribution function are given, respectively, by

$$f(x)=\frac{\beta\gamma}{x^{\gamma+1}}\,e^{-\beta/x^{\gamma}},\ x,\ \beta,\ \gamma>0$$

$$F(x)=e^{-\beta/x^{\gamma}},\ x,\ \beta,\ \gamma>0$$

In this paper, a study of the multicomponent stress-strength reliability is introduced in presence of partially accelerated life testing and fuzziness. The study is performed considering the generalized hybrid censoring scheme under the inverse Weibull distribution. Rare articles in literature discussed the issue of fuzzy multicomponent stress-strength reliability in case of partially accelerated life testing. In the present study, a novel analysis of the multicomponent stress-strength reliability is introduced assuming partially accelerated life testing and generalized hybrid censoring scheme in presence of fuzziness. The motivation of this article is to study the behavior of the system when each unit of the system is run at either normal condition or accelerated condition. Since most tests make it simpler to maintain a stress-strength with partially accelerated life test. Additionally, for some materials and products, the accelerated test models for stress-strength under the partially accelerated life testing are better developed and data analysis for reliability estimation is well developed. Sometimes the experiment in real-world tests couldn't be fully controlled because things could accidentally break. So, the generalized progressive hybrid censoring scheme is used. The system's time is described by the inverse Weibull model, which increases the system's flexibility. An estimator of the multicomponent stress-strength reliability is obtained by using the method of maximum likelihood. An asymptotic confidence interval for the multicomponent stress-strength reliability is deduced. A simulation study is introduced to obtain numerical results for the fuzzy multicomponent stress-strength reliability for different values of the parameters and different schemes. A real data application representing the failure times for a software model is introduced to obtain the fuzzy multicomponent stress-strength reliability when applying different schemes.

Method details

Partially accelerated life testing modeling description

Consider that the stress variable Y is consisted of n components which will be divided into two groups. The first group is consisted of $n^{(1)}$ components with the normal condition case. The second group is consisted of $n^{(2)}=n-n^{(1)}$ components with the accelerated condition case. Each component in Y has a subsystem of the strength variable X with k components. Also, the subsystems of X will be divided into two groups. The first group is consisted of $n^{(1)}$ subsystems each with k components under the normal condition case. The second group is consisted of $n^{(2)}$ subsystems each with k components under the accelerated condition case. The probability density function of X and Y will be given, respectively, by

$$f(x)=\begin{cases}\dfrac{\beta_1\gamma}{x^{\gamma+1}}\,e^{-\beta_1/x^{\gamma}}, & normal\ case\\ \dfrac{\lambda\beta_1\gamma}{(\lambda x)^{\gamma+1}}\,e^{-\beta_1/(\lambda x)^{\gamma}}, & accelerated\ case\end{cases}$$

$$g(y)=\begin{cases}\dfrac{\beta_2\gamma}{y^{\gamma+1}}\,e^{-\beta_2/y^{\gamma}}, & normal\ case\\ \dfrac{\lambda\beta_2\gamma}{(\lambda y)^{\gamma+1}}\,e^{-\beta_2/(\lambda y)^{\gamma}}, & accelerated\ case\end{cases}$$

where $\lambda>1$ is the acceleration factor

Generalized progressive hybrid censoring scheme

For the stress variable Y, assume that the predetermined integers $h^{(1)}$ and $b^{(1)}$ with $1\le h^{(1)}\le b^{(1)}\le n^{(1)}$ in the normal case and h_2 and b_2 with $1\le h^{(2)}\le b^{(2)}\le n^{(2)}$ in the accelerated case. Consider the two predetermined time points $\tau^{(1)}$ and $\tau^{(2)}$ in the normal and accelerated conditions, respectively. The non-negative integers $q_1^{(1)},\ q_2^{(1)},\ \dots,\ q_{b^{(1)}}^{(1)}$ for the normal case and $q_1^{(2)},\ q_2^{(2)},\ \dots,\ q_{b^{(2)}}^{(2)}$ for the accelerated case represent prefixed numbers with $\sum_{i=1}^{b^{(1)}} q_i^{(1)}+b^{(1)}=n^{(1)}$ and $\sum_{i=1}^{b^{(2)}} q_i^{(2)}+b^{(2)}=n^{(2)}$. Let $Y^{(1)}_{i:b^{(1)}:n^{(1)}}$ and $Y^{(2)}_{i:b^{(2)}:n^{(2)}}$ denotes the failure of the i-th unit of the test in case of normal and accelerated conditions, respectively, then the testing will stop at the following points:

$$\tau^{*(\delta)}=\begin{cases}Y^{(\delta)}_{h^{(\delta)}:b^{(\delta)}:n^{(\delta)}}, & if\ \tau^{(\delta)}<Y^{(\delta)}_{h^{(\delta)}:b^{(\delta)}:n^{(\delta)}}<Y^{(\delta)}_{b^{(\delta)}:b^{(\delta)}:n^{(\delta)}}\\ \tau^{(\delta)}, & if\ Y^{(\delta)}_{h^{(\delta)}:b^{(\delta)}:n^{(\delta)}}<\tau^{(\delta)}<Y^{(\delta)}_{b^{(\delta)}:b^{(\delta)}:n^{(\delta)}}\\ Y^{(\delta)}_{b^{(\delta)}:b^{(\delta)}:n^{(\delta)}}, & if\ Y^{(\delta)}_{h^{(\delta)}:b^{(\delta)}:n^{(\delta)}}<Y^{(1)}_{b^{(1)}:b^{(1)}:n^{(1)}}<\tau^{(\delta)}\end{cases},\ \delta=1,\ 2$$

Where $\delta=1,\ 2$ represents the case of normal and accelerated conditions, respectively. The following scenario can be observed in case of the different types of failures under the generalized progressive hybrid censoring scheme:

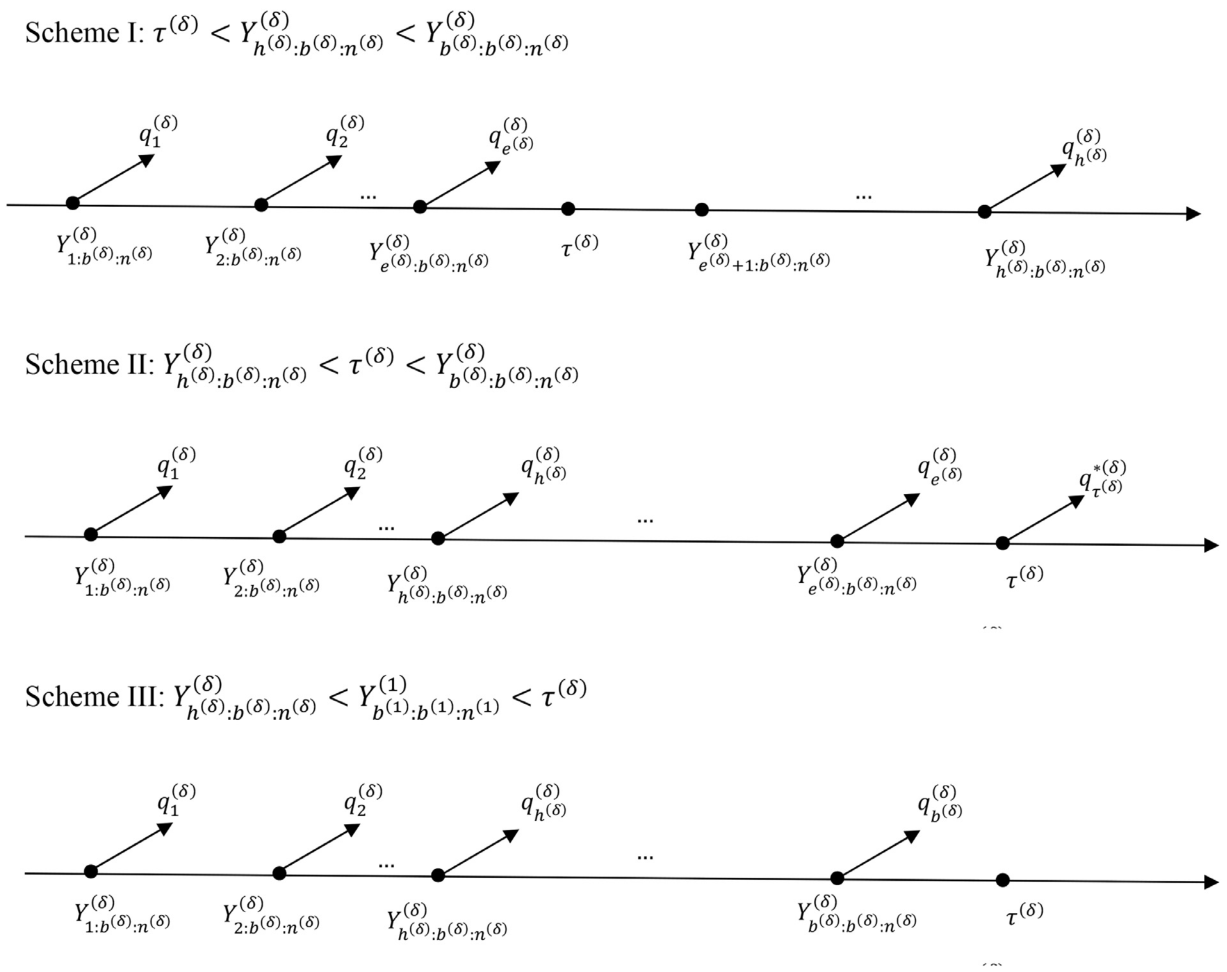

Fig. 1. Graphical presentation of the generalized progressive hybrid censoring scheme for the stress variable.

Scheme I: $Y^{(\delta)}_{1:b^{(\delta)}:n^{(\delta)}},\ Y^{(\delta)}_{2:b^{(\delta)}:n^{(\delta)}},\ \ldots,\ Y^{(\delta)}_{h^{(\delta)}:b^{(\delta)}:n^{(\delta)}},\ if\ \tau^{(\delta)} < Y^{(\delta)}_{h^{(\delta)}:b^{(\delta)}:n^{(\delta)}} < Y^{(\delta)}_{b^{(\delta)}:b^{(\delta)}:n^{(\delta)}}$
Scheme II: $Y^{(\delta)}_{1:b^{(\delta)}:n^{(\delta)}},\ Y^{(\delta)}_{h^{(\delta)}:b^{(\delta)}:n^{(\delta)}},\ \ldots,\ Y^{(\delta)}_{e^{(\delta)}:b^{(\delta)}:n^{(\delta)}},\ if\ Y^{(\delta)}_{h^{(\delta)}:b^{(\delta)}:n^{(\delta)}} < \tau^{(\delta)} < Y^{(\delta)}_{b^{(\delta)}:b^{(\delta)}:n^{(\delta)}}$
Scheme III: $Y^{(\delta)}_{1:b^{(\delta)}:n^{(\delta)}},\ Y^{(\delta)}_{h^{(\delta)}:b^{(\delta)}:n^{(\delta)}},\ \ldots,\ Y^{(\delta)}_{b^{(\delta)}:b^{(\delta)}:n^{(\delta)}},\ if\ Y^{(\delta)}_{h^{(\delta)}:b^{(\delta)}:n^{(\delta)}} < Y^{(1)}_{b^{(1)}:b^{(1)}:n^{(1)}} < \tau^{(\delta)}$

where $\delta = 1,\ 2$ for normal and accelerated case, respectively. The graphical presentation of the generalized progressive hybrid censoring scheme is shown in Fig. 1.
where

$$q^{(\delta)}_{h^{(\delta)}} = n^{(\delta)} - h^{(\delta)} - \sum_{i=1}^{e^{(\delta)}} q_i^{(\delta)}$$

$$q^{*(\delta)}_{\tau^{(\delta)}} = n^{(\delta)} - e^{(\delta)} - \sum_{i=1}^{e^{(\delta)}} q_i^{(\delta)}$$

$$q^{(\delta)}_{b^{(\delta)}} = n^{(\delta)} - b^{(\delta)} - \sum_{i=1}^{e^{(\delta)}} q_i^{(\delta)}$$

For the strength variable X, assume that the predetermined integers $l_i^{(1)}$ and $m_i^{(1)}$ for $i = 1,\ 2,\ \ldots,\ b^{(1)}$ with $1 \le l_i^{(1)} \le m_i^{(1)} \le k_i^{(1)}$ in the normal case and $l_i^{(2)}$ and $m_i^{(2)}$ for $i = 1,\ 2,\ \ldots,\ b^{(2)}$ with $1 \le l_i^{(2)} \le m_i^{(2)} \le k_i^{(2)}$ in the accelerated case. Consider the predetermined time points $T_i^{(1)}$ for $i = 1,\ 2,\ \ldots,\ b^{(1)}$ and $T_i^{(2)}$ for $i = 1,\ 2,\ \ldots,\ b^{(2)}$ in the normal and accelerated conditions, respectively. The non-negative prefixed integers $r^{(1)}_{i1},\ r^{(1)}_{i2},\ \ldots,\ r^{(1)}_{b^{(1)}m_i^{(1)}}$ for $i = 1,\ 2,\ \ldots,\ b^{(1)}$ in the normal case and $r^{(2)}_{i1},\ r^{(2)}_{i2},\ \ldots,\ r^{(2)}_{b^{(2)}m_i^{(2)}}$ for $i = 1,\ 2,\ \ldots,\ b^{(2)}$

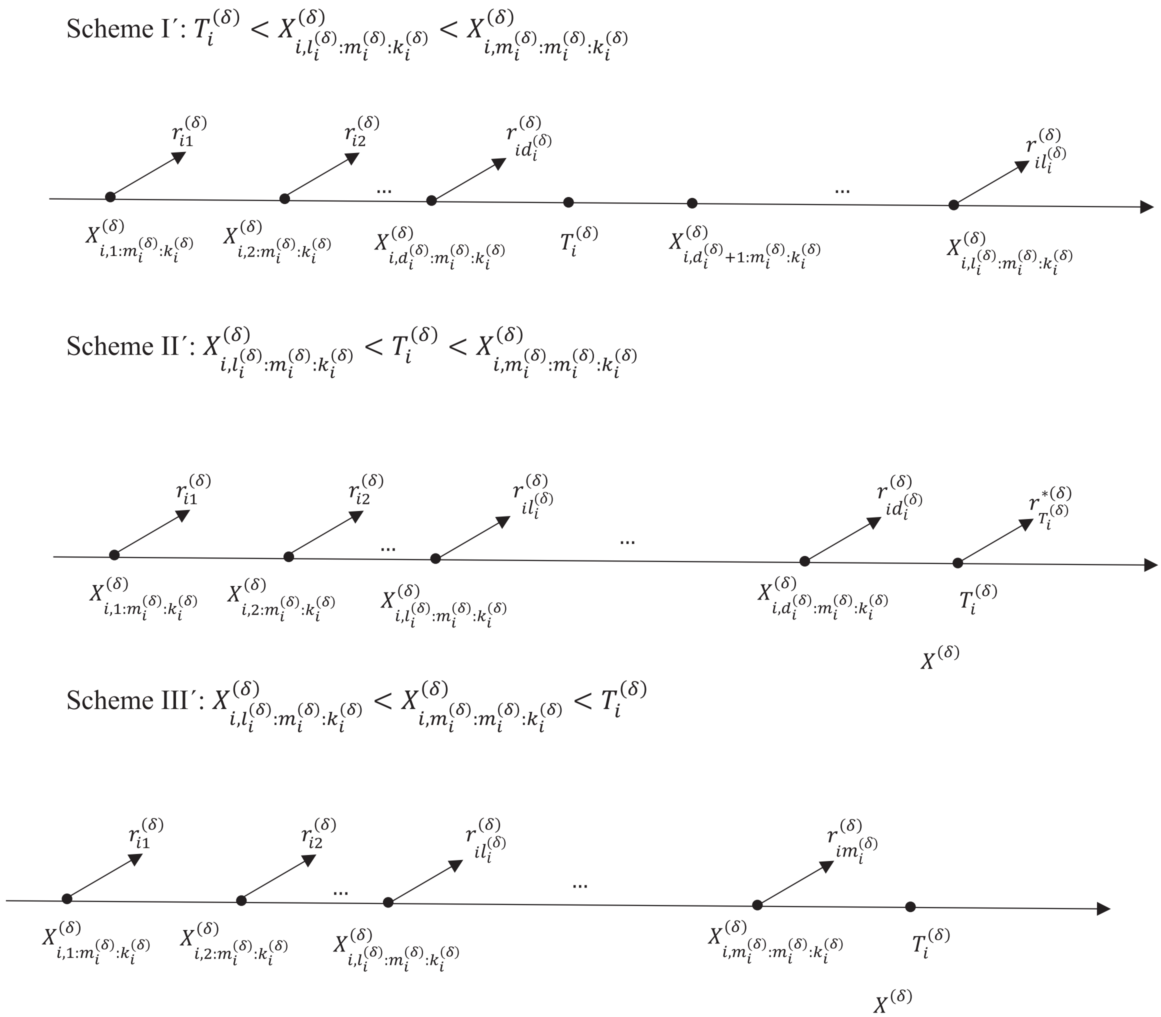

Fig. 2. Graphical presentation of the generalized progressive hybrid censoring scheme for the strength variable.

in the accelerated case where $\sum_{j=1}^{m_i^{(1)}} r_{ij}^{(1)} + m_i^{(1)} = k_i^{(1)}$ for $i = 1, 2, \ldots, b^{(1)}$ and $\sum_{j=1}^{m_i^{(2)}} r_{ij}^{(2)} + m_i^{(2)} = k_i^{(2)}$ for $i = 1, 2, \ldots, b^{(2)}$. Let $X^{(1)}_{i,j:m_i^{(1)}:k_i^{(1)}}$ for $i = 1, 2, \ldots, b^{(1)}$ and $X^{(2)}_{i,j:m_i^{(2)}:k_i^{(2)}}$ for $i = 1, 2, \ldots, b^{(2)}$ denotes the failure of the j-th element in the i subsystem of the test in case of normal and accelerated conditions, respectively, then the testing will stop at the following points:

$$T_i^{*(\delta)} = \begin{cases} X^{(\delta)}_{i,l_i^{(\delta)}:m_i^{(\delta)}:k_i^{(\delta)}}, & if\ T_i^{(\delta)} < X^{(\delta)}_{i,l_i^{(\delta)}:m_i^{(\delta)}:k_i^{(\delta)}} < X^{(\delta)}_{i,m_i^{(\delta)}:m_i^{(\delta)}:k_i^{(\delta)}} \\ T_i^{(\delta)}, & if\ X^{(\delta)}_{i,l_i^{(\delta)}:m_i^{(\delta)}:k_i^{(\delta)}} < T_i^{(\delta)} < X^{(\delta)}_{i,m_i^{(\delta)}:m_i^{(\delta)}:k_i^{(\delta)}}, \\ X^{(\delta)}_{i,m_i^{(\delta)}:m_i^{(\delta)}:k_i^{(\delta)}}, & if\ X^{(\delta)}_{i,l_i^{(\delta)}:m_i^{(\delta)}:k_i^{(\delta)}} < X^{(\delta)}_{i,m_i^{(\delta)}:m_i^{(\delta)}:k_i^{(\delta)}} < T_i^{(\delta)} \end{cases} \text{for } i = 1, \ldots, b^{(\delta)},\ \delta = 1, 2$$

where $\delta = 1, 2$ represents the case of normal and accelerated conditions, respectively. The following scenario can be observed in case of the different types of failures under the generalized progressive hybrid censoring scheme:

Scheme I' : $X^{(\delta)}_{i,1:m_i^{(\delta)}:k_i^{(\delta)}}, X^{(\delta)}_{i,2:m_i^{(\delta)}:k_i^{(\delta)}}, \ldots, X^{(\delta)}_{i,l_i^{(\delta)}:m_i^{(\delta)}:k_i^{(\delta)}}, if\ T_i^{(\delta)} < X^{(\delta)}_{i,l_i^{(\delta)}:m_i^{(\delta)}:k_i^{(\delta)}} < X^{(\delta)}_{i,m_i^{(\delta)}:m_i^{(\delta)}:k_i^{(\delta)}}$
Scheme II' : $X^{(\delta)}_{i,1:m_i^{(\delta)}:k_i^{(\delta)}}, \ldots, X^{(\delta)}_{i,l_i^{(\delta)}:m_i^{(\delta)}:k_i^{(\delta)}}, \ldots, X^{(\delta)}_{i,d_i^{(\delta)}:m_i^{(\delta)}:k_i^{(\delta)}}, if\ X^{(\delta)}_{i,l_i^{(\delta)}:m_i^{(\delta)}:k_i^{(\delta)}} < T_i^{(\delta)} < X^{(\delta)}_{i,m_i^{(\delta)}:m_i^{(\delta)}:k_i^{(\delta)}}$
Scheme III' : $X^{(\delta)}_{i,1:m_i^{(\delta)}:k_i^{(\delta)}}, X^{(\delta)}_{i,2:m_i^{(\delta)}:k_i^{(\delta)}}, \ldots, X^{(\delta)}_{i,m_i^{(\delta)}:m_i^{(\delta)}:k_i^{(\delta)}}, if\ X^{(\delta)}_{i,l_i^{(\delta)}:m_i^{(\delta)}:k_i^{(\delta)}} < X^{(\delta)}_{i,m_i^{(\delta)}:m_i^{(\delta)}:k_i^{(\delta)}} < T_i^{(\delta)}$

for $i = 1, \ldots, b^{(\delta)}, \delta = 1, 2$. the graphical presentation of the generalized progressive hybrid censoring scheme is shown in Fig. 2.

where

$$r^{(\delta)}_{il_i^{(\delta)}} = k_i^{(\delta)} - l_i^{(\delta)} - \sum_{j=1}^{d_i^{(\delta)}} r_{ij}^{(\delta)}$$

$$r^{*(\delta)}_{T_i^{(\delta)}} = k_i^{(\delta)} - d_i^{(\delta)} - \sum_{j=1}^{d_i^{(\delta)}} r_{ij}^{(\delta)}$$

$$r^{(\delta)}_{im_i^{(\delta)}} = k_i^{(\delta)} - m_i^{(\delta)} - \sum_{j=1}^{d_i^{(\delta)}} r_{ij}^{(\delta)}$$

Fuzzy multicomponent stress-strength reliability

The multicomponent reliability of a system is the probability that at least $(X_1, X_2, \dots, X_k)$ will exceed $Y$where X and Y are independent random variables. Let us consider that the units of the system will attend the normal case with probability p and the accelerated case with probability $(1-p)$. From the definition of fuzzy probability given by Zadeh [27], the fuzzy multicomponent stress-strength reliability can be formulated as follows.

$$R^F_{s,k} = \sum_{i=s}^{k} \binom{k}{i} \left\{ p \int_0^\infty \mu_A(y)\left[1 - F_1(y)\right]^i \left[F_1(y)\right]^{k-i} dG_1(y) + (1-p) \int_0^\infty \mu_A(y)\left[1 - F_2(y)\right]^i \left[F_2(y)\right]^{k-i} dG_2(y) \right\},\ 0 < p < 1$$

Where $\mu_A(y)$ is an appropriate membership function on $Y \to [0,\ 1]$, therefore in the case of inverse Weibull $\mu_A(y)$ is assumed to increase as Y is increasing which can be formulated as follows

$$\mu_A(y) = \begin{cases} 0, & y < 0 \\ e^{-c/y^\gamma}, & y > 0 \end{cases}$$

Since X and Y follow the inverse Weibull distribution, then fuzzy multicomponent stress-strength reliability will be given by

$$\begin{aligned} R^F_{s,k} = \sum_{i=s}^{k} \binom{k}{i} \Bigg\{ & p \int_0^\infty e^{-c/y^\gamma} \left[1 - e^{-\beta_1/y^\gamma}\right]^i e^{-\beta_1(k-i)/y^\gamma} \frac{\beta_2\gamma}{y^{\gamma+1}} e^{-\beta_2/y^\gamma} dy \\ & +(1-p) \int_0^\infty e^{-c/y^\gamma} \left[1 - e^{-\beta_1/(\lambda y)^\gamma}\right]^i e^{-\beta_1(k-i)/(\lambda y)^\gamma} \frac{\beta_2\gamma}{(\lambda y)^{\gamma+1}} e^{-\beta_2/(\lambda y)^\gamma} dy \Bigg\},\ 0 < p < 1,\ c > 0 \end{aligned}$$

$$R^F_{s,k} = \sum_{i=s}^{k} \sum_{j=0}^{i} \binom{k}{i} \binom{i}{j} (-1)^j \left\{ \frac{p\beta_2}{\left[c + \beta_1(k-i+j) + \beta_2\right]} + \frac{(1-p)\beta_2}{\left[c\lambda^\gamma + \beta_1(k-i+j) + \beta_2\right]} \right\} \tag{1}$$

It can be observed that when $p = 1$, we obtain the fuzzy multicomponent stress strength reliability with all units in the normal case only. When $\lambda = 0$, we obtain the fuzzy multicomponent stress strength reliability with all units in the normal case only. When $c = 0$, we obtain the classical multicomponent stress strength reliability.

Maximum likelihood estimation method

In order to obtain an estimate for the fuzzy multicomponent stress strength reliability under the generalized progressive hybrid censoring and in case of the accelerated life testing, the maximum likelihood method is applied as follows. The likelihood function for the model can be formulated as follows.

$$Y = \begin{pmatrix} Y_1^{(1)} \\ Y_2^{(1)} \\ \vdots \\ Y_{b^{(1)}}^{(1)} \\ \vdots \\ Y_1^{(2)} \\ Y_2^{(2)} \\ \vdots \\ Y_{b^{(2)}}^{(2)} \end{pmatrix},\ X = \begin{pmatrix} X_{11}^{(1)} & X_{12}^{(1)} & \cdots & X_{1m_1^{(1)}}^{(1)} \\ X_{21}^{(1)} & X_{22}^{(1)} & \cdots & X_{2m_2^{(1)}}^{(1)} \\ \vdots & \vdots & \cdots & \vdots \\ X_{b^{(1)}1}^{(1)} & X_{b^{(1)}2}^{(1)} & \cdots & X_{b^{(1)}m_{b^{(1)}}^{(1)}}^{(1)} \\ \vdots & \vdots & \cdots & \vdots \\ X_{11}^{(2)} & X_{12}^{(2)} & \cdots & X_{1m_1^{(2)}}^{(2)} \\ X_{21}^{(2)} & X_{22}^{(2)} & \cdots & X_{2m_2^{(2)}}^{(2)} \\ \vdots & \vdots & \cdots & \vdots \\ X_{b^{(2)}1}^{(2)} & X_{b^{(2)}1}^{(1)} & \cdots & X_{b^{(2)}m_{b^{(2)}}^{(2)}}^{(1)} \end{pmatrix}$$

$$L=\prod_{\delta=1}^{2}\prod_{i=1}^{e^{(\delta)}}\left\{\begin{array}{c}\prod_{j=1}^{d_i^{(\delta)}} f\left(\lambda^{\delta-1}x^{(\delta)}_{i,j:m_{i:}^{(\delta)}k_i^{(\delta)}}\right)\left[\bar{F}\left(\lambda^{\delta-1}x^{(\delta)}_{i,j:m_{i:}^{(\delta)}k_i^{(\delta)}}\right)\right]^{r_{ij}^{\delta}}\left[\bar{F}\left(\lambda^{\delta-1}t^{(\delta)}_{i:m_{i:}^{(\delta)}k_i^{(\delta)}}\right)\right]^{r^{*(\delta)}_{t_i^{(\delta)}}} g\left(\lambda^{\delta-1}y^{(\delta)}_{i:b^{(\delta)}:n^{(\delta)}}\right)\\ \left[\bar{G}\left(\lambda^{\delta-1}y^{(\delta)}_{i:b^{(\delta)}:n^{(\delta)}}\right)\right]^{q_i^{(\delta)}}\left[\bar{G}\left(\lambda^{\delta-1}\tau^{(\delta)}\right)\right]^{q^{*(\delta)}_{\tau^{(\delta)}}}\end{array}\right\}$$

where

$$e^{(\delta)}=\begin{cases}h^{(\delta)} & if\quad \tau^{(\delta)}<y^{(\delta)}_{h^{(\delta)}:b^{(\delta)}:n^{(\delta)}}<y^{(\delta)}_{b^{(\delta)}:b^{(\delta)}:n^{(\delta)}}\\ e^{(\delta)} & if\quad y^{(\delta)}_{h^{(\delta)}:b^{(\delta)}:n^{(\delta)}}<\tau^{(\delta)}<y^{(\delta)}_{b^{(\delta)}:b^{(\delta)}:n^{(\delta)}},\\ b^{(\delta)} & if\quad y^{(\delta)}_{h^{(\delta)}:b^{(\delta)}:n^{(\delta)}}<y^{(1)}_{b^{(1)}:b^{(1)}:n^{(1)}}<\tau^{(\delta)}\end{cases}$$

$$d_i^{(\delta)}=\begin{cases}l_i^{(\delta)} & if\quad t_i^{(\delta)}<x^{(\delta)}_{i,l_i^{(\delta)}:m_i^{(\delta)}:k_i^{(\delta)}}<x^{(\delta)}_{i,m_i^{(\delta)}:m_i^{(\delta)}:k_i^{(\delta)}}\\ d_i^{(\delta)} & if\quad x^{(\delta)}_{i,l_i^{(\delta)}:m_i^{(\delta)}:k_i^{(\delta)}}<t_i^{(\delta)}<x^{(\delta)}_{i,m_i^{(\delta)}:m_i^{(\delta)}:k_i^{(\delta)}}\\ m_i^{(\delta)} & if\quad x^{(\delta)}_{i,l_i^{(\delta)}:m_i^{(\delta)}:k_i^{(\delta)}}<x^{(\delta)}_{i,m_i^{(\delta)}:m_i^{(\delta)}:k_i^{(\delta)}}<t_i^{(\delta)}\end{cases}$$

$$q^{*(\delta)}_{\tau^{(\delta)}}=\begin{cases}0 & if\quad \tau^{(\delta)}<Y^{(\delta)}_{h^{(\delta)}:b^{(\delta)}:n^{(\delta)}}<Y^{(\delta)}_{b^{(\delta)}:b^{(\delta)}:n^{(\delta)}}\\ q^{*(\delta)}_{\tau^{(\delta)}} & if\quad Y^{(\delta)}_{h^{(\delta)}:b^{(\delta)}:n^{(\delta)}}<\tau^{(\delta)}<Y^{(\delta)}_{b^{(\delta)}:b^{(\delta)}:n^{(\delta)}}\\ 0 & if\quad Y^{(\delta)}_{h^{(\delta)}:b^{(\delta)}:n^{(\delta)}}<Y^{(1)}_{b^{(1)}:b^{(1)}:n^{(1)}}<\tau^{(\delta)}\end{cases}$$

$$r^{*(\delta)}_{t_i^{(\delta)}}=\begin{cases}0 & if\quad t_i^{(\delta)}<x^{(\delta)}_{i,l_i^{(\delta)}:m_i^{(\delta)}:k_i^{(\delta)}}<x^{(\delta)}_{i,m_i^{(\delta)}:m_i^{(\delta)}:k_i^{(\delta)}}\\ r^{*(\delta)}_{t_i^{(\delta)}} & if\quad x^{(\delta)}_{i,l_i^{(\delta)}:m_i^{(\delta)}:k_i^{(\delta)}}<t_i^{(\delta)}<x^{(\delta)}_{i,m_i^{(\delta)}:m_i^{(\delta)}:k_i^{(\delta)}}\\ 0 & if\quad x^{(\delta)}_{i,l_i^{(\delta)}:m_i^{(\delta)}:k_i^{(\delta)}}<x^{(\delta)}_{i,m_i^{(\delta)}:m_i^{(\delta)}:k_i^{(\delta)}}<t_i^{(\delta)}\end{cases}$$

The likelihood function after the inverse Weibull distribution will be given as follows.

$$L=\prod_{\delta=1}^{2}\prod_{i=1}^{e^{(\delta)}}\left\{\begin{array}{c}\prod_{j=1}^{d_i^{(\delta)}}\frac{\beta_1\gamma\lambda^{\delta-1}}{\left(\lambda^{\delta-1}x_{ij}^{\delta}\right)^{\gamma+1}}e^{-\frac{\beta_1}{\left(\lambda^{\delta-1}x_{ij}^{\delta}\right)^{\gamma}}}\left[1-e^{-\frac{\beta_1}{\left(\lambda^{\delta-1}x_{ij}^{\delta}\right)^{\gamma}}}\right]^{r_{ij}^{\delta}}\left[1-e^{-\frac{\beta_1}{\left(\lambda^{\delta-1}t_i^{\delta}\right)^{\gamma}}}\right]^{r^{*(\delta)}_{t_i^{(\delta)}}}\frac{\beta_2\gamma\lambda^{\delta-1}}{\left(\lambda^{\delta-1}y_i^{\delta}\right)^{\gamma+1}}\\ e^{-\frac{\beta_2}{\left(\lambda^{\delta-1}y_i^{\delta}\right)^{\gamma}}}\left[1-e^{-\frac{\beta_2}{\left(\lambda^{\delta-1}y_i^{\delta}\right)^{\gamma}}}\right]^{q_i^{\delta}}\left[1-e^{-\frac{\beta_2}{\left(\lambda^{\delta-1}\tau^{\delta}\right)^{\gamma}}}\right]^{q^{*}_{\tau^{(\delta)}}}\end{array}\right\}$$

The logarithm of the likelihood function is taken and the result is given as follows.

$$\begin{aligned}log\,L=&\sum_{\delta=1}^{2}\left\{\left(\sum_{i=1}^{e^{(\delta)}}d_i^{(\delta)}\right)\log\beta_1+\left(\sum_{i=1}^{e^{(\delta)}}d_i^{(\delta)}+e^{(\delta)}\right)\log\gamma+e^{(\delta)}\log\beta_2-\gamma\left(\sum_{i=1}^{e^{(\delta)}}d_i^{(\delta)}+e^{(\delta)}\right)\right.\\&\log\lambda^{\delta-1}-(\gamma+1)\sum_{i=1}^{e^{(\delta)}}\sum_{j=1}^{d_i^{(\delta)}}\log x_{ij}^{\delta}-\sum_{i=1}^{e^{(\delta)}}\sum_{j=1}^{d_i^{(\delta)}}\frac{\beta_1}{\left(\lambda^{\delta-1}x_{ij}^{\delta}\right)^{\gamma+1}}+\sum_{i=1}^{e^{(\delta)}}\sum_{j=1}^{d_i^{(\delta)}}r_{ij}^{\delta}\log\left[1-e^{-\frac{\beta_1}{\left(\lambda^{\delta-1}x_{ij}^{\delta}\right)^{\gamma}}}\right]\\&+\sum_{i=1}^{e^{(\delta)}}r^{*(\delta)}_{t_i^{(\delta)}}d_i^{(\delta)}\log\left[1-e^{-\frac{\beta_1}{\left(\lambda^{\delta-1}t_i^{(\delta)}\right)^{\gamma}}}\right]-(\gamma+1)\sum_{i=1}^{e^{(\delta)}}\log y_i^{\delta}-\sum_{i=1}^{e^{(\delta)}}\frac{\beta_2}{\left(\lambda^{\delta-1}y_i^{\delta}\right)^{\gamma}}+\\&\left.\sum_{i=1}^{e^{(\delta)}}q_i^{(\delta)}\log\left[1-e^{-\frac{\beta_2}{\left(\lambda^{\delta-1}y_i^{\delta}\right)^{\gamma}}}\right]+q^{*(\delta)}_{\tau^{(\delta)}}\,e^{(\delta)}\log\left[1-e^{-\frac{\beta_2}{\left(\lambda^{\delta-1}\tau^{(\delta)}\right)^{\gamma}}}\right]\right\}\end{aligned}\tag{2}$$

The maximum likelihood estimates of the parameters $\beta_1,\ \beta_2,\ \gamma$ and λ can be obtained by using Eq. (2) and the method of Newton Raphson by the aid of R software program [28]. Then the maximum likelihood estimate for the fuzzy multicomponent stress-strength reliability is obtained by substituting in Eq. (1).

Asymptotic confidence interval

In order to obtain an asymptotic confidence interval for the fuzzy multicomponent stress strength reliability, the variance of $\hat{R}^F_{s,k}$ is formulated as

$$var\left(\hat{R}^F_{s,k}\right) = \left[\frac{\partial R^F_{s,k}}{\partial\beta_1}\right]^2_{\beta_1=\hat{\beta}_1} var\left(\hat{\beta}_1\right) + \left[\frac{\partial R^F_{s,k}}{\partial\beta_2}\right]^2_{\beta_2=\hat{\beta}_2} ar\left(\hat{\beta}_2\right) + \left[\frac{\partial R^F_{s,k}}{\partial\gamma}\right]^2_{\gamma=\hat{\gamma}} var(\hat{\gamma}) + \left[\frac{\partial R^F_{s,k}}{\partial\lambda}\right]^2_{\lambda=\hat{\lambda}} var\left(\hat{\lambda}\right)$$
$$+2\left[\frac{\partial R^F_{s,k}}{\partial\beta_1}\right]_{\beta_1=\hat{\beta}_1}\left[\frac{\partial R^F_{s,k}}{\partial\beta_2}\right]_{\beta_2=\hat{\beta}_2} cov\left(\hat{\beta}_1,\hat{\beta}_2\right) + 2\left[\frac{\partial R^F_{s,k}}{\partial\beta_1}\right]_{\beta_1=\hat{\beta}_1}\left[\frac{\partial R^F_{s,k}}{\partial\gamma}\right]_{\gamma=\hat{\gamma}} cov\left(\hat{\beta}_1,\hat{\gamma}\right)$$
$$+2\left[\frac{\partial R^F_{s,k}}{\partial\beta_1}\right]_{\beta_1=\hat{\beta}_1}\left[\frac{\partial R^F_{s,k}}{\partial\lambda}\right]_{\lambda=\hat{\lambda}} cov\left(\hat{\beta}_1,\hat{\lambda}\right) + 2\left[\frac{\partial R^F_{s,k}}{\partial\beta_2}\right]_{\beta_2=\hat{\beta}_2}\left[\frac{\partial R^F_{s,k}}{\partial\gamma}\right]_{\gamma=\hat{\gamma}} cov\left(\hat{\beta}_2,\hat{\gamma}\right)$$
$$+2\left[\frac{\partial R^F_{s,k}}{\partial\beta_2}\right]_{\beta_2=\hat{\beta}_2}\left[\frac{\partial R^F_{s,k}}{\partial\lambda}\right]_{\lambda=\hat{\lambda}} cov\left(\hat{\beta}_2,\hat{\lambda}\right) + 2\left[\frac{\partial R^F_{s,k}}{\partial\gamma}\right]_{\gamma=\hat{\gamma}}\left[\frac{\partial R^F_{s,k}}{\partial\lambda}\right]_{\lambda=\hat{\lambda}} cov\left(\hat{\gamma},\hat{\lambda}\right)$$

where

$$var\left(\hat{\beta}_1\right) = E\left[-\frac{\partial^2 logL}{\partial\beta_1^2}\right]^{-1}_{\beta_1=\hat{\beta}_1},\ var\left(\hat{\beta}_2\right) = E\left[-\frac{\partial^2 logL}{\partial\beta_2^2}\right]^{-1}_{\beta_2=\hat{\beta}_2},\ var(\hat{\gamma}) = E\left[-\frac{\partial^2 logL}{\partial\gamma^2}\right]^{-1}_{\gamma=\hat{\gamma}},$$
$$var\left(\hat{\lambda}\right) = E\left[-\frac{\partial^2 logL}{\partial\lambda^2}\right]^{-1}_{\lambda=\hat{\lambda}},\ cov\left(\hat{\beta}_1,\hat{\beta}_2\right) = 0,\ cov\left(\hat{\beta}_1,\hat{\gamma}\right) = E\left[-\frac{\partial^2 logL}{\partial\beta_1\partial\gamma}\right]^{-1}_{\beta_1=\hat{\beta}_1,\gamma=\hat{\gamma}},$$
$$cov\left(\hat{\beta}_1,\hat{\lambda}\right) = E\left[-\frac{\partial^2 logL}{\partial\beta_1\partial\lambda}\right]^{-1}_{\beta_1=\hat{\beta}_1,\lambda=\hat{\lambda}},cov\left(\hat{\beta}_2,\hat{\gamma}\right) = E\left[-\frac{\partial^2 logL}{\partial\beta_2\partial\gamma}\right]^{-1}_{\beta_2=\hat{\beta}_2,\gamma=\hat{\gamma}},$$
$$cov\left(\hat{\beta}_2,\hat{\lambda}\right) = E\left[-\frac{\partial^2 logL}{\partial\beta_2\partial\lambda}\right]^{-1}_{\beta_2=\hat{\beta}_2,\lambda=\hat{\lambda}},cov\left(\hat{\gamma},\hat{\lambda}\right) = E\left[-\frac{\partial^2 logL}{\partial\gamma\partial\lambda}\right]^{-1}_{\gamma=\hat{\gamma},\lambda=\hat{\lambda}}$$

The $(1-\alpha)100\%$ asymptotic confidence interval for the fuzzy multicomponent stress strength reliability will be given by

$$\hat{R}^F_{s,k} \pm z_{1-\frac{\alpha}{2}}\sqrt{var\left(\hat{R}^F_{s,k}\right)}$$

Simulation study

Balakrishnan and Sandhu [29] presented an algorithm to simulate data for censoring scheme. The following algorithm is constructed in order to simulate data for the generalized progressive hybrid censoring scheme.

1. Set iteration =1000
2. Start with $i = 1$
3. Put initial values for the parameters $\beta_1,\ \beta_2,\ \gamma,\ \lambda$ and the constants $n, k,\ p, c$
4. Set the predetermined values for $(n^{(\delta)},\ h^{(\delta)}, b^{(\delta)}, l_i^{(\delta)},\ m_i^{(\delta)}),\ for\ i = 1,\ \dots,\ b^{(\delta)},\ \delta = 1,\ 2$
5. Define the schemes $q_i^{(\delta)}$ and $r_{ij}^{(\delta)}$ for $j = 1,\dots,\ m_i^{(\delta)}, i = 1,\dots,\ b^{(\delta)},\ \delta = 1,\ 2$
6. Generate two random samples $V_i^{(1)}$ and $V_i^{(2)}$ with sizes $b^{(1)}$ and $b^{(2)}$, respectively from the distribution $uniform(0,1)$
7. Define $Z_i^{(\delta)} = V_i^{\left(i+\sum_{j=b^{(\delta)}-i+1}^{b^{(\delta)}} q_j^{(\delta)}\right)^{-1}}$ for $i = 1, 2,\dots, b^{(\delta)},\ \delta = 1,\ 2$
8. Set $\omega_i^{(\delta)} = 1 - \prod_{j=b^{(\delta)}-i+1}^{b^{(\delta)}} Z_j^{(\delta)}$ for $i = 1, 2,\dots, b^{(\delta)},\ \delta = 1,\ 2$
9. The simulated random samples will be computed from the following relations:
   - In the normal condition
   $$Y_i^{(1)} = \left(-\frac{\beta_2}{\log\left(\omega_i^{(1)}\right)}\right)^{1/\gamma}$$
   - In the accelerated condition
   $$Y_i^{(2)} = \frac{\left(-\frac{\beta_2}{\log\left(\omega_i^{(2)}\right)}\right)^{1/\gamma}}{\lambda}$$

10. Generate random samples $Y_{ij}^{(\delta)}$ with sizes $m_i^{(\delta)}$ for $j = 1, \ldots, m_i^{(\delta)}$, $i = 1, \ldots, b^{(\delta)}$, $\delta = 1, 2$ from the distribution $uniform(0, 1)$
11. Define $\phi_{ij}^{(\delta)} = Y_{ij}^{\left(i+\sum_{j=m_i^{(\delta)}-i+1}^{m_i^{(\delta)}} r_{ij}^{(\delta)}\right)^{-1}}$ for $j = 1, \ldots, m_i^{(\delta)}$, $i = 1, \ldots, b^{(\delta)}$, $\delta = 1, 2$
12. Set $\eta_{ij}^{(\delta)} = 1 - \prod_{j=m_i^{(\delta)}-i+1}^{m_i^{(\delta)}} \phi_{ij}^{(\delta)}$ for $j = 1, \ldots, m_i^{(\delta)}$, $i = 1, \ldots, b^{(\delta)}$, $\delta = 1, 2$
13. The simulated random samples will be computed from the following relations:
    - In the normal condition

$$X_{ij}^{(1)} = \left(-\frac{\beta_1}{\log\left(\eta_{ij}^{(1)}\right)}\right)^{1/\gamma}$$

    - In the accelerated condition

$$X_{ij}^{(2)} = \frac{\left(-\frac{\beta_1}{\log\left(\eta_{ij}^{(2)}\right)}\right)^{1/\gamma}}{\lambda}$$

Two different schemes will be considered which will be given as follows.
Scheme (I)

$$q_i^{\delta} = 0 \ for \ i = 1, \ldots, b^{(\delta)} - 1, \ q_{b^{(\delta)}}^{(\delta)} = n^{(\delta)} - b^{(\delta)}, \ \delta = 1, 2$$

$$r_{ij}^{(\delta)} = 0, \ j = 1, \ldots, m_i^{(\delta)} - 1,$$

$$r_{m_i^{(\delta)} j}^{(\delta)} = k - m_i^{(\delta)}, \ i = 1, 2, \ldots, b^{(\delta)}, \ j = 1, 2, \ldots, m_i^{(\delta)}$$

Scheme (II)

$$q_1^{\delta} = n^{(\delta)} - b^{(\delta)}, \ q_i^{(\delta)} = 0 \ for \ i = 2, \ldots, b^{(\delta)}, \ \delta = 1, 2$$

$$r_{i1}^{(\delta)} = 0, \ r_{ij}^{(\delta)} = k - m_i^{(\delta)} \ for \ j = 2, \ldots, m_i^{(\delta)}, \ i = 1, 2, \ldots, b^{(\delta)}$$

Scheme (I) and (II) are applied to obtain the maximum likelihood estimates for the fuzzy multicomponent stress-strength reliability when $n = 10, s = 3$ and $k = 5$ using the given algorithm. The results are shown in Table 1. Also, the asymptotic confidence intervals for fuzzy multicomponent stress-strength reliability are obtained.

From Table 1, it can be observed that:

1. The mean squared errors (MSE) are decreasing as the values of p and c are increasing.
2. The width of the confidence intervals is decreasing as the value of p is increasing.
3. The mean squared errors (MSE) in case of scheme (I) is smaller than the mean squared errors (MSE) in case of scheme (II).
4. The confidence intervals in case of scheme (II) are wider than the confidence intervals in case of scheme (I).

Application to real life data

The following data represents the failure times for a certain software model which is discussed in Lyu [30]. Let us consider that the failures due to software faults represent the strength data (X) and the failures due to Pascal programming represent the stress data (Y). These data are presented as follows.

$$Y = \begin{pmatrix} 35.85 \\ 43.50 \\ 45.30 \\ 56.40 \\ 94.90 \\ 173.40 \\ 184.90 \\ 196.20 \\ 236.30 \\ 346.40 \end{pmatrix},$$

Table 1
The results for the maximum likelihood estimates (with mean and MSE) and the asymptotic confidence intervals for $R^F_{3,5}$ when $\gamma = 1.5,\ \beta_1 = \beta_2 = 0.5,\ \lambda = 0.1$.

Scheme	c	p	R_{true}	$\hat{R}^F_{3,5}$	$mean(\hat{R}^F_{3,5})$	MSE	$C.I.$
(I)	0.1	0.1	0.482357	0.414145	0.322448	0.038652	[0.205475, 0.622815] (0.417340)
		0.3	0.457223	0.383314	0.299661	0.035631	[0.195390, 0.571238] (0.375847)
		0.5	0.432089	0.352484	0.276873	0.032943	[0.112982, 0.434339] (0.321357)
		0.9	0.381821	0.224822	0.228663	0.028921	[0.133563, 0.316082] (0.182518)
	0.5	0.1	0.442095	0.593931	0.297679	0.032270	[0.454437, 0.733425] (0.278988)
		0.3	0.375598	0.307348	0.243921	0.024599	[0.181422, 0.433273] (0.251850)
		0.5	0.309100	0.181568	0.191241	0.018060	[0.082070, 0.281065] (0.198994)
		0.9	0.176105	0.094084	0.090582	0.008387	[0.041534, 0.146633] (0.105098)
	0.9	0.1	0.418131	0.352336	0.280098	0.028962	[0.289469, 0.415203] (0.125734)
		0.3	0.340692	0.287524	0.223747	0.019439	[0.22788, 0.347164] (0.119280)
		0.5	0.263253	0.208187	0.168474	0.012332	[0.154139, 0.262235] (0.108095)
		0.9	0.108375	0.064038	0.056849	0.003095	[0.033568, 0.094507] (0.060938)
(II)	0.1	0.1	0.482357	0.560078	0.335106	0.044607	[0.316990, 0.803166] (0.486175)
		0.3	0.457223	0.507037	0.294315	0.043893	[0.302316, 0.711758] (0.409442)
		0.5	0.432089	0.304499	0.256680	0.043646	[0.138479, 0.470519] (0.332039)
		0.9	0.381821	0.074940	0.213930	0.041229	[0.052492, 0.371992] (0.319499)
	0.5	0.1	0.442095	0.502682	0.316508	0.036764	[0.253109, 0.631081] (0.377971)
		0.3	0.375598	0.397140	0.252611	0.028131	[0.249577, 0.544703] (0.295126)
		0.5	0.309100	0.291599	0.188713	0.021479	[0.185270, 0.397928] (0.212657)
		0.9	0.176105	0.101167	0.060667	0.014254	[0.062320, 0.140013] (0.077692)
	0.9	0.1	0.418131	0.360656	0.303930	0.033514	[0.155198, 0.566113] (0.410914)
		0.3	0.340692	0.282045	0.238700	0.022878	[0.128884, 0.435207] (0.306323)
		0.5	0.263253	0.203435	0.173471	0.014526	[0.093441, 0.313429] (0.219988)
		0.9	0.108375	0.046214	0.043011	0.004668	[0.022163, 0.070265] (0.048101)

$$X = \begin{pmatrix} 4 & 6 & 14 & 14 & 15 & 22 & 42 & 84 & 221 & 303 & 758 & 760 \\ 1 & 15 & 19 & 24 & 41 & 44 & 54 & 145 & 153 & 180 & 397 & 409 \\ 1 & 5 & 8 & 16 & 17 & 19 & 29 & 36 & 54 & 87 & 163 & 1337 \\ 3 & 10 & 11 & 24 & 54 & 100 & 163 & 179 & 252 & 253 & 360 & 460 \\ 3 & 9 & 9 & 12 & 15 & 18 & 75 & 137 & 212 & 328 & 366 & 428 \\ 1 & 30 & 115 & 131 & 264 & 269 & 279 & 344 & 472 & 495 & 550 & 999 \\ 15 & 36 & 55 & 75 & 111 & 288 & 407 & 409 & 845 & 897 & 1314 & 4179 \\ 56 & 59 & 98 & 212 & 287 & 385 & 1682 & 1812 & 2439 & 3500 & 4973 & 6203 \\ 35 & 47 & 83 & 92 & 249 & 352 & 607 & 614 & 673 & 863 & 991 & 9549 \\ 10 & 19 & 20 & 20 & 24 & 60 & 79 & 250 & 338 & 1737 & 1960 & 7984 \end{pmatrix}$$

The Kolmogorov–Smirnov (K-S) test is performed to ensure that X and Y follow the inverse Weibull distribution. Also, the inverse Weibull distribution is compared with some other distributions, such as exponentiated exponential distribution and the inverse Topp-Leone distribution. The results are shown in Table 2.

Table 2
Comparison between the inverse Weibull and other distributions.

Data set	Distribution	$\hat{\beta}$	$\hat{\gamma}$	Log L	AIC	K-S	p-value
Y	Inverse Weibull	491.6597 (11.8941)	1.4473 (0.0778)	−58.6583	121.3167	0.2539	0.4647
	Exponentiated Exponential	1.904e+05 (4.194)	4.078 (6.898e-02)	−275.4576	554.9153	0.59999	0.00056
	Inverse Topp-Leone	–	0.25018 (0.07911)	−77.6907	157.3814	0.51923	0.004939
X	Inverse Weibull	6.7853 (0.9278)	0.5152 (0.0347)	−824.9238	1653.848	0.3638	0.0841
	Exponentiated Exponential	3.850e-01 (3.516e-02)	1.232e+03 (2.966)	−833.5991	1671.198	0.152	0.007816
	Inverse Topp-Leone	–	0.24527 (0.02239)	−944.7076	1891.415	0.26999	5.049e-08

The exponentiated exponential distribution [31] with shape parameter β and scale parameter γ has the cumulative distribution function and probability density function given, respectively, as follows

$$F(x) = \left(1 - e^{-\frac{x}{\gamma}}\right)^{\beta}, x, \beta, \gamma > 0$$

$$f(x) = \frac{\beta}{\gamma}\left(1 - e^{-\frac{x}{\gamma}}\right)^{\beta-1} e^{-\frac{x}{\gamma}}, x, \beta, \gamma > 0$$

The inverse Topp-Leone distribution is introduced by Hassan et al. [32] which has the probability density function and cumulative distribution function given respectively by

$$f(x) = 2\gamma x(1+x)^{-2\gamma-1}(1+2x)^{\gamma-1},\ x,\ \gamma > 0$$

$$F(x) = 1 - \left\{\frac{(1+2x)^{\gamma}}{(1+x)^{2\gamma}}\right\}$$

The results obtained in Table 2 show that the inverse Weibull model is the better distribution to model the datasets than the other distributions.

Now, considering three different schemes, to obtain the results for maximum likelihood estimates for the fuzzy multicomponent stress strength reliability when $s = 3,\ k = 5$. The different schemes are illustrated as follows.

For $n^{(1)} = n^{(2)} = 5,\ h^{(1)} = h^{(2)} = 2,\ b^{(1)} = b^{(2)} = 4,\ q_i^{(1)} = q_i^{(2)} = 0\ (for\ i = 1,\ 2,\ 4),\ q_3^{(1)} = q_3^{(2)} = 1$

Scheme I: $\tau^{(1)} = 40,\ \tau^{(1)} < Y^{(1)}_{2:4:5} < Y^{(1)}_{4:4:5}$ terminates at $Y^{(1)}_{2:4:5}$ with $q_1^{(1)} = 0, q_2^{(1)} = 3,\ Y^{(1)} = (35.85,\ 43.50)$
$\tau^{(2)} = 180,\ \tau^{(2)} < Y^{(2)}_{2:4:5} < Y^{(2)}_{4:4:5}$ terminates at $Y^{(2)}_{2:4:5}$ with $q_1^{(2)} = 0, q_2^{(2)} = 3,\ Y^{(2)} = (173.40,\ 184.90)$

Scheme II: $\tau^{(1)} = 50, Y^{(1)}_{2:4:5} < \tau^{(1)} < Y^{(1)}_{4:4:5}$ terminates at $\tau^{(1)}$ with $q_1^{(1)} = q_2^{(1)} = 0, q_3^{(1)} = 1, q^*_{\tau^{(1)}} = 1,\ E^{(1)} = 3,\ Y^{(1)} = (35.85,\ 43.50,\ 45.30)$
$\tau^{(2)} = 200, Y^{(2)}_{2:4:5} < \tau^{(2)} < Y^{(2)}_{4:4:5}$ terminates at $\tau^{(2)}$ with $q_1^{(2)} = q_2^{(2)} = 0, q_3^{(2)} = 1, q^*_{\tau^{(2)}} = 1,\ E^{(2)} = 3,\ Y^{(2)} = (173.40,\ 184.90,\ 196.20)$

Scheme III: $\tau^{(1)} = 100, Y^{(1)}_{2:4:5} < Y^{(1)}_{4:4:5} < \tau^{(1)}$ terminates at $Y^{(1)}_{4:4:5}$ with $q_1^{(1)} = q_2^{(1)} = 0, q_3^{(1)} = 1, q_4^{(1)} = 0,\ Y^{(1)} = (35.85,\ 43.50,\ 45.30,\ 56.40)$
$\tau^{(2)} = 350, Y^{(2)}_{2:4:5} < Y^{(2)}_{4:4:5} < \tau^{(2)}$ terminates at $Y^{(2)}_{4:4:5}$ with $q_1^{(2)} = q_2^{(2)} = 0, q_3^{(2)} = 1, q_4^{(2)} = 0,\ Y^{(2)} = (173.40,\ 184.90,\ 196.20,\ 236.30)$

For $k_1^{(1)} = k_1^{(2)} = 12,\ l_1^{(1)} = l_1^{(2)} = 5,\ m_1^{(1)} = m_1^{(2)} = 7,\ r_{1j}^{(1)} = 0\ (for\ j = 1,\ 3,\ 4,\ 5,\ 6,\ 7),\ r_{12}^{(1)} = 2,$
$r_{11}^{(2)} = 2,\ r_{1j}^{(2)} = 0\ (for\ j = 2,\ 3,\ 4,\ 5,\ 6,\ 7)$

Scheme I: $T_1^{(1)} = 10,\ T_1^{(1)} < X^{(1)}_{1,5:7:12} < X^{(1)}_{1,7:7:12}$ terminates at $X^{(1)}_{1,5:7:12}$ with $r_{1j}^{(1)} = 0\ (for\ j = 1,\ 3,\ 4),\ r_{12}^{(1)} = 2,\ r_{15}^{(1)} = 5,\ X_{1,j}^{(1)} = (4,\ 6,\ 14,\ 14,\ 15)$
$T_1^{(2)} = 200,\ T_1^{(2)} < X^{(2)}_{1,5:7:12} < X^{(2)}_{1,7:7:12}$ terminates at $X^{(2)}_{1,5:7:12}$ with $r_{11}^{(2)} = 2,\ r_{1j}^{(2)} = 0\ (for\ j = 2,\ 3,\ 4),\ r_{15}^{(5)} = 5,\ X_{1,j}^{(2)} = (1,\ 30,\ 115,\ 131,\ 264)$

Scheme II: $T_1^{(1)} = 20,\ X^{(1)}_{1,5:7:12} < T_1^{(1)} < X^{(1)}_{1,7:7:12}$ terminates at $T_1^{(1)}$ with $r_{1j}^{(1)} = 0\ (for\ j = 1,\ 3,\ 4,\ 5),\ r_{12}^{(1)} = 2, d_1^{(1)} = 5,\ r^*_{T_1^{(1)}} = 5,\ X_{1,j}^{(1)} = (4,\ 6,\ 14,\ 14,\ 15)$
$T_1^{(2)} = 270,\ X^{(2)}_{1,5:7:12} < T_1^{(2)} < X^{(2)}_{1,7:7:12}$ terminates at $T_1^{(2)}$ with $r_{11}^{(2)} = 2,\ r_{1j}^{(2)} = 0\ (for\ j = 2,\ 3,\ 4,\ 5,\ 6),\ d_1^{(2)} = 6, r^*_{T_1^{(2)}} = 4,\ X_{1,j}^{(2)} = (1,\ 30,\ 115,\ 131,\ 264,\ 269)$

Scheme III: $T_1^{(1)} = 50,\ X^{(1)}_{1,5:7:12} < X^{(1)}_{1,7:7:12} < T_1^{(1)}$ terminates at $X^{(1)}_{1,7:7:12}$ with $r_{1j}^{(1)} = 0\ (for\ j = 1,\ 3,\ 4,\ 5,\ 6),\ r_{12}^{(1)} = 2, r_{17}^{(1)} = 3,\ X_{1,j}^{(1)} = (4,\ 6,\ 14,\ 14,\ 15,\ 22,\ 42)$
$T_1^{(2)} = 280,\ X^{(2)}_{1,5:7:12} < X^{(2)}_{1,7:7:12} < T_1^{(2)}$ terminates at $X^{(2)}_{1,7:7:12}$ with $r_{11}^{(2)} = 2,\ r_{1j}^{(2)} = 0\ (for\ j = 2,\ 3,\ 4,\ 5,\ 6),\ r_{17}^{(2)} = 3,\ X_{1,j}^{(2)} = (1,\ 30,\ 115,\ 131,\ 264,\ 269,\ 279)$

For $k_2^{(1)} = k_2^{(2)} = 12,\ l_2^{(1)} = l_2^{(2)} = 5,\ m_2^{(1)} = m_2^{(2)} = 8,\ r_{21}^{(1)} = r_{21}^{(2)} = 2, r_{2j}^{(1)} = r_{2j}^{(2)} = 0\ (for\ j = 2,\ 3,\ 4,\ 5,\ 6,\ 7), r_{28}^{(1)} = r_{28}^{(2)} = 5$

Scheme I: $T_2^{(1)} = 30,\ T_2^{(1)} < X_{2,5:8:12}^{(1)} < X_{2,8:8:12}^{(1)}$ terminates at $X_{2,5:8:12}^{(1)}$ with $r_{21}^{(1)} = 2,\ r_{2j}^{(1)} = 0\ (for\ j = 2,\ 3,\ 4), r_{25}^{(1)} = 5,\ X_{2,j}^{(1)} = (1,\ 15,\ 19,\ 24,\ 41)$

$T_2^{(2)} = 100,\ T_2^{(2)} < X_{2,5:8:12}^{(2)} < X_{2,8:8:12}^{(2)}$ terminates at $X_{2,5:8:12}^{(2)}$ with $r_{21}^{(2)} = 2,\ r_{2j}^{(2)} = 0\ (for\ j = 2,\ 3,\ 4), r_{25}^{(2)} = 5,\ X_{2,j}^{(2)} = (15,\ 36,\ 55,\ 75,\ 111)$

Scheme II: $T_2^{(1)} = 50,\ X_{2,5:8:12}^{(1)} < T_2^{(1)} < X_{2,8:8:12}^{(1)}$ terminates at $T_2^{(1)}$ with $r_{21}^{(1)} = 2,\ r_{2j}^{(1)} = 0\ (for\ j = 2,\ 3,\ 4,\ 5,\ 6), r_{T_2^{(1)}}^{*(1)} = 4,\ X_{2,j}^{(1)} = (1,\ 15,\ 19,\ 24,\ 41,\ 44)$

$T_2^{(2)} = 200,\ X_{2,5:8:12}^{(2)} < T_2^{(2)} < X_{2,8:8:12}^{(2)}$ terminates at $T_2^{(2)}$ with $r_{21}^{(2)} = 2,\ r_{2j}^{(2)} = 0\ (for\ j = 2,\ 3,\ 4,\ 5), r_{T_2^{(2)}}^{*(2)} = 5,\ X_{2,j}^{(2)} = (15,\ 36,\ 55,\ 75,\ 111)$

Scheme III: $T_2^{(1)} = 150,\ X_{2,5:8:12}^{(1)} < X_{2,8:8:12}^{(1)} < T_2^{(1)}$ terminates at $X_{2,8:8:12}^{(1)}$ with $r_{21}^{(1)} = 2,\ r_{2j}^{(1)} = 0\ (for\ j = 2,\ 3,\ 4,\ 5,\ 6,\ 7), r_{28}^{(1)} = 3,\ X_{2,j}^{(1)} = (1,\ 15,\ 19,\ 24,\ 41,\ 44,\ 54,\ 145)$

$T_2^{(2)} = 425,\ X_{2,5:8:12}^{(2)} < X_{2,8:8:12}^{(2)} < T_2^{(1)}$ terminates at $X_{2,8:8:12}^{(2)}$ with $r_{21}^{(2)} = 2,\ r_{2j}^{(2)} = 0\ (for\ j = 2,\ 3,\ 4,\ 5,\ 6,\ 7), r_{28}^{(2)} = 3,\ X_{2,j}^{(2)} = (15,\ 36,\ 55,\ 75,\ 111,\ 288,\ 407,\ 409)$

For $k_3^{(1)} = k_3^{(2)} = 12,\ l_3^{(1)} = l_3^{(2)} = 6,\ m_3^{(1)} = m_3^{(2)} = 8,\ r_{31}^{(1)} = r_{31}^{(2)} = 1, r_{3j}^{(1)} = r_{3j}^{(2)} = 0\ (for\ j = 2,\ 4,\ 5,\ 6,\ 7), r_{33}^{(1)} = r_{33}^{(2)} = 2,\ r_{38}^{(1)} = r_{38}^{(2)} = 1$

Scheme I: $T_3^{(1)} = 18,\ T_3^{(1)} < X_{3,6:8:12}^{(1)} < X_{3,8:8:12}^{(1)}$ terminates at $X_{3,6:8:12}^{(1)}$ with $r_{31}^{(1)} = 1, r_{3j}^{(1)} = 0\ (for\ j = 2,\ 4,\ 5), r_{33}^{(1)} = 2,\ r_{36}^{(1)} = 3,\ X_{3,j}^{(1)} = (1,\ 5,\ 8,\ 16,\ 17,\ 19)$

$T_3^{(2)} = 300,\ T_3^{(2)} < X_{3,6:8:12}^{(2)} < X_{3,8:8:12}^{(2)}$ terminates at $X_{3,6:8:12}^{(2)}$ with $r_{31}^{(2)} = 1, r_{3j}^{(2)} = 0\ (for\ j = 2,\ 4,\ 5), r_{33}^{(2)} = 2,\ r_{36}^{(2)} = 3,\ X_{3,j}^{(2)} = (56,\ 59,\ 98,\ 212,\ 287,\ 385)$

Scheme II: $T_3^{(1)} = 20,\ X_{3,6:8:12}^{(1)} < T_3^{(1)} < X_{3,8:8:12}^{(1)}$ terminates at $T_3^{(1)}$ with $r_{31}^{(1)} = 1, r_{3j}^{(1)} = 0\ (for\ j = 2,\ 4,\ 5,\ 6), r_{33}^{(1)} = 2, r_{T_3^{(1)}}^{*(1)} = 3,\ X_{3,j}^{(1)} = (1,\ 5,\ 8,\ 16,\ 17,\ 19)$

$T_3^{(2)} = 450,\ X_{3,6:8:12}^{(2)} < T_3^{(2)} < X_{3,8:8:12}^{(2)}$ terminates at $T_3^{(2)}$ with $r_{31}^{(2)} = 1, r_{3j}^{(2)} = 0\ (for\ j = 2,\ 4,\ 5,\ 6), r_{33}^{(2)} = 2, r_{T_3^{(2)}}^{*(2)} = 3,\ X_{3,j}^{(2)} = (56,\ 59,\ 98,\ 212,\ 287,\ 385)$

Scheme III: $T_3^{(1)} = 40,\ X_{3,6:8:12}^{(1)} < X_{3,8:8:12}^{(1)} < T_3^{(1)}$ terminates at $X_{3,8:8:12}^{(1)}$ with $r_{31}^{(1)} = 1, r_{3j}^{(1)} = 0\ (for\ j = 2,\ 4,\ 5,\ 6,\ 7), r_{33}^{(1)} = 2, r_{38}^{(1)} = 1,\ X_{3,j}^{(1)} = (1,\ 5,\ 8,\ 16,\ 17,\ 19,\ 29,\ 36)$

$T_3^{(2)} = 2000,\ X_{3,6:8:12}^{(2)} < X_{3,8:8:12}^{(2)} < T_3^{(2)}$ terminates at $X_{3,8:8:12}^{(2)}$ with $r_{31}^{(2)} = 1, r_{3j}^{(2)} = 0\ (for\ j = 2,\ 4,\ 5,\ 6,\ 7), r_{33}^{(2)} = 2, r_{38}^{(2)} = 1,\ X_{3,j}^{(2)} = (56,\ 59,\ 98,\ 212,\ 287,\ 385,\ 1682,\ 1812)$

For $k_4^{(1)} = k_4^{(2)} = 12,\ l_4^{(1)} = l_4^{(2)} = 5,\ m_4^{(1)} = m_4^{(2)} = 7,\ r_{41}^{(1)} = r_{41}^{(2)} = 4, r_{4j}^{(1)} = r_{4j}^{(2)} = 0\ (for\ j = 2,\ 3,\ 4,\ 6,\ 7), r_{45}^{(1)} = r_{45}^{(2)} = 1$

Scheme I: $T_4^{(1)} = 35,\ T_4^{(1)} < X_{4,5:7:12}^{(1)} < X_{4,7:7:12}^{(1)}$ terminates at $X_{4,5:7:12}^{(1)}$ with $r_{41}^{(1)} = 4, r_{4j}^{(1)} = 0\ (for\ j = 2,\ 3,\ 4), r_{45}^{(1)} = 3, X_{4,j}^{(1)} = (3,\ 10,\ 11,\ 24,\ 54)$

$T_4^{(2)} = 130,\ T_4^{(2)} < X_{4,5:7:12}^{(2)} < X_{4,7:7:12}^{(2)}$ terminates at $X_{4,5:7:12}^{(2)}$ with $r_{41}^{(2)} = 4, r_{4j}^{(2)} = 0\ (for\ j = 2,\ 3,\ 4), r_{45}^{(2)} = 3, X_{4,j}^{(2)} = (35,\ 47,\ 83,\ 92,\ 249)$

Scheme II: $T_4^{(1)} = 60,\ X_{4,5:7:12}^{(1)} < T_4^{(1)} < X_{4,7:7:12}^{(1)}$ terminates at $T_4^{(1)}$ with $r_{41}^{(1)} = 4, r_{4j}^{(1)} = 0\ (for\ j = 2,\ 3,\ 4), r_{45}^{(1)} = 1, r_{T_4^{(1)}}^{*(1)} = 2, X_{4,j}^{(1)} = (3,\ 10,\ 11,\ 24,\ 54)$

$T_4^{(2)} = 260,\ X_{4,5:7:12}^{(2)} < T_4^{(2)} < X_{4,7:7:12}^{(2)}$ terminates at $T_4^{(2)}$ with $r_{41}^{(2)} = 4, r_{4j}^{(2)} = 0\ (for\ j = 2,\ 3,\ 4), r_{45}^{(2)} = 1, r_{T_4^{(2)}}^{*(2)} = 2, X_{4,j}^{(2)} = (35,\ 47,\ 83,\ 92,\ 249)$

Scheme III: $T_4^{(1)} = 180,\ X_{4,5:7:12}^{(1)} < X_{4,7:7:12}^{(1)} < T_4^{(1)}$ terminates at $T_4^{(1)}$ with $r_{41}^{(1)} = 4, r_{4j}^{(1)} = 0\ (for\ j = 2,\ 3,\ 4,\ 6,\ 7), r_{45}^{(1)} = 1, X_{4,j}^{(1)} = (3,\ 10,\ 11,\ 24,\ 54,\ 100,\ 163)$

$T_4^{(2)} = 700,\ X_{4,5:7:12}^{(2)} < X_{4,7:7:12}^{(2)} < T_4^{(2)}$ terminates at $T_4^{(2)}$ with $r_{41}^{(2)} = 4, r_{4j}^{(2)} = 0\ (for\ j = 2,\ 3,\ 4,\ 6,\ 7), r_{45}^{(2)} = 1, X_{4,j}^{(2)} = (35,\ 47,\ 83,\ 92,\ 249,\ 352,\ 607)$

The results for the values of maximum likelihood estimators for the parameters $\hat{\beta}_1, \hat{\beta}_2, \hat{\gamma}\ and\ \hat{\lambda}$ with their standard errors under different schemes are presented in Table 3. Also, the results for the fuzzy multicomponent stress strength reliability are obtained

From the results obtained in Table 3, it can be observed that scheme III is the best scheme since it gives better values for the MLE of the fuzzy multicomponent stress strength reliability.

Method validation

In this paper, a study of the multicomponent stress-strength reliability is introduced in presence of partially accelerated life testing and fuzziness and applying the generalized hybrid censoring scheme under the inverse Weibull distribution. The motivation to this study that rare papers in literature analyzed the fuzzy multicomponent stress-strength reliability in case of partially accelerated life testing and generalized hybrid censoring scheme. In the present study, a novel analysis of the multicomponent stress-strength reliability is introduced assuming partially accelerated life testing and generalized hybrid censoring scheme in presence of fuzziness.

Table 3
The results for MLE for the parameters with standard errors (in parentheses) and fuzzy multicomponent stress strength reliability for the three schemes.

Scheme	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\gamma}$	$\hat{\lambda}$	c	p	$\hat{R}^F_{3,5}$
I	4.34590	9.43523	0.46487	0.28017	0.5	0.1	0.24571
	(1.07544)	(3.77778)	(0.06269)	(0.16826)		0.3	0.24340
						0.5	0.24110
						0.9	0.23649
					2	0.1	0.20404
						0.3	0.19730
						0.5	0.19056
						0.9	0.17709
II	3.46528	7.07788	0.43777	0.27314	0.5	0.1	0.25653
	(0.68713)	(3.06453)	(0.05087)	(0.14841)		0.3	0.25352
						0.5	0.25051
						0.9	0.24450
					2	0.1	0.20118
						0.3	0.19315
						0.5	0.18511
						0.9	0.16905
III	3.93967	7.57285	0.50031	0.22713	0.5	0.1	0.27765
	(0.75342)	(3.97195)	(0.04640)	(0.09942)		0.3	0.27405
						0.5	0.27044
						0.9	0.26322
					2	0.1	0.22863
						0.3	0.21848
						0.5	0.20834
						0.9	0.18805

The inverse Weibull model is used to describe the time of the system which make the system more flexible. An estimator of the fuzzy multicomponent stress-strength reliability is obtained by using the method of maximum likelihood. An asymptotic confidence interval for the fuzzy multicomponent stress-strength reliability is deduced. A simulation study is introduced to obtain numerical results for the fuzzy multicomponent stress-strength reliability for different values of the parameters and different schemes. A real data application representing the failure times for a software model is introduced to obtain the fuzzy multicomponent stress-strength reliability when applying different schemes.

Ethics statement

Not applicable.

Data availability

all data are included in the research

CRediT authorship contribution statement

Neama Salah Youssef Temraz: Conceptualization, Methodology, Validation, Data curation, Writing – original draft, Writing – review & editing.

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Declaration of Competing Interest

The author declare that she has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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