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Multicriteria group decision making via generalized trapezoidal intuitionistic fuzzy number-based novel similarity measure and its application to diverse COVID-19 scenarios

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Abstract
Havoc, brutality, economic breakdown, and vulnerability are the terms that can be rightly associated with COVID-19, for the kind of impact it is having on the whole world for the last two years. COVID-19 came as a nightmare and it is still not over yet, changing its form factor with each mutation. Moreover, each unpredictable mutation causes more severeness. In the present article, we outline a decision support algorithm using Generalized Trapezoidal Intuitionistic Fuzzy Numbers (GTrIFNs) to deal with various facets of COVID-19 problems. Intuitionistic fuzzy sets (IFSs) and their continuous counterparts, viz., the intuitionistic fuzzy numbers (IFNs), have the flexibility and effectiveness to handle the uncertainty and fuzziness associated with real-world problems. Although a meticulous amount of research works can be found in the literature, a wide majority of them are based mainly on normalized IFNs rather than the more generalized approach, and most of them had several limitations. Therefore, we have made a sincere attempt to devise a novel Similarity Measure (SM) which considers the evaluation of two prominent features of GTrIFNs, which are their expected values and variances. Then, to establish the superiority of our approach we present a comparative analysis of our method with several other established similarity methods considering ten different profiles of GTrIFNs. The proposed SM is then validated for feasibility and applicability, by elaborating a Fuzzy Multicriteria Group Decision Making (FMCGDM) algorithm and it is supportedby a suitable illustrative example. Finally, the proposed SM approach is applied to tackle some significant concerns due to COVID-19. For instance, problems like the selection of best medicine for COVID-19 infected patients; proper healthcare waste disposal technique; and topmost government intervention measures to prevent the COVID-19 spread, are some of the burning issues which are handled with our newly proposed SM approach.

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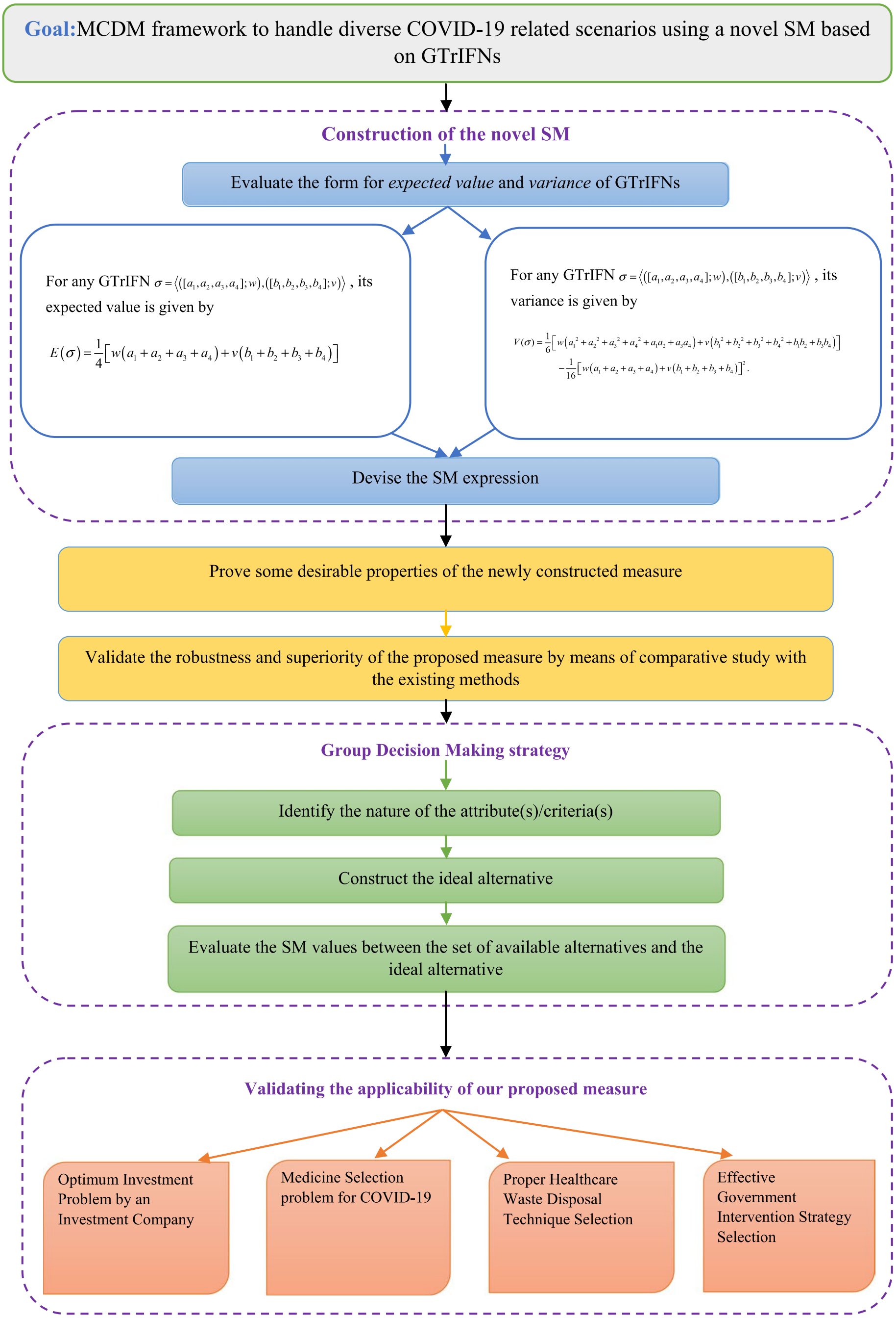
Graphical abstract



Keywords Generalized trapezoidal intuitionistic fuzzy numbers · Similarity measure · Multicriteria decision making · Uncertainty/imprecision · Health · Intervention strategy

Abbreviations

COVID-19	Corona Virus December-2019
FDA	Food and Drug Administration
FMCGDM	Fuzzy Multicriteria Decision Making
FS	Fuzzy Set
GTrIFN	Generalized Trapezoidal Intuitionistic Fuzzy Number
IFN	Intuitionistic Fuzzy Number
IFS	Intuitionistic Fuzzy Set
ISHWM	Indian Society of Hospital Waste Management
MCDM	Multicriteria Decision Making
MERS	Middle East Respiratory Syndrome
PPE	Personal Protective Equipment
SARS	Severe Acute Respiratory Syndrome
SEIR	Susceptible-Exposed-Infected-Recovered
SM	Similarity Measure
TIFN	Triangular Intuitionistic Fuzzy Number
TrIFN	Trapezoidal Intuitionistic Fuzzy Number
WHO	World Health Organization

1 Introduction

The Boolean or traditional logic which takes the value 1 (true) or 0 (false) is not always applicable in real-life problems, where the information available at hand is uncertain or imprecise. To cope up with such uncertain and vague situations, a particular class of sets proposed by Zadeh (Zadeh 1965) known as fuzzy sets (FSs) is considered suitable. In FS theory, each and every element belonging to the set is specified by a membership value in the range [0,1]. However, to deal with more complicated and uncertainty-led scenarios, it was found that, merely the concept of FSs is not enough and hence few extensions of these sets were proposed. One such major development was the advent of IFSs by Atanassov (Atanassov 1986). Due to the presence of non-membership degree function in IFSs which is absent in FSs, it is intuitive that IFSs have an upper hand over FSs by being more effective in dealing with uncertainty-based practical applications. Moreover, IFSs are capable of imitating the available data or information more accurately and realistically. IFSs can also maintain the imprecise contents of information and they can facilitate approximate reasoning behavior. Although the descriptive capability of IFSs is higher than that of the traditional FS theory due to their additional presence of non-membership and hesitancy functions, however, they have fairly higher computational complexity over FSs.

In Multicriteria Decision Making (MCDM) problems, generally, several conflicting criteria are simultaneously considered before we finally obtain a ranking order for the available set of alternatives. MCDM is considered a research hotspot for researchers and it has also become one of the vibrant areas under the domain of decision theory. Similarly, FMCGDM problems are scenarios, where a group of decision-makers is appointed to assess the available set of alternatives with respect to certain governing criteria. While dealing with numerous real-world situations, decision-maker may be unable to provide deterministic values for the alternatives but are instead able to provide fuzzy numbers. FS theory can model this kind of uncertainty, which occurs in MCDM problems. The pioneers in proposing the fuzzy decision-making model were Bellman and Zadeh (1970). Following them, a handful number of studies have been accomplished on fuzzy MCDM (Hwang and Yoon 1981; Chen and Hwang 1992; Xu 2004, 2007; Wang and Parkan 2005; Wu and Chen 2007), which has given birth to several efficient methodologies till date. Chen (2009) developed a methodology using the technique for order preference by similarity to ideal solution (TOPSIS) for dealing with fuzzy multicriteria group decision-making (MCGDM) problems, by extending a very well-known classical MCDM approach. Ye (2009) handled intuitionistic fuzzy cross entropy-based MCDM problems, where IFSs represent the characteristics of alternatives and the criteria weights are indicated by fuzzy values. Furthermore, Xu (2010) presented a deviation-based approach, considering the score function and the accuracy function, to solve FMCGDM problems, where the information is an intuitionistic fuzzy one. Ye (Ye 2010) devised an entropy weight-based correlation coefficient for solving fuzzy decision-making problems under an intuitionistic fuzzy environment. Li (2011) proposed new extension principles considering interval-IFSs and also defined a few algebraic operators. Farhadinia and Ban (2013) devised some novel SMs for both generalized intuitionistic and generalized interval-valued fuzzy numbers by extending the existing SMs on generalized fuzzy numbers. Wang and Zhang (2009) devised a method based on aggregation operators for solving MCDM problems, considering trapezoidal intuitionistic fuzzy numbers (TrIFNs). Zhang and Liu (2010) considered triangular intuitionistic fuzzy information and applied them in decision-making problems by devising a method to aggregate them. Wang et al. (2013) developed triangular intuitionistic fuzzy operators and applied them in the fault analysis of systems.

The SM concept is undoubtedly a significant one in human cognition-induced thought processes. As humans are natural decision-makers, therefore in a variety of decision making instances, may it be in psychology, ecology, case-based reasoning, or information retrieval, the importance of SMs cannot be ignored. SMs developed particularly for dealing with IFSs, form an integral part of numerous decision making instances. The determination of the similarity coefficient between IFSs is considered crucial in IFS theory since it acts as a measure of distinction between those sets. The development of new and efficient similarity methodologies has always been an area of significant interest and there have been some remarkable developments under the framework of SMs for IFSs, attracting the research community globally. In this context, several papers have been devoted to evaluating the SM between GTrIFNs, some of which have been mentioned here. For instance, Chen and Chen (2009) proposed a few SMs for interval-valued fuzzy numbers and they carried out a risk analysis based on their proposed measures. Consequently, Wei and Chen (Wei and Chen 2009) proposed a novel similarity method for fuzzy risk analysis as well. Chen (2011) proposed some quadratic operator-based SMs for the fuzzy recommendation process and obtained some fascinating conclusions from them. Farhadinia (2012) on the other hand, utilized the concept of geometric distance and perimeter, in devising an SM for generalized fuzzy numbers. Also, Ye (2012a) proposed a cosine vector SM for trapezoidal intuitionistic fuzzy numbers and applied it to a group decision making problem. Further, Ye (2012b) also proposed two distance-based SMs for intuitionistic trapezoidal fuzzy numbers. One such SM is based on the Hamming distance, while the other is based on the Euclidean distance. Later, Tang et al. (2017) proposed a dice similarity for generalized IFNs with applications to group decision making problems. Thereafter, Yue et al. (2019) devised an SM for triangular intuitionistic fuzzy numbers (TIFNs) to facilitate Smart Environmental

Protection. Their SM considered evaluation of the area and the perimeter of such IFNs. Dinagar and Helena (2019) also proposed a centroid-based SM for GTrIFNs.

At present, the havoc and the outrage created as a result of the novel COVID-19 virus are miserable in almost all countries of the world. Even the developed counties with the best medical facilities are struggling their way through this unprecedented situation. The majority of the countries are still absolutely clueless and they are directing their people to quarantine themselves to cutoff any probable chances of getting infected with the virus (Ren et al. 2020). Consequently, the researchers of these countries are engaging themselves to devise suitable models to predict any probable spike in the number of active cases; recovery rate of the patients; transmission rate of the virus; etc. The outcome of those surveys has greatly assisted the healthcare workers and the medical staff to be prepared for the worst scenarios that may probably arise due to the virus, and hence take necessary precautionary measures. Castillo and Melin (2020) proposed a time series model using fuzzy hybrid approaches to predict the number of confirmed cases and the deaths of patients in the respective countries of the world. Castillo and Melin (2021a) also developed a hybrid model where the fractal dimension and the fuzzy logic concepts are composed to predict the unusual trends that could take place in the COVID-19 time series data of the countries. Thereafter, Sun and Wang (2020) showed that ordinary differential equations could be helpful to devise a model that can collect the COVID-19 data from a specific location within a definite time interval. An effective hybrid model was also developed by Melin et al. (2020a), that could predict the future trends of the virus and simultaneously, alert the frontline workers about any probable uncanny situation beforehand. Chen et al. (2020) study the dynamics of SARS-CoV-2 like a protease structure. Fan et al. (2020) analyzed how the spring festival transportation in China had influenced the further spread of the novel coronavirus epidemic. Grifoni et al. (2020) established that an approach based on sequence homology and bioinformatics is fairly capable of predicting the candidate targets showing immune response against SARS-CoV-2. He (2020) pointed out some alternative measures that could help control the COVID-19 outbreak apart from the contact tracing and quarantine measures that are being implemented. In Huang et al. (2020), the prediction and distribution (spatial–temporal) of the COVID-19 scenario in China was presented. Similarly, Ibrahim et al. (2020) obtained significant results to predict the binding site for COVID-19 spike-host cell receptor GRP78. Ivanov (2020) discussed how the global supply chains are heavily impacted due to the COVID-19 (SARS-CoV-2) outbreak. The authors in Liu et al. (2020) try to understand the factors behind the unreported cases of Wuhan, China, and they also explain the importance of appropriate public health intervention strategies to control the pandemic. In Ton et al. (2020), the authors perform a deep docking of 1.3 billion compounds to identify the prime inhibitors of the main protease of SARS-CoV-2. Das et al. (2021) presented a comparative study of several intervention strategies and developed a Susceptible-Exposed-Infected-Recovered (SEIR) model for the analysis of outputs obtained. Nabi et al. (2021) suggested certain optimal control measures in their work and also projected the fractional dynamics of COVID-19. Likewise, Castillo and Melin (2021b) introduced a novel fuzzy fractal approach to control the COVID-19 pandemic.The authors in Mishra et al. (2021), utilized the hesitant FSs to develop a suitable framework that can predict the best medicine for treating the COVID-19 patients showing mild symptoms. Majumder et al. (2020) identified the COVID-19 infected population with the help of a decision making strategy under a fuzzy rough set-theoretic environment. In Si et al. (2021), the authors have proposed a picture fuzzy set-based decision strategy and applied it to identify the set of best medicines for COVID-19 which are available in the Indian market. Certain important aspects associated with the coronavirus such as staying away from the virus, dominating the virus, and ensuring complete protection against the virus are time-consuming projects (Ghosh et al. 2020). Therefore, at best the scientists and the researchers can do is to facilitate an immediate medical service to the coronavirus-affected patients, so that maximum lives could be saved.

1.1 Motivation of our work

Most of the real-life phenomena that take place naturally are based upon some imprecise nature of information or data. Often human beings use vague linguistic expressions to convey their message to other individuals. Thus, uncertainty is an inseparable element of human beings as well as nature. In this regard, the use of fuzzy set theory or fuzzy logic helps us to deal with such types of complex and ambiguous situations. The main advantage of fuzzy logic is that it has some rigid set of rules with the help of which we can manage the uncertainty in an efficient manner. With fuzzy logic, we can develop models which are capable of arriving at a final decision by smoothly handling the uncertainty arising during the process.

Likewise, for expressing the degree of similarity that exists between objects around us, we use an important tool in the form of SM. Various functions measure the degree of similarity between objects or sets and are applied in the field of ecology, psychology, information retrieval, numerical taxonomy, citation analysis, and physical anthropology. The degree of similarity or dissimilarity between objects among the various extensions of FSs has gathered remarkable attention and it has become an area of significant interest in fuzzy mathematics. In this regard, several term-term vector SMs like the Jaccard SM, Dice SM, and Cosine SM (Jaccard 1901; Dice 1945; Salton and McGill 1987; Kima and Choi 1999) and their robust extensions are widely in practice. However, these SMs have a major disadvantage in that they are considered discrete and which results in the loss of data or information. In other words, during the process of information integration, the continuous sets always tend to preserve the integrity of the information, while the discrete sets tend to lose some of that information partially. Hence, the importance and merits of the continuous sets are higher than the discrete ones in the broader research community. Noteworthy is that, in the literature, we can find numerous examples of discrete or non-continuous IFS-based SMs, but only a handful of SMs exists based on continuous TrIFNs (Nehi 2010) or GTrIFNs.

Some of notable existing SMs are by Chen and Chen (2009), Wei and Chen (2009), Chen (2011), Farhadinia (2012), Ye (2012a), (2012b), Tang et al. (2017), Yue et al. (2019), Dinagar and Helena (2019). But most of these measures have a "blind spot", as in some particular situations, they are unable to yield proper classification results or make justifiable distinctions. For instance, suppose we consider two GTrIFNs and let the components of one fuzzy number representing its respective membership function are greater than the other fuzzy number, with heights being the same for both, then the approach by Wei and Chen (2009) and Farhadinia (2012), produces absurd results. Even for exactly similar GTrIFNs, the approaches by Ye (2012a), Tang et al. (2017), and Dinagar and Helena (2019), fail to obtain unit similarity value which depicts a major structural setback. Also, whenever the height of the fuzzy numbers depicting the non-membership functions of both GTrIFNs is unity (1) and accordingly if the height of the fuzzy number depicting the membership function is zero (0), then Chen's (2011) approach fails to be applied. In addition, there are several other drawbacks of these existing similarity methods and for a detailed explanation of the same, please refer to Sect. 4, where some other conflicting scenarios are discussed. Furthermore, some real-life applications of these measures are not found. The drawbacks of these existing approaches greatly motivate us to construct a novel SM for GTrIFNs that could possibly overcome those discrepancies.

Further, several novel research ideas which have not been attempted earlier, inspire us to conduct this study. Some of which are listed below:

- Defining an SM for GTrIFNs by incorporating their expected values and variances has not been studied before.
- The use of α-cut technique to deduce the mathematical form for expected value & variance is a novel attempt in this article, whereasthe earlier researchers had explored the possibilistic theory concepts only.
- Defining group decision making algorithms with GTrIFNs-based SM has not been investigated earlier.
- Moreover, none of the earlier researchers had attempted to study GTrIFNs in problems of COVID-19 medicine selection, healthcare waste disposal technique, and selection of government intervention strategy.

Towards the latter part of the article, it is established that the obtained decision results with our proposed method are at par with analytical output and human intuition.

1.2 Problem statement

Discrete or non-continuous IFS-based similarity measures are very commonly found in the literature, while their continuous counterparts are not found in abundance because to deal with the continuous fuzzy numbers one must be prepared for the computational complexity associated with them while performing the arithmetic operations. In this context, although a small number of continuous IFS-based SMs exist, yet most of these existing measures fail under some specific situations and often lead to overestimation or underestimation of similarity results. This inconsistent nature of the existing SMs encourages us to propose an efficient, rational, and novel SM. Our proposed measure considers two prominent/deterministic features of any fuzzy number, which are "expected value" and "variance" of GTrIFNs. Our article explores the α-cut technique in evaluating their corresponding expressions which is an entirely new venture and has not been attempted earlier by researchers for GTrIFNs. Although, possibility theory-based concepts for expected value/mean, variance, standard deviation, etc., are widely in practice, as an alternative approach, our α-cut method for evaluation stands out from the rest and hence the novelty of this article. We then incorporate the expected values, variances, heights, and respective membership & non-membership components of GTrIFNs, into devising an SM and thus establish certain desirable properties of its own. Once we have proved the standard and acceptance of our newly constructed measure, we then illustrate an FMCGDM procedure. To validate the applicability of our measure we present a numerical example of "Optimum Investment by an Investment Company". Furthermore, the broader objective which is being served by our proposed measure is its application to various complex scenarios as a result of the COVID-19 pandemic that calls for efficient decision making. One such issue is the selection of medicines for COVID-19 infected patients. At the time of writing this paper, only a miniature number of medicines are being approved for usage to coronavirus-infected patients. Thus, with the help of information available at hand and the knowledge gathered from the concerned experts in the field, we undertake our study considering only those medicines which are acceptable and comparatively better than the others. Then, it is established that our proposed SM approach is capable of determining the appropriate medicines for COVID-19 when the preference information for medicines and their symptoms are expressed with the help of GTrIFNs.However, the selection of medicines is not an easy task. With the intake of different types of medicines, different people might react to it differently, so far as the side effects and the effectiveness of the medicines are concerned. Moreover, there have been cases reported of patients showing some unfamiliar symptoms due to the medicines. Therefore, our paper proposes a GTrIFN-based approach to fill up this research gap and hence, determine the most suitable drug that can be recommended to COVID-19 affected patients. The intention is to save maximum lives as possible and hopefully, slow down the devastating nature of this pandemic.

Another issue of healthcare waste disposal becomes a major concern in a country like India where it is already struggling to manage such a large population of 1.3 billion people. COVID-19 has further increased the number of infectious wastes being produced each day, which may include- used needles, personal protective equipment(s) (PPEs), gloves, chemicals, etc. Hence, we discuss a case study in one of the COVID-19 hotspot state of India: Maharashtra. The reason for selecting such a state is that more the number of infections per day would imply more production of healthcare wastes so that we can demonstrate how our measure can be helpful in selecting the best technique for disposing of such hazardous wastes.

While the third and final issue which we shall discuss in our article is the deployment of an efficient government strategy to help minimize the transmission rate of the disease and also which is acceptable to the citizens of our country India. We have observed in other countries of the world too that timely implementation of appropriate COVID safety measures has slowed the virus spread, while on the other hand, some countries have paid the price by losing the lives of people for showing the slightest of carelessness. Thus, it is a vital issue, and we show how we can apply our proposed SM approach to determine the topmost efficient and acceptable government intervention strategy thereby preventing the mass transmission of the virus. In addition, we also obtain the priority ordering of the available intervention measures so that when one of them fails, the other can be applied (in decreasing order of preference).

1.3 Objectives of our work

The main objectives of this study are listed down below:

- to construct an efficient, feasible, and rational SM for GTrIFNs that can overcome the limitations of most of the existing SM methods
- to evaluate the "expected value" and "variance" expressions for GTrIFNs, using the α-cut technique
- to elaborate an FMCGDM procedure with the help of the newly proposed SM for GTrIFNs
- to effectively select the best medicine available in the market for ensuring the maximum protection against the COVID-19 virus
- to select the best healthcare waste disposal strategy in one of the Indian states
- to determine the most effective and robust intervention strategy that should be adopted by the Indian government to control the pandemic.

1.4 Organization of the paper

The rest of the paper is structured as follows. In Sect. 2, we discuss a few definitions related to fuzzy sets, GTrIFNs and SMs. Section 3, discusses the technique of evaluating the expected value and the variance for GTrIFNs, and then we explain our newly proposed SMin detail. Moreover, we prove certain desirable properties that are satisfied by our measure. In Sect. 4, we present a comparative analysis of our present approach with several other existing approaches, considering ten different profiles of fuzzy numbers (GTrIFNs). The superior ability of our present approach over the others is clearly visible in this section. In Sect. 5, we elaborate an FMCGDM procedure based on our designed SM. Moreover, we present a practical real-life scenario, where our proposed method finds its application. In Sect. 6, our proposed method is being applied threefold- first in COVID-19 medicine selection, second in evaluating the best healthcare waste disposal technique in an Indian state, and third in determining the most suitable government intervention strategy to reduce the rate of virus spread, in the Indian context. And lastly, we come up with the key points and concluding remarks in Sect. 7.

2 Preliminaries

Some basic concepts and essential backdrops which will be necessitated in the follow-up are presented below.

Definition 1 (Fuzzy Set) (Zadeh 1965).

Let us consider Λ to be the universe of discourse and let ζ be a fuzzy subset on Λ, then its membership function $\mu_\zeta : \Lambda \rightarrow [0, 1]$, assigns a real number in the interval $[0, 1]$, say $\mu_\zeta(x)$ to each $x \in \zeta$.

Definition 2 : (Height) (Zadeh 1965).

Height of a FS is the highest membership grade that can be attained by any arbitrary element in the set. The denotation for height is $h(\zeta)$, where

$$h(\zeta) = \sup_{x\in\Lambda} \mu_\zeta(x) \tag{1}$$

Definition 3 (Support) (Zadeh 1965).

Support of a fuzzy subset ζ in the universe of discourse Λ, is itself the crisp subset of Λ consisting of all elements having non zero membership grades in ζ.

That is, sup

$$(\zeta) = \left\{x \in \Lambda : \mu_\zeta(x) > 0\right\} \tag{2}$$

Definition 4 (Intuitionistic Fuzzy Set (IFS)) (Atanassov 1986).

An IFS ζ in Λ is defined by Atanassov (1986) as,

$A = \left\{ \left\langle \left(x, \mu_{\zeta}(x), \nu_{\zeta}(x) \right) \right\rangle | x \in \Lambda \right\}$, where $\mu_{\zeta}(x) : \Lambda \rightarrow [0,1]$ and $\nu_{\zeta}(x) : \Lambda \rightarrow [0,1]$, with the condition that, $0 \leq \mu_{\zeta}(x) + \nu_{\zeta}(x) \leq 1$. Here,$\mu_{\zeta}(x)$ and $\nu_{\zeta}(x)$ are the membership and non-membership grades for the element x in the set ζ.

Moreover, for each IFS ζ in Λ,$\pi_{\zeta}(x)$ is known as the Atanassov's intuitionistic index of the element x in the set Λ, and is defined as

$$\pi_{\zeta}(x) = 1 - \mu_{\zeta}(x) - \nu_{\zeta}(x), x \in \Lambda \tag{3}$$

It is also referred to as the hesitancy degree of x to Λ. Clearly, $0 \leq \pi_{\zeta}(x) \leq 1, x \in \Lambda$. But, IFSs are discrete sets. Hence, Grzegrorzewski (2003) introduced an IFN with a motive to extend the discrete concept of IFSs to a continuous one.

Definition 5 (Intuitionistic Fuzzy Number (IFN)) (Grzegrorzewski 2003).

Let ζ be an IFN in the real domain of numbers ($\mathbb{R}$), having its membership and non-membership mappings defined as,

$$\mu_{\zeta}(x) = \begin{cases} 0, x < \lambda_1, \\ p_{\zeta}(x), \lambda_1 \leq x < \lambda_2, \\ 1, \lambda_2 \leq x < \lambda_3, \\ q_{\zeta}(x), \lambda_3 \leq x < \lambda_4, \\ 0, x \geq \lambda_4. \end{cases} ,\nu_{\zeta}(x) = \begin{cases} 1, x < \eta_1, \\ r_{\zeta}(x), \eta_1 \leq x < \eta_2, \\ 0, \eta_2 \leq x < \eta_3, \\ s_{\zeta}(x), \eta_3 \leq x < \eta_4, \\ 1, x \geq \eta_4. \end{cases} \tag{4}$$

where $0 \leq \mu_{\zeta}(x) + \nu_{\zeta}(x) \leq 1$ and $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \eta_1, \eta_2, \eta_3, \eta_4 \in \mathbb{R}$ such that, $\eta_1 \leq \lambda_1 \leq \eta_2 \leq \lambda_2 \leq \lambda_3 \leq \eta_3 \leq \lambda_4 \leq \eta_4$, and four functions $p_{\zeta}, q_{\zeta}, r_{\zeta}, s_{\zeta} : \mathbb{R} \rightarrow [0,1]$, are known as the sides of a fuzzy number. Out of them, the functions p_{ζ} and q_{ζ} are non-decreasing continuous functions and the functions r_{ζ} and s_{ζ} are non-increasing continuous functions.

Definition 6 (Trapezoidal Intuitionistic Fuzzy Number (TrIFN)) (Nehi 2010).

A TrIFN ζ with parameters $\eta_1 \leq \lambda_1 \leq \eta_2 \leq \lambda_2 \leq \lambda_3 \leq \eta_3 \leq \lambda_4 \leq \eta_4$ is denoted as.
$\zeta = \left\langle \left(\lambda_1, \lambda_2, \lambda_3, \lambda_4 \right), \left(\eta_1, \eta_2, \eta_3, \eta_4 \right) \right\rangle$, in the set of real numbers $\mathbb{R}$.
Here, the membership and non-membership functions have the form

$$\mu_{\zeta}(x) = \begin{cases} 0, x < \lambda_1, \\ \frac{x-\lambda_1}{\lambda_2-\lambda_1}, \lambda_1 \leq x < \lambda_2, \\ 1, \lambda_2 \leq x < \lambda_3, \\ \frac{x-\lambda_4}{\lambda_3-\lambda_4}, \lambda_3 \leq x < \lambda_4, \\ 0, x \geq \lambda_4. \end{cases} , \quad \nu_{\zeta}(x) = \begin{cases} 1, x < \eta_1, \\ \frac{x-\eta_2}{\eta_1-\eta_2}, \eta_1 \leq x < \eta_2, \\ 0, \eta_2 \leq x < \eta_3, \\ \frac{x-\eta_3}{\eta_4-\eta_3}, \eta_3 \leq x < \eta_4, \\ 1, x \geq \eta_4. \end{cases} \tag{5}$$

When $\eta_2 = \eta_3$(hence $\lambda_2 = \lambda_3$) in a TrIFN ζ, the TrIFN becomes triangular intuitionistic fuzzy number, which is considered as a special case of the TrIFN.

Definition 7 (Generalized Trapezoidal Intuitionistic Fuzzy Numbers (GTrIFN)) (Ye 2012a).

The membership and non-membership function of GTrIFN,

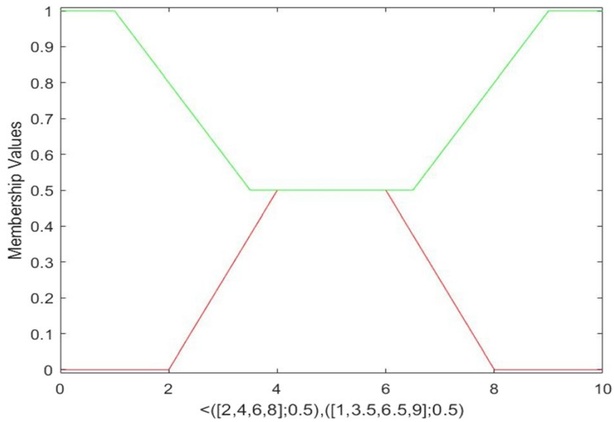


Fig. 1 GTrIFN, $\lambda = \langle([2, 4, 6, 8];0.5),\rangle$ $\langle([1, 3.5, 6.5, 9];0.5)\rangle$

$\sigma = \Big\langle \big([\lambda_1, \lambda_2, \lambda_3, \lambda_4];w_1\big), \big([\eta_1, \eta_2, \eta_3, \eta_4];w_2\big) \Big\rangle$, where $\eta_1 \le \lambda_1 \le \eta_2 \le \lambda_2 \le \lambda_3 \le \eta_3 \le \lambda_4 \le \eta_4$ and $0 \le w_1, w_2 \le 1$ is givenby

$$\mu_\sigma(x) = \begin{cases} 0, x < \lambda_1, \\ w_1 \frac{x-\lambda_1}{\lambda_2-\lambda_1}, \lambda_1 \le x < \lambda_2, \\ w_1, \lambda_2 \le x < \lambda_3, \\ w_1 \frac{x-\lambda_4}{\lambda_3-\lambda_4}, \lambda_3 \le x < \lambda_4, \\ 0, x \ge \lambda_4. \end{cases}, \quad \nu_\sigma(x) = \begin{cases} w_2, x < \eta_1, \\ w_2 \frac{x-\eta_2}{\eta_1-\eta_2}, \eta_1 \le x < \eta_2, \\ 0, \eta_2 \le x < \eta_3, \\ w_2 \frac{x-\eta_3}{\eta_4-\eta_3}, \eta_3 \le x < \eta_4, \\ w_2, x \ge \eta_4. \end{cases} \tag{6}$$

Here, σ is called a GTrIFN and w_1, w_2 are the heights of the membership and non-membership function respectively. If $w_1 = 1, w_2 = 1$, then GTrIFN σ is a normal TrIFN. If $\lambda_2 = \lambda_3$ and $w_1, w_2 < 1$, then σ is a generalized TIFN, otherwise normal TIFN.

A GTrIFN, $\lambda = \langle([2, 4, 6, 8];0.5), ([1, 3.5, 6.5, 9];0.5)\rangle$ is illustrated in **Fig. 1**.

Definition 8 (α-level or α-cut or cut worthy set) (Dutta et al. 2011).

For any given a FS ζ in Λ, and any real number $\alpha \in [0, 1]$, we define the α-cut of ζ, ${}^\alpha\zeta$ as the crisp subset,

$$^\alpha\zeta = \big\{x \in \Lambda : \mu_\zeta(x) \ge \alpha\big\} \tag{7}$$

Say, for example, ζ be a FScharacterized by its membership function,

$$\mu_\zeta(x) = \begin{cases} \frac{x-p}{q-p}, p \le x \le q \\ \frac{s-x}{s-r}, r \le x \le s \end{cases}$$

Finding the α-cut of ζ, demands setting both left and right reference functions of ζ to $\alpha \in [0, 1]$.

Thus, $\alpha = \frac{x-p}{q-p}$ and $\alpha = \frac{s-x}{s-r}$.

Now, we express x in terms of α as,

$x = (q - p)\alpha + p$ and $x = s - (s - r)\alpha$, giving the α-cut of ζ to be,

$$^{\alpha}\zeta = [(q-p)\alpha + p, s - (s-r)\alpha]$$

Definition 9 (Similarity Measure) (Liu 1992).

A real function $D_S : \mathbb{R} \times \mathbb{R} \to [0,1]$, satisfying the following properties, is called a SM,
P-1: $0 \le D_S(\zeta_1, \zeta_2) \le 1$.
P-2: $D_S(\zeta_1, \zeta_2) = D_S(\zeta_2, \zeta_1)$.
P-3: $D_S(\zeta_1, \zeta_2) = 1 \Leftrightarrow \zeta_1 = \zeta_2$, where $\zeta_1, \zeta_2 \in \mathbb{R}$.

3 The proposed novel SM approach

Before we begin our discussion about our proposed SM approach for GTrIFNs, we initially illustrate the procedure for calculating the expected value and variance of GTrIFNs. We then explain our newly constructed SM and establish some desirable properties. The following sequel begins the discussion in this regard.

3.1 Expected value of a GTrIFN

The concept of expected value in statistical terms refers to the midpoint or center of the distribution of any random variable. In other words, it is the average value of the variable which can be anticipated in quite a longterm. Expected values enable decision-makers in the selection of suitable scenarios, which are likely to give optimum benefit and as such, they can be evaluated for both continuous and discrete fuzzy numbers. It is to be noted that, one may also refer to the expected value as the mean value or expectation or the first moment.

Consequently, the expected value of a GTrIFN, $\sigma = \left\langle \left([\lambda_1, \lambda_2, \lambda_3, \lambda_4; w_1] \right), \left([\eta_1, \eta_2, \eta_3, \eta_4; w_2] \right) \right\rangle$ characterized by the membership and non-membership function defined as, $\mu_\sigma(x) = \begin{cases} 0, x < \lambda_1, \\ w_1 \frac{x-\lambda_1}{\lambda_2-\lambda_1}, \lambda_1 \le x < \lambda_2, \\ w_1, \lambda_2 \le x < \lambda_3, \\ w_1 \frac{x-\lambda_4}{\lambda_3-\lambda_4}, \lambda_3 \le x < \lambda_4, \\ 0, x \ge \lambda_4. \end{cases}$, $\nu_\sigma(x) = \begin{cases} w_2, x < \eta_1, \\ w_2 \frac{x-\eta_2}{\eta_1-\eta_2}, \eta_1 \le x < \eta_2, \\ 0, \eta_2 \le x < \eta_3, \\ w_2 \frac{x-\eta_3}{\eta_4-\eta_3}, \eta_3 \le x < \eta_4, \\ w_2, x \ge \eta_4. \end{cases}$ is determined using the α-cut method and the following theorem vividly explains this concept.

Theorem 1: Let $\sigma = \left\langle \left([\lambda_1, \lambda_2, \lambda_3, \lambda_4; w_1] \right), \left([\eta_1, \eta_2, \eta_3, \eta_4; w_2] \right) \right\rangle$ be a GTrIFN, and. $\eta_1 \le \lambda_1 \le \eta_2 \le \lambda_2 \le \lambda_3 \le \eta_3 \le \lambda_4 \le \eta_4$ and $0 \le w_1, w_2 \le 1$, whose expected value is evaluated as

$$E(\sigma) = \frac{1}{4}\left[w_1 \left(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 \right) + w_2 \left(\eta_1 + \eta_2 + \eta_3 + \eta_4 \right) \right] \tag{8}$$

Proof: We first calculate the α-cut of the GTrIFN σ.
So, we have.
$\alpha = w_1 \frac{x-\lambda_1}{\lambda_2-\lambda_1}$ and $\alpha = w_1 \frac{x-\lambda_4}{\lambda_3-\lambda_4}$ (**For membership function**).

$\Rightarrow x = \lambda_1 + \frac{1}{w_1}\left\{\alpha(\lambda_2 - \lambda_1)\right\}$ and $\Rightarrow x = \lambda_4 + \frac{1}{w_1}\left\{\alpha(\lambda_3 - \lambda_4)\right\}$.
$\alpha = w_2\frac{x-\eta_2}{\eta_1-\eta_2}$ and $\alpha = w_2\frac{x-\eta_3}{\eta_4-\eta_3}$ (**For non-membership function**).
$\Rightarrow x = \eta_2 + \frac{1}{w_2}\left\{\alpha(\eta_1 - \eta_2)\right\}$ and $\Rightarrow x = \eta_3 + \frac{1}{w_2}\left\{\alpha(\eta_4 - \eta_3)\right\}$.

$$\text{Therefore, } {}^{\alpha}\sigma = \begin{cases} \left[\lambda_1 + \frac{1}{w_1}\left\{\alpha(\lambda_2 - \lambda_1)\right\}, \lambda_4 + \frac{1}{w_1}\left\{\alpha(\lambda_3 - \lambda_4)\right\}\right] (Membership) \\ \left[\eta_2 + \frac{1}{w_2}\left\{\alpha(\eta_1 - \eta_2)\right\}, \eta_3 + \frac{1}{w_2}\left\{\alpha(\eta_3 - \eta_4)\right\}\right] (Non-membership) \end{cases}$$

It is concept-wise clear that,α-cut of a fuzzy set/number always produces an interval. Say for an interval of the type $[p, q]$, its expected/mean value is given to be $\frac{p+q}{2}$.Similar intuition works here too. Thus,

$$\text{Expected value of } \sigma, E(\sigma) = \frac{1}{2}\begin{bmatrix} \int_0^{w_1} \left[\lambda_1 + \frac{1}{w_1}\left\{\alpha(\lambda_2 - \lambda_1)\right\}\right] d\alpha + \int_0^{w_1} \left[\lambda_4 + \frac{1}{w_1}\left\{\alpha(\lambda_3 - \lambda_4)\right\}\right] d\alpha \\ + \int_0^{w_2} \left[\eta_2 + \frac{1}{w_2}\left\{\alpha(\eta_1 - \eta_2)\right\}\right] d\alpha + \int_0^{w_2} \left[\eta_3 + \frac{1}{w_2}\left\{\alpha(\eta_4 - \eta_3)\right\}\right] d\alpha \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} \left[\lambda_1\alpha + \frac{1}{w_1}\left\{\frac{\alpha^2}{2}(\lambda_2 - \lambda_1)\right\}\right]_0^{w_1} + \left[\lambda_4\alpha + \frac{1}{w_1}\left\{\frac{\alpha^2}{2}(\lambda_3 - \lambda_4)\right\}\right]_0^{w_1} \\ + \left[\eta_2\alpha + \frac{1}{w_2}\left\{\frac{\alpha^2}{2}(\eta_1 - \eta_2)\right\}\right]_0^{w_2} + \left[\eta_3\alpha + \frac{1}{w_2}\left\{\frac{\alpha^2}{2}(\eta_4 - \eta_3)\right\}\right]_0^{w_2} \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} \lambda_1 w_1 + \frac{w_1}{2}(\lambda_2 - \lambda_1) + \lambda_4 w_1 + \frac{w_1}{2}(\lambda_3 - \lambda_4) \\ +\eta_2 w_2 + \frac{w_2}{2}(\eta_1 - \eta_2) + \eta_3 w_2 + \frac{w_2}{2}(\eta_4 - \eta_3) \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} \frac{w_1\left(2\lambda_1 + \lambda_2 - \lambda_1 + 2\lambda_4 + \lambda_3 - \lambda_4\right)}{2} \\ + \frac{w_2\left(2\eta_2 + \eta_1 - \eta_2 + 2\eta_3 + \eta_4 - \eta_3\right)}{2} \end{bmatrix}$$

$$\Rightarrow E(\sigma) = \frac{1}{4}\left[w_1\left(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4\right) + w_2\left(\eta_1 + \eta_2 + \eta_3 + \eta_4\right)\right]$$

Hence, the proof.

3.2 Variance of a GTrIFN

Variance is often regarded as a deterministic parameter of fuzzy numbers, and in statistical terms, it gives us an idea about the variability measure which exists in a dataset. Variance is an indicator of positive and negative; or useful and unnecessary fluctuations of the fuzzy numbers, from their standard expected value. The variance value is always non-negative. Accordingly, the square root of the variance value gives the standard deviation of fuzzy numbers, which is however exempted in this study. Depending upon the trend followed by variance values for a particular dataset, the decision-makers can modify their requirements to yield the maximum benefit. The direction of deviation among variables in a dataset is not a matter of concern in determining the final variance results.

Clearly the variance of any set $A \in \mathbb{R}$, is defined as $V(A) = E(A^2) - [E(A)]^2$.

Thus, even in case of a continuous fuzzy number σ, we have, $V(\sigma) = E(\sigma^2) - [E(\sigma)]^2$.

The mathematical expression for it using GTrIFNs is illustrated by the following theorem.

Theorem 2: Let $\sigma = \left\langle \left([\lambda_1, \lambda_2, \lambda_3, \lambda_4]; w_1 \right), \left([\eta_1, \eta_2, \eta_3, \eta_4]; w_2 \right) \right\rangle$ be a GTrIFN, where.

$\eta_1 \le \lambda_1 \le \eta_2 \le \lambda_2 \le \lambda_3 \le \eta_3 \le \lambda_4 \le \eta_4, 0 \le w_1, w_2 \le 1$ and whose expected value is given by.

$\Rightarrow E(\sigma) = \frac{1}{4}\left[w_1 \left(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 \right) + w_2 \left(\eta_1 + \eta_2 + \eta_3 + \eta_4 \right) \right]$, then its variance has the form

$$V(\sigma) = \frac{1}{6}\left[w_1 \left(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 + \lambda_4^2 + \lambda_1\lambda_2 + \lambda_3\lambda_4 \right) + w_2 \left(\eta_1^2 + \eta_2^2 + \eta_3^2 + \eta_4^2 + \eta_1\eta_2 + \eta_3\eta_4 \right) \right] - \frac{1}{16}\left[w_1 \left(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 \right) + w_2 \left(\eta_1 + \eta_2 + \eta_3 + \eta_4 \right) \right]^2. \tag{9}$$

Proof: The proof is given in (**Appendix A**).

3.3 Proposed approach

Having established the importance and the mathematical forms for the expected value and variance of GTrIFNs with the help of the α- cut technique, we are now in a firm position to explain the idea behind constructing our novel SM. Firstly, we select GTrIFNs in our study because they are capable of representing the uncertain information in a suitable manner, and the height parameters present in both the membership and the non-membership functions of GTrIFNs, help in reflecting the confidence degree of judgments made by the decision-makers. Precisely speaking, GTrIFNs can better handle the fuzziness, cognitive restrictions of the decision-makers, and the criticality of practical decision making problems. Secondly, we encounter several existing approaches in the past, where the researchers have utilized the possibility or credibility theory to devise the mathematical forms for expected value, variance, covariance, standard deviation, etc., of fuzzy numbers. But one cannot find applications of the more traditional, fuzzy theoretic α- cut technique to explore such parameters, which might surely bring more comprehensive implications. Therefore, as an alternative approach, we utilize the α- cut method in evaluating the mathematical expressions for the expected value and variance of GTrIFNs.

At present, no such established work explores this direction of GTrIFNs, hence it reflects the novel venture of this article. Generally, for any interval whenever we discuss its expected value or mean, we then mean about its mid-point. Further, the α- cut always produces an interval irrespective of the type of the fuzzy number. As a combination of both these ideas, if we first evaluate the α- cut of a GTrIFN which produces an interval, and then if we calculate its corresponding mid-point, we get the required expected value for the GTrIFN considered. Similarly, we explore the form for the variance of GTrIFNs, and the detailed derivations are already discussed in the previous sequel. Broadly, this sums up the idea behind the formulation process of our proposed SM expression. Thus, our newly constructed SM has the following definition.

Let us consider,$\lambda = \left\langle \left([\lambda_{11}, \lambda_{12}, \lambda_{13}, \lambda_{14}]; w_1 \right), \left([\lambda_{21}, \lambda_{22}, \lambda_{23}, \lambda_{24}]; w_2 \right) \right\rangle$ and.

$\eta = \left\langle \left([\eta_{11}, \eta_{12}, \eta_{13}, \eta_{14}]; v_1 \right), \left([\eta_{21}, \eta_{22}, \eta_{23}, \eta_{24}]; v_2 \right) \right\rangle$ are two non-zero GTrIFNs, where.

$\lambda_{21} \le \lambda_{11} \le \lambda_{22} \le \lambda_{12} \le \lambda_{13} \le \lambda_{23} \le \lambda_{14} \le \lambda_{24}, 0 \le w_1, w_2 \le 1$ and.

$\eta_{21} \le \eta_{11} \le \eta_{22} \le \eta_{12} \le \eta_{13} \le \eta_{23} \le \eta_{14} \le \eta_{24}, 0 \le v_1, v_2 \le 1.$

Then, the SM between λ and η is proposed as,

$$S_{TIFN}(\lambda, \eta) = \frac{2\left[\sum_{p=1}^{4} \lambda_{1p}\eta_{1p} + \sum_{p=1}^{4} \lambda_{2p}\eta_{2p} + \sum_{i=1}^{2} w_i v_i + E(\lambda)E(\eta) + V(\lambda)V(\eta)\right]}{\sum_{p=1}^{4} \lambda_{1p}^2 + \sum_{p=1}^{4} \lambda_{2p}^2 + \sum_{p=1}^{4} \eta_{1p}^2 + \sum_{p=1}^{4} \eta_{2p}^2 + \sum_{i=1}^{2} w_i^2 + \sum_{i=1}^{2} v_i^2 + E(\lambda)^2 + E(\eta)^2 + V(\lambda)^2 + V(\eta)^2} \tag{10}$$

where,$\lambda_{ip}, \eta_{ip} (i = 1, 2; p = 1, 2, 3, 4)$, w_1, w_2- membership and non-membership heights of GTrFN λ,v_1, v_2- membership and non-membership heights of GTrFN η,$E(\lambda)$- expected value of GTrFN λ,$E(\eta)$- expected value of GTrFN η,$V(\lambda)$- variance of GTrFN λ,$V(\eta)$- variance of GTrFN η.

To prove it as a SM, it must satisfy the following properties:

P-1: $0 \le S_{TIFN}(\lambda, \eta) \le 1.$

P-2: $S_{TIFN}(\lambda, \eta) = S_{TIFN}(\eta, \lambda).$

P-3: $S_{TIFN}(\lambda, \eta) = 1 \Leftrightarrow \lambda = \eta.$

Proof: (**P-1**) Clearly by definition, $S_{TIFN}(\lambda, \eta) \ge 0.$

Now, $2\left[\sum_{p=1}^{4} \lambda_{1p}\eta_{1p} + \sum_{p=1}^{4} \lambda_{2p}\eta_{2p} + \sum_{i=1}^{2} w_i v_i + E(\lambda)E(\eta) + V(\lambda)V(\eta)\right]$

$$\le \sum_{p=1}^{4} \lambda_{1p}^2 + \sum_{p=1}^{4} \lambda_{2p}^2 + \sum_{p=1}^{4} \eta_{1p}^2 + \sum_{p=1}^{4} \eta_{2p}^2 + \sum_{i=1}^{2} w_i^2 + \sum_{i=1}^{2} v_i^2 + E(\lambda)^2 + E(\eta)^2 + V(\lambda)^2 + V(\eta)^2$$

$\Rightarrow 0 \le S_{TIFN}(\lambda, \eta) \le 1.$

(**P-2**) Here,

$$S_{TIFN}(\lambda, \eta) = \frac{2\left[\sum_{p=1}^{4} \lambda_{1p}\eta_{1p} + \sum_{p=1}^{4} \lambda_{2p}\eta_{2p} + \sum_{i=1}^{2} w_i v_i + E(\lambda)E(\eta) + V(\lambda)V(\eta)\right]}{\sum_{p=1}^{4} \lambda_{1p}^2 + \sum_{p=1}^{4} \lambda_{2p}^2 + \sum_{p=1}^{4} \eta_{1p}^2 + \sum_{p=1}^{4} \eta_{2p}^2 + \sum_{i=1}^{2} w_i^2 + \sum_{i=1}^{2} v_i^2 + E(\lambda)^2 + E(\eta)^2 + V(\lambda)^2 + V(\eta)^2}$$

$$= \frac{2\left[\sum_{p=1}^{4} \eta_{1p}\lambda_{1p} + \sum_{p=1}^{4} \eta_{2p}\lambda_{2p} + \sum_{i=1}^{2} v_i w_i + E(\eta)E(\lambda) + V(\eta)V(\lambda)\right]}{\sum_{p=1}^{4} \eta_{1p}^2 + \sum_{p=1}^{4} \eta_{2p}^2 + \sum_{p=1}^{4} \lambda_{1p}^2 + \sum_{p=1}^{4} \lambda_{2p}^2 + \sum_{i=1}^{2} v_i^2 + \sum_{i=1}^{2} w_i^2 + E(\eta)^2 + E(\lambda)^2 + V(\eta)^2 + V(\lambda)^2}$$

$$= S_{TIFN}(\eta, \lambda)$$

(**P-3**) For $\lambda = \eta$, we have $\lambda_{ip} = \eta_{ip}, w_i = v_i, E(\lambda) = E(\eta), V(\lambda) = V(\eta)$, then clearly,$S_{TIFN}(\lambda, \eta) = 1.$

Conversely, let $S_{TIFN}(\lambda, \eta) = 1$, then

$$2\left[\sum_{p=1}^{4}\lambda_{1p}\eta_{1p}+\sum_{p=1}^{4}\lambda_{2p}\eta_{2p}+\sum_{i=1}^{2}w_iv_i+E(\lambda)E(\eta)+V(\lambda)V(\eta)\right]$$

$$=\sum_{p=1}^{4}\lambda_{1p}^2+\sum_{p=1}^{4}\lambda_{2p}^2+\sum_{p=1}^{4}\eta_{1p}^2+\sum_{p=1}^{4}\eta_{2p}^2+\sum_{i=1}^{2}w_i^2+\sum_{i=1}^{2}v_i^2+E(\lambda)^2+E(\eta)^2+V(\lambda)^2+V(\eta)^2$$

$$\Rightarrow\sum_{p=1}^{4}\left(\lambda_{1p}-\eta_{1p}\right)^2+\sum_{p=1}^{4}\left(\lambda_{2p}-\eta_{2p}\right)^2+\sum_{i=1}^{2}\left(w_i-v_i\right)^2+(E(\lambda)-E(\eta))^2+(V(\lambda)-V(\eta))^2=0$$

$\Rightarrow \lambda_{ip}=\eta_{ip}, w_i=v_i, E(\lambda)=E(\eta), V(\lambda)=V(\eta)$, for $i=1,2;p=1,2,3,4$, otherwise $S_{TIFN}(\lambda,\eta)\neq 1$, which is a contradiction.

$$\Rightarrow \lambda=\eta$$

Hence, $S_{TIFN}(\lambda,\eta)$ is an SM.

Remark 1: Since,$0\leq S_{TIFN}(\lambda,\eta)\leq 1$, so $S_{TIFN}(\lambda,\eta)=0$, means complete dissimilarity between λ and η. Whereas,$S_{TIFN}(\lambda,\eta)=1$, means complete/total similarity between λ and η. Maximum $S_{TIFN}(\lambda,\eta)$ value means maximum similarity between λ and η.

Here are some of the desirable properties that are being satisfied by our newly developed measure.

Property 3.3.1 *For any three non-zero GTrIFNs* $\lambda=\left\langle\left([\lambda_{11},\lambda_{12},\lambda_{13},\lambda_{14}];w_{\lambda1}\right),\left([\lambda_{21},\lambda_{22},\lambda_{23},\lambda_{24}];w_{\lambda2}\right)\right\rangle$,

$\eta=\left\langle\left([\eta_{11},\eta_{12},\eta_{13},\eta_{14}];w_{\eta1}\right),\left([\eta_{21},\eta_{22},\eta_{23},\eta_{24}];w_{\eta2}\right)\right\rangle$, *and* $\sigma=\left\langle\left([\sigma_{11},\sigma_{12},\sigma_{13},\sigma_{14}];w_{\sigma1}\right),\left([\sigma_{21},\sigma_{22},\sigma_{23},\sigma_{24}];w_{\sigma2}\right)\right\rangle$, *if* $\lambda\subseteq\eta\subseteq\sigma$, *then* $S_{TIFN}(\lambda,\eta)\geq S_{TIFN}(\lambda,\sigma)$ *and* $S_{TIFN}(\eta,\sigma)\geq S_{TIFN}(\lambda,\sigma)$.

Proof: *We are given the condition that*, $\lambda\subseteq\eta\subseteq\sigma$ *and we are required to prove that*,

$$S_{TIFN}(\lambda,\eta)\geq S_{TIFN}(\lambda,\sigma)\text{ and }S_{TIFN}(\eta,\sigma)\geq S_{TIFN}(\lambda,\sigma)\tag{11}$$

From the given condition we get, $\lambda_{1p}\leq\eta_{1p}\leq\sigma_{1p}$, $\lambda_{2p}\leq\eta_{2p}\leq\sigma_{2p}$ (*for* $p=1,2,3,4$), *and* $w_{\lambda1}\leq w_{\eta1}\leq w_{\sigma1}, w_{\lambda2}\leq w_{\eta2}\leq w_{\sigma2}$.

To establish our claim, we take help of a well-known result which is given below.

Result: *If*

$$\frac{1}{2}\geq\frac{a}{A}\geq\frac{b}{B}\text{ and }\frac{1}{2}\geq\frac{c}{C}\geq\frac{d}{D},\text{ then }\frac{a+c}{A+C}\geq\frac{b+d}{B+D},\text{ provided }a,b,c,d,A,B,C,D\text{ are real positive numbers}\tag{12}$$

Now, the GTrIFNs being non-zero and the given information implies.

$\lambda_{1p}\geq 0, \sigma_{1p}-\eta_{1p}\geq 0$, *and* $\eta_{1p}\sigma_{1p}-\left(\lambda_{1p}\right)^2\geq 0$.

Clearly, $\left(\sigma_{1p}-\eta_{1p}\right)\left(\eta_{1p}\sigma_{1p}-\left(\lambda_{1p}\right)^2\geq 0\right)\lambda_{1p}\geq 0$

$$\Rightarrow \lambda_{1p}^3\left(\eta_{1p} - \sigma_{1p}\right) + \lambda_{1p}\eta_{1p}\sigma_{1p}\left(\sigma_{1p} - \eta_{1p}\right) \geq 0$$

$$\Rightarrow \lambda_{1p}\eta_{1p}\left(\lambda_{1p}^2 + \sigma_{1p}^2\right) \geq \lambda_{1p}\sigma_{1p}\left(\lambda_{1p}^2 + \eta_{1p}^2\right)$$

$$\Rightarrow \frac{\lambda_{1p}\eta_{1p}}{\lambda_{1p}^2 + \eta_{1p}^2} \geq \frac{\lambda_{1p}\sigma_{1p}}{\lambda_{1p}^2 + \sigma_{1p}^2}, \quad (for\, p = 1, 2, 3, 4)$$

By similar argument we also have, $\frac{\lambda_{2p}\eta_{2p}}{\lambda_{2p}^2+\eta_{2p}^2} \geq \frac{\lambda_{2p}\sigma_{2p}}{\lambda_{2p}^2+\sigma_{2p}^2}$, $(for\, p = 1, 2, 3, 4)$.

For any real numbers a, b, *we know that* $a^2 + b^2 \geq 2ab \ \Rightarrow \frac{1}{2} \geq \frac{ab}{a^2+b^2}$.

So, *we have*,

$$\frac{1}{2} \geq \frac{\lambda_{1p}\eta_{1p}}{\lambda_{1p}^2 + \eta_{1p}^2} \geq \frac{\lambda_{1p}\sigma_{1p}}{\lambda_{1p}^2 + \sigma_{1p}^2} \text{ and } \frac{1}{2} \geq \frac{\lambda_{2p}\eta_{2p}}{\lambda_{2p}^2 + \eta_{2p}^2} \geq \frac{\lambda_{2p}\sigma_{2p}}{\lambda_{2p}^2 + \sigma_{2p}^2} \tag{13}$$

Similarly, *we can obtain the other inequalities as follows*,

$$\frac{1}{2} \geq \frac{w_{\lambda 1}w_{\eta 1}}{w_{\lambda 1}^2 + w_{\eta 1}^2} \geq \frac{w_{\lambda 1}w_{\sigma 1}}{w_{\lambda 1}^2 + w_{\sigma 1}^2} \tag{14}$$

$$\frac{1}{2} \geq \frac{E(\lambda)E(\eta)}{E(\lambda)^2 + E(\eta)^2} \geq \frac{E(\lambda)E(\sigma)}{E(\lambda)^2 + E(\sigma)^2} \tag{15}$$

$$\frac{1}{2} \geq \frac{V(\lambda)V(\eta)}{V(\lambda)^2 + V(\eta)^2} \geq \frac{V(\lambda)V(\sigma)}{V(\lambda)^2 + V(\sigma)^2} \tag{16}$$

Combining Eqs. (13)–(16) *and the result as mentioned above*, *we obtain*

$$\frac{2\left[\sum_{p=1}^{4}\lambda_{1p}\eta_{1p} + \sum_{p=1}^{4}\lambda_{2p}\eta_{2p} + w_{\lambda 1}w_{\eta 1} + w_{\lambda 2}w_{\eta 2} + E(\lambda)E(\eta) + V(\lambda)V(\eta)\right]}{\sum_{p=1}^{4}\lambda_{1p}^2 + \sum_{p=1}^{4}\lambda_{2p}^2 + \sum_{p=1}^{4}\eta_{1p}^2 + \sum_{p=1}^{4}\eta_{2p}^2 + w_{\lambda 1}^2 + w_{\eta 1}^2 + w_{\lambda 2}^2 + w_{\eta 2}^2 + E(\lambda)^2 + E(\eta)^2 + V(\lambda)^2 + V(\eta)^2} \geq$$

$$\frac{2\left[\sum_{p=1}^{4}\lambda_{1p}\sigma_{1p} + \sum_{p=1}^{4}\lambda_{2p}\sigma_{2p} + w_{\lambda 1}w_{\sigma 1} + w_{\lambda 2}w_{\sigma 2} + E(\lambda)E(\sigma) + V(\lambda)V(\sigma)\right]}{\sum_{p=1}^{4}\lambda_{1p}^2 + \sum_{p=1}^{4}\lambda_{2p}^2 + \sum_{p=1}^{4}\sigma_{1p}^2 + \sum_{p=1}^{4}\sigma_{2p}^2 + w_{\lambda 1}^2 + w_{\sigma 1}^2 + w_{\lambda 2}^2 + w_{\sigma 2}^2 + E(\lambda)^2 + E(\sigma)^2 + V(\lambda)^2 + V(\sigma)^2}$$

$$\Rightarrow S_{TIFN}(\lambda, \eta) \geq S_{TIFN}(\lambda, \sigma)$$

Likewise, *the other inequality*, $S_{TIFN}(\eta, \sigma) \geq S_{TIFN}(\lambda, \sigma)$ *can be proved. Hence the proof.*

Property 3.3.2 *For any two GTrIFNs* $\lambda = \left\langle \left([\lambda_{11}, \lambda_{12}, \lambda_{13}, \lambda_{14}]; w_{\lambda 1} \right), \left([\lambda_{21}, \lambda_{22}, \lambda_{23}, \lambda_{24}]; w_{\lambda 2} \right) \right\rangle$ *and.*

$\eta = \left\langle \left([\eta_{11}, \eta_{12}, \eta_{13}, \eta_{14}]; w_{\eta 1} \right), \left([\eta_{21}, \eta_{22}, \eta_{23}, \eta_{24}]; w_{\eta 2} \right) \right\rangle$, *our proposed measure satisfies.*

$S_{TIFN}(\lambda, \eta) = S_{TIFN}(\lambda \cup \eta, \lambda \cap \eta).$

Proof: *For the given GTrIFNs, we evaluate the mathematical forms for* $\lambda \cup \eta$ *and* $\lambda \cap \eta$ *as follows,*

$$\lambda \cup \eta = \left\langle \begin{array}{l} \left([\max(\lambda_{11}, \eta_{11}), \max(\lambda_{12}, \eta_{12}), \max(\lambda_{13}, \eta_{13}), \max(\lambda_{14}, \eta_{14})]; \max(w_{\lambda 1}, w_{\eta 1}) \right), \\ \left([\min(\lambda_{21}, \eta_{21}), \min(\lambda_{22}, \eta_{22}), \min(\lambda_{23}, \eta_{23}), \min(\lambda_{24}, \eta_{24})]; \min(w_{\lambda 2}, w_{\eta 2}) \right) \end{array} \right\rangle \tag{17}$$

$$\lambda \cap \eta = \left\langle \begin{array}{l} \left([\min(\lambda_{11}, \eta_{11}), \min(\lambda_{12}, \eta_{12}), \min(\lambda_{13}, \eta_{13}), \min(\lambda_{14}, \eta_{14})]; \min(w_{\lambda 1}, w_{\eta 1}) \right), \\ \left([\max(\lambda_{21}, \eta_{21}), \max(\lambda_{22}, \eta_{22}), \max(\lambda_{23}, \eta_{23}), \max(\lambda_{24}, \eta_{24})]; \max(w_{\lambda 2}, w_{\eta 2}) \right) \end{array} \right\rangle \tag{18}$$

Without loss of generality, we set $\max(\lambda_{1p}, \eta_{1p}) = \lambda_{1p}$, $\max(\lambda_{2p}, \eta_{2p}) = \lambda_{2p}$, $\max(w_{\lambda i}, w_{\eta i}) = w_{\lambda i}$, *for* $p = 1, 2, 3, 4$ *and* $i = 1, 2$.

Therefore,

$$S_{TIFN}(\lambda \cup \eta, \lambda \cap \eta) = \frac{2\left[\sum_{p=1}^{4} \lambda_{1p}\eta_{1p} + \sum_{p=1}^{4} \lambda_{2p}\eta_{2p} + \sum_{i=1}^{2} w_{\lambda i} w_{\eta i} + E(\lambda \cup \eta)E(\lambda \cap \eta) + V(\lambda \cup \eta)V(\lambda \cap \eta) \right]}{\sum_{p=1}^{4} \lambda_{1p}^2 + \sum_{p=1}^{4} \lambda_{2p}^2 + \sum_{p=1}^{4} \eta_{1p}^2 + \sum_{p=1}^{4} \eta_{2p}^2 + \sum_{i=1}^{2} w_{\lambda i}^2 + \sum_{i=1}^{2} w_{\eta i}^2 + E(\lambda \cup \eta)^2 + E(\lambda \cap \eta)^2 + V(\lambda \cup \eta)^2 + V(\lambda \cap \eta)^2}$$

$$= \frac{2\left[\sum_{p=1}^{4} \lambda_{1p}\eta_{1p} + \sum_{p=1}^{4} \lambda_{2p}\eta_{2p} + \sum_{i=1}^{2} w_{\lambda i} w_{\eta i} + E(\lambda)E(\eta) + V(\lambda)V(\eta) \right]}{\sum_{p=1}^{4} \lambda_{1p}^2 + \sum_{p=1}^{4} \lambda_{2p}^2 + \sum_{p=1}^{4} \eta_{1p}^2 + \sum_{p=1}^{4} \eta_{2p}^2 + \sum_{i=1}^{2} w_{\lambda i}^2 + \sum_{i=1}^{2} w_{\eta i}^2 + E(\lambda)^2 + E(\eta)^2 + V(\lambda)^2 + V(\eta)^2}$$

$$= S_{TIFN}(\lambda, \eta)$$

Hence, the proof.

Property 3.3.3 *For any two GTrIFNs* $\lambda = \left\langle \left([\lambda_{11}, \lambda_{12}, \lambda_{13}, \lambda_{14}]; w_{\lambda 1} \right), \left([\lambda_{21}, \lambda_{22}, \lambda_{23}, \lambda_{24}]; w_{\lambda 2} \right) \right\rangle$ *and.*

$\eta = \left\langle \left([\eta_{11}, \eta_{12}, \eta_{13}, \eta_{14}]; w_{\eta 1} \right), \left([\eta_{21}, \eta_{22}, \eta_{23}, \eta_{24}]; w_{\eta 2} \right) \right\rangle$, *our proposed measure satisfies the following property.*

$S_{TIFN}(\lambda, \lambda \cap \eta) = S_{TIFN}(\eta, \lambda \cup \eta).$

Proof: *With similar consideration as in property* 3.3.2, *we set* $\max(\lambda_{1p}, \eta_{1p}) = \lambda_{1p}$, $\max(\lambda_{2p}, \eta_{2p}) = \lambda_{2p}$, $\max(w_{\lambda i}, w_{\eta i}) = w_{\lambda i}$, *for* $p = 1, 2, 3, 4$ *and* $i = 1, 2$.

Then we obtain,

$$S_{TIFN}(\lambda,\lambda\cap\eta)=\frac{2\left[\sum\limits_{p=1}^{4}\lambda_{1p}\eta_{1p}+\sum\limits_{p=1}^{4}\lambda_{2p}\eta_{2p}+\sum\limits_{i=1}^{2}w_{\lambda i}w_{\eta i}+E(\lambda)E(\lambda\cap\eta)+V(\lambda)V(\lambda\cap\eta)\right]}{\sum\limits_{p=1}^{4}\lambda_{1p}^{2}+\sum\limits_{p=1}^{4}\lambda_{2p}^{2}+\sum\limits_{p=1}^{4}\eta_{1p}^{2}+\sum\limits_{p=1}^{4}\eta_{2p}^{2}+\sum\limits_{i=1}^{2}w_{\lambda i}^{2}+\sum\limits_{i=1}^{2}w_{\eta i}^{2}+E(\lambda)^{2}+E(\lambda\cap\eta)^{2}+V(\lambda)^{2}+V(\lambda\cap\eta)^{2}}$$

$$=\frac{2\left[\sum\limits_{p=1}^{4}\lambda_{1p}\eta_{1p}+\sum\limits_{p=1}^{4}\lambda_{2p}\eta_{2p}+\sum\limits_{i=1}^{2}w_{\lambda i}w_{\eta i}+E(\lambda)E(\eta)+V(\lambda)V(\eta)\right]}{\sum\limits_{p=1}^{4}\lambda_{1p}^{2}+\sum\limits_{p=1}^{4}\lambda_{2p}^{2}+\sum\limits_{p=1}^{4}\eta_{1p}^{2}+\sum\limits_{p=1}^{4}\eta_{2p}^{2}+\sum\limits_{i=1}^{2}w_{\lambda i}^{2}+\sum\limits_{i=1}^{2}w_{\eta i}^{2}+E(\lambda)^{2}+E(\eta)^{2}+V(\lambda)^{2}+V(\eta)^{2}}$$

$$=\frac{2\left[\sum\limits_{p=1}^{4}\eta_{1p}\lambda_{1p}+\sum\limits_{p=1}^{4}\eta_{2p}\lambda_{2p}+\sum\limits_{i=1}^{2}w_{\eta i}w_{\lambda i}+E(\eta)E(\lambda)+V(\eta)V(\lambda)\right]}{\sum\limits_{p=1}^{4}\eta_{2p}^{2}+\sum\limits_{p=1}^{4}\eta_{1p}^{2}+\sum\limits_{p=1}^{4}\lambda_{1p}^{2}+\sum\limits_{p=1}^{4}\lambda_{2p}^{2}+\sum\limits_{i=1}^{2}w_{\eta i}^{2}+\sum\limits_{i=1}^{2}w_{\lambda i}^{2}+E(\eta)^{2}+E(\lambda)^{2}+V(\eta)^{2}+V(\lambda)^{2}}$$

$$=\frac{2\left[\sum\limits_{p=1}^{4}\eta_{1p}\lambda_{1p}+\sum\limits_{p=1}^{4}\eta_{2p}\lambda_{2p}+\sum\limits_{i=1}^{2}w_{\eta i}w_{\lambda i}+E(\eta)E(\lambda\cup\eta)+V(\eta)V(\lambda\cup\eta)\right]}{\sum\limits_{p=1}^{4}\eta_{2p}^{2}+\sum\limits_{p=1}^{4}\eta_{1p}^{2}+\sum\limits_{p=1}^{4}\lambda_{1p}^{2}+\sum\limits_{p=1}^{4}\lambda_{2p}^{2}+\sum\limits_{i=1}^{2}w_{\eta i}^{2}+\sum\limits_{i=1}^{2}w_{\lambda i}^{2}+E(\eta)^{2}+E(\lambda\cup\eta)^{2}+V(\eta)^{2}+V(\lambda\cup\eta)^{2}}$$

$$=S_{TIFN}(\eta,\lambda\cup\eta)$$

Property 3.3.4 *For GTrIFNs* $\lambda=\left\langle\left([\lambda_{11},\lambda_{12},\lambda_{13},\lambda_{14}];w_{\lambda1}\right),\left([\lambda_{21},\lambda_{22},\lambda_{23},\lambda_{24}];w_{\lambda2}\right)\right\rangle$ *and.*

$\eta=\left\langle\left([\eta_{11},\eta_{12},\eta_{13},\eta_{14}];w_{\eta1}\right),\left([\eta_{21},\eta_{22},\eta_{23},\eta_{24}];w_{\eta2}\right)\right\rangle$, *our proposed measure satisfies.*

$S_{TIFN}(\lambda,\lambda\cup\eta)=S_{TIFN}(\eta,\lambda\cap\eta)$.

Proof: *The approach of the proof is similar to property* 3.3.3 *and so it is not illustrated here.*

Property 3.3.5 *For any three non-zero GTrIFNs* λ,η, *and* σ (*with the notations as defined earlier*), *if* $\lambda\subseteq\eta\subseteq\sigma$, *then we have* $S_{TIFN}(\lambda,\sigma)\le\min\left(S_{TIFN}(\lambda,\eta),S_{TIFN}(\eta,\sigma)\right)$.

Proof: *We have already obtained that,* $\lambda\subseteq\eta\subseteq\sigma$ *implies* $S_{TIFN}(\lambda,\eta)\ge S_{TIFN}(\lambda,\sigma)$ *and.*

$S_{TIFN}(\eta,\sigma)\ge S_{TIFN}(\lambda,\sigma)$. *Since,* $S_{TIFN}(\lambda,\sigma)$ *is less than or equal to either of* $S_{TIFN}(\lambda,\eta)$ *and* $S_{TIFN}(\eta,\sigma)$. *Therefore, it is intuitive that,*

$S_{TIFN}(\lambda,\sigma)\le\min\left(S_{TIFN}(\lambda,\eta),S_{TIFN}(\eta,\sigma)\right)$.

Hence the proof.

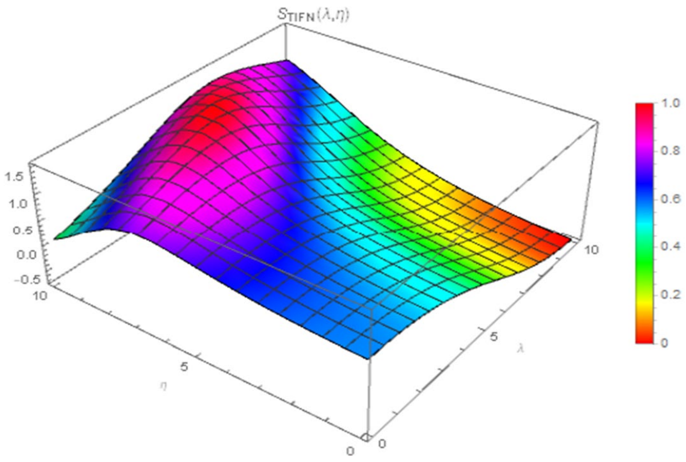


Fig. 2 3D surface of proposed measure for $a = 2$

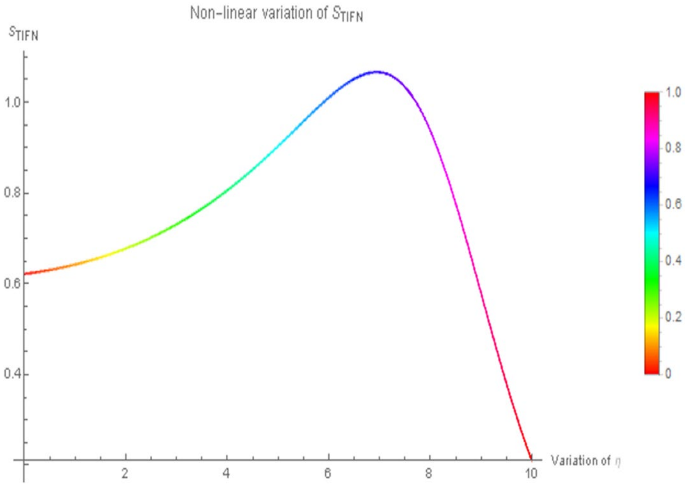


Fig. 3 2D plot of proposed measure for $a = 2$

3.4 Boundedness and non-linearity of our proposed measure

We consider two GTrIFNs λ and η in such a manner that we can obtain the variation of our proposed similarity function S_{TIFN} with all permissible values of λ and η. Next, we also portray the non-linear variation of our measure with different values of η, for a particular value of λ(say, 0.5). The heights of both the GTrIFNs are given some particular values because otherwise the figures cannot be generated for varying values of λ and η.

Let us consider λ and η to be as follows,

$$\lambda = \langle([\lambda - 2a, \lambda - a, \lambda + a, \lambda + 2a];0.7), ([\lambda - 3a, \lambda - 2a, \lambda + 2a, \lambda + 3a];0.3)\rangle$$

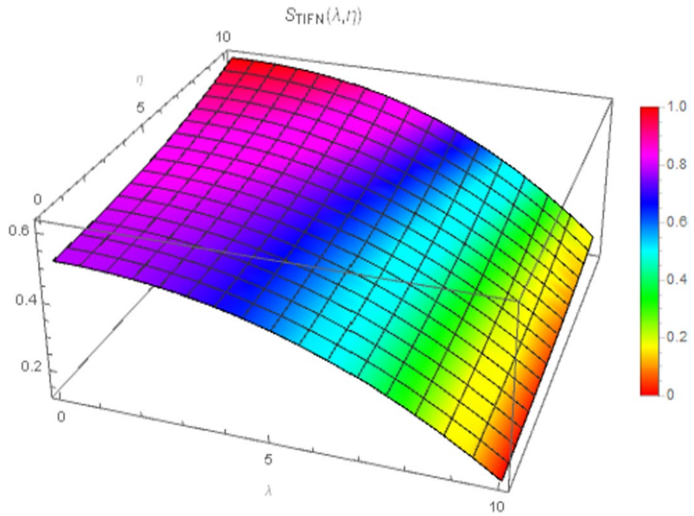


Fig. 4 3D surface of proposed measure for $a = 5$

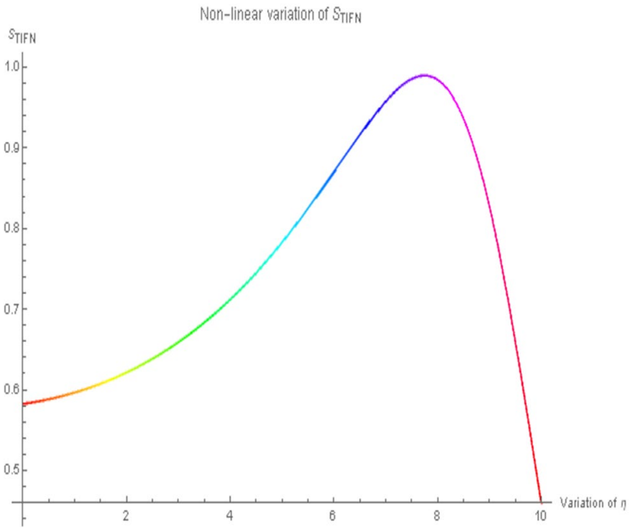


Fig. 5 2D plot of proposed measure for $a = 5$

and, $\eta = \langle([\eta - 4a, \eta - 2a, \eta + 2a, \eta + 4a];0.5), ([\eta - 5a, \eta - 3a, \eta + 3a, \eta + 5a];0.5)\rangle$, where '$a$' is any positive real number. The GTrIFNs are considered in such a way so that by changing the value of a, we can generate the corresponding 3-D and 2-D representation of our proposed similarity function as shown in Figs. 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 below.

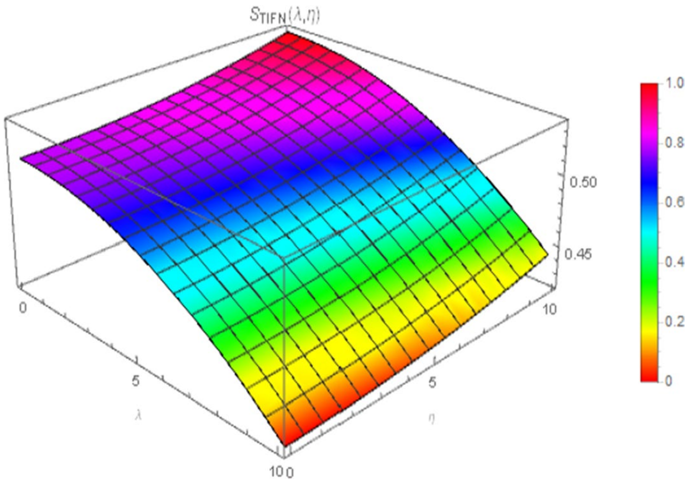


Fig. 6 3D surface of proposed measure for $a = 10$

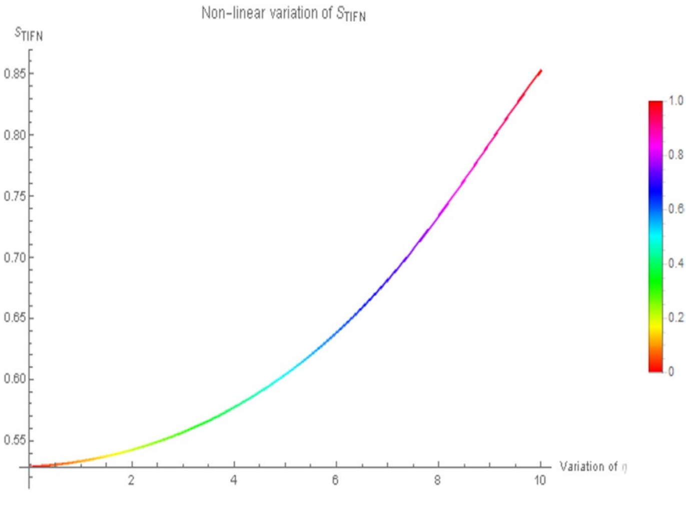


Fig. 7 2D plot of proposed measure for $a = 10$

It is evident from the figures plotted above that our proposed similarity function is bounded between the values of 0 and 1. Moreover, the nonlinearity of the proposed measure is apparent in all the figures.

3.5 Propositions

Some suitable propositions that can be derived from our newly proposed SM are presented below.

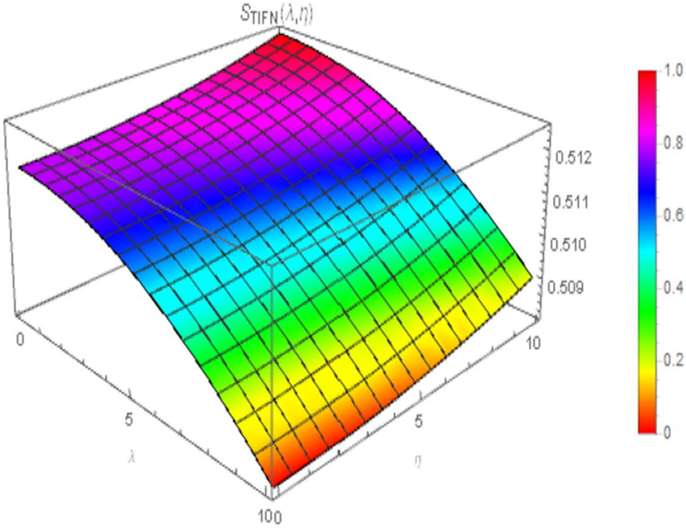


Fig. 8 3D surface of proposed measure for $a = 50$

Proposition 3.5.1 *If* $\lambda = \left\langle \left([\lambda_{11}, \lambda_{12}, \lambda_{13}, \lambda_{14}];1 \right), \left([\lambda_{21}, \lambda_{22}, \lambda_{23}, \lambda_{24}];1 \right) \right\rangle$ *and.*

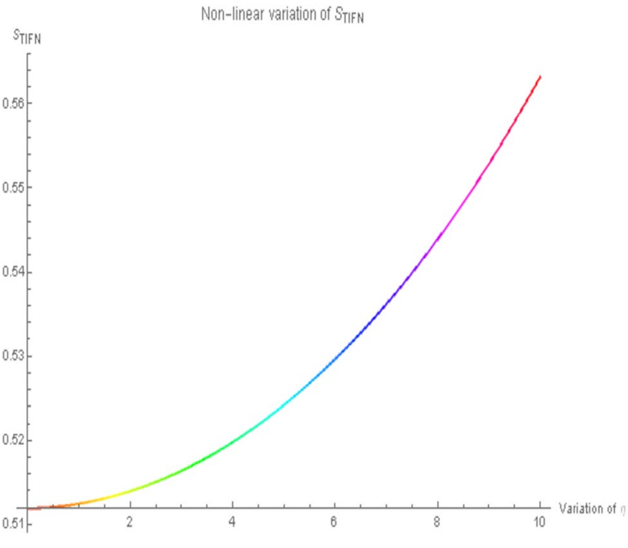


Fig. 9 2D plot of proposed measure for $a = 50$

$\eta = \left\langle \left([\eta_{11}, \eta_{12}, \eta_{13}, \eta_{14}];1 \right), \left([\eta_{21}, \eta_{22}, \eta_{23}, \eta_{24}];1 \right) \right\rangle$ *be two normal TrIFNs, then*

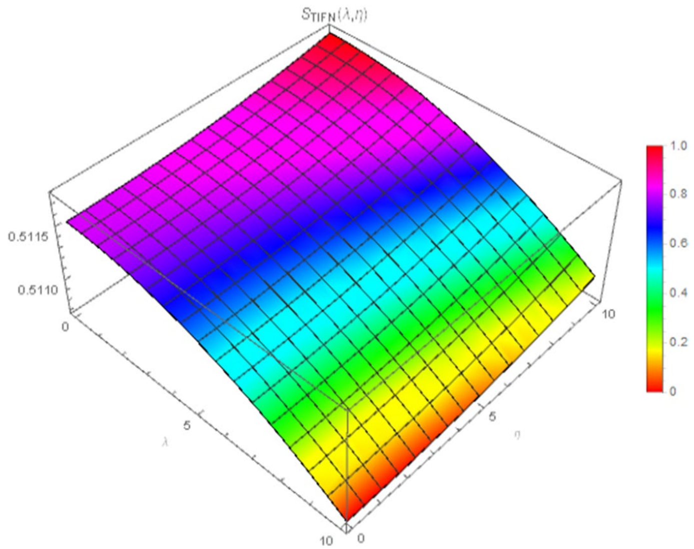


Fig. 10 3D surface of proposed measure for $a = 100$

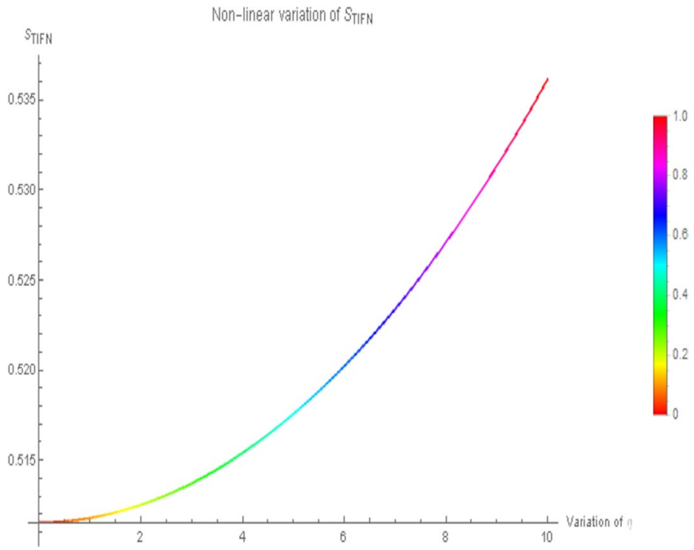


Fig. 11 2D plot of proposed measure for $a = 100$

$$S_{TIFN}(\lambda,\eta) = \frac{2\left[\sum_{p=1}^{4}\lambda_{1p}\eta_{1p} + \sum_{p=1}^{4}\lambda_{2p}\eta_{2p} + 2 + E(\lambda)E(\eta) + V(\lambda)V(\eta)\right]}{\sum_{p=1}^{4}\lambda_{1p}^{2} + \sum_{p=1}^{4}\lambda_{2p}^{2} + \sum_{p=1}^{4}\eta_{1p}^{2} + \sum_{p=1}^{4}\eta_{2p}^{2} + 4 + E(\lambda)^{2} + E(\eta)^{2} + V(\lambda)^{2} + V(\eta)^{2}} \tag{19}$$

Proof: *From our proposed SM expression in Eq.* (10), *for the given normal TrIFNs we have*,

$$S_{TIFN}(\lambda,\eta)=\frac{2\left[\begin{pmatrix}\lambda_{11}\eta_{11}+\lambda_{12}\eta_{12}+\lambda_{13}\eta_{13}+\lambda_{14}\eta_{14}+\\ \lambda_{21}\eta_{21}+\lambda_{22}\eta_{22}+\lambda_{23}\eta_{23}+\lambda_{24}\eta_{24}\end{pmatrix}+1.1+1.1+E(\lambda)E(\eta)+V(\lambda)V(\eta)\right]}{\begin{pmatrix}\left(\lambda_{11}^2+\lambda_{12}^2+\lambda_{13}^2+\lambda_{14}^2\right)+\left(\eta_{11}^2+\eta_{12}^2+\eta_{13}^2+\eta_{14}^2\right)+\\ \left(\lambda_{21}^2+\lambda_{22}^2+\lambda_{23}^2+\lambda_{24}^2\right)+\left(\eta_{21}^2+\eta_{22}^2+\eta_{23}^2+\eta_{24}^2\right)\end{pmatrix}+1+1+1+1+E(\lambda)^2+E(\eta)^2+V(\lambda)^2+V(\eta)^2}$$

$$=\frac{2\left[\sum_{p=1}^{4}\lambda_{1p}\eta_{1p}+\sum_{p=1}^{4}\lambda_{2p}\eta_{2p}+2+E(\lambda)E(\eta)+V(\lambda)V(\eta)\right]}{\sum_{p=1}^{4}\lambda_{1p}^2+\sum_{p=1}^{4}\lambda_{2p}^2+\sum_{p=1}^{4}\eta_{1p}^2+\sum_{p=1}^{4}\eta_{2p}^2+4+E(\lambda)^2+E(\eta)^2+V(\lambda)^2+V(\eta)^2}$$

Hence, *the proof.*

Proposition 3.5.2 *If* $\lambda=\left\langle\left([\lambda_{11},\lambda_{12},\lambda_{13}];1\right),\left([\lambda_{21},\lambda_{22},\lambda_{23}];1\right)\right\rangle$ *and.*

$\eta=\left\langle\left([\eta_{11},\eta_{12},\eta_{13}];1\right),\left([\eta_{21},\eta_{22},\eta_{23}];1\right)\right\rangle$ *be two normal TIFNs, then*

$$S_{TIFN}(\lambda,\eta)=\frac{2\left[\begin{pmatrix}\lambda_{11}\eta_{11}+2\lambda_{12}\eta_{12}+\lambda_{13}\eta_{13}+\\ \lambda_{21}\eta_{21}+2\lambda_{22}\eta_{22}+\lambda_{23}\eta_{23}\end{pmatrix}+2+E(\lambda)E(\eta)+V(\lambda)V(\eta)\right]}{\begin{pmatrix}\lambda_{11}^2+2\lambda_{12}^2+\lambda_{13}^2+\lambda_{21}^2+2\lambda_{22}^2+\lambda_{23}^2+\\ \eta_{11}^2+2\eta_{12}^2+\eta_{13}^2+\eta_{21}^2+2\eta_{22}^2+\eta_{23}^2\end{pmatrix}+4+E(\lambda)^2+E(\eta)^2+V(\lambda)^2+V(\eta)^2} \tag{20}$$

Proof: *From our proposed SM expression in Eq.* (10), *for the given normal TIFNs we have*,

$$S_{TIFN}(\lambda,\eta)=\frac{2\left[\begin{pmatrix}\lambda_{11}\eta_{11}+\lambda_{12}\eta_{12}+\lambda_{12}\eta_{12}+\lambda_{13}\eta_{13}+\\ \lambda_{21}\eta_{21}+\lambda_{22}\eta_{22}+\lambda_{22}\eta_{22}+\lambda_{23}\eta_{23}\end{pmatrix}+1.1+1.1+E(\lambda)E(\eta)+V(\lambda)V(\eta)\right]}{\begin{pmatrix}\left(\lambda_{11}^2+\lambda_{12}^2+\lambda_{12}^2+\lambda_{13}^2\right)+\left(\eta_{11}^2+\eta_{12}^2+\eta_{12}^2+\eta_{13}^2\right)+\\ \left(\lambda_{21}^2+\lambda_{22}^2+\lambda_{22}^2+\lambda_{23}^2\right)+\left(\eta_{21}^2+\eta_{22}^2+\eta_{22}^2+\eta_{23}^2\right)\end{pmatrix}+1+1+1+1+E(\lambda)^2+E(\eta)^2+V(\lambda)^2+V(\eta)^2}$$

$$=\frac{2\left[\begin{pmatrix}\lambda_{11}\eta_{11}+2\lambda_{12}\eta_{12}+\lambda_{13}\eta_{13}+\\ \lambda_{21}\eta_{21}+2\lambda_{22}\eta_{22}+\lambda_{23}\eta_{23}\end{pmatrix}+2+E(\lambda)E(\eta)+V(\lambda)V(\eta)\right]}{\begin{pmatrix}\lambda_{11}^2+2\lambda_{12}^2+\lambda_{13}^2+\lambda_{21}^2+2\lambda_{22}^2+\lambda_{23}^2+\\ \eta_{11}^2+2\eta_{12}^2+\eta_{13}^2+\eta_{21}^2+2\eta_{22}^2+\eta_{23}^2\end{pmatrix}+4+E(\lambda)^2+E(\eta)^2+V(\lambda)^2+V(\eta)^2}$$

Hence, *the proof.*

Proposition 3.5.3 *Suppose* $\lambda=\left\langle\left([0,0,0,0];w_1\right),\left([0,0,0,0];w_2\right)\right\rangle$ *and* $\eta=\left\langle\left([0,0,0,0];v_1\right),\left([0,0,0,0];v_2\right)\right\rangle$ *be two GTrIFNs with heights* $w_1(membership), w_2(non-membership) \& v_1(membership)\ \ v_2(non-membership)$ *respectively. Then*,

$$S_{TIFN}(\lambda,\eta)=\frac{2\left(w_1v_1+w_2v_2\right)}{w_1^2+w_2^2+v_1^2+v_2^2} \tag{21}$$

Proof: *For the above GTrIFNs* λ *and* η, *the values of their expected values and variances are identically zero which is clear from the expressions presented in subsection* 3.1 *and* 3.2, *respectively*.

Therefore, we have $E(\lambda) = 0 = E(\eta)$; $V(\lambda) = 0 = V(\eta)$. *Hence, from our proposed SM expression in Eq.* (10), *we have,*

$$S_{TIFN}(\lambda,\eta) = \frac{2\left[\begin{pmatrix} 0.0+0.0+0.0+0.0+ \\ 0.0+0.0+0.0+0.0 \end{pmatrix} + w_1.v_1 + w_2.v_2 + 0.0 + 0.0\right]}{\begin{pmatrix} (0+0+0+0)+(0+0+0+0)+ \\ (0+0+0+0)+(0+0+0+0) \end{pmatrix} + w_1^2 + v_1^2 + w_2^2 + v_2^2 + 0 + 0 + 0 + 0}$$
$$= \frac{2\left(w_1 v_1 + w_2 v_2\right)}{w_1^2 + w_2^2 + v_1^2 + v_2^2}$$

Hence, the proof.

Proposition 3.5.4 *If* $\lambda = \langle([0,0,0,0];1),([0,0,0,0];1)\rangle$ *and.*

$\eta = \left\langle\left([\eta_{11},\eta_{12},\eta_{13},\eta_{14}];1\right),\left([\eta_{21},\eta_{22},\eta_{23},\eta_{24}];1\right)\right\rangle$ *are two normal TrIFNs, then*

$$S_{TIFN}(\lambda,\eta) = \frac{4}{\sum_{p=1}^{4}\eta_{1p}^2 + \sum_{p=1}^{4}\eta_{2p}^2 + 4 + E(\eta)^2 + V(\eta)^2} \tag{22}$$

Proof: *For the given normal TrIFN* λ, *we have* $E(\lambda) = 0$; $V(\lambda) = 0$ *as we have already explained. Now, from our proposed SM expression in Eq.* (10), *for the given normal TIFNs we have,*

$$S_{TIFN}(\lambda,\eta) = \frac{2\left[\begin{pmatrix} 0.\eta_{11}+0.\eta_{12}+0.\eta_{12}+0.\eta_{13}+ \\ 0.\eta_{21}+0.\eta_{22}+0.\eta_{22}+0.\eta_{23} \end{pmatrix} + 1.1 + 1.1 + 0.E(\eta) + 0.V(\eta)\right]}{\begin{pmatrix} (0+0+0+0)+\left(\eta_{11}^2+\eta_{12}^2+\eta_{12}^2+\eta_{13}^2\right)+ \\ (0+0+0+0)+\left(\eta_{21}^2+\eta_{22}^2+\eta_{22}^2+\eta_{23}^2\right) \end{pmatrix} + 1+1+1+1+0+E(\eta)^2+0+V(\eta)^2}$$
$$= \frac{2(2)}{\begin{pmatrix} \left(\eta_{11}^2+\eta_{12}^2+\eta_{12}^2+\eta_{13}^2\right)+ \\ \left(\eta_{21}^2+\eta_{22}^2+\eta_{22}^2+\eta_{23}^2\right) \end{pmatrix} + 4 + E(\eta)^2 + V(\eta)^2}$$
$$= \frac{4}{\sum_{p=1}^{4}\eta_{1p}^2 + \sum_{p=1}^{4}\eta_{2p}^2 + 4 + E(\eta)^2 + V(\eta)^2}$$

Hence, the proof.

Proposition 3.5.5 *Suppose* $\lambda = \left\langle\left([0,0,0,0];w_1\right),\left([0,0,0,0];w_2\right)\right\rangle$ *and.*

$\eta = \left\langle\left([0,0,0,0];w_1\right),\left([0,0,0,0];w_2\right)\right\rangle$ *are two GTrIFNs with same heights* w_1*(membership),* w_2*(non − membership). Then,*

$$S_{TIFN}(\lambda,\eta) = 1 \tag{23}$$

Proof: *For the above GTrIFNs* λ *and* η, *we have* $E(\lambda) = 0 = E(\eta)$; $V(\lambda) = 0 = V(\eta)$. *Hence, from our proposed SM expression in Eq.* (10), *we have,*

$$S_{TIFN}(\lambda,\eta)=\frac{2\left[\begin{pmatrix}0.0+0.0+0.0+0.0+\\0.0+0.0+0.0+0.0\end{pmatrix}+w_1.w_1+w_2.w_2+0.0+0.0\right]}{\begin{pmatrix}(0+0+0+0)+(0+0+0+0)+\\(0+0+0+0)+(0+0+0+0)\end{pmatrix}+w_1^2+w_1^2+w_2^2+w_2^2+0+0+0+0}$$
$$=\frac{2\left(w_1^2+w_2^2\right)}{w_1^2+w_2^2+w_1^2+w_2^2}$$
$$=\frac{2\left(w_1^2+w_2^2\right)}{2\left(w_1^2+w_2^2\right)}$$
$$=1$$

Hence, the proof.

Remark 2: In particular, if $\lambda=\langle([0,0,0,0];1),([0,0,0,0];1)\rangle$ and $\eta=\langle([0,0,0,0];1),([0,0,0,0];1)\rangle$, then clearly, $S_{TIFN}(\lambda,\eta)=1$.

Proposition 3.5.6 *For two GTrIFNs* $\lambda=\langle([1,1,1,1];1),([1,1,1,1];1)\rangle$ *and* $\eta=\langle([0,0,0,0];0),([0,0,0,0];0)\rangle$, *the following equalities hold*:

(i) $S_{TIFN}(\lambda,\eta)=0$
(ii) $S_{TIFN}(\lambda,\lambda^c)=0$ *and* $S_{TIFN}(\eta,\eta^c)=0$
(iii) $S_{TIFN}(\lambda,\eta)=S_{TIFN}(\lambda^c,\eta^c)$

Proof: *Given that,* $\lambda=\langle([1,1,1,1];1),([1,1,1,1];1)\rangle$ *and* $\eta=\langle([0,0,0,0];0),([0,0,0,0];0)\rangle$.*So, we have their respective complements as,* $\lambda^c=\langle([0,0,0,0];0),([0,0,0,0];0)\rangle$ *and* $\eta^c=\langle([1,1,1,1];1),([1,1,1,1];1)\rangle$.

(i) *We can observe that,*

$$\lambda\cap\eta=\langle([\min(1,0),\min(1,0),\min(1,0),\min(1,0)];\min(1,0)),$$
$$([\min(1,0),\min(1,0),\min(1,0),\min(1,0)];\min(1,0))\rangle$$
$$=\phi$$

Clearly, $S_{TIFN}(\lambda,\eta)=0$.

(ii) $S_{TIFN}(\lambda,\lambda^c)=S_{TIFN}(\langle([1,1,1,1];1),([1,1,1,1];1)\rangle,\langle([0,0,0,0];0),([0,0,0,0];0)\rangle)=0$

Similarly, we can prove that, $S_{TIFN}(\eta,\eta^c)=0$.

(iii)
$$\begin{aligned}S_{TIFN}(\lambda^c,\eta^c)&=S_{TIFN}(\langle([0,0,0,0];0),([0,0,0,0];0)\rangle,\langle([1,1,1,1];1),([1,1,1,1];1)\rangle)\\&=S_{TIFN}(\langle([1,1,1,1];1),([1,1,1,1];1)\rangle,\langle([0,0,0,0];0),([0,0,0,0];0)\rangle)\\&=S_{TIFN}(\lambda,\eta)\end{aligned}$$

Proposition 3.5.7 *For two GTrIFNs of the form* $\lambda=\langle([-a,-a,-a,-a];1),([-a,-a,-a,-a];1)\rangle$ *and.*

$\eta=\langle([a,a,a,a];1),([a,a,a,a];1)\rangle$ *and* $a\in\mathbb{R}$, *we obtain* $S_{TIFN}(\lambda,\eta)=\frac{2a^4-6a^2+1}{2a^4+6a^2+1}$.

Proof: *For the given GTrIFNs, we obtain* $E(\lambda)=-2a$, $E(\eta)=2a$, $V(\lambda)=-2a^2$, $V(\eta)=-2a^2$.

$$S_{TIFN}(\lambda,\eta) = \frac{2[-8a^2+2-4a^2+4a^4]}{8a^2+8a^2+4+8a^2+8a^4} = \frac{2[-12a^2+2+4a^4]}{24a^2+4+8a^4} = \frac{4}{4}\frac{-6a^2+1+2a^4}{6a^2+1+2a^4} = \frac{2a^4-6a^2+1}{2a^4+6a^2+1}.$$

Proposition 3.5.8 *For two GTrIFNs of the form* $\lambda = \langle([0,0,0,0];w),([0,0,0,0];w)\rangle$ *and.*

$\eta = \langle([1,1,1,1];w),([1,1,1,1];w)\rangle$, *we obtain* $S_{TIFN}(\lambda,\eta) = \frac{w^2}{4w^4-4w^2+3w^2+2}$.

Proof: *For the given GTrIFNs, we obtain* $E(\lambda)=0$, $E(\eta)=2w$, $V(\lambda)=0$, $V(\eta)=2w-4w^2$. *Then,*

$$S_{TIFN}(\lambda,\eta) = \frac{2[0+2w^2+0+0]}{0+8+4w^2+4w^2+\left(2w-4w^2\right)^2} = \frac{4w^2}{8+8w^2+4\left(w-2w^2\right)^2} = \frac{w^2}{2+2w^2+\left(w-2w^2\right)^2}$$
$$= \frac{w^2}{2+2w^2+w^2+4w^4-4w^3}$$
$$= \frac{w^2}{4w^4-4w^3+3w^2+2}$$

Proposition 3.5.9 *For two GTrIFNs of the form* $\lambda = \left\langle\left(\left[\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}\right];w\right),\left(\left[\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}\right];w\right)\right\rangle$ *and.*

$\eta = \left\langle\left(\left[-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}\right];w\right),\left(\left[-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}\right];w\right)\right\rangle$, *then from our SM we obtain.*

$S_{TIFN}(\lambda,\eta) = \frac{\left(w-2w^2\right)^2+4w^2-8}{\left(w-2w^2\right)^2+12w^2+8}$.

Proof: *For the given GTrIFNs, we obtain* $E(\lambda)=w$, $E(\eta)=-w$, $V(\lambda)=\frac{w}{2}-w^2$, $V(\eta)=\frac{w}{2}-w^2$.

$$S_{TIFN}(\lambda,\eta) = \frac{2\left[-2+2w^2-w^2+\left(\frac{w}{2}-w^2\right)\left(\frac{w}{2}-w^2\right)\right]}{4+4w^2+2w^2+\left(\frac{w}{2}-w^2\right)^2+\left(\frac{w}{2}-w^2\right)^2} = \frac{2\left[-2+w^2+\left(\frac{w-2w^2}{2}\right)\left(\frac{w-2w^2}{2}\right)\right]}{4+6w^2+\frac{\left(w-2w^2\right)^2}{4}+\frac{\left(w-2w^2\right)^2}{4}}$$
$$= \frac{2\left[-8+4w^2+\left(w-2w^2\right)^2\right]}{16+24w^2+2(w-2w^2)^2}$$
$$= \frac{\left(w-2w^2\right)^2+4w^2-8}{(w-2w^2)^2+12w^2+8}$$

Hence the proof.

4 Comparative analysis

In this section, we present a comparative analysis of our present proposed approach with the existing SM approaches considering ten different profiles of GTrIFNs, to reveal the efficiency of our approach over the others. Now, in the literature, we come across few SM approaches on GTrIFNs which are listed down below.

Here, we have presented the existing formulas according to our definitions of GTrIFNs for enhanced understanding, i.e., let $\lambda = \left\langle\left([\lambda_{11},\lambda_{12},\lambda_{13},\lambda_{14}];w_1\right),\left([\lambda_{21},\lambda_{22},\lambda_{23},\lambda_{24}];w_2\right)\right\rangle$ and.

$\eta = \langle ([\eta_{11}, \eta_{12}, \eta_{13}, \eta_{14}]; v_1), ([\eta_{21}, \eta_{22}, \eta_{23}, \eta_{24}]; v_2) \rangle$ are two non-zero GTrIFNs,where, $\lambda_{21} \leq \lambda_{11} \leq \lambda_{22} \leq \lambda_{12} \leq \lambda_{13} \leq \lambda_{23} \leq \lambda_{14} \leq \lambda_{24}, 0 \leq w_1, w_2 \leq 1$ and. $\eta_{21} \leq \eta_{11} \leq \eta_{22} \leq \eta_{12} \leq \eta_{13} \leq \eta_{23} \leq \eta_{14} \leq \eta_{24}, 0 \leq v_1, v_2 \leq 1$.

4.1 SM approach (Chen and Chen 2009)

Chen and Chen in 2009, defined the SM between two GTrIFNs λ and η as,

$$S_{CCH}(\lambda, \eta) = \left(1 - \frac{1}{4}\sum_{p=1}^{4}(\lambda_{2p} - \eta_{2p})\right) \times \left(1 - \frac{1}{4}\sum_{p=1}^{4}\left|(\lambda_{2p} - \lambda_{1p}) - (\eta_{2p} - \eta_{1p})\right|\right) \times \frac{(1 - |w_2 - v_2|) \times (1 - |w_1 - v_1|)}{1 + \left|\sqrt{\frac{1}{4}\sum_{p=1}^{4}\left(\lambda_{2p} - \frac{1}{4}\sum_{p=1}^{4}\lambda_{2p}\right)^2} - \sqrt{\frac{1}{4}\sum_{p=1}^{4}\left(\eta_{2p} - \frac{1}{4}\sum_{p=1}^{4}\eta_{2p}\right)^2}\right|}$$

4.2 SM approach (Wei and Chen 2009)

Wei and Chen presented their version of SM as,

$$S_{WC}(\lambda, \eta) = \left(1 - \frac{1}{4}\left(\sum_{p=1}^{4}\left|\lambda_{1p} - \eta_{1p}\right| + \sum_{p=1}^{4}\left|\lambda_{2p} - \eta_{2p}\right|\right)\right) \times \frac{[\min(P(\lambda), P(\eta)) + \min(w_i, v_i)]}{[\max(P(\lambda), P(\eta)) + \max(w_i, v_i)]},$$

where,

$$P(\lambda) = \sqrt{(\lambda_{11} - \lambda_{12})^2 + w_1^2} + \sqrt{(\lambda_{13} - \lambda_{14})^2 + w_1^2} + \sqrt{(\lambda_{21} - \lambda_{22})^2 + w_2^2} + \sqrt{(\lambda_{23} - \lambda_{24})^2 + w_2^2} + (\lambda_{13} - \lambda_{12}) + (\lambda_{14} - \lambda_{11}) + (\lambda_{23} - \lambda_{22}) + (\lambda_{24} - \lambda_{21})$$

$$P(\eta) = \sqrt{(\eta_{11} - \eta_{12})^2 + v_1^2} + \sqrt{(\eta_{13} - \eta_{14})^2 + v_1^2} + \sqrt{(\eta_{21} - \eta_{22})^2 + v_2^2} + \sqrt{(\eta_{23} - \eta_{24})^2 + v_2^2} + (\eta_{13} - \eta_{12}) + (\eta_{14} - \eta_{11}) + (\eta_{23} - \eta_{22}) + (\eta_{24} - \eta_{21})$$

4.3 SM approach (Chen 2011)

Similarly, Chen in 2011 defined the SM to be,

$$S_{CH}(\lambda, \eta) = \frac{1}{2}\left(1 - \sqrt{\frac{1}{4}\sum_{p=1}^{4}(\lambda_{2p} - \eta_{2p})^2}\right) \times \frac{1 - \max(w_2, v_2)}{1 - \min(w_2, v_2)} \times \left(1 + \left(1 - \sqrt{\tilde{s}(\lambda, \eta)}\right) \times (1 - |w_1 - v_1|)\right),$$ where,

$$\tilde{s}(\lambda, \eta) = \frac{1}{4}\left((\lambda_{11} - \lambda_{21} - \eta_{11} + \eta_{21})^2 + (\lambda_{12} - \lambda_{22} - \eta_{12} + \eta_{22})^2 + (\lambda_{23} - \lambda_{13} - \eta_{23} + \eta_{13})^2 + (\lambda_{24} - \lambda_{14} - \eta_{24} + \eta_{14})^2\right)$$

4.4 (SM approach (Farhadinia 2012)

According to Farhadinia, the SM has the following definition,

$$S_{FB}(\lambda,\eta)=\left(1-\frac{1}{4}\left(\sum_{p=1}^{4}\left|\lambda_{1p}-\eta_{1p}\right|+\sum_{p=1}^{4}\left|\lambda_{2p}-\eta_{2p}\right|\right)\right)\times\frac{[\min\left(P_e(\lambda),P_e(\eta)\right)+\min\left(w_i,v_i\right)]}{[\max\left(P_e(\lambda),P_e(\eta)\right)+\max\left(w_i,v_i\right)]},$$

where,

$$P_e(\lambda)=e^{\left[\sqrt{(\lambda_{11}-\lambda_{12})^2+w_1^2}+\sqrt{(\lambda_{13}-\lambda_{14})^2+w_1^2}+\sqrt{(\lambda_{21}-\lambda_{22})^2+w_2^2}+\sqrt{(\lambda_{23}-\lambda_{24})^2+w_2^2}+(\lambda_{13}-\lambda_{12})+(\lambda_{14}-\lambda_{11})+(\lambda_{23}-\lambda_{22})+(\lambda_{24}-\lambda_{21})\right]}$$

$$P_e(\eta)=e^{\left[\sqrt{(\eta_{11}-\eta_{12})^2+v_1^2}+\sqrt{(\eta_{13}-\eta_{14})^2+v_1^2}+\sqrt{(\eta_{21}-\eta_{22})^2+v_2^2}+\sqrt{(\eta_{23}-\eta_{24})^2+v_2^2}+(\eta_{13}-\eta_{12})+(\eta_{14}-\eta_{11})+(\eta_{23}-\eta_{22})+(\eta_{24}-\eta_{21})\right]}$$

4.5 Cosine SM-based approach (Ye 2012a)

Ye in 2012 defined the cosine SM between two GTrIFNs λ and η as,

$$S_{YC}(\lambda,\eta)=\frac{\sum_{p=1}^{4}\lambda_{1p}\eta_{1p}+\sum_{p=1}^{4}\lambda_{2p}\eta_{2p}+\sum_{i=1}^{2}w_iv_i}{\sqrt{\sum_{p=1}^{4}\lambda_{1p}^2+\sum_{p=1}^{4}\lambda_{2p}^2+\sum_{i=1}^{2}w_i^2}\sqrt{\sum_{p=1}^{4}\eta_{1p}^2+\sum_{p=1}^{4}\eta_{2p}^2+\sum_{i=1}^{2}v_i^2}}$$

4.6 Distance-Based SM Approach (Ye 2012b)

Ye in 2012, also defined the Hamming and Euclidean distance-based SMs between two GTrIFNs λ and η, which are given below,

$$S_{YH}(\lambda,\eta)=1-\frac{1}{12}\left(\sum_{p=1}^{4}\left|\lambda_{1p}-\eta_{1p}\right|+\sum_{p=1}^{4}\left|\lambda_{2p}-\eta_{2p}\right|+\sum_{i=1}^{2}\left|w_i-v_i\right|\right)$$

and, $$S_{YE}(\lambda,\eta)=1-\sqrt{\frac{1}{12}\left(\sum_{p=1}^{4}\left(\lambda_{1p}-\eta_{1p}\right)^2+\sum_{p=1}^{4}\left(\lambda_{2p}-\eta_{2p}\right)^2+\sum_{i=1}^{2}\left(w_i-v_i\right)^2\right)}.$$

4.7 SM approach (Tang et al. 2017)

Tang et al., presented their version of SM as,

$$S_T(\lambda,\eta)=\frac{2\left(\sum_{p=1}^{4}\lambda_{1p}\eta_{1p}+\sum_{p=1}^{4}\lambda_{2p}\eta_{2p}+\sum_{i=1}^{2}w_iv_i\right)}{\left(\sum_{p=1}^{4}\lambda_{1p}^2+\sum_{p=1}^{4}\lambda_{2p}^2+\sum_{i=1}^{2}w_i^2\right)+\left(\sum_{p=1}^{4}\eta_{1p}^2+\sum_{p=1}^{4}\eta_{2p}^2+\sum_{i=1}^{2}v_i^2\right)}$$

4.8 SM approach (Yue et al. 2019)

Yue et al., defined the SM between two GTrIFNs λ and η as,
$S_{QY}(\lambda,\eta) = \frac{S_{QY1}(\lambda,\eta)+S_{QY2}(\lambda,\eta)}{2}$, where,

$$S_{QY1}(\lambda,\eta) = 2^{-\frac{|\lambda_{11}-\eta_{11}+\lambda_{12}-\eta_{12}+\lambda_{13}-\eta_{13}+\lambda_{14}-\eta_{14}|}{4}} \times \frac{\min\left(Ar(\lambda'),Ar(\eta')\right)}{\max\left(Ar(\lambda'),Ar(\eta')\right)} \times \frac{\min\left(Per(\lambda'),Per(\eta')\right)+\min\left(w_1,v_1\right)}{\max\left(Per(\lambda'),Per(\eta')\right)+\max\left(w_1,v_1\right)}$$

$$S_{QY2}(\lambda,\eta) = 2^{-\frac{|\lambda_{21}-\eta_{21}+\lambda_{22}-\eta_{22}+\lambda_{23}-\eta_{23}+\lambda_{24}-\eta_{24}|}{4}} \times \frac{\min\left(Ar(\lambda''),Ar(\eta'')\right)}{\max\left(Ar(\lambda''),Ar(\eta'')\right)} \times \frac{\min\left(Per(\lambda''),Per(\eta'')\right)+\min\left(w_2,v_2\right)}{\max\left(Per(\lambda''),Per(\eta'')\right)+\max\left(w_2,v_2\right)}$$

and,

$$Ar(\lambda') = \frac{1}{2}(\lambda_{14}-\lambda_{11})w_1\,;\ Ar(\eta') = \frac{1}{2}(\eta_{14}-\eta_{11})v_1\,;\ Ar(\lambda'') = \frac{1}{2}(\lambda_{24}-\lambda_{21})w_2\,;\ Ar(\eta'') = \frac{1}{2}(\eta_{24}-\eta_{21})v_2$$

$$Per(\lambda') = \sqrt{(\lambda_{12}-\lambda_{11})^2+w_1^2}+\sqrt{(\lambda_{14}-\lambda_{13})^2+w_1^2}\,;\ Per(\eta') = \sqrt{(\eta_{12}-\eta_{11})^2+v_1^2}+\sqrt{(\eta_{14}-\eta_{13})^2+v_1^2}$$

$$Per(\lambda'') = \sqrt{(\lambda_{22}-\lambda_{21})^2+w_2^2}+\sqrt{(\lambda_{24}-\lambda_{23})^2+w_2^2}\,;\ Per(\eta'') = \sqrt{(\eta_{22}-\eta_{21})^2+v_2^2}+\sqrt{(\eta_{24}-\eta_{23})^2+v_2^2}$$

4.9 SM approach (Dinagar and Helena 2019)

Dinagar and Helena presented their version of SM between two GTrIFNs λ and η as follows,

$$S_{DH}(\lambda,\eta) = [|1-(R(\lambda)-R(\eta))|]\times\left[1-\left|\min\left(w_1,v_1\right)-\max\left(w_2,v_2\right)\right|\right]\times\frac{\min\left(S(\lambda'),S(\eta')\right)+\min\left(S(\lambda''),S(\eta'')\right)}{\max\left(S(\lambda'),S(\eta')\right)+\max\left(S(\lambda''),S(\eta'')\right)}$$

where, $R(\lambda) = \frac{w_1.S(\lambda')+w_2.S(\lambda'')}{w_1+w_2}$; $R(\eta) = \frac{v_1.S(\eta')+v_2.S(\eta'')}{v_1+v_2}$, and

$$S(\lambda') = \left(\frac{2\lambda_{11}+7\lambda_{12}+7\lambda_{13}+2\lambda_{14}}{18}\right)\left(\frac{7w_1}{18}\right)\ ;\ S(\lambda'') = \left(\frac{2\lambda_{21}+7\lambda_{22}+7\lambda_{23}+2\lambda_{24}}{18}\right)\left(\frac{11+7w_2}{18}\right)$$

Table 1 Profiles of GTrIFNs

Ser. No	Profiles	λ	η
1	Profile 1	⟨([4, 6, 8, 9];0.3), ([3, 5, 8, 9];0.4)⟩	⟨([3, 5, 7, 9];0.3), ([2, 4, 7, 9];0.4)⟩
2	Profile 2	⟨([4, 6, 6, 8];0.2), ([3, 5, 7, 9];0.6)⟩	⟨([4, 6, 6, 8];0.2), ([3, 5, 7, 9];0.6)⟩
3	Profile 3	⟨([4, 6, 6, 8];0.3), ([3, 5, 7, 9];0.5)⟩	⟨([3, 4, 5, 7];0.5), ([2, 4, 6, 8];0.2)⟩
4	Profile 4	⟨([4, 6, 8, 9];0.1), ([3, 5, 8, 10];0.6)⟩	⟨([5, 9, 9, 11];0.4), ([3, 7, 11, 13];0.3)⟩
5	Profile 5	⟨([2, 4, 6, 8];0), ([1, 3, 6, 8];1)⟩	⟨([4, 6, 8, 9];1), ([3, 5, 8, 9];0)⟩
6	Profile 6	⟨([2, 4, 6, 7];1), ([1, 3, 6, 8];0)⟩	⟨([4, 6, 7, 9];0), ([3, 5, 7, 9];1)⟩
7	Profile 7	⟨([4, 5, 5, 7];0), ([4, 5, 6, 7];1)⟩	⟨([3, 4, 4, 6];0), ([3, 4, 5, 6];1)⟩
8	Profile 8	⟨([2, 3, 4, 5];0.1), ([1.5, 2.5, 4.5, 5.5];0.8)⟩	⟨([2.5, 3.5, 4.5, 5.5];0.2), ([1, 2, 5, 6];0.6)⟩
9	Profile 9	⟨([2, 4, 6, 7];0.1), ([1, 3, 6, 7];0.9)⟩	⟨([2, 4, 6, 7];0.9), ([1, 3, 6, 8];0.1)⟩
10	Profile 10	⟨([2, 4, 6, 7];0.1), ([1, 3, 6, 8];0.9)⟩	⟨([4, 6, 7, 9];0.1), ([3, 5, 7, 9];0.9)⟩

Table 2 Comparative analysis of the proposed SM approach with existing methods

Ser No	Profiles	Chen and Chen (2009) (S_{CCH})	Wei and Chen (2009) (S_{WC})	Chen (2011) (S_{CH})	Farhadinia (2012) (S_{FB})	Ye (2012a) (S_{YC})	Ye (2012b) (S_{YH})
1	Profile 1	0 (#)	−0.992 (#)	0 (#)	−0.98 (#)	0.9946	0.5000
2	Profile 2	1	1	1	1	0.9988 (#)	1
3	Profile 3	0 (#)	−1.205 (#)	0 (#)	−1.14 (#)	0.9948	0.2083
4	Profile 4	−0.1711 (#)	−1.966 (#)	−0.2348 (#)	−0.0010 (#)	0.9954	−0.3000 (#)
5	Profile 5	0 (#)	−1.9888 (#)	0 (#)	−0.0547 (#)	0.9827	−0.3333 (#)
6	Profile 6	0 (#)	−1.8973 (#)	0 (#)	−0.2548 (#)	0.9826	−0.2500 (#)
7	Profile 7	0 (#)	−1 (#)	N/A	−1 (#)	0.9939	0.3333
8	Profile 8	0.2432	0 (#)	0.1455	0 (#)	0.9916	0.6417
9	Profile 9	0.0287 (#)	0.6476	0.0083 (#)	0 (#)	0.9943	0.7833
10	Profile 10	1.2873 (#)	−2.0726 (#)	−0.4359 (#)	−0.3037 (#)	0.9833	−0.0833 (#)
Ser No	Profiles	Ye (2012b) (S_{YE})	Tang et al. (2017) (S_T)	Yue et al. (2019) (S_{QY})	Dinagar and Helena (2019) (S_{DH})	Proposed approach (S_{TIFN})	
1	Profile 1	0.2929	0.9906	0.4719	0.4394	0.9842	
2	Profile 2	1	0.9988 (#)	1	0.6000 (#)	1	
3	Profile 3	0.0369 (#)	0.9783	0.3515	0.3812	0.9766	
4	Profile 4	−0.7602 (#)	0.9642	0.1507	0.0835 (#)	0.9475	
5	Profile 5	−0.5275 (#)	0.9539	0.1221	0 (#)	0.9395	
6	Profile 6	−0.4434 (#)	0.9556	0.1588	0 (#)	0.9575	
7	Profile 7	0.1835	0.9754	0.5000	0 (#)	0.9804	
8	Profile 8	0.5867	0.9878	0.7613	0.1075	0.9830	
9	Profile 9	0.5641	0.9937	0.4715	0.1033	0.9906	
10	Profile 10	−0.3844 (#)	0.9562	0.2819	0.3456	0.9562	

Here, (N/A)- means not applicable, (#)- denotes absurd result, or which cannot be determined by the approach.

$$S(\eta') = \left(\frac{2\eta_{11}+7\eta_{12}+7\eta_{13}+2\eta_{14}}{18}\right)\left(\frac{7v_1}{18}\right)\ ;\ S(\eta'') = \left(\frac{2\eta_{21}+7\eta_{22}+7\eta_{23}+2\eta_{24}}{18}\right)\left(\frac{11+7v_2}{18}\right).$$

4.10 Inferences from the analysis

We now consider ten different profiles of GTrIFNs, which are shown in Tables 1 and 2 presents the SMs which are evaluated for the corresponding profiles of fuzzy numbers. The graphical representations for the considered profiles are depicted in Figs 16–25 (See Appendix B).

The highlights of the present SM approach with other approaches are listed as under,

- **With** (Chen and Chen 2009) **approach**:
  As evident from Table 2, the SM for profile 2 should be one since two identical GTrIFNs are considered. Consequently, the unit similarity value is obtained with both Chen & Chen's approach and our proposed SM approach, which is logical. For profiles 1, 3, 5, 6, and 7, Chen & Chen's approach found total dissimilarity between the pairs of GTrIFNs which were considered. However, by observing the profiles it is clear that a considerable amount of similarity should exist for them rather than being completely dissimilar. In this context, our present approach obtained the similarity values as 0.9842, 0.9766, 0.9395, 0.9575, and 0.9804 respectively for the above-mentioned profiles. Further, Chen & Chen's approach obtained a negative similarity value (for profile 4), an extremely low similarity value (for profile 9), and an exceedingly high similarity value (for profile 10), which indicated a major setback of the approach. On the other hand, our approach obtained logical similarity values of 0.9475 (for profile 4), 0.9906 (for profile 9), and 0.9562 (for profile 10), respectively.
- **With** (Wei and Chen 2009) **approach**:
  With Wei and Chen's approach, negative and unacceptable values of similarity are obtained for a bunch of profiles, which are- 1, 3, 4, 5, 6, 7, and 10. The said method also evaluated 0 (zero) as similarity value for profile 8, which is illogical as we can notice from Table 1, that pairs of GTrIFNs considered in profile 8 are almost similar. Only for profiles 2 and 9, Wei & Chen's approach is capable of obtaining acceptable similarity values. On the other hand, from Table 1, the superiority of our newly constructed approach is tangible.
- **With** (Chen 2011) **approach**:
  With characteristics almost similar to the (Chen and Chen 2009) approach, Chen's (2011) approach determined the similarity value as 0 (zero) for profiles 1, 3, 5, and 6. Also, for profiles 4 and 10, the method obtained negative similarity values; for profile 7, it could not be applied; and for profile 9, it obtained a sufficiently low value of similarity. In short, there are several instances where the inefficacy of Chen's approach is demonstrated. Therefore, our present approach without these limitations outperforms this scenario as well.
- **With** (Farhadinia 2012) **approach**:
  With the approach by Farhadinia, the only instance when a legitimate value of similarity is obtained is for profile 2, where two identical GTrIFNs are considered and it evaluated the similarity value as 1 (one). Other than that, for the rest of the profiles, the evaluated values of similarity with Farhadinia's approach are totally absurd. For profiles 1, 3, 4, 5, 6, 7, 8, 9, and 10, it evaluated the similarity values as -0.98, -1.14, -0.0010, -0.0547, -0.2548, -1, 0, 0, and -0.3037, respectively. Whereas with our pro-

posed SM approach, the corresponding values of similarity are 0.9842, 0.9766, 0.9475, 0.9395, 0.9575, 0.9804, 0.9830, 0.9906, and 0.9562.

- **With** (Ye 2012a) **cosine similarity-based approach**:

  With Ye's cosine similarity-based approach, quite logical values of similarity are obtained for the majority of the profiles considered. However, there still exists a significant setback to this approach. We know that, for any newly developed SM, it should obtain unit similarity value for two identical quantities/objects considered and which is a fundamental property that it should satisfy. However, Ye's approach did not obtain a unit similarity value for profile 2, where two identical GTrIFNs are considered, rather it calculated the similarity value to be 0.9988 which is close to 1 (one), but not exactly 1. Hence, our proposed approach holds an upper hand in this case as well.
- **With** (Ye 2012b) **Hamming distance-based SM approach**:

  With Ye's Hamming distance-based similarity approach, no noticeable discrepancy is observed in the evaluated similarity values for profiles 1, 3, 7, 8, and 9. However, it fails in obtaining positive and logical values of similarity for profiles 4, 5, 6, and 10, which is evident from Table 2. On the contrary, our present approach calculates the respective similarity values as 0.9475 (profile 4), 0.9395 (profile 5), 0.9575 (profile 6), and 0.9562 (profile 10), which are in tone with common human intuition.
- **With** (Ye 2012b) **Euclidean distance-based SM approach**:

  With traits almost similar to Ye's Hamming distance-based SM, even in this case too, negative values of similarity are obtained for profiles 4, 5, 6, and 10. Hereby, from Table 2, one can notice the efficacy of our proposed approach in obtaining rational similarity values. Moreover, Ye's Euclidean distance-based approachevaluated the similarity value as 0.0369 for profile 3, which is significantly low. But we can observe the resemblance between the pairs of GTrIFNs considered in profile 3, and as such, our proposed approach is once again successful in proving its proficiency.
- **With** (Tang et al. 2017) **approach**:

  Almost similar to Ye's cosine similarity-based approach, with Tang et al.'s approach as well, we do not obtain the similarity value as 1 (one) for profile 2, which is counter-intuitive as we have already explained. However, the similarity values obtained for the remaining profiles are reasonable.
- **With** (Yue et al. 2019) **approach**:

  Yue et al.'s approach is the only existing approach encountered so far, where no such unavoidable discrepancy is observed in the values of similarity which are determined for the profiles. Here, the obtained similarity values could have been more distinctive, but they are more or less acceptable. However, in this case, the computational time involved in calculating the results of similarity is very high due to the complexity of the expressions involved in their SM. Whereas, our approach involved considerably less amount of time.
- **With** (Dinagar and Helena 2019) **approach**:

  Dinagar and Helena SM approach also failed on many occasions. For instance, the measure obtained a similarity result of 0.6000 for profile 2, which is illogical as it should have been 1 (one). For profiles 5, 6, and 7, it obtained the similarity value as 0 (zero), meaning a total dissimilarity between the GTrIFNs considered in those profiles. But it is again not true, since, from Table 1, we can see that the fuzzy numbers considered in those profiles must have some positive similarity value. However, our presented approach overcomes those flaws and obtains more logical and more legitimate values of similarity for each of the profiles considered.

Therefore, our proposed approach outperforms all the existing SM methods on a number of occasions, thus proving its efficacy, feasibility, and rationality. Hence, our newly constructed approach is worthy of due consideration.

5 FMCGDM based on the proposed approach

In this section, we provide a method for handling FMCGDM problems. Thus, the novelty and applicability of the newly proposed SM approach are illustrated in detail in this section.

The process of selecting the best alternative or calculating a preference ordering of alternatives, amongst a set of alternatives governed by some set of predefined multicriteria, is referred to as MCDM. Generally, two kinds of criteria(s) are considered in an MCDM problem, one is theb enefit/profit criteria which are to be maximized and the other is the cost criteria, which is to be minimized. Cost criteria(s) also can be converted into profit ones (Zizovic et al. (Zizovic and DamljanovicN 2017)). Now a solution that minimizes all the cost criteria and maximizes all the profit criteria is called an idealone(Xu & Yang (Xu and Yang 2001)).

Now, let $A = \{A_1, A_2, ..., A_p\}$ be a set of alternatives and let $C = \{C_1, C_2, ..., C_q\}$ be a set of predefined criteria(s) respectively. Let k experts are invited to make the judgment and we consider the criteria(s) to be of equal weightage. Let us assume that the experts give the values of the alternatives in the form of a judgment matrix, $J_{ij} = [d_{ij}(k)]_{p\times q}$, using GTrIFNs as,

$d_{ij}(k) = \left\langle \left([a_{ij1}(k), a_{ij2}(k), a_{ij3}(k), a_{ij4}(k)]; w_{ij}(k) \right), \left([b_{ij1}(k), b_{ij2}(k), b_{ij3}(k), b_{ij4}(k)]; v_{ij}(k) \right) \right\rangle$,
where.

$a_{ijm}, b_{ijm} \in \mathbb{R}\,(m = 1, 2, 3, 4); 0 \le b_{ij1} \le a_{ij1} \le b_{ij2} \le a_{ij2} \le a_{ij3} \le b_{ij3} \le a_{ij4} \le b_{ij4} \le 1; 0 < w_{ij}(k), v_{ij}(k) \le 1$.

The values of the entries in the judgment matrix indicate the degree that an alternative A_i satisfies the criterion C_j or not. Thus, the procedure for selecting the best suitable alternative or preference ordering of alternatives is presented as follows.

Step-1: Now, we construct the normalized judgment matrix, $\overline{J}_{ij} = [t_{ij}]_{p\times q}$, based on the values given by the experts as follows-

$$t_{ij} = \left\langle \left(\left[\frac{\sum_{s=1}^{k} a_{ij1}(s)}{k}, \frac{\sum_{s=1}^{k} a_{ij2}(s)}{k}, \frac{\sum_{s=1}^{k} a_{ij3}(s)}{k}, \frac{\sum_{s=1}^{k} a_{ij4}(s)}{k} \right]; \frac{\sum_{s=1}^{k} w_{ij}(s)}{k} \right), \left(\left[\frac{\sum_{s=1}^{k} b_{ij1}(s)}{k}, \frac{\sum_{s=1}^{k} b_{ij2}(s)}{k}, \frac{\sum_{s=1}^{k} b_{ij3}(s)}{k}, \frac{\sum_{s=1}^{k} b_{ij4}(s)}{k} \right]; \frac{\sum_{s=1}^{k} v_{ij}(s)}{k} \right) \right\rangle$$

Thus, we can elicit the decision matrix, $D = [t_{ij}]_{p\times q}$, which is represented by GTrIFNs.

Step-2: For determining the best suitable alternative, SM is to be evaluated between the available alternatives with an ideal alternative.But, in the real world, although it is hard to realize an ideal alternative, we construct it in order to provide a useful theoretical base for

Table 3 Judgment matrix, $J_{ij} = [d_{ij}]_{12\times3}$

	k	C_1	C_2	C_3
A_1	1	$\langle([0.3,0.5,0.5,0.7];0.2),([0.2,0.4,0.6,0.8];0.6)\rangle$	$\langle([0.2,0.4,0.6,0.8];0.2),([0.1,0.3,0.6,0.9];0.5)\rangle$	$\langle([0.25,0.35,0.45,0.55];0.2),([0.1,0.2,0.5,0.6];0.6)\rangle$
	2	$\langle([0.2,0.3,0.4,0.6];0.4),([0.1,0.3,0.5,0.7];0.3)\rangle$	$\langle([0.2,0.4,0.6,0.8];0.3),([0.1,0.3,0.6,0.8];0.6)\rangle$	$\langle([0.2,0.3,0.4,0.5];0.4),([0.15,0.25,0.45,0.55];0.3)\rangle$
	3	$\langle([0.3,0.5,0.6,0.8];0.1),([0.2,0.4,0.6,0.9];0.6)\rangle$	$\langle([0.2,0.3,0.3,0.5];0.1),([0.2,0.3,0.4,0.5];0.9)\rangle$	$\langle([0.3,0.5,0.6,0.8];0.6),([0.2,0.4,0.7,0.9];0.4)\rangle$
A_2	1	$\langle([0.4,0.5,0.5,0.7];0.2),([0.4,0.5,0.6,0.7];0.7)\rangle$	$\langle([0.3,0.5,0.7,0.8];0.2),([0.2,0.4,0.7,0.9];0.3)\rangle$	$\langle([0.6,0.7,0.7,0.9];0.1),([0.6,0.7,0.8,0.9];0.9)\rangle$
	2	$\langle([0.3,0.4,0.4,0.6];0.4),([0.3,0.4,0.5,0.6];0.3)\rangle$	$\langle([0.3,0.5,0.7,0.8];0.4),([0.2,0.4,0.7,0.8];0.6)\rangle$	$\langle([0.5,0.6,0.6,0.8];0.3),([0.5,0.6,0.7,0.8];0.7)\rangle$
	3	$\langle([0.2,0.3,0.4,0.5];0.5),([0.15,0.25,0.45,0.55];0.5)\rangle$	$\langle([0.4,0.5,0.5,0.7];0.7),([0.4,0.5,0.6,0.7];0.1)\rangle$	$\langle([0.4,0.5,0.6,0.7];0.4),([0.35,0.45,0.65,0.75];0.6)\rangle$
A_3	1	$\langle([0.3,0.5,0.5,0.7];0),([0.2,0.4,0.6,0.8];1)\rangle$	$\langle([0.5,0.7,0.8,1.0];0.9),([0.4,0.6,0.8,1.0];0.1)\rangle$	$\langle([0.35,0.45,0.55,0.65];0.9),([0.2,0.3,0.6,0.7];0.1)\rangle$
	2	$\langle([0.2,0.3,0.4,0.6];1),([0.1,0.3,0.5,0.7];0)\rangle$	$\langle([0.3,0.5,0.7,0.8];0.1),([0.2,0.4,0.7,0.9];0.7)\rangle$	$\langle([0.3,0.5,0.7,0.8];0.8),([0.2,0.4,0.7,0.8];0.2)\rangle$
	3	$\langle([0.3,0.4,0.5,0.6];0.4),([0.25,0.35,0.55,0.65];0.4)\rangle$	$\langle([0.5,0.7,0.9,1.0];0),([0.4,0.6,0.9,1.0];1)\rangle$	$\langle([0.3,0.5,0.7,0.8];0.6),([0.2,0.4,0.7,0.9];0.4)\rangle$
A_4	1	$\langle([0.5,0.7,0.9,1.0];0.6),([0.4,0.6,1.0,1.0];0.6)\rangle$	$\langle([0.6,0.8,0.9,1.0];1),([0.5,0.7,0.9,1.0];0)\rangle$	$\langle([0.7,0.9,0.9,0.9];0.1),([0.6,0.8,1.0,1.0];0.3)\rangle$
	2	$\langle([0.35,0.45,0.55,0.65];0.1),([0.2,0.3,0.6,0.7];0.8)\rangle$	$\langle([0.6,0.8,1.0,1.0];1),([0.5,0.7,1.0,1.0];0)\rangle$	$\langle([0.7,0.9,0.9,0.9];0.6),([0.6,0.8,1.0,1.0];0.1)\rangle$
	3	$\langle([0.3,0.5,0.6,0.9];0.9),([0.2,0.4,0.6,1.0];0.1)\rangle$	$\langle([0.4,0.6,0.8,0.9];1),([0.2,0.5,0.8,0.9];0)\rangle$	$\langle([0.5,0.7,0.9,1.0];0.3),([0.4,0.6,0.9,1.0];0.3)\rangle$

evaluating the alternatives. Therefore, we define an ideal GTrIFN for a profit criterion as, $A^* = \langle([1,1,1,1];1),([0,0,0,0];0)\rangle$ and for a cost criterion, we assume the ideal alternative to be as, $A^* = \langle([0,0,0,0];0),([1,1,1,1];1)\rangle$.

Step-3: Then, consequently we obtain the SMexpression between the set of alternatives A_i and the ideal alternative, A^* using the following technique.

$$S_{TIFN}(A_i, A^*) = \sum_{j=1}^{x} \frac{2\left[a_{ij1} + a_{ij2} + a_{ij3} + a_{ij4} + w_{ij} + E(A_i)\right]}{\begin{pmatrix} a_{ij1}^2 + a_{ij2}^2 + a_{ij2}^2 + a_{ij3}^2 + \\ b_{ij1}^2 + b_{ij2}^2 + b_{ij3}^2 + b_{ij4}^2 \end{pmatrix} + 6 + w_{ij}^2 + v_{ij}^2 + E(A_i)^2 + V(A_i)^2} + \sum_{j=1}^{y} \frac{2\left[b_{ij1} + b_{ij2} + b_{ij3} + b_{ij4} + v_{ij} + E(A_i)\right]}{\begin{pmatrix} a_{ij1}^2 + a_{ij2}^2 + a_{ij2}^2 + a_{ij3}^2 + \\ b_{ij1}^2 + b_{ij2}^2 + b_{ij3}^2 + b_{ij4}^2 \end{pmatrix} + 6 + w_{ij}^2 + v_{ij}^2 + E(A_i)^2 + V(A_i)^2} \tag{24}$$

where, $x \in X$ (set of profit criteria(s), say); $y \in Y$ (set of cost criteria(s), say).

Table 4 Normalized judgment matrix, $\overline{J_{ij}} = [t_{ij}]_{4\times3}$

	C_1	C_2	C_3
A_1	$\left\langle \begin{array}{c}([0.267, 0.434, 0.5, 0.7];0.23),\\ ([0.167, 0.367, 0.567, 0.8];0.5)\end{array}\right\rangle$	$\left\langle \begin{array}{c}([0.2, 0.367, 0.5, 0.7];0.2),\\ ([0.134, 0.3, 0.534, 0.734];0.67)\end{array}\right\rangle$	$\left\langle \begin{array}{c}([0.25, 0.383, 0.483, 0.617];0.4),\\ ([0.15, 0.283, 0.55, 0.683];0.32)\end{array}\right\rangle$
A_2	$\left\langle \begin{array}{c}([0.3, 0.4, 0.434, 0.6];0.37),\\ ([0.283, 0.383, 0.517, 0.617];0.5)\end{array}\right\rangle$	$\left\langle \begin{array}{c}([0.334, 0.5, 0.634, 0.767];0.43),\\ ([0.267, 0.434, 0.667, 0.8];0.34)\end{array}\right\rangle$	$\left\langle \begin{array}{c}([0.5, 0.6, 0.634, 0.8];0.27),\\ ([0.483, 0.583, 0.717, 0.817];0.73)\end{array}\right\rangle$
A_3	$\left\langle \begin{array}{c}([0.267, 0.4, 0.467, 0.634];0.47),\\ ([0.183, 0.35, 0.55, 0.717];0.47)\end{array}\right\rangle$	$\left\langle \begin{array}{c}([0.434, 0.634, 0.8, 0.934];0.34),\\ ([0.334, 0.534, 0.8, 0.967];0.6)\end{array}\right\rangle$	$\left\langle \begin{array}{c}([0.317, 0.483, 0.65, 0.75];0.77),\\ ([0.2, 0.367, 0.667, 0.8];0.23)\end{array}\right\rangle$
A_4	$\left\langle \begin{array}{c}([0.383, 0.55, 0.683, 0.85];0.53),\\ ([0.267, 0.434, 0.734, 0.9];0.5)\end{array}\right\rangle$	$\left\langle \begin{array}{c}([0.534, 0.734, 0.9, 0.967];1),\\ ([0.4, 0.634, 0.9, 0.967];0)\end{array}\right\rangle$	$\left\langle \begin{array}{c}([0.634, 0.834, 0.9, 0.934];0.34),\\ ([0.534, 0.667, 0.967, 1.0];0.23)\end{array}\right\rangle$

The above formula helps evaluate the SM for each alternative(s) based on all criteria. These similarity values help to determine the best alternative among the set of alternatives and also helps us simultaneously obtain the preference ordering of alternatives. SM values between every alternative and the ideal alternative determine the best suitable alternative and finally, we obtain a preference ordering of alternatives.

5.1 Numerical illustration

Here, in this section, we demonstrate a realistic scenario, to show the application of our proposed SM approach in an FMCGDM problem of alternatives.

5.1.1 Optimum investment by an investment company

There is a panel with four possible alternatives to invest the money (adapted from, Herrera and Herrera-Viedma 2000): (1) A_1 is a car company; (2) A_2 is a food company; (3) A_3 is a computer company; (4) A_4 is a television company. The investment company must decide according to the following three criteria: (1) C_1 is the social benefit; (2) C_2 is the economic benefit; (3) C_3 is the environmental impact, where C_1 and C_2 are the benefit criteria and C_3 is the cost criterion. The four possible alternatives are to be evaluated under the above three criteria by corresponding to the judgment matrix as shown in Table 3.

Step-1: Suppose we invite k experts (say, $k = 3$) to make the judgment. The values of the experts in the form of GTrIFNs are presented in Table 3. Then the preference values of an alternative $A_i (i = 1, 2, 3, 4)$ on a criterion $C_j (j = 1, 2, 3)$ are normalized by the method discussed in the previous section and the normalized judgment matrix is shown by Table 4.

Table 5 Ranking order comparisons obtained with the existing similarity methods and the proposed approach

Existing similarity methods	Similarity measure value between alternatives				Ranking order	Best alternative
	(A_1, A^*)	(A_2, A^*)	(A_3, A^*)	(A_4, A^*)		
S_{CCH}Chen and Chen (2009)	−0.0012	0.0036	−0.0006	0.0037	$A_4 > A_2 > A_3 > A_1$	A_4
S_{WC}Wei and Chen (2009)	−0.0007	0.0036	−0.0008	0.0108	$A_4 > A_2 > A_1 > A_3$	A_4
S_{CH}Chen (2011)	0.0147	0.0363	0.0278	0.0590	$A_4 > A_2 > A_3 > A_1$	A_4
S_{FB}Farhadinia (2012)	−0.0011	0.0015	−0.0007	−0.0013	$A_2 > A_3 > A_1 > A_4$	A_2
S_{YC}Ye (2012a)	0.5318	0.5660	0.5398	0.5813	$A_4 > A_2 > A_3 > A_1$	A_4
S_{YH}Ye (2012b)	0.5605	0.5969	0.5603	0.6153	$A_4 > A_2 > A_1 > A_3$	A_4
S_{YE}Ye (2012b)	0.4851	0.5328	0.4804	0.4961	$A_2 > A_4 > A_1 > A_3$	A_2
S_TTang et al. (2017)	0.4909	0.5424	0.5227	0.5785	$A_4 > A_2 > A_3 > A_1$	A_4
S_{QY}Yue et al. (2019)	0.0952	0.1882	0.1319	0.2344	$A_4 > A_2 > A_3 > A_1$	A_4
S_{DH}Dinagar and Helena (2019)	0.0236	0.0688	0.0661	0.0524	$A_2 > A_3 > A_4 > A_1$	A_2
S_{TIFN}(Proposed)	0.5670	0.6806	0.6360	0.7168	$A_4 > A_2 > A_3 > A_1$	A_4

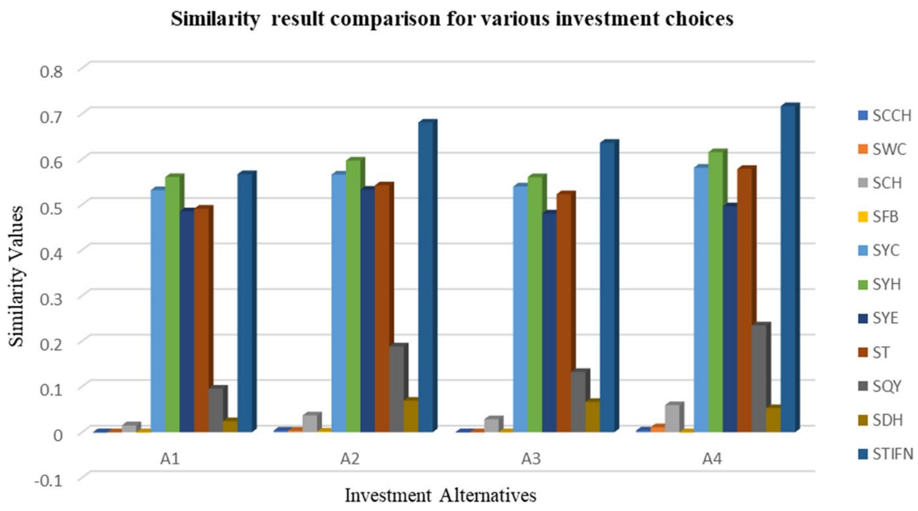


Fig. 12 Ranking comparisons of various investment alternatives under different methods

Step-2: The ideal alternative is adopted here as,
For profit criterion, $A^* = \langle([1, 1, 1, 1];1), ([0, 0, 0, 0];0)\rangle$.
For cost criterion, $A^* = \langle([0, 0, 0, 0];0), ([1, 1, 1, 1];1)\rangle$.
Step-3: We evaluate, $S_{TIFN}(A_i, A^*)$, using Eq. (7), and we obtain

$$S_{TIFN}(A_1, A^*) = 0.5670,\ S_{TIFN}(A_2, A^*) = 0.6806,\ S_{TIFN}(A_3, A^*) = 0.6360,\ S_{TIFN}(A_4, A^*) = 0.7168$$

As SM of the alternative A_4 with A^* is obtained to be maximum, so we can declare A_4 as the best suitable alternative and correspondingly, the required preference ordering is.

$$A_4 > A_2 > A_3 > A_1$$

Therefore, the maximum profitable choice that can be made by the Investment Company is to invest the money in the Television Company (A_4).

5.1.2 Comparative study

The SM values for each alternative with the ideal alternative obtained with each of the existing measures are demonstrated in Table 5. When we observe the ranking order with each of these methods we find that the result for the best alternative is the same (i.e., A_4) for most of the methods- S_{CCH}(Chen and Chen 2009;S_{WC}Wei and Chen 2009;S_{CH} Chen 2011;S_{YC}Ye 2012a;S_{YH}Ye 2012b;S_TTang et al. 2017;S_{QY}Yue et al. 2019) and our proposed method. However, the approaches by- S_{FB}(Farhadinia 2012);S_{YE} Ye 2012b and S_{DH}Dinagar and Helena 2019), evaluated the best alternative as A_2(Food Company).

For better visualization, a graphical representation of the ranking order comparisons of various alternatives is shown in Fig. 12 below.

6 Burning issues resulting from COVID-19 pandemic era

This section is dedicated to validating the veracity and legitimacy of our proposed SM in efficiently handling the various facets of complex COVID-19 problems. In Sect. 6.1we discuss the problem of COVID-19 medicine selection, Sect. 6.2 undertakes a real-case study from an Indian state (Maharashtra) and deals with the problem of proper healthcare waste disposal measures therein, Sect. 6.3 discusses the most effective government intervention strategy against COVID-19 in the Indian context, and in Sect. 6.4 we present a formal analysis of our results and its managerial implications.

6.1 COVID-19 medicine selection

In these prevailing times of the COVID-19 pandemic, health care officials, medical practitioners, and other frontline workers are facing a huge dilemma regarding which medicine is suited best for the treatment of COVID-19 infected patients since there is no such trustworthy and accurate available information with them.From the onset of this pandemic and until now, unfortunately only a few medicines are given authorization/license for usage to coronavirus-positive patients. COVID-19 which first broke out in Wuhan city of China towards the latter part of the year 2019, usually belongs to a larger family of viruses (coronaviruses), which targets humans as well as some other animals. The spreading and infectious ability of this virus is exceptionally high like that of MERS (Middle East Respiratory Syndrome) and SARS (Severe Acute Respiratory Syndrome) virus. Generally, there are two main modes of transmission of the virus, which are, through air transmissions of small droplets and by physical contact with the infected person. With the kind of impact this virus is having on the entire human population of the world, the World Health Organization (WHO) declared it as a pandemic on March 11, 2020 (Clinical Management Protocol 2020).

Respiratory complications like shortness of breath due to the filling of the lung sacs with mucus; are often experienced by the COVID-19 active patients. Moreover, they also experience some other symptoms like sore throat, loss of smell (anosmia), loss of taste (agues), cough, nausea, chills, fever, etc. However, some specific symptoms like loss of appetite, fatigue, myalgia, decreased neural response, and reduced mobility is more common in the older group of active patients. Depending upon the symptoms manifested by the coronavirus-affected patients, they are classified into three categories, viz., mild cases, moderate or intermediate cases, and severe or critical cases. Now, since specific treatments are not at hand and firm medical evidence is unavailable, we can only rely upon certain therapies for the treatment of active patients. And as such, few drugs are available in the market that serves this purpose.

The Food and Drug Administration (FDA) has primarily approved some medicines such as *Hydroxychloroquine* (M_1), *Remdesivir* (M_2), *Tocilizumab* (M_3), and *Convalescent Plasma* (M_4), for the treatment of coronavirus positive patients only under emergency situations. The FDA has strongly emphasized that the past history of the patient, and whether undergoing an immune-suppressed condition or not, will be the key factors that will determine the administration of the drug's dosage to the patient. Clinical studies have found that *Hydroxychloroquine* (M_1) has some significant advantages and lowers down the intensity of coronavirus symptoms. But, some cases of side-effects due to the medicine were also reported in human trials which were conducted. *Remdesivir* (M_2)

Table 6 Judgment matrix, $J_{ij} = [d_{ij}]_{16\times4}$

	k	C_1	C_2	C_3	C_4
M_1	1	⟨([0.20, 0.30, 0.50, 0.60];0.10), ([0.10, 0.30, 0.60, 0.70];0.60)⟩	⟨([0.17, 0.29, 0.38, 0.46];0.42), ([0.10, 0.21, 0.40, 0.50];0.53)⟩	⟨([0.91, 0.93, 0.95, 0.97];0.42), ([0.89, 0.92, 0.96, 0.98];0.37)⟩	⟨([0.62, 0.73, 0.86, 0.93];0.42), ([0.56, 0.68, 0.88, 0.93];0.26)⟩
	2	⟨([0.26, 0.35, 0.61, 0.80];0.61), ([0.21, 0.28, 0.63, 0.83];0.31)⟩	⟨([0.61, 0.82, 0.88, 0.92];0.46), ([0.59, 0.76, 0.90, 0.96];0.54)⟩	⟨([0.83, 0.87, 0.91, 0.96];0.28), ([0.78, 0.86, 0.91, 0.98];0.69)⟩	⟨([0.38, 0.40, 0.43, 0.47];0.69), ([0.36, 0.40, 0.44, 0.47];0.22)⟩
	3	⟨([0.42, 0.59, 0.63, 0.90];0.72), ([0.36, 0.50, 0.69, 0.90];0.11)⟩	⟨([0.59, 0.71, 0.82, 0.92];0.71), ([0.46, 0.63, 0.88, 0.93];0.18)⟩	⟨([0.12, 0.19, 0.28, 0.43];0.61), ([0.10, 0.17, 0.28, 0.46];0.14)⟩	⟨([0.69, 0.70, 0.72, 0.76];0.81), ([0.69, 0.70, 0.72, 0.77];0.17)⟩
	4	⟨([0.33, 0.42, 0.51, 0.79];0.13), ([0.29, 0.39, 0.60, 0.80];0.56)⟩	⟨([0.46, 0.57, 0.68, 0.79];0.23), ([0.45, 0.49, 0.69, 0.79];0.77)⟩	⟨([0.40, 0.50, 0.60, 0.70];0.80), ([0.31, 0.43, 0.68, 0.80];0.12)⟩	⟨([0.53, 0.59, 0.64, 0.67];0.18), ([0.43, 0.56, 0.66, 0.69];0.68)⟩
M_2	1	⟨([0.49, 0.51, 0.68, 0.81];0.41), ([0.39, 0.49, 0.69, 0.89];0.51)⟩	⟨([0.19, 0.23, 0.29, 0.35];0.43), ([0.15, 0.20, 0.30, 0.38];0.51)⟩	⟨([0.61, 0.63, 0.69, 0.77];0.86), ([0.44, 0.62, 0.71, 0.77];0.11)⟩	⟨([0.16, 0.26, 0.46, 0.66];0.40), ([0.10, 0.26, 0.56, 0.76];0.20)⟩
	2	⟨([0.61, 0.68, 0.72, 0.76];0.34), ([0.41, 0.62, 0.73, 0.81];0.61)⟩	⟨([0.10, 0.30, 0.40, 0.50];0.11), ([0.09, 0.26, 0.46, 0.56];0.14)⟩	⟨([0.18, 0.28, 0.38, 0.48];0.11), ([0.10, 0.20, 0.40, 0.50];0.72)⟩	⟨([0.90, 0.92, 0.94, 0.98];0.11), ([0.88, 0.92, 0.96, 0.99];0.49)⟩
	3	⟨([0.44, 0.63, 0.79, 0.91];0.64), ([0.35, 0.59, 0.84, 0.92];0.27)⟩	⟨([0.46, 0.51, 0.63, 0.72];0.19), ([0.45, 0.49, 0.69, 0.76];0.24)⟩	⟨([0.42, 0.44, 0.48, 0.56];0.64), ([0.40, 0.43, 0.49, 0.59];0.32)⟩	⟨([0.81, 0.88, 0.92, 0.97];0.63), ([0.78, 0.88, 0.92, 0.97];0.14)⟩
	4	⟨([0.27, 0.33, 0.42, 0.57];0.52), ([0.16, 0.28, 0.48, 0.64];0.48)⟩	⟨([0.51, 0.61, 0.68, 0.73];0.34), ([0.48, 0.58, 0.72, 0.79];0.48)⟩	⟨([0.28, 0.31, 0.38, 0.44];0.68), ([0.21, 0.29, 0.40, 0.45];0.15)⟩	⟨([0.64, 0.71, 0.88, 0.89];0.43), ([0.56, 0.69, 0.89, 0.89];0.27)⟩
M_3	1	⟨([0.40, 0.52, 0.63, 0.76];0.69), ([0.38, 0.48, 0.66, 0.76];0.28)⟩	⟨([0.71, 0.74, 0.77, 0.79];0.53), ([0.70, 0.73, 0.75, 0.80];0.14)⟩	⟨([0.82, 0.85, 0.89, 0.90];0.16), ([0.81, 0.85, 0.90, 0.96];0.23)⟩	⟨([0.73, 0.80, 0.84, 0.89];0.64), ([0.72, 0.76, 0.89, 0.92];0.10)⟩
	2	⟨([0.34, 0.39, 0.44, 0.49];0.56), ([0.24, 0.35, 0.49, 0.50];0.10)⟩	⟨([0.54, 0.59, 0.69, 0.79];0.49), ([0.49, 0.55, 0.70, 0.80];0.56)⟩	⟨([0.84, 0.86, 0.88, 0.94];0.82), ([0.74, 0.86, 0.90, 0.95];0.18)⟩	⟨([0.34, 0.40, 0.49, 0.57];0.23), ([0.30, 0.38, 0.50, 0.59];0.73)⟩
	3	⟨([0.49, 0.54, 0.63, 0.77];0.72), ([0.42, 0.51, 0.64, 0.79];0.22)⟩	⟨([0.34, 0.38, 0.44, 0.56];0.29), ([0.28, 0.36, 0.51, 0.80];0.34)⟩	⟨([0.76, 0.78, 0.80, 0.83];0.36), ([0.66, 0.78, 0.83, 0.94];0.64)⟩	⟨([0.64, 0.69, 0.72, 0.79];0.28), ([0.56, 0.66, 0.78, 0.84];0.73)⟩
	4	⟨([0.72, 0.82, 0.86, 0.92];0.43), ([0.68, 0.78, 0.88, 0.94];0.08)⟩	⟨([0.18, 0.28, 0.38, 0.49];0.54), ([0.15, 0.20, 0.40, 0.60];0.40)⟩	⟨([0.14, 0.18, 0.29, 0.39];0.51), ([0.10, 0.18, 0.33, 0.50];0.18)⟩	⟨([0.79, 0.81, 0.82, 0.89];0.28), ([0.75, 0.80, 0.84, 0.90];0.29)⟩

Table 6 (continued)

	k	C_1	C_2	C_3	C_4
M_4	1	$\langle ([0.14, 0.19, 0.26, 0.38];0.40), ([0.10, 0.15, 0.23, 0.41];0.60) \rangle$	$\langle ([0.42, 0.46, 0.49, 0.52];0.31), ([0.39, 0.46, 0.50, 0.59];0.34) \rangle$	$\langle ([0.79, 0.81, 0.88, 0.96];0.29), ([0.69, 0.80, 0.89, 0.96];0.74) \rangle$	$\langle ([0.14, 0.18, 0.22, 0.24];0.40), ([0.13, 0.18, 0.23, 0.26];0.60) \rangle$
	2	$\langle ([0.28, 0.38, 0.46, 0.56];0.89), ([0.26, 0.36, 0.48, 0.58];0.10) \rangle$	$\langle ([0.89, 0.90, 0.92, 0.94];0.73), ([0.86, 0.90, 0.93, 0.94];0.19) \rangle$	$\langle ([0.64, 0.68, 0.81, 0.84];0.64), ([0.64, 0.67, 0.82, 0.88];0.32) \rangle$	$\langle ([0.19, 0.29, 0.31, 0.49];0.23), ([0.15, 0.28, 0.36, 0.59];0.42) \rangle$
	3	$\langle ([0.17, 0.29, 0.39, 0.44];0.62), ([0.15, 0.25, 0.39, 0.46];0.34) \rangle$	$\langle ([0.76, 0.78, 0.82, 0.86];0.49), ([0.74, 0.77, 0.84, 0.88];0.51) \rangle$	$\langle ([0.69, 0.79, 0.89, 0.99];0.29), ([0.59, 0.70, 0.90, 0.99];0.17) \rangle$	$\langle ([0.27, 0.37, 0.47, 0.57];0.72), ([0.25, 0.35, 0.55, 0.65];0.21) \rangle$
	4	$\langle ([0.18, 0.29, 0.36, 0.42];0.58), ([0.11, 0.22, 0.38, 0.44];0.36) \rangle$	$\langle ([0.64, 0.69, 0.72, 0.81];0.10), ([0.56, 0.68, 0.79, 0.88];0.19) \rangle$	$\langle ([0.62, 0.68, 0.86, 0.88];0.39), ([0.61, 0.64, 0.88, 0.90];0.21) \rangle$	$\langle ([0.30, 0.50, 0.70, 0.90];0.44), ([0.20, 0.40, 0.80, 0.90];0.26) \rangle$

Table 7 Normalized judgment matrix, $\overline{J_{ij}} = [t_{ij}]_{4\times4}$

	C_1	C_2	C_3	C_4
M_1	$\left\langle ([0.30, 0.41, 0.56, 0.77];0.39), ([0.24, 0.37, 0.63, 0.81];0.44) \right\rangle$	$\left\langle ([0.46, 0.60, 0.69, 0.77];0.45), ([0.40, 0.52, 0.72, 0.79];0.55) \right\rangle$	$\left\langle ([0.56, 0.62, 0.68, 0.76];0.53), ([0.52, 0.59, 0.71, 0.80];0.38) \right\rangle$	$\left\langle ([0.55, 0.60, 0.66, 0.71];0.52), ([0.51, 0.58, 0.67, 0.71];0.46) \right\rangle$
M_2	$\left\langle ([0.45, 0.54, 0.65, 0.76];0.38), ([0.33, 0.49, 0.68, 0.81];0.57) \right\rangle$	$\left\langle ([0.31, 0.41, 0.50, 0.57];0.27), ([0.29, 0.38, 0.54, 0.62];0.37) \right\rangle$	$\left\langle ([0.37, 0.41, 0.48, 0.56];0.55), ([0.29, 0.38, 0.50, 0.58];0.45) \right\rangle$	$\left\langle ([0.63, 0.69, 0.80, 0.87];0.39), ([0.58, 0.69, 0.83, 0.90];0.27) \right\rangle$
M_3	$\left\langle ([0.49, 0.57, 0.64, 0.73];0.52), ([0.43, 0.53, 0.67, 0.75];0.37) \right\rangle$	$\left\langle ([0.44, 0.50, 0.57, 0.66];0.49), ([0.40, 0.46, 0.59, 0.75];0.46) \right\rangle$	$\left\langle ([0.64, 0.67, 0.71, 0.76];0.46), ([0.58, 0.67, 0.74, 0.84];0.33) \right\rangle$	$\left\langle ([0.62, 0.67, 0.72, 0.78];0.41), ([0.58, 0.65, 0.75, 0.81];0.49) \right\rangle$
M_4	$\left\langle ([0.19, 0.29, 0.37, 0.45];0.67), ([0.15, 0.24, 0.37, 0.47];0.32) \right\rangle$	$\left\langle ([0.68, 0.71, 0.74, 0.78];0.41), ([0.64, 0.70, 0.76, 0.82];0.31) \right\rangle$	$\left\langle ([0.68, 0.74, 0.86, 0.92];0.43), ([0.63, 0.70, 0.87, 0.93];0.43) \right\rangle$	$\left\langle ([0.22, 0.33, 0.42, 0.55];0.50), ([0.18, 0.30, 0.48, 0.60];0.37) \right\rangle$

can be recommended for usage in patients showing very mild symptoms of the virus and the oxygen supply in them is gradually increasing. Likewise, *Tocilizumab* (M_3) is prescribed to patients who are constantly under ventilation and the virus inside their body is inducing some moderate symptoms. While the therapy *Convalescent Plasma* (M_4) is given to patients who are under the use of steroids, the virus shows some moderate to severe symptoms, and the demand for oxygen in such patients goes higher with each passing day. These drugs are governed by certain performance evaluation factors or criteria, which are *Ease Breathing* (C_1), *Coolify* (C_2), *Antiviral activity* (C_3), and *Side effects* (C_4). These evaluation factors corresponding to each of the drugs are expressed in the form of GTrIFNs. It is to be noted that, criteria C_1, C_2, and C_3, may be regarded as benefit criteria, while C_4 is the cost criterion for our present scenario.

The preference information for each of the medicines, with respect to each of the performance evaluation factors or criteria, are presented in the judgment matrix as shown in Table 6. There are four such criteria in total, for each of the four medicines available and they are expressed with the help of GTrIFNs.

Step-1: In the present scenario, we have k experts (where, $k = 4$) to make the judgment. The information provided by the experts in the form of GTrIFNs is visible in Table 6. These health experts take help of the limited available evidence at present and the past history of those medicines in dealing with such a family of viruses; in providing as much accurate data as they can. Having already illustrated the normalization procedure, the preference values of each medicine $M_i(i = 1, 2, 3, 4)$ with respect to every criterion $C_j(j = 1, 2, 3, 4)$ are normalized accordingly and the normalized judgment matrix is shown in Table 7.

Step-2: The ideal medicine which is constructed for the sake of evaluation is considered as,

$M^* = \langle([1, 1, 1, 1];1), ([0, 0, 0, 0];0)\rangle$, for profit criterion.

$M^* = \langle([0, 0, 0, 0];0), ([1, 1, 1, 1];1)\rangle$, for cost criterion.

Table 8 Ranking order comparisons for the COVID-19 medicines

Existing similarity methods	Similarity measure value between medicines				Ranking order	Best medicine
	(M_1, M^*)	(M_2, M^*)	(M_3, M^*)	(M_4, M^*)		
S_{CCH}Chen and Chen (2009)	−0.0005	0.0010	0.0019	−0.0007	$M_3 > M_2 > M_1 > M_4$	M_3
S_{WC}Wei and Chen (2009)	0.0008	−0.0015	0.0040	0.0062	$M_4 > M_3 > M_1 > M_2$	M_4
S_{CH}Chen (2011)	0.0414	0.0442	0.0489	0.0354	$M_3 > M_2 > M_1 > M_4$	M_3
S_{FB}Farhadinia (2012)	0.0002	−0.0023	0.0009	0.0046	$M_4 > M_3 > M_1 > M_2$	M_4
S_{YC}Ye (2012a)	0.5742	0.5627	0.5845	0.5558	$M_3 > M_1 > M_2 > M_4$	M_3
S_{YH}Ye (2012b)	0.5848	0.5837	0.5908	0.5914	$M_4 > M_3 > M_1 > M_2$	M_4
S_{YE}Ye (2012b)	0.5218	0.5161	0.5277	0.5094	$M_3 > M_1 > M_2 > M_4$	M_3
S_TTang et al. (2017)	0.5641	0.5398	0.5769	0.5208	$M_3 > M_1 > M_2 > M_4$	M_3
S_{QY}Yue et al. (2019)	0.1835	0.1693	0.1974	0.1704	$M_3 > M_1 > M_4 > M_2$	M_3
S_{DH}Dinagar and Helena (2019)	0.0914	0.0935	0.1067	0.0967	$M_3 > M_4 > M_2 > M_1$	M_3
S_{TIFN}(Proposed)	0.7011	0.6743	0.7181	0.6538	$M_3 > M_1 > M_2 > M_4$	M_3

For the present scenario, the criteria C_1 (*Ease Breathing*), C_2 (*Coolify*) &C_3 (*Antiviral Activity*) are the benefit criteria and C_4 (*Side Effect*) is the cost criterion.

Step-3: Finally, we evaluate the SM of each medicine with the ideal medicine, that is $S_{TIFN}(M_i, M^*)$, and we obtain the results with our approach as,

$$S_{TIFN}(M_1, M^*) = 0.7011,\ S_{TIFN}(M_2, M^*) = 0.6743,\ S_{TIFN}(M_3, M^*) = 0.7181,\ S_{TIFN}(M_4, M^*) = 0.6538$$

Here, the similarity results suggest that the SM of the medicine M_3 with M^* is the highest, and therefore we can conclude that M_3 as the best medicine available in the market that can be recommended to COVID-19 infected patients. Moreover, the ordering for the medicines is obtained as,

$$M_3 > M_1 > M_2 > M_4$$

Therefore, the medicine *Tocilizumab* (M_3) will prove to be the most effective one than the others, during treatment of the virus-infected patients. With the usage of the medicine *Tocilizumab* (M_3), the patient recovers sooner than the other medicines if administered at the same time. Thus, it reflects the proficiency and effectiveness of the medicine *Tocilizumab*.

6.1.1 Comparative study

Here, Table 8 presents the SM values for each medicine with the ideal medicine obtained with each of the existing measures. For most of the existing methods- S_{CCH}(Chen and Chen 2009; S_{CH}Chen 2011; S_{YC}Ye 2012a; S_{YE}Ye 2012b; S_TTang et al. 2017; S_{QY}Yue et al. 2019; S_{DH}Dinagar and Helena 2019),and our proposed method, interestingly we observe that the result for the best medicine is the same (i.e., M_3 (*Tocilizumab*)).However, certain approaches which for instance- S_{WC}(Wei and Chen 2009; S_{FB}Farhadinia 2012, and S_{YH}Ye 2012b), predicted that the best medicine should be M_4 (*Convalescent Plasma*).Although

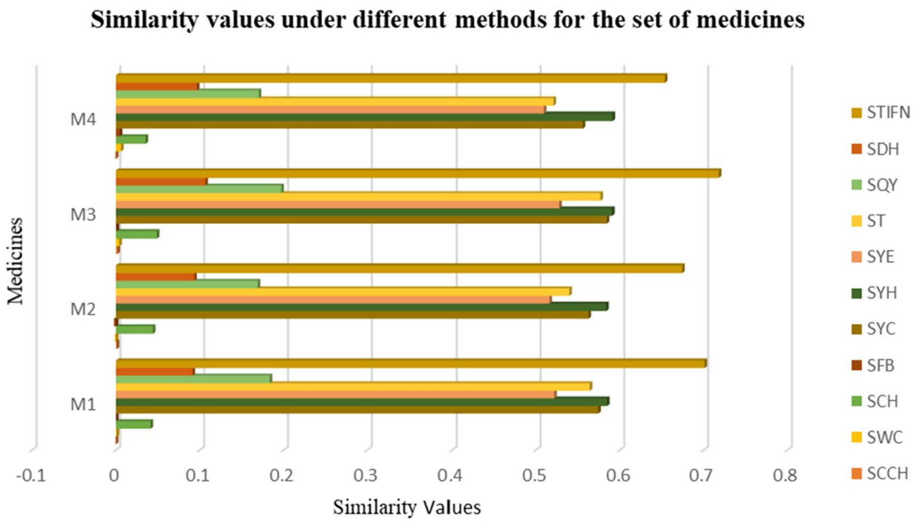


Fig. 13 Efficacy of different medicines under different similarity methods

a difference in the ranking order is attained with our proposed approach and some of the existing approaches, it is noteworthy that most of these approaches had certain drawbacks which were demonstrated very clearly in Sect. 4. Moreover, the final result for the best medicine obtained with our approach is identical with several other existing measures which indicate that our approach is more trustworthy, more efficient, more intuitive, and more logical.

The graphical illustration for the similarity results of various medicine alternatives is presented in Fig. 13 below.

6.2 Best strategy for healthcare waste disposal: a case study in Maharashtra, India

In a developing nation like India, the selection of an efficient strategy for the disposal of healthcare wastes is quite a tedious job. And as such the municipal authorities have run out of resources for disposing of the ever-increasing production of hazardous wastes each day since COVID-19 has hit the country. Although there exist certain waste disposal measures, the selection of an optimum one amongst them requires evaluation of different performance indices or criteria. The COVID-19 pandemic has worsened the already inefficient healthcare waste disposal strategies because managing such an enormous quantity of wastes that are being produced at the end of each day is fairly challenging. During this COVID era, the needles and syringes administered to COVID infected patients, dressings/bandages with bloodstains, personal protective equipment(s) (PPEs) worn by healthcare experts, dead bodies of patients, and other hazardous materials form a major portion of such wastes. Moreover, certain harmful chemicals, radioactive substances, and medical devices are also found that pose a serious threat to the soil and the grazing animals if they are to intake it. Thus, a proper waste management scheme is of utmost importance, particularly during these tough times of the pandemic. Because if such wastes are left untouched and are not properly disposed of, then this might further accelerate the rate of COVID-19 spread. In absence of proper preventive measures, the people with the responsibility of saving others' lives will themselves be at the risk of getting infected with the virus. Therefore, for the betterment of humankind and to avoid environmental pollution, all healthcare wastes should be accumulated and disposed of at a distant location/abandoned land.

India is a country where improper waste disposal measures are often in practice. Huge dumps of wastes can be seen blocking the passage of underground drainage systems and at the river banks, which become breeding ground for insects causing deadly diseases in humans. Being the second-most populous country in the world, the government of India needs to take this matter seriously by investing money and human labor so that the wastes produced each day are being disposed of during the same day itself or on a weekly basis. According to the data obtained by the Indian Society of Hospital Waste Management (ISHWM), it is estimated that at least 1–2 kg of waste is being generated for each bed in a hospital during a span of 24 h; while the amount in a practitioner's clinic is 600 g per bed (Manupati et al. 2021). The WHO in 2020 issued a few guidelines on how to treat COVID-19 wastes in an efficient manner (World Health Organization 2020). Some of them are:

- Wastes should be separated at the source depending upon their nature
- Irresistible trash cans should be used for dumping COVID-19 wastes having different colored liners

- COVID-19 wastes should be treated like other infectious healthcare waste and they should be collected at the end of each day and then be transported safely in puncture-proof and leakproof bins
- The place for storing COVID-19 wastes should be sanitized and properly maintained with good hygiene
- The waste disposal techniques should be tested and validated at regular intervals.

In our case, we consider a case study from Maharashtra, one of the most badly affected Indian states due to COVID-19. Maharashtra can be considered as one of the major business and industrial hubs of India, with an average population of almost 112 million. Most of the country's export–import business takes place from the ports located at Maharashtra via the Arabian sea to the other parts of the world. But this financial advantage and the easy access to people from all parts of the world, paved the gateway for a transmissible virus like COVID-19 to seep into the state. Soon the virus started infecting people of the state and with each passing day, the number of cases increased exponentially. The state government and the medical facilities were not prepared for such an uncanny situation which left them in a state of shock. Maharashtra reported 6,611,078 COVID infected cases (highest in India) out of which 6,450,585 people were cured and almost 140,216 people died of the virus. (Source: https://www.covid19india.org, as accessed on December 21, 2021).

More infections increased the demand for more protective equipment and more production of healthcare waste each day. In India, before the COVID-19 era had started, people were already struggling to get rid of the healthcare wastes in an efficient manner and since the pandemic has started, the problem got even worse. Therefore, addressing this issue is the current need of the hour and accordingly, we consider 6 (six) different healthcare waste disposal techniques in our study. Those are H_1: *Microwave*, H_2: *Landfill*,H_3: *Incineration*,H_4: *Plasma pyrolysis*,H_5: *Integrated steam sterilization system*,H_6: *Chemical disinfection system*. Now, to determine the best disposal strategy among them, we would require certain criteria or attributes based on which the healthcare waste disposal alternatives shall be ranked in order of preference. We consider 5 (five) such criteria in our study viz.,

- C_1(*Operational safety*): This measure takes into account the risk factors and the precautionary measures required for the functioning of a waste disposal alternative.
- C_2(*Annual operating cost*): The operation of certain healthcare waste disposal techniques comes with a maintenance cost. This measure estimates the cost annually.
- C_3(*Reliability*): This measure assesses how will a disposal technique perform in the long run or in other words, its durability.
- C_4(*Treatment efficiency*): The long-term suitability and capability to eradicate the waste at minimum cost and maximum efficiency are measured by this criterion.
- C_5(*Toxic emissions and health effects*): The risk factors associated with a particular healthcare waste disposal alternative, for instance, the harmful emissions affecting the environment and also the health complications to humans in the vicinity of the operation are measured by this criterion.

Also, to arrive at an accurate decision we consider the difference of opinions of 3(three) experts in the field comprising a waste management field expert, a doctor, and an environmentalist. The group decision algorithm is discussed step-wise as follows:

Step-1: The three experts provide their preference values for set of the alternatives $H_i(i = 1, 2, 3, 4, 5, 6)$ with respect to each of the criteria $C_j(i = 1, 2, 3, 4, 5)$ which is

Table 9 Judgment matrix, $J_{ij} = [d_{ij}]_{18\times5}$

	k	C_1	C_2	C_3	C_4	C_5
H_1	1	⟨([0.10, 0.20, 0.30, 0.40];0.32), ([0.08, 0.17, 0.40, 0.50];0.61)	([0.20, 0.24, 0.26, 0.28];0.41), ([0.15, 0.21, 0.27, 0.32];0.39)	([0.42, 0.48, 0.52, 0.58];0.46), ([0.40, 0.44, 0.52, 0.58];0.21)	([0.36, 0.37, 0.38, 0.39];0.32), ([0.35, 0.36, 0.38, 0.40];0.19)	([0.19, 0.22, 0.26, 0.29];0.54), ([0.17, 0.23, 0.28, 0.31];0.40)⟩
	2	⟨([0.61, 0.62, 0.63, 0.64];0.29), ([0.54, 0.56, 0.58, 0.60];0.36)	([0.32, 0.34, 0.36, 0.38];0.10), ([0.29, 0.32, 0.37, 0.40];0.30)	([0.31, 0.33, 0.35, 0.37];0.31), ([0.28, 0.29, 0.35, 0.37];0.60)	([0.25, 0.30, 0.35, 0.40];0.25), ([0.20, 0.25, 0.35, 0.45];0.30)	([0.60, 0.61, 0.62, 0.63];0.40), ([0.55, 0.60, 0.65, 0.70];0.50)⟩
	3	⟨([0.24, 0.28, 0.32, 0.36];0.60), ([0.23, 0.28, 0.32, 0.37];0.40)	([0.15, 0.17, 0.19, 0.21];0.15), ([0.13, 0.17, 0.20, 0.24];0.23)	([0.28, 0.29, 0.30, 0.31];0.50), ([0.24, 0.28, 0.31, 0.36];0.50)	([0.10, 0.15, 0.20, 0.25];0.40), ([0.10, 0.15, 0.20, 0.25];0.50)	([0.72, 0.74, 0.76, 0.78];0.70), ([0.68, 0.72, 0.76, 0.80];0.10)⟩
H_2	1	⟨([0.20, 0.23, 0.26, 0.29];0.42), ([0.15, 0.20, 0.26, 0.30];0.39)	([0.36, 0.38, 0.40, 0.42];0.72), ([0.32, 0.36, 0.42, 0.44];0.22)	([0.49, 0.52, 0.55, 0.58];0.34), ([0.47, 0.50, 0.55, 0.59];0.58)	([0.31, 0.34, 0.37, 0.40];0.12), ([0.30, 0.34, 0.38, 0.42];0.36)	([0.92, 0.94, 0.96, 0.98];0.10), ([0.90, 0.92, 0.96, 0.99];0.20)⟩
	2	⟨([0.30, 0.31, 0.32, 0.34];0.50), ([0.28, 0.30, 0.32, 0.34];0.10)	([0.52, 0.54, 0.56, 0.58];0.52), ([0.50, 0.54, 0.58, 0.62];0.43)	([0.22, 0.23, 0.24, 0.25];0.08), ([0.20, 0.22, 0.25, 0.28];0.20)	([0.34, 0.35, 0.36, 0.37];0.43), ([0.33, 0.35, 0.37, 0.39];0.21)	([0.82, 0.84, 0.86, 0.88];0.40), ([0.80, 0.82, 0.86, 0.89];0.30)⟩
	3	⟨([0.45, 0.47, 0.49, 0.51];0.30), ([0.40, 0.45, 0.50, 0.55];0.45)	([0.16, 0.20, 0.22, 0.26];0.48), ([0.14, 0.18, 0.23, 0.27];0.51)	([0.26, 0.28, 0.30, 0.36];0.66), ([0.24, 0.28, 0.32, 0.36];0.32)	([0.38, 0.39, 0.40, 0.41];0.20), ([0.36, 0.37, 0.40, 0.42];0.30)	([0.64, 0.67, 0.70, 0.73];0.70), ([0.60, 0.64, 0.71, 0.75];0.25)⟩
H_3	1	⟨([0.82, 0.83, 0.84, 0.85];0.55), ([0.80, 0.82, 0.84, 0.86];0.40)	([0.12, 0.13, 0.14, 0.15];0.80), ([0.10, 0.12, 0.14, 0.16];0.20)	([0.86, 0.87, 0.88, 0.89];0.24), ([0.85, 0.86, 0.89, 0.90];0.68)	([0.90, 0.92, 0.94, 0.96];0.40), ([0.88, 0.90, 0.94, 0.98];0.50)	([0.22, 0.24, 0.26, 0.28];0.30), ([0.20, 0.22, 0.26, 0.28];0.50)⟩
	2	⟨([0.90, 0.92, 0.94, 0.96];0.40), ([0.88, 0.89, 0.94, 0.98];0.60)	([0.08, 0.10, 0.12, 0.14];0.60), ([0.08, 0.10, 0.12, 0.14];0.40)	([0.79, 0.80, 0.82, 0.83];0.22), ([0.78, 0.80, 0.82, 0.84];0.49)	([0.85, 0.87, 0.89, 0.92];0.52), ([0.84, 0.86, 0.89, 0.94];0.40)	([0.18, 0.20, 0.24, 0.26];0.42), ([0.15, 0.20, 0.25, 0.30];0.28)⟩
	3	⟨([0.96, 0.97, 0.98, 0.99];0.24), ([0.95, 0.96, 0.98, 0.99];0.76)	([0.05, 0.06, 0.07, 0.08];0.70), ([0.04, 0.05, 0.07, 0.09];0.30)	([0.90, 0.91, 0.92, 0.93];0.42), ([0.88, 0.90, 0.93, 0.96];0.10)	([0.92, 0.93, 0.94, 0.95];0.40), ([0.90, 0.92, 0.94, 0.96];0.50)	([0.10, 0.14, 0.18, 0.20];0.62), ([0.09, 0.14, 0.18, 0.22];0.34)⟩
H_4	1	⟨([0.42, 0.45, 0.48, 0.51];0.50), ([0.39, 0.42, 0.48, 0.52];0.50)	([0.64, 0.65, 0.67, 0.68];0.50), ([0.62, 0.64, 0.68, 0.70];0.50)	([0.16, 0.17, 0.18, 0.19];0.20), ([0.15, 0.16, 0.19, 0.20];0.30)	([0.30, 0.31, 0.32, 0.33];0.40), ([0.29, 0.30, 0.32, 0.34];0.52)	([0.82, 0.83, 0.84, 0.85];0.50), ([0.80, 0.81, 0.85, 0.86];0.20)⟩
	2	⟨([0.28, 0.29, 0.30, 0.31];0.30), ([0.26, 0.28, 0.30, 0.32];0.26)	([0.56, 0.58, 0.60, 0.62];0.18), ([0.55, 0.56, 0.62, 0.64];0.24)	([0.18, 0.19, 0.20, 0.21];0.64), ([0.16, 0.18, 0.20, 0.22];0.36)	([0.41, 0.43, 0.45, 0.47];0.24), ([0.39, 0.42, 0.46, 0.48];0.33)	([0.62, 0.64, 0.66, 0.68];0.70), ([0.60, 0.64, 0.66, 0.68];0.30)⟩
	3	⟨([0.22, 0.24, 0.26, 0.28];0.42), ([0.20, 0.23, 0.27, 0.30];0.56)	([0.72, 0.74, 0.76, 0.78];0.32), ([0.70, 0.71, 0.76, 0.79];0.21)	([0.27, 0.28, 0.29, 0.30];0.60), ([0.25, 0.27, 0.29, 0.31];0.40)	([0.24, 0.26, 0.28, 0.30];0.42), ([0.21, 0.25, 0.29, 0.32];0.56)	([0.90, 0.91, 0.92, 0.94];0.56), ([0.88, 0.90, 0.94, 0.96];0.43)⟩

Table 9 (continued)

	k	C_1	C_2	C_3	C_4	C_5
H_5	1	⟨([0.36, 0.37, 0.38, 0.39];0.27), ([0.31, 0.33, 0.39, 0.42];0.10)⟩	⟨([0.72, 0.73, 0.74, 0.76];0.50), ([0.68, 0.72, 0.76, 0.78];0.50)⟩	⟨([0.16, 0.18, 0.20, 0.22];0.80), ([0.15, 0.17, 0.20, 0.22];0.20)⟩	⟨([0.52, 0.54, 0.56, 0.58];0.62), ([0.50, 0.54, 0.57, 0.59];0.39)⟩	⟨([0.68, 0.69, 0.70, 0.71];0.27), ([0.65, 0.68, 0.71, 0.72];0.66)⟩
	2	⟨([0.28, 0.29, 0.30, 0.31];0.80), ([0.24, 0.28, 0.31, 0.36];0.20)⟩	⟨([0.68, 0.69, 0.70, 0.71];0.40), ([0.66, 0.68, 0.71, 0.73];0.10)⟩	⟨([0.19, 0.21, 0.23, 0.25];0.43), ([0.15, 0.18, 0.24, 0.27];0.30)⟩	⟨([0.62, 0.64, 0.66, 0.68];0.71), ([0.60, 0.63, 0.67, 0.68];0.18)⟩	⟨([0.78, 0.79, 0.80, 0.82];0.10), ([0.75, 0.78, 0.81, 0.84];0.46)⟩
	3	⟨([0.34, 0.35, 0.36, 0.37];0.70), ([0.32, 0.34, 0.36, 0.40];0.20)⟩	⟨([0.78, 0.80, 0.82, 0.84];0.41), ([0.76, 0.78, 0.82, 0.86];0.55)⟩	⟨([0.42, 0.44, 0.46, 0.48];0.27), ([0.40, 0.43, 0.47, 0.49];0.71)⟩	⟨([0.43, 0.45, 0.47, 0.49];0.23), ([0.41, 0.44, 0.47, 0.50];0.12)⟩	⟨([0.86, 0.87, 0.88, 0.89];0.40), ([0.84, 0.86, 0.88, 0.90];0.60)⟩
H_6	1	⟨([0.42, 0.43, 0.44, 0.45];0.60), ([0.40, 0.42, 0.44, 0.47];0.40)⟩	⟨([0.82, 0.83, 0.84, 0.85];0.20), ([0.80, 0.82, 0.84, 0.86];0.30)⟩	⟨([0.52, 0.54, 0.56, 0.58];0.27), ([0.50, 0.53, 0.57, 0.59];0.76)⟩	⟨([0.48, 0.49, 0.50, 0.51];0.16), ([0.45, 0.49, 0.51, 0.53];0.19)⟩	⟨([0.92, 0.93, 0.94, 0.96];0.40), ([0.90, 0.92, 0.94, 0.98];0.50)⟩
	2	⟨([0.52, 0.54, 0.56, 0.58];0.10), ([0.50, 0.52, 0.56, 0.59];0.10)⟩	⟨([0.72, 0.74, 0.77, 0.81];0.27), ([0.69, 0.74, 0.79, 0.86];0.52)⟩	⟨([0.49, 0.51, 0.53, 0.57];0.72), ([0.45, 0.50, 0.54, 0.58];0.19)⟩	⟨([0.29, 0.31, 0.33, 0.35];0.28), ([0.26, 0.30, 0.34, 0.36];0.66)⟩	⟨([0.62, 0.63, 0.64, 0.65];0.40), ([0.60, 0.62, 0.65, 0.66];0.50)⟩
	3	⟨([0.71, 0.72, 0.73, 0.74];0.20), ([0.68, 0.71, 0.74, 0.78];0.30)⟩	⟨([0.54, 0.56, 0.58, 0.60];0.34), ([0.52, 0.54, 0.58, 0.62];0.62)⟩	⟨([0.36, 0.38, 0.40, 0.42];0.27), ([0.35, 0.37, 0.41, 0.43];0.41)⟩	⟨([0.80, 0.81, 0.82, 0.83];0.16), ([0.78, 0.80, 0.83, 0.84];0.27)⟩	⟨([0.64, 0.66, 0.68, 0.70];0.40), ([0.63, 0.66, 0.69, 0.72];0.60)⟩

Table 10 Normalized judgment matrix, $\overline{J_{ij}} = [t_{ij}]_{6\times5}$

	C_1	C_2	C_3	C_4	C_5
H_1	$\langle([0.32, 0.37, 0.42, 0.47];0.50), ([0.28, 0.34, 0.44, 0.49];0.42)\rangle$	$\langle([0.22, 0.25, 0.27, 0.29];0.22), ([0.19, 0.24, 0.28, 0.32];0.31)\rangle$	$\langle([0.34, 0.37, 0.39, 0.42];0.56), ([0.31, 0.34, 0.39, 0.44];0.37)\rangle$	$\langle([0.24, 0.27, 0.31, 0.35];0.39), ([0.22, 0.25, 0.31, 0.37];0.39)\rangle$	$\langle([0.50, 0.52, 0.55, 0.57];0.55), ([0.47, 0.52, 0.56, 0.60];0.44)\rangle$
H_2	$\langle([0.32, 0.34, 0.36, 0.38];0.41), ([0.28, 0.32, 0.36, 0.39];0.31)\rangle$	$\langle([0.35, 0.37, 0.39, 0.42];0.74), ([0.32, 0.36, 0.41, 0.44];0.25)\rangle$	$\langle([0.32, 0.34, 0.36, 0.39];0.46), ([0.30, 0.33, 0.37, 0.41];0.47)\rangle$	$\langle([0.34, 0.36, 0.37, 0.39];0.25), ([0.33, 0.35, 0.38, 0.41];0.29)\rangle$	$\langle([0.79, 0.82, 0.84, 0.86];0.40), ([0.77, 0.79, 0.84, 0.88];0.25)\rangle$
H_3	$\langle([0.89, 0.91, 0.92, 0.93];0.53), ([0.88, 0.89, 0.92, 0.94];0.42)\rangle$	$\langle([0.08, 0.09, 0.11, 0.12];0.70), ([0.07, 0.09, 0.11, 0.13];0.27)\rangle$	$\langle([0.85, 0.86, 0.87, 0.88];0.53), ([0.84, 0.85, 0.88, 0.90];0.39)\rangle$	$\langle([0.89, 0.91, 0.92, 0.94];0.44), ([0.87, 0.89, 0.92, 0.96];0.47)\rangle$	$\langle([0.17, 0.19, 0.23, 0.25];0.45), ([0.15, 0.19, 0.23, 0.27];0.47)\rangle$
H_4	$\langle([0.31, 0.33, 0.35, 0.37];0.41), ([0.28, 0.31, 0.35, 0.38];0.51)\rangle$	$\langle([0.64, 0.66, 0.68, 0.69];0.34), ([0.62, 0.64, 0.68, 0.71];0.35)\rangle$	$\langle([0.20, 0.21, 0.22, 0.23];0.48), ([0.19, 0.20, 0.23, 0.24];0.42)\rangle$	$\langle([0.32, 0.34, 0.35, 0.37];0.35), ([0.30, 0.32, 0.35, 0.38];0.50)\rangle$	$\langle([0.78, 0.79, 0.81, 0.82];0.59), ([0.76, 0.78, 0.82, 0.84];0.34)\rangle$
H_5	$\langle([0.33, 0.34, 0.35, 0.36];0.59), ([0.29, 0.32, 0.35, 0.39];0.33)\rangle$	$\langle([0.73, 0.74, 0.75, 0.77];0.44), ([0.70, 0.73, 0.76, 0.79];0.45)\rangle$	$\langle([0.26, 0.28, 0.30, 0.32];0.50), ([0.24, 0.26, 0.30, 0.33];0.44)\rangle$	$\langle([0.52, 0.54, 0.56, 0.58];0.52), ([0.50, 0.54, 0.57, 0.59];0.26)\rangle$	$\langle([0.77, 0.78, 0.79, 0.81];0.26), ([0.75, 0.77, 0.80, 0.82];0.67)\rangle$
H_6	$\langle([0.55, 0.57, 0.58, 0.59];0.30), ([0.53, 0.55, 0.58, 0.61];0.30)\rangle$	$\langle([0.69, 0.71, 0.73, 0.75];0.27), ([0.67, 0.70, 0.74, 0.78];0.51)\rangle$	$\langle([0.46, 0.48, 0.50, 0.52];0.42), ([0.44, 0.47, 0.51, 0.54];0.45)\rangle$	$\langle([0.52, 0.54, 0.55, 0.56];0.20), ([0.50, 0.53, 0.56, 0.58];0.44)\rangle$	$\langle([0.73, 0.74, 0.75, 0.77];0.40), ([0.71, 0.74, 0.76, 0.79];0.60)\rangle$

Table 11 Ranking order ofavailable set of waste disposal measures

Existing similarity methods	SM values for healthcare waste disposal alternatives						Ranking order	Best
	(H_1, H^*)	(H_2, H^*)	(H_3, H^*)	(H_4, H^*)	(H_5, H^*)	(H_6, H^*)		
S_{CCH}Chen and Chen (2009)	0.0006	0.0004	0.1025	0.0006	0.0008	0.0006	$H_3 > H_5 > H_1 \approx H_4 \approx H_6 > H_2$	H_3
S_{WC}Wei and Chen (2009)	0.0012	0.0006	0.0028	0.0011	0.0016	0.0009	$H_3 > H_5 > H_1 > H_4 > H_6 > H_2$	H_3
S_{CH}Chen (2011)	0.0443	0.0260	0.0487	0.0458	0.0458	0.0305	$H_3 > H_5 \approx H_4 > H_1 > H_6 > H_2$	H_3
S_{FB}Farhadinia (2012)	0.0013	0.0006	0.0192	0.0009	0.0020	0.0009	$H_3 > H_5 > H_1 > H_4 \approx H_6 > H_2$	H_3
S_{YC}Ye (2012a)	0.5483	0.4345	0.6004	0.5383	0.5533	0.4891	$H_3 > H_5 > H_1 > H_4 > H_6 > H_2$	H_3
S_{YH} Ye (2012b)	0.5803	0.5719	0.5940	0.5770	0.5868	0.5723	$H_3 > H_5 > H_1 > H_4 > H_6 > H_2$	H_3
S_{YE}Ye (2012b)	0.5210	0.4373	0.5363	0.5094	0.5255	0.4999	$H_3 > H_5 > H_1 > H_4 > H_6 > H_2$	H_3
S_TTang et al. (2017)	0.4802	0.4426	0.5808	0.4721	0.5117	0.4675	$H_3 > H_5 > H_1 > H_4 > H_6 > H_2$	H_3
S_{QY}Yue et al. 2019)	0.1971	0.1415	0.2005	0.1581	0.1860	0.1353	$H_3 > H_1 > H_5 > H_4 > H_2 > H_6$	H_3
S_{DH}Dinagar and Helena (2019)	0.0466	0.0296	0.0882	0.0385	0.0601	0.0344	$H_3 > H_5 > H_1 > H_4 > H_6 > H_2$	H_3
S_{TIFN}(Proposed)	0.5990	0.4869	0.6853	0.5938	0.6659	0.5733	$H_3 > H_5 > H_1 > H_4 > H_6 > H_2$	H_3

presented in Table 9. The decision matrix being generated is further normalized to obtain a normalized judgment matrix as shown in Table 10.

Step-2: For our present case study, criteria C_1(*Operational safety*), C_3(*Reliability*), and C_4(*Treatment efficiency*) are profit-type, whereas C_2(*Annual operating cost*) and C_5(*Toxic emissions and health effects*) are cost-type criteria. It is noteworthy that no healthcare waste disposal strategy is completely full-proof and flawless. Every such strategy has its own pros and cons. But in order to achieve our desired goal of ranking the available set of disposal techniques in our country, we construct the form for an ideal disposal strategy (H^*) depending upon the nature of the criteria. Thus, we have.

$H^* = \langle([1, 1, 1, 1];1), ([0, 0, 0, 0];0)\rangle$, for profit criterion.

$H^* = \langle([0, 0, 0, 0];0), ([1, 1, 1, 1];1)\rangle$,for cost criterion.

Step-3: We then obtain our proposed similarity measure results evaluated for various healthcare waste disposal alternatives as,

$$S_{TIFN}(H_1, H^*) = 0.5990,\ S_{TIFN}(H_2, H^*) = 0.4869,\ S_{TIFN}(H_3, H^*) = 0.6853,$$
$$S_{TIFN}(H_4, H^*) = 0.5938,\ S_{TIFN}(H_5, H^*) = 0.6659,\ S_{TIFN}(H_6, H^*) = 0.5733$$

Thus, we obtain the highest value of similarity for H_3(*Incineration*) which makes it the most desirable technique for disposing the infectious waste. Consequently, ranking order for the set of alternatives is obtained as,

H_3(***Incineration***) > H_5(***Integrated steam sterilization system***) > H_1(***Microwave***) > H_4 (***Plasma pyrolysis***) > H_6(***Chemical disinfection system***) > H_2(***Landfill***)

The main advantage of the *Incineration* (H_3) technique is that it can get rid of the pathogens and completely destroy the harmful waste organics. Moreover, its treatment capacity is higher than most of the remaining methods and it requires very little space for operation.

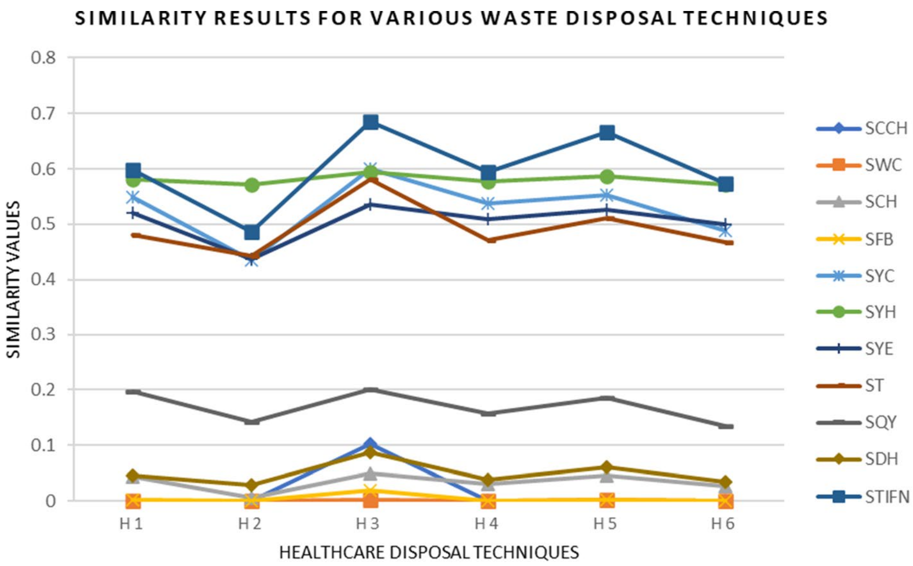


Fig. 14 Comparison of similarity outputs obtained under different similarity methods

6.2.1 Comparative study

The SM outcomes obtained for different healthcare waste disposal alternatives are shown in Table 11 below. It is observed that all the existing SM methods along with our proposed method determine the technique of *Incineration* (H_3) as the most preferable one. However, certain anomalies were observed with some of the existing approaches, for instance S_{CCH} (Chen and Chen 2009) method could not distinguish between the alternatives H_1, H_4, and H_6; S_{CH} (Chen 2011) methodcould not distinguish between the alternatives H_5 and H_4;S_{FB} [22]method could not distinguish between the alternatives H_4 and H_6. However, our proposed measure is free from such discrepancies and it is capable of arriving at a final decision systematically.

A pictorial representation showing the best waste disposal technique under different SM methods is presented in Fig. 14 below.

6.3 Government intervention strategies for fighting against COVID-19: a case study of India

The determination of an effective governmental strategy is of prime importance in this COVID-19 era considering how big of a threat this virus has become to the whole world. The medical workers, researchers, and scientists are doing their part to possibly eradicate the virus, and simultaneously it is the duty of the state/central governments to adopt certain measures to help minimize the spread of the virus. Such measures may be in the form of certain intervention strategies or guidelines that the people must strictly adhere to, for their own well-being. However, such measures involve considering various criteria and operating with vague or uncertain nature of information, which suggests that suitable governmental intervention can be considered as an MCDM scenario. Moreover, the medical organizations and governments track the effectiveness of any intervention strategy in a particular country and measure its mass acceptance before implementing it in their own countries respectively. While formulating such strategies, the government addresses the more severe issues first and then the less severe ones, and so on. COVID-19 outbreak has greatly affected the mental health of people and it has jeopardized the financial condition of countries from all around the globe.

Here, we undertake a case study in the Indian context. Initially, when COVID-19 affected some of the Indian states and the cases were small in number, the people of India followed the guidelines as issued by WHO, which were physical/social distancing, good hygiene, and wearing of face-masks. But, by the time this message was conveyed to each and every part of the country, people already started getting infected with the virus. In a country with the second-largest population, it becomes very susceptible to a contagious virus-like COVID-19 to maximize its spread. Therefore, some solid interventions should be implemented to slow down the virus from infecting more and more individuals of the population. At present, India ranks second in the world with respect to the number of COVID-19 infected cases. More precisely, India records nearly 34 million (34,752,164) COVID cases in the country since this pandemic has first started (Source: https://www.worldometers.info/coronavirus/ as accessed on December 22, 2021).

In our present case study, we outline 8 (eight) such government intervention strategies or alternatives, which are.

- G_1(*Curfew*): In India, when curfew was first imposed very strictly nationwide, the COVID-19 cases per day showed a steep decline. But, at later stages of the pandemic, due to lack of proper management, unavailability of an adequate number of resources for living, and unpreparedness for such a catastrophic pandemic, the curfew rules got relaxed and the COVID-19 infected graphs showed exponential increase again.
- G_2(*External border restrictions*): Indian government practiced this strategy at the initial stages of the pandemic when there was limited available information about the virus. This technique was fairly effective in countries where it was being implemented at the earliest. Once the virus has already been mitigated, then this measure loses its significance.
- G_3(*Social distancing*): This strategy is among the most important ones whose objective is to reduce the interaction with people as much as we can because we are unsure who might act as a carrier of this virus. In this way, we reduce the chances of getting infected with the virus.
- G_4(*Quarantining patients and those suspected of infection*): Quarantining or self-isolation is one of the preventive measures practiced by people at the early stages of the pandemic. In absence of proper medicinal treatment or due to lack of vaccines, quarantining showed positive results in slowing the virus transmission to a certain extent.
- G_5(*Closure of schools*): Schools and other educational institutions in India were immediately closed when the government sensed the potential danger of the small children and the youth of the country getting infected with the virus and transmitting the same to their parents. This measure was also effective to a certain extent.
- G_6(*Vaccinations*): Recently with the advent of vaccines curing the virus, the entire human population saw this as a blessing. But, in order to be able to administer the vaccine doses to the whole population would further require a considerable amount of time. Moreover, the shortage of vaccines and urgent importing of vaccine raw materials posed further concerns. However, the Indian government has been fairly active in ensuring that every individual gets access to at least one dose of the vaccine, which is still better than being deprived of it. Recently, India has remotely started production of some foreign vaccines at certain reputed virology institutes of the country as well.
- G_7(*Restriction of mass gatherings*): As directed by WHO, the Indian government imposed a certain set of guidelines regarding the permissible number of people allowed to gather at a particular place. This restriction helped to decline the transmission rate of the virus.
- G_8(*Internal border restrictions*): This strategy is effective in preventing the infected symptomatic and infected asymptomatic patients from traveling and transmitting it onto the non-infected population of the country.

To test the effectiveness and applicability of our intervention strategy alternatives, a set of 4 (four) criteria(s) are chosen viz., C_1: *Ease of implementation*, C_2: *Total cost*,C_3: *Effectiveness to prevent COVID-19 spread*,C_4: *High acceptability to citizens*. We also consider 3(three) decision makers for smoothly conducting the study by arriving at a final decision. We proceed step-wise as follows.

Step-1: We construct the decision matrix for the intervention strategy alternatives $G_i(i = 1, 2, 3, 4, 5, 6, 7, 8)$ with respect to the set of criteria $C_j(i = 1, 2, 3, 4)$ and it is presented in Table 12. The entries of the decision (judgment) matrix are then normalized to get rid of any physical dimensions and the normalized judgment matrix is obtained in Table 13 below.

Table 12 Judgment matrix, $J_{ij} = [d_{ij}]_{24\times4}$

	k	C_1	C_2	C_3	C_4
G_1	1	$\left\langle \begin{matrix}([0.16,0.18,0.20,0.22];0.10),\\([0.15,0.16,0.21,0.23];0.20)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.81,0.82,0.83,0.84];0.50),\\([0.80,0.82,0.83,0.86];0.50)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.62,0.63,0.64,0.65];0.60),\\([0.60,0.62,0.64,0.66];0.40)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.51,0.52,0.53,0.54];0.80),\\([0.50,0.51,0.53,0.54];0.20)\end{matrix}\right\rangle$
	2	$\left\langle \begin{matrix}([0.22,0.24,0.26,0.28];0.30),\\([0.21,0.23,0.27,0.29];0.40)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.92,0.93,0.94,0.95];0.70),\\([0.90,0.92,0.94,0.96];0.30)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.51,0.53,0.55,0.57];0.50),\\([0.50,0.52,0.56,0.58];0.50)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.52,0.54,0.56,0.58];0.80),\\([0.50,0.53,0.56,0.59];0.20)\end{matrix}\right\rangle$
	3	$\left\langle \begin{matrix}([0.41,0.42,0.43,0.44];0.70),\\([0.40,0.42,0.44,0.46];0.10)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.86,0.87,0.88,0.89];0.50),\\([0.84,0.86,0.88,0.90];0.50)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.10,0.20,0.30,0.40];0.20),\\([0.10,0.15,0.35,0.45];0.30)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.42,0.43,0.44,0.45];0.60),\\([0.40,0.42,0.45,0.47];0.30)\end{matrix}\right\rangle$
G_2	1	$\left\langle \begin{matrix}([0.26,0.28,0.30,0.32];0.60),\\([0.24,0.26,0.30,0.32];0.40)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.78,0.79,0.80,0.81];0.20),\\([0.76,0.78,0.80,0.82];0.30)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.48,0.49,0.50,0.51];0.60),\\([0.47,0.48,0.50,0.52];0.40)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.39,0.40,0.41,0.42];0.50),\\([0.38,0.39,0.42,0.44];0.20)\end{matrix}\right\rangle$
	2	$\left\langle \begin{matrix}([0.31,0.32,0.33,0.34];0.40),\\([0.30,0.32,0.34,0.36];0.60)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.88,0.89,0.90,0.91];0.40),\\([0.86,0.89,0.90,0.92];0.50)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.51,0.53,0.55,0.57];0.70),\\([0.50,0.53,0.56,0.59];0.30)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.61,0.64,0.67,0.70];0.90),\\([0.60,0.62,0.68,0.72];0.10)\end{matrix}\right\rangle$
	3	$\left\langle \begin{matrix}([0.42,0.43,0.44,0.45];0.80),\\([0.40,0.42,0.44,0.46];0.20)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.96,0.97,0.98,0.99];0.30),\\([0.95,0.96,0.98,0.99];0.50)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.27,0.29,0.31,0.33];0.80),\\([0.25,0.27,0.32,0.34];0.10)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.18,0.20,0.22,0.24];0.70),\\([0.16,0.18,0.24,0.26];0.30)\end{matrix}\right\rangle$
G_3	1	$\left\langle \begin{matrix}([0.32,0.33,0.34,0.35];0.60),\\([0.30,0.32,0.34,0.36];0.40)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.92,0.94,0.96,0.98];0.20),\\([0.90,0.94,0.96,0.98];0.10)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.39,0.40,0.41,0.42];0.30),\\([0.36,0.40,0.42,0.46];0.70)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.20,0.24,0.26,0.28];0.60),\\([0.18,0.22,0.26,0.28];0.40)\end{matrix}\right\rangle$
	2	$\left\langle \begin{matrix}([0.34,0.36,0.38,0.40];0.40),\\([0.30,0.34,0.38,0.42];0.20)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.82,0.84,0.86,0.88];0.40),\\([0.80,0.84,0.86,0.88];0.50)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.43,0.45,0.47,0.49];0.10),\\([0.42,0.44,0.48,0.50];0.10)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.30,0.32,0.34,0.36];0.10),\\([0.30,0.31,0.34,0.38];0.20)\end{matrix}\right\rangle$
	3	$\left\langle \begin{matrix}([0.42,0.44,0.46,0.48];0.50),\\([0.40,0.44,0.47,0.49];0.50)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.86,0.87,0.88,0.89];0.60),\\([0.84,0.86,0.88,0.90];0.10)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.31,0.33,0.35,0.37];0.50),\\([0.30,0.32,0.36,0.38];0.40)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.10,0.15,0.20,0.25];0.60),\\([0.10,0.15,0.24,0.29];0.40)\end{matrix}\right\rangle$
G_4	1	$\left\langle \begin{matrix}([0.51,0.52,0.53,0.54];0.80),\\([0.50,0.51,0.54,0.56];0.20)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.78,0.79,0.80,0.81];0.40),\\([0.76,0.78,0.81,0.83];0.30)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.52,0.53,0.54,0.55];0.70),\\([0.50,0.52,0.54,0.56];0.30)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.46,0.49,0.52,0.55];0.10),\\([0.45,0.48,0.53,0.57];0.20)\end{matrix}\right\rangle$
	2	$\left\langle \begin{matrix}([0.29,0.30,0.32,0.34];0.70),\\([0.28,0.29,0.33,0.35];0.20)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.72,0.73,0.74,0.75];0.30),\\([0.70,0.72,0.74,0.76];0.40)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.28,0.30,0.32,0.34];0.50),\\([0.26,0.30,0.33,0.38];0.40)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.52,0.55,0.58,0.61];0.90),\\([0.50,0.54,0.59,0.62];0.10)\end{matrix}\right\rangle$
	3	$\left\langle \begin{matrix}([0.31,0.33,0.35,0.37];0.80),\\([0.28,0.32,0.36,0.38];0.20)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.90,0.91,0.92,0.93];0.70),\\([0.90,0.91,0.92,0.94];0.30)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.42,0.44,0.46,0.48];0.50),\\([0.40,0.44,0.48,0.50];0.30)\end{matrix}\right\rangle$	$\left\langle \begin{matrix}([0.33,0.36,0.39,0.42];0.10),\\([0.32,0.36,0.40,0.44];0.20)\end{matrix}\right\rangle$

Table 12 (continued)

	k	C_1	C_2	C_3	C_4
G_5	1	$\left\langle ([0.22, 0.23, 0.24, 0.25];0.10), ([0.20, 0.22, 0.24, 0.26];0.20) \right\rangle$	$\left\langle ([0.68, 0.69, 0.70, 0.71];0.20), ([0.66, 0.68, 0.70, 0.72];0.40) \right\rangle$	$\left\langle ([0.38, 0.39, 0.40, 0.41];0.50), ([0.36, 0.38, 0.40, 0.42];0.50) \right\rangle$	$\left\langle ([0.52, 0.53, 0.54, 0.55];0.70), ([0.50, 0.52, 0.54, 0.56];0.30) \right\rangle$
	2	$\left\langle ([0.32, 0.35, 0.38, 0.41];0.50), ([0.31, 0.34, 0.38, 0.42];0.30) \right\rangle$	$\left\langle ([0.56, 0.57, 0.58, 0.59];0.60), ([0.54, 0.56, 0.58, 0.60];0.40) \right\rangle$	$\left\langle ([0.52, 0.54, 0.57, 0.61];0.30), ([0.50, 0.53, 0.58, 0.64];0.30) \right\rangle$	$\left\langle ([0.31, 0.33, 0.37, 0.40];0.10), ([0.29, 0.32, 0.38, 0.44];0.40) \right\rangle$
	3	$\left\langle ([0.36, 0.37, 0.38, 0.39];0.10), ([0.35, 0.37, 0.39, 0.41];0.20) \right\rangle$	$\left\langle ([0.71, 0.74, 0.77, 0.80];0.70), ([0.70, 0.72, 0.78, 0.81];0.30) \right\rangle$	$\left\langle ([0.41, 0.43, 0.45, 0.47];0.50), ([0.38, 0.42, 0.46, 0.49];0.40) \right\rangle$	$\left\langle ([0.21, 0.22, 0.23, 0.24];0.30), ([0.20, 0.21, 0.24, 0.26];0.70) \right\rangle$
G_6	1	$\left\langle ([0.88, 0.89, 0.90, 0.91];0.30), ([0.86, 0.88, 0.90, 0.94];0.70) \right\rangle$	$\left\langle ([0.21, 0.22, 0.23, 0.24];0.20), ([0.19, 0.21, 0.24, 0.26];0.60) \right\rangle$	$\left\langle ([0.94, 0.95, 0.96, 0.97];0.30), ([0.93, 0.94, 0.97, 0.99];0.10) \right\rangle$	$\left\langle ([0.87, 0.88, 0.89, 0.90];0.40), ([0.86, 0.88, 0.89, 0.91];0.60) \right\rangle$
	2	$\left\langle ([0.90, 0.92, 0.94, 0.96];0.90), ([0.89, 0.91, 0.95, 0.97];0.10) \right\rangle$	$\left\langle ([0.10, 0.14, 0.18, 0.22];0.70), ([0.10, 0.13, 0.19, 0.23];0.30) \right\rangle$	$\left\langle ([0.89, 0.91, 0.93, 0.95];0.50), ([0.88, 0.90, 0.94, 0.96];0.30) \right\rangle$	$\left\langle ([0.91, 0.93, 0.96, 0.98];0.40), ([0.90, 0.93, 0.96, 0.99];0.10) \right\rangle$
	3	$\left\langle ([0.96, 0.97, 0.98, 0.99];0.30), ([0.90, 0.94, 0.98, 0.99];0.30) \right\rangle$	$\left\langle ([0.12, 0.14, 0.16, 0.18];0.20), ([0.10, 0.14, 0.18, 0.22];0.20) \right\rangle$	$\left\langle ([0.91, 0.92, 0.93, 0.94];0.10), ([0.90, 0.92, 0.94, 0.96];0.10) \right\rangle$	$\left\langle ([0.90, 0.92, 0.94, 0.96];0.50), ([0.88, 0.92, 0.96, 0.98];0.50) \right\rangle$
G_7	1	$\left\langle ([0.32, 0.36, 0.40, 0.44];0.30), ([0.28, 0.32, 0.40, 0.48];0.20) \right\rangle$	$\left\langle ([0.67, 0.68, 0.69, 0.70];0.10), ([0.65, 0.68, 0.69, 0.72];0.70) \right\rangle$	$\left\langle ([0.23, 0.25, 0.27, 0.29];0.20), ([0.22, 0.25, 0.28, 0.30];0.60) \right\rangle$	$\left\langle ([0.46, 0.47, 0.48, 0.49];0.50), ([0.45, 0.46, 0.49, 0.50];0.40) \right\rangle$
	2	$\left\langle ([0.14, 0.17, 0.20, 0.23];0.50), ([0.13, 0.17, 0.21, 0.24];0.10) \right\rangle$	$\left\langle ([0.72, 0.73, 0.74, 0.75];0.70), ([0.70, 0.72, 0.75, 0.77];0.30) \right\rangle$	$\left\langle ([0.17, 0.19, 0.21, 0.23];0.90), ([0.15, 0.18, 0.22, 0.26];0.10) \right\rangle$	$\left\langle ([0.71, 0.73, 0.75, 0.77];0.30), ([0.70, 0.73, 0.76, 0.79];0.10) \right\rangle$
	3	$\left\langle ([0.55, 0.57, 0.59, 0.61];0.40), ([0.55, 0.57, 0.59, 0.61];0.60) \right\rangle$	$\left\langle ([0.85, 0.87, 0.89, 0.91];0.60), ([0.84, 0.87, 0.90, 0.94];0.40) \right\rangle$	$\left\langle ([0.27, 0.28, 0.29, 0.30];0.80), ([0.26, 0.27, 0.30, 0.31];0.10) \right\rangle$	$\left\langle ([0.69, 0.72, 0.73, 0.74];0.60), ([0.68, 0.71, 0.74, 0.76];0.40) \right\rangle$

Table 12 (continued)

	k	C_1	C_2	C_3	C_4
G_8	1	$\left\langle \begin{array}{l} ([0.62, 0.64, 0.67, 0.71];0.20), \\ ([0.60, 0.63, 0.68, 0.74];0.40) \end{array} \right\rangle$	$\left\langle \begin{array}{l} ([0.91, 0.92, 0.93, 0.94];0.30), \\ ([0.89, 0.91, 0.94, 0.96];0.40) \end{array} \right\rangle$	$\left\langle \begin{array}{l} ([0.34, 0.38, 0.42, 0.46];0.40), \\ ([0.33, 0.37, 0.43, 0.49];0.40) \end{array} \right\rangle$	$\left\langle \begin{array}{l} ([0.31, 0.34, 0.36, 0.38];0.60), \\ ([0.29, 0.33, 0.37, 0.39];0.40) \end{array} \right\rangle$
	2	$\left\langle \begin{array}{l} ([0.41, 0.42, 0.44, 0.46];0.70), \\ ([0.40, 0.41, 0.45, 0.47];0.30) \end{array} \right\rangle$	$\left\langle \begin{array}{l} ([0.88, 0.89, 0.90, 0.91];0.90), \\ ([0.86, 0.89, 0.91, 0.92];0.10) \end{array} \right\rangle$	$\left\langle \begin{array}{l} ([0.47, 0.49, 0.50, 0.52];0.80), \\ ([0.45, 0.48, 0.51, 0.54];0.10) \end{array} \right\rangle$	$\left\langle \begin{array}{l} ([0.27, 0.28, 0.29, 0.30];0.10), \\ ([0.25, 0.27, 0.29, 0.30];0.20) \end{array} \right\rangle$
	3	$\left\langle \begin{array}{l} ([0.38, 0.40, 0.42, 0.44];0.20), \\ ([0.36, 0.38, 0.42, 0.46];0.30) \end{array} \right\rangle$	$\left\langle \begin{array}{l} ([0.84, 0.86, 0.88, 0.90];0.70), \\ ([0.82, 0.84, 0.88, 0.92];0.30) \end{array} \right\rangle$	$\left\langle \begin{array}{l} ([0.57, 0.58, 0.59, 0.60];0.60), \\ ([0.56, 0.58, 0.60, 0.62];0.10) \end{array} \right\rangle$	$\left\langle \begin{array}{l} ([0.42, 0.46, 0.50, 0.54];0.40), \\ ([0.38, 0.45, 0.52, 0.58];0.40) \end{array} \right\rangle$

Table 13 Normalized judgment matrix, $\overline{J_{ij}} = [t_{ij}]_{8\times4}$

	C_1	C_2	C_3	C_4
G_1	$\left\langle \begin{array}{l}([0.26, 0.28, 0.30, 0.31];0.37),\\ ([0.25, 0.27, 0.31, 0.33];0.23)\end{array} \right\rangle$	$\left\langle \begin{array}{l}([0.86, 0.87, 0.88, 0.89];0.57),\\ ([0.85, 0.87, 0.88, 0.91];0.47)\end{array} \right\rangle$	$\left\langle \begin{array}{l}([0.41, 0.45, 0.50, 0.54];0.44),\\ ([0.40, 0.43, 0.52, 0.56];0.57)\end{array} \right\rangle$	$\left\langle \begin{array}{l}([0.48, 0.50, 0.51, 0.52];0.74),\\ ([0.47, 0.49, 0.51, 0.53];0.20)\end{array} \right\rangle$
G_2	$\left\langle \begin{array}{l}([0.33, 0.34, 0.36, 0.37];0.60),\\ ([0.31, 0.34, 0.36, 0.38];0.30)\end{array} \right\rangle$	$\left\langle \begin{array}{l}([0.87, 0.88, 0.89, 0.90];0.30),\\ ([0.86, 0.88, 0.89, 0.91];0.44)\end{array} \right\rangle$	$\left\langle \begin{array}{l}([0.42, 0.44, 0.45, 0.47];0.70),\\ ([0.41, 0.43, 0.46, 0.48];0.24)\end{array} \right\rangle$	$\left\langle \begin{array}{l}([0.39, 0.41, 0.44, 0.45];0.70),\\ ([0.38, 0.40, 0.45, 0.47];0.30)\end{array} \right\rangle$
G_3	$\left\langle \begin{array}{l}([0.36, 0.38, 0.39, 0.41];0.50),\\ ([0.34, 0.37, 0.40, 0.42];0.40)\end{array} \right\rangle$	$\left\langle \begin{array}{l}([0.87, 0.88, 0.90, 0.92];0.40),\\ ([0.85, 0.88, 0.90, 0.92];0.24)\end{array} \right\rangle$	$\left\langle \begin{array}{l}([0.38, 0.39, 0.41, 0.43];0.30),\\ ([0.36, 0.38, 0.42, 0.45];0.47)\end{array} \right\rangle$	$\left\langle \begin{array}{l}([0.20, 0.24, 0.27, 0.30];0.44),\\ ([0.19, 0.23, 0.28, 0.32];0.57)\end{array} \right\rangle$
G_4	$\left\langle \begin{array}{l}([0.37, 0.38, 0.40, 0.42];0.77),\\ ([0.35, 0.37, 0.41, 0.43];0.17)\end{array} \right\rangle$	$\left\langle \begin{array}{l}([0.80, 0.81, 0.82, 0.83];0.47),\\ ([0.79, 0.80, 0.82, 0.84];0.40)\end{array} \right\rangle$	$\left\langle \begin{array}{l}([0.41, 0.42, 0.44, 0.46];0.57),\\ ([0.39, 0.42, 0.45, 0.48];0.44)\end{array} \right\rangle$	$\left\langle \begin{array}{l}([0.44, 0.47, 0.50, 0.53];0.37),\\ ([0.42, 0.46, 0.51, 0.54];0.43)\end{array} \right\rangle$
G_5	$\left\langle \begin{array}{l}([0.30, 0.32, 0.34, 0.35];0.24),\\ ([0.29, 0.31, 0.34, 0.36];0.24)\end{array} \right\rangle$	$\left\langle \begin{array}{l}([0.65, 0.67, 0.68, 0.70];0.50),\\ ([0.64, 0.65, 0.69, 0.71];0.50)\end{array} \right\rangle$	$\left\langle \begin{array}{l}([0.44, 0.45, 0.47, 0.50];0.44),\\ ([0.41, 0.44, 0.48, 0.52];0.44)\end{array} \right\rangle$	$\left\langle \begin{array}{l}([0.35, 0.36, 0.38, 0.40];0.37),\\ ([0.33, 0.35, 0.39, 0.42];0.60)\end{array} \right\rangle$
G_6	$\left\langle \begin{array}{l}([0.91, 0.93, 0.94, 0.95];0.50),\\ ([0.88, 0.91, 0.94, 0.97];0.50)\end{array} \right\rangle$	$\left\langle \begin{array}{l}([0.14, 0.17, 0.19, 0.21];0.37),\\ ([0.13, 0.16, 0.20, 0.24];0.47)\end{array} \right\rangle$	$\left\langle \begin{array}{l}([0.91, 0.93, 0.94, 0.95];0.30),\\ ([0.90, 0.92, 0.95, 0.97];0.17)\end{array} \right\rangle$	$\left\langle \begin{array}{l}([0.89, 0.91, 0.93, 0.95];0.44),\\ ([0.88, 0.91, 0.94, 0.96];0.47)\end{array} \right\rangle$
G_7	$\left\langle \begin{array}{l}([0.34, 0.37, 0.40, 0.43];0.40),\\ ([0.32, 0.35, 0.40, 0.44];0.37)\end{array} \right\rangle$	$\left\langle \begin{array}{l}([0.75, 0.76, 0.77, 0.79];0.47),\\ ([0.73, 0.76, 0.78, 0.81];0.50)\end{array} \right\rangle$	$\left\langle \begin{array}{l}([0.22, 0.24, 0.26, 0.27];0.64),\\ ([0.21, 0.24, 0.27, 0.29];0.30)\end{array} \right\rangle$	$\left\langle \begin{array}{l}([0.62, 0.64, 0.65, 0.67];0.47),\\ ([0.61, 0.64, 0.66, 0.68];0.37)\end{array} \right\rangle$
G_8	$\left\langle \begin{array}{l}([0.47, 0.49, 0.51, 0.54];0.37),\\ ([0.45, 0.47, 0.52, 0.56];0.50)\end{array} \right\rangle$	$\left\langle \begin{array}{l}([0.88, 0.89, 0.90, 0.92];0.64),\\ ([0.86, 0.88, 0.91, 0.94];0.30)\end{array} \right\rangle$	$\left\langle \begin{array}{l}([0.46, 0.48, 0.50, 0.53];0.60),\\ ([0.45, 0.48, 0.51, 0.55];0.20)\end{array} \right\rangle$	$\left\langle \begin{array}{l}([0.34, 0.36, 0.38, 0.41];0.37),\\ ([0.31, 0.35, 0.39, 0.42];0.44)\end{array} \right\rangle$

Table 14 Ranking order comparison of various government intervention strategies

Existing similarity methods	Similarity measure values for the government intervention strategies considered								Ranking order	Best
	(G_1, G^*)	(G_2, G^*)	(G_3, G^*)	(G_4, G^*)	(G_5, G^*)	(G_6, G^*)	(G_7, G^*)	(G_8, G^*)		
S_{CCH}Chen and Chen (2009)	0.0001	0.0001	0.0002	0.0002	0	0.0003	0	0	$G_6 > G_4 \approx G_3 > G_1 \approx G_2 > G_8 \approx G_7 \approx G_5$	G_6
S_{WC}Wei and Chen (2009)	0.0003	0.0002	0.0003	0.0005	0	0.0013	0	0	$G_6 > G_4 > G_3 \approx G_1 > G_2 > G_8 \approx G_7 \approx G_5$	G_6
S_{CH}Chen 2011)	0.0554	0.0525	0.0579	0.0620	0.0061	0.0742	0.0406	0.0513	$G_6 > G_4 > G_3 > G_1 > G_2 > G_8 > G_7 > G_5$	G_6
S_{FB}Farhadinia (2012)	0.0003	0.0002	0.0004	0.0005	0	0.0014	0.0001	0.0002	$G_6 > G_4 > G_3 > G_1 > G_2 \approx G_8 > G_7 > G_5$	G_6
S_{YC}Ye (2012a)	0.5546	0.5507	0.5649	0.5656	0.5275	0.5764	0.5434	0.5498	$G_6 > G_4 > G_3 > G_1 > G_2 > G_8 > G_7 > G_5$	G_6
S_{YH}Ye (2012b)	0.5865	0.5860	0.5869	0.5917	0.5754	0.5981	0.5767	0.5854	$G_6 > G_4 > G_3 > G_1 > G_2 > G_8 > G_7 > G_5$	G_6
S_{YE}Ye (2012b)	0.5179	0.5145	0.5179	0.5225	0.4362	0.5252	0.4929	0.5092	$G_6 > G_4 > G_3 \approx G_1 > G_2 > G_8 > G_7 > G_5$	G_6
S_TTang et al. (2017)	0.4992	0.4981	0.5177	0.5187	0.4756	0.5320	0.4941	0.4971	$G_6 > G_4 > G_3 > G_1 > G_2 > G_8 > G_7 > G_5$	G_6
S_{QY}Yue et al. (2019)	0.1867	0.1843	0.1995	0.2057	0.1258	0.2157	0.1360	0.1802	$G_6 > G_4 > G_3 > G_1 > G_2 > G_8 > G_7 > G_5$	G_6
S_{DH}Dinagar and Helena (2019)	0.0681	0.0564	0.0714	0.0757	0.0416	0.0964	0.0424	0.0502	$G_6 > G_4 > G_3 > G_1 > G_2 > G_8 > G_7 > G_5$	G_6
S_{TIFN}(Proposed)	**0.6540**	**0.6501**	**0.6638**	**0.6754**	**0.5873**	**0.6772**	**0.6107**	**0.6275**	$G_6 > G_4 > G_3 > G_1 > G_2 > G_8 > G_7 > G_5$	G_6

Step-2: The ideal solution or the ideal intervention strategy (G^*) depending upon the criteria-type takes the following form,

$G^* = \langle([1, 1, 1, 1];1), ([0, 0, 0, 0];0)\rangle$, for profit criterion.

$G^* = \langle([0, 0, 0, 0];0), ([1, 1, 1, 1];1)\rangle$, for cost criterion.

From the criteria(s) we have considered, we can say that criteria C_1(*Ease of implementation*), C_3(*Effectiveness to prevent COVID-19 spread*), and C_4(*High acceptability to citizens*) are profit-type, whereas C_2(*Total cost*) is a cost-type criterion.

Step-3: Finally, our proposed measure evaluates the SM results for various intervention strategy alternatives as

$$S_{TIFN}(G_1, G^*) = 0.6540,\ S_{TIFN}(G_2, G^*) = 0.6501,\ S_{TIFN}(G_3, G^*) = 0.6638,\ S_{TIFN}(G_4, G^*) = 0.6754,$$
$$S_{TIFN}(G_5, G^*) = 0.5873,\ S_{TIFN}(G_6, G^*) = 0.6772,\ S_{TIFN}(G_7, G^*) = 0.6107,\ S_{TIFN}(G_8, G^*) = 0.6275,$$

Thus, our proposed measure determines G_6(*Vaccinations*) as the most preferable and suitable solid intervention strategy that can be implemented on a large scale. Consequently, the preference ranking order (decreasing) for the available set of alternatives is obtained as,

G_6(Vaccinations) > G_4 (Quarantining patients and those suspected of infection) > G_3 (Social distancing) > G_1 (Curfew) > G_2 (External border restrictions) > G_8 (Internal border restrictions) > G_7 (Restriction of mass gatherings) > G_5 (Closure of schools)

The more the people get vaccinated, the more lives could be saved. Vaccination drives to the remote areas and spreading general awareness among people about the importance of vaccines would be really helpful in this regard.

6.3.1 Comparative study

Table 14 presents a clear comparison of the SM results obtained for variousgovernment intervention strategy alternatives. Although, all of the similarity methods under studyobtain the optimal intervention strategy as '*Vaccination* (G_6)', yet certain irregularities are

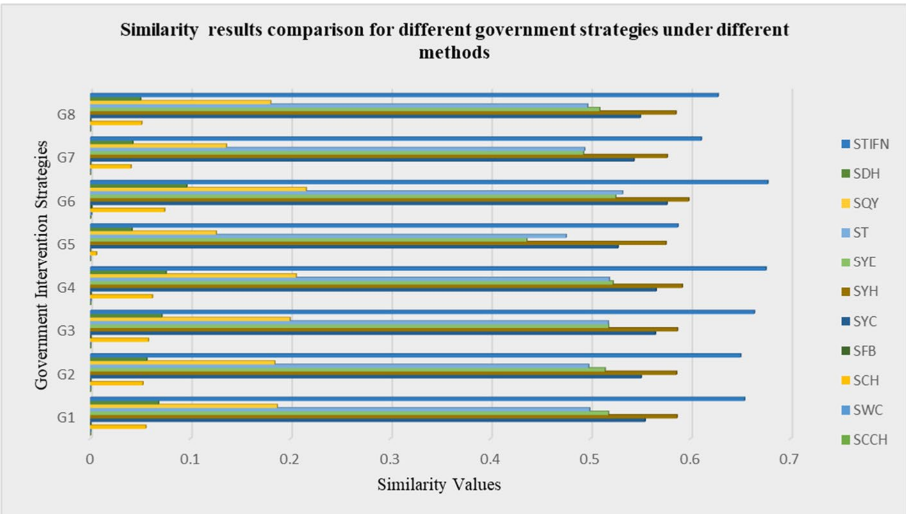


Fig. 15 Optimal intervention strategy evaluated under different methods

Table 15 Works done previously on GTrIFNs

Authors	Work done	Application
Chen and Chen(2009)	SM based on degree of difference of spread between interval-valued GTrIFNs	Fuzzy Risk Analysis Problem
Wei and Chen (2009)	Proposed a SM based on the concepts of geometric distance, perimeter, and heights of GTrIFNs	Fuzzy Risk Analysis Problem
Chen (2011)	Proposed a SM between GTrIFNs based on the quadratic mean operator	Fuzzy Recommendation problems
Farhadinia (2012)	Proposed SM based on geometric distance and perimeter concepts of GTrIFNs	Fuzzy Risk Analysis
Ye (2012a)	Proposed SM based on vector SMs of Jaccard, Dice, and cosine for GTrIFNs	Optimum Investment Problem
Ye (2012b)	Proposed few distance-based SMs for GTrIFNs utilizing the expected value concept	Management of Mega Manufacturing Companies
Farhadinia and Ban (2013)	Proposed new SMs for GTrIFNs and interval-valued GTrIFNs based on confidence degree	Fuzzy Risk Analysis
Chakraborty et al. (2015)	Proposed new arithmetic operations based on (α, β)-cut method, vertex method, and extension principle method for GTrIFNs	Fuzzy Transportation Problem
Mondal and Roy (2014)	Proposed non-linear arithmetic operations for GTrIFNs	Bank Accounts Management Problem
Gani and Mohamed (2015)	Proposed ranking scheme for GTrIFNs	Intuitionistic Fuzzy Assignment Problem
Jamkhaneh (2016)	Proposed ranking scheme based on value and ambiguity of GTrIFNs	-NIL-
Bărbăcioru (2016)	Applied Maple software to evaluate the ranking of GTrIFNs	-NIL-
Garai et al. (2018)	Proposed possibility-necessity-credibility measures on GTrIFNs	Multiproduct Manufacturing System
Uthra et al. (2017)	Devised a new ranking scheme for GTrIFNs	Fuzzy Transportation Problem
Tang et al. (2017)	Proposed Dice SMs for GTrIFNs	Enterprise Resource Planning Systems
Hunwisai et al. (2019)	Devised a ranking scheme incorporating the cut-set technique for GTrIFNs	Fuzzy Transportation Problem
Yue et al. (2019)	Developed a two-sided matching model/algorithm for GTrIFNs	Smart Environment Protection
Dinagar and Helena (2019)	Proposed a SM for GTrIFNs with the help of notions such as “centroid of centroids” and “area between the centroid of centroids” and also devised a ranking function	-NIL-

observed with some of those methods. For instance,S_{CCH}[10]method fails to distinguish between the pairs of alternatives $\{G_4$ and $G_3\}$,$\{G_1$ and $G_2\}$, $\{G_8, G_7,$ and $G_5\}$; S_{WC}Wei and Chen 2009) method could not distinguish between the pairs of alternatives $\{G_3$ and $G_1\}$, $\{G_8, G_7,$ and $G_5\}$;S_{FB}[22]method could not distinguish between the alternatives G_2 and G_8. Thus, the usefulness and versatility of our proposed measure are demonstrated once again.

Here, we provide graphical illustration in Fig. 15, for better visualizing the final results obtained for the most effective government intervention measure to control the pandemic spread.

6.4 Results and its managerial implications

From the results obtained for the case studies which we have carried out, it can be asserted that our proposed measure is capable of performing well and achieving logical outcomes in diverse environments. Whereas the lacunas of the existing measures were exposed on a number of occasions. For example, for the optimum investment problem, some of the existing measures obtained negative values of similarity which is absurd. Even for the COVID-19 related case studies, similar type of outcomes are observed. For pairs of alternatives where it is intuitive that there exists a certain value of similarity between them, in those situations some of the existing methods found total a dissimilarity between them. Also, for distinct and non-similar pairs of GTrIFNs, several existing measures failed to distinguish between them. Some measures even overestimated and underestimated the values of similarity between GTrIFNs under study. However, our proposed measure is free from such discrepancies as mentioned above and it is far more efficient than the others. This suggests the robustness, feasibility, and rationality of our newly defined SM. Thus, our proposed SM may be utilized in a variety of decision scenarios considering the ability of GTrIFNs to efficiently handle the ambiguous and vague nature of the information. In the literature, although one can find a humongous amount of research works being carried out on IFSs or normalized TrIFNs, there is a dearth of GTrIFN-based measures. Some of the works pertinent to this field of research are listed below in Table 15.

For the COVID-19 medicine selection problem, the best medicine alternative is *Tocilizumab* (M_3), followed by *Hydroxychloroquine* (M_1), *Remdesivir* (M_2), and *Convalescent Plasma* (M_4). *Tocilizumab* has better working efficiency and is free from notable side effects, but if there occurs a shortage of this medicine, then *Hydroxychloroquine* can be recommended. Similarly, *Remdesivir* and *Convalescent Plasma* therapy can be suggested as per convenience.

In the case of the healthcare waste disposal problem, our proposed measure evaluated ***Incineration*** (H_3), ***Integrated steam sterilization system*** (H_5), and ***Microwave*** (H_1) as the top three waste disposal techniques. The ***Incineration*** technique however has some environmental concerns when practiced over a long period of time. Because in India, people are already experiencing the adverse effects of air pollution and further it should not be worsened by incinerating or burning tons and tons of hazardous wastes. Therefore, certain environment-friendly alternative procedures can be introduced.

For the final case study of solid governmental intervention strategy selection, our proposed measure determined the top five alternatives in the Indian context, which are ***Vaccinations*** (G_6), ***Quarantining patients and those suspected of infection*** (G_4), ***Social distancing*** (G_3), ***Curfew*** (G_1), and ***External border restrictions*** (G_2). ***Vaccinations*** that are developed for a certain genomic sequence of the virus will surely help in fighting that particular variant, but it is cumbersome to predict that it will be effective against all mutated versions of the virus. Therefore, the fight against COVID-19 is not over yet and more robust intervention strategies can be suggested in the future.

7 Conclusion

The role of SM is very vital in solving a wide variety of decision making scenarios and it is one of the crucial concepts in human cognition-based thought processes. In our article, we have devised a novel SM for GTrIFNs and it is then applied to a medicine selection procedure, for the treatment of patients infected with the COVID-19 virus. GTrIFNs have several advantages due to which they have been increasingly applied to MCDM problems, some of which are- they are capable of representing the ill-known quantities at ease, they can assess the available information more holistically, and they also add several dimensions to the decision information at hand. Our proposed measure very effortlessly incorporates the concepts of expected values, variances, and heights of GTrIFNs into its expression. Also, it is capable of outperforming the deficiencies exhibited by most of the existing approaches, which is validated through a meticulous comparative study. The final results obtained with our proposed measure are intuitive, logical, commonsensical, and rational. Moreover, a robust group decision making algorithm presented together with a numerical illustrative example indicates the veracity and applicability of our designed SM. Consequently, the effectiveness of our measure in selecting the optimum medicine for COVID-19 treatment is worth noticing. Being able to attempt a COVID-19 related scenario is in itself a daunting task to accomplish and which brings in numerous challenges due to the lack of limited available evidence on the virus causing this outbreak.Also, the outcome for the best suitable medicine coincides with several other existing methods, which is vivid from the comparative study presented. Therefore, the key results deduced in this article are affirmative to benefit the health workers and other frontline workers in making the optimum choice for the speedy recovery of patients laid down with the COVID-19 virus.Thus, it can be inferred that our newly devised measure has enormous potentiality and strong inherent capability, which enables it to be applied to a wide range of decision making instances. In addition, our proposed measure is also effective in handling two more serious concerns resulting from the pandemic era: first being the problem of a worthwhile waste disposal strategy for eradicating the hazardous wastes produced each day and second is the determination of solid intervention measures by the government for controlling the virus spread. Thus, the proficiency of our newly constructed measure is even further justified.

The contributions and originality of our article can be pinned down to the following points:

- Firstly, we defined the concept of “expected value” and “variance” for GTrIFNs and deduced their mathematical forms. Both these quantities are trait-defining parameters that describe the behavior of a fuzzy number/fuzzy set and how they would behave over a certain period of time. Evaluating such crucial indices have become a topic of significant concern among researchers. In the literature, several mathematicians have determined the mathematical form for expected value, variance, standard deviation, etc., with the help of possibility theoretic concepts. But, none of them utilized the α -cut technique for evaluating the same. Hence, as a novel venture, we have added a new direction by exploiting the more conventional α-cut procedure for evaluation.

- Secondly, we have constructed a novel SM for GTrIFNs which involves the operations between membership and non-membership components of GTrIFNs, their heights, their expected values, and also their variances. We have shown that our similarity function satisfies all the basic properties and axioms of an SM. Thereafter, we illustrate certain desirable properties of it. The main advantage of our SM expression is that it is relatively simpler than most of the other methods, and it is also capable of yielding precise and efficient outcomes.
- Thirdly, we discussed a group decision making procedure based on our newly developed measure. Decision making in groups has always been a perplexing job since it involves a difference of opinions among various decision makers, different levels of expertise, a difference of perception, etc. Therefore, to accommodate the highs and lows of opinions from decision makers and arriving at a final conclusion is much more challenging and time taking. In this regard, our proposed measure can be fruitfully applied in tackling such complex scenarios. For better visualization, a suitable numerical illustration of an investment company is contemplated.
- Fourthly, we have shown how our proposed measure can also handle various COVID-19 associated problems. In the first problem, we discuss the selection of the best medicines for the treatment of COVID-19 infected patients. Initially, when the information about the virus was very limited, there existed no such prescribed or specific treatment to handle the virus. Healthcare officials resorted to certain therapies to minimize the complications caused by the virus. Thus, our approach helped to select one such optimal therapy or drug that can be recommended to COVID-19 active patients. Our second problem discusses how in a developing country like India, the COVID-19 pandemic has expedited the number of hazardous wastes produced in a day. Those wastes if not disposed of with the help of proper disposal techniques can cause major health concerns. In this context, we demonstrate how our proposed measure can be used to determine the optimum healthcare disposal technique so that chances of COVID-19 and other associated deadly diseases (resulting from waste disposals), may be further minimized. Thirdly, we determine the best government intervention strategy that can be adopted for the safety of the citizens in the Indian context. Such strategies have been proven to be greatly effective in reducing the transmission rate of the virus.

Some key advantages of our article are:

- One major advantage of our proposed method is that, although it is developed specifically for GTrIFNs, it is not only confined to them. We can deduce results for the normalized fuzzy numbers, trapezoidal fuzzy numbers, triangular fuzzy numbers, etc., from our proposed SM expression, as per our need. Moreover, certain other higher extensions of the IFNs can be derived accordingly with the same proficiency by following our detailed evaluation procedure.
- Our evaluated mathematical forms for the “expected value” and “variance” of GTrIFNs are simple, logical, and theoretically correct since the proposed definitions do not conflict with the corresponding expressions obtained under the possibility theory environment. The calculative procedure might be different, but the final result for the expressions must be the same, which is a fundamental property and

our proposed measure satisfies that aspect comprehensively. This indicates that our proposed measure is structurally stable and well formulated.

- Our proposed measure tends to overcome most of the deficiencies exhibited by the existing measures, which is evident in several sections of the manuscript. Moreover, our SM expression is easy to understand and is capable of producing rational outcomes.
- Our proposed measure handles various issues related to COVID-19 very effectively and at ease. The detailed evaluation procedure is logical and the ability to reach a conclusion in such a daunting task surely indicates the legitimacy and reliability of our proposed SM approach.

However, a few limitations of our work are:

- This article is deprived of the possibility theoretic concepts, and in the literature, one can find articles in abundance, where possibilistic concepts bring interesting implications. Moreover, there is a scope to establish a bridge between the "possibility theoretic" and "fuzzy set theoretic" concepts, so that we develop some hybrid definitions from the best of two domains.
- Also, the normalization procedure deployed in this article, particularly in the group decision making algorithm is more of a conventional one, and therefore, there is room for improvement in the normalization technique as well.

In the future direction, we shall try to extend our proposed measure to interval-valued IFNs to handle problems related to complex group decision making, medical diagnosis, risk analysis, pattern recognition, and image processing. We shall also evaluate the forms for covariance and standard deviation of such fuzzy numbers. We shall seek potential applications of our newly developed SM into reaching a consensus among the experts involved in group decision making scenarios. Moreover, we shall think of some bio-medical waste disposal techniques which will posea minimal threat to the environment. We shall also utilize different MCDM methods to introduce novel and robust intervention strategies in near future. Furthermore, some recent relevant works related to COVID-19 like forecasting with the help of time-series models like in Bocaletti et al. (2020); Chakraborty and Ghosh (2020); Li and Feng (2020); Mandal et al. (2020); Roosa et al. (2020), and studying the spatial and temporal patterns of COVID-19 in various countries like in Melin et al. (2020b); Melin and Castillo (2021), can be considered. We also envision working with type-2 fuzzy logic that would provide better flexibility to handle uncertainty in decision making problems like in Gonzalez et al. (2016); Ontiveros et al. (2018).

Appendix A

Proof: We have already obtained the expression for expected value of GTrIFN, which is

$$\Rightarrow E(\sigma) = \frac{1}{4}\left[w_1\left(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4\right) + w_2\left(\eta_1 + \eta_2 + \eta_3 + \eta_4\right)\right]$$

and also, $V(\sigma) = E(\sigma^2) - [E(\sigma)]^2$.

So, we proceed by evaluating first, $E\left(\sigma^2\right)$.

$$E(\sigma) = \frac{1}{2}\left[\begin{array}{l} \int\limits_0^{w_1}\left[\lambda_1 + \frac{1}{w_1}\left\{\alpha(\lambda_2 - \lambda_1)\right\}\right]^2 d\alpha + \int\limits_0^{w_1}\left[\lambda_4 + \frac{1}{w_1}\left\{\alpha(\lambda_3 - \lambda_4)\right\}\right]^2 d\alpha \\ + \int\limits_0^{w_2}\left[\eta_2 + \frac{1}{w_2}\left\{\alpha(\eta_1 - \eta_2)\right\}\right]^2 d\alpha + \int\limits_0^{w_2}\left[\eta_3 + \frac{1}{w_2}\left\{\alpha(\eta_4 - \eta_3)\right\}\right]^2 d\alpha \end{array}\right]$$

$$= \frac{1}{2}\left[\begin{array}{l} \int\limits_0^{w_1}\left[\lambda_1^2 + \frac{\alpha^2}{w_1^2}(\lambda_2 - \lambda_1)^2 + \frac{2\lambda_1(\lambda_2 - \lambda_1)}{w_1}\alpha\right] d\alpha + \int\limits_0^{w_1}\left[\lambda_4^2 + \frac{\alpha^2}{w_1^2}(\lambda_3 - \lambda_4)^2 + \frac{2\lambda_4(\lambda_3 - \lambda_4)}{w_1}\alpha\right] d\alpha \\ + \int\limits_0^{w_2}\left[\eta_2^2 + \frac{\alpha^2}{w_2^2}(\eta_1 - \eta_2)^2 + \frac{2\eta_2(\eta_1 - \eta_2)}{w_2}\alpha\right] d\alpha + \int\limits_0^{w_2}\left[\eta_3^2 + \frac{\alpha^2}{w_2^2}(\eta_4 - \eta_3)^2 + \frac{2\eta_3(\eta_4 - \eta_3)}{w_2}\alpha\right] d\alpha \end{array}\right]$$

$$= \frac{1}{2}\left[\begin{array}{l} \left[\lambda_1^2\alpha + \frac{\alpha^3}{3w_1^2}(\lambda_2 - \lambda_1)^2 + \frac{2\lambda_1(\lambda_2 - \lambda_1)}{w_1}\frac{\alpha^2}{2}\right]_0^{w_1} + \left[\lambda_4^2\alpha + \frac{\alpha^3}{3w_1^2}(\lambda_3 - \lambda_4)^2 + \frac{2\lambda_4(\lambda_3 - \lambda_4)}{w_1}\frac{\alpha^2}{2}\right]_0^{w_1} \\ + \left[\eta_2^2\alpha + \frac{\alpha^3}{3w_1^2}(\eta_1 - \eta_2)^2 + \frac{2\eta_2(\eta_1 - \eta_2)}{w_1}\frac{\alpha^2}{2}\right]_0^{w_2} + \left[\eta_3^2\alpha + \frac{\alpha^3}{3w_1^2}(\eta_4 - \eta_3)^2 + \frac{2\eta_3(\eta_4 - \eta_3)}{w_1}\frac{\alpha^2}{2}\right]_0^{w_2} \end{array}\right]$$

$$= \frac{1}{2}\left[\begin{array}{l} w_1\left(\frac{3\lambda_1^2 + \lambda_1^2 + \lambda_2^2 - 2\lambda_1\lambda_2 + 3\lambda_1\lambda_2 - 3\lambda_1^2 + 3\lambda_4^2 + \lambda_3^2 + \lambda_4^2 - 2\lambda_3\lambda_4 + 3\lambda_3\lambda_4 - 3\lambda_4^2}{3}\right) \\ + w_2\left(\frac{3\eta_2^2 + \eta_1^2 + \eta_2^2 - 2\eta_1\eta_2 + 3\eta_1\eta_2 - 3\eta_2^2 + 3\eta_3^2 + \eta_4^2 + \eta_3^2 - 2\eta_3\eta_4 + 3\eta_3\eta_4 - 3\eta_3^2}{3}\right) \end{array}\right]$$

$$\Rightarrow E\left(\sigma^2\right) = \frac{1}{6}\left[w_1\left(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 + \lambda_4^2 + \lambda_1\lambda_2 + \lambda_3\lambda_4\right) + w_2\left(\eta_1^2 + \eta_2^2 + \eta_3^2 + \eta_4^2 + \eta_1\eta_2 + \eta_3\eta_4\right)\right]$$

Since, $V(\sigma) = E(\sigma^2) - [E(\sigma)]^2$.

$$V(\sigma) = \frac{1}{6}\left[w_1\left(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 + \lambda_4^2 + \lambda_1\lambda_2 + \lambda_3\lambda_4\right) + w_2\left(\eta_1^2 + \eta_2^2 + \eta_3^2 + \eta_4^2 + \eta_1\eta_2 + \eta_3\eta_4\right)\right] - \frac{1}{16}\left[w_1\left(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4\right) + w_2\left(\eta_1 + \eta_2 + \eta_3 + \eta_4\right)\right]^2.$$

Hence, the proof.

Appendix B

See Figs. 16, 17, 18, 19, 20, 21, 22, 23, 24, 25

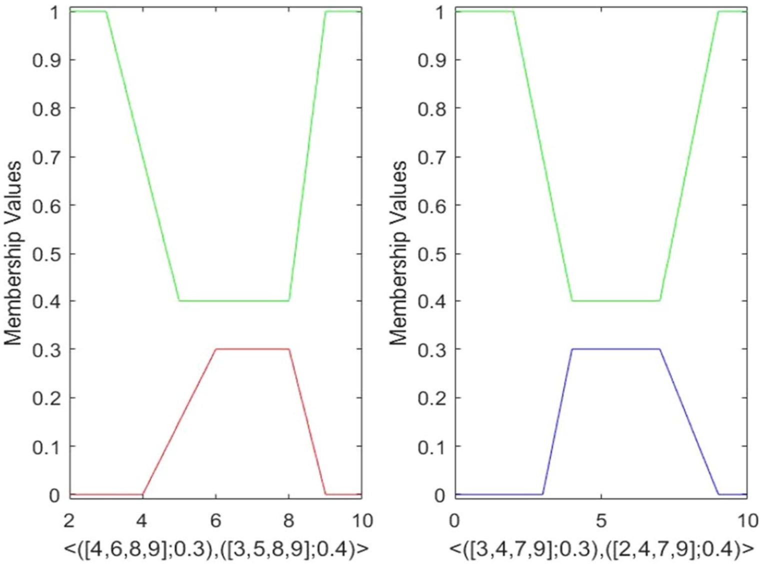


Fig. 16 Profile 1

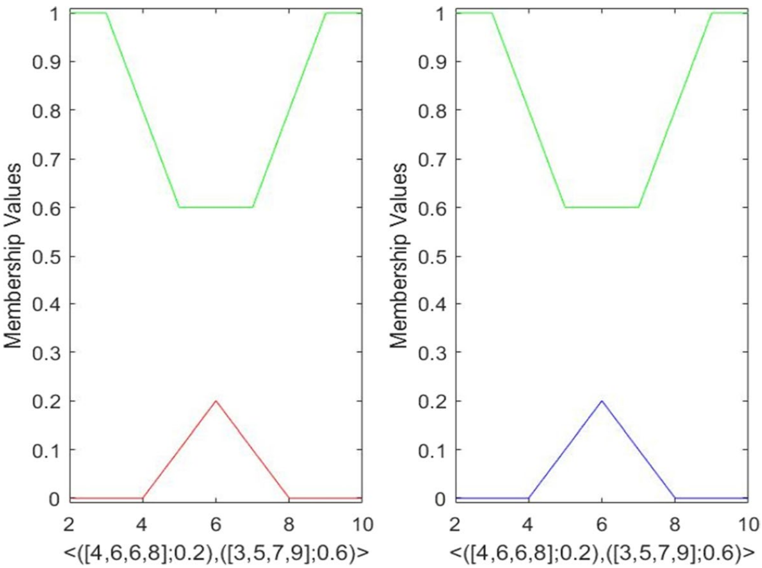


Fig. 17 Profile 2

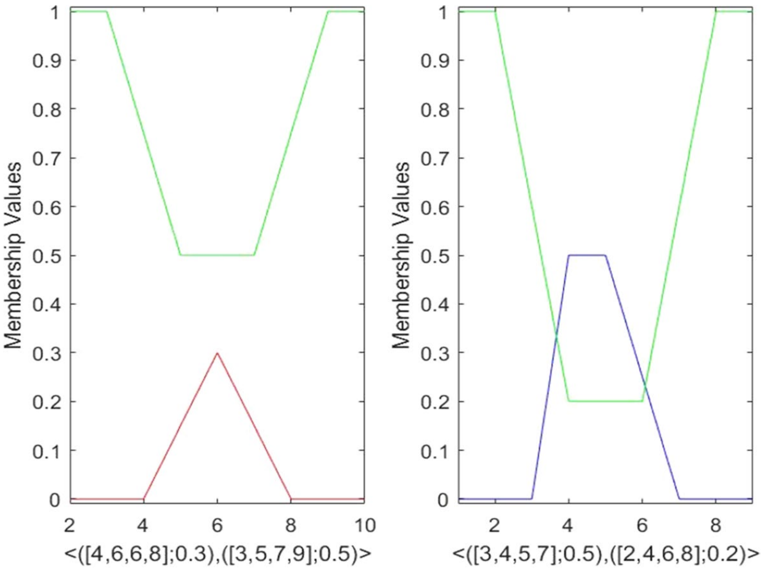


Fig. 18 Profile 3

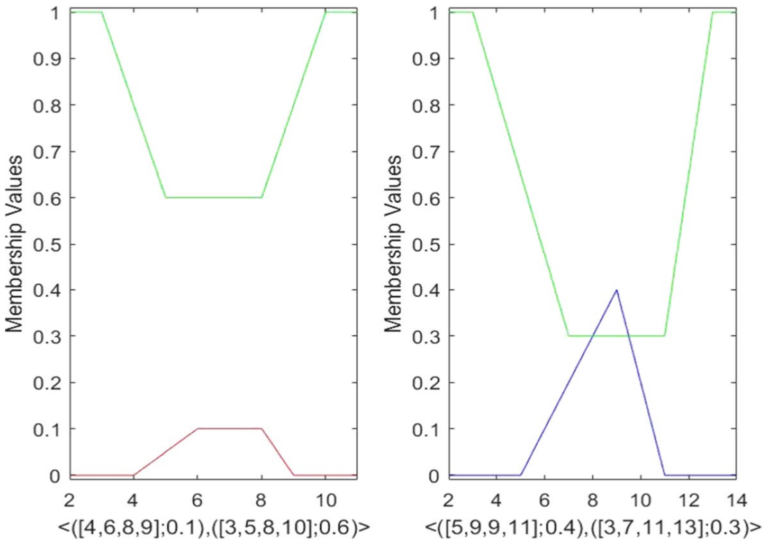


Fig. 19 Profile 4

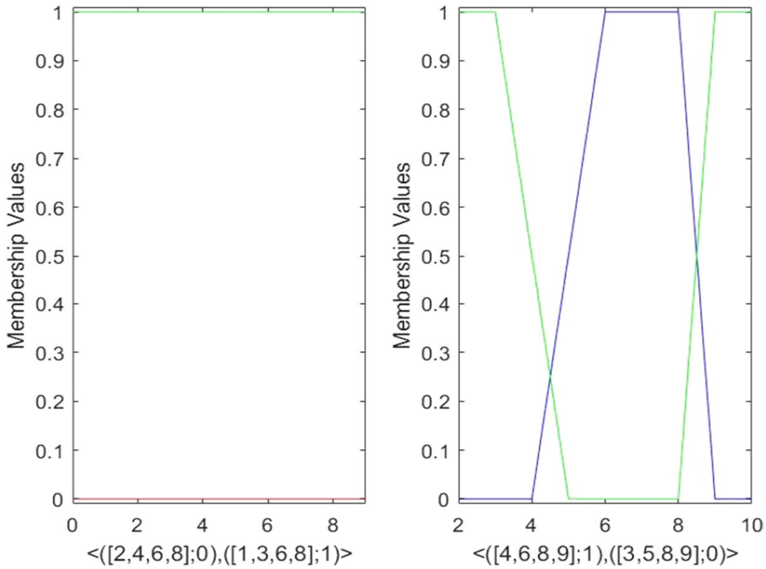


Fig. 20 Profile 5

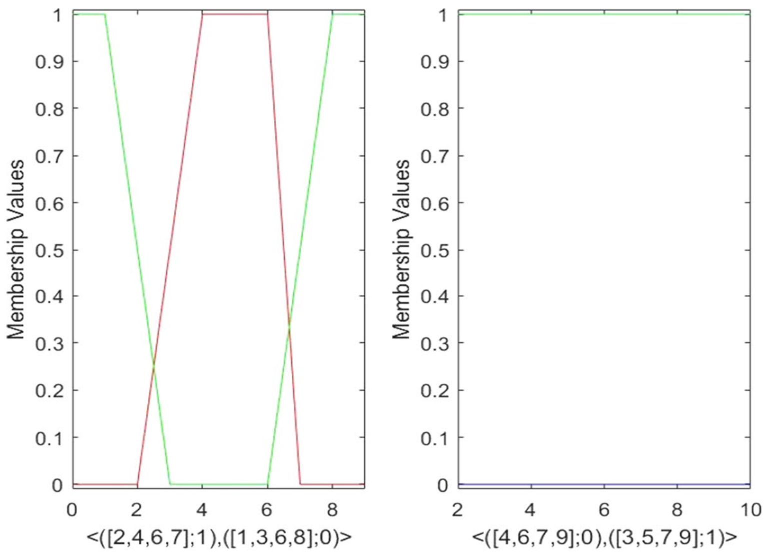


Fig. 21 Profile 6

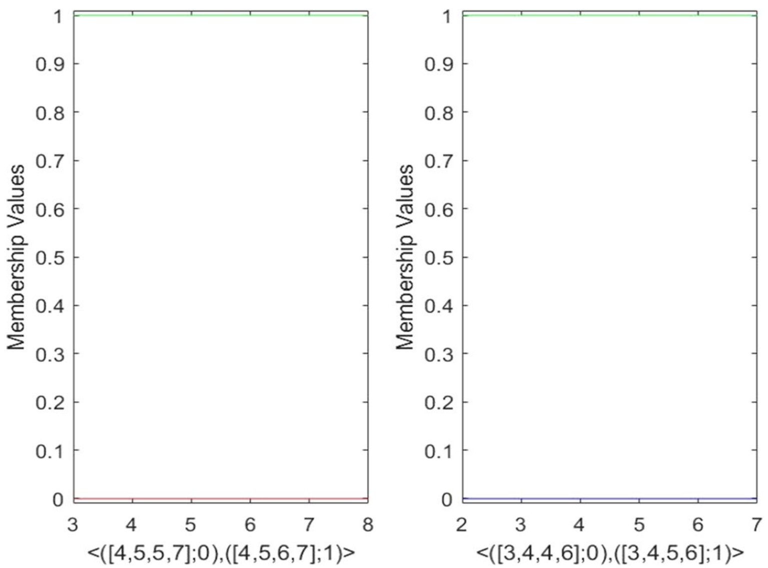


Fig. 22 Profile 7

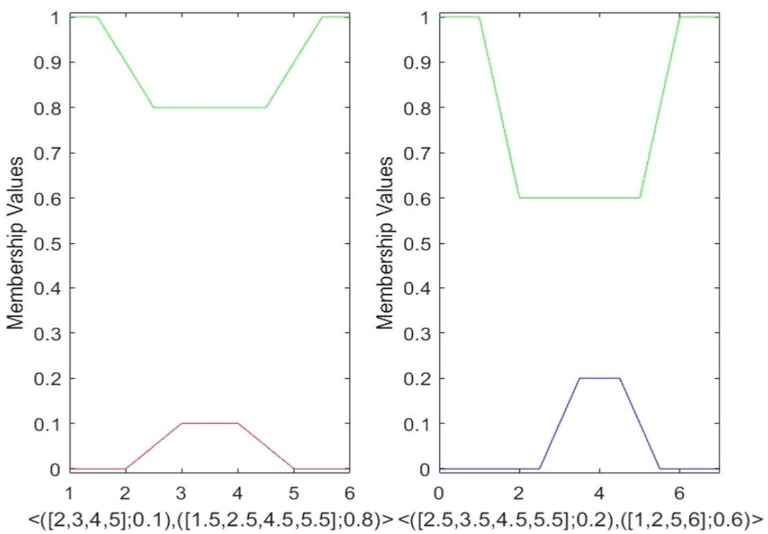


Fig. 23 Profile 8

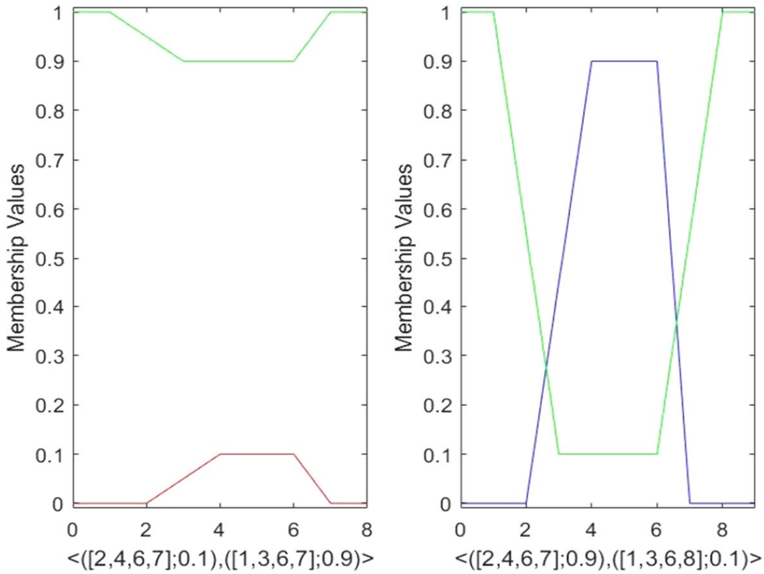


Fig. 24 Profile 9

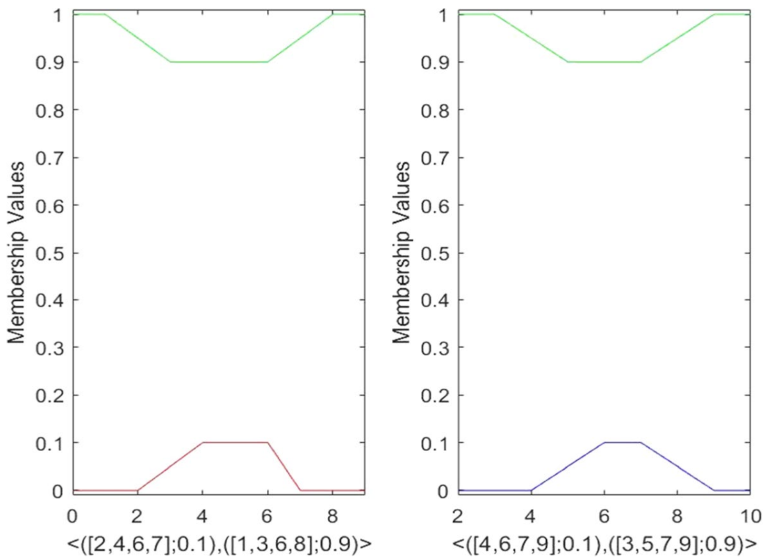


Fig. 25 Profile 10

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Data availability No data were used to support the findings of the study performed.

Declarations

Conflict of interest Authors declare that they have no potential conflict of interest.

Ethical approval This article does not contain any studies with animals performed by any of the authors.

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