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Neuro-Computing for Hall Current and MHD Effects on the Flow of Micro-Polar Nano-Fluid Between Two Parallel Rotating Plates

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Abstract

The present works focus on the effects of electric and magnetic fields on the flow of micro-polar nano-fluid between two parallel plates with rotation under the impact of Hall current (EMMN-PPRH) has considered by using Artificial Neural Networks with the scheme of Levenberg–Marquardt backpropagation (ANN-SLMB). The nonlinear PDEs are transformed into nonlinear ODEs by employing similarity variables. By varying different parameters such as coupling parameter, electric parameter, rotation parameter, viscosity parameter, Prandtl number and the Brownian motion parameter, a dataset for recommended ANN-SLMB is produced for numerous scenarios through utilizing homotopy analysis method (HAM). The ANN-SLMB training, testing and validation technique have been used to analyze the approximate solution of individual cases, and the recommended model has matched for confirmation. After that, regression analysis, MSE, and histogram investigations were utilized to validate the proposed ANN-SLMB. The recommended technique is distinguished nearest of the suggested and reference findings, with an accuracy level ranging from 10^{-09} to 10^{-11} .

List of Symbols

Keywords Nano-fluid · HAM · Levenberg–Marquardt Scheme · Artificial neural network

\bowtie	Hakeem Ullah hakeemullah 1@gmail.com	\overline{B}	Magnetic field
\bowtie	Muhammad Asif Zahoor Raja	C D_B	Fluid concentration Brownian diffusion of nano-fluids
\bowtie	Kottakkaran Sooppy Nisar n sooppy@psau.edu.sa	$\overline{E} \\ c_p$	Electric field intensity Specific heat
	Muhammad Shoaib dr.shoaib@cuiatk.edu.pk	D_T C_f	Thermophoretic diffusion of nano-fluids Skin friction coefficient
	Ajed Akbar ajedakbar@gmail.com	$h = \frac{J_w}{I}$	Distance between the plates Mass flux
	Saeed Islam saeedislam@awkum.edu.pk	J Kr	Rotation parameter
1	Department of Mathematics, Abdul Wali Khan University, Mardan 23200, Khyber Pakhtunkhwa, Pakistan	$\frac{M}{N}$	Magnetic parameter Micro-rotation angular velocity
2	Department of Mathematics, COMSATS University Islamabad, Attock Campus, Attock 43600, Pakistan	k n _e	The boundary parameter Number density of electron
3	Future Technology Research Center, National Yunlin University of Science and Technology, 123 University Road,	N_1 Nb	Coupling parameter Brownian motion
4	Section. 3, Douliou, Yunlin 64002, Taiwan, R.O.C. Department of Mathematics, College of Arts and Sciences,	O(0, 0, 0) N_2	Origin Spin gradient viscosity parameter
	Prince Sattam Bin Abdulaziz University, Wadi Aldawaser, Saudi Arabia	Nt	Thermophoretic parameter



Nu	Nusselt number
N_3	Micro-polar fluid constant
Pr	Prandtl number
Q_w	Heat flux
Re	Viscosity parameter
EI	Electric parameter
ANN	Artificial Neural Network
SLMB	Scheme Levenberg-Marquardt backpropaga-
	tion
AE	Absolute error
f,g	Velocity distributions
G	Micro-rotation profile
Re_{x}	Local Reynolds number
\overline{S}_{xy}	Cauchy stress tensor
Sc	Schmidt number
Sh	Sherwood number
t_e	Times of electron Collision
Т	Fluid temperature
$(\overline{u}, \overline{v}, \overline{w})$	Velocities components
$\overline{\upsilon}_0$	Suction/Injection velocity
е	Charge of electron
(x, y, z)	Coordinates
P_e	Pressure of electron
Greek	Letters
$ ho_f$	Base fluid density
δ	Transpiration parameter
v	Kinematic coefficient of viscosity
$\widehat{\kappa}$	Vertex viscosity
η	Similarity variable
μ	Dynamic viscosity
Ω	Angular velocity
τ^*	Nano-particles and heat capacity ratio
ω_e	Electron Oscillating frequency
Θ	Dimensional temperature profile
Φ	Dimensional concentration profile
α	Stretching parameter
$lpha^*$	Thermal diffusivity
σ_{nf}	Electrical conductivity

1 Introduction

Artificial neural networks (ANNs) are interesting and significant paradigms associated with artificial intelligence (AI). ANNs can evolve in a variety of ways, depending on the data that feeds through the network throughout the learning process, whether externally or internally. To increase the performance of a multilayer perceptron (MLP) network, an artificial neural network uses the backpropagation (BP) technique to undertake simultaneous training. It is the most widely used, effective, and easy-to-learn paradigm



for complex multi-layered networks. The Levenberg–Marquardt methodology/algorithm (LMA) is a ground-breaking, convergent, reliable technique for artificial neural networks (ANNs) that provides a numerical solution to a wide range of fluid flow problems. Dreyfus employed backpropagation in 1973 to modify controller settings in proportion to error gradients. Werbos' (1975) backpropagation technique made multi-layer network training practicable. In 1982, he popularized Linnainmaa's AD approach by applying it to neural networks [1, 2]. Backpropagation has been widely used as a learning mechanism in feed-forward multilayer neural networks.

Certain researchers have recently used this technique to explore mass and heat transport aspects, as well as non-Newtonian fluid flow systems. Ahmad et al. [3] looked at intelligent computing approaches for analyzing nonlinear reactive fluid transport models in soft-tissues and microvessels. Shoaib et al. [4] used neural networks to explore the generation of entropy under the influence of MHD and thermal radiation. Sabir et al. [5] used a computational intelligence approach using Levenberg-Marquardt backpropagation neural networks to describe the fourth-order nonlinear system of the Emden–Fowler model. Uddin et al. [6] showed how to integrate magnetic and radiation impacts to comprehend the research of a Maxwell Nano-liquid thin film stream over a stretchable and spinning disk using LM-NN-based computational intelligence. Khan et al. [7] used a BNN-LMS to investigate heat transport between two permeable parallel plates of steady Nano-fluids applying Thermophoresis and Brownian impacts. Shah et al. [8] propose using neural networks to investigate the design of neural network-based intelligent calculation for the numerical behavior of unsteady 3D flow in the Powell-Eyring magneto-nanofluidic model.

The study of the magnetic characteristics and performance of electrically conducting fluids is known as magnetohydrodynamics (MHD). Magnetofluids include electrolytes, plasmas, saltwater, and liquid metals. The Magnetohydrodynamics field was initially described by Hannes Alfven [9]. Magnetohydrodynamics has a wide range of applications in engineering and technology, including crystal growth, plasma, electromagnetic casting, liquid-metal cooling of reactors, MHD sensors, magnetic drug targeting, and MHD power production. Magnetohydrodynamics is affected by the intensity of magnetic generations. Whenever the magnetic force increases, the Hall Effect caused by the Hall current cannot be ignored. Edwin Hall [10] is the forerunner to introduce the idea of Hall current. It is important and intriguing to study hydrodynamic issues. The effect of Hall currents improves hydrodynamical problems. Ahmed and Zueco [11] used the Hall effects to study the heat and mass transfer with a rotating and porous medium and found an accurate solution to the simulated problem. Pop and Soundalgekar [12] investigated the Hall effect in the time-dependent magnetohydrodynamic viscous flowing fluid. Sulochana [13] examined unsteady flow across a porous material in a rotating parallel plate, while taking Hall effects into consideration. Awais et al. [14] studied the effect of viscous dissipation on convection Jeffery fluid flow under the impacts of ion slip and Hall current. Abdel Aziz [15] explored the Hal effects on viscid and nano-fluid flow, as well as heat transfer via a stretched sheet.

Micro-polar fluids are such types of fluids that have a micro-structure and an irregular stress tensor. Eringen [16, 17] was the first one to propose the concept of micro-polar fluid. Lukaszewicz [18] discusses detailed research and a wider variety of applications for micro-polar fluids. Physically, micro-polar fluids are defined as fluids with irregularly orientated liquids floating in a viscous channel. All such fluids are utilized in the study of blood, colloidal suspensions flow, exotic emollients, liquefied crystals, brain fluid, paints, and turbulent shear flows. Mohammeadein and Gorla [19] performed the first research of micro-polar fluids on a horizontal plate of mass dissemination under the effect of an oblique magnetic field. Kasivishwanathan et al. [20] studied the flow of magnetohydrodynamics micro-polar fluid and accomplished a set of accurate solutions. Bhargava et al. [21] explored the mixed convection flow micro-polar fluid flow through a permeable stretched sheet using a finite element solution. Agarwal and Dhanapal [22] investigated the flows of micro-polar fluid with free convection between two parallel permeability upright plates using numerical solutions. Srinivasacharya et al. [23] studied the unsteady stokes micropolar fluid between permeable and parallel plates. Ziabakhsh et al. [24] used a homotopic approach to solve the problem of micro-polar nano-fluid in a permeable surface through the influence of heat and mass transmission. Nazar et al. [25] and Ishak et al. [26] considered the stagnation point flow of micro-polar fluid across a strained surface. Nadeem et al. [27, 28] examined the micro-polar nano-fluid in two parallel and horizontal rotating plates. The problem's analytical solution had been determined, and the included parameters had been investigated in their research.

For the unusual aspects of nano-liquids that make them efficient in a variety of applications, nano-fluids are exploited in medicinal processes, hybrid-powered engines, fuel cells, and micro-electronics, now these are primarily utilized in nanotechnologies fields [29]. Wang et al. [30] provided a quick overview of nano-fluids based on their research and application areas. The flow of nano-fluid across parallel plates is a frequent and traditional problem in several applications in the petroleum industry, crude oil purification, accelerators, aerodynamic heating, MHD power generators and pumps, the design of liquid metal cooling systems, and various vehicle sprays. Goodman [31] was the pioneer to use parallel plates to analyze viscous fluids. Sheikholeslami et al. [32, 33] used magnetohydrodynamics (MHD) effects to explore the flow of nano-fluid of viscous fluids in three dimensions across rotating parallel plates. Attia et al. [34] investigated magnetohydrodynamically influenced viscous flows across parallel plates. Borkakoti and Bharali [35] studied magnetohydrodynamics viscous flows between parallel plates with a stretched sheet as one of the plates. They utilized numerical approaches to solve the simulated problems and detailed descriptions of the impacts of various parameters. Sheikholeslami's current work on nano-fluids and their applications with various performance, characteristics, and impacts using various numerical and analytical approaches may be found in [36-39]. Rokni et al. [40] and Tauseef et al. [41] deliberated the impacts of magnetohydrodynamics and temperature on nano-fluid flow on rotating parallel plates. Mahmoodi and Kandelousi [42] utilized the differential transformation technique to study the hydromagnetic influence of Kerosene-Alumina nano-fluid flow in the occurrence of heat transfer analysis.

The flow of fluid in a rotating system is a naturally occurring phenomenon. Such rotation takes place throughout the fluid particles and increases as the fluid starts to flow. As a result, rotations play a role in natural fluid flow phenomena. Greenspan [43] explored in-depth the topic of the flow of fluid in a system of rotation. Geoffrey and Taylor [44] introduced the experimental notion of the motion of viscous fluid in a system of rotation. Vajravelua and Kumar [45] looked at the flow of magnetohydrodynamics viscous fluid between two rotating parallel and horizontal plates, one of which was stretched and the other porous. They have developed a numerical solution and examined the effects of various physical factors. Mehmood and Asif [46] extended their work. Hayat et al. [47, 48] extended their study into two and three dimensions by using several models to study the flow of non-Newtonian fluid with rotation.

Nano-fluids have enhanced thermophysical characteristics and might be used as heat transfer fluids. In general, such fluids are made up of nanometer-sized particles known as nanoparticles. Such fluids are colloidal suspensions of nanoparticles in a base fluid that are being manufactured. Nanoparticles are utilized in nano-fluids that include metals, carbides, oxides, or carbon nanotubes. Choi [49] praised the fact that thermal conductivity and convective heat transfer of nanoparticles are significantly increased proportionally to heat transmission. The heat flow process is increased, which improves heat transmission. Many researchers and scientists from all around the world were drawn to the nano-fluids topic. These fluids are used in a variety of technical and industrial applications, such as high-power lasers, cooling nuclear systems, solar water heating, microwave tubes, biomedical applications and engine transmission oil [50].

The homotopy analysis method has been used to solve an increasing number of nonlinear ordinary/partial differential equations in science, economics, and engineering during



the last two decades. [51] For example, the wave resonance criteria of an arbitrary number of travelling gravity waves were used to find numerous steady-state resonant waves in deep and finite water depth [52], which corresponded with Phillips' criterion for four waves of modest amplitude. Debnath and Das [53] use an artificial neural network to predict the power and torque characteristics of a three-bucket Savonius rotor. Das et al. [54] use a hybrid differential evolution approach to estimate essential dimensions for a trapezoidalshaped steel fin. Singla and Das [55] developed a differential evolution (DE)-based inverse analysis for optimizing the heat transfer rate from a rectangular stepped finned surface fulfilling a given volume. Das et al. [56] used the artificial bee colony (ABC) optimization method to perform an inverse study of a double-glazed flat-plate solar collector. Panda et al. [57] describe an analytical solution for a rectangular fin with simultaneous heat and mass transmission between the fin surface and the fin tip, and use inverse heat transfer analysis to determine the unknown thermal and geometrical configurations of the fin.

As obtaining an exact analytical solution to a problem might be complicated, the researcher uses a collection of numerical and semi-numerical techniques to tackle the problem. Keller Box Technique [58], HPM [59], Galerkin finite element method [60], Spectral Relaxation Method [61], as well as many others. All of the mentioned literature on nanofluid flow for several fluidic systems was invented using numerous numerical and semi numerical techniques; however, because of their advantage and efficiency, intelligence numerical computing models are essential to use the EMMN-PPRH model, i.e., the effects of electric and magnetic field on the micro-polar nano-fluid flow between two parallel plates with rotation under the influence of Hall current. Computational intelligence techniques have been utilized in stochastic numerical calculating solutions linked to artificial neural networks to obtain the results/outcomes of differential equations for linear and nonlinear, displaying varying capabilities under diverse conditions. The arrangement of such these techniques contains COVID-19 Models [56-63], electromagnetic [64], dust density model [65], atomic physics [66], entropy generation system [67], nonlinear Painlevé-I transcendent model [68], electrohydrodynamic pump flow system [69], nonlinear corneal shape model [60], Thomas–Fermi model [71], mosquito dispersal model [72], second grade nanofluidic system [73], and Emden-Fowler systems [74-76]. All of these motivating characteristics encourage researchers to use a dependable and accurate AI algorithm-based numerical computational model for numerical analysis of the Non-Newtonian Nano-fluid mathematical model by employing numerical and graphical evaluates to explore the effects of all alternatives or physical measures on the velocities, temperature, and concentration profiles. Package of Mathematica and MATLAB has been used to improve numerical accuracy.

The following are the key perspectives of the suggested design-computing methodology:

The (EMMN-PPRH) model, a new AI-based intelligent computing technique, was studied using an artificial neural network via the Levenberg–Marquardt backpropagation system (ANN-SLMB).

The mathematical modeling of the innovative scheme EMMN-PPRH in terms of PDEs has been translated to nonlinear ODEs by giving the appropriate analogous adjustment. The HAM is used to construct a dataset for the suggested ANN-SLMB as an alternative to (EMMN-PPRH) to using the rotation parameter, the viscosity parameter, the coupling parameter, the Brownian motion parameter, the Prandtl number, and the electric parameter

The ANN-SLMB testing, validation, and training processes are used to model EMMN-PPRH for different scenarios, and evaluation with orientation outcomes rationalizes the accuracy of the recommended ANN-SLMB.

Convergence graphs of estimated MSE, fitness, histograms and regression metrics demonstrate the suitability of the suggested ANN-SLMB to appropriately describe the EMMN-PPRH model.

The following study is categorized as follows: in section two, the design and effects of the EMMN-PPRH model problem are presented. The solution approach, as well as the effects of the proposed ANN-SLMB on various scenarios of EMMN-PPRH, is presented in section three, and the final comments and potential future study are defined in the final unit.

2 Mathematical Modeling

Let us consider the movement of micro-polar, electrically conducting nano-fluid between both horizontal and parallel plates. The coordinate system is designed so that the plate and the fluid rotate around the y-axis at the same angular velocity Ω . The distance between the top and lower plates is denoted by h. In order to extend the bottom plate down the x-axis, while maintaining the origin O(0, 0, 0) constant, the two forces have to be of equal magnitude but opposite in direction. The upper plate has the ability to achieve constant wall suction velocity/injection \overline{v}_0 . The fluid flow heat transfer is considered to be in a steady condition that is incompressible, laminar, and steady. In y-direction, a magnetic field \overline{B}_0 and an electric field \overline{E}_0 are operating. Furthermore, the micro-polar nano-fluid model considers the influence of Hall current. Figure 1 depicts the physical model. Whenever the magnetic field grows stronger, the fluids become electrically conducting, causing a Hall current to be generated, which influences the micro-polar nano-fluids. This

action increases the *z*-direction force, generating a flow distribution in the same direction and deflecting the micro-polar nano-fluid flows into 3 dimensions. The generalized Ohm's law, which includes the Hall current, is expressed as follows:

$$\overline{J} + \frac{\omega_e t_e}{B_0} \times \left(\overline{J} \times \overline{B}\right) - \frac{P_e \sigma_{nf}}{e n_e} - (\sigma_{nf}) \left(\overline{V} \times \overline{B} + \overline{E}\right) = 0,$$
(1)

where $\overline{J} = (J_x, J_y, J_z)$ denotes current density and $\overline{J} = (\sigma_{nf})(\overline{V} \times \overline{B} + \overline{E})$, \overline{E} denotes the electric field intensity $(0, E_0, 0) = \overline{E}$, $\overline{V}(\overline{u}, \overline{v}, \overline{w})$ denotes components of velocity, $\overline{B} = (0, B_0, 0)$ indicates magnetic field, ω_e indicates oscillating frequency of the electron, t_e denotes the times of electron collision, P_e is the pressure of the electron, σ_{nf} represent electrical conductivity, n_e is the number density of electron and e is stand for charge of electron. Considering the inherent parameters, the Ohm law in generalized form offers $(J_y = 0)$ in the flow field for weakly ionized molecules. Using these assumptions, we get J_x and J_z as [10-17]:

$$J_x = \frac{\sigma_{nf} \overline{B}_0^2}{1+m^2} (m\overline{u} - \overline{w}), \qquad (2)$$

$$J_z = \frac{\sigma_{nf} \overline{B}_0^2}{1 + m^2} (\overline{u} + m\overline{w}).$$
(3)

Here, $m = t_e \omega_e$ represents the Hall parameter.

By suppositions stated above, the continuity and momentum equations are reduced to [19–28]:

$$\frac{\partial \overline{v}}{\partial y} + \frac{\partial \overline{u}}{\partial x} = 0, \tag{4}$$

$$\frac{\partial \overline{u}}{\partial y}.\overline{v} + \frac{\partial \overline{u}}{\partial x} \cdot \overline{u} = -\frac{1}{\rho_f} \frac{\partial \widehat{p}}{\partial x} + \left(\frac{\widehat{\kappa}}{\rho_f} + v\right) \left(\frac{\partial^2 \overline{u}}{\partial x^2} + \frac{\partial^2 \overline{u}}{\partial y^2}\right) - 2\overline{w} \Omega + \frac{\widehat{\kappa}}{\rho_f} \frac{\partial \overline{N}}{\partial y} - \frac{\overline{B}_0^2 \sigma_{nf}}{m^2 + 1} \frac{1}{\rho_f} (m\overline{w} + \overline{u}) - \frac{\sigma_{nf} \overline{B}_0 \overline{E}_0}{\rho_f},$$
(5)

$$\frac{\partial \overline{v}}{\partial x} \cdot \overline{u} + \frac{\partial \overline{v}}{\partial y} \cdot \overline{v} = -\frac{1}{\rho_f} \frac{\partial \widehat{p}}{\partial x} + \left(\frac{\partial^2 \overline{v}}{\partial y^2} + \frac{\partial^2 \overline{v}}{\partial x^2}\right) + \frac{\widehat{\kappa}}{\rho_f} \frac{\partial \overline{N}}{\partial x},$$
(6)

$$\frac{\partial \overline{w}}{\partial x} \cdot (\overline{u}) + \frac{\partial \overline{w}}{\partial y} \cdot (\overline{v}) = \left(\frac{\widehat{\kappa}}{\rho_f} + v\right) \left(\frac{\partial^2 \overline{w}}{\partial x^2} + \frac{\partial^2 \overline{w}}{\partial y^2}\right) + 2\overline{u}\Omega + \frac{\sigma_{nf}\overline{B}_0^2}{1 + m^2} \frac{1}{\rho_f} (m\overline{u} - \overline{w}) + \frac{\sigma_{nf}\overline{B}_0\overline{E}_0}{\rho_f}.$$
 (7)

From Eqs. (4)–(7), v, μ denotes the kinematic and dynamic viscosities coefficient, respectively, Ω represents angular velocity, $\hat{\kappa}$ denotes vertex viscosity, ρ_f indicates the base fluid density, and \overline{N} is stand for the micro-rotation angular velocity.

The equation of energy is become as [36-39]:

$$\frac{\partial T}{\partial x} \cdot (u) + \frac{\partial T}{\partial y} \cdot (v) + \frac{\partial T}{\partial z} \cdot (w) = \alpha^* \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) \tau^* \left[D_B \left\{ \frac{\partial C}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{\partial C}{\partial z} \frac{\partial T}{\partial z} \right\} + \frac{D_T}{T_C} \left\{ \left(\frac{\partial T}{\partial x} \right)^2 + \left(\frac{\partial T}{\partial z} \right)^2 + \left(\frac{\partial T}{\partial y} \right)^2 \right\} \right].$$
(8)

The equation of micro-rotation angular velocity is become as [19-28]:

$$\frac{\partial \overline{N}}{\partial x} \cdot (\overline{u}) + \frac{\partial \overline{N}}{\partial y} \cdot (\overline{v}) = -\gamma \cdot \frac{1}{\rho_j} \left(\frac{\partial^2 \overline{N}}{\partial x^2} + \frac{\partial^2 \overline{N}}{\partial y^2} \right)
+ \frac{\widehat{\kappa}}{\rho_j} \left(\frac{\partial \overline{v}}{\partial x} - \frac{\partial \overline{u}}{\partial y} \right) - \frac{2\widehat{\kappa}\overline{N}}{\rho_j}.$$
(9)

Mass transfer equation is become as [36–39]:

$$\frac{\partial C}{\partial x} \cdot (u) + \frac{\partial C}{\partial y} \cdot (v) + \frac{\partial C}{\partial z} \cdot (w) = (D_B)$$
$$\cdot \left(\frac{\partial^2 C}{\partial z^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial x^2}\right)$$
$$+ D_T \cdot \frac{1}{T_0} \cdot \left(\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial x^2}\right).$$
(10)

Here in Eq. (8), α^* denotes thermal diffusivity, *T* denotes temperature, $\tau^* = (\rho c)_p / (\rho c)_f$ is defined as the ratios of nanoparticles and effective heat capacity, D_B denotes the Brownian diffusion coefficients, D_T denotes thermophoretic diffusion coefficient, c_p denotes specific heat, and *C* is stand for the fluid concentration of the fluids particles. The boundary conditions for the state problem are as follows:

$$\overline{v} = 0, \ \overline{u} = \alpha x, \ \overline{w} = 0, \ T = T_h, \ C = C_h, \ \overline{N} = -k \frac{\partial \overline{u}}{\partial y}, \ \text{at } y = 0,$$

 $\overline{u} = \overline{v} = \overline{w} = 0, \ T = T_0, \ C = C_0, \ \overline{N} = k \frac{\partial \overline{u}}{\partial y} \ \text{when } y = h.$ (11)

The boundary parameter is denoted by k. The fluid flow is the turbulent, weak, and strong concentration when the valve of k = 1.0, k = 1/2 and k = 0, respectively. The dimensionless variables are represented as follows:

$$\overline{N} = -\frac{\alpha x G(\eta)}{h}, \ \overline{u} = \alpha x f'(\eta), \ \overline{w} = \alpha x g(\eta), \ \overline{v} = -\alpha h f(\eta),$$
$$\Theta(\eta) = \frac{T - T_h}{T_0 - T_h}, \ \Phi(\eta) = \frac{C - C_h}{C_0 - C_h}, \ \text{where } \eta = \frac{y}{h}.$$
(12)







when the dimensionless variables from Eq. (12) are replaced by Eqs. (4)–(10), (4) remains identical, and the other governing Eqs. (5)–(10) are simplified to the form of

$$(1+N_1)f^{iv} - \operatorname{Re}(f'f'' - ff''') + \frac{M}{1+m^2}(f'' + mg') - 2Krg' + N_1G'' + MEI = 0,$$
(13)

$$(1 + N_1)g'' - \operatorname{Re}(fg' - gf') + 2Krf' + \frac{M}{1 + m^2}(mf' - g) - MEI = 0,$$
(14)

$$\Theta'' + \Pr\left(\operatorname{Re} f \Theta' + Nb \Phi' \Theta' + Nt \Theta'^2\right) = 0, \tag{15}$$

$$N_2 G'' - N_1 (2G + f'') - N_3 \operatorname{Re} (Gf' - G'f) = 0, \quad (16)$$

$$\Phi'' + \operatorname{Re}Scf \Phi' + \frac{Nt}{Nb}\Theta'' = 0.$$
(17)

When the dimensionless variable from Eq. (12) is utilized in it, Eq. (11) is simplified to the following form:

$$f = 0, \ f' = 1, \ g = 0, \ \Theta = 1, \ G = -kf'', \ \Phi = 1, \ \text{at } \eta = 0$$

$$f = \delta, \ f' = 0, \ g = 0, \ \Theta = 0, \ G = kf'', \ \Phi(1) = 0, \ \text{when } \eta = 1.$$
(18)

The dimensional physical parameters after simplification are as follows:

$$\operatorname{Re} = \frac{ah^2}{v}, \text{ EI} = \frac{E_0}{B_0 ax}, \text{ Kr} = \frac{2\Omega h^2}{v},$$

$$M = \frac{\sigma_{nf}h^2 B_0^2}{\rho v}, \quad N_1 = \frac{\hat{\kappa}}{\mu}, \quad N_2 = \frac{v_s}{vh^2}, \quad N_3 = \frac{j}{h^2},$$

$$\Pr = \frac{\mu}{\rho_f \alpha}, \quad Nb = \frac{(\rho c)_p D_B C_h}{\alpha^* (\rho c)_f}, \quad Nt = \frac{(\rho c)_p D_T T_0}{(\rho c)_f T_c},$$

$$Sc = \frac{\mu}{D\rho_f}, \quad \delta = \frac{v_0}{h}.$$
(19)

where N_1 denotes the coupling parameter, Kr denotes rotation parameter, M is stand for magnetic parameter, N_2 represents spin gradient viscosity parameter, EI is stand for electric parameter, δ denotes transpiration parameter, N_3 is stand for micro-polar fluid constant, Re is stand for viscosity parameter, PR indicates Prandtl number, Nt denotes thermophoretic parameter, Sc indicates the Schmidt number, and Nb indicates the Brownian motion parameter.

Here, $C_f = \frac{(\overline{S}_{xy})_{y=0}}{\rho \overline{u}_w^2}$ represents the skin friction, where $\overline{S}_{xy} = \left((\mu + \widehat{\kappa})\frac{\partial u}{\partial y} + \widehat{\kappa}N\right), Q_w$ is the heat flux and $Q_w = \left(\frac{\partial T}{\partial y}\right)_{y=0}(-\overline{k})$, the Nusselt number is expressed as $Nu = \frac{hQ_w}{\widehat{k}(T_0 - T_h)}$. The number of Sherwood is expressed as $Sh = \frac{hJ_w}{\widehat{k}(T_0 - T_h)}$, J_w is the mass flux and $J_w = (-D_B)\left(\frac{\partial C}{\partial y}\right)_{y=0}$. C_f , Nu and Sh are achieved in the dimensionless form as

$$C_f \sqrt{\text{Re}_x} = N_1 G(0) + (1 + N_1) f''(0),$$

$$Nu = -\Theta'(0), \text{ Sh} = -\Phi'(0).$$
(20)

wherever Re_x stands for local Reynolds number and is denoted by $\operatorname{Re}_x = \frac{\overline{u}_w x}{v}$.





Fig. 3 (a) Configuration of a specific neural framework.(b) EMMN-PPRH Mathematical Model



(a) Configuration of a specific neural framework



(b) EMMN-PPRH Mathematical Model

3 Solution Approach and Assessment

The recommended ANN-SLMB in the configuration of a neural network is depicted in Fig. 2. The recommended ANN-SLMB is conducted using the 'nf tool,' which is a technique for fitting NN tools in MATLAB's neural network (NN) toolbox, while backpropagation of Levenberg–Marquardt is used to determine the weight of neural networks. The proposed ANN-SLMB framework is shown in Fig. 3a, and the

mathematical model is shown in Fig. 3b, while the overall extension of the flow is shown in Fig. 4.

Figures 5, 6, 7, 8, 9, 10, 11, 12, 13 show the ANN-SLMB implications for the EMMN-PPRH model in different scenarios (1 to 6). Figures 5 and 6 show the outcomes of six scenarios M, kr, R, Rd, Sc and Nt in form of execution and transition phases. The fitting plots and error histograms in Figs. 7, 8 are discussed in expressions of solution with error for four distinct cases, whereas regression assessments





Fig. 4 The suggested ANN-SLMB workflow procedure for the EMMN-PPRH model

in Figs. 9, 10, 11 are exhibited for four distinct cases of the EMMN-PPRH model. Furthermore, for each EMMN-PPRH model scenario, the convergence achieves parameter in form of MSE, execution, performance duration, executed period, and gauges of backpropagation and behavioral convolution are included in Tables (2–7) for all 6 scenarios, respectively.

Figures 5, 6a, c, e for test procedures indicated the MSE convergence, training, and validation improvements for six scenarios of the EMMN-PPRH model. The best network performance was achieved with MSE close to 2.19×10^{-10} , 2.12×10^{-10} , 2.47×10^{-10} , 2.37×10^{-10} , 2.56×10^{-10} and 1.82×10^{-10} . The reduced the MSE figure, the more precise and accurate the execution of the recommended approach. The concerning values of step size Mu and gradient of Levenberg–Marquardt are finely $[1.00 \times 10^{-08}$, 1.00×10^{-08} , 1.00×10^{-08} , 1.00×10^{-08} , 1.00×10^{-08} and 1.00×10^{-08}] and $[9.97 \times 10^{-08}$, 9.97×10^{-08} , 9.97×10^{-08} , 9.96×10^{-08} , 9.97×10^{-08}] are existent in Figs. 5, 6b, d,

f. The results and graphical illustrations above demonstrate that ANN-SLMB is competent, accurate, and convergent for case 4 of the EMMN-PPRH model.

Figures 7, 8 assess the efficiency of the EMMN-PPRH model by analyzing the created consequences of six distinct scenarios for inputs ranging from 0 to 1 with a step range of 0.01 with reference numerical results of HAM and related outcomes, as well as the design of error dynamics. The greatest error for the test, train, and validation statistics achieved via recommended ANN-SLMB is less than 3.70×10^{-10} , 3.34×10^{-10} , 7.18×10^{-10} , 2.90×10^{-10} , 4.16×10^{-10} and 2.85×10^{-10} , whereas the error dynamics and results of the EMMN-PPRH model for 6 different scenarios are also analyzed for each input point apart from the error histograms are demonstrated in Figs. 7, 8a, b, c, respectively. The average value of error bin via compared zero line has errors about 7.29×10^{-08} , 7.29×10^{-08} , -1.9×10^{-06} , 5×10^{-08} , 2.04



Fig. 5 Performance solution and State transition of Recommended ANN-SBLM for solving EMMN-PPRH model for case 4 of scenarios 1, 2 and 3





Fig. 6 Performance solution and State transition of Recommended ANN-SBLM for solving EMMN-PPRH model for case 4 of scenarios 4, 5 and 6



(a) Effects of Fitness and E.H for C4 of S1



(b) Effects of Fitness and E.H for C4 of S2



(c) Effects of Fitness and E.H for C4 of S3

Fig. 7 Solution of Fitness and Error analysis of Recommended ANN-SBLM for solving EMMN-PPRH model for case 4 of scenarios 1, 2 and 3





(a) Effects of Fitness and E.H for C4 of S4



(b) Effects of Fitness and E.H for C4 of S5



(c) Effects of Fitness and E.H for C4 of S6

Fig. 8 Solution of Fitness and Error analysis of Recommended ANN-SBLM for solving EMMN-PPRH model for case 4 of scenarios 4, 5 and 6



Fig. 9 Regression effectiveness of the Recommended ANN-SBLM for solving EMMN-PPRH model for case 4 of scenarios 1 and 2



Fig. 10 Regression effectiveness of the Recommended ANN-SBLM for solving EMMN-PPRH model for case 4 of scenarios 3 and 4

 $\times 10^{-06}$ and $- 1.6 \times 10^{-06}$ for case 4 of all 6 distinct scenarios of the model, EMMN-PPRH. Co-relation analyses are commonly used to classify the investigation inside regression analyses. Figures 9, 10, 11 show the effects of the EMMN-PPRH model's related six scenarios. Correlation values are constantly near to unity, indicating that this is the optimal value for validation, training, and testing for precise modeling, which indicates how effectively ANN-SLMB resolves the EMMN-PPRH model.

Furthermore, for various EMMN-PPRH model parameters (1–6), the associated numerical data in Table 2, 3, 4, 5, 6, 7 demonstrate that MSE efficiency for the proposed ANN-SLMB technique is close 1E-10. The numerical results in Tables 2, 3, 4, 5, 6, 7 demonstrate that ANN-SLMB solves the EMMN-PPRH model well.

The results of ANN-SLMB are validated for velocities $f'(\eta)$ and $g(\eta)$, and temperature $\Theta(\eta)$ profiles for scenarios 1–6 of the EMMN-PPRH paradigm are presented in Figs. 12 and 13, respectively. The influence of velocity profiles $f'(\eta)$





Fig. 11 Regression effectiveness of the Recommended ANN-SBLM for solving EMMN-PPRH model for case 4 of scenarios 5 and 6

Scenarios	Case Interest physical quantities														
		Kr	EI	Re	N_1	Nb	Pr	М	m	Nt	N_2	N_3	Sc	δ	k
S 1	C1	0.5	0.6	1	1.5	0.8	1.5	1	0.6	0.3	1.5	1	0.8	0.1	1
	C2	1.0	0.6	1	1.5	0.8	1.5	1	0.6	0.3	1.5	1	0.8	0.1	1
	C3	1.5	0.6	1	1.5	0.8	1.5	1	0.6	0.3	1.5	1	0.8	0.1	1
	C4	2.0	0.6	1	1.5	0.8	1.5	1	0.6	0.3	1.5	1	0.8	0.1	1
S2	C1	0.8	1	1	2	0.8	1.5	1	0.6	0.3	2	1	0.8	0.1	1
	C2	0.8	2	1	2	0.8	1.5	1	0.6	0.3	2	1	0.8	0.1	1
	C3	0.8	3	1	2	0.8	1.5	1	0.6	0.3	2	1	0.8	0.1	1
	C4	0.8	4	1	2	0.8	1.5	1	0.6	0.3	2	1	0.8	0.1	1
S 3	C1	0.8	0.6	1	2	0.8	1.5	1	0.6	0.3	2	2	0.8	0.1	1
	C2	0.8	0.6	2	2	0.8	1.5	1	0.6	0.3	2	2	0.8	0.1	1
	C3	0.8	0.6	3	2	0.8	1.5	1	0.6	0.3	2	2	0.8	0.1	1
	C4	0.8	0.6	4	2	0.8	1.5	1	0.6	0.3	2	2	0.8	0.1	1
S4	C1	0.8	0.6	1	0.1	0.8	1.5	1	0.6	0.3	1	1	0.8	0.1	1
	C2	0.8	0.6	1	1.0	0.8	1.5	1	0.6	0.3	1	1	0.8	0.1	1
	C3	0.8	0.6	1	1.5	0.8	1.5	1	0.6	0.3	1	1	0.8	0.1	1
	C4	0.8	0.6	1	2.0	0.8	1.5	1	0.6	0.3	1	1	0.8	0.1	1
S5	C1	0.8	0.6	1	1	0.1	1.5	1	0.6	0.3	2	1	0.8	0.1	1
	C2	0.8	0.6	1	1	0.3	1.5	1	0.6	0.3	2	1	0.8	0.1	1
	C3	0.8	0.6	1	1	0.5	1.5	1	0.6	0.3	2	1	0.8	0.1	1
	C4	0.8	0.6	1	1	0.7	1.5	1	0.6	0.3	2	1	0.8	0.1	1
S6	C1	0.8	0.6	1	1	0.5	1	1	0.6	0.3	2	1	0.8	0.1	1
	C2	0.8	0.6	1	1	0.5	2	1	0.6	0.3	2	1	0.8	0.1	1
	C3	0.8	0.6	1	1	0.5	3	1	0.6	0.3	2	1	0.8	0.1	1
	C4	0.8	0.6	1	1	0.5	4	1	0.6	0.3	2	1	0.8	0.1	1

Table 1 Depiction of all scenarios along with 4 different cases for the EMMN-PPRH model



Table 2ANN-SLMBconsequence of theEMMN-PPRH model forScenario 1

Case	MSE			Execution	Gradient	Mu	Epoch	Time
	MSE training	MSE validation	MSE testing					
C1	1.6506E-11	2.5701E-11	2.0446E-10	1.65E-11	9.97E-08	1.00E-09	441	1
C2	2.2740E-10	2.4744E-10	3.4024E-10	2.27E-10	9.99E-08	1.00E-08	504	1
C3	1.9755E-11	2.1231E-11	2.6966E-11	1.98E-11	9.97E-08	1.00E-09	409	1
<u>C4</u>	2.1852E-10	3.7083E-10	3.0959E-10	2.19E-10	9.97E-08	1.00E-08	512	1
Case	MSE			Execution	Gradient	Mu	Epoch	Time
	MSE training	MSE validation	MSE testing					
C1	2.0306E-10	2.9670E-10	3.7321E-10	1.73E-10	9.97E-08	1.00E-08	492	1
C2	1.7326E-10	2.6433E-10	2.8741E-10	1.30E-10	9.97E-08	1.00E-08	472	1
C3	1.9992E-10	3.6165E-10	5.4846E-10	2.00E-10	9.99E-08	1.00E-08	459	1
C4	2.1177E-10	3.3445E-10	1.5116E-10	2.12E-10	9.97E-08	1.00E-08	486	1
Case	MSE			Execution	Gradient	Mu	Epoch	Time
	MSE training	MSE validation	MSE testing					
C1	2.9319E-10	3.1247E-10	4.4236E-10	2.93E-10	9.99E-08	1.00E-08	670	2
C2	2.3887 E-10	3.2580E-10	1.0593E-09	2.39E-10	9.97E-08	1.00E-08	542	2
C3	2.0165E-10	4.3343E-10	3.9637E-10	2.02E-10	9.97E-08	1.00E-08	507	1
C4	2.4663E-10	5.3626E-10	7.1892E-10	2.47E-10	9.97E-08	1.00E-08	480	1
Case	MSE			Execution	Gradient	Mu	Epoch	Time
	MSE training	MSE validation	MSE testing				1	
C1	2.1338E-10	3.1629E-10	4.0416E-10	2.13E-10	9.97E-08	1.00E-08	479	1
C2	2.1338E-10	3.5543E-10	4.1233E-10	2.40E-10	9.96E-08	1.00E-08	435	1
C3	1.7297E-10	1.6901E-10	7.6699E-10	1.73E-10	9.99E-08	1.00E-08	448	1
C4	2.3679E-10	1.9375E-10	2.9072E-10	2.37E-10	9.96E-08	1.00E-08	430	1
Case	MSE			Execution	Gradient	Mu	Epoch	Time
	MSE training	MSE validation	MSE testing					
C1	1.3223E-11	1.5951E-11	1.3657E-10	1.32E-11	1.00E-07	1.00E-09	432	1
CI								1
C2	1.3786E-11	2.1300E-11	2.0248E-11	1.38E-11	9.98E-08	1.00E - 09	444	1
C2 C3	1.3786E-11 1.0380E-11	2.1300E-11 1.5965E-11	2.0248E-11 1.1869E-11	1.38E-11 1.04E-11	9.98E-08 9.96E-08	1.00E-09 1.00E-09	444 481	1

EMMN-PPRH model for Scenario 2

Table 3ANN-SLMBconsequence of the

Table 4ANN-SLMBconsequence of theEMMN-PPRH model forScenario 3

Table 5 ANN-SLMB
consequence of the
EMMN-PPRH model for
Scenario 4

Table 6 ANN-SLMB
consequence of the
EMMN-PPRH model for
Scenario 5

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Table 7ANN-SLMBconsequence of theEMMN-PPRH model forScenario 6

Case	MSE			Execution	Gradient	Mu	Epoch	Time
	MSE training	MSE validation	MSE testing					
C1	1.1750E-11	3.0851E-11	2.3592E-11	1.18E-11	9.96E-08	1.00E-09	428	1
C2	1.1815E-10	2.0001E-10	3.7015E-10	1.18E-10	9.98E-08	1.00E-08	495	1
C3	1.0830E-10	2.0475E-10	1.7171E-10	1.08E-10	1.00E-07	1.00E-08	499	1
C4	1.8210E-10	2.1362E-10	2.8512E-10	1.82E-10	9.97E-08	1.00E-08	475	1

and $g(\eta)$ is enumerated for the variations of rotation parameter Kr, viscosity parameter Re and electric parameter Kr in sub Fig. 12a, c, e for case 4 of scenarios 1, 2, and 3 of the EMMN-PPRH paradigm, Moreover, in sub Fig. 12b, d, f, the corresponding AE values are included in order to execute the EMMN-PPRH model technique. The impact of the rotation parameter Kr on the velocity profile is illustrated in Fig. 12a. While can be observed, as the rotation parameter rises, that increasing the motion of fluid, with the impact being more evident towards the stretching plate. Consequently, raising the rotation parameter increases the carioles force, which increases the rotational velocity and kinetic energy increases due to this rotation in fluid, which raises the motion of the flow. The electric parameter EI impacts on velocity profile is shown in Fig. 12c. It is observed that, when the values of the electric parameter increase, the velocity increases along with y and z directions, respectively. This is because a significant quantity of electric field causes rapid ionization in micro-polar nano-fluid, which causes nanoparticle motility to increase. The effect of the viscosity parameter (Re), on the velocity profile, is exposed in Fig. 12e. The velocity distribution is found to decrease as the viscosity parameter (Re) increases. This is because increasing the value of Re decreases the viscid forces that produce the well-built inertial forces, causing the velocity field to slow down.

It is feasible to see the overlapping of references and recommended solutions. As a result, for the EMMN-PPRH model, Fig. 13a, c, e demonstrate the results different magnitudes for coupling parameter N_1 , Brownian motion parameter Nb and Prandtl number Pr for velocity $f'(\eta)$ and temperature $\theta(\eta)$ profiles for case 4 of scenarios 4, 5, and 6, respectively. Whereas in Fig. 13b, d, f the appropriate values of AE are computed. The influence of coupling parameter (N_1) , on velocity, is indicated in Fig. 13a. It can be demonstrated that the coupling parameter (N_1) decreases the velocity profile towards the lower plate and raises the velocity profile from the center to the top plate. The features of the Brownian motion parameter on Temperature $\Theta(\eta)$ are depicted in Fig. 13c. The Brownian motion is the random movement of particles in nano-fluids. It is also seen that the Brownian motion of micro-polar fluid at the level of molecular is a crucial determinant in nano-fluid thermal conductivity.



Growing is considered to raise the temperature profile. Furthermore, raising the Brownian motion (Nb), rises the kinetic energy of the micro polar nano-fluid within the fluid, raising the rate of heat transfer and the thickness of the boundary layer, culminating in temperature profile rises. The effect of the Prandtl number (Pr) on Temperature $\Theta(\eta)$ is presented in Fig. 13e. The temperature distribution is inversely related to Prandtl number (Pr). Temperature distribution reduces with rising Prandtl number (Pr) values and increases with decreasing Prandtl number (Pr) values. Physically, fluids with a low Prandtl number have highly thermal diffusivity, whereas fluids with a high Prandtl number Pr have the opposite effect. Because of this, a high Pr value causes the thermal boundary layer to collapse.

The results of ANN-SLMB related with HAM for all 6 scenarios, so that the absolute error was decided to approach the exactness measures from suggestion solutions, and the consequences are demonstrated in sub Figs. 12b, d, f, 13b, d, f for case 4 of scenarios 1, 2, 3, 4, 5, and 6. The AE accomplish values for scenarios 1,2,3 for velocity profiles are 10^{-07} to 10^{-04} , 10^{-08} to 10^{-04} and 10^{-07} to 10^{-04} are shown in sub Fig. 12b, d, f, whereas for velocity and temperature profiles, the AE achieve values for scenarios 4, 5, and 6 are approximately 10^{-07} to 10^{-04} , 10^{-08} to 10^{-04} and 10^{-08} to 10^{-04} are indicated in sub Fig. 9b, d, f, respectively. The ANN-SLMB computing methodology solves EMMN-PPRH model variations with sufficient convergent and dynamic efficiency in all of these numerical and graphical examples.

4 Conclusions

In this paper, the EMMN-PPRH model was analytically and numerically determined the effects of electric and magnetic fields on the micro-polar nano-fluid flow between two parallel rotating plates under the impact of Hall current.

• The solution of a mathematical model presenting EMMN-PPRH with a variation of convinced scenarios is examined by calculating an artificial neural network system using the Levenberg–Marquardt approach with backpropagation.



Fig. 12 Analysis of recommended ANN-SBLM through reference data set results for case 4 of EMMN-PPRH model scenarios 1, 2, and 3





Fig. 13 Analysis of recommended ANN-SBLM through reference dataset results for case 4 of EMMN-PPRH model scenarios 4, 5, and 6

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- The dataset for the EMMN-PPRH model is generated using the homotopy analysis analytical approach, which includes deviations from several physical measurements such as the rotation parameter, the electric parameter, the viscosity parameter, the coupling parameter, the Brownian motion parameter, and the Prandtl number.
 - In the present research, the proposed methodology ANN-SBLM based on Levenberg–Marquardt scheme is used to obtain the convergence of various parameters for velocity, temperature and concentration profiles which is closed to the solution obtain by HAM method.
 - Furthermore, we obtain some statistical data of the model that is mean square error, gradient, performance, Mu.
 - The EMMN-PPRH reference dataset is formed by modifying numerous variants, with 90%, 05%, and 05% of the dataset used for ANN-SBLM training, testing, and validation, respectively.
 - The scheme's achievement authenticates a level of 10^{-09} to 10^{-11} for both reference and recommended results, and this is further supported by graphical and numerical demonstrations of error-histogram graphs of convergence, regression dynamics, and mean square errors.

In the future, new ANN-SBLM type solvers and its hybrid versions will be designed to further investigate and address fluid mechanics challenges [77–82].

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