

Optimizing Workover Rig Fleet Sizing and Scheduling Using Deterministic and Stochastic Programming Models

Miguel A. Fernández Pérez,^{†,‡} Fabricio Oliveira,^{*,§} and Silvio Hamacher[†]

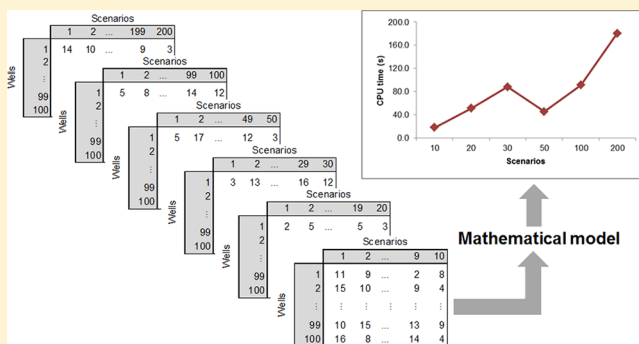
[†]Industrial Engineering Department, Pontifical Catholic University of Rio de Janeiro, Rio de Janeiro, RJ 22451-900, Brazil

[‡]Department of Engineering, Pontifical Catholic University of Peru, Lima 32, Peru

[§]Systems Analysis Laboratory, Department of Mathematics and Systems Analysis, School of Science, Aalto University, FI-00076 AALTO, Finland

Supporting Information

ABSTRACT: We present deterministic and stochastic programming models for the workover rig problem, one of the most challenging problems in the oil industry. In the deterministic approach, an integer linear programming model is used to determine the rig fleet size and schedule needed to service wells while maximizing oil production and minimizing rig usage cost. The stochastic approach is an extension of the deterministic method and relies on a two-stage stochastic programming model to define the optimal rig fleet size considering uncertainty in the intervention time. In this approach, different scenario-generation methods are compared. Several experiments were performed using instances based on real-world problems. The results suggest that the proposed methodology can be used to solve large instances and produces quality solutions in computationally reasonable times.



1. INTRODUCTION

Before oil wells can begin operation, equipment must be installed to bring the oil to the surface. Various artificial lifting techniques can be employed to bring oil to the surface, including mechanical pumping, progressive cavity pumping, and gas lift. As the equipment develops faults over time, interventions must be performed in wells to maintain production or improve well productivity. The aim of these interventions may be, among others, recompletion, restoration, cleaning, and stimulation.¹

Interventions are performed by workover rigs, a scarce resource with high operating costs of which there are usually not enough to service all the wells that require maintenance.² The rigs can be classified according to their *service level*, which is derived from the matching of technical specifications of the rig and the technical requirements for the service, such as technical limitations related to the depth of the wells, depth from water levels, the traction capacity, pressure, and others. The rigs with higher service levels are capable of performing services in wells with lower levels (i.e., requirements). For example, the tubing limits of the rigs vary by depth, from 2590 to 8839 m. While a rig with 2590 m specification for tubing limit would not be capable to service a well with depth of 5000 m, any rig with tubing longer than that (for example, those with 8839 m) would.³

When defining the service schedule, various factors in addition to the service level must be considered, such as well production, the duration of the intervention, the time horizon within which the services can be performed, the geographical location of the rig in relation to the well, the environmental risk, and safety issues. A

delay in the interventions can lead to substantial production losses.⁴

The workover rig problem can be classified as the workover rig scheduling problem (WRSP) and the workover rig routing problem (WRRP). The WRSP can be seen as a problem of scheduling jobs on parallel machines, where the times required to move the rigs between the wells are disregarded because they are not significant compared to the intervention times. The WRRP can be seen as a problem of routing and scheduling with multiple vehicles, where the travel times are significant and thus must be considered. According to Gouvêa et al.⁵ the adoption of the WRSP as a modeling standpoint is appropriate when the times required to move the rigs between the wells are on the order of minutes or hours and the rig intervention time is on the order of days or weeks.

Various studies in which a deterministic approach to the WRSP is adopted can be found in the literature. Costa⁶ proposed a 0–1 integer linear programming model and constructed a group of instances based on real-world problems in Brazil. The paper by Costa⁶ has been used as a reference in many other papers as the basis for a range of metaheuristics such as genetic algorithms, the greedy randomized adaptive search procedure, and the simulated annealing algorithm.^{7–9} Although metaheur-

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istics yield good results for small and medium-sized instances (with less than 75 wells), they do not produce optimal values for large instances (with more than 100 wells). Pérez et al.¹⁰ proposed an improvement to the model described by Costa⁶ and managed to solve optimally all the instances he constructed in short computational times, including large instances whose solutions were not previously known. The model proved to be more effective than the other metaheuristics tested.

A deterministic approach is also reported in the literature to solve the WRRP. Aloise et al.⁴ proposed a variable neighborhood search heuristic and tested it on real-world cases in Brazil. Ribeiro et al.¹¹ proposed the clustering search and adaptive large neighborhood search heuristics using instances generated by Neves.¹² Ribeiro et al.¹³ proposed a mathematical model and a branch-price-and-cut algorithm and generated a group of random instances. Duhamel et al.¹⁴ proposed three mixed-integer programming models: the first is an improvement of the model proposed by Aloise et al.;⁴ the second is based on the open vehicle routing model; and the third is based on the set covering model. Ribeiro et al.¹⁵ developed a branch-price-and-cut heuristic and a hybrid genetic algorithm. Monemi et al.¹⁶ proposed a mixed-integer programming model that includes a hyper-heuristic.

Few studies in the literature deal with the rig fleet sizing problem. Irgens et al.¹⁷ used a local search algorithm and a formulation that minimizes production loss and included as a constraint the condition that the cost of using the rigs should not exceed the designated budget. Bissoli et al.¹⁸ incorporated rig rental cost into the mathematical model proposed by Ribeiro et al.¹³ and implemented an adaptive large neighborhood search metaheuristic. The first use of a stochastic approach was described by Bassi et al.,¹⁹ who considered the probability distribution of the intervention time and analyzed the trade-off between rig rental cost and the expected cost of lost production. They developed a simulation–optimization method that involved sampling the uncertain parameter (generating scenarios) with the Monte Carlo method and then solving the problem using a greedy algorithm.

Even though mathematical programming techniques have also been applied for optimizing production operations, the majority of studies available in the literature tackle daily oil production optimization problems using heuristic rules that are incorporated into software tools known as well management routines.²⁰ The present work involves studying the workover rig problem from a perspective that reflects real-world operations and takes into account the variety of rigs in a fleet and the uncertainty inherent in the well intervention time. The main contribution of our study to the existing literature is the development of a thorough framework that can be employed to solve large challenging instances in reasonable computational times for the workover rig problem under uncertainty. To this end, we extend the mathematical model proposed by Pérez et al.,¹⁰ which was originally developed to determine a schedule for a homogeneous fleet of rigs servicing a set of wells with a deterministic intervention time. In the stochastic approach, the uncertainty in the intervention time is represented using scenario-generation methods. The Monte Carlo, scenario reduction, and quasi-Monte Carlo methods are compared to determine the method that produces the best results in terms of approximating the uncertainty satisfactorily with the fewest scenarios. The paper also contributes to the existing literature by providing data that can be used to build instances for the workover rig problem (based on the review of various papers).

The remainder of this paper is organized as follows. Section 2 describes the workover rig fleet sizing and scheduling problem based on a deterministic approach, while section 3 describes the workover rig fleet sizing problem under uncertainty using a stochastic approach. Section 4 reviews the scenario-generation methods used for the stochastic approach. In section 5, computational experiments are performed to assess the performance of the proposed mathematical models and the scenario-generation methods. Section 6 provides conclusions and suggestions for further studies.

2. THE WORKOVER RIG FLEET SIZING AND SCHEDULING PROBLEM

In the deterministic approach, the workover rig fleet sizing and scheduling problem (WRFSSP) can be formulated as follows: a set of wells $i = 1, \dots, J$ needs to be maintained by a fleet of rigs $n = 1, \dots, N$ in a planning time T . The rigs are grouped into classes $m = 1, \dots, M$, there are a certain number of rigs available in each class M_m , and each class has a nominal service level w_m and hourly rig cost of β_m . Each rig n has a certain service level v_n derived from the rig class service level w_m of a given class of rig m . These service levels serve as a proxy to represent the technical specifications of the rig and their compatibility with operational requirements of the well. Each well i is associated with an intervention time d_i , an oil flow rate p_i , and a required service level r_i . A well can only be serviced by rigs with service levels greater than or equal to the service level required for the well. Wells that are not serviced are left for another round of planning together with new wells that may need to be serviced. The problem is considered from the point of view of production companies that own the right of exploring the wells but do not possess their own rig fleet and must thus rent them for a fixed time period. The problem consists of determining the size of a fleet of heterogeneous rigs required to service a set of wells while minimizing the cost of lost oil production and the rig rental over a time horizon. The cost of lost production is calculated using the oil price α . The rig displacement, assembly, and disassembly costs are assumed to be included in the cost of rig rental.

Pérez et al.¹⁰ developed an efficient reformulation of the mathematical model proposed by Costa⁶ that allows posterior decomposition of the model for the WRSP. The mathematical models proposed by Pérez et al.¹⁰ and Costa⁶ are called the decomposed mathematical model (DMM) and original mathematical model (OMM), respectively. The decomposition procedure reduces the number of variables and constraints considered in the OMM. The decomposition procedure allows the model to be developed without considering the problem of well allocation to rigs, making it even smaller in size. The formulations of the DMM and OMM are provided in Appendix A and Appendix B of the [Supporting Information](#), respectively.

Next, we show how the DMM can be extended such that it can be used to determine the optimal size of a heterogeneous fleet of rigs while minimizing the cost of lost production and the rig rental. We refer to this extended mathematical model as DMM'. The notation used for the sets, parameters, and variables is defined in the Nomenclature section.

$$\begin{aligned} \text{(DMM')} \text{ Min } & \alpha \left(\sum_{i=1}^J \sum_{n=1}^N \sum_{t=1}^T (t + d_i - 1) p_i S_{int} \right. \\ & \left. + T \sum_{i=1}^J p_i \left(1 - \sum_{n=1}^N \sum_{t=1}^T S_{int} \right) \right) + T \sum_{m=1}^M SU_m \beta_m \end{aligned} \quad (1)$$

subject to

$$\sum_{n=1}^N \sum_{t=1}^T S_{int} \leq 1 \quad \forall i \quad (2)$$

$$\sum_{i=1}^J \sum_{h=t-d_i+1}^t S_{inh} \leq SA_n \quad \forall t, n \quad (3)$$

$$\sum_{n=1|v_n=w_m}^N SA_n = SU_m \quad \forall m \quad (4)$$

$$SU_m \leq M_m \quad \forall m \quad (5)$$

$$S_{int} \in \{0, 1\} \quad \forall i, n, t | 1 \leq t \leq T - d_i + 1 \text{ and } r_i \leq v_n \quad (6)$$

$$SA_n \in \{0, 1\} \quad \forall n \quad (7)$$

$$SU_m \in \mathbb{Z}^+ \quad \forall m \quad (8)$$

In this mathematical model, the objective function 1 represents minimization of the cost of lost production and the cost of rig rental over the time horizon. The first component of lost production corresponds to the loss from wells that have been selected for intervention, and the second to the loss from wells that have not been selected. Lost production in the wells not selected occurs over the entire time horizon. Notice that the index t is used also as a scalar to determine the beginning of the interventions in each well. Constraint 2 ensures that the start of an intervention in each well (S_{int}) occurs no more than once for each rig at a specific time. Constraint 3 ensures that at a given time, the rigs that have been rented (SA_n) perform no more than one intervention and that there is no interference between interventions in different wells; that is, if a well i is being serviced by a rig n in the range $[t - d_i + 1, t]$, no other well can be serviced in that range by the same rig n . Constraint 4 specifies the number of rigs rented per class (SU_m), i.e., rigs with a service level v_n equal to the service level w_m of rig class m are grouped together. Constraint 5 ensures that the number of rigs rented in each class does not exceed the number of rigs available in that class. Constraints 6–8 define the decision variable domains. Constraint 6 ensures that the interventions in the wells start within the time horizon and that each well is serviced by a rig with an appropriate service level.

The DMM' is reformulated into the deterministic mathematical model (DTMM), which is composed by two separated parts (DTMM1 and DTMM2). The DTMM includes the binary variable SD_{imt} which represents whether a rig of class m starts servicing well i at time t . The variable SD_{imt} is used to replace S_{int} to reduce the number of variables and constraints, as the number of rig classes is generally smaller than the number of available rigs ($M < N$). We determine SD_{imt} as follows:

$$SD_{imt} = \sum_{n=1|v_n=w_m}^N S_{int} \quad \forall i, m, t \quad (9)$$

Equation 9 associates the rig class m with the rig n allocated to the well i according to the service level ($v_n = w_m$). As rigs in the same class m have a service level w_m , the sum $\forall n | v_n = w_m$ can be used in constraint 3:

$$\sum_{n=1|v_n=w_m}^N \sum_{i=1}^J \sum_{h=t-d_i+1}^t S_{inh} \leq \sum_{n=1|v_n=w_m}^N SA_n \quad \forall t, m \quad (10)$$

$$\sum_{i=1}^J \sum_{h=t-d_i+1}^t SD_{imh} \leq SU_m \quad \forall t, m \quad (11)$$

Constraint 11 ensures that no more than SU_m interventions are performed by rigs of class m at a given time t and that there is no interference between the interventions in the wells. The DTMM is then formulated as follows:

$$\begin{aligned} \text{(DTMM1)} \text{ Min } & \alpha \left(\sum_{i=1}^J \sum_{m=1}^M \sum_{t=1}^T (t + d_i - 1) p_i SD_{imt} \right. \\ & \left. + T \sum_{i=1}^J p_i \left(1 - \sum_{m=1}^M \sum_{t=1}^T SD_{imt} \right) \right) + T \sum_{m=1}^M SU_m \beta_m \end{aligned} \quad (12)$$

subject to

eqs 5, 8, and 11

$$\sum_{m=1}^M \sum_{t=1}^T SD_{imt} \leq 1 \quad \forall i \quad (13)$$

$$SD_{imt} \in \{0, 1\} \quad \forall i, m, t | 1 \leq t \leq T - d_i + 1 \text{ and } r_i \leq w_m \quad (14)$$

In this mathematical model, the objective function 12 represents minimization of the cost of lost production and the cost of rig rental over the time horizon. Constraint 13 ensures that the start of an intervention in each well occurs no more than once for each rig class at a specific time. Constraint 14 defines the domain of the decision variable SD_{imt} . It ensures that interventions in the wells start within the time horizon and that each well is serviced by a rig in a class with an appropriate service level.

The optimal solutions for SD_{imt}^* and SU_m^* determined with the DTMM1 can be used to solve the problem of allocating wells to the rented rigs. This configuration is shown below:

$$\begin{aligned} \text{(DTMM2)} \text{ Min } & \alpha \left(\sum_{i=1}^J \sum_{n=1}^N \sum_{t=1}^T (t + d_i - 1) p_i S_{int} \right. \\ & \left. + T \sum_{i=1}^J p_i \left(1 - \sum_{n=1}^N \sum_{t=1}^T S_{int} \right) \right) + T \sum_{m=1}^M SU_m^* \beta_m \end{aligned} \quad (15)$$

subject to

eqs 2, 3, and 7

$$\sum_{n=1|v_n=w_m}^N SA_n = SU_m^* \quad \forall m \quad (16)$$

$$S_{int} \in \{0, 1\} \quad \forall i, n, t \quad \left| \quad \sum_{m=1|v_n=w_m}^M SD_{imt}^* = 1 \quad (17)$$

In this mathematical model, the objective function 15 represents minimization of the cost of lost production and the

cost of rig rental over the time horizon. Constraint 16 ensures that the total numbers of rigs rented in each class are exactly the same as the values of SU_m^* found. Constraint 17 limits the domain of the decision variable S_{mt} and ensures that the start of the interventions and the allocation of rigs in each class are the same as the values found in SD_{imt}^* .

3. THE WORKOVER RIG FLEET SIZING PROBLEM UNDER UNCERTAINTY

In the stochastic approach, the workover rig fleet sizing problem is analyzed under uncertainty. The main difference in relation to the problem described in section 2 is that uncertainty in the well intervention time is considered. Each well i is associated with a finite set of intervention-time scenarios d_i^k , $k = 1, \dots, K$, generated from a random sample with a probability of occurrence π_k , such that $\sum_k \pi_k = 1$. Thus, the stochastic problem can be written in its deterministic equivalent form and can be solved by commercial optimization softwares.

Under uncertainty, the workover rig fleet sizing problem consists of determining the size of the rig fleet needed to service the wells while minimizing rig rental cost and the expected cost of lost production over a time horizon. The aim is to support decision-making by reducing the impact of lost production due to unexpected events that can delay or prolong interventions.

In the following description, the stochastic mathematical model (STMM) is presented as an extension of the DTMM. It is formulated as a two-stage stochastic programming model in which the first-stage variables represent the fleet of rigs rented by class (SU_m), while the second-stage variables correspond to the start of an intervention by a particular class of rigs in a particular scenario (SD_{imt}^k). The notation used for the sets, parameters, and decision variables is defined in the Nomenclature section. The mathematical model is

$$\begin{aligned} \text{(STMM) Min } & \alpha \left(\sum_{k=1}^K \pi_k \left(\sum_{i=1}^J \sum_{m=1}^M \sum_{t=1}^T (t + d_i^k - 1) p_i SD_{imt}^k \right. \right. \\ & \left. \left. + T \sum_{i=1}^J p_i \left(1 - \sum_{m=1}^M \sum_{t=1}^T SD_{imt}^k \right) \right) \right) + T \sum_{m=1}^M SU_m \beta_m \end{aligned} \quad (18)$$

subject to
eqs 5 and 8

$$\sum_{m=1}^M \sum_{t=1}^T SD_{imt}^k \leq 1 \quad \forall i, k \quad (19)$$

$$\sum_{i=1}^J \sum_{h=t-d_i^k+1}^t SD_{imh}^k \leq SU_m \quad \forall m, t, k \quad (20)$$

$$\begin{aligned} SD_{imt}^k & \in \{0, 1\} \quad \forall i, m, t, k | 1 \leq t \leq T - d_i^k + 1 \\ \text{and } r_i & \leq w_m \end{aligned} \quad (21)$$

In this mathematical model, the objective function 18 represents minimization of the expected cost of lost production and the cost of rig rental over the time horizon. Constraint 19 ensures that the start of an intervention in each well in each scenario occurs no more than once for each rig class at a specific time. Constraint 20 ensures that for each scenario, each rig class starts no more than SU_m interventions at a given time and that the interventions in the wells do not interfere with each other. Constraint 21 defines the domain of the decision variable SD_{imt}^k .

Notice that the STMM has the fundamental nature of providing first-stage decisions that are supposed to be made before the uncertainty is realized. For decisions regarding the second-stage (i.e., the scheduling decisions), the deterministic version (DTMM, which is equivalent to STMM with a single scenario) can be used considering as fixed the previously decided rig fleet and the scenario observed for the intervention times.

4. SCENARIO-GENERATION METHODS

In particular, the two-stage stochastic programming model presents two sources of difficulty when formulated with integer variables:²¹

- The exact evaluation of the expected value of the second-stage for a given first-stage decision. For continuous probability distributions of the uncertain parameter, the exact evaluation of the expected value of the second-stage involves the calculation of a multidimensional integral in the objective function, which is practically impossible. For discrete probability distributions, the expected value requires solving all possible realizations of the uncertain parameter and can be computationally intractable.
- The optimization of the expected value of the second-stage on the first-stage decisions. Consequently, the problem of optimization brings serious computational difficulties.

In such conditions, it is not practical to solve a stochastic optimization problem directly. However, one can use sampling techniques (scenario-generation) that consider a random subset of the probability distribution to obtain approximate results. The approximation of the stochastic optimization problem with this sampling strategy is known as sample average approximation (SAA).²²

By defining a scenario-generation method that ensures a good approximation to the random variable with a minimum number of scenarios, the computational resources required to solve the problem can be reduced significantly.

Kaut and Wallace²³ discuss how to evaluate the quality of scenario-generation methods for a given stochastic programming model. This approach establishes the number of scenarios required to accurately solve a stochastic optimization problem and is based on testing the stability of the several scenario-generation methods and choosing the one that is best suited for the given decision model. The stability tests have been widely used in several areas of research, for example, finance,^{24–27} power generation,^{28–32} transport,^{33–35} and petroleum supply management.³⁶

Several scenario-generation methods can be found in the literature. Brief descriptions of some of the main techniques are provided below.

4.1. Monte Carlo. The Monte Carlo method is the most widely used approach for dealing with stochastic optimization problems. It consists of generating a pseudorandom series (i.e., a series that imitates randomness) of independent numbers uniformly distributed in the interval $[0-1]$ and then constructing a sample by means of an appropriate transformation of the probability distribution. The convergence rate associated with the Monte Carlo method is $O(1/\sqrt{K})$,³⁷ i.e., the rate at which the error of the estimator decreases as the sample size K increases.

4.2. Scenario Reduction. The scenario reduction method was developed by Dupačová et al.³⁸ and Heitsch and Römischn,^{39,40} and consists of finding a subset of scenarios that approximates an initial set of scenarios in terms of a probabilistic

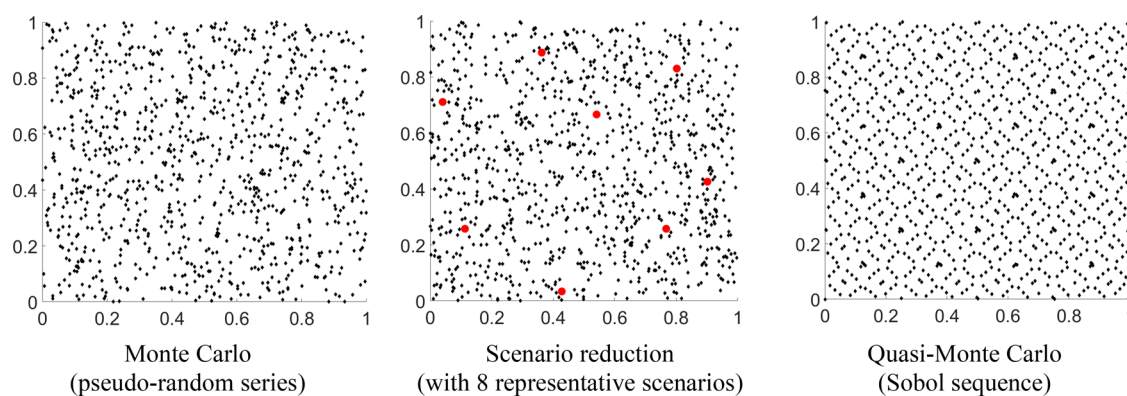


Figure 1. Scenarios for a two-dimensional uniform distribution on the open interval (0–1).

Table 1. Computational Results for the OMM' and DTMM

Inst.	No. of rigs used			Cost (US\$)	Time (s)		Inst.	No. of rigs used			Cost (US\$)	Time (s)	
	C3	C4	C5		OMM'	DTMM		C3	C4	C5		OMM'	DTMM
50/1	3	0	2	804228	95.7	1.5	150/1	5	5	2	2655603	–	0.7
50/2	2	1	2	1002968	88.3	0.6	150/2	5	5	2	2821940	–	1.8
50/3	4	1	1	980735	95.8	0.5	150/3	5	5	2	2741773	–	1.9
50/4	4	0	1	911815	85.9	0.3	150/4	5	5	3	2654760	–	1.9
50/5	3	1	1	848755	94.2	0.1	150/5	5	5	4	3139315	–	4.9
100/1	4	4	1	1798595	–	1.4	200/1	5	5	5	4600353	–	1.9
100/2	4	4	1	1712425	–	1.2	200/2	5	5	5	3869185	–	1.1
100/3	4	4	2	2065730	–	0.7	200/3	5	5	5	3720820	–	0.6
100/4	4	2	2	1967948	–	1.4	200/4	5	5	5	4077768	–	0.7
100/5	4	3	2	1836493	–	1.5	200/5	5	5	5	3955583	–	1.7

distance metric. New probabilities are assigned to the preserved scenarios, and a probability of zero is assigned to the eliminated scenarios. The most commonly used metric is the Fortet–Mourier metric.

The scenario reduction method is frequently used in problems that involve many scenarios and require long computational times. When used in problems with high-dimensional random variables, the method suffers from the disadvantage that the error between the approximate distribution and initial distribution increases, producing suboptimal values of the objective function.⁴¹

4.3. Quasi-Monte Carlo. The quasi-Monte Carlo method generates samples known as low-discrepancy sequences or quasi-random numbers, which are intended to increase the accuracy of the estimator by generating highly uniform points.

Low-discrepancy sequences fill the spaces in the interval [0–1] uniformly up to a specific density and have the potential to accelerate the convergence rate associated with the Monte Carlo method $O(1/\sqrt{K})$ to approximately $O(1/K)$. The quasi-Monte Carlo method is generally considered suitable for problems with high-dimensional random variables. The most widely used sequence is the Sobol sequence as it is more effective and produces accurate results for various problems.³⁷

A graphical example illustrating the differences between Monte Carlo, scenario reduction, and quasi-Monte Carlo methods with 1000 scenarios for a two-dimensional random variable is given in Figure 1.

4.4. Assessment of the Scenario-Generation Method. To establish the number of scenarios to be considered and assess the stability of the scenario-generation method to be chosen to model the uncertainty, we apply the in-sample and out-of-sample stability tests proposed by Kaut and Wallace.²³

In-sample stability determines whether similar or equal values of the objective function are obtained for various replications when the problem is solved with a set of scenarios generated to represent an uncertain parameter. Out-of-sample stability checks whether the optimal first-stage solutions obtained in each replication produce similar or equal values of the objective function when they are evaluated with the true distribution.

To evaluate out-of-sample stability, a set of reference scenarios large enough to approximate the true distribution is usually generated. The approximation to the true distribution is used for the sake of practical convenience, as in most cases there is no historical data for the uncertain parameter.

5. COMPUTATIONAL EXPERIMENTS

To evaluate the performance of the mathematical models developed to solve the workover rig problem, various medium-sized and large instances (with 50 and more than 100 wells, respectively) were generated. The large instances generated are difficult to solve and more complex than similar instances available in the literature. Details of how the values of the parameters used in the models were defined are given in Appendix D of the Supporting Information. All data generated is available in the Supporting Information zip file. Also, all data and models implemented can be made available upon request to the authors.

The time horizon considered is 15 days, broken into uniformly sized time steps of half-day length (12 h). The oil flow rate, the rig rental cost, and the oil price are reported in m^3/day , $\text{US}\$/\text{h}$, and $\text{US}\$/\text{m}^3$, respectively. We highlight that the time-indexed parameters have been scaled accordingly to the length of one time period.

Table 2. Computational Results for the Monte Carlo Method

Test		In-sample			Out-of-sample		
#Scenarios	#Sol.	Cost (US\$)	Std. Dev. (US\$)	Time (s)	Cost (US\$)	Std. Dev. (US\$)	Time (s)
10	3	1661725	25943	18.2	1662778	1139	4451.5
20	3	1659951	18579	50.1	1662792	1135	4448.4
30	2	1671085	16175	70.3	1662350	71	4530.1
50	2	1663545	10553	45.2	1662371	89	4525.5
100	2	1660835	7579	90.5	1662406	103	4517.7
200	2	1661481	5720	181.5	1662420	105	4514.6

Table 3. Computational Results for the Scenario Reduction Method

Test		In-sample			Out-of-sample		
#Scenarios	#Sol.	Cost (US\$)	Std. Dev. (US\$)	Time (s)	Cost (US\$)	Std. Dev. (US\$)	Time (s)
10	3	1615774	19184	17.7	1662507	689	4500.8
20	2	1619829	15102	50.2	1662357	78	4528.6
30	2	1622818	13097	81.2	1662343	63	4531.7
50	2	1625307	11594	44.7	1662350	71	4530.1
100	2	1630082	10312	90.7	1662350	71	4530.1
200	2	1636628	8878	183.5	1662350	71	4530.1

The models were implemented in AIMMS 3.14 using the CPLEX 12.6 solver, and all experiments were performed on a computer equipped with an Intel Core i7-3960X 3.3 GHz processor and 64 GB of RAM.

5.1. Assessment of the Performance of the DMM. In this first experiment, the DTMM is compared with the extension of the OMM for the WRFSSP. The extended model (OMM') is described in Appendix C of the [Supporting Information](#). This assessment is intended to compare the running time required by the DTMM for the different instances compared with a model normally used in the literature as a benchmark. We highlight that we use DTMM to refer to the process of solving DTMM1 and subsequently solving DTMM2.

Table 1 gives the results for the OMM' and DTMM for instances with 50, 100, 150, and 200 wells. In this table are presented five randomly generated instances for each number of wells. A gap of 0% and running time of 300 s were set as the stopping criteria for the solver. The table shows the number of rigs used in classes 3 (C3), 4 (C4), and 5 (C5) as well as the total cost and computational time for the OMM' and DTMM (i.e., the total computational time for DTMM1 and DTMM2 combined).

Table 1 shows that the DTMM required less time to solve all the instances, including large ones. In fact, the majority of the time reported for DTMM is taken by DTMM1, as DTMM2 takes negligible computational times to be solved (less than 0.1 s in all cases). The mean running time for all the instances was 1.3 s. The OMM' failed to solve the large instances before the stopping criteria were met (as indicated by “–” in Table 1). A comparison of the results shows that the DTMM can solve large-scale deterministic problems efficiently.

We highlight that a comparison between the OMM and the DMM for the WRSP has already been presented in Pérez et al.,¹⁰ showing similar results. However, one should notice the increment of the complexity of the problem when solving WRFSSP. For example, the OMM for the WRSP takes nearly 36 s for an instance with 50 wells, while for WRFSSP the solution time is approximately 96 s (considering the same computational setting), that is, an increase of 167%. The DTMM presented a reduced computational time without a significant increase from the WRSP to the WRFSSP.

5.2. Selection of a Scenario-Generation Method. We describe the experiment in which the STMM is used with a large 100-well instance to determine which of the following three scenario-generation methods is best suited to the problem: Monte Carlo (MC), scenario reduction (SR), and quasi-Monte Carlo (QMC). The intervention-time scenarios are generated based on the probability curve constructed by Costa⁶ using historical intervention times.

The MC and QMC methods were implemented in MATLAB R2013a. For the QMC, the Sobol low-discrepancy sequence was used with the scramble option. The SR method was implemented with the SCENRED2 tool in GAMS configured with the Fortet–Mourier metric, the forward reduction algorithm, the first-order norm, and an initial set of 1000 scenarios. The parameters for the SR method were defined based on preliminary experiments considering several possible combinations for these parameters and took into account the time required and efficiency of the forward reduction algorithm.

To test the out-of-sample stability, a reference set of 10000 scenarios representing an approximation to the true distribution was generated. For the in-sample test, 30 replications with different numbers of scenarios were generated.

Initially, a preliminary analysis is performed based on 10, 20, and 30 scenarios in each replication to determine first-stage solutions for the in-sample test close to the “true” first-stage optimal solution, which is obtained by solving the problem using the set of reference scenarios. The first-stage solutions found in the preliminary analysis can be used to solve the problem for a larger number of scenarios in the in-sample test. Progressive increases in the number of scenarios will cause the in-sample and out-of-sample standard deviation to decrease, and the results can be expected to converge to a single solution. To assess the results of the in-sample test after the preliminary analysis, the STMM described in section 3 is used. The variable SU_m is assigned the value found in each of the previous solutions, thereby reducing the search space for the problem.

A stopping criterion of a 0.01% gap was established for the solver in the preliminary analysis. This was necessary because the CPLEX solver took a long time to converge from the lower limit to the optimal value even when the best value found was known to be the optimal value or one very close to it, as confirmed in the

Table 4. Computational Results for the Quasi-Monte Carlo Method

Test	In-sample				Out-of-sample		
	#Scenarios	#Sol.	Cost (US\$)	Std. Dev. (US\$)	Time (s)	Cost (US\$)	Std. Dev. (US\$)
10	2	1666383	14411	17.7	1662392	99	4520.8
20	2	1660811	5733	51.0	1662392	99	4520.8
30	2	1662049	3679	87.9	1662385	96	4522.4
50	2	1661405	2507	45.1	1662399	101	4519.3
100	2	1662049	1351	91.3	1662336	52	4533.2
200	1	1662256	713	180.7	1662322	0	4536.3

experiments. To assess the in-sample test after the preliminary analysis and the out-of-sample test, a gap of 0% was used as the stopping criterion for the solver.

Tables 2, 3, and 4 present the computational results for different numbers of scenarios for the MC, SR, and QMC methods, respectively. The in-sample columns show the number of different first-stage solutions found in 30 replications (#Sol.), the expected total cost, the standard deviation of the total cost, and the mean running time. The out-of-sample columns show the expected total cost, the standard deviation of the total cost, and the running time using the solutions found in the in-sample test.

Tables 2 and 3 show that the MC and SR methods initially found three first-stage solutions in the 30 replications. As the number of scenarios increases to the maximum of 200, the number of solutions decreases to two. Table 4 shows that the QMC found two solutions initially and that with 200 scenarios it converges to a single solution. In all cases the in-sample and out-of-sample standard deviations decrease. The reduction is most significant with the QMC and reaches zero in the out-of-sample test. Notice that, although the in-sample test converges to a single solution, the evaluation is performed considering 30 distinct sets of 200 scenarios, and thus a small variation (standard deviation) is expected. The mean running time in the in-sample test for 10, 20, 30, 50, 100, and 200 scenarios is around 18, 50, 80, 45, 91, and 182 s, respectively. For the out-of-sample test, the running time to assess the set of reference scenarios is 4516 s on average.

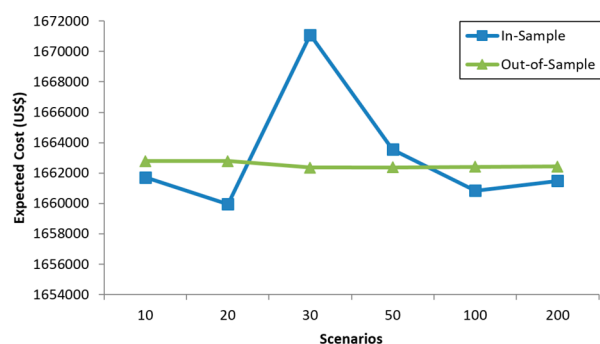


Figure 2. Expected cost of the Monte Carlo method.

Figures 2, 3, and 4 show the expected values, and Figure 5 shows the standard deviations of the MC, SR, and QMC scenario-generation methods. The figures can be used to assess the stability of each method.

These figures show that the QMC method is more stable than either the MC or SR method in the in-sample and out-of-sample tests. The in-sample standard deviation for the QMC method decreases linearly after 20 scenarios, and the out-of-sample

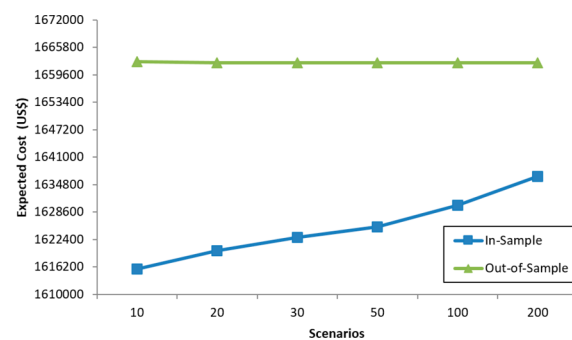


Figure 3. Expected cost of the scenario reduction method.

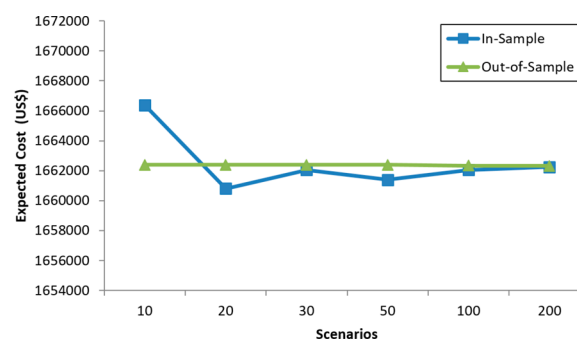


Figure 4. Expected cost of the quasi-Monte Carlo method.

standard deviation starts to decrease after 50 scenarios, reaching US\$0 for 200 scenarios.

The first-stage solution found by the QMC in Table 4 suggests that this is the “true” first-stage optimal solution, with an optimal value of US\$1,662,322, as shown in the out-of-sample test. The stability tests therefore indicate that the QMC method is the most suitable scenario-generation method and that replications with at least 100 scenarios should be generated to solve the proposed instance.

5.3. Applications with Instances with Larger Numbers of Scenarios. In this section, we present the results of experiments with (a) 150 wells and (b) 200 wells, using the STMM and the QMC scenario-generation method. These experiments were intended to confirm that the STMM could solve highly complex instances efficiently. The methodology is the same as that used in section 5.2. Details of the computational results for the in-sample and out-of-sample tests for these instances with different numbers of scenarios are given in Appendix E of the Supporting Information.

5.3.1. Instance with 150 Wells. Figures 6 and 7 show the stability of the QMC method for an instance with 150 wells. Figure 6 shows that the problem stabilizes after 100 scenarios. In Figure 7, the standard deviation decreases almost linearly between 20 and 200 scenarios in the in-sample test and between

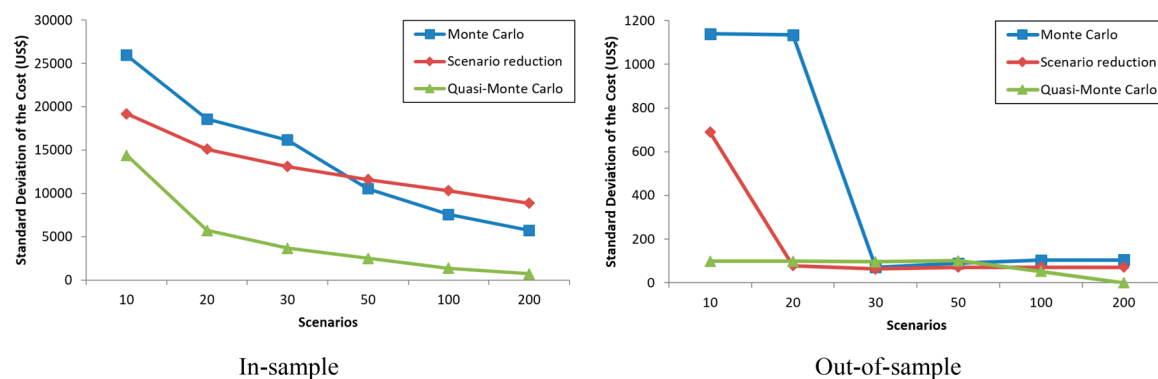


Figure 5. Standard deviation of the cost.

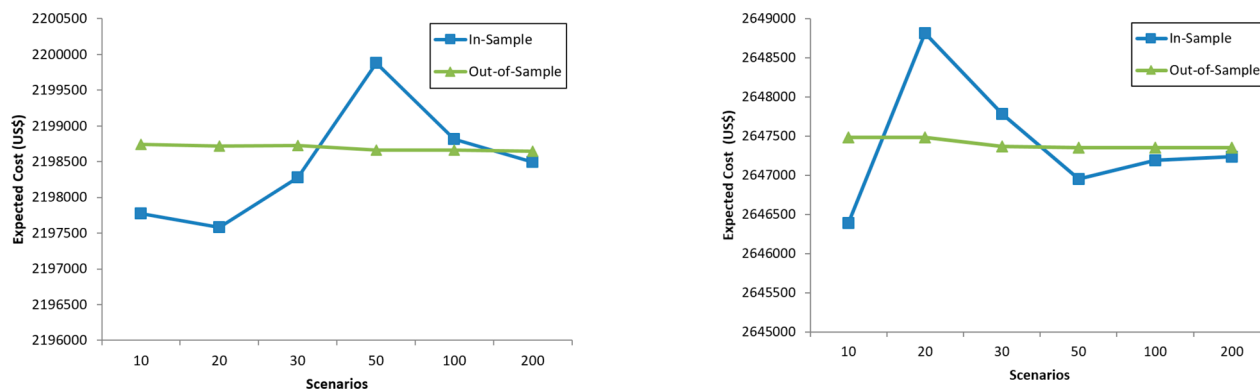


Figure 6. Expected cost.

Figure 8. Expected cost.

30 and 200 scenarios in the out-of-sample test. The results suggest that replications with fewer than 200 scenarios should not be generated, as this number of scenarios produces lower bounds for the optimal value for the problem, as shown in Figure 6.

5.3.2. Instance with 200 Wells. Figures 8 and 9 show the stability of the QMC method for an instance with 200 wells. Figure 8 shows that the problem is reasonably stable above 50 scenarios, and the second shows that the standard deviation falls almost linearly in the in-sample and out-of-sample tests, decreasing to zero with 50 scenarios and remaining at this value as the number of scenarios increases. Again, the results suggest that replications with at least 100 scenarios should be considered.

In general, the three instances considered yielded good results with 100 scenarios and there was little difference between the

expected cost in the in-sample and out-of-sample tests. Above a certain number of scenarios, the standard deviation of the cost in the QMC method decreased. The decrease appears to follow a nearly constant rate as the number of scenarios increases. This information could be useful when designing experiments with different levels of variability.

6. CONCLUSIONS AND FURTHER RESEARCH

In this paper, we presented two mathematical programming models for deterministic and stochastic cases of the workover rig problem. We have also generated various instances to evaluate the performance of the proposed models.

First, the deterministic version of the WRFSSP was considered. The model was formulated as an extension of the model proposed by Pérez et al.¹⁰ applied to the WRSP. The computational experiments performed considering medium-

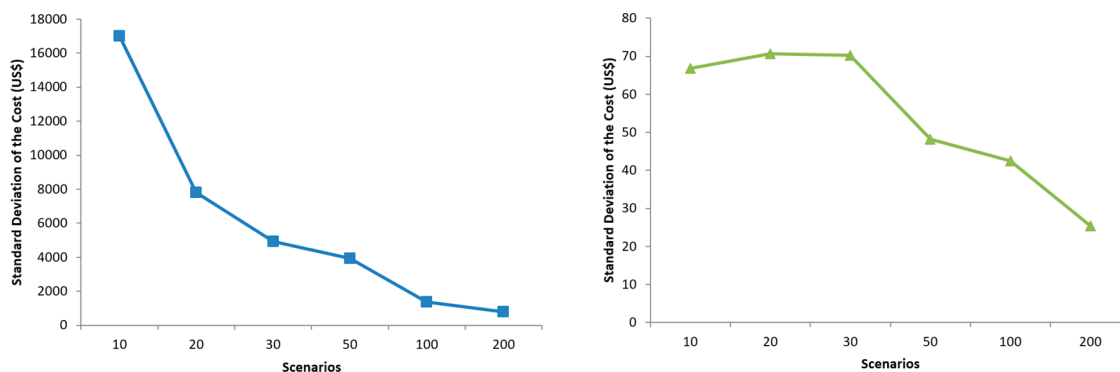


Figure 7. Standard deviation of the cost: in-sample (left) and out-of-sample (right).

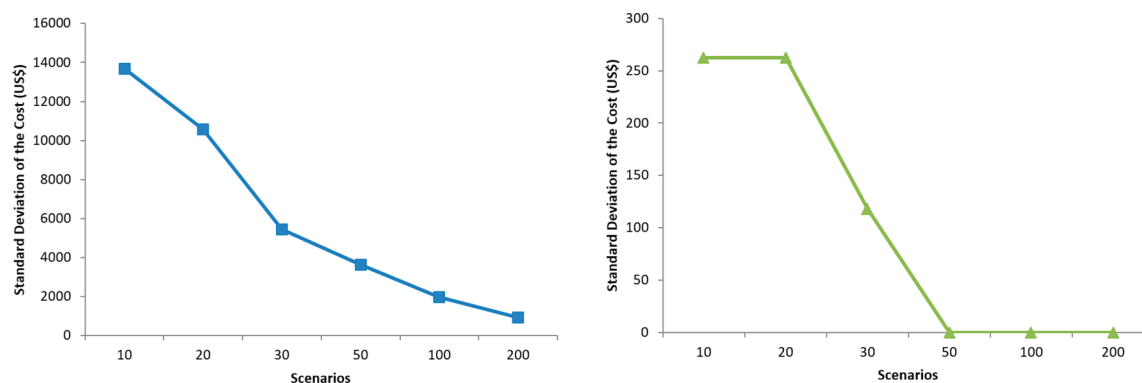


Figure 9. Standard deviation of the cost: in-sample (left) and out-of-sample (right).

sized and large instances were efficiently solved in shorter computational times (1.3 s on average). The results confirmed that the proposed model is more efficient than the model described in the literature.

Second, the problem was formulated as a two-stage stochastic programming model in which the uncertainty associated with the well intervention times was considered. The assessment of alternative scenario-generation methods suggested that the quasi-Monte Carlo method was the most suitable for representing the uncertainties considered, which involves a high-dimensional random variable (the well intervention time). The experiments revealed that the Monte Carlo method does not converge as quickly as the quasi-Monte Carlo method and that the scenario reduction method converges slowly because information is lost when the number of scenarios is reduced and new probabilities are assigned to the scenarios. The efficiency of the stochastic model combined with the quasi-Monte Carlo method was assessed by experimenting with large size instances. Using the proposed methodology, the three instances considered were solved satisfactorily.

Three areas could profitably be explored in future research. First, different intervention-time probability distributions based on real-world cases could be considered. Second, other scenario-generation methods, such as the moment matching method, could be used. Finally, efficient models for offshore oil fields including new operating characteristics could usefully be developed.

■ ASSOCIATED CONTENT

📄 Supporting Information

The Supporting Information is available free of charge on the ACS Publications website at DOI: 10.1021/acs.iecr.7b04500.

Appendix A: Decomposed mathematical model (DMM).
Appendix B: Original mathematical model (OMM).
Appendix C: Extension of the OMM (OMM'). Appendix D: Construction of random instances. Appendix E: Computational results for larger instances (PDF)
Experimental data from section 5 (ZIP)

■ AUTHOR INFORMATION

Corresponding Author

*E-mail: fabricio.oliveira@aalto.fi.

ORCID

Fabricio Oliveira: 0000-0003-0300-9337

Notes

The authors declare no competing financial interest.

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■ NOMENCLATURE

Sets

i, j : Well index, $i, j = \{1, 2, \dots, J\}$
 n : Available rigs index, $n = \{1, 2, \dots, N\}$
 m : Rig class index, $m = \{1, 2, \dots, M\}$
 t, h : Time index, $t, h = \{1, 2, \dots, T\}$
 k : Scenario index, $k = \{1, 2, \dots, K\}$

Parameters

J : Number of wells
 N : Number of rigs
 M : Number of rig classes
 M_m : Number of rigs per class m
 T : Time horizon
 K : Number of scenarios generated
 p_i : Oil flow rate in well i
 d_i : Duration of the intervention in well i
 d_i^k : Duration of the intervention in well i in scenario k
 π_k : Probability of scenario k occurring
 r_i : Service level required for well i
 v_n : Service level of rig n
 w_m : Service level of rig class m
 α : Price of oil
 β_m : Hourly cost of rig class m

Variables

$S_{it} = 1$, if the service in well i starts at time t ; 0, otherwise
 $S_{int} = 1$, if rig n starts to perform maintenance services on well i at time t ; 0, otherwise
 $SD_{int} = 1$, if a rig of class m starts to perform maintenance services on well i at time t ; 0, otherwise
 $SD_{int}^k = 1$, if a rig of class m starts to perform maintenance services on well i at time t in scenario k ; 0, otherwise
 $SA_n = 1$, if rig n is rented; 0, otherwise
 SU_m = Number of rigs of class m rented

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