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Research article

Dynamical interaction of solitary, periodic, rogue type wave solutions and multi-soliton solutions of the nonlinear models

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A R T I C L E I N F O

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A B S T R A C T

This study presents a modification form of modified simple equation method, namely new modified simple equation method. Multiple waves and interaction of soliton solutions of the Phi-4 and Klein-Gordon models are investigated via the scheme. Consequently, we derive various novels and more general interaction, and multiple wave solutions in term of exponential, hyperbolic, and trigonometric, rational function solutions combining with some free parameters. Taking special values of the free parameters, interaction of two dark bells, interaction of two bright bells, two kinks, two periodic waves, kink and soliton, kink-rogue wave solutions are obtained which is the key significance of this method. Properties of the achieved solutions have many useful descriptions of physical behavior, correlated to the solutions are attained in this work through plentiful 3D figures, density plot and 2D contour plots. The results derived may increase the prospect of performing significant experimentations and carry out probable applications.

1. Introduction

Perplexing phenomena customarily turn into the nonlinear differential equations (NLDEs). Consequently, the study of nonlinear differential equations (NLDEs) has sustained to attract much in last few years. Many scientific experimental models are employed in nonlinear differential equations (NLDEs) form including nonlinear fibers optics large-amplitude wave motions, fluids, plasma, solid-state physics etc. therefore, in the previous several times, many scientists and researchers worked to discover new effective methods [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19] for explaining (NLDEs) which are significant to elucidate different intricate problem such as F-expansion method and exp-expansion method [1], modified (g'/g)-expansion method [2], improved differential transform method [3], modified double sub-equation method [4], extended (g'/g)-expansion method [5], generalized (g'/g)-expansion method [6], new generalized (g'/g)-expansion method [7], discrete algebraic framework [8], modified simple equation method [9], hirota differential operator scheme [10], IRM-CG method [11], tanh method [12], tanh and the sine–cosine methods [13], hirota bilinear [14, 15, 16], EMSE method [17], generalized Riccati equation mapping method [18], nonlinear capacity method [19] and so on.

Thereupon, divergent skills have been implemented to find out multi-soliton solutions and interact soliton of nonlinear evolution equations, such as bilinear formalism and various ansatze's function [20], novel transformation method [21], direct rational exponential scheme [22] and multi expansion function method [23] etc. To establish the interaction and multi-soliton solution via Hirota's bilinear method, different ansatze functions was used in [24, 25, 26, 27, 28]. In recent, new modified simple equation method implemented to investigate multi-soliton solution form nonlinear evolution equation [29]. Owing to the importance of multi-soliton and various interactions of solitons in nonlinearity as well as getting innovative idea of new modified simple equation method for deriving solitonic collision, we willing to applied it on complex nonlinear models.

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The present paper focuses on the Phi-04 equation which has the following form [30, 31]

$$u_{tt} - u_{xx} + m^2 u + \lambda u^3 = 0. \quad (1)$$

Where m and λ are real valued parameters and u is realistic wave function that raised in nuclear and particle physics over decades. If we considered $m^2 = \alpha$ then the equation represents the Klein-Gordon equation in the following form

$$u_{tt} - u_{xx} + \alpha u + \beta u^3 = 0. \quad (2)$$

Where α and β are non-zero constant. Eq. (2) played significant role in solid state physics, nonlinear optics and quantum field theory. The only difference between Eqs. (1) and (2) is that coefficient of u obviously positive in Eq. (1), but in (2), it can take any value of positive or negative.

In this study, the properties of soliton, multi-soliton and interact soliton solution have been existed by implementing new modified simple equation [29] for Phi-4 equation. A variety of authoritative approaches has been applied to study phi 4. Such as the Phi-4 equation was studied also in [30] where using modified residual power series method was used to obtain analytical solution. Akter and Akbar [31] executed the modified simple equation method to construct exact solutions to Phi4 model. Envelope solitons was established for Phi-4 equation in [32] by using sech and tanh functions. Bifurcation and new exact traveling wave solutions was executed for time-space fractional Phi-4 equation [33]. The Klein-Gordon equation was studied [34] by improved approximation scheme to bound-state solutions. Analytical solutions were found for Klein–Gordon equation with a combined potential in [35]. Bright, Dark and rogue wave soliton solution existed from quadratic nonlinear Klein–Gordon equation [36].

2. Basic explanation of the method

To describe these essential steps of the new modified simple equation method, we deliberate a general form of a higher dimensional partial differential equation

$$H[u] = H(f, f_{xx}, f_t, f_{xt} \dots \dots \dots), \quad (3)$$

where $f = f(x,t)$ and H is a polynomial about f and its derivatives.

The NMSE method can be articulated in the following step

Step-01: Suppose that the solution of eq. (3) has the following form

$$f = \sum_{r=0}^{M} \sum_{i+j=r}^{M} \alpha_i \alpha_j \left(\frac{\varphi_1'(\eta_1)}{\acute{\varphi}_1(\eta_1)} \right)^i \left(\frac{\acute{\varphi}_2(\eta_2)}{\acute{\varphi}_2(\eta_2)} \right)^j . \quad (4)$$

Where $\eta_i = k_i x - \omega_i t,\ i = 1,2$ and α_i, α_j where $i, j = 0, 1, 2 \dots$ are arbitrary constant to be resolute later such that $\alpha_N \neq 0$.

Step-02: To control the positive integer M, we usually balance the highest-order derivatives and the nonlinear term in eq. (3).

Step-03: Substituting all necessary derivatives of $\mathrm{f}(x)$ in eq. (3). Then we obtain a polynomial of $\varphi_1^{-\mathrm{i}}, \varphi_2^{-\mathrm{j}}$ and $\varphi_1^{-\mathrm{i}}\varphi_2^{-\mathrm{j}}$.

Step-05: Now equating to zero of the co-efficient of same power of $\varphi_1^{-\mathrm{i}}\varphi_2^{-\mathrm{j}}$ and solve them to estimate of the values $\alpha_\mathrm{i}, \alpha_\mathrm{j}, \varphi_1$ and φ_2. Consequently, the deserved solutions are obtained.

3. Application of new modified simple equation method

In this section, we bring to bear a new form of modified simple equation to construct new solitary wave solution for some nonlinear model. The main advantage of this method is investigating novel multi-soliton and interact-soliton solution. The tanh function method, G'/G expansion method, exp-expansion method has predefined solution but new MSE method is direct method and has no predefined solution.

We start with phi 4 equation in the form:

$$u_{tt} - u_{xx} + m^2 u + \lambda u^3 = 0, \quad (5)$$

which was derived in the study of the propagation of nuclear and particle physics over decades. So far, there have been few studies on exact solutions of eq. (5), and we intend to construct multi-wave interaction solutions for this equation in this section. For eq. (5) the balance between u_{tt} and u^3 is $l = 1$. Following the new MSE method, we assume the solution of eq. (5) is

$$u = \alpha_0 + \alpha_1 \left(\frac{\varphi_1'(\eta_1)}{\varphi_1(\eta_1)} \right) + \alpha_2 \left(\frac{\varphi_2'(\eta_2)}{\varphi_2(\eta_2)} \right). \quad (6)$$

Where $\eta_r = k_r x - \omega_r t;\ r = 1, 2 . k_r,\ r = 1, 2$ are the angular speed and $\omega_r,\ r = 1, 2$ wave frequencies? The constraints $\alpha_i,\ i = 0, 1, 2$ are unknown to be determined later such that $\alpha_l \neq 0$.

To govern the unknown constraints, we substitute eq. (6) and its derivative term in eq. (5). Then equating the co-efficient of all power of $\varphi_1(\eta_1)^{-i}, \varphi_2(\eta_2)^{-j}$ and $\varphi_1(\eta_1)^{-i}\varphi_2(\eta_2)^{-j}$ zero.

$$\lambda \alpha_0^3 + \mathrm{m}^2 \alpha_0 = 0. \quad (7)$$

$$\alpha_1 \left(\omega_1^2 - \mathrm{k}_1^2\right) \varphi_1''' + \left(3\lambda\alpha_0^2 + \mathrm{m}^2\right) \varphi_1' = 0. \quad (8)$$

$$\alpha_2 \left(\omega_2^2 - \mathrm{k}_2^2\right) \varphi_2''' + \left(3\lambda\alpha_0^2 + \mathrm{m}^2\right) \varphi_2' = 0. \quad (9)$$

$$\alpha_1 \left(\mathrm{k}_1^2 - \omega_1^2\right) \varphi_1''\varphi_1' + 3\lambda\alpha_0\alpha_1^2\varphi_1'\varphi_1' = 0. \quad (10)$$

$$\alpha_2 \left(\mathrm{k}_2^2 - \omega_2^2\right) \varphi_2''\varphi_2' + 3\lambda\alpha_0\alpha_2^2\varphi_2'\varphi_2' = 0. \quad (11)$$

$$\lambda\alpha_1^3 - 2\alpha_1\mathrm{k}_1^2 + 2\alpha_1\omega_1^2 = 0. \quad (12)$$

$$\lambda\alpha_2^3 - 2\alpha_2\mathrm{k}_2^2 + 2\alpha_2\omega_2^2 = 0. \quad (13)$$

$$3\lambda\alpha_2\alpha_1^2\varphi_1'\varphi_1'\varphi_2' = 0. \quad (14)$$

$$3\lambda\alpha_1\alpha_2^2\varphi_2'\varphi_2'\varphi_1' = 0. \quad (15)$$

$$6\lambda\alpha_1\alpha_0\alpha_2\varphi_1'\varphi_2' = 0. \quad (16)$$

To estimate the values of all constraints, we solve the eq. (6) and eq. (12) to eq. (16).

We get

$$\alpha_0 = 0, +\frac{m}{\sqrt{-\lambda}}, -\frac{m}{\sqrt{-\lambda}}, \qquad \alpha_1 = \sqrt{\frac{2(\mathrm{k}_1^2 - \omega_1^2)}{\lambda}}, -\sqrt{\frac{2(\mathrm{k}_1^2 - \omega_1^2)}{\lambda}},$$

$$\alpha_2 = \sqrt{\frac{2\left(\mathrm{k}_2^2 - \omega_2^2\right)}{\lambda}}, -\sqrt{\frac{2\left(\mathrm{k}_2^2 - \omega_2^2\right)}{\lambda}}.$$

Case-01: When $\alpha_0 = 0$ then the remaining eq. (10) and eq. (11) becomes,

$$\varphi_1'' = 0.$$

$$\varphi_2'' = 0.$$

From eq. (8) and (9), we can write

$$\varphi_1 = a_0 + a_1 e^{p\eta_1} + a_2 e^{-p\eta_1}. \quad (17)$$

$$\varphi_1 = b_0 + b_1 e^{q\eta_2} + b_2 e^{-q\eta_2}. \quad (18)$$

Making the use eq. (17) and (18) in eq. (6), we get

$$u = p\alpha_1 \frac{a_1 e^{p\eta_1} + a_2 e^{-p\eta_1}}{a_0 + a_1 e^{p\eta_1} + a_2 e^{-p\eta_1}} + q\alpha_2 \frac{b_1 e^{q\eta_2} + b_2 e^{-q\eta_2}}{b_0 + b_1 e^{q\eta_2} + b_2 e^{-q\eta_2}}, \quad (19)$$

where $p = \frac{m}{\sqrt{\mathrm{k}_1^2 - \omega_1^2}},\ q = \frac{m}{\sqrt{\mathrm{k}_2^2 - \omega_2^2}}, \alpha_1 = \pm\sqrt{\frac{2(\mathrm{k}_1^2 - \omega_1^2)}{\lambda}}, \alpha_2 = \pm\sqrt{\frac{2(\mathrm{k}_1^2 - \omega_1^2)}{\lambda}}$.

If conditions $\mathrm{k}_1^2 > \omega_1^2$ and $\mathrm{k}_2^2 > \omega_2^2$ apply to the solution of eq. (19) then the solution can express as hyperbolic function.

For $a_0 = a_1 = a_2, b_0 = b_1 = b_2$ eq. (19) develops

$$u = m\sqrt{\frac{2}{\lambda}} \frac{2\cosh(p\eta_1)}{1 + 2\cosh(p\eta_1)} + m\sqrt{\frac{2}{\lambda}} \frac{2\cosh(q\eta_2)}{1 + 2\cosh(q\eta_2)}, \quad (20)$$

where $p = \frac{m}{\sqrt{\mathrm{k}_1^2 - \omega_1^2}},\ q = \frac{m}{\sqrt{\mathrm{k}_2^2 - \omega_2^2}}$.

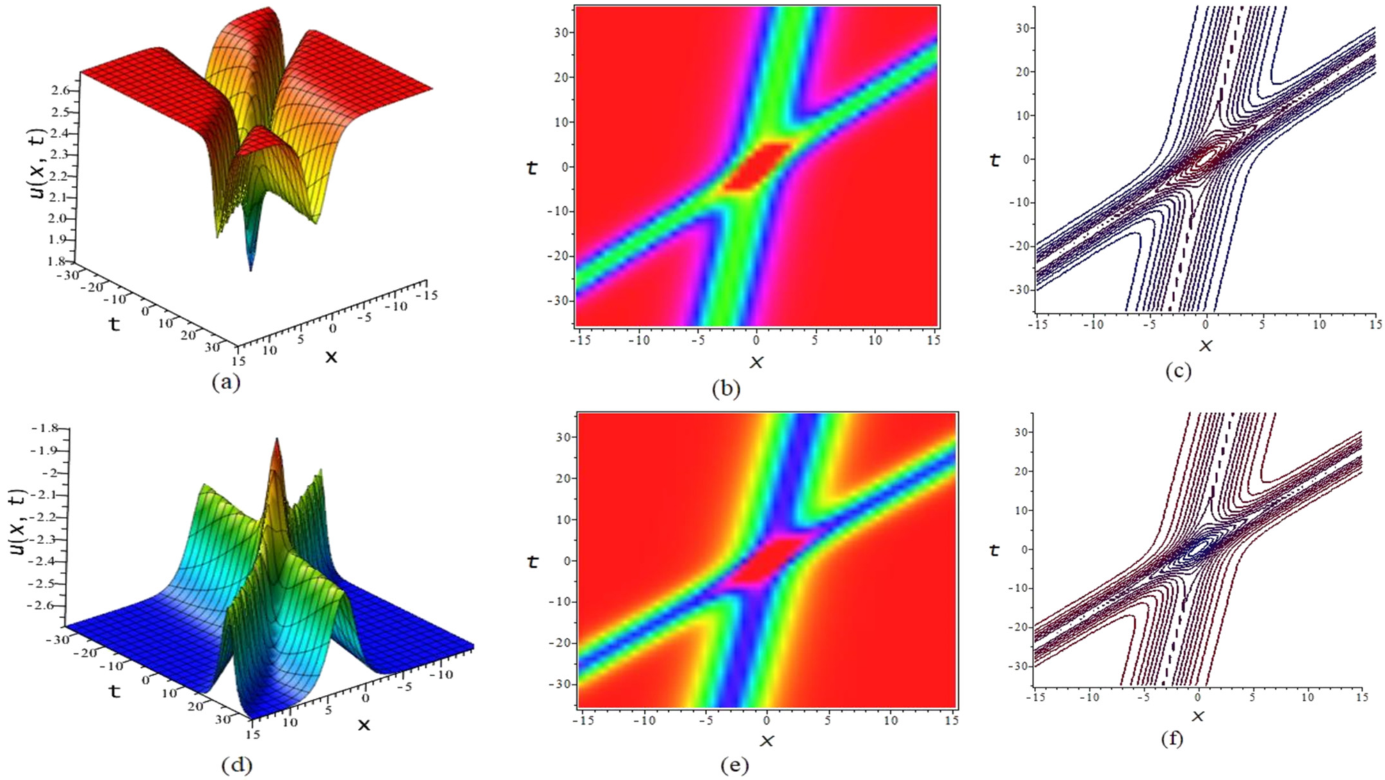

Fig. 1. Profile of interaction solution of eq. (20) for $k_1 = 5.5$, $k_2 = 2.5$, $\omega_1 = 0.5$, $\omega_2 = 1.5$, $m = 0.807$, $\lambda = 0707$. Where image (a) 3D plot, (b) density plot, (c) contour plot for $m = 0.807 > 0$. When $m = -0.807 < 0$ dark type interaction solution is shown in image (d) 3D plot and the corresponding (e) density plot, (f) contour plot.

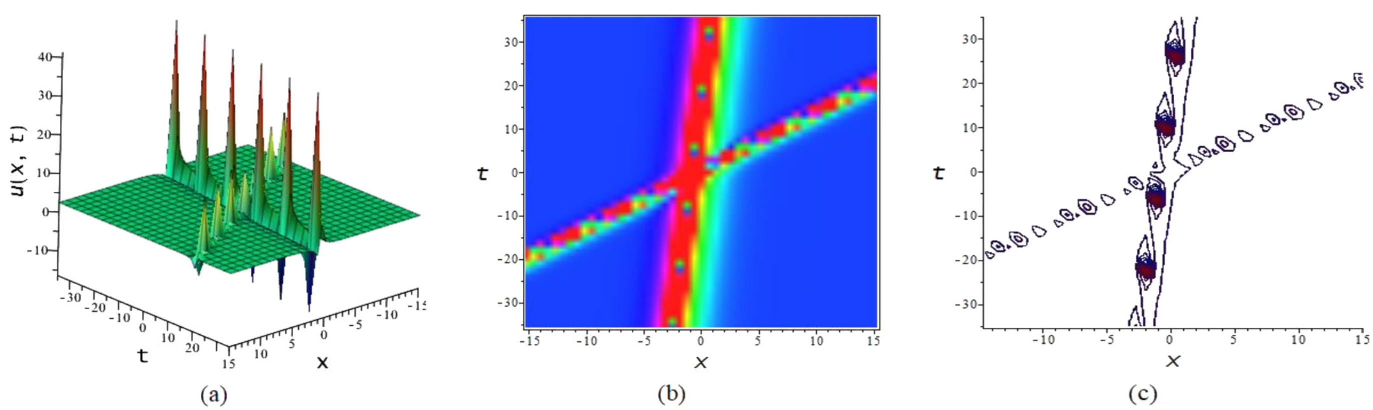

Fig. 2. Profile of interaction of soliton solution of eq. (21) for $k_1 = 10.5$, $k_2 = 5.5$, $\omega_1 = 0.5$, $\omega_2 = 1.5$, $m = 1, \lambda = 1.3$, where image (a) 3D plot, (b) density plot, (c) contour plot.

If we consider $a_0 = a_1 = -a_2, b_0 = b_1 = -b_2$ then the solution of eq. (19) converts

$$u = m\sqrt{\frac{2}{\lambda}}\frac{2\sinh(p\eta_1)}{1+2\sinh(p\eta_1)} + m\sqrt{\frac{2}{\lambda}}\frac{2\sinh(q\eta_2)}{1+2\sinh(q\eta_2)}, \tag{21}$$

where $p = \frac{m}{\sqrt{k_1^2-\omega_1^2}}$, $q = \frac{m}{\sqrt{k_2^2-\omega_2^2}}$.

Now we customary $a_0 = a_1 = a_2, b_0 = b_1 = -b_2$ under some condition $k_1^2 > \omega_1^2$ and $k_2^2 < \omega_2^2$ then the solution of eq. (19) represents trigonometric function solution below.

$$u = m\sqrt{\frac{2}{\lambda}}\frac{2\cosh(p\eta_1)}{2cosh\left(p\eta_1\right)+1} + m\sqrt{\frac{2}{\lambda}}\frac{2\sin(q\eta_2)}{2\sin\left(q\eta_2\right)-i}, \tag{22}$$

where $p = \frac{m}{\sqrt{k_1^2-\omega_1^2}}$, $q = \frac{m}{\sqrt{\omega_2^2-k_2^2}}$.

Now we set $a_0 = a_1 = -a_2, b_0 = b_1 = b_2$ under some condition $k_1^2 < \omega_1^2$ and $k_2^2 > \omega_2^2$ then the solution of eq. (19) converts.

$$u = m\sqrt{\frac{2}{\lambda}}\frac{2\sin(p\eta_1)}{2\sin\left(p\eta_1\right)-i} + m\sqrt{\frac{2}{\lambda}}\frac{2\cos(q\eta_2)}{2\cos\left(q\eta_2\right)+1}, \tag{23}$$

where $p = \frac{m}{\sqrt{2\left(\omega_2^2-k_2^2\right)}}$, $q = \frac{m}{\sqrt{2\left(\omega_2^2-k_2^2\right)}}$.

Now we set $a_0 = a_1 = -a_2, b_0 = b_1 = -b_2$ under some condition $k_1^2 < \omega_1^2$ and $k_2^2 < \omega_2^2$ then

$$u = m\sqrt{\frac{2}{\lambda}}\frac{2\sin(p\eta_1)}{2sin\left(p\eta_1\right)-i} + m\sqrt{\frac{2}{\lambda}}\frac{2\sin(q\eta_2)}{2\sin\left(q\eta_2\right)-i}, \tag{24}$$

Case-02: When $\alpha_0 = +\frac{m}{\sqrt{-\lambda}}$, $\alpha_{1=} \pm\sqrt{\frac{2(k_1^2-\omega_1^2)}{\lambda}}$, $\alpha_2 = \pm\sqrt{\frac{2(k_1^2-\omega_1^2)}{\lambda}}$ then the residual eq. (8)–(11) becomes

$$\alpha_1\left(\omega_1^2-k_1^2\right)\varphi_1''' + 4m^2\varphi_1' = 0. \tag{25}$$

$$\alpha_2\left(\omega_2^2-k_2^2\right)\varphi_2''' + 4m^2\varphi_2' = 0. \tag{26}$$

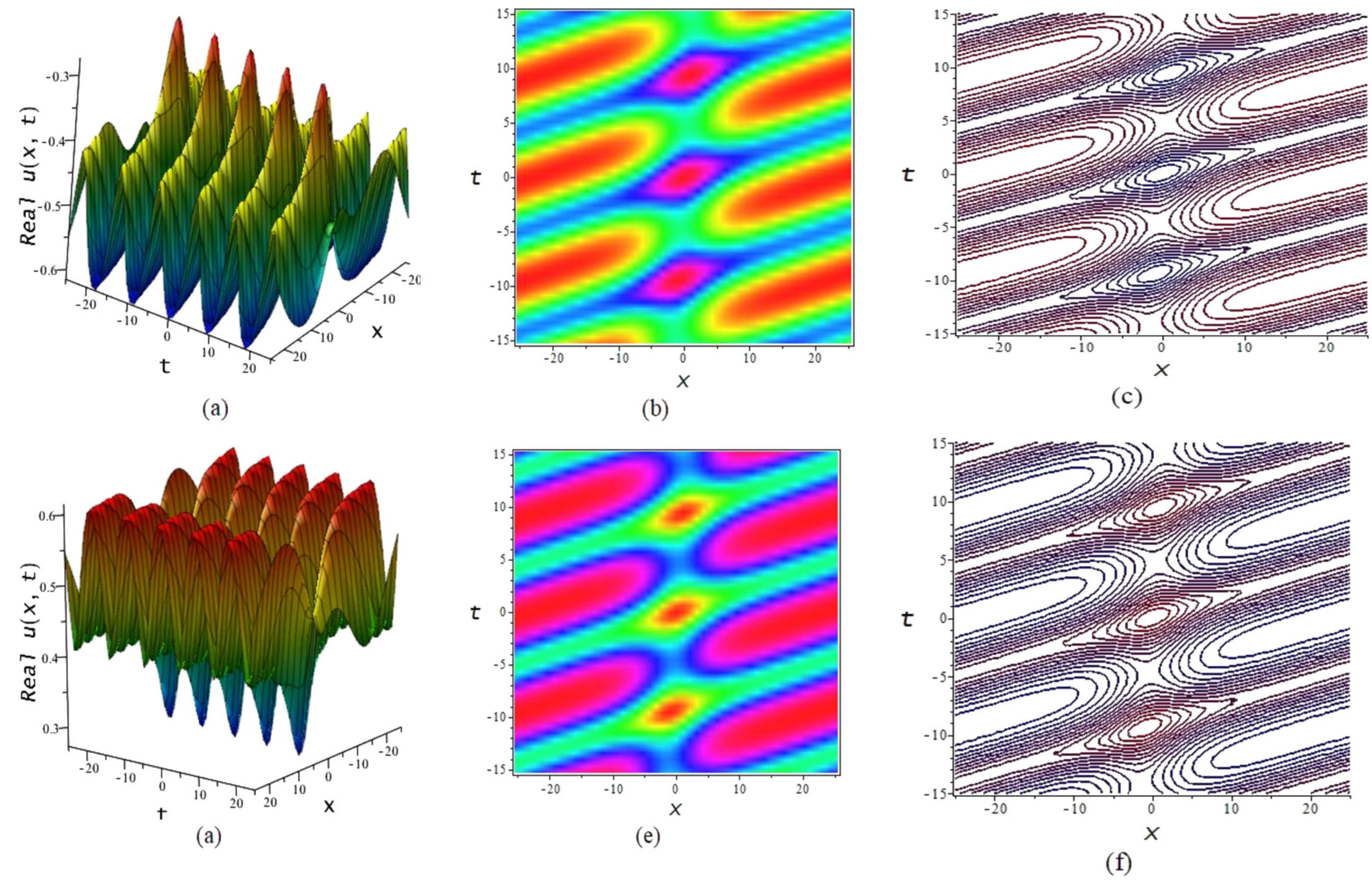

Fig. 3. Profile of interaction of lump and periodic wave solution of eq. (22) for $k_1 = k_2 = 0.5$, $\omega_1 = 0.05$, $\omega_2 = 2.5$, $m = 0.33$, $\lambda = 1.32$, where, $m = 0.33 > 0$, we get the dark type interaction shown in image (a) 3D plot, (b) density plot, (c) contour plot. For $m = -0.33 < 0$, bright type interaction exists in image (d) 3D plot, (e) density plot, (f) contour plot.

$$3\alpha_1\left(k_1^2-\omega_1^2\right)\varphi_1''\varphi_1'+3m\sqrt{-\lambda}\alpha_1^2\varphi_1'\varphi_1'=0. \tag{27}$$

$$3\alpha_2\left(k_2^2-\omega_2^2\right)\varphi_2''\varphi_2'+3m\sqrt{-\lambda}\alpha_2^2\varphi_2'\varphi_2'=0 \tag{28}$$

From eq. (27) and (28), we get

$$\varphi_1'=-\frac{\left(k_1^2-\omega_1^2\right)\varphi_1''}{\alpha_1 m\sqrt{-\lambda}}. \tag{29}$$

$$\varphi_2'=-\frac{\left(k_2^2-\omega_2^2\right)\varphi_2''}{\alpha_2 m\sqrt{-\lambda}}. \tag{30}$$

Now insert eq. (29) in eq. (25) and eq. (30) in eq. (26) then we obtain,

$$\varphi_1''=a_3e^{r\eta_1}. \tag{31}$$

$$\varphi_2''=b_3e^{l\eta_2}, \tag{32}$$

where a_3, b_3 are free parameters.

Now insert eq. (31) in eq. (29) and eq. (32) in eq. (30) then we obtain,

$$\varphi_1=a_4-\frac{\left(k_1^2-\omega_1^2\right)a_3e^{r\eta_1}}{r\alpha_1 m\sqrt{-\lambda}}. \tag{33}$$

$$\varphi_2=b_4-\frac{\left(k_2^2-\omega_2^2\right)b_3e^{l\eta_1}}{r\alpha_2 m\sqrt{-\lambda}}, \tag{34}$$

where a_3, a_4, b_3, b_4 are free parameters.

To obtain the require solution of eq. (5), we insert the above values in eq. (6)

$$u=\frac{m}{\sqrt{-\lambda}}+\frac{\left(\omega_1^2-k_1^2\right)}{m\sqrt{-\lambda}}\frac{a_3e^{r\eta_1}}{a_4+\frac{a_3\left(\omega_1^2-k_1^2\right)}{2m^2}e^{r\eta_1}}+\frac{\left(\omega_2^2-k_2^2\right)}{m\sqrt{-\lambda}}\frac{b_3e^{l\eta_2}}{b_4+\frac{b_3\left(\omega_2^2-k_2^2\right)}{2m^2}e^{l\eta_2}}, \tag{35}$$

where $r=\frac{2m}{\sqrt{2\left(\omega_1^2-k_1^2\right)}}$, $l=\frac{2m}{\sqrt{2\left(\omega_2^2-k_2^2\right)}}$.

For $\alpha_{1}= \pm\sqrt{\frac{2(k_1^2-\omega_1^2)}{\lambda}}, \alpha_2=\pm\sqrt{\frac{2(k_1^2-\omega_1^2)}{\lambda}}$.

If we set $a_5=b_5=1$, $a_3=\frac{2m^2}{\left(\omega_1^2-k_1^2\right)}$, $b_3=\frac{2m^2}{\left(\omega_2^2-k_2^2\right)}$. Then we obtain,

$$u=\frac{m}{\sqrt{-\lambda}}+\frac{m}{\sqrt{-\lambda}}\operatorname{sech}\left(\frac{g\eta_1}{2}\right)e^{\frac{g\eta_1}{2}}+\frac{m}{\sqrt{-\lambda}}\operatorname{sech}\left(\frac{h\eta_2}{2}\right)e^{\frac{h\eta_2}{2}}. \tag{36}$$

If we set $a_5=b_5=-1$, $a_4=\frac{-2m^2}{\left(\omega_1^2-k_1^2\right)}$, $b_4=\frac{-2m^2}{\left(\omega_2^2-k_2^2\right)}$, then we obtain,

$$u=\frac{m}{\sqrt{-\lambda}}-\frac{m}{\sqrt{-\lambda}}\operatorname{cosech}\left(\frac{g\eta_1}{2}\right)e^{\frac{-g\eta_1}{2}}-\frac{m}{\sqrt{-\lambda}}\operatorname{cosech}\left(\frac{h\eta_2}{2}\right)e^{\frac{-h\eta_2}{2}}. \tag{37}$$

Case-03: when $\alpha_0=-\frac{m}{\sqrt{-\lambda}}$, $\alpha_{1}=\pm\sqrt{\frac{2(k_1^2-\omega_1^2)}{\lambda}}, \alpha_2=\pm\sqrt{\frac{2(k_1^2-\omega_1^2)}{\lambda}}$ then the residual eq. (8)–(11) becomes,

$$\alpha_1\left(\omega_1^2-k_1^2\right)\varphi_1'''+4m^2\varphi_1'=0. \tag{38}$$

$$\alpha_2\left(\omega_2^2-k_2^2\right)\varphi_2'''+4m^2\varphi_2'=0. \tag{39}$$

$$3\alpha_1\left(k_1^2-\omega_1^2\right)\varphi_1''\varphi_1'+3m\sqrt{-\lambda}\alpha_1^2\varphi_1'\varphi_1'=0. \tag{40}$$

$$3\alpha_2\left(k_2^2-\omega_2^2\right)\varphi_2''\varphi_2'+3m\sqrt{-\lambda}\alpha_2^2\varphi_2'\varphi_2'=0. \tag{41}$$

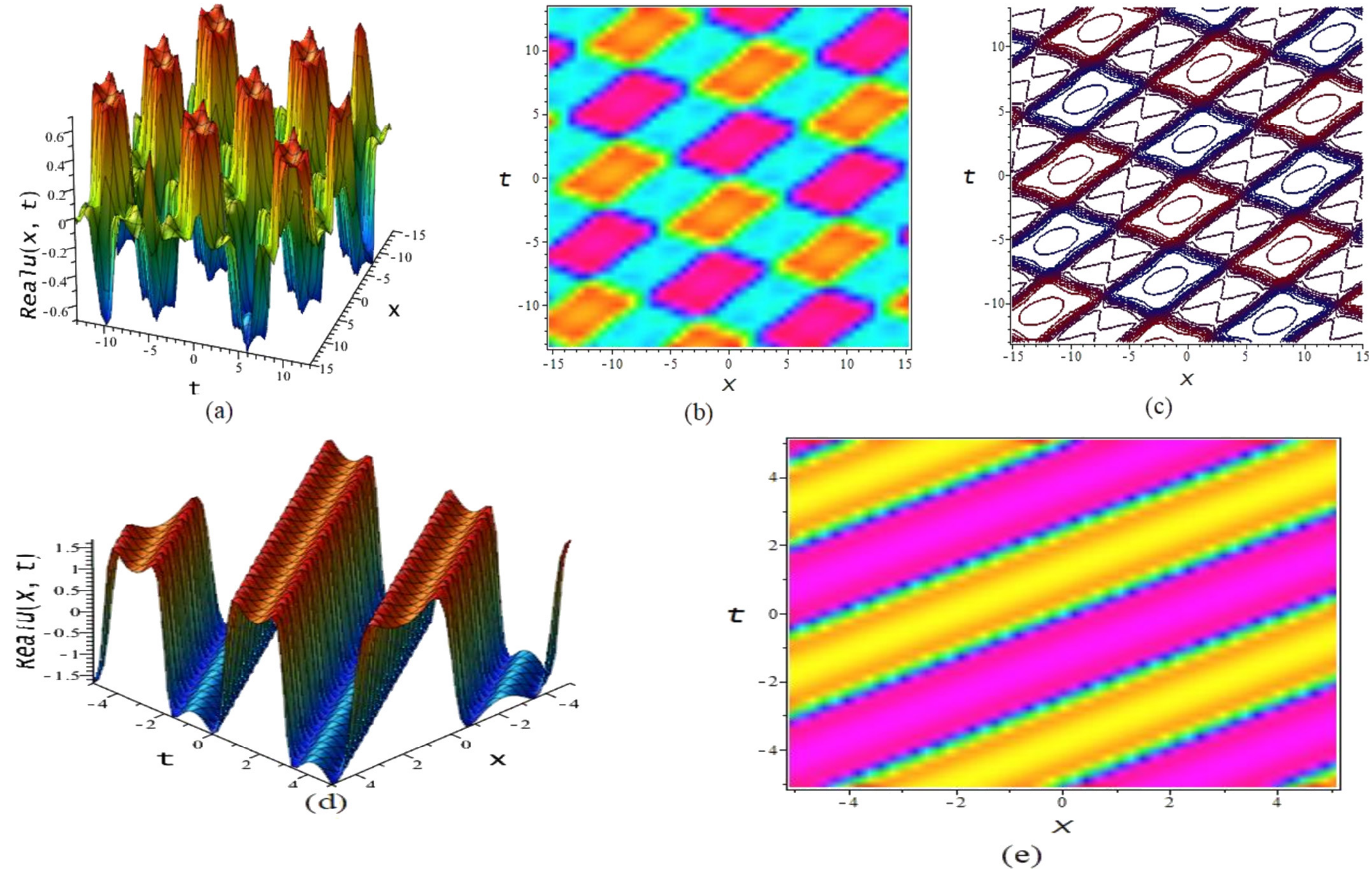

Fig. 4. Profile of periodic solution of eq. (24) for $k_1 = 1$, $k_2 = 0.5$, $\omega_1 = \omega_2 = 1.5$, $m = 0.5$, $\lambda = -1$ in image (a) 3D plot, (b) density plot, (c) contour plot. For $k_1 = k_2 = 1.5$, $\omega_1 = \omega_2 = 2.5$, $m = 1$, $\lambda = 0.707$ another periodic soliton obtained in graph (d) 3D plot, (e) density plot.

From eq. (40) and (41), we get

$$\varphi_1' = \frac{\left(k_1^2 - \omega_1^2\right)\varphi_1''}{m\sqrt{2\left(\omega_1^2 - k_1^2\right)}}. \tag{42}$$

$$\varphi_2' = \frac{\left(k_2^2 - \omega_2^2\right)\varphi_2''}{m\sqrt{2\left(\omega_2^2 - k_2^2\right)}}. \tag{43}$$

Now insert eq. (42) in eq. (38) and eq. (43) in eq. (39) then we obtain,

$$\varphi_1'' = a_4 e^{g\eta_1}. \tag{44}$$

$$\varphi_2'' = b_4 e^{h\eta_2}, \tag{45}$$

where $g = \frac{-2m}{\sqrt{2\left(\omega_1^2 - k_1^2\right)}}$ and $h = \frac{-2m}{\sqrt{2\left(\omega_2^2 - k_2^2\right)}}$.

Now insert eq. (44) in eq. (42) and eq. (45) in eq. (43), then we obtain,

$$\varphi_1 = a_5 + \frac{a_4\left(\omega_1^2 - k_1^2\right)}{2m^2} e^{g\eta_1}. \tag{46}$$

$$\varphi_2 = b_5 + \frac{b_4\left(\omega_2^2 - k_2^2\right)}{2m^2} e^{lh}, \tag{47}$$

where a_4, a_5, b_4, b_5 are free parameters.

To obtain the require solution of eq. (5), we insert the Eqs. (42), (43), (46), (47) in eq. (6)

$$u = -\frac{m}{\sqrt{-\lambda}} - \frac{a_4\left(\omega_1^2 - k_1^2\right)}{m\sqrt{-\lambda}} \frac{e^{g\eta_1}}{a_5 + \frac{a_4\left(\omega_1^2 - k_1^2\right)}{2m^2} e^{g\eta_1}.} - \frac{b_4\left(\omega_2^2 - k_2^2\right)}{m\sqrt{-\lambda}} \frac{e^{h\eta_2}}{b_5 + \frac{b_4\left(\omega_2^2 - k_2^2\right)}{2m^2} e^{h\eta_2}.}. \tag{48}$$

If we set $a_5 = b_5 = 1$, $a_4 = \frac{2m^2}{\left(\omega_1^2 - k_1^2\right)}$, $b_4 = \frac{2m^2}{\left(\omega_2^2 - k_2^2\right)}$. Then we obtain,

$$u = -\frac{m}{\sqrt{-\lambda}} - \frac{m}{\sqrt{-\lambda}} \operatorname{sech}\left(\frac{g\eta_1}{2}\right) e^{\frac{g\eta_1}{2}} - \frac{m}{\sqrt{-\lambda}} \operatorname{sech}\left(\frac{h\eta_2}{2}\right) e^{\frac{h\eta_2}{2}}. \tag{49}$$

If we set $a_5 = b_5 = -1$, $a_4 = \frac{-2m^2}{\left(\omega_1^2 - k_1^2\right)}$, $b_4 = \frac{-2m^2}{\left(\omega_2^2 - k_2^2\right)}$. Then we obtain,

$$u = -\frac{m}{\sqrt{-\lambda}} + \frac{m}{\sqrt{-\lambda}} \operatorname{cosech}\left(\frac{g\eta_1}{2}\right) e^{\frac{-g\eta_1}{2}} + \frac{m}{\sqrt{-\lambda}} \operatorname{cosech}\left(\frac{h\eta_2}{2}\right) e^{\frac{-h\eta_2}{2}}. \tag{50}$$

4. Influence of new modified simple equation method on phi-04 equation

In this segment, we will discuss the influence of NMSE technique to construct multi-soliton and different type of interaction solutions to phi-4 model. By investigating this method, it is possible to obtain the dynamical behavior and characters of multi-soliton and interaction solutions. If we substituting $m^2 = \alpha$ them we get same type solution for Klein-Gordon equation.

If conditions $k_1^2 > \omega_1^2$ and $k_2^2 > \omega_2^2$ apply to the solution of eq. (19) then the solution can express as hyperbolic function in eq. (20) and eq. (21). From these solutions the interaction soliton solution established in Fig. 1(a-f) and 2(a-c) for the values of parameters $k_1 = 5.5$, $k_2 = 2.5$, $\omega_1 = 0.5$, $\omega_2 = 1.5$, $\lambda = 0707$.

Now we insert $a_0 = a_1 = a_2, b_0 = b_1 = -b_2$ under some condition $k_1^2 > \omega_1^2$ and $k_2^2 < \omega_2^2$ and insert $a_0 = a_1 = -a_2, b_0 = b_1 = b_2$ under some condition $k_1^2 < \omega_1^2$ and $k_2^2 > \omega_2^2$ then the solution of eq. (19) represents

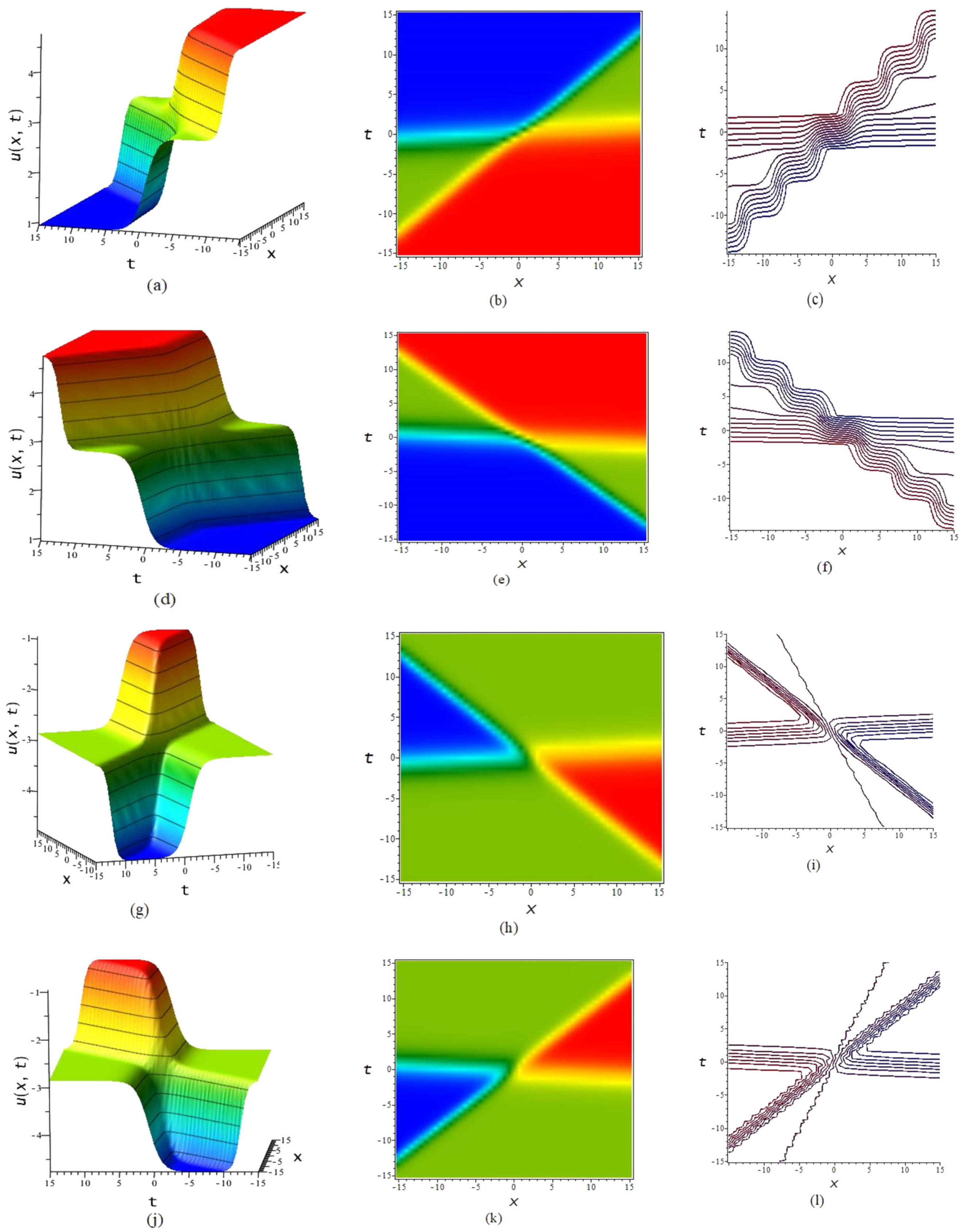

Fig. 5. Profile of multi-soliton solution of eq. (36) for $k_1 = k_2 = 1.5$, $\omega_1 = 0.5$, $\omega_2 = 1.5$, $m = 0.33$, $\lambda = 1.32$, where α_1 and α_2 be positive then we get the complexions (double kink) multi-soliton solution shown in image (a) 3D plot, (b) density plot, (c) contour plot, and α_1 and α_2 be negative then we get the complexions (double anti-kink) multi-soliton solution shown in image (d) 3D plot, (e) density plot, (f) contour plot. For $\omega_1 = -0.6 < 0$, the complexions (double kink) multi-soliton solution exists in image (j) 3D plot, (k) density plot, (l) contour plot. If we set $\omega_2 = -10 < 0$, then the profile represents the complexions (double anti-kink) multi-soliton solution shown in image (j) 3D plot, (h) density plot, (i) contour plot.

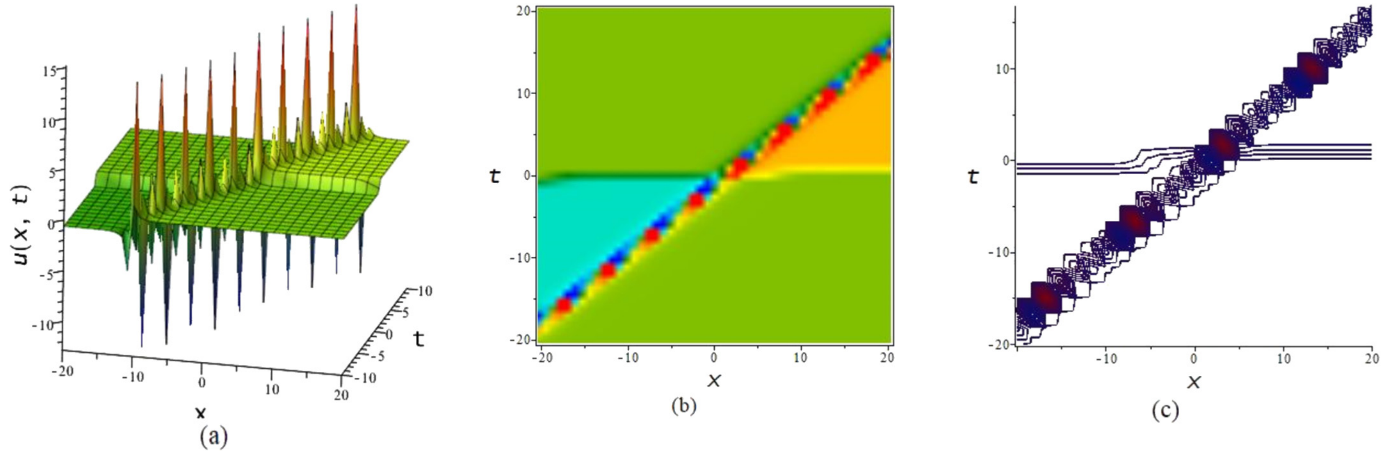

Fig. 6. The profile represents interaction of kink and soliton solution of the eq. (37) for $k_1 = k_2 = 0.5$, $\omega_1 = 0.6$, $\omega_2 = 10$, $m = 0.807$, $\lambda = -0707$. Here (a) 3D plot, (b) density plot and (c) contour plot.

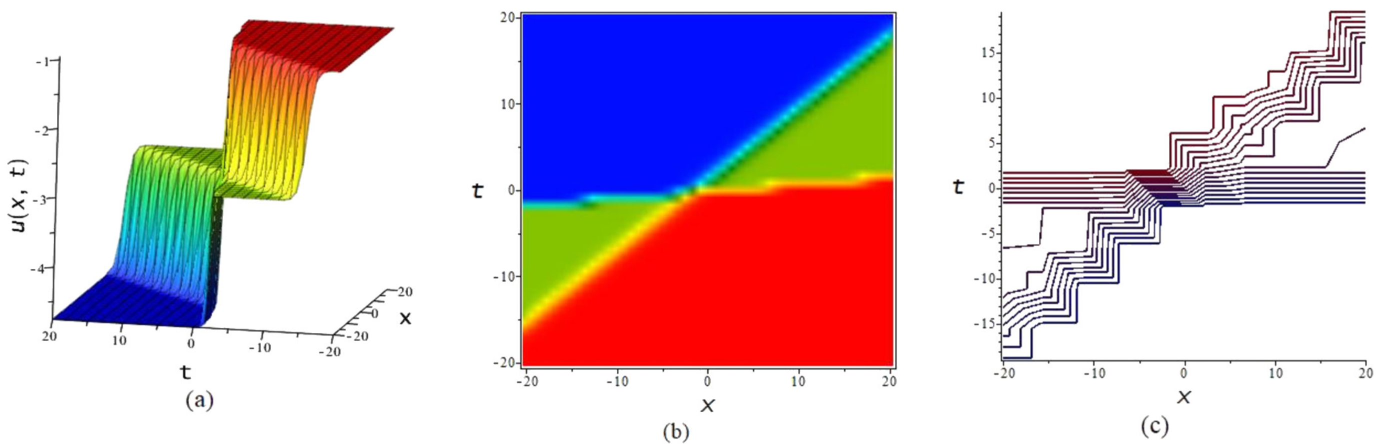

Fig. 7. The solution of eq. (49) for the parameters $k_1 = k_2 = 0.5$, $\omega_1 = 0.6$, $\omega_2 = 6$, $m = 0.8$, $\lambda = -0.707$. Here (a) 3D plot, (b) density plot and (c) contour plot.

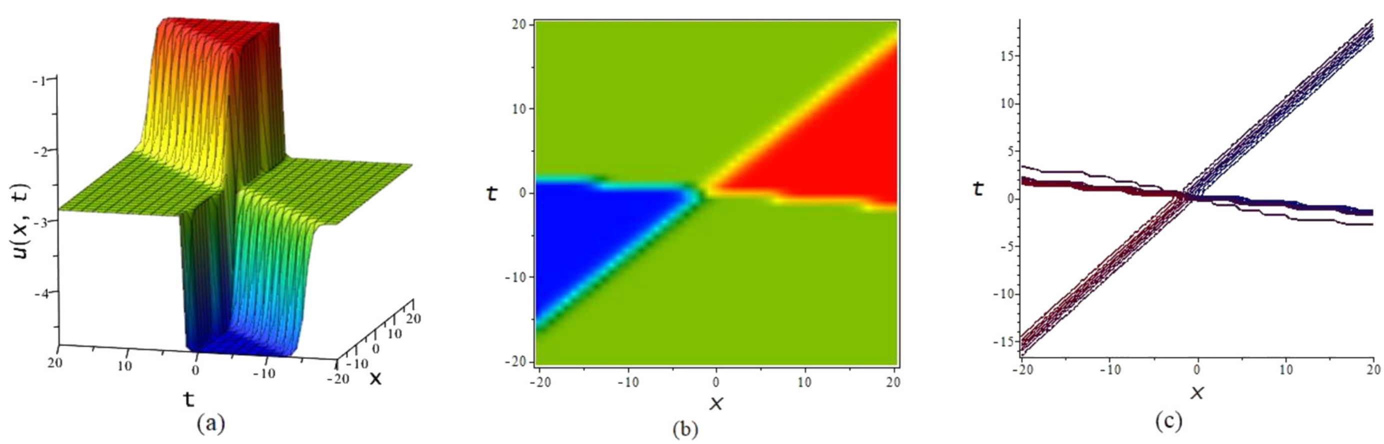

Fig. 8. The solution of eq. (49) for the parameters $k_1 = k_2 = 0.5$, $\omega_1 = 0.6$, $\omega_2 = -6$, $m = 0.8$, $\lambda = -0.707$. Here (a) 3D plot, (b) density plot and (c) contour plot.

a combination of hyperbolic and trigonometric function solution in eq. (22) and eq. (23) respectively. From the solution of eq. (22) and eq. (23) the interaction of double periodic and lump solution found in Fig. 3(a-f).

Now we customary $a_0 = a_1 = -a_2, b_0 = b_1 = -b_2$ under some condition $k_1^2 < \omega_1^2$ and $k_2^2 < \omega_2^2$ then the eq. (19) converts into trigonometry function solution in eq. (24). From the solution of eq. (22) periodic solution obtained in Fig. 4(a-e).

If inserting $a_5 = b_5 = 1$, $a_3 = \frac{2m^2}{\left(\omega_1^2 - k_1^2\right)}$, $b_3 = \frac{2m^2}{\left(\omega_2^2 - k_2^2\right)}$ or $a_5 = b_5 = -1$, $a_4 = \frac{-2m^2}{\left(\omega_1^2 - k_1^2\right)}$, $b_4 = \frac{-2m^2}{\left(\omega_2^2 - k_2^2\right)}$ in eq. (35) then the hyperbolic function solutions are instigated in eq. (36) and eq. (37) respectively. From the solution of eq. (36) multi soliton solutions are reconfiguring in Fig. 5(a-l). From the solution of eq. (37) the interaction soliton solutions are reconfiguring in Fig. 6(a-c).

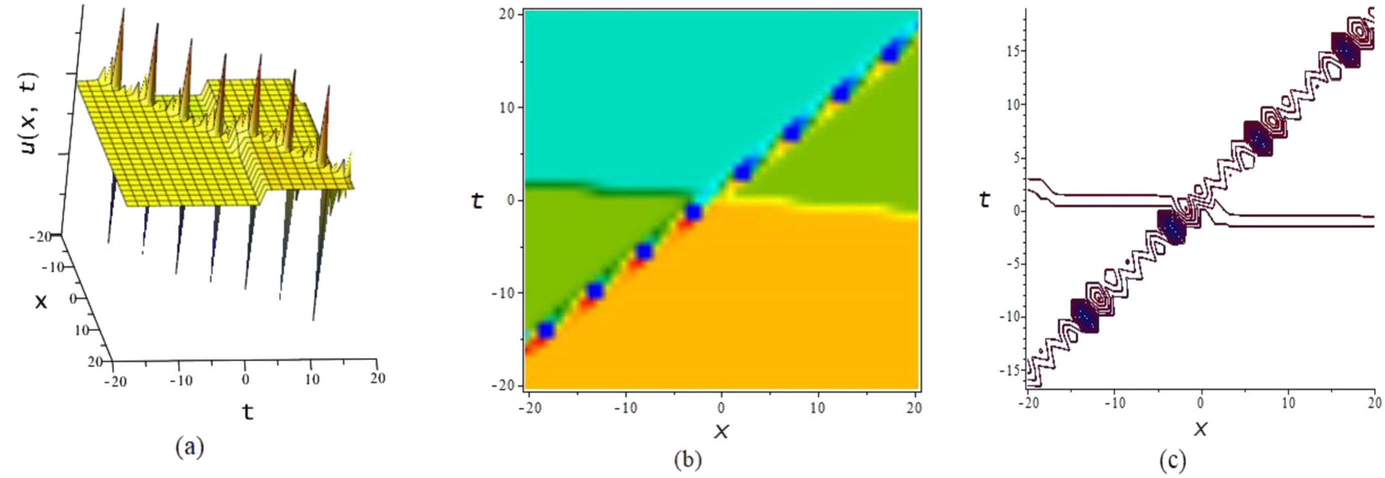

Fig. 9. The solution of eq. (50) for the parameters $k_1 = k_2 = 0.5$, $\omega_1 = 0.6$, $\omega_2 = 6$, $m = 0.8$, $\lambda = -0.707$. Here (a) 3D plot, (b) density plot and (c) contour plot.

If we set $a_5 = b_5 = \pm 1$, $a_4 = \frac{\pm 2m^2}{\left(\omega_1^2 - k_1^2\right)}$, $b_4 = \frac{\pm 2m^2}{\left(\omega_2^2 - k_2^2\right)}$ in eq. (48) then the hyperbolic function solutions are instigated in eq. (49) and eq. (50) respectively. From the solution of eq. (49) multi soliton solutions are reconfiguring in Fig. 7(a-c) and 8(a-c). From the solution of eq. (37) the interaction soliton solutions are reconfiguring in Fig. 9(a-c).

5. Conclusions

A new modified simple equation method was introduced and implemented to two nonlinear model Phi-4 and Klein-Gordon equations in this work. As a result, we erected multi-soliton solution, complexion solution, interaction of soliton and solitary solution of the phi-4 models. We have presented interaction of two dark bells, interaction of two bright bells, two kinks, double periodic waves, interaction kink and soliton, kink-rogue wave solutions of the nonlinear models. Our treatment shows that the new form of modified simple equation method provided weighty features for the ascertainment of multiple-soliton solutions and different type interactions solutions which will be effective for a wide classis of complex nonlinear models. This study had also shown the revel power of new modified simple equation method on nonlinear models. It remains a vital point for us to establish the obtained result in nuclear and particle physics over decades. In further, we expose the stimulating observation about the multiple-soliton solution and interaction solution in different fields of mathematical engineering.

Declarations

Author contribution statement

Md Mamunur Roshid: Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper. A. Aldurayhim; M. M. Rahman; H.O. Roshid: Performed the experiments. Fahad Sameer Alshammari: Conceived and designed the experiments.

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Data availability statement

Data included in article/supp. material/referenced in article.

Declaration of interests statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

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