Research article

Pattern selection in coupled neurons under high-low frequency electric field

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A B S T R A C T

Biological neurons exposed to an external electric field can induce polarization and charge fluctuation. Indeed, the exchange of calcium, sodium and potassium ions across the cell membrane can induce an electric field as a result of a time-varying electromagnetic field set up. This field could further modulate the cell electrical activity by inducing multiple firing modes. Based on the physical law of electric field, an improved model which includes an additive electrical field variable is constructed for a network of neurons to study wave propagation and mode transition by exploring the longtime dynamics of slightly perturbed plane waves in the network. Wave pattern and mode transition dependence on the different parameters of external electric field are discussed. It is found that the plane wave propagating in the coupled system breaks down to localized structures under the activation of modulational instability. The network under high-low external electric field supports bursting synchronization. This could be a fruitful avenue to discern the occurrence of paroxysmal epilepsy.

1. Introduction

The brain is a major part of the central nervous system, dedicated to information encoding, processing, storage and exchange. It's largely regarded to be constituted by a network of neurons, which collectively participate in maintaining the normal physiological activity of the nervous system [1, 2, 3, 4]. Information in the form of bioelectric signal is communicated from one neuron to another via electric and chemical synapses. Hodgkin and Huxley (HH) [5] derived the first mathematical model to model the electrical activity of a neuron, using data from electrophysiological experiments on the giant squid axon. From their contribution, the bioelectric signal flowing in neurons as information has the form of an impulse, resulting from the potential difference between the cell membrane. Many other models were developed from the HH after some simplifications notably the FitzHugh Nagumo (FHN) [6, 7], Morris Lecar (ML) [8] and Hindmarsh Rose (HR) [9] models. The FHN model is widely used to detect excitable dynamics of media and hence has contributed greatly in neurodynamics. As well, physicists and engineers have widely build nonlinear circuits in order that the processing of electric signal from biological cells could be reproduced. Ideas, concepts and approaches of nonlinear dynamics are applied to comprehend the physiology of the nervous system. Thus, appropriate parameters setting in dynamical equations and maps could be effective

in generating time series with rhythms same as obtained from biological experiments.

Neuronal networks with varieties of topological connections are constructed in order to study some collective behaviors such as wave propagation, pattern selection and synchronization. For example, Rulkov studied pattern formation in a network of cells to understand the role and effect of the coupling strength between adjacent cells [10]. Gosak et al. [11], reported the formation of continuous pulse from neuronal networks with autapses, which regulates the collective behavior of the network as pacemaker. Ma et al. [12], reported target wave breakup in excitable media exposed to high intensity electromagnetic radiation. Mvogo et al. [13], reported various dynamical motifs including breathing and swimming patterns in diffusive excitable media under magnetic flow. The investigation of these collective behaviors helps to discern the potential mechanism of information processing and related disease emergence within the nervous system.

During the dynamical analysis on most reliable models, many physical and biological factors are included. This permits results from these improved models to be consistent with biological experiments. For example magnetic flux variable is included in the standard neuronal models and memristor used to achieve the coupling between magnetic flux variable and membrane potential hence induction current from the physical effect of electromagnetic induction is considered [14, 15, 16].
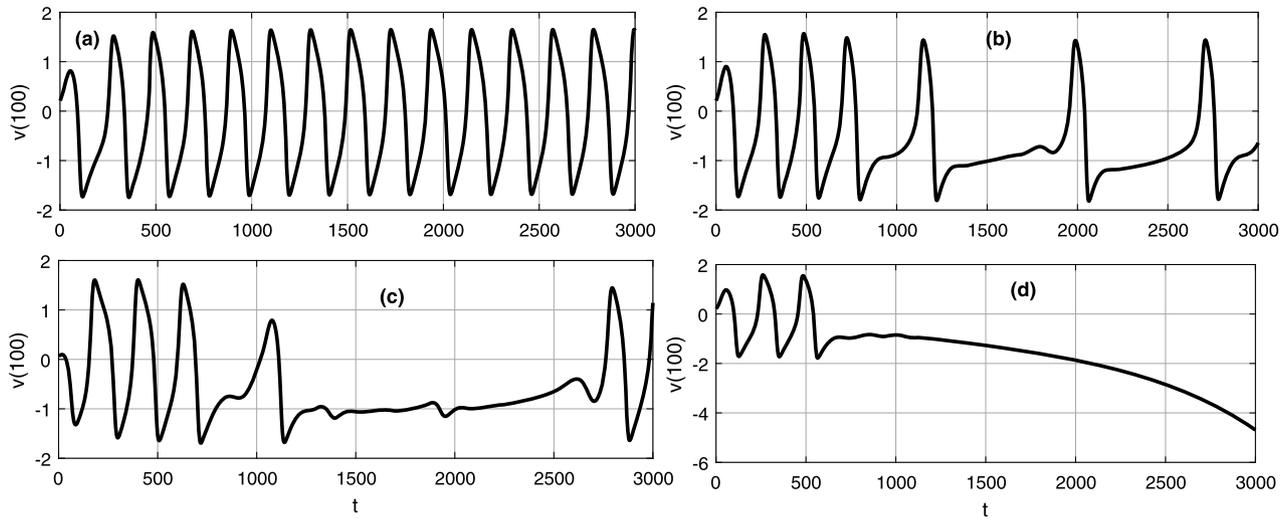
Fig. 1. Sampled time series for membrane action potential, calculated by setting different intensities of stimulation currents at fixed values of $\omega = 0.20$, for (a) $I_0 = 0.01$, (b) $I_0 = 0.4$, (c) $I_0 = 0.4$, (d) $I_0 = 1.0$. The effect of electric field is included by setting $k_0 = 0.1$.

It's further verified that electromagnetic radiation imposed on the magnetic flux variable as an external forcing induces multiple modes in electrical activities, consistent with biological experiments [17, 18, 19]. Fluctuation in charge distribution and polarization of the media occurs when exposed to electric field. Magnetization also occurs when the media is exposed to time-varying magnetic field due to induced electric field generated [20]. Therefore, during dynamical analysis on reliable model, the effect of electric field distribution becomes more distinct and should be included.

In this paper, a new neuronal network is presented, with a new field variable introduced to include the effect of electric field. Using this improved model, we show that the new field can enhance spatiotemporal information exchange under the activation of modulational instability. Modulational Instability technique is a phenomenon well discussed in diverse domain of nonlinear physics involving the formation of soliton in dissipative media [21, 22, 23]. External electric field in the form of high and low periodic frequency signal is imposed on the electric field variable and mode transition in electrical activity studied. We reveal the possibility of stimulating and enhancing signal exchange in the network in the absence of direct simulation current and electrical synapse.

2. Model, scheme and discussion

Hodgkin and Huxley (HH) related the shape and speed of the action potential to the observed changes in the membrane permeability. Indeed, the dynamics of nerve impulse (action potential) is modeled by a complex system of nonlinear partial differential equations with no tractable analytical investigation. Followed from this work, FitzHugh and Nagumo proposed some relevant changes to these systems of nonlinear equations which retained its simplicity but allow the reliable modeling of nerve impulse propagation. The simple two variables FHN model has become a prototype equation used in describing the electrical activities of excitable media. However, the physical effect of electric field is not considered because polarization and variation in charge distribution in the media occur when exposed to electric field. An improved model is designed to reproduce the electrical activities of neurons by introducing the additive electric field variable. The new three variables FHN dynamical equations for a system of nth coupled neurons read

$$\begin{cases} \epsilon \dot{v}_n = & v_n - \frac{v_n^3}{3} - w_n + I_{ext} + D(v_{n+1} - 2v_n + v_{n-1}), \\ \dot{w}_n = & av_n + bw_n + d + k_0 E_n, \\ \dot{E}_n = & k_1 w_n + k_2 E_n + E_{ext}, \end{cases} \quad (1)$$

with $n = 1, 2, ..., N$, where N represents the position of node in the network. v, w and E are the variables for membrane action potential, transmembrane current and electric field. a, b and d are constant model parameters adjusted to fit real biological situations. k_0, k_1 and k_2 are feedback gain; k_0 bridges the coupling and modulation on transmembrane current from electric field, k_1 describes the degree of polarization of the media and k_2 is thought as a physical parameter adjusting saturation of E. ϵ is the parameter for time scale for membrane potential and transmembrane current. I_{ext} is the transmembrane mapped from external forcing and highly determinant to different dynamical regimes including quiescent, bursting, spiking and chaotic series. Based on the improved model, electric field will be applied to check activation of quiescent state as well as mode transition between various dynamical states under subthreshold stimulus. We will also investigate the possibilities such activation and mode transition in the absence of a direct current stimulus. D is the wiring or coupling strength of the gap junction in the nearest neighbor interaction regime. E_{ext} is the external electric field.

3. Numerical results and discussion

To study the long time evolution of the membrane action potential in the network given by Eq. (1), we find the numerical solution. We carried out the numerical simulation of the Eq. (1) via fourth-order Runge Kulta Computational scheme with time step $h = 0.01$. The initial conditions are carefully selected to correspond with slightly modulated plane waves, having the wave number 0.12π and perturbed wave number 0.5π. Constant parameters in Eq. (1) are chosen as $\epsilon = 0.1, D = 0.05, a = 3.0, b = -3.0, d = 0.5, k_1 = 15.0$, and $k_2 = 0.0$. Firstly, in other to detect the mode response of electrical activities, different transmembrane intensities current I_0 is selected at a fix value of the angular frequency $\omega = 0.20$. The results are plotted in Fig. 1.

The results of the plot in Fig. 1 confirm that the discharge mode in electrical activities of the neurons are dependent on the settings of the transmembrane current in the presence of the electric field. Extensive numerical simulation representing the sampled time series for membrane potential show how the electrical activities are changed from periodic type to spiking and then bursting. Indeed, electrical activities are clearly controlled by the intensity of external stimuli. As well, different values for angular frequency of transmembrane current, for a fix intensity $I_0 = 1.0$ are selected. The results are plotted in Fig. 2.

It is found in Fig. 2 that the electrical activities of the neurons can present multiple types of mode, when the angular frequency is increased. Chaotic-like mode is also detected in the sampled time series for membrane potential as the angular frequency is carefully increased.
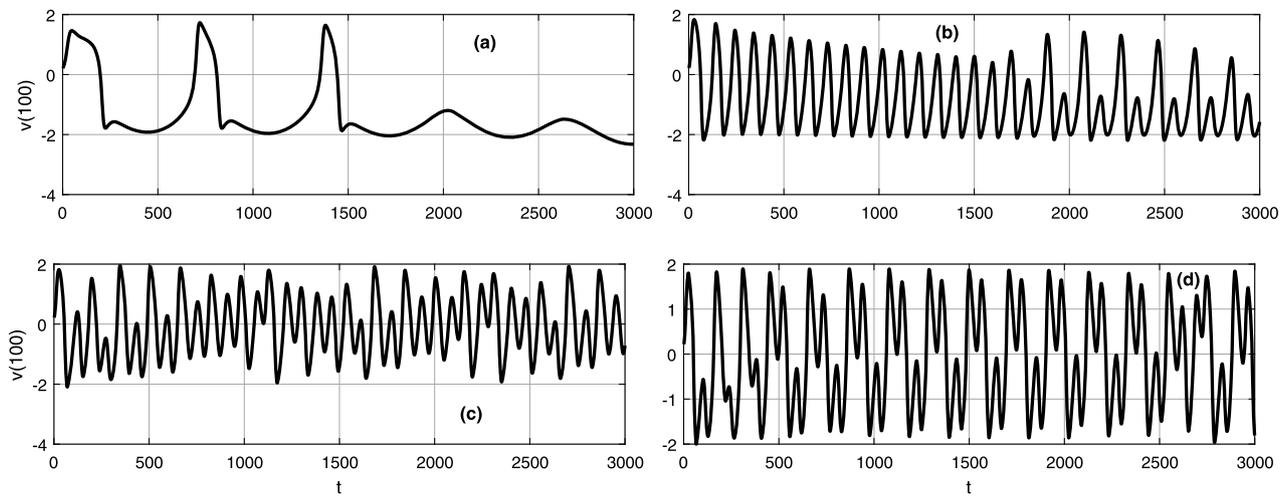
Fig. 2. Sampled time series for membrane action potential, calculated by setting different angular frequencies of stimulation currents at fixed values of $I_0 = 1.0$, for (a) $\omega = 2.0$, (b) $\omega = 13.0$, (c) $\omega = 16.0$, (d) $\omega = 18.0$. The effect of electric field is included by setting $k_0 = 0.1, k_1 = 15$.
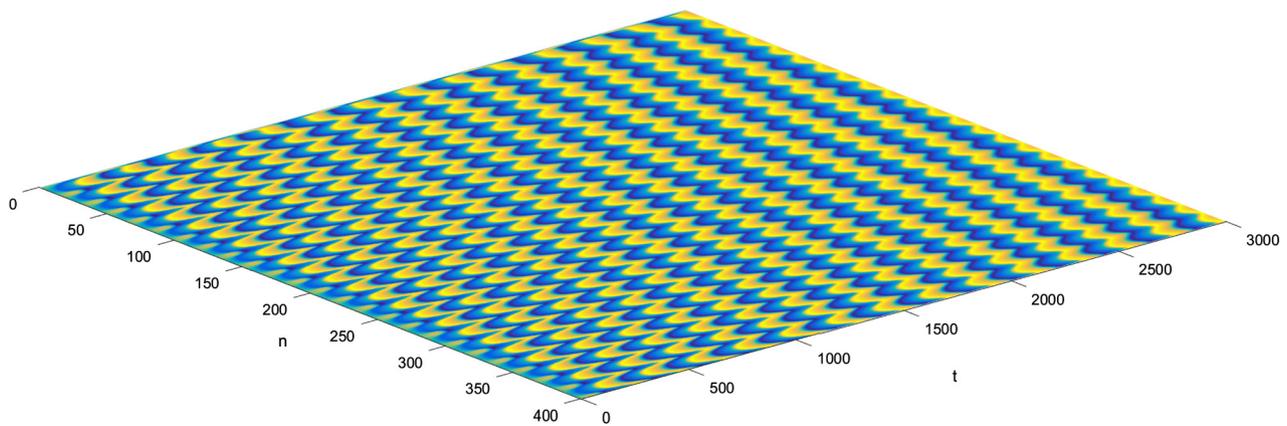

Fig. 3. Developed spatiotemporal pattern calculated for $I_{ext} = 0.01\sin(0.2t)$. The effect of electric field is included by setting $k_0 = 0.1, k_1 = 15$.

The sampled time series of membrane potential presented above is limited to two variables in the time dimension. Useful information regarding the status of the network can be discerned by probing the spatiotemporal patterns. Indeed, the proper analysis of the spatiotemporal patterns is vital for discerning a wide range of naturally occurring and pathological phenomena. The form of the impulse propagating in the network is very relevant because it could stand as a signature to certain pathologies [24, 25]. By selecting the appropriate electric field back $k_0 = 0.1$, the subsequent spatiotemporal of membrane potential is plotted in Fig. 3.

The spatiotemporal pattern presented in Fig. 3 shows periodic patterns of wave, which are localized in space and time. The existence of localized wave pattern confirms the theory of modulational instability (MI) in the network lattice. Indeed, it's reiterates MI as a mechanism underlying the formation of wave pattern in discrete systems. It's known that depending on the value of the electric feedback $k_0 = 0.1$ selected, one should expect different behaviors of the system. By setting different electric field feedbacks, the spatiotemporal patterns obtained are plotted in Fig. 4.

It is found in Fig. 4 that the pattern of wave is modified as the electric field gain is increased to $k_0 = 0.47$. This confirms that during the continuous pump and exchange of charged ions in the nerve cell, the intrinsic electric field set up modifies the physical properties (nonlinearity) of the media. This modification in the nonlinearity of the media promotes the mechanism of modulational instability (MI). This in accordance with the theory of MI, where due to the concomitant effects of dispersion, nonlinearity and dissipation within the media, a small

perturbation (intrinsic electric field) on the nerve impulse plane wave result in instability. The impulse plane wave could as a result subsequently breakdown, during propagation into wave trains with localized patterns.

It is also important to discuss the dynamical response due to $k_0 = 0.47$, when an external electric field E_{ext} is applied. The calculated spatiotemporal patterns obtained are plotted in Fig. 5

It is found in Fig. 5 that the patterns are modified as k_0 is progressively increased in the presence of an external high-low frequency electric field. The corresponding sampled time series is plotted in Fig. 6.

It is confirmed in Fig. 6 that the discharge mode and electrical activities present various firing modes when the value of the intrinsic electric field is increased. Indeed, an increased in k_0 promotes intermittent bursting. This results from the polarization of the excitable media under subthreshold stimulus.

Henceforth, to discern the effect of this external electric field, we set the transmembrane current to zero. The corresponding time series and spatiotemporal patterns are plotted in Figs. 7 and 8.

It is confirmed in Figs. 7 and 8 that the localized wave pattern is modified as we increase the value of k_0. The sampled time series indicates various discharge modes in electrical activity. The existence of various firing modes in the absent of transmembrane suggests the possibility of stimulating and enhancing nerve impulse transmission through the application of an external electric field. To predict the possibility of achieving synchronization in the network under high-low electric field, we plot the sampled time series at various nodes of the lattice. The result is presented in Fig. 9.
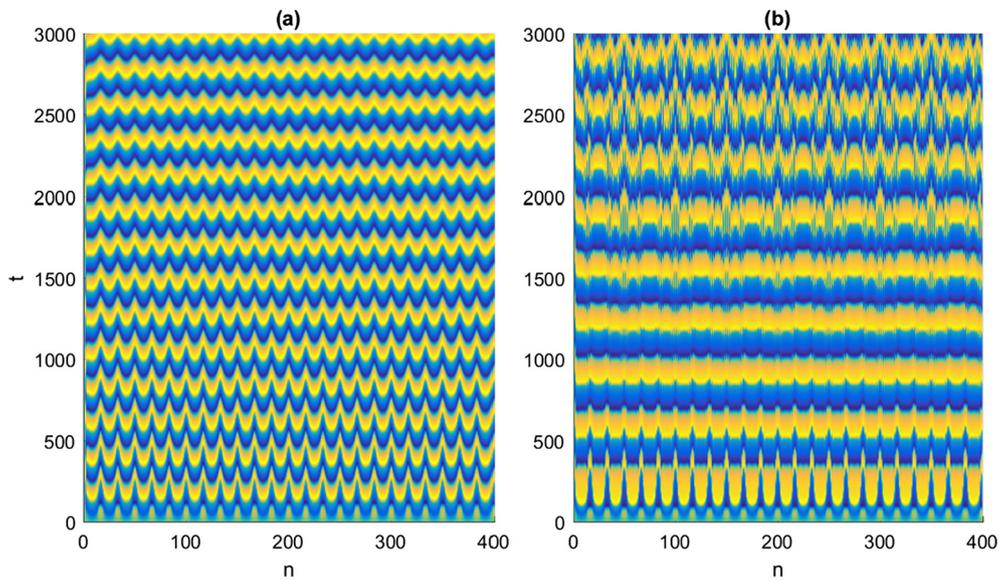
Fig. 4. Developed spatiotemporal patterns calculated under different electric feedback parameter for (a) $k_0 = 0.30$, (b) $k_0 = 0.47$, with $I_{ext} = 0.01\sin(0.2t)$.
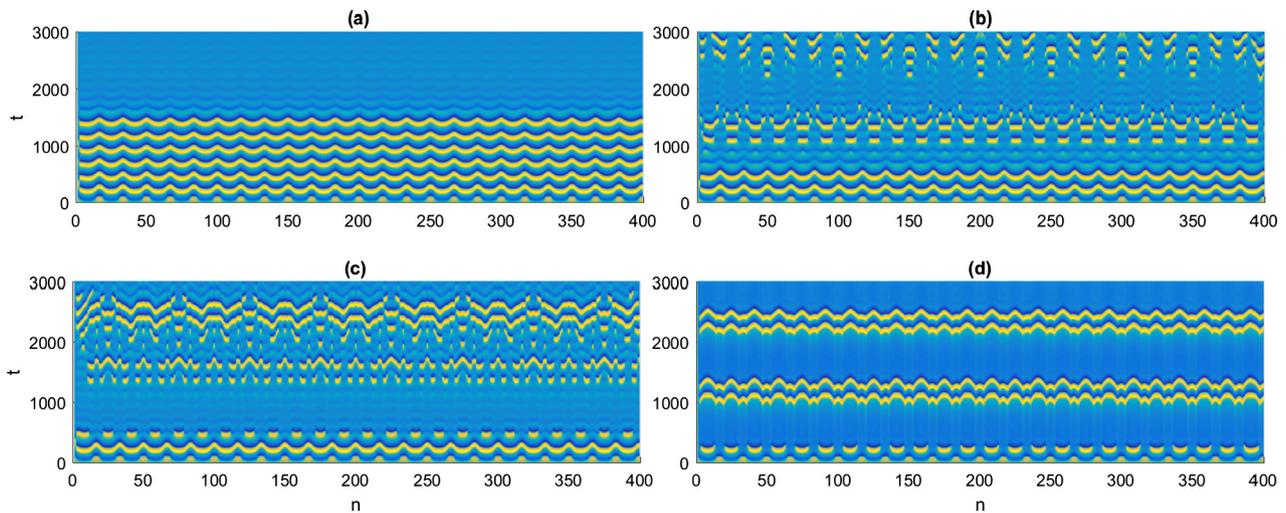

Fig. 5. Developed spatiotemporal patterns calculated under different electric feedback parameter for (a) $k_0 = 0.0030$, (b) $k_0 = 0.005$, (c) $k_0 = 0.008$, (d) $k_0 = 0.04$, with $I_{ext} = 0.01\sin(0.2t)$. The effect of electric field is included by setting $E_{ext} = 6.50\sin(0.05t) + 6.20\cos(0.2t)$.
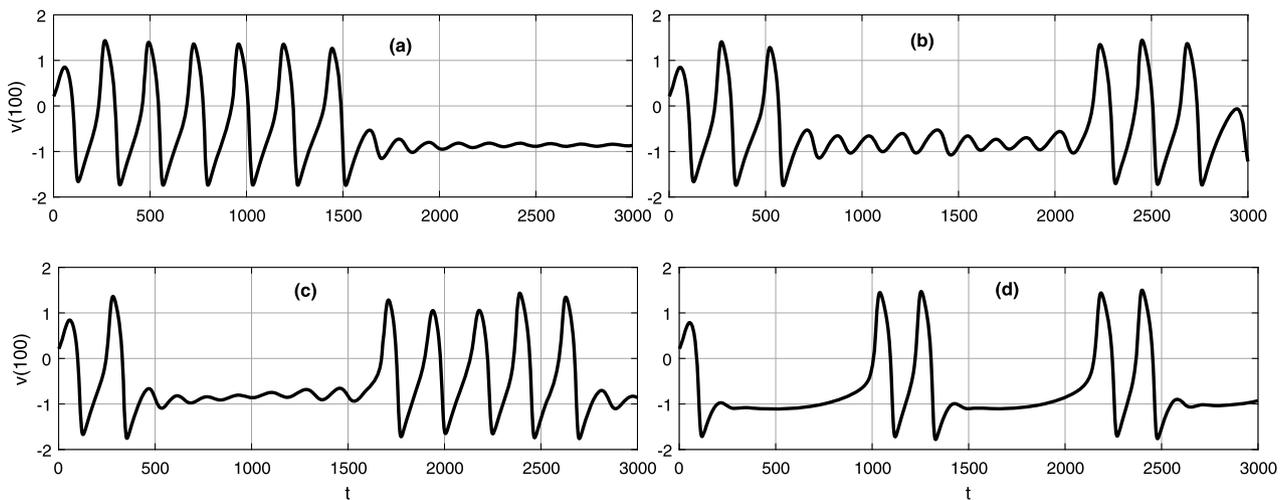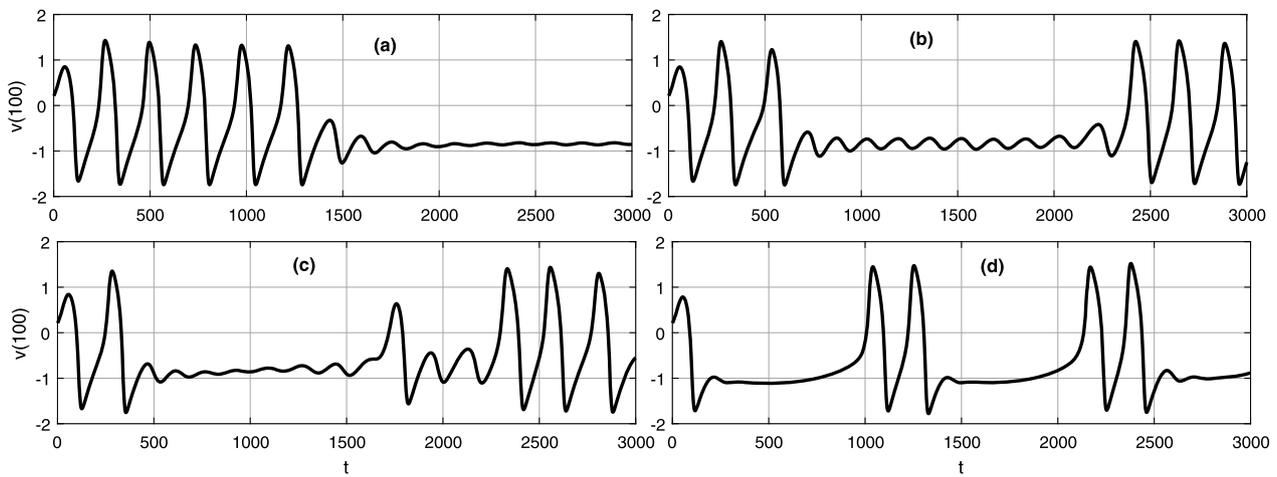

Fig. 6. Sampled time series for membrane action potential, calculated by setting different electric feedback parameter for (a) $k_0 = 0.0030$, (b) $k_0 = 0.005$, (c) $k_0 = 0.008$, (d) $k_0 = 0.04$, with $I_{ext} = 0.01\sin(0.2t)$. The effect of electric field is included by setting $E_{ext} = 6.50\sin(0.05t) + 6.20\cos(0.2t)$.
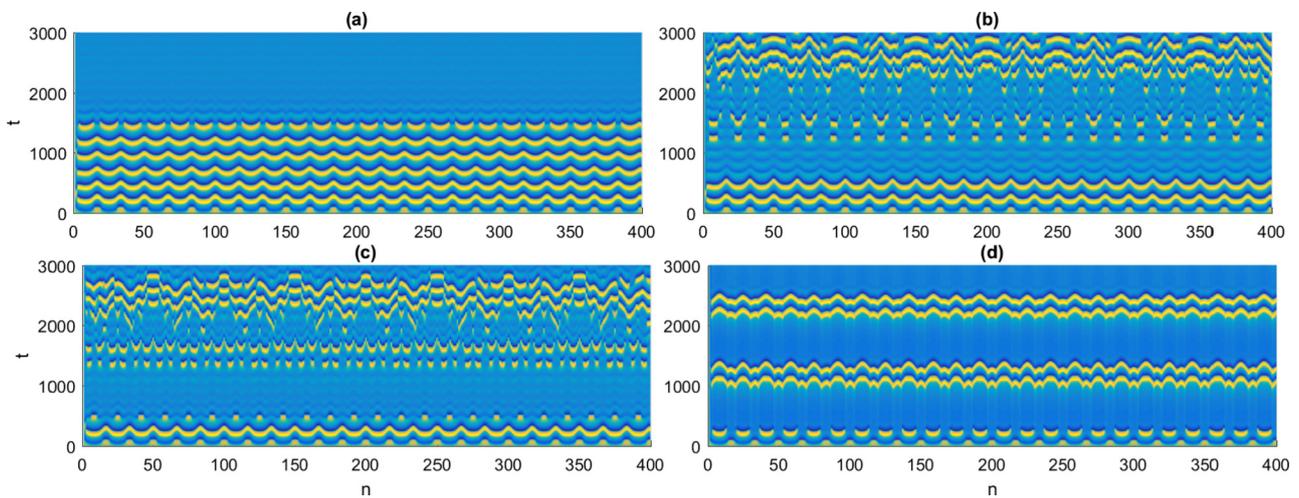
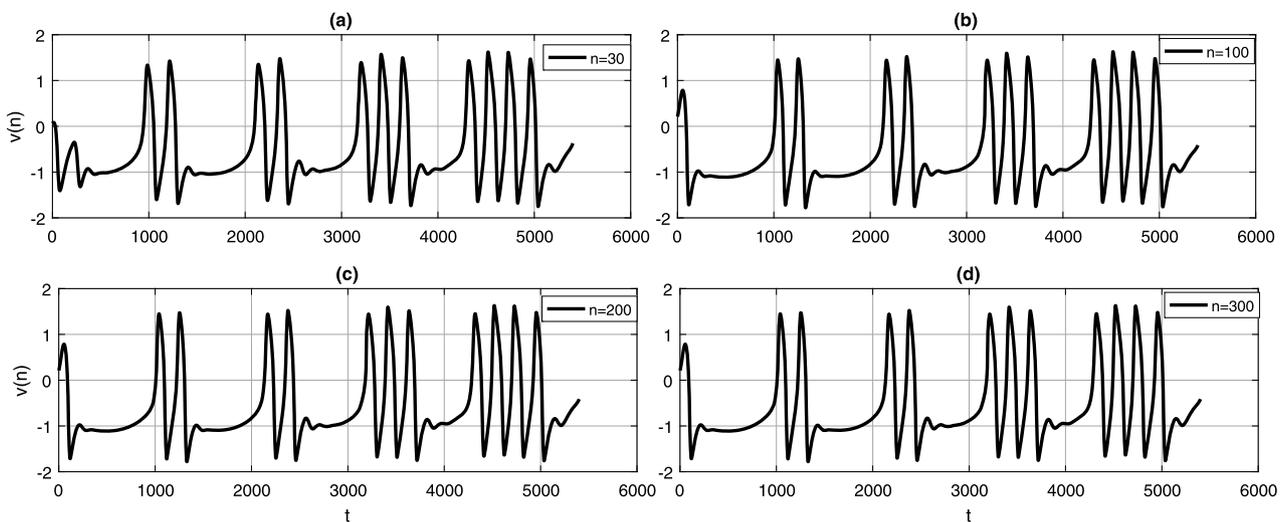**Fig. 7.** Sampled time series for membrane action potential, calculated by setting different electric feedback parameter for (a) $k_0 = 0.0030$, (b) $k_0 = 0.005$, (c) $k_0 = 0.008$, (d) $k_0 = 0.04$, with $I_{ext} = 0.0$. The effect of the external electric field is included by setting $E_{ext} = 6.50\sin(0.05t) + 6.20\cos(0.2t)$.

Fig. 8. Developed spatiotemporal patterns calculated under different electric feedback parameter for (a) $k_0 = 0.0030$, (b) $k_0 = 0.005$, (c) $k_0 = 0.008$, (d) $k_0 = 0.04$, with $I_{ext} = 0.0$. The effect of the external electric field is included by setting $E_{ext} = 6.50\sin(0.05t) + 6.20\cos(0.2t)$.

Fig. 9. Sampled time series for membrane action potential, calculated at different nodes for fixed values of $k_0 = 0.04$, $I_{ext} = 0.0$. The effect of the external electric field is included by setting $E_{ext} = 6.50\sin(0.05t) + 6.20\cos(0.2t)$.

Fig. 9 reveals bursting firing mode at the node 10, 100, 200 and 300. In other words, the oscillations at different positions along the cell lattice exhibit the same dynamics. This confirms the possibility of obtaining synchronization in the coupled neuron under an external field.

Indeed, synchronous firing activity of cortical interconnected neurons is reported to be highly relevant for higher brain functions including attention and sleep-wake state switching [26]. This result suggests the potential ability of high-low electric field to induce paroxysmal epilepsy, usually associated with bursting synchronization [27].

4. Conclusion

Based on the law of static electric field, an additive electric field variable is used to model the effect of electric field in neuronal network considering the complex exchange of calcium, sodium and potassium ions across the cell membrane. As such, an improved neuronal network is built to discuss pattern selection in electrical activities in the presence of high-low frequency electric field. It is found that the activation of modulational instability, an envelope impulse plane wave within the cell lattice exposed to electric field breaks down into wave train with localized pattern. Also, extensive dynamical analysis via numerical simulations revealed that in the absent of the transmembrane current, the high-low electric field can stimulate and assist signal propagation in the network. Finally, we predict the realization of bursting synchronization in the neuronal network, which is usually characteristics of paroxysmal epilepsy.

Declarations

Author contribution statement

Clovis Ntahkie Takembo: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Wrote the paper.

Michael Ekonde Sone: Contributed reagents, materials, analysis tools or data; Wrote the paper.

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Data availability statement

Data will be made available on request.

Declaration of interests statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

References

[1] Y. Qian, F. Liu, et al., Spatiotemporal dynamics in excitable homogeneous random networks composed of periodically self-sustained oscillation, Sci. Rep. 7 (2017) 11885.

[2] F.M. Moukam, E.M. Inack, E.M. Yamakou, Localized nonlinear excitations in diffusive Hindmarsh-Rose neural networks, Phys. Rev. E 89 (2014) 052919.

[3] A.S. Eteme, C.B. Tabi, A. Mohamadou, Long-range patterns in Hindmarsh-Rose networks, Commun. Nonlinear Sci. Numer. Simul. 43 (2017) 211.

[4] C.J. Chen, H. Bao, M. Chen, et al., Non-ideal memristor synapse-coupled bi-neuron Hopfied neural networks: numerical simulations and breadboard experiments, AEÜ, Int. J. Electron. Commun. 111 (2019) 152894.

[5] A.L. Hodgkin, A.F. Huxley, A quantitative description of membrane current and its application to conduction and excitation in nerve, J. Physiol. 117 (1952) 500–544.

[6] R. FitzHugh, Impulses and physiological states in theoretical models of nerve membrane, Biophys. J. 1 (6) (1961) 445–466.

[7] J. Nagumo, S. Arimoto, S. Yoshizawa, et al., An active pulse transmission line simulating nerve axon, Proc. IRE 50 (1962) 2061–2070.

[8] C. Morris, H. Lecar, Voltage oscillations in the barnacle giant muscle fiber, Biophys. J. 35 (1981) 193–213.

[9] J.L. Hindmarsh, R.M. Rose, A model of the nerve impulse using two first-order differential equations, Nature (London) 296 (1982) 162–164.

[10] N.F. Rulkov, Regularization of synchronized chaotic bursts, Phys. Rev. Lett. 86 (2001) 183.

[11] M. Gosak, M. Marhl, M. Perc, Pacemaker-guided noise-induced spatial periodicity in excitable media, Physica D 238 (2009) 506–515.

[12] J. Ma, F. Wu, et al., Electromagnetic induction and radiation-induced abnormality of wave propagation in excitable media, Physica A 486 (2017) 508–516.

[13] A. Mvogo, C.N. Takembo, H.P. Ekobena Fouda, T.C. Kofane, Pattern formation in diffusive excitable systems under magnetic flow effects, Phys. Lett. A 381 (2017) 2264–2271.

[14] M. Lv, C.N. Wang, G.D. Ren, et al., Model of electrical activity in a neuron under magnetic flow effect, Nonlinear Dyn. 85 (2016) 1479–1490.

[15] C.N. Takembo, A. Mvogo, H.P. Ekobena Fouda, T.C. Kofane, Modulated wave formation in myocardial cells under electromagnetic radiation, Int. J. Mod. Phys. B 32 (2018) 1850165.

[16] C.N. Takembo, A. Mvogo, H.P. Ekobena, T.C. Kofane, Localized modulated wave solution of diffusive FitzHugh-Nagumo cardiac networks under magnetic flow effect, Nonlinear Dyn. 95 (2) (2019) 1079–1098.

[17] J. Ma, Y. Wang, C. Wang, et al., Mode selection in electrical activities of myocardial cell exposed to electromagnetic radiation, Chaos Solitons Fractals 89 (2017) 219–225.

[18] C.N. Takembo, A. Mvogo, H.P. Ekobena, T.C. Kofane, Effect of electromagnetic radiation on the dynamics of spatiotemporal patterns in memristor-based neuronal network, Nonlinear Dyn. 95 (2) (2019) 1067–1078.

[19] C.N. Takembo, A. Mvogo, H.P. Ekobena, T.C. Kofane, Wave pattern stability of neurons coupled by memristive electromagnetic induction, Nonlinear Dyn. 96 (2019) 1083–1093.

[20] J. Ma, Y. Zang, et al., Model of electrical activity of neuron under electric field, Nonlinear Dyn. 95 (1) (2018) 1–14.

[21] T.B. Benjamin, J.E. Feir, The disintegration of wave trains on deep water Part I; Theory, J. Fluid Mech. 27 (1967) 417.

[22] Y.S. Kivshar, M. Peyrard, Modulational instability in discrete lattices, Phys. Rev. A 46 (1992) 3198.

[23] A.S. Tankou, C.N. Takembo, G.H. Ben-Bolie, P. Owona Ateba, Localized nonlinear excitations in diffusive memristor-based neuronal network, PLoS ONE 14 (6) (2019) e0214989.

[24] S. Brunak, B. Lautrup, Neural Networks, World Scientific Publishing, Singapore, 1990.

[25] C.N. Takembo, H.P. Ekobena Fouda, Effect of temperature fluctuation on the localized pattern of action potential in cardiac tissue, Sci. Rep. 10 (2020) 15087.

[26] E. Ioka, K. Tsumoto, H. Kitajima, Synchronous firing frequency dependence in unidirectional coupled neuronal networks with chemical synapse, Neurocomputing 350 (2019) 202–211.

[27] J. Ma, F.Q. Wu, C.N. Wang, Synchronization behaviors of coupled neurons under electromagnetic radiation, Int. J. Mod. Phys. B 30 (2016) 1650251.