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Modeling and optimal control applied to reduce the effects of greenhouse gases emitted from the coal-based power plant in Bangladesh

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ABSTRACT

The coal-fired power station is believed to be one of the major emitters of air pollutants, particularly carbon dioxide (CO2), which is the main sensitive driver of climate change due to global warming, consequently causing significant intimidation for the Sundarbans, the world's largest mangrove forest and nearby due to high emissions of air pollutants such as Carbon-Di-Oxide (CO2). Here, we used a compartmental mathematical model with 3 compartments to study the dynamics of greenhouse gas emissions, concentration, and uptake, which we can control by installing a chemical reactor system near the power plant and naturally afforesting the regions. The model was built from scratch to study these types of problems. First, we formulated the optimal control problem by connecting two control measurement systems: a chemical reactor system and natural afforestation. For this purpose, Pontryagin's maximum principle is used. The novelty of this work is the investigation of optimal strategies to minimize the impact of gases emitted by Coal based power plants on neighboring regions. More realistic facts such as system damage from excess emissions, most absorbers, and other facts are covered here. The numerical solution obtained illustrates the outcome of the system with initial values and theoretical parameters that best represent reality. By evaluating the performance index scores, and objective function values, we found that both controls (the chemical reactor system and natural afforestation) help minimize air pollution. We then simulated our model with 5 different control strategies to observe its performance in reducing pollutants. Once we determine that two control strategies are equally effective in reducing pollution, let's compare them by looking at the costs associated with each strategy. Therefore, using both control systems (chemical reactor and natural afforestation) with a higher reaction rate, we suggested chemical reactor system control as the best strategy.

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[1] A crucial contribution of the authors is a formulation of the Mathematical Model to study the dynamics of greenhouse gas emissions. The Mathematical model (1) is a major contribution of the authors and is a very unique and novel approach to reducing GHGs emission-related issues in coastal areas. This model (**The Paul-Biswas Model**) can be used to solve similar problems by extending, specializing or generalizing to solve problems in this domain effectively.

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1. Introduction

Due to the opportuneness of uninterrupted power generation, thermal powers are an obvious necessity. However, the number of harmful gases and radiation of these power plants poses a great risk to life and the climate. Various technologies are used to reduce pollution in the power industry. However, underdeveloped countries cannot develop and maintain all these technologies. As a result, pollution from coal-based power plants impact the environmental change negatively. The process of a coal-based power plant is to generate electricity from heat in a boiler that first produces steam. Under high pressure, the steam produced is put into a turbine, which produces electricity using a revolving generator. The method is to cool the steam, heat it with water and return it to the boiler to start the generation process. The main cause of air pollution, global warming, and other air pollution is burning. A Coal based power plant emits approximately one year: (1) 3,700,000 tons of Carbon-di-Oxide (CO2); (2) 10,000 tons of sulfur dioxide (SO2); (3) 500 tons of small airborne material; (4) 10,200 tons of nitrogen oxide (NO); (5) 720 tons of carbon monoxide (CO); (6) 220 tons of hydrocarbons, volatile organic compounds (VOC); (7) 170 pounds of mercury; (8) Arsenic 225 pounds; and (9) 114 pounds of lead, 4 pounds of cadmium, other toxic metals, and traces of uranium [1], which are harmful to global health and the environment [2]). Fossil fuel burning is considered a major contributor to Carbon-di-Oxide (CO2) emissions as well as to global warming. In addition, the burning of coal produces exhaust gases, acid rain, and various gases that damage the environment. Other information about this project and power plant can be found in our previous study where we described the circulation of pollutants emitted from the plant [2]. Due to the negative impact on all living things and the environment, the design of the Rampal port is considered one of the most controversial decisions. But the important discussion about the environmental protection of the Sundarbans has not ended. With that in mind, let's take a closer look at the natural ability of the Sundarbans to resist the environmental changes caused by coal-based power plants. To this end, we have attempted to calculate the air pollutants including carbon dioxide (CO2) emissions of these fires and calculated the absorption levels of the Sundarbans themselves. We also consider technologies that can protect the Sundarbans and nearby areas from degradation [3]. Therefore, Figs. 1 and 2 show how this power station can pose a serious threat to the resources of the Sundarbans and the neighboring coastal areas of Bangladesh. Some of the statistical results of CO2 emissions and combustion are shown in Figs. 1 and 2, and Table 1 below with the parameter speciations in Table 2.

Table 1 shows that industries (electricity producers) produce large amount of greenhouse gases worldwide. Since 2014, the number of coal-fired power stations in Bangladesh has increased significantly. Therefore, it is very important and poses a major threat to all living beings. Fig. 3 shows the distances of the various areas neighboring the plants likely to be harmed by the air pollutants emission from the plant.

There are many mathematical models to investigate and study the emission and absorption of pollutants. Shukla et al. [4] proposed and analyzed a five-dimensional non-linear system of ODEs to study the pollutants control mechanisms and two different particles from city's atmosphere by rain. In Ref. [3]. Mazumder and others examined the Rampal plant model and concluded that the Sundarbans have an estimated carbon sequestration capacity of 4.2 million kg per day, which is very low for the design, and recommended that the process be CCS (Carbon Capture and Storage) is environmentally friendly. Readers are referred to Refs. [5,6,7,8,9–12,13,14,15,16], [17–19], and these references provide further details on carbon dioxide (CO2) emissions and some recent advances in mathematical modeling and control strategies. But here we have examined an entirely new and different problem of Emission, diffusion, and uptake of air pollutants depending on different conditions. In this case, we also proposed new control strategies to reduce air pollution spread and validated the result with numerical simulation. Diffusion models are commonly used in processes where air pollutants are assumed to be generated and spread from a source. Again, the direction of the airflow plays a big role as we assumed the airflow area is much larger. It is therefore possible to express this process using the compartment model.

Since we have considered two problems together, diffusion and airflow, we have proposed a new kind of model for the mathematical study of this problem. In addition to this complexity, we found the whole process quite complex to understand, so we had to modify the mathematical model accordingly. After expressing the entire system in mathematical equations, we solved the model

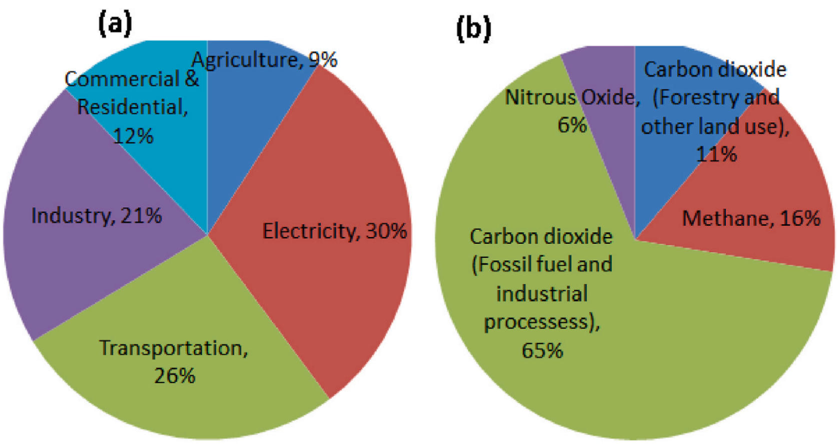

Fig. 1. The volume (in percentage) of air pollutants discharging from various fields **(a)** and the usages of various GHG's (in percent) **(b).**

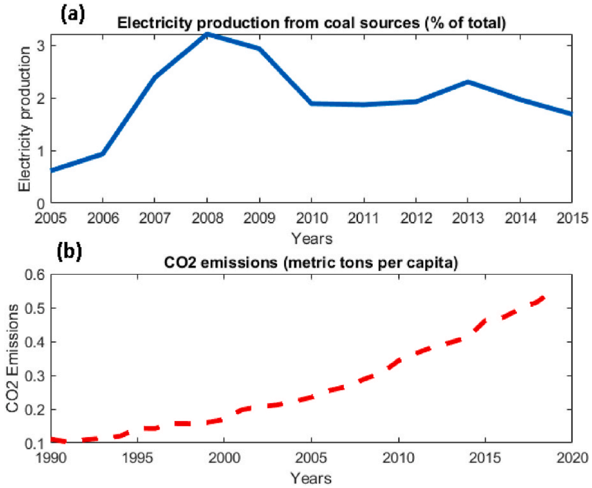

Fig. 2. Total coal used for electricity production (a) and per capita carbon dioxide (CO2) discharged due to the utilization of energy over time (b).

Table 1
Installing of the power source in Bangladesh (In MW).

The Year of Installation	2011	2012	2013	2014	2015	2016
Types of Power Stations						
Gas	544	460	1114	1408	450	1500
Coal				550	1900	1300
Fuel Oil (FO[1])	1248	1145				
Diesel	100					
Gas/Fuel Oil (FO)	302	390	520	365		
Gas/Diesel		150	0			
Wind			100			
Solar		12				

Table 2
Specification of variables and parameters in the model (1).

Symbols	Description	Unit
$P(t)$	Quantity of air pollutants at plant's environment	Kilo tons
$S(t)$	Quantity air pollutants in Sundarbans	Kilo tons
$C(t)$	Quantity air pollutants in Khulna-Bagerhat Area	Kilo tons
γ	Rate of air pollutants transportation towards Sundarbans	Kilo tons/year
β	Rate of air pollutants transportation to Khulna-Bagerhat area	Kilo tons/year
ν_1, ν_2, ν_3	Total air pollutants absorber trees at each area.	Dimensionless
k_2	Quantity of air pollutants flow toward Bay of Bengal	Kilo tons/year
α	Rate of air pollutants taking up by trees	Kilo tons/year
a	Total quantity of air pollutants emitting at a constant rate from the power plant	Kilo tons/year
δ	Rate of air pollutants absorption by rain water and water resources	Kilo tons/year
ρ	Rate of uncertain air pollutants transportation to Sundarbans	Kilo tons/year
φ	Rate of uncertain air pollutants transportation to Khulna-Bagerhat area	Kilo tons/year
$u_1(t)$	Rate of dissolved pollutants after reacting with external species will be sprayed from our control system	Kilo tons/year
$u_2(t)$	Rate of removal of pollutants by afforestation	Kilo tons/year

analytically and performed all the necessary analyses to demonstrate its validity and efficiency in solving this case and other similar cases. This makes it much easier to fix the problem. Also, we used an optimal control strategy to know how to control the whole process with two control variables as a solution. In fact, using optimal control theory, we concluded that such problems can be solved quite economically by appropriate methods. Our research provides solid guidance on how to identify solutions to such problems and evaluate the effectiveness of the solutions.

2. Mathematical model formulation

We have proposed a new model to control the air pollutant emissions and circulation based on the previous mathematical model and analysis of air pollutant developed by Paul and Biswas [2]. Here we introduced two control variables into our developed model to

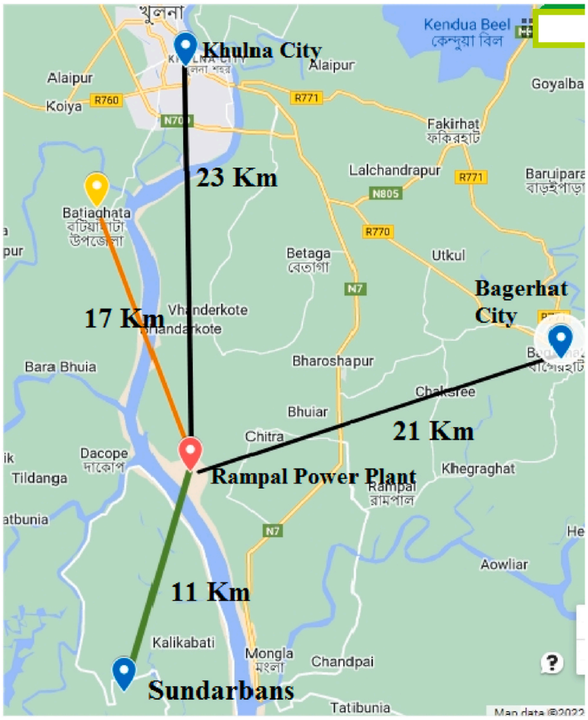

Fig. 3. Position of the power station with respect to the vulnerable regions.

control the emission and concentration of air pollutants. First, we proposed to build a control system in the power plant that sprays various external chemicals to react with air pollutants and separate pollutants from them. Secondly, to minimize the concentration and storage of air pollutants in the Khulna-Bagerhat region, we have proposed a tree farming project that can minimize the quantity of pollutants and their harmful effects. Because the Sundarbans are adequately capable of minimizing and absorbing air pollutants, mainly carbon dioxide (CO2), we believe there is no need to artificially control and reduce pollutants. Fig. 4 describes the dynamics of carbon dioxide (CO2) and other air pollutant diffusion and removal strategies below.

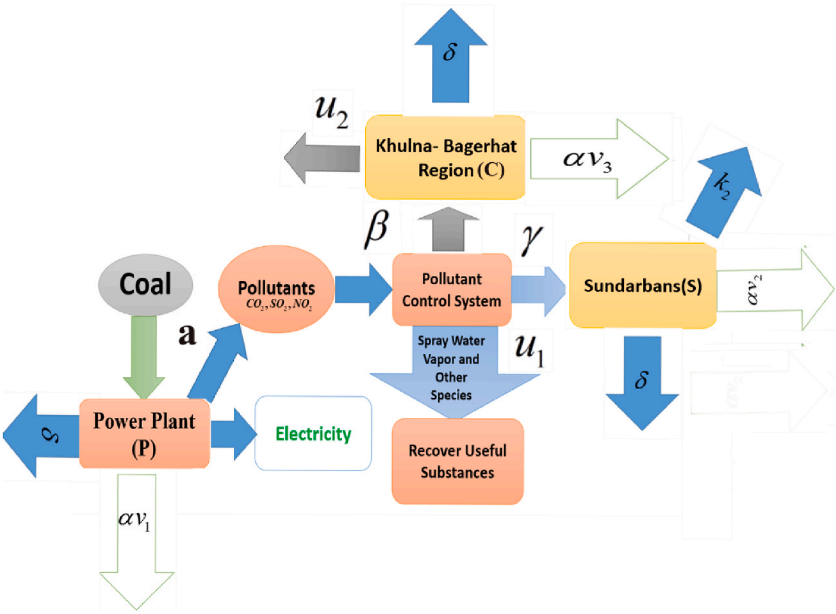

Fig. 4. The diagram of proposed model to prevent the spread of air pollutants in different directions from power plants.

The pollutants emitted from the power station are transmitted to the air and instantly start spreading to other areas. One is the Sundarbans, which is the world's largest mangrove forest, and other is the vast Khulna-Bagerhat region. These two regions are located on the opposite sides of the power plant, so the flow of air plays a vital role in the transportation of air pollutants. Due to the abundance of plants in the Sundarbans, the absorption capacity of other pollutants, including carbon dioxide (CO2), is extremely high, despite being the nearest region, air pollutants in the Sundarbans cannot be stored very often. Besides, for nine months in a year, the air flows from the Southern to the northern part of Bangladesh, carrying most of the air pollutants to the opposite side of the Sundarbans. As a result of this type of airflow, most of the pollutants emitted from the power station run to the Khulna-Bagerhat area and spread across a wide region. In the Khulna-Bagerhat area, the density of the green resources is relatively low, so their ability to absorb contaminants is lower naturally. Thus, a large portion of the transmitted contaminants is stored in the region, consequently

Quantity is gradually increased day by day. Moreover, a small part of whole contaminants that are transported towards the Sundarbans is absorbed by its water resources and the remaining part is transported and absorbed in the Bay of Bengal. There are some air pollutants at each compartment initially. Air pollutants emitting from plant are transported to the plant's surrounding environment at a constant rate a, then it is subtracted from this compartment and added to another two more compartments at γ and β rate respectively. Pollutants of this compartment is also absorbed by trees of this region at αv_1 rate. Absorption by trees depends on the total area of trees v_1 at each compartment. Sulfur dioxide and nitrogen oxides are considered as the biggest threat to green resources. The vital phytotoxic (harmful to plants) air pollutants are ozone (O3), sulfur dioxide (SO2) and nitrogen oxides (NO).

Burning coal to generate electricity produces huge amount of sulfur dioxide and nitrogen oxides along with carbon dioxide (CO2). So, if excessive number of pollutants starts emitting from the power plant, then the reserve of sulfur dioxide and nitrogen oxides will increase highly and will be transported to neighboring areas too. There are many types of plants and trees which are very sensitive to sulfur dioxide and nitrogen oxides. Hence, the extra amount of sulfur dioxide and nitrogen oxides will obviously do harm to the green assets of any area, and the air pollutants absorption capacity of those areas will be reduced gradually. We have considered this fact while modeling the air pollutants' circulation and transportation.

The extra air pollutants will restrict the absorption power by the green area. We considered that when the excessive quantity of air pollutants will be reserved in both areas, the absorption rate by trees and green plants will be plunged and presented by the term $\frac{\alpha v_2 S}{P\rho}$ for Sundarbans and $\frac{\alpha v_3 C}{P\varphi}$ for the Khulna-Bagerhat region. The term $P\rho$ specifies the extra quantity of harmful pollutants for Sundarbans and $P\varphi$ represents the extra quantity of harmful pollutants in the Khulna-Bagerhat region. Since, the species of plants of the two regions are different and differently sensitive to the harmful pollutants, we have used different rates for two varieties of regions. When quantity of pollutants in power plant area P will be very high, then the terms $P\rho$ and $P\varphi$ will be high too. As a result, the terms $\frac{\alpha v_2 S}{P\rho}$ and $\frac{\alpha v_3 C}{P\varphi}$ will be less than natural.

The height of the exhaust pipes of any power plant is usually so high that the pollutants emitted from the plant cannot do any harm to the surrounding environment. Air pollutants usually leave the surrounding area very quickly due to high air flow at higher altitudes and are transported to other remote areas. However, most air pollutants are eliminated by interacting with various elements of the environment before being transported to other remote areas. In the case of any industry located next to any mangrove forest, air pollutants are transported to the surrounding area very quickly due to high air flow. As a result, a small quantity of air pollutants can reside in the environment around the power plant. At a time, it is also true that, due to environmental and health issues, power plants or industries with high power generation capacity are usually located in areas where the quantity of green resources is higher or the population density is very low. Considering all these factors, we have already assumed that the area around the power plant is small and the volume of forest in the vicinity of the power plant is low.

Now by introducing two control variables (chemical reactor system and natural afforestation) in the model (presented in Fig. 4), we can illustrate the whole system by the following system of non-linear ODEs described by model (1):

$$\begin{aligned}
\frac{dP}{dt} &= a - \beta P - \delta P - \gamma P - \alpha v_1 P - u_1 P \\
\frac{dS}{dt} &= \gamma P - \delta S - k_2 S - \frac{\alpha v_2 S}{\rho P} \\
\frac{dC}{dt} &= \beta P - \delta C - \frac{\alpha v_3 C}{\varphi P} - u_2 C
\end{aligned} \tag{1}$$

where, the initial conditions are given by equation (2)

$$\begin{aligned}
P(0) &= P_0 \geq 0, \\
S(0) &= S_0 \geq 0, \\
C(0) &= C_0 \geq 0,
\end{aligned} \tag{2}$$

The first control system, chemical reactor system u_1 should be installed at the power plant so that various chemicals and water can react with gases emitted from the power plant and absorb them instantly by producing some useful sediment too. We are proposing to use water mainly because of its availability and low cost, also it has no effect on health and the environment. Afforestation is proposed as the second method of controlling the pollutants at the Khulna-Bagerhat region. Afforestation has a very low cost and will save the environment in the future too. Fast-growing green resources that have the capacity to absorb air pollutants emitted from burning coal will be cultivated in this region.

The initial conditions from equation (2) indicate that, there exists some carbon dioxide (CO2) at every compartment previously. In plant's environment, the amount of carbon dioxide (CO2) is very high due to excess carbon dioxide (CO2) emission from the power station. Sundarbans contain a very little amount of carbon dioxide (CO2) due to its high absorption capacity, and the Khulna region contains carbon dioxide (CO2), which is bigger than Sundarbans because its absorption power is not as high as Sundarbans. So, $P_0 \geq C_0 \geq S_0$ and $P_0 \geq 0, C_0 \geq 0, S_0 \geq 0$.

The above model must be studied aiming to understand the dynamics of circulation and absorption of air pollutants. The aim of this study is to measure and minimize the quantity of air pollutants generated by Rampal power plant. To that end, a closed set $\Omega = \{(P(t), S(t), C(t)) \in \mathbb{R}^3_+\}$ has been taken where the initial conditions are $P_0 \geq 0, S_0 \geq 0, C_0 \geq 0$.

3. Analytical analysis

In this section, we find analytic solution and the equilibrium of model (1) for the constant value of control parameters (i.e. $u_1(t)$ and $u_2(t)$) with the help of ([20–22,23,11]). For the constant value of the control parameters, we perform the boundedness analysis of the solutions of the model (1).

3.1. Analytical solutions

The analytical solutions (S, P, C) have been performed by solving the model (1). For this purpose, we solve the set of equation (1) and get the following values of (S, P, C):

$$P(t) = \frac{a + e^{-t\,(\theta+u_1)}\left(P_0 - \frac{a}{\theta+u_1}\right)(\theta+u_1)}{\theta+u_1}$$

$$S(t) = \frac{e^{-t\,(\delta+k_2)}\left(S_0\,(-P_0\,(\theta+u_1))^{\frac{\alpha\,v_2}{a\,\rho}} + \frac{\gamma\int_0^t e^{-x\,(\beta+\gamma-k_2+u_1+\alpha\,v_1)}\left(a-P_0\,(\theta+u_1)-a\,e^{x\,(\theta+u_1)}\right)^{\frac{\alpha\,v_2}{a\,\rho}}\left(P_0\,(\theta+u_1)-a+a\,e^{x\,(\theta+u_1)}\right)dx}{\theta+u_1}\right)}{\left(a-P_0\,(\theta+u_1)-a\,e^{t\,(\theta+u_1)}\right)^{\frac{\alpha\,v_2}{a\,\rho}}}$$

$$C(t) = \frac{e^{-t\,(\delta+u_2)}\left(C_0\,(-P_0\,(\theta+u_1))^{\frac{\alpha\,v_3}{a\,\varphi}} + \frac{\beta\int_0^t e^{-x\,(\beta+\gamma+u_1-u_2+\alpha\,v_1)}\left(a-P_0\,(\theta+u_1)-a\,e^{x\,(\theta+u_1)}\right)^{\frac{\alpha\,v_3}{a\,\varphi}}\left(P_0(\theta+u_1)-a_0+a\,e^{x\,(\theta+u_1)}\right)dx}{\theta+u_1}\right)}{\left(a-P_0\,(\theta+u_1)-a\,e^{t\,(\theta+u_1)}\right)^{\frac{\alpha\,v_3}{a\,\varphi}}}$$

were, $\beta + \delta + \gamma + \alpha\,v_1 = \theta$.

4. Equilibrium analysis

We can find the equilibrium for the model (1) by considering,

$$\frac{dP^*(t)}{dt} = \frac{dS^*(t)}{dt} = \frac{dC^*(t)}{dt} = 0$$

So, we can write from the model (1) as

$$\begin{aligned}
\frac{dP}{dt} &= a - \beta\,P - \delta\,P - \gamma\,P - \alpha\,v_1\,P - u_1 P = 0 \\
\frac{dS}{dt} &= \gamma P - \delta\,S - k_2\,S - \frac{\alpha\,v_2\,S}{\rho\,P} = 0 \\
\frac{dC}{dt} &= \beta P - \delta\,C - \frac{\alpha\,v_3\,C}{\varphi\,P} - u_2 C = 0
\end{aligned} \tag{3}$$

Solving the above system of equation (3), we get the set $E^*(P^*, S^*, C^*)$, were

$$p^*(t) = \frac{a}{\beta+\delta+\gamma+u_1+\alpha\,v_1},$$

$$S^*(t) = \frac{a^2\,\gamma\,\rho}{(\beta+\delta+\gamma+u_1+\alpha\,v_1)\,(\alpha^2\,v_1\,v_2 + a\,\delta\,\rho + \alpha\,\beta\,v_2 + \alpha\,\delta\,v_2 + a\,k_2\,\rho + \alpha\,\gamma\,v_2 + \alpha\,u_1\,v_2)}$$

$$C^*(t) = \frac{a^2\,\beta\,\varphi}{(\beta+\delta+\gamma+u_1+\alpha\,v_1)\,(\alpha^2\,v_1\,v_3 + a\,\delta\,\varphi + \alpha\,\beta\,v_3 + \alpha\,\delta\,v_3 + \alpha\,\gamma v_3 + a\,\varphi\,u_2 + \alpha\,u_1\,v_3)}$$

5. The optimal control problem characterization

We have introduced two control variables in the developed model, symbolically (u_1, u_2): (a) u_1 is used for dissolving pollutants by reacting with external species and water that will be sprayed from our control system. (b) u_2 is the rate of absorption of air pollutants by natural forests of fast-growing green resources that have the capacity to absorb air pollutants at high rates in the Khulna-Bagerhat region. Herein, the two control variables are considered as the function of time (t) i.e., $u_1(t)$ and $u_2(t)$. In the present paper, our target is to minimize the cost owing to building up a maintained control system and reducing the extra quantity of pollutants naturally in the Khulna-Bagerhat region with the natural afforestation project. Since, the production process of the power plant has not been started yet, green resources with high absorption capacity can easily absorb air pollutants transported from power plants.

Hence, the aim of adopting optimal controls is to control air pollution, climate change and other problems in the most cost-effective manner in all regions. Hence, using Pontryagin's maximum principle (Lenhart and Workman [24]), we have constructed the objective functional as minimization type as follows:

$$\text{Min } J(u_1, u_2) = \int_0^T \left(P(t) + C(t) + Au_1^2 + Bu_2^2\right) dt \tag{4}$$

where, the constants A and B present the costs involved with developing and maintaining the reactor at the power plant and forestry project respectively. Further, we consider $[0, T]$ as the execution time duration. Now, we will find the optimal solutions $(u_1{}^*(t), u_2{}^*(t))$ such that, $J(u_1^*, u_2^*) = \min\{(u_1^*, u_2^*) \in U\}$, where $U = \{(u_1, u_2) / u_i(t)\}$ is Lebesgue measurable on $0 \le u_1(t), u_2(t) \le 1, t \in [0, T]$.

We can again formulate model (1) as the standard form of OCP described by equation (5) with equation (4) as

$$\begin{cases} \text{Minimize } J(y, u) = \int_0^T L(t, y, u) dt \\ \text{subject to} \\ \dot{y}(t) = f(y) + g(y)u, \forall t \in [0, T] \\ u \in U, \forall t \in [0, T] \\ y(0) = y_0 \end{cases} \tag{5}$$

were,

$$y(t) = \begin{pmatrix} P(t) \\ S(t) \\ C(t) \end{pmatrix}, g(y) = \begin{pmatrix} -P & 0 \\ 0 & 0 \\ 0 & -C \end{pmatrix},$$

$$f(y) = \begin{pmatrix} a - \beta P - \delta P - \gamma P - \alpha v_1 P - u_1 P \\ \gamma P - \delta S - k_2 S - \dfrac{\alpha v_2 S}{\rho P} \\ \beta P - \delta C - \dfrac{\alpha v_3 C}{\varphi P} - u_2 C \end{pmatrix}$$

$$U(t) = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

The integrand of objective functional can be denoted by equation no. (6),

$$L(y, u) = P + C + Au_1^2 + Bu_2^2 \tag{6}$$

Theorem 1. There are optimal controls u_1^*, u_2^* such that $J(u_1^*, u_2^*) = \min\{J(u_1^*, u_2^*), (u_1^*, u_2^*) \in U\}$ [25].

Proof: The integrand of the objective functional $J(u_1, u_2)$ is a convex function of $u_1$and u_2. Since the solution of the model (1) is bounded, hence the system satisfies the Lipchitz property with respect to the variables P,S,C. Therefore, there exists an optimal pair u_1^*, u_2^*. Hence the theorem is proved.

From equation (6) we can write the Lagrangian of the problem as $L = P + C + Au_1^2 + Bu_2^2$ [22]. Now, we can form the Hamiltonian H for the problem given by the following equation (7) [22].

$$H(P(t), S(t), C(t), u_1(t), u_2(t), \lambda_1(t), \lambda_2(t), \lambda_3(t)) = P + C + Au_1^2 + Bu_2^2 - \lambda_1(t)\{\beta P - a + \delta P + \gamma P + Pu_1 + \alpha P v_1\}$$

$$-\lambda_2(t)\left\{\delta S - \gamma P + \frac{\alpha S v_2}{P\rho} + k_2 S\right\} - \lambda_3(t)\left\{C\delta - \beta P + Cu_2 + \frac{\alpha C v_3}{P\varphi}\right\} \tag{7}$$

By using the Maximum Principle, we can determine the adjoins system and transversality conditions. So, from equation (7), we get adjoins system are $\frac{\partial\lambda}{\partial t} = -\frac{\partial H}{\partial P}, \frac{\partial\lambda_2}{\partial t} = -\frac{\partial H}{\partial S}, \frac{\partial\lambda_3}{\partial t} = -\frac{\partial H}{\partial C}$ with the transversality conditions $\lambda_i(T) = 0, i = 1,2,3$.

Thus, we have the conditions $\lambda_1(T) = 0, \lambda_2(T) = 0, (T) = 0$.

Now, using the optimality conditions $\frac{\partial H}{\partial u_1} = 0, \frac{\partial H}{\partial u_2} = 0$, we get the control variables as equations (8) and (9),

$$u_1 = \frac{\lambda_1 P}{2A} \tag{8}$$

$$u_2 = \frac{C\lambda_3}{2B} \tag{9}$$

And the Adjoint system is given by the following equation (10) -(12) by solving the system consists of equations (8) and (9).

$$\frac{d\lambda_1}{dt} = \frac{\alpha c v_3}{p^2\varphi} - u_1 - \alpha v_1 - \delta + \frac{\alpha s v_2}{p^2\rho} \tag{10}$$

$$\frac{\partial\lambda_2}{\partial t} = -\delta - k_2 - \frac{\alpha v_2}{p\rho} \tag{11}$$

$$\frac{\partial\lambda_3}{\partial} = 1 - u_2 - \frac{\alpha v_3}{p\varphi} - \delta \tag{12}$$

By solving the above system of equations 10–12, we get the values of $\{\lambda_1^*.\lambda_2^*, \lambda_3^*\}$ having initial conditions $\lambda_1(0) = \lambda_p$, $\lambda_2(0) = \lambda_S$, . $\lambda_3(0) = \lambda_C$. We get equations 13–15 as below,

$$\lambda_1^* = \lambda_p - t\left(\delta + u_1 + \alpha v_1 - \frac{\alpha c v_3}{p^2\varphi} - \frac{\alpha s v_2}{p^2\rho} - 1\right) \tag{13}$$

$$\lambda_2^* = \lambda_S - t\left(\delta + k_2 + \frac{\alpha v_2}{p\rho}\right) \tag{14}$$

$$\lambda_3^* = \lambda_C - t\left(\delta + u_2 + \frac{\alpha v_3}{p\varphi} - 1\right) \tag{15}$$

Putting the Values of $\{\lambda_1^*, \lambda_2^*, \lambda_3^*\}$ in the equations (8) and (9), we get the values of controls as the following form in equations (16) and (17),

$$u_1^* = \frac{Ap\left(\lambda_p - t\left(\delta + u_1 + \alpha v_1 - \frac{\alpha c v_3}{p^2\varphi} - \frac{\alpha s v_2}{p^2\rho} - 1\right)\right)}{2} \tag{16}$$

$$u_2^* = \frac{B\lambda_C\left(\lambda_C - t\left(\delta + u_2 + \frac{\alpha v_3}{p\varphi} - 1\right)\right)}{2} \tag{17}$$

We can clearly see that the condition of optimality is satisfied here [24].

$$\frac{\partial^2 H}{\partial u_1^2} > 0, \frac{\partial H^2}{\partial u_2^2} > 0$$

$$\frac{\partial^2 H}{\partial u_1^2}\frac{\partial H^2}{\partial u_2^2} - \left(\frac{\partial H^2}{\partial u_1 \partial u_2}\right)^2 > 0$$

6. Numerical simulations and efficiency analysis

The remarkable feature of the Neilan and Lenhart model is that they assumed that the reaction of chemicals and water vapor with pollutants in the chemical reactor control system is limited. To handle such a situation, they introduced an extra variable R which denotes the quantity of pollutants removed for limiting the reaction. Assuming that R_M is the total quantity of removed pollutants by chemical reactor control system during the whole period, the constraint can be represented by

$$\dot{R}(t) = \theta up, ...R(0) = 0, ...R(T) = R_M$$

Considering the above two conditions and combining them together, we can study our problem as:

$$\frac{dP}{dt} = a - \beta P - \delta P - \gamma P - \alpha v_1 P - \theta P u_1$$

$$\frac{dS}{dt} = \gamma P - \delta S - k_2 S - \frac{\alpha v_2 S}{\rho P} \tag{18}$$

$$\frac{dC}{dt} = \beta P - \delta C - \frac{\alpha v_3 C}{\varphi P} - u_2 C$$

$$\frac{dR}{dt} = \theta P u_1$$

in fact, for our new problem described by equation (18), variable R does not appear either in the cost in any other differential equations or in the mixed constraints. So, we have removed it from our system in analytical analysis.

On the other hand, it is possible to collect some valuable compounds from pollutants removed by chemical reactor methods. If it is not possible to collect all of them as useful compounds, then we are assuming that $\frac{1}{3}$ of the total removed pollutants can be collected as sediment during reaction. Thus, we can study our optimal control problem as the following form in equation (19):

$$\begin{cases} \text{Minimize } J(y,u) = \int_0^T L(t,y,u)dt \\ \text{subject to} \\ \dot{y}(t) = f(y) + g(y,u), \forall t \in [0,T] \\ u \in U, \forall t \in [0,T] \\ y(0) = y_0 \end{cases} \tag{19}$$

were,

$$y(t) = \begin{pmatrix} P(t) \\ S(t) \\ C(t) \\ R(t) \end{pmatrix}, f(y) \begin{pmatrix} a - \beta P - \delta P - \alpha v_1 P - \theta P u_1 \\ \gamma P - \delta S - k_2 S - \dfrac{\alpha v_2 S}{\rho P} \\ \beta P - \delta C - \dfrac{\alpha v_3 C}{\varphi} - u_2 C \\ \theta P u_1(t) \end{pmatrix}, u(t) = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

where, the objective given by equation (20) is to be minimized,

$$L(y,u) = P + C + A u_1^2 + B u_2^2 \tag{20}$$

where, $\theta = rate$ of compounds are recovered from the chemical reactor system. In this case, we have considered two controls: chemical reactor control (u_1) and natural afforestation (u_2), both two controls are necessary for reducing the quantity of air pollutants in all regions.

In this section, we simulate the model using Open-OCL (Open Optimal Control Problem Library) with MATLAB programming languages. This library uses IPOPT Optimal Control Solver, which is well-recognized worldwide for solving these types of problems. Here, we have simulated our problem for fixed 30 years, which is the total lifetime of the power plant so both two controls are stopped after 30 years. The simulation was performed using values presented in Table 3 and considering initial values $P(0) = 50, S(0) = 10$ and $C(0) = 100$. All the initial values have been used in this research are hypothetical but very close to the real values. In addition, we have simulated our model with different initial conditions to examine the behavior of the system and illustrate outcomes under all possible conditions. In many cases, the quantity of initial pollutants in the power plant region has been taken to 0 and the number of initial pollutants in the Khulna Bagerhat area has been taken to 50. Because, the source of pollutants is much higher in the Khulna Bagerhat area. Besides, the quantity of pollutants also increases and decreases due to airflow, population growth and various reasons. So, we have depicted the situation by varying the initial quantity of pollutants in each area to examine the behavior of the system. By using these parameter values and initial values, we will be able to understand the whole situation, or this result will always maintain a ratio with the original results, which will be found after initializing power production of the plant. Most of the parameter values were collected from Refs. [3,26,27]) and some values were assumed based on our assumptions.

Values of weighting parameters A and B are determined based on the priority of control strategies. Using a larger parameter value for parameter A, when A is being multiplied with the 1st control variable, means the 1st control strategy is considered more important

Table 3
Numerical values of Parameters in model (1).

Parameters	a	γ	β	δ	α	k_2	v_1	v_2	v_3	ρ	φ	A	B
Values	675	0.25	0.4	0.1	1.5	0.2	1	50	5	1	1	2000	150

and essential in minimizing the objective functional. In selecting these two parameters, we must have given more importance to our first control method and therefore multiplied with the variable expressing the first control method. Since we must control air pollution as soon as possible and we must keep in mind the cost of doing so, we made such a decision. Since afforestation is a relatively long-term activity and it is never possible to achieve success by spending more money only for afforestation, less cost has been fixed for afforestation. We know that if it is possible to eliminate air pollutants directly from the atmosphere or from the environment of the power plant through various chemicals, then it is possible to protect the nature from the harmful effects of air pollution very quickly. But the chemical reaction method is certainly expensive and difficult enough to maintain. So, logically the cost of this method is higher, but it is highly effective and can be deployed initially.

We have also calculated the efficiency of each individual control for evaluating their performance on controlling the effects. Unless otherwise stated, we have used the formula to calculate the efficiency index is outlined below.

$$\text{Efficiency Index} = \left(1 - \frac{P^c}{P^o}\right) \times 100$$

where P^c and P^o are the cumulated number of pollutants at each state with and without control respectively. Here, having bigger efficiency index will be considered as the best strategy [25]. The cumulated number of pollutants at each region (such as Power Plant, Khulna-Bagerhat, Sundarbans) with the time duration [0, 30] is denoted by

$$\int_0^{30} P(t)dt, \int_0^{30} S(t)dt, \int_0^{30} C(t)dt$$

We specify the two control policies: STR-1 where $u_1 \neq 0$, $u_2 = 0$ and STR-2 where $u_1 = 0$, $u_2 \neq 0$ and presented in Table 4. Hence, for evaluating the performances of each strategy, we have defined those using acronyms and they are mentioned bellow.

WOC = Without any control measures.
WBC = With both controls' measures.
W1C = With only 1st control and another is disabled.
W2C = With only 2nd control and another is disabled.
WCC = Using both controls with increased chemical reaction rate.

We have simulated our optimal control problem numerically for finding other important properties and efficiencies of each control and to observe the air pollutants level before and after applying control.

We simulated our model without using any type of control measures for 30 years of time intervals and the simulation is presented in Fig. 5. When pollutants are emitted from the power plant, they are transported to the other two regions (Sundarbans and Khulna-Bagerhat) immediately. Therefore, the quantity of pollutants does not increase highly in the Power Plant region. Furthermore, some pollutants are absorbed by the trees and water resources of the power plant region. Most of the pollutants transported to Sundarbans are absorbed due to the presence of sufficient trees in the Sundarbans area. That is why Sundarbans always have environmentally low pollutants content. However, due to the flow of air towards the Khulna during the 9 months of a year, most of the pollutants go towards this area. Again, due to the absence of sufficient trees, the contaminants can never be properly absorbed and the number of contaminants are being constantly increasing in the Khulna-Bagerhat region.

As the chemical reactor control system removes a larger quantity of contaminants from the power plant before transportation, the number of pollutants transportation reduces significantly. So, we can see from Fig. 6 that when only the chemical reactor control system is working, the quantity of contaminant in each area decreases to a certain degree. From Table 5, we see that after 30 years, the quantity of pollutants in Sundarbans reduces from 84 to 80 kilo-tons, for the Khulna-Bagerhat region the number of pollutants reduces from 843 to 767 kilo-tons and for the power plants region it reduces from 338 to 333 kilo-tons. Since the transportation of pollutants towards the Sundarbans is very limited and the absorption capacity of Sundarbans is very high, this Chemical reaction control system has had a very limited effect on the Sundarbans. From this, we can conclude that without the chemical reactor control system, the number of pollutants in the Sundarbans would be approximately the same. But the chemical reaction control system has significantly impacted the Khulna-Bagerhat region. However, from the chemical reaction control system, it is possible to recover some valuable and effective compounds degraded by the reaction of various chemicals and water with the pollutants, which can be used in other industrial factories later.

Since airflow is directed towards Khulna-Bagerhat for 9 months of a year, controlling it only by chemical reaction system in this is not possible. With chemical reaction control, we can remove a certain number of contaminants from the power plant before it begins transporting to other regions, but there is still a need for another control system for the Khulna-Bagerhat region, as we have confirmed from Figs. 5 and 6.

Table 4
Efficiency index of 3 types of controls for in each state.

E.I of	U_1 (Chemical Reactor System)	U_2 (Natural afforestation)	U (WCC)
P	5.1769	0	8.2790
S	15.1540	0	20.6843
C	26.2489	82.6679	83.6900

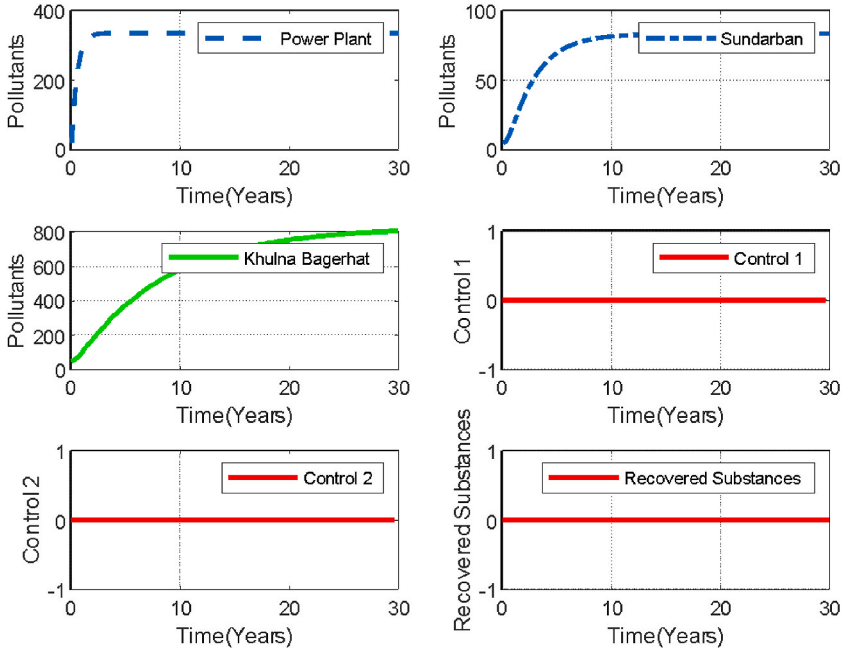

Fig. 5. Dynamic behavior of stored pollutants in Power plant, Sundarbans and Khulna-Bagerhat regions in the absence of any control system (Control 1 and Control 2, i.e., chemical reactor system, $u_1 = 0$ and natural afforestation, $u_2 = 0$) show an increasing number of pollutant and the number of Recovered substances from the chemical reactor is zero.

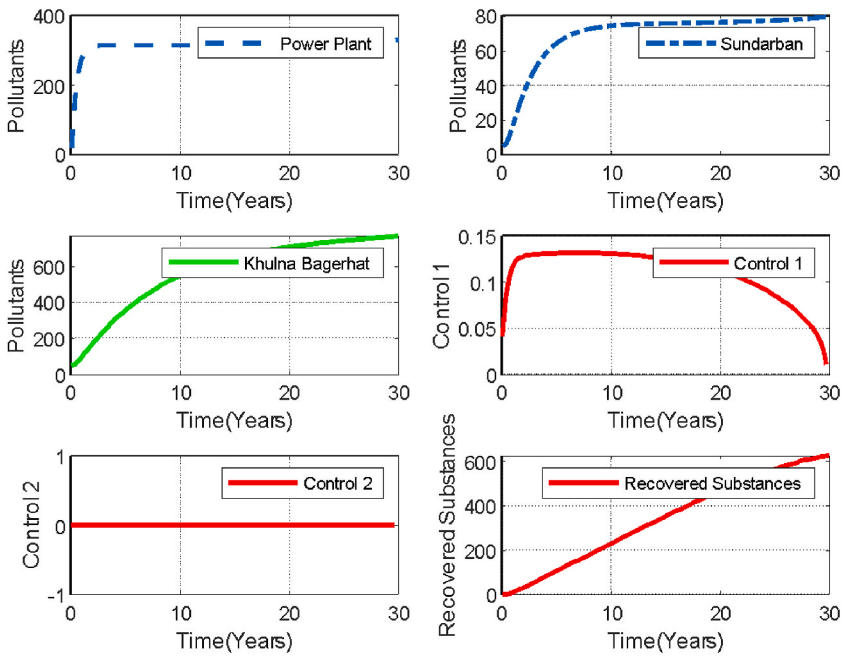

Fig. 6. Dynamic behavior of stored pollutants in power plant, Sundarbans and Khulna-Bagerhat regions in the presence of chemical reactor system (control 1 and the control 2, i.e., natural afforestation, $u_2 = 0$ and Control) and the number of Recovered substances from the chemical reactor system Only the implementation of the first control measures has reduced the amount of air pollutants of the power plant, the amount of air pollutants of the Sundarbans has decreased slightly and the amount of Khulna region has come down from 800 kilotons to 600 or 650 kilotons and at the same time the various compounds produced as a result of chemical reactions have accumulated 600 kilotons.

Since chemical reactions are very expensive and can be used to remove a certain amount of contaminant daily, afforestation is a good and long-term control system for removing contaminants transported to the Khulna-Bagerhat region. As we can see from Fig. 8, it is possible to reduce the number of air pollutants in the Khulna region only by the afforestation project. Although it has no role in

Table 5
Summary of cost functional, end time values, quantity of removed pollutants for each type of controls at end states.

Cases	Iterations	Costs	P (30)	S (30)	C (30)	Removed Pollutants	R (30)
WCC	17	4732	184	90	116	1650	550
WBC	34	2559	333	80	190	1593	531
W1C	9	4559	335	81	767	3117	1039
W2C	84	15550	337	83	193	17538	0
WOC	2	33239	337	84	843	0	0

reducing the contamination of other areas, it does more than any other control system in decreasing the contamination of a large area like the Khulna-Bagerhat area. Where the contaminants in the Khulna-Bagerhat area were 767 kilo-tons at the end of 30 years through the chemical reactor system, it was only 193 kilotons in the 30th year only through the afforestation project without using chemical reactor control. It is clear from Fig. 7 that, by applying the Afforestation control system, it is possible to reduce the contamination only in the Khulna-Bagerhat area. This control has no effect on the number of pollutants in other regions, and the number of contaminants remains the same after adopting this control in other regions over a period of 30 years. Fig. 7 shows that with the adoption of this control system, the number of pollutants soars slightly over the course of five years and booms to 120 kilo-tons from the initial state. But then it does not increase until 20 years later and remained the same. From this, we can understand that if the afforestation program had been adopted in advance, the Khulna-Bagerhat region would have been prepared to protect itself from the contamination of the Power Plant region. However, one of the good aspects of the chemical reaction system is that it produces many efficient and expensive compounds that can be used later in many cases but not through only the Afforestation control system.

When applying both controls together, we can see from Fig. 8 that the optimal solution for each region is the same in the shape of the curves. But looking closely at the curves and values in Table 4, we can see that applying both control measures together reduce the cost of control and the amount of air pollution in each region. Particularly, due to the application of two control systems together, the necessity of the chemical reaction system has been reduced to some extent. Because previously controlled by chemical reaction, the number of air pollutants needed to be reduced was able to meet most of the control through the afforestation control system. In fact, there is no need to cut down on transportation to minimize the number of pollutants transported to the Khulna-Bagerhat area. Instead of focusing on reducing transportation to the Khulna-Bagerhat area, it is possible to restrict the number of pollutants only through the afforestation control system. We can see from Fig. 8 that, in Khulna-Bagerhat, Sundarbans and Power plant regions, the maximum reduction of pollutants has been possible by applying the two control systems at the lowest cost. Since we assume that the chemical reaction system produces about 30% of the effective compounds of the removed contaminants. We also observe that the number of

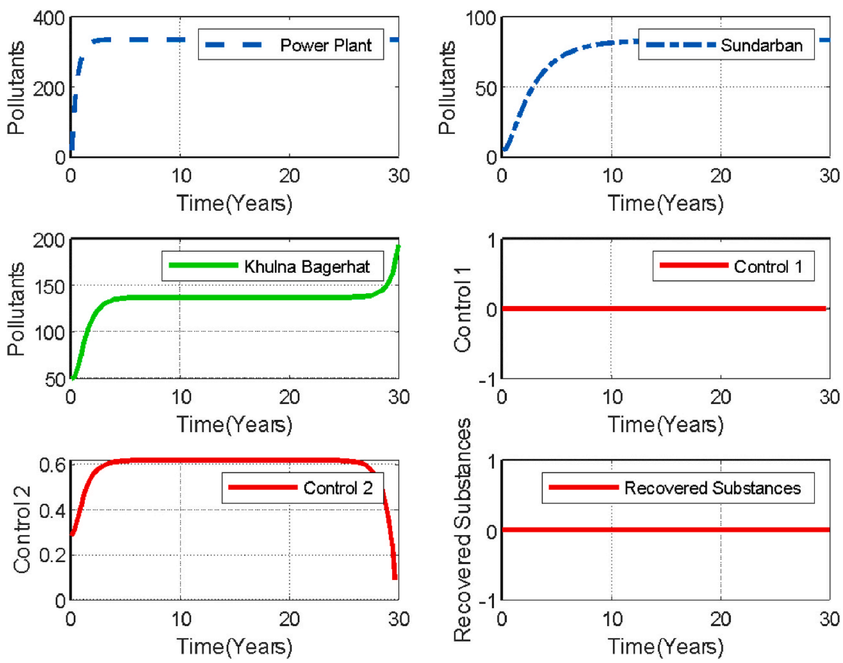

Fig. 7. Dynamic behavior of stored pollutants in Power plant, Sundarbans, Khulna-Bagerhat regions in the presence of natural afforestation (Control 2), u_2 (i.e., chemical reactor system, $u_1 = 0$) and the number of Recovered substances from the chemical reactor system. Only the implementation of the second control system resulted in a very small reduction in air pollutants in Sundarbans and Power Plant areas, but air pollutants in Khulna Bagerhat area came down from 800 kilotons to 200 kilotons. In this case, since the first control mechanism i.e. chemical reaction is not initiated, no compound is produced.

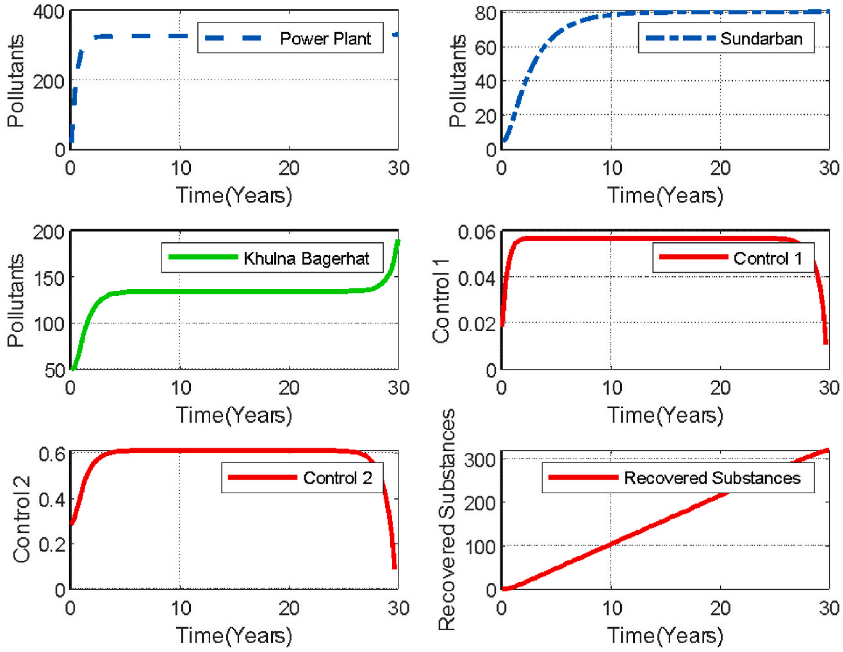

Fig. 8. Dynamic behavior of stored pollutants in the power plant, Sundarbans, Khulna-Bagerhat in the presence of both control measures (Control 1 and Control 2, i.e., chemical reactor system, $u_1 \neq 0$ and natural afforestation, $u_2 \neq 0$) and the number of Recovered substances from the chemical reactor system. Simultaneous implementation of both the control measures i.e., chemical reaction and afforestation projects has resulted in significant reduction of air pollution in Sundarbans and power plant areas and no impact on air pollution in Khulna Bagerhat area. In this case, the amount of compound produced as a result of chemical reaction was found to be around 300 kilotons. The implementation of the first control system is reduced to that of the second control system to reduce cost since the first control system is quite expensive.

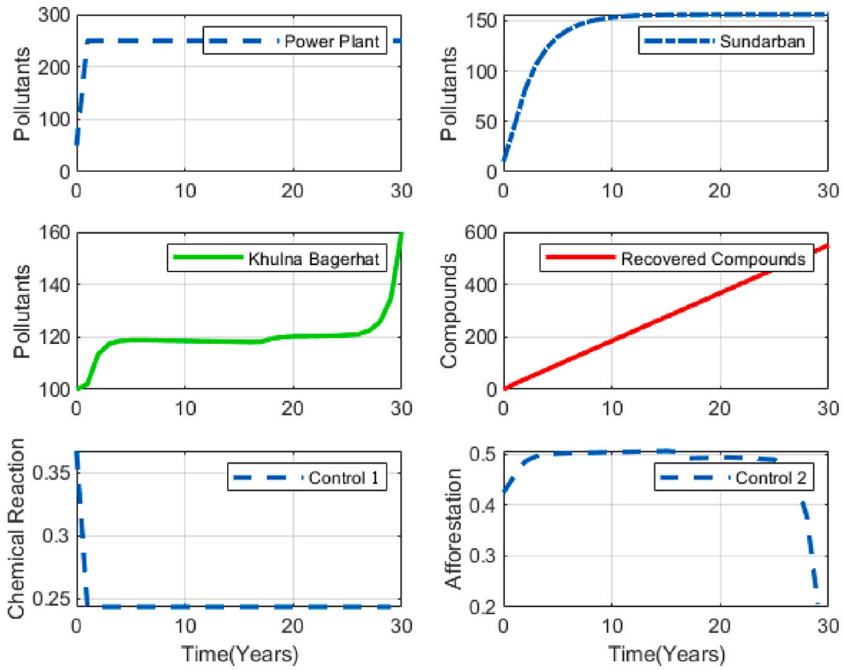

Fig. 9. The optimal trajectories of stored pollutants in the **Power Plant, Sundarbans and Khulna-Bagerhat** regions when both control measures (**Control 1 and Control 2**) are activated simultaneously with increased chemical reaction rate keeping $S(t) \leq 170$, $\theta u_1(t)P(t) \leq 550$ and the number of **Recovered substances** from the chemical reactor system. By speeding up the rate of chemical reactions and quantifying the compounds produced, we find that air pollutants in the Khulna Bagerhat area have decreased by a certain amount and the Recovered compounds produced have reached close to 600 kilotons.

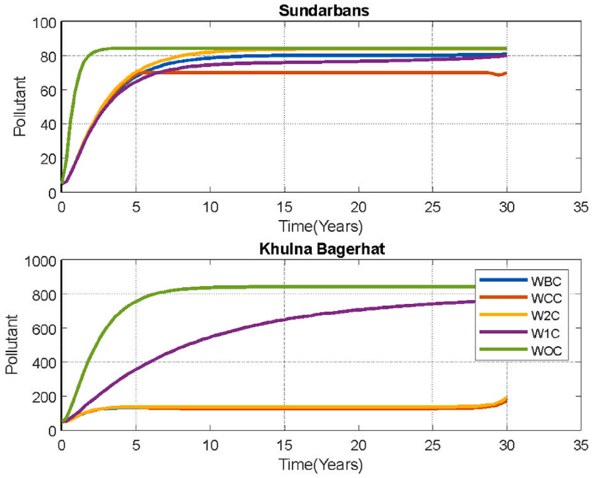

Fig. 10. Comparison of pollutants spread over time in **Sundarbans** and **Khulna-Bagerhat** region for 5 different combinations of control strategies (WBC, WCC, W2C, W1C, WOC). Here it is clear that all control strategies affect the **Sundarbans** equally and indicate that no specific control measures are required for the Sundarbans. But it is not possible to reduce the air pollution **in Khulna Bagerhat** area without implementation of any control system and only by implementation of first control system. But the remaining 3 strategies i.e., using second control, using both controls, and using both control by controlling the reaction rate work equally well to reduce air pollutants in Khulna Bagerhat area.

effective compounds produced in the 30th year is 531 kilo-tons, so the chemical reactor control system can remove a total amount of 1593 kilo-tons of contaminants. The level of contamination in the Sundarbans dropped slightly to 80 kilo-tons in the 30th year, and in the power plant area, it was 330 kilo-tons.

Now, for reducing the effects of air pollutants, from Fig. 9 we can see that it is the best strategy to minimize most of the pollutants is increasing the reaction rate of the chemical reactor system (WCC). Because this can influence all the states at a significant rate and using both two controls simultaneously over time. We can see from Fig. 9 that, after applying this strategy, the pollutants of Sundarbans reduce to 70 kilo-tons, Khulna-Bagerhat decreases by 174 kilo-tons, and the amount of totally removed pollutants is 1652 kilo-tons. From Fig. 10, we can observe that chemical reactor system control can minimize the transportation of pollutants by maximizing the reaction rate, and this is most effective for the Sundarbans. But using both controls (chemical reactor system control and afforestation) can reduce drastically the same amount of pollution from those areas without increasing reaction rates. Especially, afforestation with a chemical reactor control system (WBC) can reduce the pollutants from the Sundarbans region at a lower rate than that of using a chemical reactor control system and afforestation with an increasing reaction rate (WCC). But using both controls (chemical reactor method and afforestation) without increased reaction rate (WBC) in chemical reactor system is less expensive and reduces pollution equally in the Khulna-Bagerhat region.

After calculating the efficiency index of two control variables in each state, we found that the chemical reactor control system is efficient for each state and contributed minimizing the air pollutants from each state at an average level overall. Afforestation only works on state **C** (Khulna-Bagerhat) greatly. We have found the efficiency index is 82.6679 of afforestation here which means afforestation has a very significant effect to minimize the air pollutants in this region. The chemical reactor control system has a significant effect on the state S (Sundarbans) too.

From Table 4 and Fig. 10, we can say that using both controls simultaneously with an increased reaction rate (WCC) is the best policy in terms of reducing pollutants. On the other hand, using both controls without increasing the reaction rate (WBC) of chemical reactor system is the best in terms of minimizing costs. In fact, both strategies (WBC and WCC) perform almost the same here and differ only in terms of costs a little bit.

The efficiency index and, the cost functional, types of controls and value at end states are given at Tables 4 and 5

7. Conclusions

After calculating the efficiency of two control variables in each state, we have found that chemical reactor system control is efficient for each region and contributes to minimize the air pollutants from each region at an average level. Natural afforestation control only works on state C (Khulna-Bagerhat) and intended to do so. We have found that in this region (Khulna-Bagerhat), the efficiency index of natural afforestation control is 82.6679, which means natural afforestation has a very significant effect in minimizing the air pollutants in this region. The chemical reactor control system also has a significant effect on the state S (Sundarbans) because of its efficiency index (15.1540). From Table 4, we can say that the simultaneous activation of both controls is the best strategy with respect to reducing the pollutants at the lowest cost. From Tables 4 and 5 and Fig. 10, we can conclude that both controls (chemical reactor system and afforestation) without any condition and both controls with increased reaction rate perform almost the same. If we want a solution with minimum cost, we must select 1st case (WBC). But, if cost is not a concern, both controls should be adopted with enhanced reaction rate (WCC) it can remove more pollutants than (WBC) and produce some useful substances for other industries.

From Fig. 10, we see that both controls with increased reaction rate (WCC) and without increased reaction rate (WBC) are not effective in Khulna-Bagerhat Region and only the second control (natural afforestation) is sufficient for reducing pollutants here. So, considering another two regions Sundarbans and the power plant, we can propose both controls with increased reaction rate (WCC) as the best strategy for serving the purposes. Since we have been encouraging the use of excessive water as a reactor in the chemical reactor control system, a small quantity of chemicals will be used, and the health problems of the workers will not be a will not become a matter of concern. From this, we will be able to extract a significant number of effective compounds which can be used in other production sectors later.

How to decrease or increase the absorption rate of air pollutants by trees is basically described in the numerical simulations and efficiency analysis section. Because of using two controls at the same time in a serial based on their starting time of impact, increasing, or reducing the rate of the 1st control variable will automatically determine how the green resources will be used in order to reduce air pollutants.

By studying the model with two control strategies, we have observed that if the first control mechanism is inactive or reduced, then the necessity of the reduction of air pollutants through the second control mechanism (i.e., planting trees/afforestation) will be increased alarmingly. On the other hand, if the first control system (i.e., chemical reactor system) is active, it is possible to remove a large part of pollutants, so in that case, the removal rate of air pollutants through the second control (i.e., natural afforestation) will be reduced automatically. The method of reducing air pollutants through tree plantation is more important because of the low cost of this method and, also in the long run afforestation will contribute to the environmental development of the area.

Here, the proposed mathematical model and optimal control strategies are quite new for studying the circulation and reduction of pollutants which are produced by coal-based power plants. To represent the dynamic system of pollutants circulation and reduction, we have developed the model and designed the optimal strategies to find out the best control policies to protect Sundarbans and other neighboring areas from being ruined. For studying and solving the actual and realistic cases, this model and analysis paradigm will obviously bring a better outcome and understanding of the dynamics of the system. It is possible to extend this model to explain new facts and modified for different cases.

At the outset of this work, when we studied the previous works, we felt the need to devise a new method for measuring environmental issues by air pollutants originating from any large industry, especially power plants, located in the vicinity of mangrove forests. Because of the limitations of previous studies and not considering all the factors, it was never possible to accurately measure the environmental impact of air pollutants produced by power plants or to formulate specific methods on how to control these environmental impacts as much as possible. Then, we abandon all hopes of solving this problem by following or imitating all the conventional methods and further developing them. After that, we formulate a completely new mathematical model by understanding the whole system and considering all the elements related to the e-system. Afterwards, we analyze how the system can be controlled in the best possible way using optimal control techniques, and we conclude.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

We, the authors and our immediate family members, have no positions to declare and are not members of the journal's advisory board.

We, the authors and our immediate family members, have no related patents to declare.

The authors declare no competing interests.

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