



# Method for phase space reconstruction to estimate the short-term future behavior of pressure signals in pipelines



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## ABSTRACT

In this study, we propose a method based on phase space reconstruction to estimate the short-term future behavior of pressure signals in pipelines. The pressure time series data were obtained from an IoT experimental model conducted in the laboratory. The proposed hydraulic system demonstrated the presence of traces of weak chaos in the time series of the pressure signal. Fractal dimension analysis revealed a complex fractal structure in the data, indicating the existence of nonlinear dynamics. Similarly, Lyapunov coefficients, divergent trajectories, and autocorrelation analysis confirmed the presence of weak chaos in the time series. The results demonstrated the existence of apparently chaotic patterns that follow the theory proposed by Kolmogorov for deterministic dynamic systems that exhibit apparently random behaviors. Phase space reconstruction allowed us to show the dynamic characteristics of the signal so that short-term predictions were stable. Finally, the study of strange attractors in pipeline pressure time series can have significant contributions to anomaly detection.

- A methodology is proposed for the reconstruction of the phase space to estimate the short-term future behavior of pressure signals in pipelines in real time.
- The analysis of the proposed hydraulic system revealed some indications of weak chaos in the time series of the pressure signal obtained experimentally.
- The methodology implemented and the results of this study showed that the short-term predictions were very accurate and consistent; Chaotic patterns were also identified that support the theory proposed by Kolmogorov.

## Specifications table

Subject area:	
More specific subject area:	Hydraulic
Name of your method:	Phase space reconstruction
Name and reference of original method:	N/A
Resource availability:	<a href="https://www.edgarladino.com/transitorios-iot">https://www.edgarladino.com/transitorios-iot</a> <a href="https://www.youtube.com/watch?v=eQ_YCv4IoHw">https://www.youtube.com/watch?v=eQ_YCv4IoHw</a>

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## Introduction

The study of time series behavior in hydrological terms is a challenge for researchers in the hydraulics and hydrology fields. The differentiation between deterministic chaos and non-chaotic dynamics has been a topic of research in time series analysis [1]. From this approach, a reassessment of the presumed unpredictability of specific time series has emerged, i.e., the existence of the random component in time series is questioned [2]. On the other hand, time series represents the evolution of water behavior over time, which is fundamental to understanding hydraulic and hydrological processes for decision-making focused on water management. Rigorously establishing the provenance of a time series involves detecting different behaviors, trends, and patterns. First, it is necessary to identify whether the series shows seasonality and whether it shows predictable patterns and trends that are repeated at regular time intervals. In addition, it is relevant to analyze whether the time series exhibits stochastic behavior. Therefore, if the fluctuations in the pressure data for a hydraulic system are random and come from some distribution associated with time, we would possibly be dealing with a stochastic process.

### Method details

The time series may show signs of chaotic deterministic behavior; this implies that the fluctuations in the data are the result of a complex dynamical system, where there is high sensitivity of the system to the variation of the initial conditions. For example, deterministic chaos behavior has been observed in growth from plug flow to slow flow [3]. Furthermore, the presence of weak chaos in nonlinear dynamic models implies the existence of mathematical models that describe the implicit patterns in the time series, a theory proposed by Kolmogorov. Therefore, the presence of regularities at different scales contributes to the prediction of the observed variables in the short term. Similarly, the reconstruction of the phase space of dynamic systems allows for visualizing and analyzing the underlying structure of the system and the identification of patterns or attractors to try to predict the dynamics of complex systems. It is possible to predict failure for aviation hydraulic pumps from the pressure signal and chaos theory. For example, it is possible to employ a genetic algorithm that uses phase space reconstruction to anticipate possible failures in the hydraulic system of an aircraft. In this case, the reconstructed matrix would be used as the training and testing set in the support vector regression (SVR) model of the algorithm [4]. The chaotic properties of pressure fluctuations in the flow field, which are generated by the vibration of the tubular bulb pump unit. The results showed that the highest value of the Lyapunov exponent for the pressure fluctuation signal at a specific location of the pump unit was positive [5].

Likewise, the detection of failures in hydraulic pumps in aircraft, combining artificial intelligence with chaos theory, is possible through a neural network [6]. However, multidimensional support vector machines are an alternative tool to establish the internal relationship in time series [7]. Support vector machines (SVMs) can efficiently extract implicit features in time series, identifying trends, seasonal patterns, and anomalies. On the other hand, it has been shown that the pressure signals of a hydraulic pump are characterized by nonlinear and non-stationary behaviors [8]. Consequently, there may be evidence of weak chaos in the hydrodynamic behavior of flow in pressurized pipes; this implies that modest variations in the initial conditions can generate significant and unpredictable changes in the flow along the pipeline. Although the flow may present patterns that appear chaotic, it has been observed that these patterns follow underlying mathematical models. Even though the apparent randomness of the observed patterns in the flow, studies have shown that these patterns follow mathematical dynamics. On the one hand, sharp chaos in the flow implies strange attractors in the system phase space, considering the attractor's set of non-periodic fractals. On the other hand, weak chaos, or stochastic chaos, focuses on trajectories that exhibit a mixture of deterministic and stochastic behavior; these trajectories may show specific patterns or regularities despite their apparent unpredictability. However, it is relevant to establish the incidence of noise on the signal; these factors exogenous to the system can introduce fluctuations or disturbances that increase the uncertainty in the evolution of the dynamic component of the sign. Likewise, the study of the time series of pressures in pipelines can be approached in the same way as the Ergodic theory based on the long-term average shown by the dynamic system. Studying the dynamic evolution of the pressure flow in time implies finding how the system states are distributed in the phase space and their progress as time progresses, under the hypothesis that the long-term average value frequencies will be the same regardless of the initial system state. However, if the flow shows invariant measures, such as mean and variance concerning time, it is under a stationary scenario. Similarly, the reconstruction of attractors of chaotic dynamical systems can be approached from deep learning [9]. Whereby a nonlinear dynamical system is defined from nonlinear differential equations, in which chaotic behaviors, bifurcations, and attractors may be implicit, this system can be described as follows,

$$\frac{dx_1}{dt} = f_1(x_1, x_2, \dots, x_n), \frac{dx_2}{dt} = f_2(x_1, x_2, \dots, x_n), \dots, \frac{dx_n}{dt} = f_n(x_1, x_2, \dots, x_n)$$

Where  $x_1, x_2, \dots, x_n$  are the state variables of the system that evolve in time (t), and  $f_1, f_2, \dots, f_n$  are nonlinear functions that describe how the state variables relate to each one. Thus, the chaotic behavior of bubbles under pressure has been detected experimentally [10]. Similarly, there is chaos in free-flowing systems with pressure fluctuations [11]. The reconstruction of the state space from the technique called C-C method to establish the possible existence of a complex behavior of a daily wastewater flow is possible through the reconstructed phase spaces of a vibratory displacement of points on the basis of a pump centrifuge, which identifies the attractors of the dynamic system [12].

These attractors show the tendency of the system to oscillate around specific values, confirming the existence of controlled dynamics [13]. The detection of chaos in a time series poses significant difficulties. However, some methods and tests allow us to infer its presence. One example is fractal dimension analysis, which looks for self-similar patterns in the time series to determine whether

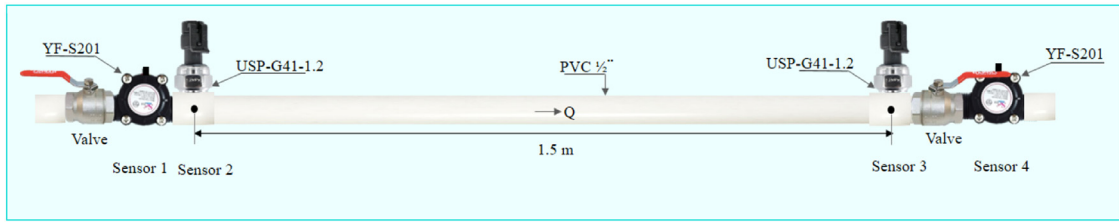


Fig. 1. Hydraulic design.

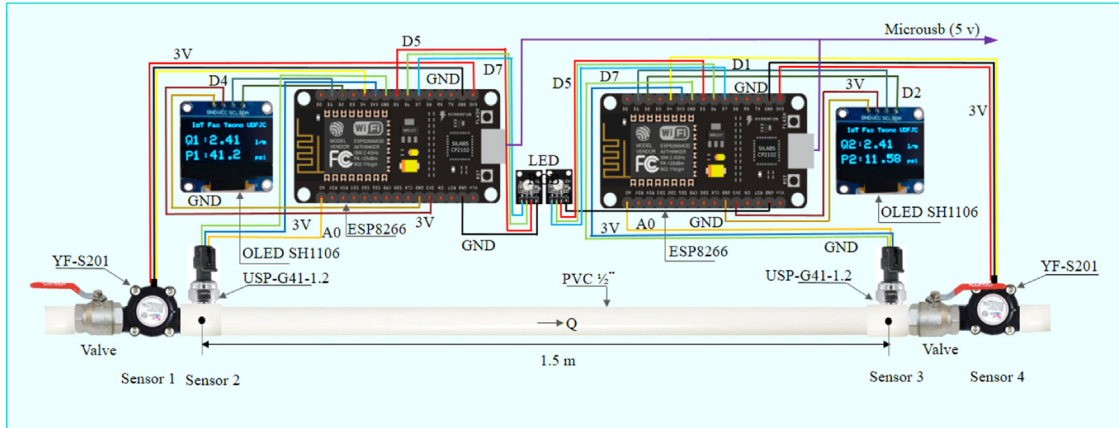


Fig. 2. Electronic system.

chaotic behavior is present. Another approach is spectral analysis, which looks for harmonic and nonlinear components in the series to detect the presence of chaos. On the other hand, Lyapunov coefficients allow for establishing the degree of susceptibility of the dynamic system to the change of initial conditions; these coefficients provide a qualitative and quantitative characterization of the dynamic behavior [14]. Consequently, this study aims to perform the phase space reconstruction for the short-term estimation of the pressure signal in real-time obtained in an experimental model. In the development of this research, we used two ESP8266 microcontrollers, which are connected to the network via WiFi to send the signal to the @Matlab server (@ThingSpeak). In addition, we used two USP-G41-1.2 pressure sensors calibrated from a standard average obtained from an SCJN series pressure gauge with an accuracy  $\pm 0.5\%$  FS. Also, two YF-S201 flow sensors were used to measure the amount of fluid transiting through the system, and for the calibration process, we used the LUXQ series precession smart vortex flow meter with a two-wire 4–20 mA output. Finally, two SH1106 OLED monitors are used.

*Experimental model*

The experimental hydraulic model is composed of 2 ESP8266 microcontrollers, 2 USP-G41-1.2 sensors (pressure), 2 YF-S201 sensors (flow rate), and 2 SH1106 OLED monitors. The objective was to create a controlled environment in the laboratory to analyze the behavior of pressure and flow in the system in real-time. The sensors send the signal to the microcontrollers, which transmit the data via Wi-Fi to the @Matlab server [15]. Previously, each microcontroller was programmed by implementing @Arduino; the proposed code establishes the communication between the sensor, the microcontroller, and the server. Similarly, the calibration equation for each sensor was incorporated into the code, generating a better approximation to the actual reading of the observed variables. Fig. 1 shows the graphical representation of the experimental hydraulic system.

The configuration proposed for the experimental hydraulic model provides efficient and accurate support to analyze and study the behavior of pressure and flow in the system under study. However, the model presented different types of noise produced mainly by fluctuations in the input signal and electromagnetic interference *in situ*, so multiple signal processing techniques were implemented to improve the quality of the captured data [16] from sensor calibration, signal filtering, and correct connection of the electronic components. Cross-validation of the data generated by the sensors was performed concerning precise measurement equipment (standards), thus minimizing the effect of the signals' noise. Fig. 2 presents the electronic model designed for this study, showing the schematic of the proposed circuit, where the different components used are shown.

Calibration of the USP-G41-1.2 sensor (pressure) and the YF-S201 sensor (flow) was carried out. In the pressure case, a SCJN series pressure manometer with an accuracy of  $\pm 0.5\%$  FS was used. For the flow rate, the LUXQ series precession smart vortex flow meter with a 4–20 mA two-wire output was used. Twenty measurements were developed under a constant temperature, the data were recorded in a table with the value of the signal produced by the sensor and the value measured by the reference equipment (standard).

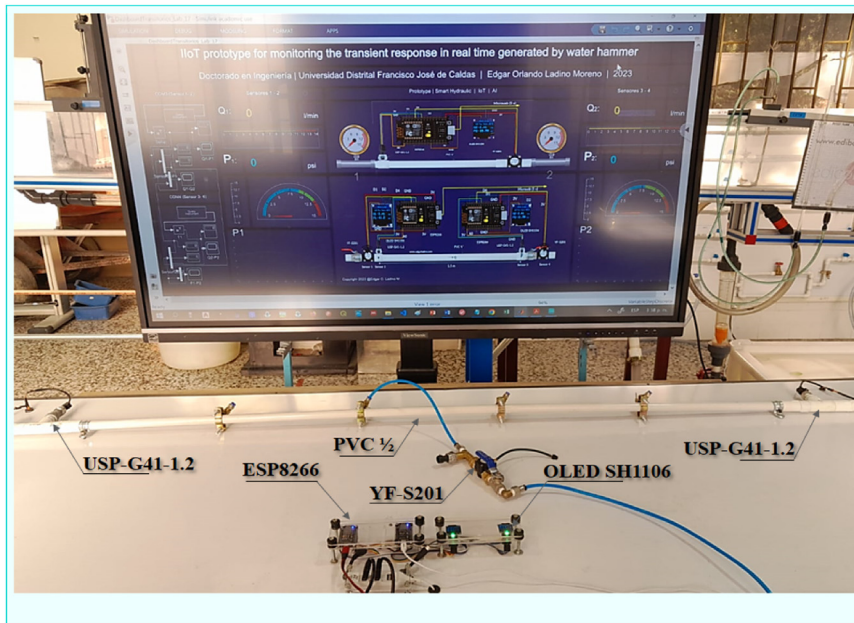


Fig. 3. Experimental design (Hydraulics Laboratory, Universidad Distrital Francisco José de Caldas).

Table 1  
Pressure signal statistics.

Variable	Mean	Standard Error	Median	Standard Deviation	Variance	Kurtosis	Coefficient of Asymmetry	Mínimum	Maximum
Pressure (psi)	12.099	0.004	12.120	0.108	0.012	2.759	-0.164	11.770	12.420

Source: Authors.

The data obtained were plotted, establishing a linear relationship between the sensor output signal and the variable measured by the reference instrument for both variables (flow and pressure). From the graph, the slope and the cut with the y-axis of the regression line were determined; then, these parameters were used to convert the pressure and flow sensor readings into actual values of the observed variables. The linear response obtained was encoded in the ESP8266 microcontroller. Fig. 3 shows the hydraulic experimental design used in this study, two control panels were developed for the visualization and monitoring of flow and pressure in real-time (*video experimental model*, <https://www.youtube.com/shorts/ZhDoXC6u3i8?app=desktop>).

### Signal processing

Fig. 4 presents the signals obtained in real-time without processing. Significant disturbances (peaks) are evident at different time intervals. These peaks generate different types of noise that affect the quality of the data obtained. Therefore, it is necessary to perform diverse signal analyses for the correction and elimination of noise, such as statistical analysis of the signals, frequency analysis, determination of the type of noise, and finally, the type of filter to implement. By erasing the noise from the signal, it is possible to obtain a more accurate representation of the flow and pressure data. For this study, several filters were analyzed and contrasted, such as the low pass filter, which allows the low frequencies to pass and reduce the high frequencies; the high pass filter, and the moving average filter. The objective of comparing these different filters was to determine which optimally removed noise from the flow and pressure signals. In Fig. 4, the moving average filter was applied with the purpose of attenuating the peaks present in the pressure signal. This filter managed to smooth the signal and was incorporated into the programming of the ESP8266 microcontroller.

Fig. 5 displays the methodology used to identify the type of signal and noise in the system. This methodology is a combination of signal analysis and data processing techniques. sequentially, an exploratory analysis of the data and statistics shown in Table 1 was performed. For this purpose, the signals were plotted in the time and frequency domain, which made it possible to identify trends and patterns in the time series. In statistical terms, the skewness value of 0.7468 indicates that the distribution of the pressure data presents a slight rightward tendency, i.e., there is a longer tail on the right side of the distribution. Regarding the dispersion of the values in the series, the standard error was found to be 0.004, indicating the variability of the signal. On the other hand, the kurtosis value of the pressure signal is 2.759, indicating that the distribution presents a leptokurtic behavior concerning the standard normal distribution. In other words, the distribution of the pressure signal tends to be sharper and more concentrated around the mean compared to a normal distribution.

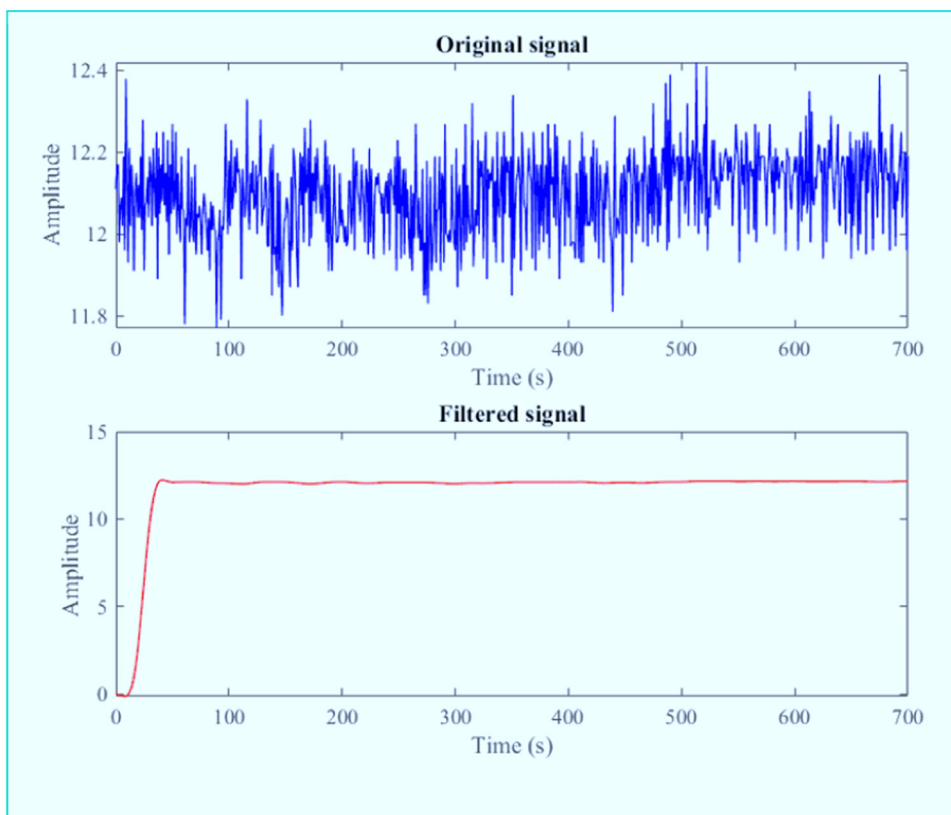


Fig. 4. Original signal - filtered signal (Pressure).

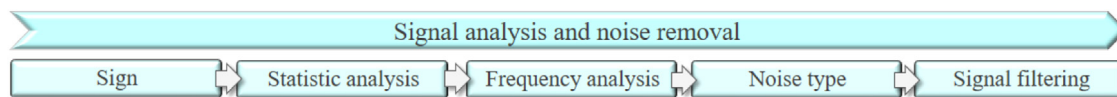


Fig. 5. Signal analysis methodology.

Table 2  
Implemented filter statistics.

Filter	Mean	Standard Error	Median	Standard Deviation	Variance	Kurtosis	Coefficient of Skewness	Minimum	Maximum
Low Pass	12.099	0.002	12.107	0.045	0.002	2.933	-0.688	11.968	12.175
High Pass	0.001	0.003	0.009	0.099	0.009	2.729	-0.061	-0.259	0.283
Moving Average	12.099	0.004	12.120	0.108	0.012	2.759	-0.164	11.770	12.420

Additionally, the sign has a standard deviation equal to 0.108, which indicates that the series values have a low dispersion concerning its mean. Thus, once the exploratory analysis of the pressure time series has been performed, it can be stated that the time series presents some noise. These findings are fundamental to understanding the behavior of the signal over time and being able to address the selection of possible filters to be used to reduce the noise present in the series.

The variation of the power concerning different frequencies as a function of time is presented in Fig. 6; this graph identifies the intensity changes of the pressure signal, allowing the establishment of the dominant frequency patterns. Thus, it is possible to identify the color changes in the occurrence of transient events in the pipeline. In fact, the sampling rate was one second (1 Hz), according to the Nyquist-Shannon sampling theorem, the maximum frequency that the signal can contain should be half the sampling frequency, therefore, the frequency maximum of the pressure signal was less than 0.5 Hz.

Low-pass and high-pass filters were analyzed with frequency up to 0.4. For the comparison of the filters (FIR), three filters were used. The first filter corresponds to the low pass filter of order 50 and cutoff frequency 0.4. Similarly, a high-pass filter with a cutoff frequency of 0.4 and order 50. Finally, a moving average filter with 5 delay samples. The comparison of different filters aims to improve signal quality by attenuating unwanted or noisy components. The filters offer the possibility to focus on specific frequency ranges and their spectral response. Based on Table 2, the statistical behavior of the filtered data shows significant information about the



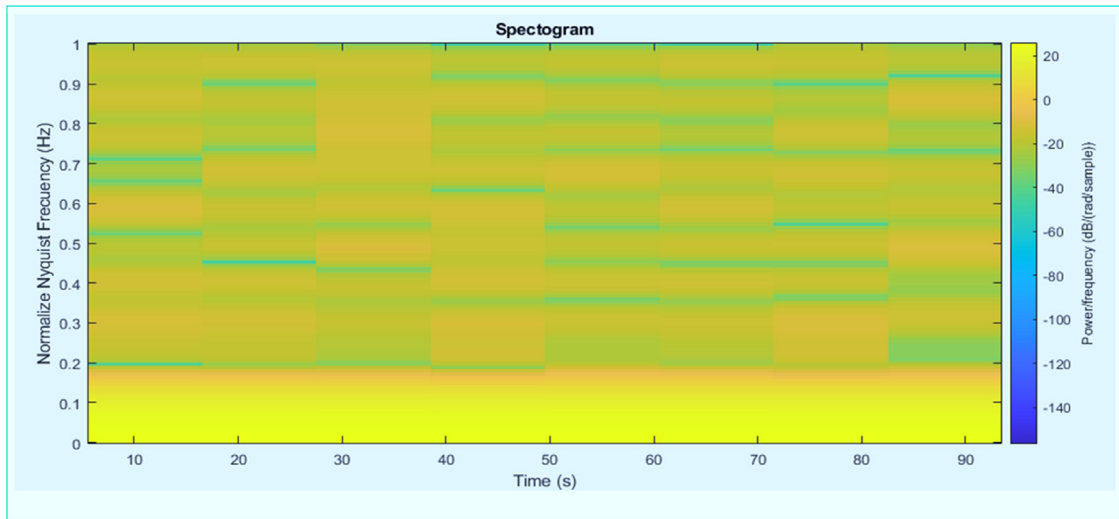


Fig. 6. Spectrogram.

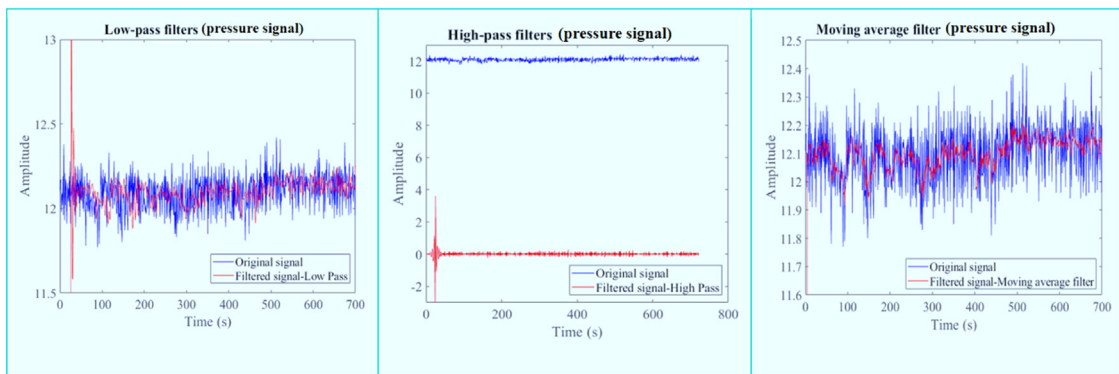


Fig. 7. Filters pressure signal filters.

characteristics of the pressure signal. In the case of the low pass filter, a mean of 12.0986 is observed. The median of 12.1065 suggests that the central value of the distribution is also close to this number. The standard deviation of 0.045076 shows the dispersion of the data concerning the mean, and the variance of 0.0020318 indicates the variability of the pressure-filtered data. Thus, the kurtosis of 2.9331 reveals a leptokurtic distribution with heavier tails and higher peaks than a normal distribution. The skewness of  $-0.68766$  indicates a skew to the left. On the other hand, the high-pass filter shows a mean of 0.00055147, indicating that the filtered values are close to zero on average. The standard deviation of 0.098535 displays the dispersion of the data concerning the mean and variance of 0.0097091. The kurtosis of 2.7289 indicates a leptokurtic distribution, while the skewness of  $-0.061348$  indicates a slight leftward trend. Finally, the moving average filter shows similar statistics to the low-pass filter, with a mean of 12.0993, a median of 12.106, a standard deviation of 0.050461, a variance of 0.0025463, a kurtosis of 3.2514 and a skewness of  $-0.61858$ . These statistics provide a detailed view of the characteristics of the filtered data and are valuable to analyze and compare the results obtained with each type of filter. For this study, the moving average filter was implemented to smooth the signal; Fig. 7 shows the filter behavior. The moving average filter removes fast high-frequency disturbances from the calculation of the mean of a data set of the pressure signal; the window of the data set determines the smoothing level when implementing the filter. However, applying the filter to the time series may introduce delays in the signal so that the signal estimates may become time-shifted. Given this, it is relevant to perform a spectral analysis of the sign to evaluate the impact of the filter on the data series. The spectral analysis also allows the evaluation of the filter effect on the signal-to-noise ratio for specific frequencies.

Thus, the pressure signal was affected by implementing the algorithm for the moving signals average filter, which highlights the underlying trends and smoothest the signal. The equation that describes this process is as follows.

$$y[n] = (x[n] + x[n - 1] + x[n - 2] + \dots + x[n - M + 1])/M \tag{1}$$

Equation (1) determines, for time instant  $n$ , the previous values of the signal  $M$  and calculates the average. The averaging of the pressure signal establishes a smoothing level of the time series, introducing a signal delay. Therefore,  $y[n]$  is the smoothed value at time instant  $n$ ;  $x[n]$  is the signal value at time instant  $n$ , and  $M$  corresponds to the window size used for the moving average.

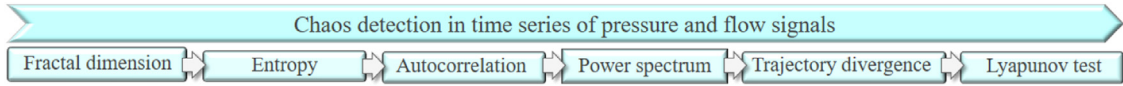


Fig. 8. Detection of chaotic behavior.

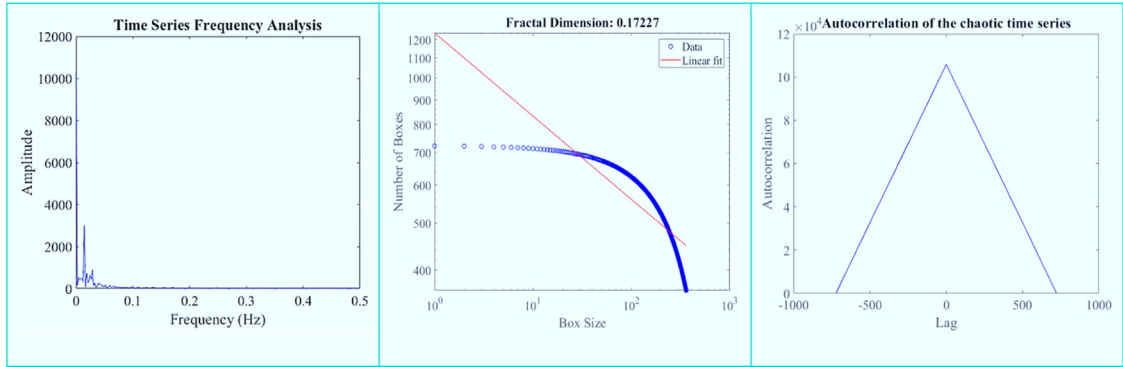


Fig. 9. Autocorrelation and spectral density.

Chaos detection in pressure time series

The study of time series behavior aims to identify patterns, trends, and dependencies of the data as a function of time. For this, it is essential to implement advanced techniques such as spectral analysis, frequency analysis, statistical analysis, and signal processing. The purpose of implementing these tools is to determine whether the data correspond to stationary or non-stationary processes, whether the series presents a stochastic behavior or arises from a deterministic chaotic behavior. To point out the existence of chaotic behavior in time series is a complex and challenging process, which depends on the thresholds established according to the dynamic behavior of the studied phenomenon. This study used different methods and techniques to analyze the time series obtained from pressure sensor observations, such as fractal dimension, entropy, autocorrelation, power spectrum, divergent trajectories, and Lyapunov tests. The main objective was to detect if the pressure signal comes from a chaotic behavior developing the methodology proposed in Fig. 8 to address the chaotic analysis for the pressure signal to determine if the series presents a highly unpredictable behavior associated with nonlinear dynamics.

The frequency analysis shown in Fig. 9 was performed using the Fast Fourier Transform (FFT) to decompose the signal into frequencies, that is, to go from a time domain to a frequency domain; this algorithm reduces the computational cost concerning the Discrete Fourier Transform (DFT) [17]. The FFT divides the signal into segments, combining the results to obtain the complete transform considering the signal symmetry and periodicity properties. The general FFT equation to calculate the DFT of a discrete signal  $x[n]$  of length  $N$  is given by

$$x[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N} \tag{2}$$

Where  $x[k]$ : frequency at index  $k$  of the transform;  $x[n]$ : signal value for index  $n$ ;  $j$ : imaginary unit and  $N$ : signal length. The pressure signal from the experimental model was subjected to a frequency spectrum analysis (Fig. 9), which shows the amplitude of each frequency component present in the signal. It is observed that the amplitude stabilizes around 0.2 Hertz (Hz), at which point the amplitude tends to zero. In terms of the fractal dimension, it is evident that the structure and variability of the pressure signal in the pipe indicate the distribution of points in space. In this case, a fractal dimension equal to 0.17227 was obtained, which suggests that the pressure signal has a highly irregular and complex fractal structure, indicating that the signal can present a deep spectrum of patterns at different scales and frequencies. In addition, the autocorrelation analysis of the time series shows the existence of dependence or recurrent patterns in the pressure data allowing to determine the possible relationship between past and present data of the experimental data to establish the level of the seasonality of the series. The autocorrelation is calculated using the autocorrelation function (®Matlab), which measures the similarity between signal values at different time observations. A high autocorrelation value indicates a strong positive correlation between time observations, which means that the values tend to be similar at those times. However, a low or negative autocorrelation value indicates a weak or inverse correlation between signal values. For example, if the series presents a significant positive autocorrelation, the possible presence of periodic patterns of the signal. On the other hand, if an autocorrelation close to zero is observed, it may indicate a lack of correlation or dependence in the time series. The triangular plot of autocorrelation presented in Fig. 9 reveals a positive relationship between past and present values in the pressure signal evidencing the effect of past values on current values significantly. Also, this autocorrelation shows a periodic pattern due to daily or seasonal fluctuations in the observed pressure values. The autocorrelation involves determining the covariance between the observed and lagged values of the time series and then dividing it by the product of the variances of both sets of data;

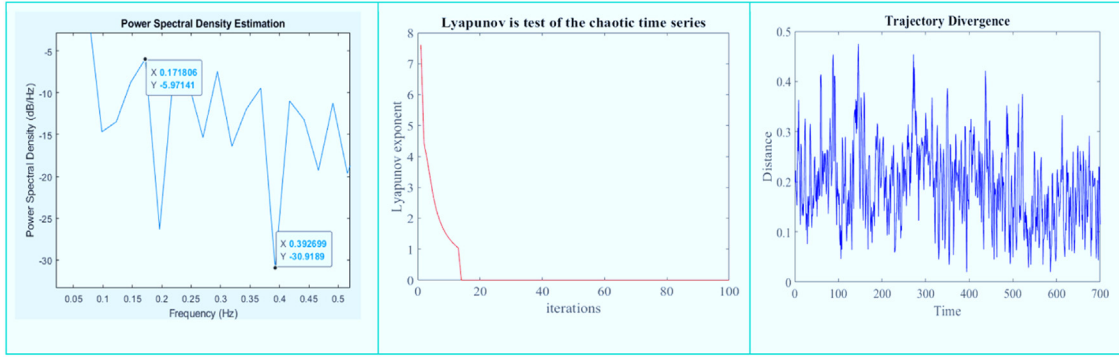


Fig. 10. Indications of the existence of chaotic behavior.

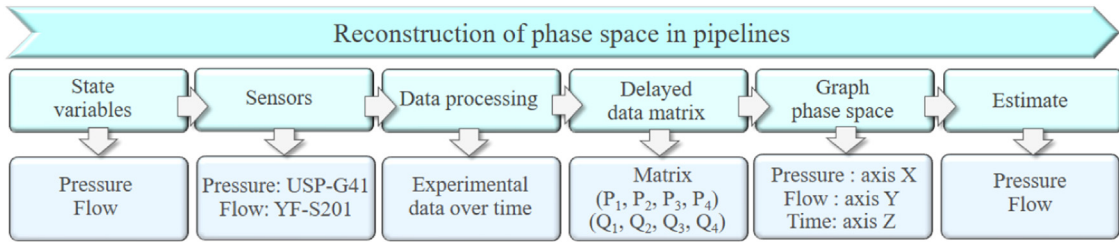


Fig. 11. Phase space reconstruction methodology.

this parameter is expressed as follows.

$$\rho(k) = \frac{\sum_{t=k+1}^T [(x_t - \mu)(x_{t-k} - \mu)]}{\sum_{t=1}^T (x_t - \mu)^2}$$

Where:  $\rho(k)$ : autocorrelation coefficient at lag  $k$ ;  $x(t)$ : observation at time  $t$ ;  $\mu$ : mean of the time series, and  $k$ : time lag.

The pressure time series presents a power concentration in a range of 0.17 to 0.39 Hz, with a maximum amplitude of  $-5$  dB/Hz and a minimum amplitude of  $-30$  dB/Hz (Fig. 10). As the frequency increases, the power concentration tends to stabilize, so the pressure signal for this time window presents a more uniform power distribution at frequencies outside the dominant range. However, this behavior could indicate the existence of other signals or background noise in the pressure signals [18]. In addition, calculations of Lyapunov coefficients were performed to analyze the response of the dynamic system to small-scale variations in the initial conditions [19].

These Lyapunov coefficients allowed us to evaluate the possible presence of chaotic behavior in the time series. In fact, for the first 15 iterations, the Lyapunov coefficients were positive, which indicates a possible exponential sensitivity to the initial conditions, which implies that the dynamical system may show chaotic behavior. This study found that the trajectories established from the Lyapunov coefficients presented a divergent behavior. It was evident that the trajectories began to move away from each other, which shows that the minimum variation of the initial conditions of the system originates significantly different results, demonstrating the instability of the dynamic system [20]. In attempting to quantify the degree of disorder and randomness of the time series, the entropy was found to be 2.1679, indicating the existence of fewer predictable patterns or structures in the long-term data. Indeed, small-scale variations in the initial conditions generated trajectories that separate rapidly and nonlinear, indicating the possible existence of strange attractors for the pressure signal in the experimental model. The study of strange attractors in pressure or flow time series in pressure pipes can have practical contributions, such as the detection of anomalies, leaks, overpressure, or water hammers. The prediction of unexpected events in the system contributes to the stability and safety of the system and provides valuable information about the system dynamics.

*Phase space reconstruction and short-term estimate*

From the phase space reconstruction, it is possible to extract chaotic features to implement them in the prediction models [21]. Thus, the one-dimensional time series obtained by the pressure sensor, a higher dimensional space was constructed, implementing time delays of the initial series, which contributes to the visualization of the spatial structure of the data with one- and two-time delays. Short-term prediction in nonlinear dynamical systems, such as pipe pressure time series, can be challenging due to the sensitivity to initial conditions and the possible presence of chaotic behavior. Fig. 11 shows the methodology applied to achieve these results.

First, the matrix of the delayed state variables at different time points was constructed [22]. For example, for the pressure data, 723 observations were used. Subsequently, the phase space was plotted using the lagged state variable (pressure) data.



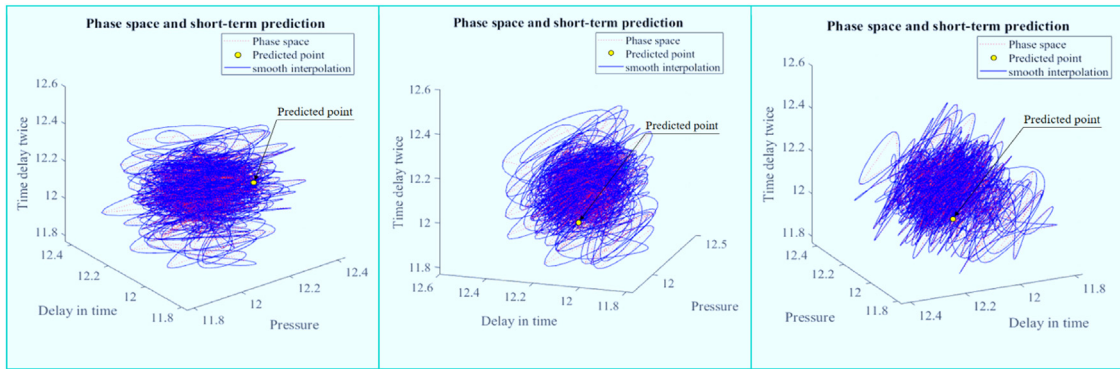


Fig. 12. Phase space and short-term prediction.

Table 3  
Prediction point.

Pressure	Time Delayed	Time delayed twice
11.96	12.19	12.17

The three-dimensional representation in Fig. 12 illustrates the pressure in the Y axis as a function of time, with one delay in the X axis and two delays in the Z axis. Using this analysis, pressure prediction was carried out, resulting in a specific value of 11.96 with delay times of 12.19 and 12.17, respectively. These parameters are essential to understand the temporal dynamics of pressure in the system, offering valuable information for the prediction and understanding of its behavior over time. The three-dimensional representation provides a visual view of temporal relationships and emerging structures in the pressure time series, framed in the context of chaos to identify non-linear patterns and recurring behaviors. The prediction based on time delays suggests the possible presence of complex and chaotic dynamics in the pressure signal of the analyzed hydraulic system, contributing to unraveling the complexity inherent in its dynamics and providing valuable information on its temporal behavior (Table 3).

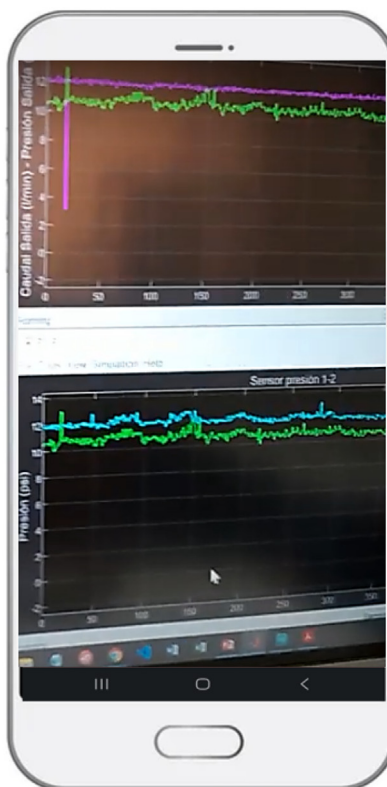
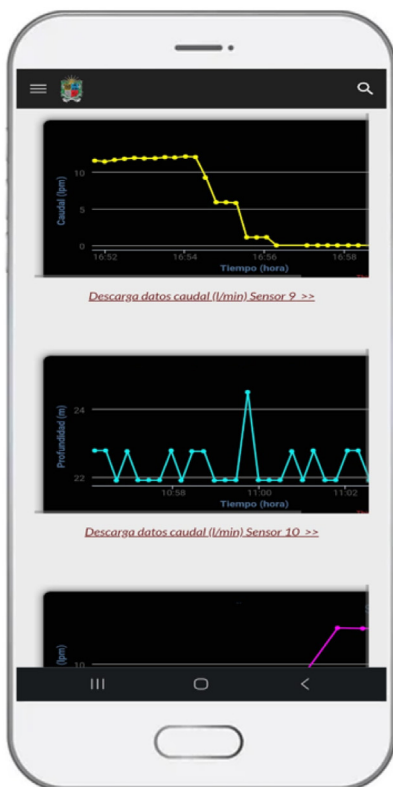
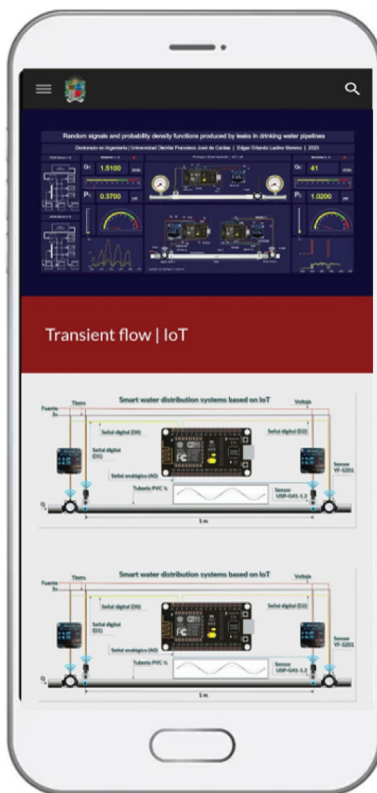
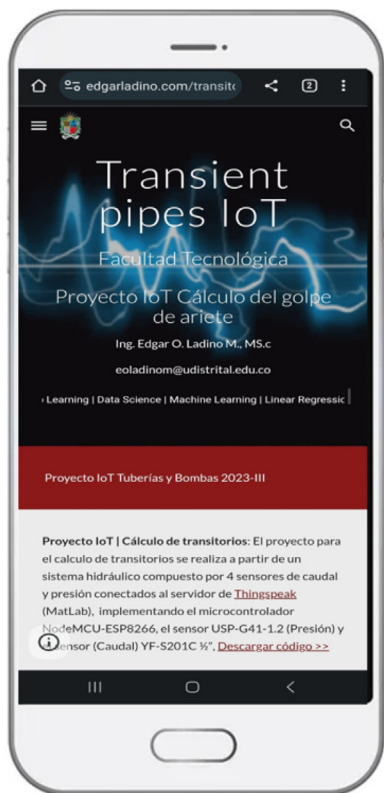
**Conclusions**

The analysis of the proposed hydraulic system revealed some indications of weak chaos in the time series of the pressure signal obtained at the experimental level. Through the application of nonlinear analysis techniques, it was found that the pressure signal exhibited a possible complex and nonlinear dynamic behavior. The identification of the weak chaos of the pressure signal suggests that certain aspects of the hydraulic system behavior can be understood and predicted within certain short-term limits. In particular, the study of the fractal dimension showed the existence of a complex fractal structure in the data, indicating the presence of nonlinear dynamics. The ability to predict the future behavior of pressure in pipelines could have significant implications in the early detection of disturbances in water distribution networks.

The exhaustive analysis of the pressure signal in the experimental model reveals a series of fundamental characteristics. The frequency spectrum highlights a stabilization around 0.2 Hertz, indicating a point at which the amplitude tends to zero. The fractal dimension, calculated at 0.17227, suggests a highly irregular and complex structure, pointing out the presence of varied patterns at various scales and frequencies in the pressure signal.

Autocorrelation analysis, for its part, confirms the existence of dependencies and recurring patterns, providing valuable information about the relationship between past and present data. These findings contribute significantly to the understanding of the pipeline pressure signal dynamics of the experimental system, establishing solid foundations for future research and practical applications in the monitoring and control of pipeline systems.

The study of strange attractors in pressure or flow time series in pipelines can have practical contributions such as anomaly detection, which contributes to reducing the uncertainty associated with the hydrodynamic behavior of the flow in the presence of invasive events such as leaks or illegal connections in the pipelines. Thus, it is possible to make real-time decisions based on the chaotic nature of the pressure signals. Furthermore, this information could be used to optimize network maintenance and improve water distribution efficiency. Finally, the study results showed that the short-term predictions were highly accurate and consistent; there were also identified chaotic patterns following underlying mathematical rules. These findings support the theory proposed by Kolmogorov for deterministic dynamical systems that exhibit apparently random behaviors, under the principles of sensitivity to initial conditions, strange attractors, fractal dimension, temporal recurrences and principles of Entropy.



## Ethics statements

The method discussed in this scientific article did not involve studies with living beings.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## CRedit authorship contribution statement

**Edgar Orlando Ladino-Moreno:** Validation, Methodology, Supervision, Software, Formal analysis, Writing – review & editing. **César Augusto García-Ubaque:** Methodology, Conceptualization, Data curation, Formal analysis, Writing – review & editing. **Eduardo Zamudio-Huertas:** Methodology, Conceptualization, Data curation, Formal analysis, Writing – review & editing.

## Data availability

Data will be made available on request.

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