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S.I.: DEEP LEARNING FOR TIME SERIES DATA

A convolutional neural network based approach to financial time series prediction

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Abstract
Financial time series are chaotic that, in turn, leads their predictability to be complex and challenging. This paper presents a novel financial time series prediction hybrid that involves Chaos Theory, Convolutional neural network (CNN), and Polynomial Regression (PR). The financial time series is first checked in this hybrid for the presence of chaos. The chaos in the series of times is later modeled using Chaos Theory. The modeled time series is input to CNN to obtain initial predictions. The error series obtained from CNN predictions is fit by PR to get error predictions. The error predictions and initial predictions from CNN are added to obtain the final predictions of the hybrid model. The effectiveness of the proposed hybrid (Chaos+CNN+PR) is tested by using three types of Foreign exchange rates of financial time series (INR/USD, JPY/USD, SGD/USD), commodity prices (Gold, Crude Oil, Soya beans), and stock market indices (S&P 500, Nifty 50, Shanghai Composite). The proposed hybrid is superior to Auto-regressive integrated moving averages (ARIMA), Prophet, Classification and Regression Tree (CART), Random Forest (RF), CNN, Chaos+CART, Chaos+RF and Chaos+CNN in terms of MSE, MAPE, Dstat, and Theil's *U*.

Keywords Deep learning · Time series prediction · CNN · Chaos · Polynomial regression · Exchange rate · Stock market index · Commodity price

1 Introduction

The Financial Time Series is a collection of observations of Financial Variable(s) recorded regularly. E.g., daily exchange rates, daily stock market index values, and daily commodity prices are financial time series. In general, The financial time series is chaotic and noisy [39]. A chaotic time series is not linear and sensitive to initial conditions. [7]. Financial Time series are also noisy, and their statistical properties vary with time. This property makes the prediction impossible [11, 19]. Building the right prediction model that can capture nonlinearity present in the time series is always challenging. It reveals, therefore, that the prediction of financial time series is a difficult and complex task.

Several researchers have demonstrated that an ensemble or hybrid forecasting model for time series can perform better in comparison with stand-alone forecasting models [4, 32]. A hybrid combines two or more stand-alone forecasting models into a mixed model to improve prediction accuracy and overcome the deficiencies of stand-alone models.

Chaos theory [26, 38] models nonlinear financial time series by using lag and embedding dimension in which a lag is the time delay, and embedding dimension is the number of variables required to capture the nonlinear dynamics of financial time series.

Applying deep learning approaches can help achieve better prediction accuracy [3, 6]. Deep learning, a subset of machine learning, allows Artificial Neural Networks (ANNs) to learn multi-level abstraction data representations (hierarchical learning) [10, 16]. The ANNs can construct a nonlinear and complex function that maps inputs to output. These are applied to solve various financial problems such as prediction of stock markets, optimization of

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portfolios, processing, and execution of trade information [43]. This field is still relatively unexplored, however.

A CNN [14] is a special case of the neural network that consists of one or more convolutional layers, often with a subsampling layer, which are followed by one or more fully connected layers as in a standard neural network. The CNNs are a type of neural network developed for two-dimensional image data. However, they can be used for one-dimensional data such as sequences of text and time series [15].

This paper presents a hybrid model involving Chaos Theory, CNN, and PR to predict financial time series as follows. The financial time series in this hybrid is checked for chaos. The chaotic modeled time series is input to CNN to obtain initial predictions. The error series obtained from CNN predictions is fit by PR to obtain error predictions. To get final forecasts from the hybrid model, CNN error predictions and initial predictions are added. Our goal is to build a more accurate model to predict different financial time series such as exchange rates, commodity prices and stock prices.

Though there are Chaos-based hybrids, such as Chaos+MLP+PSO [27], Chaos+MLP+MOPSO and Chaos+MLP+NSGA-II [31], present in the literature (see Table 1), the second-stage of the approaches modeling error series aforementioned are complex and time consuming as there are more parameters to be tuned. So, we used a simple PR to model error series as it can capture nonlinearity present in error series very well. In addition, no approach is comprehensively tested for its efficacy on three types of financial time series.

The contributions of this paper include:

- Two novel chaotic hybrids, Chaos+CNN and Chaos+CNN+PR, are proposed for prediction over 30 years of financial data.
- Solutions to three different financial time series prediction problems, including predicting exchange rates, predicting stock index, and predicting commodity prices.
- Comparative study of proposed hybrids with standalone time series prediction models including ARIMA, Prophet, CART, RF and CNN.
- Comparative study of proposed hybrids with other related chaos-based hybrids such as Chaos+CART [28] and Chaos+RF [28] found in literature.

The remainder of the paper is arranged accordingly: The related literature is presented by Sect. 2. Subsequently, Sect. 3 describes in detail the approach proposed. Next, Sect. 4 describes the experimental design, and Sect. 5 discusses the results. Finally, the paper is concluded.

2 Related literature

There are numerous hybrids for Time series in financial literature and are summarized in Cavalcante et al. [6], Huang et al. [13], Pfeiffer and Hohmann [25], Mochón et al. [22], Li and Ma [17], Bahrammirzaee [2], and Pradeepkumar and Ravi [30]. The deep learning hybrids for financial time series prediction are also found in last two decades of literature and are recently well summarized by Durairaj and Mohan [9] and deep learning approaches for financial time series forecasting are reviewed by Sezer et al. [34].

2.1 CNN-based hybrids

This section presents various related CNN-based hybrids and chaos-based hybrids proposed for financial time series prediction connected with the works mentioned above. The CNN-based hybrids are as follows:

Livieris et al. [18] proposed a CNN–LSTM model for gold price time series forecasting in which CNN is used for learning an internal representation of time series and Long

Table 1 Chaos-based hybrids for prediction of financial time series found in literature

Year	Author(s)	Chaos-based hybrids
2003	Pavlidis et al. [24]	Chaos theory hybrid methodology, ANN,Cluster, and PSO/DE
2010	Huang et al. [12]	Chaos + SVR
2014	Pradeepkumar and Ravi [27]	Chaos+ANN+PSO* Chaos+PSO+ANN*
2016	Pradeepkumar and Ravi [28]	Chaos+QRRF*, Chaos+QR, Chaos+RF
2017	Pradeepkumar and Ravi [29]	Chaos+CART, Chaos+CART-EB, Chaos+TreeNet, Chaos+LASSO, Chaos+RFTE, Chaos+MARS*
2017	Ravi et al. [31]	Chaos+MLP+MOPSO, Chaos+MLP+NSGA-II*

ANN artificial neural network, *QR* quantile regression, *QRRF* quantile regression random forest, *CART-EB* CART ensemble, *RFTE* RF tree ensemble, *PSO* particle swarm optimization, *DE* differential evolution, *MOPSO* multi-objective PSO, *MARS* multivariate adaptive regression splines, *LASSO* least absolute shrinkage selection operator, *NSGA-II* non-dominated sorting genetic algorithm-II

*Winner Hybrid

Short Term Memory (LSTM) is used for identifying short-term and long-term dependencies. Similarly, Vidal and Kristjanpoller [37] proposed another CNN-LSTM hybrid model, which could include images as input which provides a wide variety of information associated with both static and dynamic characteristics of the series. The authors utilized this approach for predicting gold price volatility. Selvin et al. [33] applied a sliding window approach and proposed a new CNN-based hybrid, namely the CNN-Sliding Window model, in which a sliding window is used for predicting future values on a short-term basis.

2.2 Chaos-based hybrids

Table 1 presents the Hybrids based on chaos theory found in the literature to predict financial time series. All of these concluded that the proposed chaos-based hybrids outperformed stand-alone models.

3 Proposed approach

In the proposed hybrid, a financial time series is checked for the presence of chaos. Lyapunov exponent [31] is used for this purpose. Chaos theory is then employed to build the scalar time series phase space [23, 35]. Optimum lag and optimal dimensional values are required for building phase space. Akaike Information Criterion (AIC) [1] It is used for optimal time series lag selection. Method of Cao's [5] is used for the optimal dimensions of embedding. Once optimal lag and optimal embedding dimension are obtained from time series, phase space can be reconstructed using Chaos Theory. Later, CNN is used for obtaining initial predictions, and finally, PR is used to fine-tune predictions. The proposed hybrid is compared with ARIMA [21], Prophet (https://facebook.github.io/prophet/), CNN, CART, RF, Chaos+CART [28], Chaos+RF [28] and Chaos+CNN.

Table 2 Notations used in proposed approach

Notation	Interpretation
l	Optimal lag
m	Optimal embedding dimension
y_t	Actual observation at time t
e_t	Error in time achievement t
$\dot{e}_t$	Error prediction in due course t
$\dot{y}_t$	Prediction at the beginning time t
$\ddot{y}_t$	Time to finish prediction t
$f_1(.)$	Nonlinear function used by CNN to obtain predictions
$f_2(.)$	Linear function used by PR to obtain predictions

Table 2 presents the notations along with their interpretations used in the proposed approach.

The proposed hybrid approach is described as follows. Let $Y = \{y_1, y_2, y_3, \ldots, y_k, y_{k+1}, \ldots, y_N\}$ be a time series with N Comments sometimes recorded $t = \{1, 2, 3, \ldots, k, k+1, \ldots N\}$. Then perform the following:

1. For chaos to occur, check Y. When there is chaos, get optimum lag (l) and optimum embedding dimensions (m) from Y.
2. Once optimal lag and embedding dimension values are obtained, reconstruct phase space from Y.
3. After phase space is reconstructed, partition Y into $Y_{\mathrm{Train}} = \{y_t;\ t = lm+1, lm+2, \ldots, k\}$ and $Y_{\mathrm{Test}} = \{y_t;\ t = k+1, k+2, \ldots, N\}$.
4. Input Y_{Train} to CNN, train CNN to get initial predictions of training set using Eq. 1.

$$\begin{aligned} \dot{y}_t &= f_1(y_{t-l}, y_{t-2l} \ldots, y_{t-ml}) \\ \text{where} \quad t &= lm+1, lm+2, \ldots, k \end{aligned} \tag{1}$$

5. Obtain initial test set predictions by input Y_{Test} to trained CNN by replacing $t = \{k+1, k+2, \ldots N\}$ in Eq. 1.
6. Compute training set of prediction errors using Eq. 2 and test set of prediction errors by replacing $t = \{k+1, k+2, \ldots N\}$ in Eq. 2.

$$\begin{aligned} e_t &= y_t - \dot{y}_t \\ \text{where} \quad t &= lm+1, lm+2, \ldots, k \end{aligned} \tag{2}$$

7. Fit Polynomial Regression to training set of errors and obtain training set error predictions using Eq. 3. Similarly fit PR to test set of errors and obtain test set error predictions by replacing $t = \{k+1, k+2, \ldots N\}$ in Eq. 3.

$$\begin{aligned} \dot{e}_t &= f_2(e_t) \\ \text{where} \quad t &= lm+1, lm+2, \ldots, k \end{aligned} \tag{3}$$

8. Add training set initial predictions and training set error predictions to obtain final training set predictions using Eq 4. Similarly, add test set initial predictions and test set error predictions to obtain final test set predictions by replacing $t = \{k+1, k+2, \ldots N\}$ in Eq. 4.

$$\begin{aligned} \ddot{y}_t &= \dot{y}_t + \dot{e}_t \\ \text{where} \quad t &= lm+1, lm+2, \ldots, k \end{aligned} \tag{4}$$

4 Experimental design

4.1 Datasets used

Various Datasets are used in this paper to observe the effectiveness of proposed hybrids. These daily datasets of 30 years approximately include:

- Three exchange rates are collected from the Federal Reserve: Indian Rupees(INR)/USD, Japanese Yen(JPY)/USD, Singapore Dollar (SGD)/USD.
- The Composite Index of Investing.com collects three stock market indicators, Standard & Poor (S&P)500, Nifty 50, and Shanghai.
- Three commodity prices in US Dollars namely Crude Oil Price, Gold Price, and Soyabeans price are collected from Investing.com

Table 3 presents these datasets along with corresponding dates, number of observations, training set, and test set. Here, the financial time series prediction problem is modeled as a supervised learning problem. Thus, each dataset is divided into a training set (80%) and a test set (20%) of observations. First, all of these datasets are checked for chaos, and it is found that chaos is present in each dataset. Later, phase space is reconstructed with the corresponding optimum lag and ideal insertion dimensions from each dataset (Fig. 1).

Table 4 presents various descriptive statistical measures of the datasets such as minimum, mean, median, maximum, standard deviation, skewness, and kurtosis. The prices of Crude Oil (USD) are in the range of (– 37.63, 145.29), Gold (USD) are in the range of (253,2069.4), and Soyabeans (USD) are in the range of (410,1764.75). The stock prices of Nifty 50 are in the range of (788.15, 14730.95), Shanghai Composite Index are in the range of (104.39,6092.06), and S&P 500 are in the range of (295.450012,3862.959961). The ranges of both commodity prices and stock prices are too much varied because of COVID-19's impact. The exchange rates of INR/USD are in the range of (16.8, 76.975), JPY/USD are in the range of

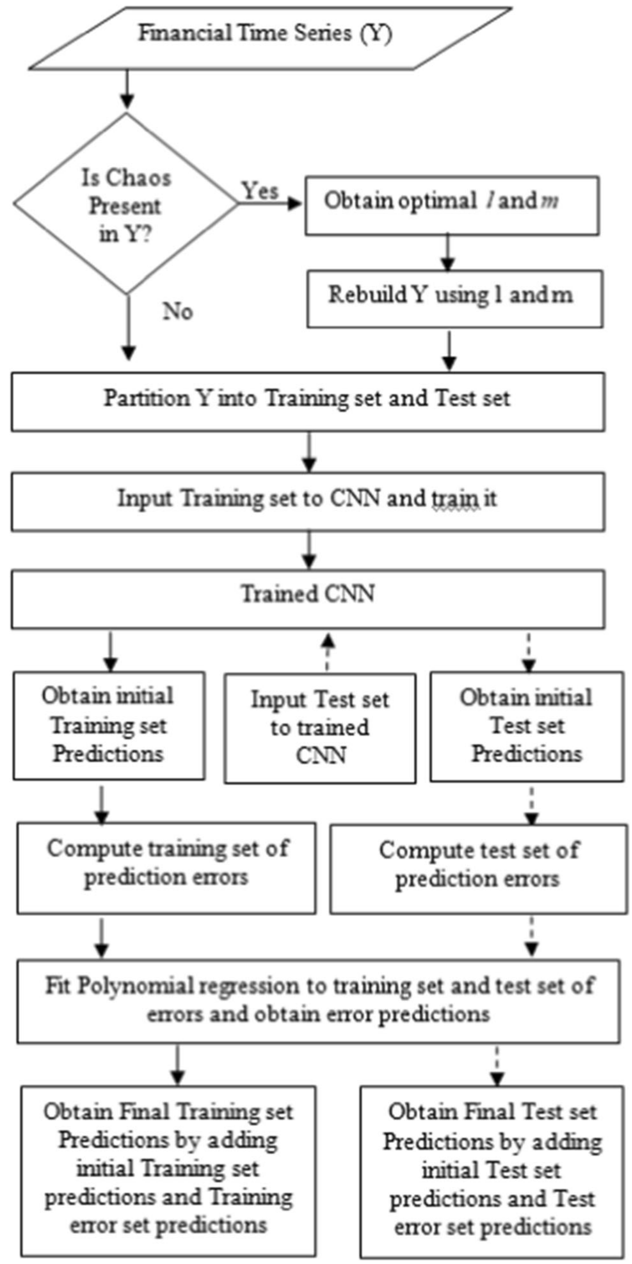

Fig. 1 Architecture of the proposed hybrid

Table 3 Datasets used

Data set	Dates	Count	Training set	Test set
Crude oil price (USD)	02-Jan-1990 to 29-Jan-2021	7890	6312	1578
Gold price (USD)	02-Jan-1990 to 29-Jan-2021	7907	6326	1581
Soybeans price (USD)	02-Jan-1990 to 31-Jan-2021	8063	6451	1612
Nifty 50 stock price	06-Nov-1995 to 29-Jan-2021	6281	5025	1256
Shanghai composite index	20-Dec-1990 to 29-Jan-2021	7362	5890	1472
S&P 500 stock index	02-Jan-1990 to 29-Jan-2021	7831	6265	1566
INR/USD	02-Jan-1990 to 29-Jan-2021	8093	6475	1618
JPY/USD	02-Jan-1990 to 29-Jan-2021	8101	6481	1620
SGD/USD	02-Jan-1990 to 29-Jan-2021	8101	6481	1620

Table 4 Descriptive statistics for all datasets

Data	Count	Min	Mean	Median	Max	SD	Skewness	Kurtosis
1.Crude oil price (USD)								
All data	7890	− 37.63	47.70264	41.405	145.29	28.83285	0.73005	− 0.49341
Training set	6312	10.72	46.91822	30.35	145.29	31.72228	0.75695	− 0.81044
Test set	1578	− 37.63	50.84032	50.915	76.41	10.91546	− 0.63922	2.93409
2.Gold price (USD)								
All data	7907	253	797.95469	465	2069.4	515.3496	0.53530	− 1.27063
Training set	6326	253	648.62458	388.1	1888.7	457.3733	1.17048	− 0.13432
Test set	1581	1070.8	1395.46405	1324.2	2069.4	212.11469	1.33137	0.90218
3.Soybeans price (USD)								
All data	8063	410	833.65716	775	1764.75	301.45914	0.764002	− 0.32651
Training set	6451	410	803.83398	658.25	1764.75	327.03820	0.977824	− 0.32905
Test set	1612	803.5	953.00537	936.75	1430	93.48183	1.718141	4.81299
4.Nifty 50 stock price								
All data	6281	788.15	4719.14243	4332.95	14730.95	3542.53831	0.62894	− 0.77501
Training set	5025	788.15	3330.31720	2598.05	8996.25	2335.60170	0.60453	− 0.87574
Test set	1256	6970.6	10275.54908	10495.65	14730.95	1530.64147	0.01037	− 0.27984
5.Shanghai composite index								
All data	7362	104.39	1994.61469	1924.3	6092.06	1075.78886	0.50195	0.08418
Training set	5890	104.39	1702.09134	1526.139	6092.06	989.68912	1.13437	2.17665
Test set	1472	2464.36	3165.10552	3114.73	5166.35	395.55923	1.82079	5.53021
6. S&P 500 stock index								
All data	7831	295.450012	1335.66791	1210.930054	3862.959961	757.17977	0.93029	0.42257
Training set	6265	295.450012	1023.83847	1110.469971	2032.359985	418.18596	− 0.09975	− 0.8005
Test set	1566	1833.40002	2583.18476	2577.915039	3862.959961	471.20240	0.49377	− 0.57165
7. INR/USD								
All data	8093	16.8	46.88723	45.5	76.975	14.00687	0.14052	− 0.49158
Training set	6475	16.8	41.56766	43.73	68.805	10.00020	− 0.42131	0.207695
Test set	1618	61.3580	68.17536	67.41149	76.975	3.83363	0.35588	− 0.88452
8. JPY/USD								
All data	8101	75.82	110.50040	109.98	159.88	15.10520	0.03674	0.43326
Training set	6481	75.82	110.26933	109.54	159.88	16.648432	0.06609	− 0.09937
Test set	1620	99.89	111.42484	110.62	125.62	5.5728100	0.54998	− 0.33050
9.SGD/USD								
All data	8101	1.2006	1.51456	1.4703	1.9085	0.18029	0.16609	− 1.246123
Training set	6481	1.2006	1.55079	1.5905	1.9085	0.18393	− 0.26470	− 1.123532
Test set	1620	1.2976	1.36961	1.3644	1.4598	0.03072	0.327661	− 0.215345

(75.82,109.98), and SGD/USD are in the range of (1.2006,1.9085).

The skewness measures asymmetry of data. The value Zero indicates the data is perfectly symmetric. The positive value indicates the tail of the distribution is more stretched on the side above mean. The negative value indicates that the tail of the distribution is more stretched on the side below the mean. The tails of the distribution of all commodity prices, stock prices and exchange rates are more stretched on the side above the mean.

The Kurtosis characterizes the relative peakedness or flatness of a distribution compared with the normal distribution. Positive kurtosis indicates a relatively peaked distribution and a negative kurtosis indicates a relatively flat distribution. The datasets of all commodity prices, Nifty 50 Stock Price, INR/USD and SGD/USD have relatively flat distribution. The stock prices such as Shanghai Composite Index and S&P 500 and JPY/USD have relatively peaked distribution.

4.2 Tasks performed and tools employed

Various tasks are carried out during the experimentation. Such tasks, as well as the tools used to conduct them, are presented in the Table 5. The Lyapunov Exponent (*lambda*) is used to check for chaos, the AIC is used to achieve optimal lag, and Cao's technique is used to provide optimal embedding dimension, as shown in Table 5. For additional information on the descriptions of the tasks aforementioned, readers are suggested to refer to [31].

While experimenting with the datasets, various parameters are obtained, and some parameters are utilized in common. Table 6 presents the optimal values for chaotic parameters obtained. $\lambda \geq 0$ denotes the presence of chaos. From the table, it is clear that all of the datasets have chaos. The optimal chaotic parameters such as lag (l) and embedding dimension (m) are also presented in Table 6. The estimateEmbeddingDim(.) method from "nonlinearTimeseries" package implemented Cao's method [5].

The optimal parameters for ARIMA (p, d, q) will be presented in respective sections. The optimal p, d, and q values of the ARIMA model are obtained using *auto_arima*(.) from "pmdarima" module of Python. The commonly used parameters for all datasets are as follows. The CNN architecture used here consists of one fully connected dense layer of 50 nodes. Each node is with the activation function of ReLU. For the CNN to be trained for 500 epochs, adam optimizer is used with MSE as a loss function. It also consists of a convolutional layer and a pooling layer. Scaled values using MinMaxScaler are input to CNN, Chaos+CNN, and Chaos+CNN+PR. While modeling errors using PR, second-degree polynomial regression is used.

Table 6 Chaotic parameters

Dataset	λ	l	m
Crude oil price	0.001618635	4	9
Gold price	0.000222457	10	8
Soyabeans price	0.003601366	10	8
Nifty 50	0.002267289	10	8
Shanghai composite	0.003585269	8	7
S&P 500	0.001685243	1	9
INR/USD	0.00099022	6	10
JPY/USD	0.003709193	1	8
SGD/USD	0.002180623	2	8

4.3 Performance measures used

The suggested hybrid's performance is measured using four performance measures: Mean Squared Error (MSE), Mean Absolute Percentage Error (MAPE), Directional Change Statistic (Dstat), and Theil's Inequality Coefficient (Theil's U).

By measuring the average of squared errors, the MSE (see Eq. 5) determines how well the model predicts the response [20]. The MAPE [20] calculates the absolute numbers of errors in percentage terms to determine how well the model predicts the response. An MSE/MAPE score near 0 suggests that the suggested model could produce predictions that are more accurate than the observed data.

Table 5 Tasks performed and tools employed

Task	Package/module	Function/measure/class	Tool used
Checking for the presence of chaos	nolds	lyap_r(.)	Python
Finding optimal lag	–	AIC	Gretl
Finding optimal embedding dimension	nonlinearTseries	estimateEmbeddingDim(.)	R
Importing data	pandas	read_csv(.)	Python
Partitioning data	scikit-learn	train_test_split(.)	Python
Fitting ARIMA to data	statsmodels	ARIMA(.).fit(), forecast(.)	Python
Fitting Prophet to data	fbprophet	Prophet(.).fit(), predict(.)	Python
Fitting CNN to data	keras	CNN(.), predict(.),	Python
Fitting PR to data	scikit-learn	PolynomialFeatures(.) LinearRegression(.).predict(.)	Python
Computing MSE	scikit-learn	mean_squared_error(.)	Python
Computing Dstat	–	–	Python
Computing Theil's U	–	–	Python
Checking for statistical significance	forecast	dm.test(.)	R

$$\mathrm{MSE} = \frac{\sum_{t=1}^{N}(y_t - \ddot{y}_t)^2}{N} \tag{5}$$

$$\mathrm{MAPE} = \frac{1}{N}\sum_{t=1}^{N}\left|\frac{(y_t - \ddot{y}_t)}{y_t}\right| \tag{6}$$

Yao and Tan [39] developed a measure (expressed in percentages) namely Dstat (see Eq. 7) to measure the directional change of time series. Higher the value of Dstat, better the movements of time series are captured by the model.

$$\begin{aligned} &\mathrm{Dstat} = \frac{1}{N}\sum_{i=1}^{N} a_t * 100\% \\ &\text{where} \quad a_t = \begin{cases} 1, & \text{if } (y_{t+1} - y_t) * (\ddot{y}_{t+1} - \ddot{y}) \geq 0 \\ 0, & \text{Otherwise} \end{cases} \end{aligned} \tag{7}$$

Theil's U indicates how near a projected time series is to the actual time series [20, 36]. The value of U (see Eq. (8)) is usually somewhere between 0 and 1. $U = 0$ indicates that $y_t = \ddot{y}_t$ for all observations and a perfect fit exists, whereas $U = 1$ indicates that the performance is poor. A Theil's U value that is closer to 0 suggests that the suggested model could produce more accurate predictions.

$$U = \frac{\sqrt{\frac{1}{N}\sum_{t=1}^{N}(y_t - \ddot{y}_t)^2}}{\sqrt{\frac{1}{N}\sum_{t=1}^{N}(y_t)^2} + \sqrt{\frac{1}{N}\sum_{t=1}^{N}(\ddot{y}_t)^2}} \tag{8}$$

In all of the related equations of these performance measures, y_t is the actual value at time t, $\ddot{y}_t$is the predicted value obtained using the proposed approach at time t and N is the number of predicted values.

5 Results and discussion

The results of each dataset are described as follows. It is important to note that, for each dataset, the proposed hybrid (Chaos+CNN+PR) is compared with ARIMA, Prophet, CNN, CART, RF, Chaos+CART [28], Chaos+RF [28] and Chaos+CNN in terms of MSE, MAPE, Dstat, and Theil's U.

5.1 INR/USD

The INR/USD test set results of prediction approaches are presented in Table 7. The table reveals that the proposed hybrid, Chaos+CNN+PR, outperformed all other approaches in terms of MSE, MAPE, Dstat and Theil's U. The performance measures MSE, MAPE and Theil's U are very much closer to 0 indicate that predictions are very much closer to actual values. Dstat value 100 indicates that directional change is fully captured by the proposed hybrid.

Among the standard prediction approaches (ARIMA (3,1,2), Prophet, CNN, CART, RF), CNN could yield better predictions in terms of MSE, MAPE and Theil's U. However, it could not capture the direction change better. In this context, CART could perform better.

Similarly, among Chaos-based hybrids (Chaos+CART, Chaos+RF, Chaos+CNN), the novel hybrid, Chaos+CNN, could yield better predictions in terms of MSE, MAPE and Theil's U. However, it could not capture the direction change better than dominative Chaos+CART.

Figure 2 depicts predictions of the test set of INR/USD. The predictions are obtained from CNN, Chaos+CNN, and Chaos+CNN+PR. From the figure, it can be observed that the predictions obtained using Chaos+CNN+PR are very much closer to actual values. It is also worth noting that the predictions obtained using CNN are better than that of Chaos+CNN.

5.2 JPY/USD

Table 8 shows the JPY/USD test set results of prediction techniques. The suggested hybrid, Chaos+CNN+PR, beat all previous techniques in terms of MSE, MAPE, Dstat, and Theil's U, as shown in the table. The MSE, MAPE, and Theil's U performance metrics are all extremely close to 0, indicating that predictions are very close to actual values. Dstat value 100 shows that the suggested hybrid completely captures directional change.

Among the conventional prediction techniques (ARIMA (0,1,0), Prophet, CNN, CART, RF), CNN followed by RF could produce superior forecasts in terms of MSE, MAPE, and Theil's U. However, it fell short of capturing the change in direction. CART may function better in this situation.

Similarly, among Chaos-based hybrids (Chaos+CART, Chaos+RF, Chaos+CNN), the proposed hybrid, Chaos+CNN, could provide superior forecasts in terms of MSE, MAPE, and Theil's U. It could not, however, record the direction change better than Chaos+CART.

The predictions of the test set of JPY/USD are shown in Fig. 3. CNN, Chaos+CNN, and Chaos+CNN+PR are used to obtain the predictions and they are shown in the figure. The predictions achieved using Chaos+CNN+PR are significantly closer to real values as seen in the figure. It's also worth mentioning that the forecasts made with CNN are superior to those made using Chaos+CNN.

Table 7 Test set results of INR/USD

Forecasting model	MSE	MAPE	DStat	Theil U
ARIMA	4.682253	2.644728	50.587507	0.000507
Prophet	19.351410	4.903262	52.690166	0.001971
CNN	0.059231	0.260106	49.969078	6.353172e − 06
CART	18.622667	4.976640	91.774891	0.002088
RF	18.081244	4.503860	74.582560	0.002023
Chaos+CART	20.243653	5.208711	90.661719	0.002264
Chaos+RF	18.119208	4.499512	73.902288	0.002028
Chaos+CNN	0.625058	0.893032	50.834879	6.718151e − 05
Chaos+CNN+PR	1.109410e − 08	0.000154	100.0	1.189694e − 12

Fig. 2 Predictions of proposed hybrid for test set of INR/USD

5.3 SGD/USD

The SGD/USD test set results of prediction approaches are presented in Table 9. The table reveals that the proposed hybrid, Chaos+CNN+PR, outperformed all other approaches in terms of MSE, MAPE, Dstat and Theil's U. The performance measures MSE, MAPE and Theil's U are very much closer to 0 indicate that predictions are very much closer to actual values. Dstat value 100 indicates that directional change is fully captured by the proposed hybrid.

Among the standard prediction approaches (ARIMA (1,0,0), Prophet, CNN, CART, RF), CNN followed by RF could yield better predictions in terms of MSE, MAPE and Theil's U. However, it could not capture the direction change better. In this context, CART could perform better.

Similarly, among Chaos-based hybrids (Chaos+CART, Chaos+RF, Chaos+CNN), the novel hybrid, Chaos+CNN, could yield better predictions in terms of MSE, MAPE and

Table 8 Test set results of JPY/USD

Forecasting model	MSE	MAPE	DStat	Theil U
ARIMA	20.522047	3.029495	50.833848	0.000817
Prophet	123.833278	8.199826	49.289684	0.005242
CNN	0.359494	0.374147	48.054354	1.443902e − 05
CART	0.903238	0.635285	51.760345	3.629257e − 05
RF	0.464442	0.441553	49.845583	1.865966e − 05
Chaos+CART	0.892972	0.633950	50.895614	3.587788e − 05
Chaos+RF	0.464889	0.445031	48.857319	1.867801e − 05
Chaos+CNN	0.361535	0.375804	48.424953	1.451573e − 05
Chaos+CNN+PR	1.312878e − 08	0.000103	100.0	5.274069e − 13

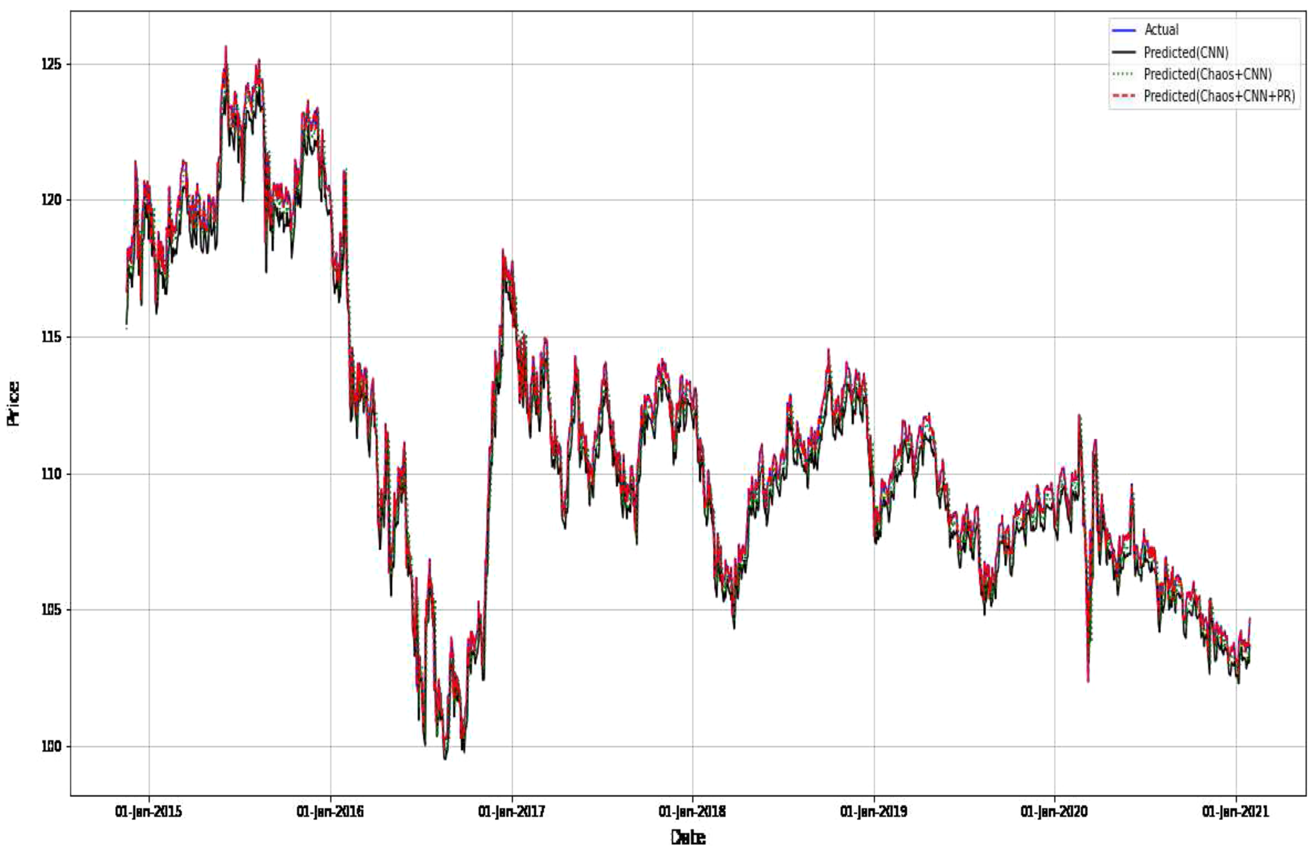

Fig. 3 Predictions of proposed hybrid for test set of JPY/USD

Theil's U. However, it could not capture the direction change better than Chaos+CART.

Figure 4 depicts predictions of the test set of SGD/USD. The predictions are obtained from CNN, Chaos+CNN, and Chaos+CNN+PR. From the figure, it can be observed that the predictions obtained using Chaos+CNN+PR are very much closer to actual values. It is also worth noting that the predictions obtained using CNN are better than that of Chaos+CNN.

5.4 S&P 500 stock index

The S&P 500 Stock Index test set results of prediction approaches are presented in Table 10. The table reveals that the proposed hybrid, Chaos+CNN+PR, outperformed all other approaches in terms of MSE, MAPE, Dstat and Theil's U. The performance measures MSE, MAPE and Theil's U are very much closer to 0 indicate that predictions are very much closer to actual values. Dstat value 100

Table 9 Test set results of SGD/USD

Forecasting model	MSE	MAPE	DStat	Theil U
ARIMA	0.024654	12.260339	51.822112	0.007313
Prophet	0.049610	19.039075	51.142680	0.015482
CNN	2.378308e − 05	0.270992	50.833848	6.345399e − 06
CART	4.499225e − 05	0.384234	55.095738	1.198772e − 05
RF	2.754348e − 05	0.291158	48.857319	7.339461e − 06
Chaos+CART	4.544950e − 05	0.381212	56.948733	1.210897e − 05
Chaos+RF	2.695724e − 05	0.287179	49.351451	7.183174e − 06
Chaos+CNN	3.842139e − 05	0.339434	50.216182	1.023644e − 05
Chaos+CNN+PR	3.175296e − 12	0.000130	100.0	8.459333e − 13

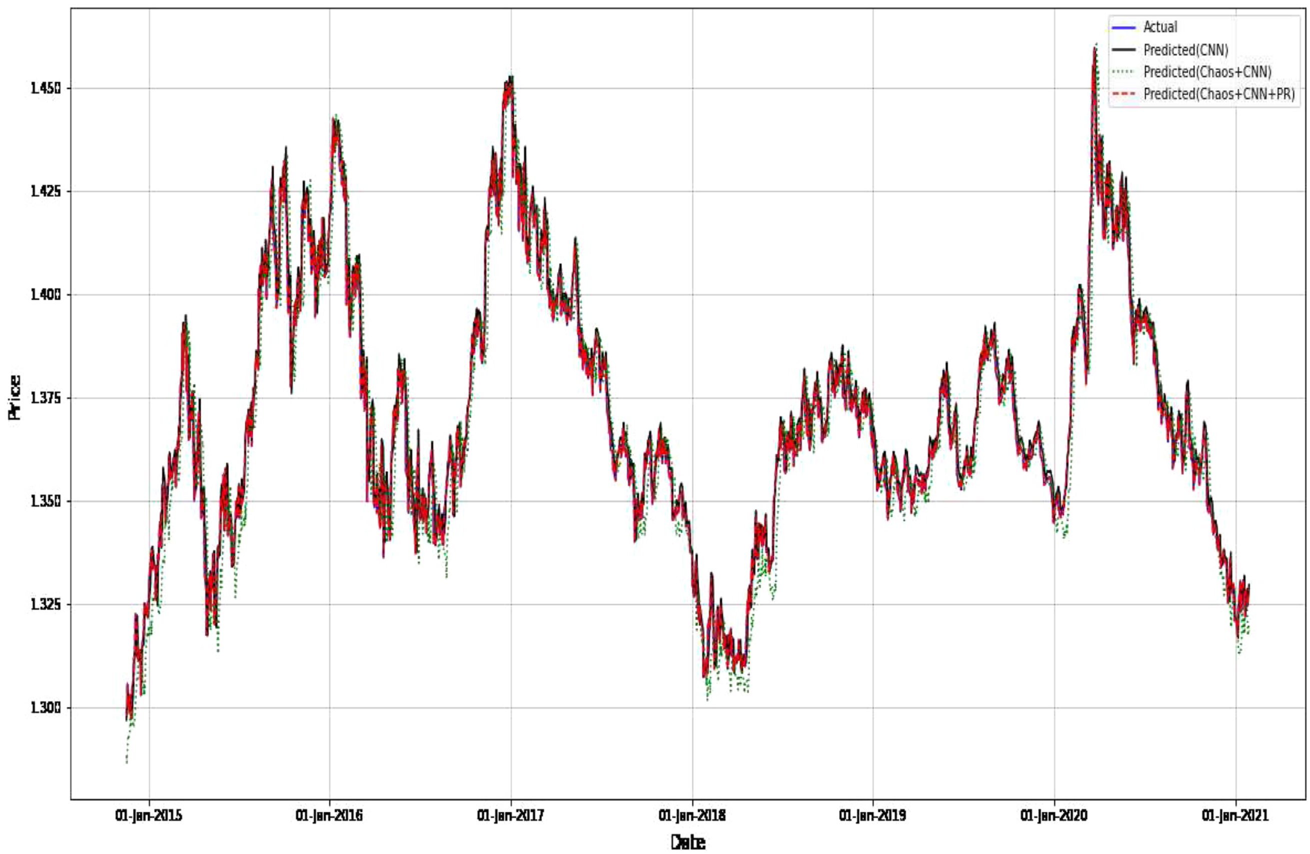

Fig. 4 Predictions of proposed hybrid for test set of SGD/USD

indicates that directional change is fully captured by the proposed hybrid.

Among the standard prediction approaches (ARIMA (1,1,1), Prophet, CNN, CART, RF), CNN could yield better predictions in terms of MSE, MAPE and Theil's U. However, it could not capture the direction change better. In this context, CART could perform better.

Similarly, among Chaos-based hybrids (Chaos+CART, Chaos+RF, Chaos+CNN), the novel hybrid, Chaos+CNN, could yield better predictions in terms of MSE, MAPE and Theil's U. However, it could not capture the direction change better than Chaos+CART.

Figure 5 depicts predictions of the test set of S&P 500 Stock Index. The predictions are obtained from CNN, Chaos+CNN, and Chaos+CNN+PR. From the figure, it can be observed that the predictions obtained using Chaos+CNN+PR are very much closer to actual values. It is also worth noting that the predictions obtained using Chaos+CNN are better than that of CNN.

Table 10 Test set results of S&P 500 stock index

Forecasting model	MSE	MAPE	DStat	Theil U
ARIMA	246490.773162	15.918238	57.444089	0.020656
Prophet	145978.130974	11.909248	54.376996	0.011875
CNN	841.164380	0.803366	51.437699	6.114755e − 05
CART	524902.738023	27.451097	96.549520	0.047702
RF	531831.296643	27.811611	95.846645	0.048434
Chaos+CART	524906.627397	27.453308	96.613418	0.047702
Chaos+RF	533627.715480	27.905680	95.846645	0.048624
Chaos+CNN	708.470136	0.690652	51.309904	5.137796e − 05
Chaos+CNN+PR	0.015496	0.004973	100.0	1.123851e − 09

Fig. 5 Predictions of proposed hybrid for test set of S&P500

5.5 Nifty 50 stock index

The Nifty 50 Stock Index test set results of prediction approaches are presented in Table 11. The table reveals that the proposed hybrid, Chaos+CNN+PR, outperformed all other approaches in terms of MSE, MAPE, Dstat and Theil's U. The MSE value of proposed hybrid is better than the remaining approaches. The MAPE and Theil's U are very much closer to 0 indicate that predictions are very much closer to actual values. Dstat value 100 indicates that directional change is fully captured by the proposed hybrid.

Among the standard prediction approaches (ARIMA (0,1,1), Prophet, CNN, CART, RF), CNN followed by ARIMA could yield better predictions in terms of MSE, MAPE and Theil's U. However, it could not capture the direction change better. In this context, CART could perform better.

Similarly, among Chaos-based hybrids (Chaos+CART, Chaos+RF, Chaos+CNN), the novel hybrid, Chaos+CNN,

Table 11 Test set results of Nifty 50 stock index

Forecasting model	MSE	MAPE	DStat	Theil U
ARIMA	3400655.703089	17.260416	55.139442	0.018258
Prophet	849534.677585	6.739681	52.270916	0.003937
CNN	16129.754511	0.850068	50.836653	7.499417e − 05
CART	4511147.427545	19.200982	87.808764	0.024733
RF	4188601.943174	18.087281	87.808764	0.022805
Chaos+CART	4510898.369836	19.194321	87.569721	0.024733
Chaos+RF	4232360.770306	18.233875	87.888446	0.023067
Chaos+CNN	1560419.525616	10.837223	50.677290	0.008003
Chaos+CNN+PR	2.639324	0.016042	100.0	1.222894e − 08

could yield better predictions in terms of MSE, MAPE and Theil's U. However, it could not capture the direction change better than Chaos+CART.

Figure 6 depicts predictions of the test set of Nifty 50 Stock Index. The predictions are obtained from CNN, Chaos+CNN, and Chaos+CNN+PR. From the figure, it can be observed that the predictions obtained using Chaos+CNN+PR are very much closer to actual values. It is also worth noting that the predictions obtained using CNN are better than that of Chaos+CNN.

5.6 Shanghai composite index

The Shanghai Composite Index test set results of prediction approaches are presented in Table 12. The table reveals that the proposed hybrid, Chaos+CNN+PR, outperformed all other approaches in terms of MSE, MAPE, Dstat and Theil's U. The MSE value of Chaos+CNN+PR is very much better than the remaining approaches. And also, the MAPE and Theil's U values are very much closer to 0 indicate that predictions are very much closer to actual

Fig. 6 Predictions of proposed hybrid for test set of Nifty 50 stock index

Table 12 Test set results of Shanghai composite index

Forecasting model	MSE	MAPE	DStat	Theil *U*
ARIMA	668777.360368	19.187506	53.840924	0.027243
Prophet	2265404.326675	87.251734	51.597552	0.172626
CNN	2956.134975	1.181762	48.130523	0.000146
CART	8076.550985	1.950799	53.840924	0.000397
RF	3623.417136	1.256007	49.490142	0.000178
Chaos+CART	7433.170556	1.879328	55.268524	0.000365
Chaos+RF	3598.312050	1.246783	49.558123	0.000176
Chaos+CNN	35844.767975	3.705842	49.694085	0.001777
Chaos+CNN+PR	0.047778	0.006898	100.0	2.348129e − 09

values. Dstat value 100 indicates that directional change is fully captured by the proposed hybrid.

Among the standard prediction approaches (ARIMA (3,1,3), Prophet, CNN, CART, RF), CNN followed by RF could yield better predictions in terms of MSE, MAPE and Theil's *U*. However, it could not capture the direction change better. In this context, CART and ARIMA could perform better.

Similarly, among Chaos-based hybrids (Chaos+CART, Chaos+RF, Chaos+CNN), the hybrid, Chaos+RF, could yield better predictions in terms of MSE, MAPE and Theil's *U*. However, it could not capture the direction change better than Chaos+CART.

Figure 7 depicts predictions of the test set of Shanghai Composite Index. The predictions are obtained from CNN, Chaos+CNN, and Chaos+CNN+PR. From the figure, it can be observed that the predictions obtained using Chaos+CNN+PR are very much closer to actual values. It is also worth noting that the predictions obtained using CNN are better than that of Chaos+CNN.

5.7 Crude oil price

The Crude Oil Price test set results of prediction approaches are presented in Table 13. The table reveals that the proposed hybrid, Chaos+CNN+PR, outperformed all other approaches in terms of MSE, MAPE, Dstat and Theil's *U*. The performance measures MSE, MAPE and Theil's *U* are very much closer to 0 indicate that predictions are very much closer to actual values. Dstat value 100 indicates that directional change is fully captured by the proposed hybrid.

Among the standard prediction approaches (ARIMA (2,1,0), Prophet, CNN, CART, RF), RF followed by CNN could yield better predictions in terms of MSE, MAPE and Theil's *U*. However, it could not capture the direction change better. In this context, ARIMA could perform better.

Similarly, among Chaos-based hybrids (Chaos+CART, Chaos+RF, Chaos+CNN), the hybrid, Chaos+RF, could yield better predictions in terms of MSE, MAPE and Theil's *U*. However, it could not capture the direction change better than Chaos+CNN.

Figure 8 depicts predictions of the test set of Crude Oil Price. The predictions are obtained from CNN, Chaos+CNN, and Chaos+CNN+PR. From the figure, it can be observed that the predictions obtained using Chaos+CNN+PR are very much closer to actual values. It is also worth noting that the predictions obtained using CNN are better than that of Chaos+CNN.

5.8 Gold price

The Gold Price test set results of prediction approaches are presented in Table 14. The table reveals that the proposed hybrid, Chaos+CNN+PR, outperformed all other approaches in terms of MSE, MAPE, Dstat and Theil's *U*. The performance measures MSE, MAPE and Theil's *U* are very much closer to 0 indicate that predictions are very much closer to actual values. Dstat value 100 indicates that directional change is fully captured by the proposed hybrid.

Among the standard prediction approaches (ARIMA (2,1,1), Prophet, CNN, CART, RF), CNN followed by RF could yield better predictions in terms of MSE, MAPE and Theil's *U*. However, it could not capture the direction change better. In this context, CART could perform better.

Similarly, among Chaos-based hybrids (Chaos+CART, Chaos+RF, Chaos+CNN), the hybrid, Chaos+RF, could yield better predictions in terms of MSE, MAPE and Theil's *U*. However, it could not capture the direction change better than Chaos+CART.

Figure 9 depicts predictions of the test set of Gold Price. The predictions are obtained from CNN, Chaos+CNN, and

Fig. 7 Predictions of proposed hybrid for test set of Shanghai composite index

Table 13 Test set results of crude oil price

Forecasting model	MSE	MAPE	DStat	Theil U
ARIMA	122.1701375	15.62913992	52.75840203	0.022626621
Prophet	2079.515013	43.66922359	50.15852885	0.181741948
CNN	3.917877221	2.501259507	49.58782498	0.00072788
CART	5.981146247	3.441771962	51.68040583	0.001107432
RF	3.906917266	2.439084782	48.82688649	0.000722327
Chaos+CART	5.859357558	3.400616935	51.61699429	0.001084689
Chaos+RF	3.89888222	2.432118422	49.39759036	0.000720849
Chaos+CNN	10.43330561	6.553971389	52.69499049	0.00194532
Chaos+CNN+PR	1.55E − 07	0.000819526	100	2.87E − 11

Chaos+CNN+PR. From the figure, it can be observed that the predictions obtained using Chaos+CNN+PR are very much closer to actual values. It is also worth noting that the predictions obtained using CNN are better than that of Chaos+CNN.

5.9 Soya beans price (USD)

The Gold Price test set results of prediction approaches are presented in Table 15. The table reveals that the proposed hybrid, Chaos+CNN+PR, outperformed all other approaches in terms of MSE, MAPE, Dstat and Theil's U. The performance measures MSE, MAPE and Theil's U are very much closer to 0 indicate that predictions are very much closer to actual values. Dstat value 100 indicates that directional change is fully captured by the proposed hybrid.

Among the standard prediction approaches (ARIMA (0,1,0), Prophet, CNN, CART, RF), CNN followed by RF could yield better predictions in terms of MSE, MAPE and Theil's U. However, it could not capture the

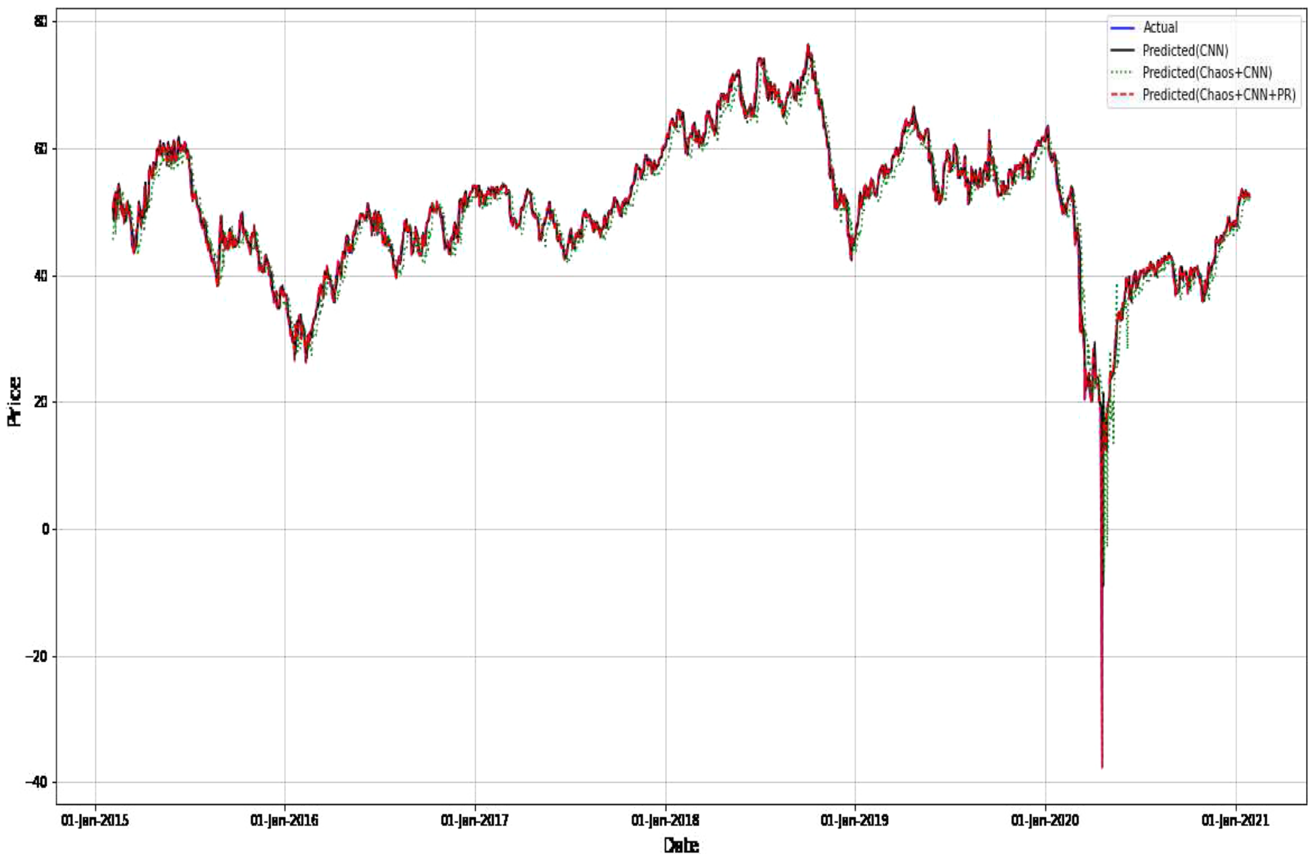

Fig. 8 Predictions of proposed hybrid for test set of crude oil price

Table 14 Test set results of gold price

Forecasting model	MSE	MAPE	DStat	Theil U
ARIMA	27802.054680	9.048922	51.708860	0.006927
Prophet	67822.883460	15.076782	50.569620	0.015280
CNN	347.445571	0.966208	46.582278	8.670003e − 05
CART	2650.180641	1.867481	58.037974	0.000671
RF	1389.370580	1.279991	51.455696	0.000351
Chaos+CART	2660.907862	1.888391	57.341772	0.000674
Chaos+RF	1530.644763	1.329603	50.506329	0.000387
Chaos+CNN	4267.047461	2.839465	51.202531	0.001080
Chaos+CNN+PR	0.003165	0.004098	100.0	7.944620e − 10

direction change better. In this context, CART could perform better.

Similarly, among Chaos-based hybrids (Chaos+−CART, Chaos+RF, Chaos+CNN), the hybrid, Chaos+RF, could yield better predictions in terms of MSE, MAPE and Theil's U. However, it could not capture the direction change better than Chaos+CART.

Figure 10 depicts predictions of the test set of Gold Price. The predictions are obtained from CNN, Chaos+CNN, and Chaos+CNN+PR. From the figure, it can be observed that the predictions obtained using Chaos+CNN+PR are very much closer to actual values. It is also worth noting that the predictions obtained using CNN are better than that of Chaos+CNN.

Finally, the Diebold and Mariano test [8] is used to officially test the statistical difference between Chaos+CNN+PR and other forecast models on average. The test of statistical signficance accepts the predictions obtained from two approaches as inputs. Table 16 shows

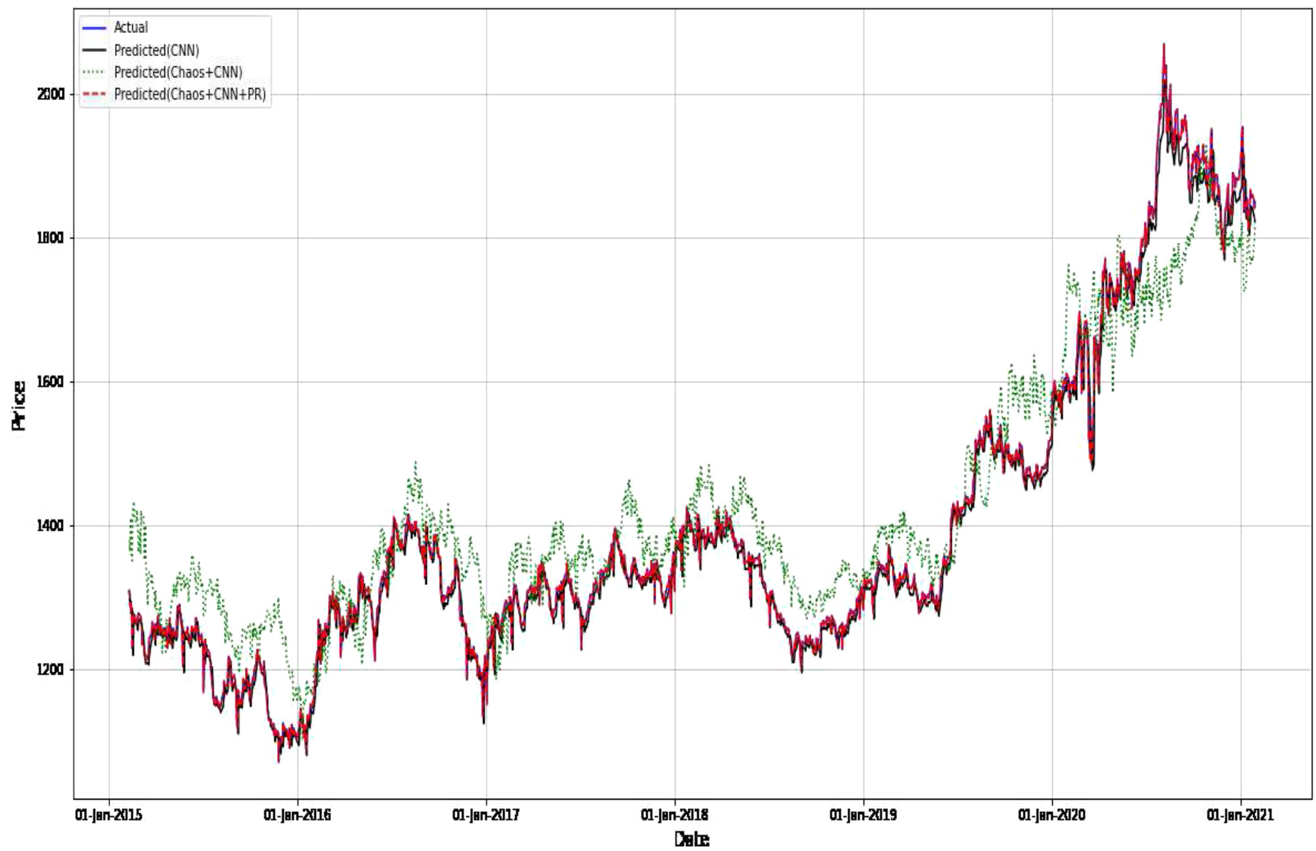

Fig. 9 Predictions of proposed hybrid for test set of gold price

Table 15 Test set results of soya beans price

Forecasting model	MSE	MAPE	DStat	Theil U
ARIMA	36277.433250	15.161464	50.775915	0.016791
Prophet	317374.435389	36.564604	50.900062	0.099628
CNN	124.167554	0.817279	51.024208	6.779597e − 05
CART	492.448338	1.732297	55.307262	0.000268
RF	186.908517	1.072228	51.707014	0.000101
Chaos+CART	483.229258	1.730958	55.493482	0.000263
Chaos+RF	186.237605	1.065448	52.203600	0.000101
Chaos+CNN	1627.197407	3.230644	51.893234	0.000890
Chaos+CNN+PR	1.336608e − 05	0.000306	100.0	7.288284e − 12

the absolute values of the Diebold-Mariano test statistic for each of the nine datasets. If the absolute value of the test statistic is less than or equal to 1.96, the corresponding model is equivalent to Chaos+CNN+PR. The table clearly shows that Chaos+CNN+PR outperforms every model for every dataset as all of the absolute values of test statistic are greater than 1.96.

6 Conclusion

A novel hybrid model, Chaos+CNN+PR, is presented in this paper to resolve to predict financial time series. The financial time series in this Hybrid is first checked for chaos. Later on, Chaos Theory can model chaos in the time series. Input to CNN is used to create initial predictions for the model time series. The CNN predictions error series is input to PR to get error predictions. The error predictions and initial CNN predictions are added to produce final

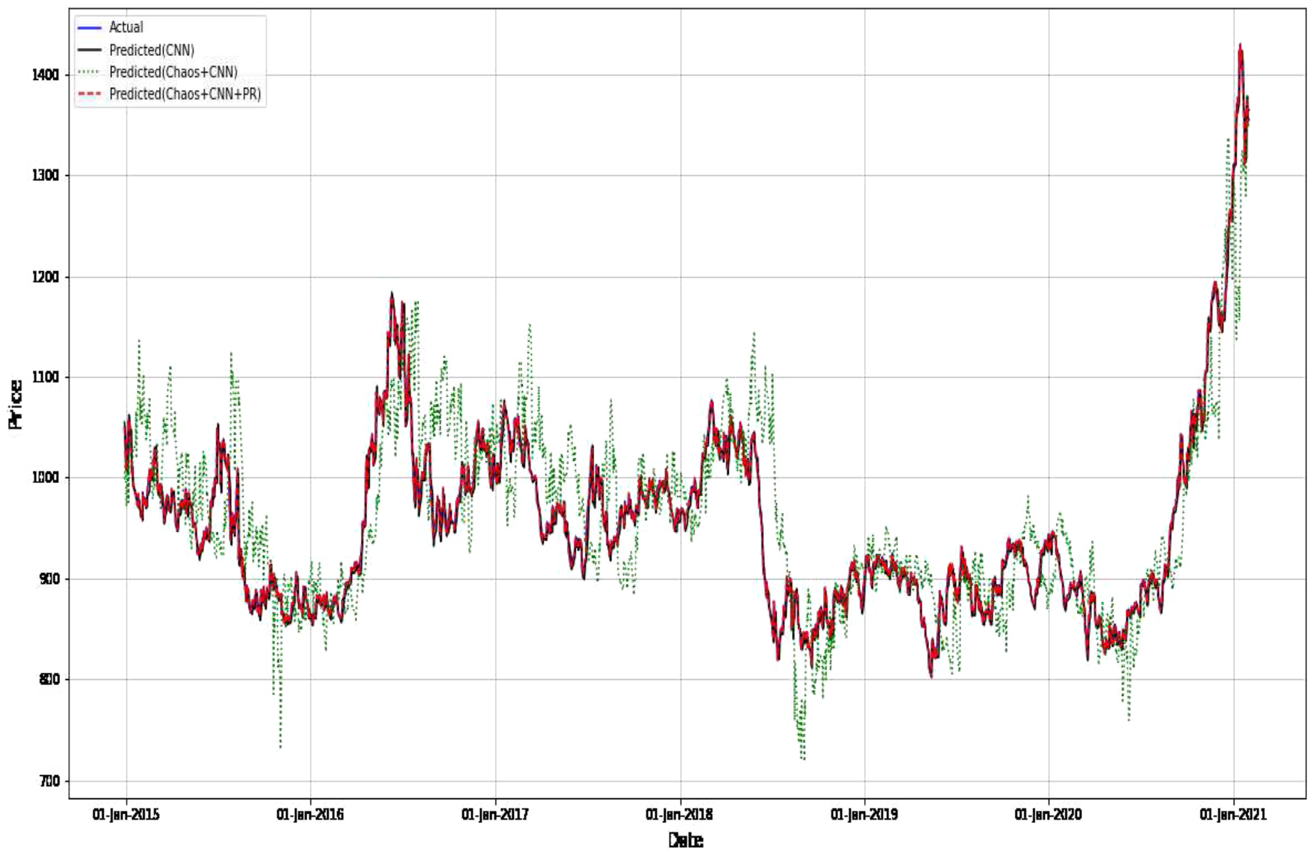

Fig. 10 Predictions of proposed hybrid for test set of soya beans price

Table 16 Diebold-Mariano test results of all datasets

DataSet	Chaos+CNN+PR Vs							
	ARIMA	Prophet	CNN	CART	RF	Chaos+CART	Chaos+RF	Chaos+CNN
Crude oil price	16.608861	59.504484	1.990288	2.906652	1.963100	2.932861	1.972510	4.379796
Gold price	25.667276	53.150327	18.387257	11.848790	10.268100	11.800267	10.247176	25.126406
US soybeans price	49.529394	104.816448	16.969270	18.889736	20.470820	19.370143	20.536260	23.143132
Nifty 50	31.629498	19.525119	23.690134	28.111042	27.348897	28.109362	27.376126	27.900677
Shanghai composite	46.510314	69.048449	10.681938	15.978978	13.188594	16.276035	12.951655	8.534146
S&P500	26.878357	25.526355	33.057314	30.817214	31.053992	30.817189	30.983468	30.201127
INR/USD	35.152535	41.220187	34.186538	27.481151	26.506402	27.166384	26.399686	37.007372
JPY/USD	24.003617	27.278746	28.290340	18.441021	15.672886	18.530120	15.560446	18.578038
SGD/USD	64.889492	101.946745	20.722893	23.810335	22.002273	24.728638	21.799964	23.933521

predictions. Three kinds of financial time series, such as foreign exchange, commodity, and stock market indices, are used to test the proposed Hybrid's effectiveness. The proposed hybrid, in terms of MSE, MAPE, Dstat, and Theil's U outperformed ARIMA, Prophet, CNN, CART, RF, Chaos+CNN, Chaos+CART, and Chaos+RF. It is also possible to extend the proposed Hybrid to various financial and non-financial time series. The regression problem solved here can also be converted into a classification problem. In this context, the approaches proposed by [40–42] are very much helpful.

Declarations

Conflict of interest The authors declare that they have no conflict of interest with any author, or organization.

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