

# Resonant Tunneling in Photonic Double Quantum Well Heterostructures

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**Abstract** Here, we study the resonant photonic states of photonic double quantum well (PDQW) heterostructures composed of two different photonic crystals. The heterostructure is denoted as B/A/B/A/B, where photonic crystals A and B act as photonic wells and barriers, respectively. The resulting band structure causes photons to become confined within the wells, where they occupy discrete quantized states. We have obtained an expression for the transmission coefficient of the PDQW heterostructure using the transfer matrix method and have found that resonant states exist within the photonic wells. These resonant states occur in split pairs, due to a coupling between degenerate states shared by each of the photonic wells. It is observed that when the resonance energy lies at a bound photonic state and the two photonic quantum wells are far away from each other, resonant states appear in the transmission spectrum of the PDQW as single peaks. However, when the wells are brought closer together, coupling between bound photonic states causes an energy-splitting effect, and the transmitted states each have two peaks. Essentially, this means that the system can be switched between single and double transparent states. We have also observed that the total number of resonant states can be controlled by varying the width of the photonic wells, and the quality factor of transmitted peaks can be drastically improved by increasing the thickness of the outer photonic barriers. It is anticipated that the resonant states described here can be used to develop new types of photonic-switching devices, optical filters, and other optoelectronic devices.

**Keywords** Photonic crystal heterostructures · Resonant tunneling · Photonic devices · Photonic quantum well · Nanophotonics

## Introduction

During the past two decades, photonic crystals [1, 2] have attracted a great deal of attention due to their ability to localize and control the flow of light within their structure. Considerable effort has been placed into finding ways to harness their potential for developing new photonic devices. Toward that end, photonic crystal heterostructures have provided a promising means for turning raw photonic crystals into functional devices [3, 4].

Photonic crystals are materials with a periodically modulating dielectric constant, which may vary in one, two, or three spatial dimensions. Due to this periodic dielectric modulation, a band gap forms in the structure's photonic dispersion relation. Light possessing a frequency within this gap is prevented from propagating in the photonic crystal. Photonic crystal heterostructures, as their name implies, are formed by joining two or more photonic crystals into a single structure. Combining various photonic crystals with different band gaps into a single heterostructure gives it a more complicated band structure than that of a bulk photonic crystal, which allows for greater customization and control over how light propagates within the overall structure. Recently, photonic crystal heterostructures have been used to develop devices such as high-quality resonant cavities [4–7], low-loss waveguides [8, 9], and high efficiency add-drop filters [10].

Photonic quantum wells (PQWs) are a class of photonic crystal heterostructures that possess a distinct band

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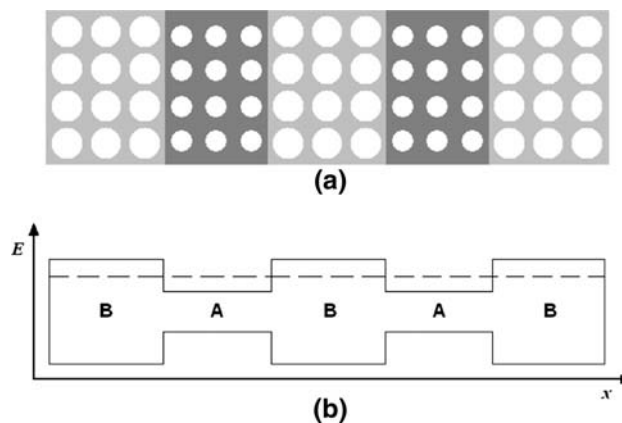
structure. A PQW consists of a photonic well imbedded between two photonic barriers. The barriers are photonic crystals with band gaps that may be regarded as potential barriers for photons, whereas the photonic well consists of either a uniform dielectric material or another photonic crystal with a different band gap than that of the barrier. Due to the photonic band mismatch between the well and the barrier, photons become confined within the well and occupy quantized states. This so-called photonic confinement effect has been observed in both theoretical [11–15] and experimental [16, 17] studies and is analogous to the electronic confinement effect that occurs in semiconductor quantum wells.

Furthermore, it has been shown that the phenomenon of resonant photonic tunneling can occur for a PQW with sufficiently thin photonic barriers [11, 12]. Resonant tunneling occurs when a photon with an energy corresponding to a bound state of the PQW tunnels through one of the barriers, where it occupies this bound state within the well for a finite period of time before escaping by tunneling back out; thus the photon is said to have occupied a *quasi-bound* or *resonant* state within the PQW [18]. As a consequence of this phenomenon, an incident photon with an energy matching a resonant state of the PQW will undergo perfect transmission through both barriers. In the transmission spectrum of a PQW, resonant states appear as sharp peaks approaching unity [11–18].

Here, we study resonant photonic states in photonic double quantum well (PDQW) heterostructures consisting of two different photonic crystals. A PDQW is formed by simply adding another photonic well and photonic barrier to a single PQW system. Using the transfer matrix method [19], we have obtained an expression for the transmission coefficient of the PDQW heterostructure. From this, we have performed numerical simulations of the transmission spectra for a number of PDQW heterostructures. In our simulations, we vary the thicknesses of the photonic wells or barriers in order to study their effect on the resonant states of the system. It is anticipated that the resonant states described here will be useful for developing new types of photonic-switching devices, optical filters, and other optoelectronic devices [3, 11–17].

### Theory

We consider two photonic crystals A and B, where A denotes the photonic well and B denotes the photonic barrier. The PDQW heterostructure is then formed by arranging these crystals in the sequence B/A/B/A/B, as illustrated in Fig. 1. Here, it is considered that both crystals are composed of air spheres embedded in a dielectric background material. Lattice constants of crystals A and B



**Fig. 1** Schematic representation of the photonic double quantum well heterostructure. **a** Cross-sectional view of the photonic crystal arrangement in the heterostructure. The *circles* represent the air spheres imbedded in each crystal, which may have different sizes as indicated. The backbone material of each crystal is shaded differently to represent a possible variation in the dielectric constant of that material. **b** Energy band structure of the double quantum well. The *dashed line* represents the energy of a possible resonant state

are denoted by  $L_A$  and  $L_B$ , respectively, while the radii of the air spheres in each crystal is given by  $a_A$  and  $a_B$ . The distances between the spheres in each crystal are given as  $b_A$  and  $b_B$ , and the refractive index of the background material of each crystal is denoted as  $n_A$  and  $n_B$ . Dispersion relations for photonic crystals A and B are obtained using the isotropic band structure model proposed by John and Wang [20], where it is assumed that each crystal possesses a three-dimensional band gap that is equal for all propagation directions. This assumption provides a simplified model which leads to qualitatively correct physics and has been widely used in photonic crystal literature [20–23]. According to this model, the dispersion relations for crystals A and B are given by

$$\cos(K_i L_i) = \frac{(n_i + 1)^2}{4n_i} \cos\left[\frac{\varepsilon_k(2n_i a_i + b_i)}{\hbar c}\right] - \frac{(n_i - 1)^2}{4n_i} \cos\left[\frac{\varepsilon_k(2n_i a_i - b_i)}{\hbar c}\right] \quad (1)$$

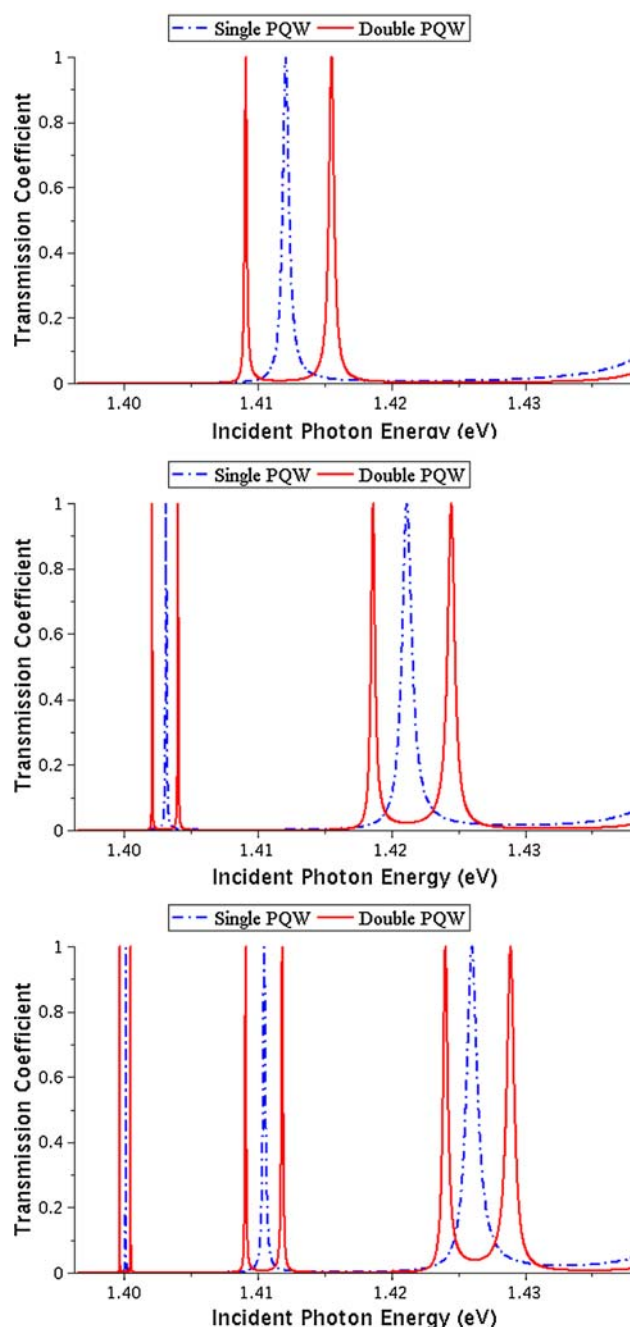
where  $i = A$  or  $B$ ,  $K_i$  denotes the Bloch wave vector in either crystal and  $\varepsilon_k$  is the incident photon energy. We consider transverse electric field waves propagating along the direction of crystal growth ( $x$ -direction) through photonic crystals A and B, and from Eq. 1, we find that band gaps appear in the photonic dispersion relations of crystals A and B. Numerical values for the parameters of photonic crystals A and B are chosen, so that the upper band edge of photonic crystal A lies within the band gap of crystal B, such that two photonic wells are formed within the heterostructure, separated by the central layer of photonic crystal B (see Fig. 1).

At this point, we denote our PDQW heterostructure as  $B_1/A_2/B_3/A_4/B_5$  in order to clarify discussions. The width of the photonic wells (photonic crystals  $A_2$  and  $A_4$ ) is given as  $d_A$ , while the width of the outer photonic barriers (crystals  $B_1$  and  $B_2$ ) is given by  $d_B$ . The width of the central photonic barrier (photonic crystal  $B_3$ , or equivalently the photonic well separation) is denoted as  $d_W$ . The transmission coefficient of the total PDQW heterostructure is calculated using the transfer matrix method, which matches the electric field and its first derivative at interfaces between adjacent layers of photonic crystals in order to relate the incident and reflected amplitudes in each layer [19]. Using the dispersion relation given in Eq. 1, the transmission coefficient is obtained as a function of incident photon energy and was plotted over the range of energies between the upper band edge of photonic crystal A to that of crystal B in order to determine the resonant states of the system.

## Results

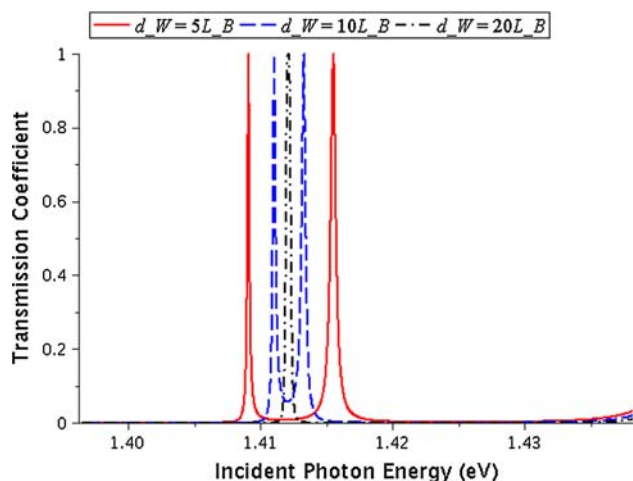
It is considered that photonic crystals A and B each consist of a silica background material (such that  $n_A = n_B = 1.45$ ) imbedded with air spheres. The radii of the air spheres in each crystal were taken as  $a_A = a_B = 125$  nm, and the lattice constants for each crystal were taken as  $L_A = 420$  nm and  $L_B = 410$  nm. The upper photonic band edges of crystals A and B were calculated using Eq. 1 as 1.3965 and 1.4382 eV, respectively. We have performed numerical simulations of the transmission coefficient of the PDQW heterostructure considered here. Transmission spectra were produced while varying parameters such as the width of the photonic wells ( $d_A$ ), the width of the central photonic barrier, i.e. the photonic well separation ( $d_W$ ), and the width of the outer photonic barriers ( $d_B$ ).

In Fig. 2, transmission spectra of the PDQW heterostructure as well as those of an equivalent single PQW heterostructure are produced while varying the photonic well width,  $d_A$ . From these spectra, we can see distinct resonant states, which occur as single peaks for the single PQW system and in split pairs for the PDQW system. We note that the split pairs of resonant peaks of the PDQW system are centered on the single peaks from the single PQW system. This illustrates the origin of the spectral splitting that occurs for the PDQW, as each isolated photonic well would contain these single, degenerate bound states. However, because the thickness of the central photonic barrier is small, the electromagnetic fields of these degenerate states overlap with one another, causing the bound states to split into symmetric and anti-symmetric pairs [17]. From Fig. 2, we observe that varying the thickness of the photonic well causes the number of bound



**Fig. 2** Transmission spectra for PQW and PDQW heterostructures where the photonic well width is varied. The *solid red curves* correspond to transmission for the PDQW, whereas the *dashed blue curves* correspond to that of a single PQW with the same outer barrier thickness and well width. In each of these plots, parameters  $d_B = 10L_B$  and  $d_W = 5L_W$  are held constant. The well widths were varied as  $d_A = 10L_A$ ,  $20L_A$ , and  $30L_A$  for the upper, middle, and lower plots, respectively. The incident photon energy was plotted from the upper band edge of photonic crystal A to that of crystal B, the range for which resonant photonic tunneling occurs

photonic states to change. It is well known that for a single PQW, increasing the well width causes more resonant states to appear [11–16], and here, we observe that this

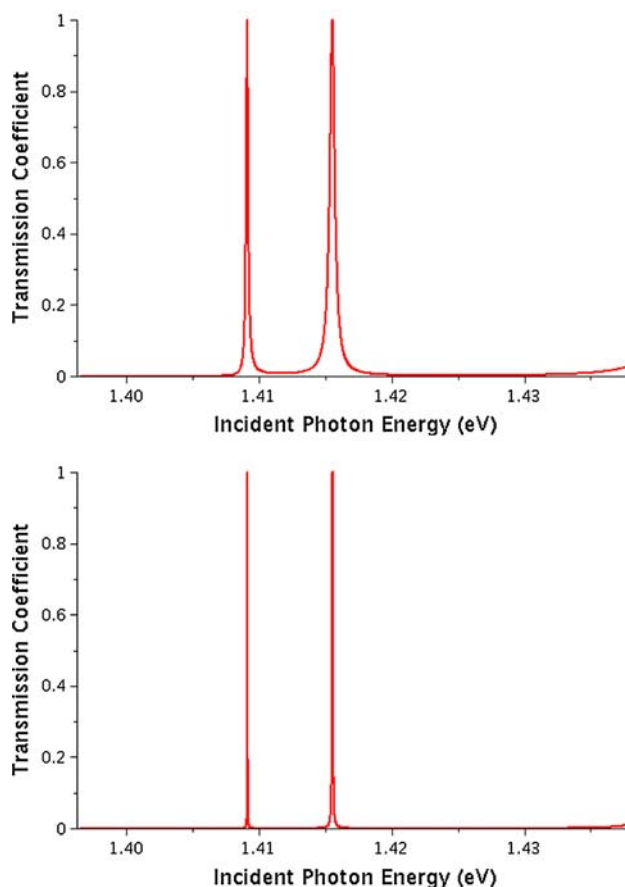


**Fig. 3** Several transmission spectra from a PDQW heterostructure with constant outer barrier thickness ( $d_B = 10L_B$ ) and well width ( $d_A = 10L_A$ ), while the inter-well separation ( $d_W$ ) is varied. The *solid red curve*, *dashed blue curve*, and *dot-dashed black curve* correspond to  $d_W = 5L_B$ ,  $10L_B$ , and  $20L_B$ , respectively. When the photonic wells are close together, strong inter-well coupling enhances the energy-splitting effect. As the wells are separated, the interaction between degenerate states diminishes, and the energy splitting cannot be resolved

occurs for the PDQW system as well. We also note that the energy-splitting effect is stronger for the degenerate states with higher energies. This is due to a larger electric field overlap between bound photonic states at higher energies [17].

In Fig. 3, we have produced transmission spectra for the PDQW heterostructure while varying only the inter-well separation,  $d_W$ , and have found that the strength of the coupling between the degenerate bound states in the two wells depends on this separation. This is to be expected, as when the photonic wells are close to one another (or equivalently when the width of the central photonic barrier is small), the overlap between electromagnetic fields of the degenerate bound states is stronger, which corresponds to a higher degree of energy splitting for those states. However, when the wells are far away from one another, the coupling strength decreases, and eventually the splitting of degenerate states cannot be resolved. At this point that the transmission spectrum of the PDQW approaches that of an equivalent single PQW, and the bound photons can be regarded as confined to identical but separate and isolated wells [15]. Thus, we observe that the system switches from two transparent states to one transparent state as the wells are brought further apart.

In Fig. 4, the transmission coefficient was plotted for the PDQW heterostructure while varying the thickness of the outer photonic barriers. We note that for all of the transmission spectra shown throughout Figs. 1, 2, 3, and 4, there is some limited degree of spectral broadening associated



**Fig. 4** Transmission spectra for a PDQW heterostructure with constant well width ( $d_A = 10L_A$ ) and inter-well separation ( $d_W = 5L_B$ ). The outer photonic barrier thickness is varied as  $d_B = 10L_B$  (*upper plot*) and  $d_B = 15L_B$  (*lower plot*). As the outer barrier thickness is increased, the quality factors of the transmitted resonant peaks increase

with each transmitted resonant peak. This is attributed to the finite periodicity of the photonic barriers [17], and results in a lower quality factor for transmitted resonant states. However, as can be seen from Fig. 4, increasing the thickness of the photonic barriers drastically enhances the spectral efficiency of the transmitted resonant states. It was found that if  $d_B$  becomes too large, then the transmission coefficient approaches zero for the range between the conduction band edges of A and B—this is expected, as incident photons cannot tunnel through the photonic barriers if they become too thick.

## Conclusions

In this paper, we have studied the resonant photonic tunneling effect in PDQW heterostructures made from two photonic crystals. By performing numerical simulations of



the transmission spectra for various PDQW heterostructures, we have demonstrated that resonant photonic tunneling occurs in these structures, whereby photons with energies corresponding to bound states of the system undergo perfect transmission through the entire structure. Our studies have shown that these states occur in split pairs, with energies centered on that of the degenerate states in each individual, isolated photonic well. This spectral splitting is attributed to the coupling of the degenerate states in each photonic well, where the finite photonic barrier separating these wells allows for an overlap of the electromagnetic fields of the degenerate states. Thus, the degree of spectral splitting can be controlled by varying the thickness of the central photonic barrier, and we have shown that the system can, therefore, be switched between one or two transparent states. The energy-splitting phenomenon described here is in qualitative agreement with reported experimental results involving a PDQW heterostructure consisting of 1-D photonic crystals [17]. Our simulations have also shown that the total number of transmitted resonant states can be controlled by modifying the width of the photonic wells in the heterostructure, and the quality factor of transmitted resonant peaks can be greatly improved by increasing the thickness of the outer photonic barriers. It is anticipated that the resonant states described in this study will be useful for developing new types of photonic-switching devices, optical filters, and other optoelectronic devices [3, 11–17].

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