



Research article

Integrating freelance models with fractional derivatives, and artificial neural networks: A comprehensive approach to advanced computation

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ABSTRACT

In order to improve results, this work investigates how the Freelance Model (FM), Fractional Derivative (FD), and Artificial Neural Network (ANN) may all function together. We suggest a new method that combines the varied skills of freelancers with the precision of fractional derivatives and the adaptability of neural networks to maximize the benefits of each. This proposed strategy provides a new perspective to the computational methodologies and holds a promising impact on diverse industries. Future developments and applications can be made possible by this promising path toward enhanced performance in complex systems and data-driven areas. Twenty neurons have been selected and data has been trained and validated in the, following manner 70 %, 15 % and 15 %. The consistency of method has been shown using the correlation/regression and histograms in order to solve the model. The results presented here not only validate the efficacy of our approach but also open avenues for further exploration and advancements in the dynamic field of advanced computation.

1. Introduction

ANN is a type of computational model that is inspired by the human brain. It is a group of algorithms that is designed to help in change the input without altering the output and gives the best result. ANN can be used in various real-life problems to predict accurate results. A detailed description of foundations of ANN, its algorithms, implementations, examples and applications are given in a book written by Zurada [1]. In another book, Daniel explained the recent leading approaches to neural networks by including the designing, execution and obtaining results of the case studies in different scenarios [2]. ANN is widely used in forecasting task [3]. give a review of research articles that utilized ANN for forecasting application and mentioned that it is adapted for forecasting in most cases over classical methods due to its nonlinearity, adaptability and their ability to perform arbitrary function mappings. In majority of the cases ANN performs better than statistical models [4]. Mostly analysts use extrapolation techniques for handling discontinuities and nonlinearity but ANN can do both well theoretically [5].

In a recent work, ANN provided a robust model for the prediction of the drought due to climate change better than statistical, physical and data-driven forecast models. Since the droughts are by nature multivariate and non-linear, and ANN can very efficiently

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and easily capture the dynamical relationships, therefore ANN is more suitable for drought prediction than contemporary techniques [6]. ANN can also be used to detect the breast cancer early and its results are way better than Naïve Bayes Algorithm [7]. [8] applied radial basis deep neural network technique based on Bayesian regularization for monkey pox transmission model and optimized the results.

[9] designed a Bayesian regularization approach to solve the fractional Layla and Majnun System and solved it accurately [10]. developed the Gudermannian neural network for solving the nonlinear multi-Pantograph delay differential equation [11]. solved the epidemic breathing transmission system using neural networks [12]. used the Heuristic computing approach for solving the hepatitis B virus model with sequential quadratic programming [13]. developed the neural network technique for solving the epidemic delay differential model [14]. utilized the neural network techniques for solving the mathematical robotic model and results showed that neural network is an accurate technique for solving robotic models [15]. utilized the neural network for solving Zika virus model and showed the efficacy of neural network. Several other researches have been done on the efficacy of ANN implementation such as [16–19].

ANN not only proven to be a very efficient method for solving ordinary and partial differential equation [20] but has also proven to be effective in solving fractional differential equations that arises in many real-life problems. A very famous telegraph equation that describes various phenomenon in engineering such as signal analysis, wave propagation, electric transmission line etc. but it is difficult to solve especially if it is of fractional order. Neural network method very efficiently solved fractional telegraph equation by considering it as an optimization problem [21]. Few other fractional differential equations with cosine basis functions and initial conditions were solved by ANN effectively [22–24].

In this research, our main objective is to develop a strategy to optimize the results of fractional freelance model, fractional derivative and artificial neural network and enhance performance of such complex system. A Freelance model was first introduced in the form of compartmental problem by Ref. [25]. They presented a model examining the impact of information on freelancing expansion, validated through numerical simulation. Freelancing after Covid 19 and lockdown have become very popular not only among Gen Z but all people who have skills to offer online. Individuals well aware of digitalization know the importance of freelancing. Some important factors that attract the digital literate person to work as freelancers are time flexibility, family time, learning opportunity, being their own boss, freedom of choosing client of their own choice, financial growth (26). (27) introduced a fractional freelance model using Atangana-Baleanu Caputo Fractional Derivative. They gave a comparison of classical, fractional mathematical model with real time data and their results depicted that the fractional model yields better prediction than the ordinary one.

Different fields can benefit from one another when the Freelance model, Fractional derivative, and artificial neural network are all brought together. The Freelance model facilitates autonomous service provision via decentralized and adaptable employment relationships. A more realistic depiction of complex systems with memory and long-range relationships can be achieved by the use of fractional derivative, which generalizes the concept of differentiation to orders other than integers. Inspired by biological neural networks, artificial neural networks are data-learning and classification-capable computing models. When combined, these parts encourage teamwork among specialists and pave the way for cutting-edge algorithm creation and insightful data analysis. Industries where complex systems and data-driven decision making are commonplace, like finance, healthcare, and engineering, stand to benefit greatly from this merger. The convergence of the Freelance model with the Fractional derivative and the artificial neural network paves the way for innovative approaches to work organization, mathematical modeling, and data analysis that can have a substantial impact in a number of different fields.

Division of the sections in this paper is as follows. In section 2, fractional derivative preliminaries used in this paper are introduced and some introduction of artificial neural network. Section 3 gives the problem formulation of fractional freelance model based on artificial neural network. Section 4 gives the results and discussion of this paper.

2. Basic theory

2.1. Fractional calculus

Since last many decades, mathematical models of integer order are being studied very successfully through numerical methods. But in recent years, a growing number of researchers from various fields of science and engineering have shown interest in dynamical systems described by system of fractional differential equations. Due to the extensive applications of FDEs in engineering and science, research in this area has grown significantly around the world. Complex fractional mathematical models are major challenge for researchers. The analysis of the conduction of fractional mathematical models requires the in-depth learning. The examples of some physical phenomenon maybe like; heat flow, wave propagation into the shallow waters, the equations governing shock wave phenomenon (28), (29).

2.1.1. Mathematical definition of fractional calculus

Some concepts that have been used in calculation of fractional calculus model are given below:

Caputo fractional derivative is given as.

$$D_t^\alpha G(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} G^{(n)}(\tau) d\tau \text{ where } n-1 < \alpha \leq n, n \in N, t > 0. \text{ If } \alpha = 1, \text{ then it becomes } D_t^\alpha G(t) = \frac{dG(t)}{dt}.$$

Some properties that will be used in calculation of fractional derivative are.

- $D_t^\alpha t^\gamma = \frac{\Gamma(1+\gamma)}{\Gamma(1+\gamma-\alpha)} t^{\gamma-\alpha}; \gamma > 0, .$

- $D_t^\alpha(cG(t)) = cD_t^\alpha G(t)$, where c is constant.
- $D_t^\alpha(uG(t) + vF(t)) = uD_t^\alpha G(t) + vD_t^\alpha F(t)$, where u and v are constant.
- $D_t^\alpha c = 0$.

Riemann-Liouville integral, is yet another definition that will be used in calculation is given as

$$J^\alpha G(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^t (t - \tau)^{\alpha-1} G(\tau) d\tau; \alpha > 0, t > 0,$$

Some important properties of Riemann-Liouville Integral are:

- $J^0 G(t) = G(t)$,
- $\frac{d}{dt} J^{\alpha+1} G(t) = J^\alpha G(t)$,
- $J^\alpha t^\gamma = \frac{\Gamma(1+\gamma)}{\Gamma(1+\gamma+\alpha)} t^{\alpha+\gamma}$,
- $J^\alpha (J^\beta G) = J^{\alpha+\beta} G$.

Benefit of using these definitions instead of any other contemporary definitions of fractional calculus is that Caputo fractional derivative requires the same initial and boundary conditions as ordinary differential equations use. These are used for modelling real life phenomenon because it is nonlocal and captures the history of mathematical models.

2.2. Artificial neural network

Intelligent computing systems called artificial neural networks (ANNs) attempt to simulate the biological nervous system. Pattern identification, recognition, and classification, electrical energy consumption predictions, and induction motor drive in pumping are only a few examples of the complicated problems that ANNs have been effectively employed to examine. The building blocks of ANNs are networks of neurons and other nodes that process data in a nonlinear fashion to produce results. The components of an ANN, including the input, hidden layers, and output, are shown in Fig. 1.

3. Problem formulation

Freelance fractional model (27) is written as

$$D_t^\alpha N(t) - \gamma + \epsilon NI + \delta N - \mu F = 0,$$

$$D_t^\alpha F(t) - \epsilon NI + \delta F + \mu F = 0,$$

$$D_t^\alpha I(t) - aF + bI = 0,$$

$$N(0) - 118.72 = 0,$$

$$F(0) - 53 = 0,$$

$$I(0) - 3 = 0,$$

(1)

Where $\gamma = 10, \epsilon = 0.0002, \delta = 0.02, \mu = 0.03, a = 0.05,$ and $b = 0.06$. Here these parameters are the transmission rates from one population to another population as mentioned in Ref. [25].

Its numerical solution with ANN will be that reduces the mean square error loss function that is exactly the same that approximately reduces the left-hand side of the sum of the mean square. There will be three trial function for each equation and their loss function by determining the weights and minimize the loss function. The input is multiplied by the connection weights, and then the bias and threshold is applied to produce the desired output in the simplest ANN model. After the model has been formulated, we use MATLAB's derived fractional Euler's approach on the reference dataset and nftool command has been used in MATLAB. A back-propagating artificial neural network (ANN) is used in conjunction with the Levenberg-Marquardt (LM) approach to create an implementation of the MATLAB software in a dataset. An intelligent LM neural network approach is utilized with twenty neurons, with 15 % of the data

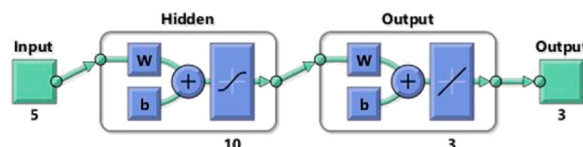


Fig. 1. ANN framework diagram.

being used for training, 15 % being utilized for validation, and 70 % being utilized for testing.

4. Results and discussion

Numerical calculations for fractional freelance model have been done using fractional Euler's method. Fig. 2 shows the comparison of numerical results of $N(t)$ calculated for classical model, fractional model at $\alpha = 1$ with real life data and it can clearly be observed that the fractional model is closer to real life data. Fig. 3 depicts the comparison of freelancer's fractional model and classical model with real life data. As it can be observed there is an increase in number of freelancers after 2019 that shows the increase in this career path after lock down.

Since this study is analyzing the impact of information on working people about freelancing so the important factor to study was the behavior of parameters a and b in Eq. (1). Fig. 4 shows the increase in non-freelancers due to the negative information ($b > a$) passed on to the workers about freelancing such as late payments, no new orders or order cancellation after submission of work. Therefore, as a result the number of freelancers will decrease and most workers will choose to work remotely instead of working online. Figs. 5 and 6 depicts the same situation for ($b > a$) that of the negative impact of freelancing is shown to the workers they will eventually prefer working remotely.

Fig. 7 shows that if the positive image of freelancing is portrayed instead of negative one then the results can be in favor of freelancers. For different values of α and ($b < a$) shows that if the freelancing platforms show the positive impact on freelancers' lives, and benefits of working as a freelancer then this reflects as ($b < a$) in Eq. (1) and therefore the increasing graph can be seen in Fig. 7. Due to this positive information the increase in number of freelancers can be seen in Fig. 9 that is due to ($b < a$). Graph for different values of α can be seen in Fig. 9 that depicts the increase in number of workers that chooses freelancing as a career path if they are shown the positive side of freelancing and minimizing the bad experiences of freelancers through the platforms, they use to provide their services. Since due to ($b < a$), $I(t)$ and $F(t)$ are increasing then it is bound to happen that $N(t)$ decreases. This decrease of $N(t)$ is shown in Fig. 8, for different values of α .

For the purpose of constructing efficient neural networks, this work demonstrates the significance of employing rigorous training and testing methodologies. If these procedures are carried out correctly, training becomes more efficient and stable as a result of effective convergence with reduced learning rates and gradients. The results of the regression analysis are depicted in Fig. 10. A regression value of $R = 1$ is discovered to exist between the calculated output and the target values. From this, it can be deduced that the results we obtain from Euler's method are in complete agreement with results shown in regression analysis of Fig. 10. These graphs of regression analysis show that there is a strong connection between increase and decrease of freelancers and non-freelancers with the information that spreads in the community. The results of testing, validation, and training for all examples are shown converging versus time periods in Fig. 11. The numbers show that the best validation performance has an MSE of $8.41E-9$ and a standard deviation of $1.35E-08$. This means that the forecast is closer to the actual values and hence this prediction is absolutely correct. The resultant histogram is shown in Fig. 12. In the graphs above, it can be observed, where the error box and zero axis are situated for all such cases. Fig. 13 compares the results of ANNs-FD with those of the reference solution, demonstrating the correctness of the suggested scheme by its demonstration of an MSE error of less than 10^{-11} .

5. Conclusion

In this work, a new framework was studied combining ANN, fractional derivative and freelance model to analyze and forecast the future of freelancing among working community. ANN has proven to be the best computational model and analysis mechanism for complex systems such as fractional freelance model. Results obtained from fractional Euler's method is in complete agreement with the predictions made through ANN. Twenty neurons were selected in this study with training, validation of 70 %, 15 % and 15 %. Reducible error performances improve the accuracy of the designed method. Regression analysis and MSE of $8.419E-9$ suggests that the model prediction is accurate and the impact of information on society about freelancing can play a major role in deciding whether this will be a future career path for GenZ or not.

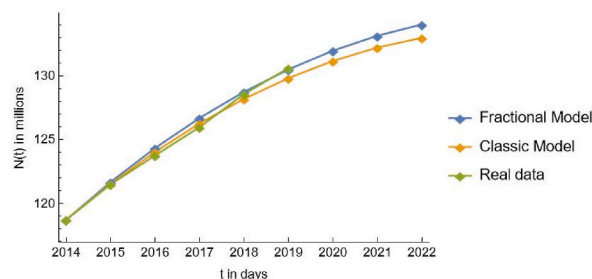


Fig. 2. Comparison of non-freelancer data with classical and fractional freelance model.

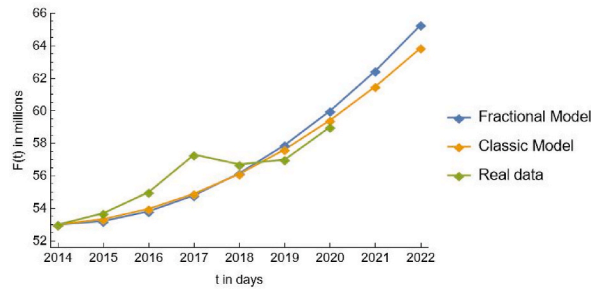


Fig. 3. Comparison of freelancer data with classical and fractional freelance model.

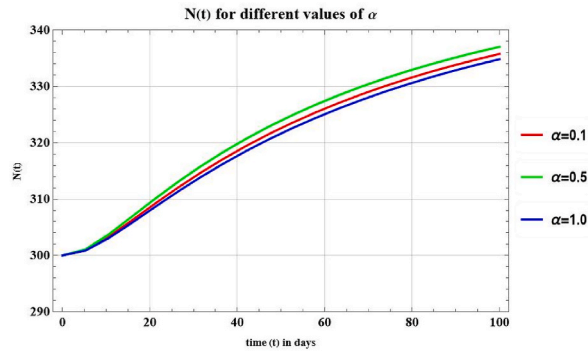


Fig. 4. Behavior of $N(t)$ for different values of α when $b > a$.

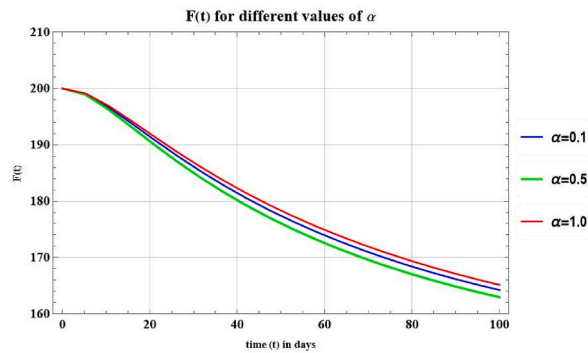


Fig. 5. Behavior of $F(t)$ for different values of α when $b > a$.

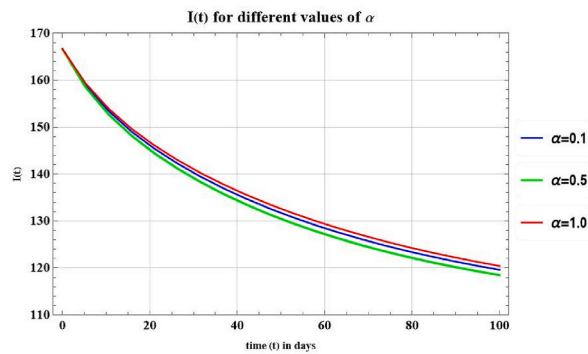


Fig. 6. Behavior of $I(t)$ for different values of α when $b > a$.

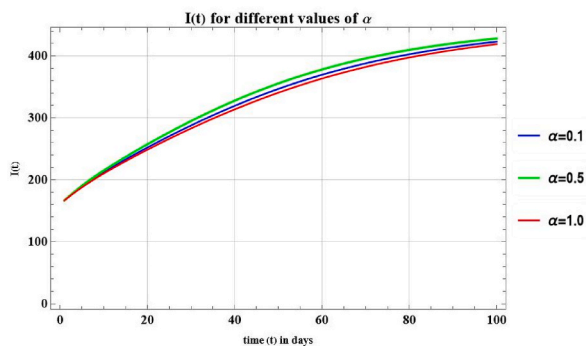


Fig. 7. Behavior of $I(t)$ for different values of α when $b < a$.

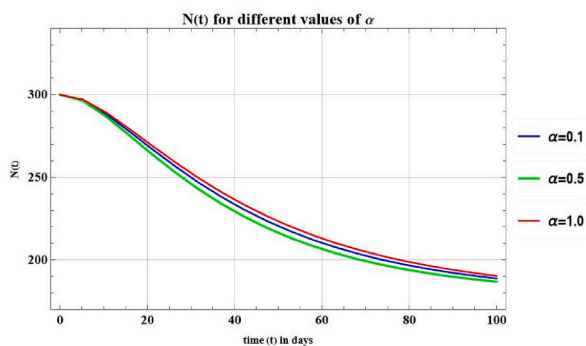


Fig. 8. Behavior of $N(t)$ for different values of α when $b < a$.

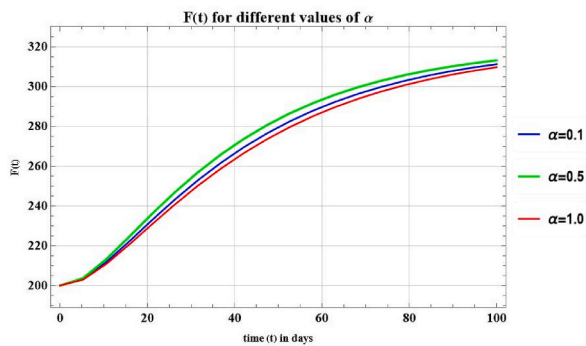


Fig. 9. Behavior of $F(t)$ for different values of α when $b < a$.

Data availability statement

The data is available on requires to the correspondence author.

Ethical approval

Not applicable.

Additional information

No additional information is available for this paper.

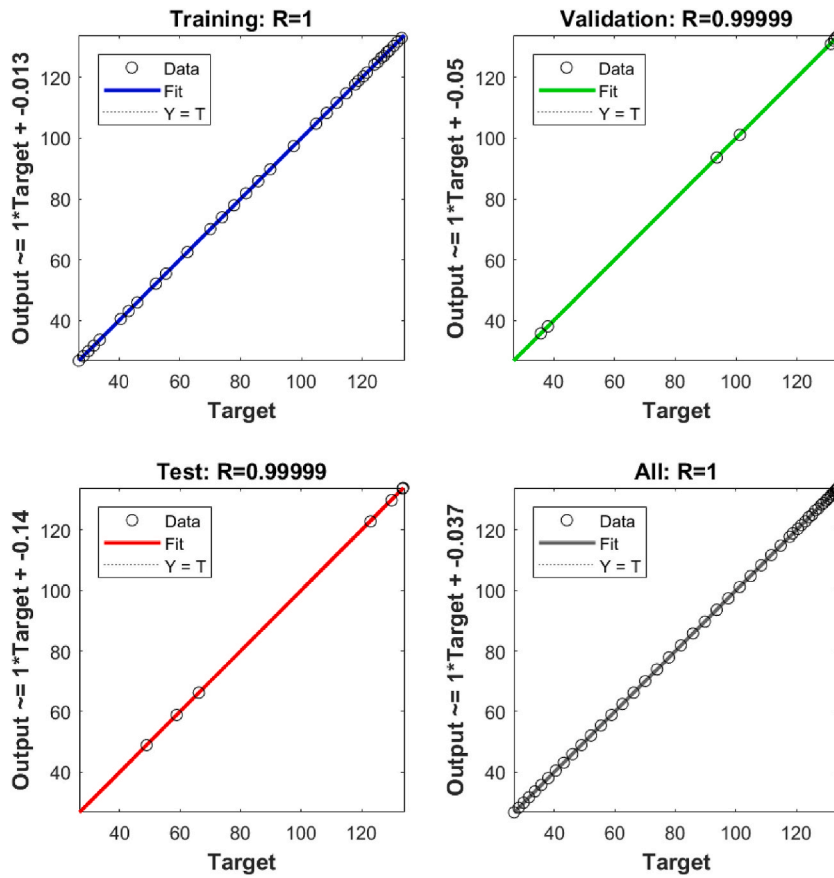


Fig. 10. Regression evaluation for $N(t)$.

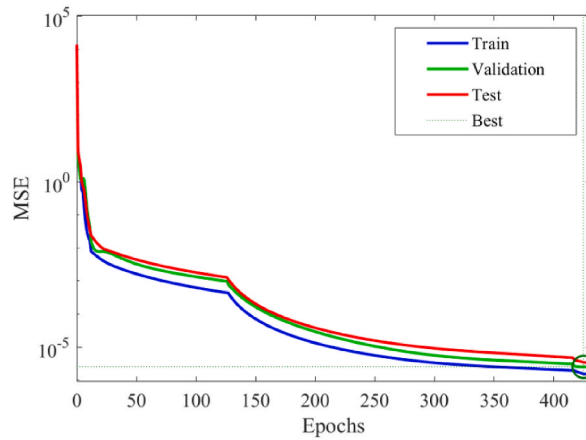


Fig. 11. MSE evaluation for $N(t)$.

CRediT authorship contribution statement

Fareeha Sami Khan: Writing – original draft, Resources, Conceptualization. **Afraz Hussain Majeed:** Writing – original draft, Visualization, Data curation, Conceptualization. **M. Khalid:** Writing – original draft, Validation, Supervision.

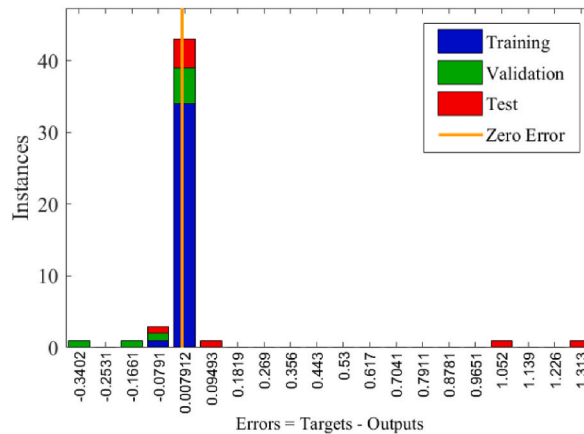


Fig. 12. Histogram error evaluation for $N(t)$.

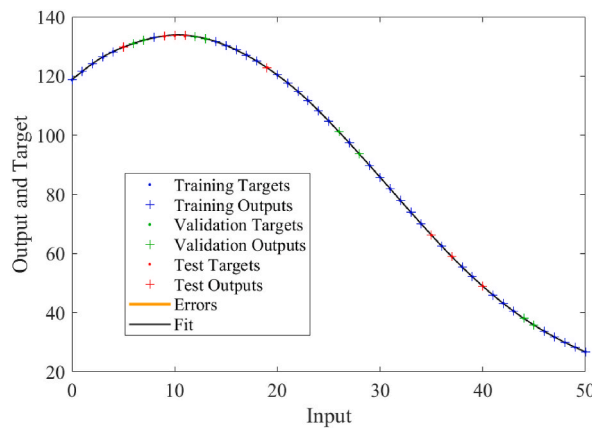


Fig. 13. Fitness curve for $N(t)$.

Declaration of generative AI and AI-assisted technologies in the writing process

Not applicable

Declaration of competing interest

The authors declare that they have no known competing interests in this paper.

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