



Picture Fuzzy Linear Assignment Method and Its Application to Selection of Pest House Location

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Abstract. The theory of picture fuzzy sets is useful for handling uncertainty in multiple attribute decision making problems by considering membership, non-membership and indeterminacy degrees independently for each element. In this paper, by extending the classical linear assignment method, we propose a novel method which is called picture fuzzy linear assignment method (PF-LAM) for solving multiple criteria group decision-making problems with picture fuzzy sets. A ranking procedure consisting of aggregation functions, score functions, accuracy functions, and weighted rank frequency and a binary mathematical model are presented to determine the priority order of various alternatives. The applicability and validity of the proposed method is shown through the selection of pest house locations. The proposed method helps managers to find the best location to construct the pest house based on the determined criteria.

Keywords: Picture fuzzy sets · Linear assignment model · Multiple criteria decision-making model · Optimization · Pest house location selection

1 Introduction

Fuzzy Sets theory, developed by Zadeh [1], is a useful and appropriate approach in order to deal with imprecise and uncertain information in vague situations. After the introduction of fuzzy sets, they have been very popular in almost all branches of science [2]. Many researchers [2–14] have introduced many extensions of ordinary fuzzy sets in the literature. These extensions have been utilized by numerous researchers in recent years in the solution of multi-attribute decision-making problems [2]. One of the latest extensions is Picture fuzzy sets (PFS). Picture Fuzzy Sets (PFS) were developed by Cuong [14] and it is a direct extension of intuitionistic fuzzy sets (IFS) that can model uncertainty using membership degree, non-membership degree, and hesitant degree independently.

Wang and Li [15] introduced the theory of picture hesitant fuzzy set based on the picture fuzzy sets and the hesitant fuzzy set. Sarwar Sindhu et al. [16] proposed a linear programming model in order to find exact weights and construct a modified distance based on similarity measure under picture fuzzy environment. Liang et al. [17] presented a MCDM method which is a combination of TODIM method with the

ELECTRE method in a picture fuzzy environment. Thao [18] developed the entropy measure for PFS and proposed the similarity measures for MCDM problems in order to select suppliers. Tian et al. [19] proposed a picture fuzzy MCDM method and introduced weighted picture fuzzy power Choquet ordered geometric operator and a weighted picture fuzzy power Shapley Choquet ordered geometric operator.

The linear assignment method (LAM) was proposed by Bernardo and Blin [20], inspiring from assignment problem in linear programming for multi-attribute decision-making [21]. The basic idea of the LAM is that the combination of the criteria-wise rankings into an overall preference ranking that produces an optimal compromise among the several component rankings. Developing an extended linear assignment method to solve multi-criteria decision-making (MCDM) problems under Pythagorean fuzzy environment was the aim of [22]. In addition, Liang et al. [23] developed the linear assignment method for interval-valued Pythagorean fuzzy sets. By extending the traditional linear assignment method, Chen [24] developed an efficient method for solving MCDM problems in the interval-valued intuitionistic fuzzy environment.

To the best of our knowledge, there is no research about extension and application of linear assignment method in picture fuzzy environment. Therefore, the aim of this paper is to develop a novel multi-attribute decision-making method based on linear assignment approach with picture fuzzy sets and also show the useful application to site selection of pest house. The proposed algorithm has the following contributions. First, judgment values are given as picture linguistic terms, which can consider the hesitancy degree of decision makers' comments about alternatives and criteria. Second, linear assignment method has been employed to rank alternatives to avoid the effect of subjectivity.

In Sect. 2, the definitions of Picture fuzzy sets are presented. In Sect. 3, the Picture fuzzy linear assignment method are detailed step by step. In Sect. 4, an application is given and in Sect. 5, the conclusion is given.

2 Picture Fuzzy Sets: Preliminaries

There are some definitions about PFS is given as follows with related equations.

Definition 2.1: A PFS on a \tilde{A}_p of the universe of discourse U is given by;

$$\tilde{A}_p = \left\{ \left\langle u, (\mu_{\tilde{A}_p}(u), I_{\tilde{A}_p}(u), \nu_{\tilde{A}_p}(u)) \mid u \in U \right\rangle \right\} \tag{1}$$

where

$$\mu_{\tilde{A}_p}(u) : U \rightarrow [0, 1], \quad I_{\tilde{A}_p}(u) : U \rightarrow [0, 1], \quad \nu_{\tilde{A}_p}(u) : U \rightarrow [0, 1]$$

and

$$0 \leq \mu_{\tilde{A}_p}(u) + I_{\tilde{A}_p}(u) + \nu_{\tilde{A}_p}(u) \leq 1 \quad \forall u \in U \tag{2}$$

Then, for each u , the numbers $\mu_{\tilde{A}_S}(u)$, $v_{\tilde{A}_S}(u)$ and $I_{\tilde{A}_S}(u)$ are the degree of membership, non-membership and indeterminacy of u to \tilde{A}_S , respectively. $\chi = 1 - (\mu_{\tilde{A}_p}(u) + v_{\tilde{A}_p}(u) + I_{\tilde{A}_p}(u))$ is called as a refusal degree [25].

Definition 2.2: Basic operators for Single-valued picture fuzzy sets;

$$\tilde{A}_p \oplus \tilde{B}_p = \left\{ \mu_{\tilde{A}_p} + \mu_{\tilde{B}_p} - \mu_{\tilde{A}_p} \mu_{\tilde{B}_p}, I_{\tilde{A}_p} I_{\tilde{B}_p}, v_{\tilde{A}_p} v_{\tilde{B}_p} \right\} \tag{3}$$

$$\tilde{A}_p \otimes \tilde{B}_p = \left\{ \mu_{\tilde{A}_p} \mu_{\tilde{B}_p}, I_{\tilde{A}_p} + I_{\tilde{B}_p} - I_{\tilde{A}_p} I_{\tilde{B}_p}, v_{\tilde{A}_p} + v_{\tilde{B}_p} - v_{\tilde{A}_p} v_{\tilde{B}_p} \right\} \tag{4}$$

$$\lambda \cdot \tilde{A}_p = \left\{ \left(1 - (1 - \mu_{\tilde{A}_p})^\lambda \right), I_{\tilde{A}_p}^\lambda, v_{\tilde{A}_p}^\lambda \right\} \text{ for } \lambda > 0 \tag{5}$$

$$\tilde{A}_p^\lambda = \left\{ \mu_{\tilde{A}_p}^\lambda, \left(1 - (1 - I_{\tilde{A}_p})^\lambda \right), \left(1 - (1 - v_{\tilde{A}_p})^\lambda \right) \right\} \text{ for } \lambda > 0 \tag{6}$$

Definition 2.3: Single-valued Picture Fuzzy Weighted Averaging operator (PFWA) with respect to, $w = (w_1, w_2, \dots, w_n)$; $w_i \in [0, 1]$; $\sum_{i=1}^n w_i = 1$, is defined as;

$$\begin{aligned} PFWA_w(\tilde{A}_1, \dots, \tilde{A}_n) &= w_1 \tilde{A}_1 + w_2 \tilde{A}_2 + \dots + w_n \tilde{A}_n \\ &= \left\{ 1 - \prod_{i=1}^n (1 - \mu_{\tilde{A}_{ii}})^{w_i}, \prod_{i=1}^n I_{\tilde{A}_{ii}}^{w_i}, \prod_{i=1}^n v_{\tilde{A}_{ii}}^{w_i} \right\} \end{aligned} \tag{7}$$

Definition 2.4: Score functions and Accuracy functions of sorting picture fuzzy numbers are defined by;

$$Score(\tilde{A}_p) = \frac{1}{2} \left(1 + 2\mu_{\tilde{A}_p} - v_{\tilde{A}_p} - I_{\tilde{A}_p} / 2 \right) \tag{8}$$

$$Accuracy(\tilde{A}_p) = \mu_{\tilde{A}_p} + v_{\tilde{A}_p} + I_{\tilde{A}_p} \tag{9}$$

Note that: $\tilde{A}_p < \tilde{B}_p$ if and only if

- (i) $Score(\tilde{A}_p) < Score(\tilde{B}_p)$ or
- (ii) $Score(\tilde{A}_p) = Score(\tilde{B}_p)$ and $Accuracy(\tilde{A}_p) < Accuracy(\tilde{B}_p)$

3 Picture Fuzzy Linear Assignment Method

The classical linear assignment method is extended to picture fuzzy linear assignment model. The proposed PF-LAM is composed of several steps as given in follows. Table 1 presents the linguistic terms and their corresponding picture fuzzy numbers. Decision matrix whose elements show the judgments values of all alternatives with respect to each criterion under picture fuzzy environment. Consider a group of k decision makers, $D = \{D_1, D_2, \dots, D_k\}$ participated in a group decision making problem, let $X = \{x_1, x_2, \dots, x_m\} (m \geq 2)$ be a discrete set of m feasible alternatives and $C = \{C_1, C_2, \dots, C_n\}$ be a finite set of n criteria and $w_j = \{w_1, w_2, \dots, w_n\}$ be the weight vector of n criteria which proves $0 \leq w_j \leq 1$ and $\sum_{j=1}^n w_j = 1$.

Table 1. Picture fuzzy linguistic terms [25]

Linguistic terms	(μ, π, ν)
Very High Importance (VHI)	(0.9, 0.0, 0.05)
High Importance (HI)	(0.75, 0.05, 0.1)
Slightly More Importance (SMI)	(0.6, 0.0, 0.3)
Equally Importance (EI)	(0.5, 0.1, 0.4)
Slightly Low Importance (SLI)	(0.3, 0.0, 0.6)
Low Importance (LI)	(0.25, 0.05, 0.6)
Very Low Importance (VLI)	(0.1, 0.0, 0.85)

Step 1: Collect the decision-makers’ evaluations for the alternatives and criteria based on Table 1.

Step 2. Aggregate the individual decision matrices based on PFWA operator as given in Eq. (7).

Step 3. Compute the elements of scored decision matrix by utilizing the picture fuzzy score function (Eq. 8).

Step 4. Establish the rank frequency non-negative matrix β_{jk} with elements that represent the frequency that A_m is ranked as the m th criterion-wise ranking.

Step 5. Calculate and establish the weighted rank frequency matrix λ , where the λ_{ik} measures the contribution of A_m to the overall ranking. Note that each entry λ_{ik} of the weighted rank frequency matrix Π is a measure of the concordance among all criteria in ranking the m th alternative k th.

$$\lambda_{ik} = w_{i1} \oplus w_{i2} \oplus \dots \oplus w_{i\beta_{mm}} \tag{10}$$

Step 6. Define the permutation matrix P as a square $(m \times m)$ matrix and set up the following linear assignment model according to the Π_{ik} value. The linear assignment model can be written in the following linear programming format:

$$\begin{aligned}
 &Max \quad \sum_{i=1}^m \sum_{k=1}^m \lambda_{ik} \cdot P_{ik} \\
 &s.t. \quad \sum_{k=1}^m P_{ik} = 1, \quad \forall i = 1, 2, \dots, m; \\
 &\quad \quad \sum_{i=1}^m P_{ik} = 1, \quad \forall k = 1, 2, \dots, m; \\
 &\quad \quad P_{ik} = 0 \text{ or } 1 \text{ for all } i \text{ and } k
 \end{aligned}$$

Step 7. Solve the linear assignment model, and obtain the optimal permutation matrix

P^* for all i and k . Calculate the multiplication of matrix $P^* \cdot X = P^* \cdot \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_m \end{bmatrix}$ and obtain

the optimal order of alternatives.

4 An Application to Pest House Location Selection

In this section, a numerical example is presented to illustrate feasibility and practical advantages of new proposed method. Nowadays, Coronavirus Disease (COVID-19) has emerged as a global problem since spread of the disease to March 2020, more than one million people have been infected by COVID-19 virus. The disease is quickly spreading between people during close contact. In the future, existing hospitals will not be enough for the patients who are suffering from Coronavirus or similar dangerous viruses. Each country has to establish pet houses. The aim of this problem is to select the best site location in order to establish pest house in Turkey. The mostly preferred five locations (X_1 : Ankara, X_2 : Izmir, X_3 : Istanbul-Atatürk Airport, X_4 : Istanbul-Sancaaktepe, and X_5 : Bursa) are evaluated as alternatives. Four criteria have been determined in order to evaluate these alternatives. Criteria are logistic support opportunities to settlements (C1), economical situations (C2), population density (C3), and proximity to settlements (C4). Three decision makers who have different significance levels such as 0.3, 0.5, 0.2, are going to evaluate the above five possible alternatives according to four criteria based on picture linguistic terms as presented in Table 2.

Aggregate the decision matrices using Eq. (7) into a single aggregated decision matrix as given in Table 3. The weight of each criterion is aggregated based on Eq. (7) and so that $w_j = [0.324, 0.091, 0.359, 0.226]$.

Table 2. Assessments of decision-makers

DM1	C ₁	C ₂	C ₃	C ₄	DM2	C ₁	C ₂	C ₃	C ₄	DM3	C ₁	C ₂	C ₃	C ₄
X ₁	HI	VHI	VLI	VHI	X ₁	SMI	HI	EI	HI	X ₁	SMI	SLI	VHI	SLI
X ₂	SLI	VHI	HI	SLI	X ₂	SLI	SLI	LI	HI	X ₂	HI	LI	VLI	LI
X ₃	VLI	HI	EI	SMI	X ₃	HI	HI	SMI	SMI	X ₃	HI	SMI	HI	HI
X ₄	SMI	HI	LI	HI	X ₄	HI	EI	LI	EI	X ₄	SMI	VHI	VLI	VHI
X ₅	HI	SLI	HI	SMI	X ₅	HI	SLI	EI	EI	X ₅	SMI	HI	LI	VLI

Table 3. Aggregated decision matrix

	C ₁	C ₂	C ₃	C ₄
X ₁	(0.65, 0.00, 0.22)	(0.77, 0.00, 0.12)	(0.57, 0.00, 0.33)	(0.77, 0.00, 0.12)
X ₂	(0.43, 0.00, 0.42)	(0.60, 0.00, 0.28)	(0.44, 0.00, 0.38)	(0.58, 0.00, 0.24)
X ₃	(0.63, 0.00, 0.19)	(0.73, 0.00, 0.12)	(0.61, 0.00, 0.26)	(0.64, 0.00, 0.24)
X ₄	(0.68, 0.00, 0.17)	(0.71, 0.00, 0.17)	(0.22, 0.00, 0.64)	(0.71, 0.00, 0.17)
X ₅	(0.73, 0.00, 0.12)	(0.43, 0.00, 0.42)	(0.56, 0.07, 0.29)	(0.47, 0.00, 0.43)

Calculate the score value of each alternative based on each criterion using Eq. (8). The results are shown in Table 4.

Table 4. The score value of each alternative

	C ₁	C ₂	C ₃	C ₄
X ₁	1.045	1.209	0.902	1.209
X ₂	0.721	0.962	0.753	0.953
X ₃	1.038	1.163	0.979	1.015
X ₄	1.097	1.119	0.401	1.119
X ₅	1.163	0.721	0.899	0.761

β_{ij} are computed and established the weighted rank frequency matrix λ_{ik} , as shown in Table 5. For example, consider λ_{12} in the following: $\lambda_{12} = w_{C2} + w_{C4} = 0.091 + 0.226 = 0.317$.

Table 5. Weighted rank frequency matrix λ_{ik}

	1st	2nd	3rd	4th	5th
X ₁	0,3170	0,0908	0,3244	0,0000	0,0000
X ₂	0,0000	0,0000	0,0000	0,6756	0,3244
X ₃	0,3586	0,0908	0,2262	0,3244	0,0000
X ₄	0,0000	0,5506	0,0908	0,0000	0,3586
X ₅	0,3244	0,0000	0,3586	0,0000	0,3170

The linear assignment model is constructed as follows. The objective function of this binary mathematical model tries to maximize the sum of the weights of alternatives by choosing the optimal order of them.

$$\begin{aligned} \text{Max } Z = & 0.3170P_{11} + 0.0908P_{12} + 0.3244P_{13} + 0.6756P_{24} + 0.3244P_{25} + 0.3586P_{31} \\ & + 0.0908P_{32} + 0.2262P_{33} + 0.3244P_{34} + 0.5506P_{42} + 0.0908P_{43} + 0.3586P_{45} \\ & + 0.3244P_{51} + 0.3586P_{53} + 0.3170P_{55} \end{aligned}$$

s.t.

$$\begin{aligned} P_{11} + P_{12} + P_{13} + P_{14} + P_{15} &= 1 \\ P_{21} + P_{22} + P_{23} + P_{24} + P_{25} &= 1 \\ P_{31} + P_{32} + P_{33} + P_{34} + P_{35} &= 1 \\ P_{41} + P_{42} + P_{43} + P_{44} + P_{45} &= 1 \\ P_{51} + P_{52} + P_{53} + P_{54} + P_{55} &= 1 \\ P_{11} + P_{21} + P_{31} + P_{41} + P_{51} &= 1 \\ P_{12} + P_{22} + P_{32} + P_{42} + P_{52} &= 1 \\ P_{13} + P_{23} + P_{33} + P_{43} + P_{53} &= 1 \\ P_{14} + P_{24} + P_{34} + P_{44} + P_{54} &= 1 \\ P_{15} + P_{25} + P_{35} + P_{45} + P_{55} &= 1 \\ P_{ik} &= 0 \text{ or } 1 \text{ for } i = 1, 2, 3, 4, 5; k = 1, 2, 3, 4, 5. \end{aligned}$$

The proposed mathematical model is solved by using GAMS 24.1.3 software and the results are obtained. After solving the model, the results are $P_{13} = 1$, $P_{24} = 1$, $P_{31} = 1$, $P_{42} = 1$ and $P_{55} = 1$. The value of objective function is $z = 2.226$. The optimal ranking order of the five alternatives is $X_3 > X_4 > X_1 > X_2 > X_5$. Best location for pest house is Istanbul-Ataturk Airport.

5 Conclusion

In the recent years, picture fuzzy sets have been very widespread in almost all branches. Picture fuzzy sets are another extension of the ordinary fuzzy sets. PFS should satisfy the condition that the sum of membership degree and non-membership degree and hesitancy degree should be equal to or less than one. In this study, the classical linear assignment model is extended to picture fuzzy linear assignment model and the novel method is applied to site selection problem for pest house. It has been successfully solved by picture fuzzy linear assignment model. The proposed PF-LAM method is performed to get the optimal preference ranking of the alternatives according to a set of criteria-wise rankings within the context of PFS.

For future studies, the proposed method can be applied to several decision support systems and the illustrative example can be extended by real data.

References

1. Zadeh, L.A.: Fuzzy sets. *Inf. Control* **8**, 338–353 (1965)
2. Kutlu Gündoğdu, F., Kahraman, C.: Spherical fuzzy sets and spherical fuzzy TOPSIS method. *J. Intell. Fuzzy Syst.* **36**(1), 337–352 (2019)
3. Zadeh, L.A.: The concept of a linguistic variable and its application to approximate reasoning. *Inf. Sci.* **8**, 199–249 (1975)
4. Atanassov, K.T.: Intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **20**(1), 87–96 (1986)
5. Atanassov, K.T.: Geometrical interpretation of the elements of the intuitionistic fuzzy objects Preprint IM-MFAIS (1989) 1-89. Sofia. Reprinted. *Int. J. Bioautomation* **20**(S1), 27–42 (2016)
6. Garibaldi, J.M., Ozen, T.: Uncertain fuzzy reasoning: a case study in modelling expert decision making. *IEEE Trans. Fuzzy Syst.* **15**(1), 16–30 (2007)
7. Grattan-Guinness, I.: Fuzzy membership mapped onto interval and many-valued quantities. *Zeitschrift für mathematische Logik und Grundlagen der Mathematik* **22**(1), 149–160 (1976)
8. Jahn, K.U.: Intervall-wertige Mengen. *Mathematische Nachrichten* **68**(1), 115–132 (1975)
9. Sambuc, R.: *Function U-Flous*. University of Marseille, Application à l'aide au Diagnostic en Pathologie Thyroïdienne (1975)
10. Smarandache, F.: Neutrosophy: Neutrosophic probability, set, and logic: Analytic synthesis & synthetic analysis (1998)
11. Torra, V.: Hesitant fuzzy sets. *Int. J. Intell. Syst.* **25**(6), 529–539 (2010)
12. Yager, R.R.: Pythagorean fuzzy subsets. In: *Joint IFSA World Congress and NAFIPS Annual Meeting*, Edmonton, pp. 57–61 (2013)
13. Yager, R.: On the theory of bags. *Int. J. Gen. Syst.* **13**(1), 23–37 (1986)
14. Cường, B.C.: Picture fuzzy sets. *J. Comput. Sci. Cybern.* **30**(4), 409 (2014)
15. Wang, R., Li, Y.: Picture hesitant fuzzy set and its application to multiple criteria decision-making. *Symmetry* **10**(7), 295 (2018)
16. Sarwar Sindhu, M., Rashid, T., Kashif, A.: Modeling of linear programming and extended TOPSIS in decision making problem under the framework of picture fuzzy sets. *PLoS ONE* **14**(8), 14 (2019)
17. Liang, W., Dai, B., Zhao, G., Wu, H.: Performance evaluation of green mine using a combined multi-criteria decision-making method with picture fuzzy information. *IEEE Access* **7**, 174139–174154 (2019)
18. Thao, N.X.: Similarity measures of picture fuzzy sets based on entropy and their application in MCDM. *Pattern Anal. Appl.* **11**, 1–11 (2019). <https://doi.org/10.1007/s10044-019-00861-9>
19. Tian, C., Peng, J., Zhang, S., Zhang, W., Wang, J.: Weighted picture fuzzy aggregation operators and their applications to multi-criteria decision-making problems. *Comput. Industr. Eng.* **137**, 106037 (2019)
20. Bernardo, J.J., Blin, J.M.: A programming model of consumer choice among multi-attributed brands. *J. Consum. Res.* **4**(2), 111 (1977)
21. Razavi Hajiagha, S.H., Shahbazi, M., Amoozad Mahdiraji, H., Panahian, H.: A bi-objective score variance based linear assignment method for group decision making with hesitant fuzzy linguistic term sets. *Technol. Econ. Dev. Econ.* **24**(3), 1125–1148 (2018)
22. Liang, D., Darko, A.P., Xu, Z., Zhang, Y.: Partitioned fuzzy measure-based linear assignment method for Pythagorean fuzzy multi-criteria decision-making with a new likelihood. *J. Oper. Res. Soc.* **15**, 1–15 (2019)
23. Liang, D., Darko, A.P., Xu, Z., Quan, W.: The linear assignment method for multicriteria group decision making based on interval-valued Pythagorean fuzzy Bonferroni mean. *Int. J. Intell. Syst.* **33**(11), 2101–2138 (2018)

24. Chen, T.Y.: The extended linear assignment method for multiple criteria decision analysis based on interval-valued intuitionistic fuzzy sets. *Appl. Math. Model.* **38**(7–8), 2101–2117 (2014)
25. Meksavang, P., Shi, H., Lin, S.M., Li, H.C.: An extended picture fuzzy VIKOR approach for sustainable supplier management and its application in the beef industry. *Symmetry* **11**(4), 468 (2019)