



# Article Numerical Study of the Environmental and Economic System through the Computational Heuristic Based on Artificial Neural Networks

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**Abstract:** In this study, the numerical computation heuristic of the environmental and economic system using the artificial neural networks (ANNs) structure together with the capabilities of the heuristic global search genetic algorithm (GA) and the quick local search interior-point algorithm (IPA), i.e., ANN-GA-IPA. The environmental and economic system is dependent of three categories, execution cost of control standards and new technical diagnostics elimination costs of emergencies values and the competence of the system of industrial elements. These three elements form a nonlinear differential environmental and economic system. The optimization of an error-based objective function is performed using the differential environmental and economic system and its initial conditions. The optimization of an error-based objective function is performed using the differential environmental and economic system and its initial conditions.

**Keywords:** environmental and economic system; interior-point; artificial neural networks; nonlinear model; statistical studies

## 1. Introduction

Management based on the supply chain has attracted the researcher's community in recent years due to wide ranging applications in industrial organizations from raw material to final product distribution to clients. Conceptual investigations on supply chain organization stressed the significance of the strategic associations between corporations in order to increase the operational and financial presentation of these companies to reduce the total inventories and cost in the supply chain. The principle of this association is concerned with matching between the participants [1–3]. Association between industry groups is an increasingly mutual avenue for these societies to maintain and find modest advantage [4,5]. The inter-firm partnering nature in the management of the supply chain was discussed by Mentzer et al. [6]. An increasing quantity of industrial groups started to realize the tactical significance of controlling, designing, and planning, supply chain



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**Copyright:** © 2021 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). systems overall rather than disconnected subsystems collection. Min et al. [7] studied the modeling supply chain processes and identified key opportunities and challenges to model the operations in supply chain networks.

To predict the growth of unified economies, the question arises whether low level economies of growth may not undergo economic losses at that time when the leading economies of the world are suffering harms in the case of economic disaster. The procedures of mathematical design are implemented for the researchers to indicate the systems as a "predator-victim" [8], which allows to form mathematical networks to define their reports. The mathematical representations of the environmental and economic system have three compartments, execution cost of control standards and new technical diagnostics (X), elimination costs of emergencies values (Y), and the competence of the system of industrial elements (Z). The construction of a mathematical differential system describes how to transform the corresponding variables per unit time in order to assume the nature of the quantities relationship, given as [9]:

$$\begin{cases} \frac{dX}{ds} = K_1 X(s)(a - X(s)) - K_2 Y(s) + K_3 Z(s), & X(0) = C_1, \\ \frac{dY}{ds} = K_4 X(s)(a - X(s)) - K_5 Y(s)(b - Y(s)) + K_6 Z(s), & Y(0) = C_2, \\ \frac{dZ}{ds} = K_7 X(s) - K_8 Y(s), & Z(0) = C_3. \end{cases}$$
(1)

In the above system,  $K_i$  shows the values of coefficient constant. The aim of the present study is to investigate the numerical performances of the environmental and economic system using the capabilities of the heuristic global search genetic algorithm (GA) and the quick local search interior-point algorithm (IPA) to optimize the artificial neural networks (ANNs) models, i.e., ANN-GA-IPA. The stochastic computing approaches have great competency to solve the nonlinear differential system. Few recent applications of the stochastic computing solvers are summarized in Table 1.

| Method/Algorithm   | Application   | References |
|--|---|------------|
| Fractional Mayer wavelet ANNs                                      | Nonlinear singular fractional models<br>in astrophysics   | [10,11]    |
| Neuro-Swarm and Neuro-Evolution integrated heuristics              | Nonlinear SITR system of COVID-19 Spread                  | [12,13]    |
| Computational intelligence of FFANN-GASQP                          | Nonlinear mosquito dispersal nonlinear system             | [14]       |
| Integrating solvers via ANN-GA-SQP, ANN-GA-ASA,<br>and ANN-PSO-IPS | Nonlinear singular Lane–Emden or Emden<br>Fowler systems  | [15–17]    |
| Hybrid intelligent mechanism with ANN, Gas, and IPA.               | Differential model of prey-predator system                | [18]       |
| Computational Heuristics of ANN-GA-SQP                             | Dengue fever model representation with nonlinear system   | [19]       |
| Intelligent computing involving FMNEICS                            | Nonlinear doubly singular differential systems            | [20,21]    |
| Self-adaptive global mine blast algorithm                          | Six different dataset representing clustering application | [22]       |
| Supervised and unsupervised Neural networks                        | Different form of ordinary/partial differential equations | [23,24]    |

Table 1. A brief literature review of the stochastic numerical solver application in a variety of fields.

By impressing the cited applications of Table 1, the authors are motivated to solve the environmental and economic nonlinear system by using the computational ANN-GA-IPA. Few novel features of the computational ANN-GA-IPA in terms of objectives of the study are briefly provided as:

• To solve the nonlinear environmental and economic system successfully by novel implementation of the computational numerical heuristics of ANN-GA-IPA.

- To certify the computational procedures of ANN-GA-IPA, the consistent, stable, and robust results with reasonable accuracy should be attained for the environmental and economic nonlinear systems.
- To verify the dependability of the computational procedure of ANN-GA-IPA, the absolute error (AE) levels lie in good ranges from reference state of the art number solution of Adams method.
- To endorsement of the scheme by calculating the different statistical inferences for solving the environmental and economic nonlinear system on multiple autonomous runs of the computational procedure of ANN-GA-IPA.

The remaining parts of the paper parts are categorized as: Section 2 describes the ANN-GA-IPA procedures using the statistical presentations. Section 3 validates the result and discussions. Section 4 shows the concluding remarks and future research reports.

#### 2. Methodology

The methodology section is presented in two phases; firstly, the design of ANN-GA-IPA, and secondly, the application procedure of designed ANN-GA-IPA to the environment and economic system (1). Meanwhile, the performance measures are also provided for better analysis of accuracy of the proposed computational design of ANN-GA-IPA for solving the nonlinear environmental and economic system.

#### 2.1. Design of ANN-GA-IPA

The design of the proposed computing ANN-GA-IPA is presented in terms of ANN topology construction involving the input, output and hidden layers, number of hidden neurons, activation function, parameters influencing the training performance relevant to the global search of GAs and local search of IPA. A good source of reference for setting the appropriate design is provided by Ojha et al., in [25].

The mathematical performances of the nonlinear environmental and economic model are selected in three phases, execution cost of control standards and new technical diagnostics, elimination costs of emergencies values and the competence of the system of industrial elements together with their derivatives in three layers' structure of ANNs in the form of continuous mapping are written as follows: [26–28]

$$[\hat{X}(s), \ \hat{Y}(s), \ \hat{Z}(s)] = \begin{bmatrix} \sum_{\substack{r=1\\k}}^{k} n_{X,r}q(w_{X,r}s + l_{X,r}), \sum_{\substack{r=1\\k}}^{k} n_{Y,r}q(w_{Y,r}s + l_{Y,r}), \\ \sum_{\substack{r=1\\k}}^{k} l_{Z,r}q(w_{Z,r}s + l_{Z,r}) \end{bmatrix},$$

$$[\hat{X}'(s), \ \hat{Y}'(s), \ \hat{Z}'(s)] = \begin{bmatrix} \sum_{\substack{r=1\\k\\r=1\\k\\r=1}}^{k} n_{X,r}q'(w_{X,r}s + l_{X,r}), \sum_{\substack{r=1\\k\\r=1}}^{k} n_{Y,r}q'(w_{Y,r}s + l_{Y,r}), \\ \sum_{\substack{r=1\\k\\r=1}}^{k} n_{Z,r}q'(w_{Z,r}s + l_{Z,r}) \end{bmatrix},$$

$$(2)$$

where *W* is an unknown weight vector with its components depend upon number of reuron *k*. The mathematical representation of *W* is shown mathematically as follows:

$$W = [W_X, W_Y, W_Z], \text{ for } W_X = [n_X, w_X, l_X], W_Y = [n_Y, \omega_Y, l_Y], \text{ and } W_Z = [n_Z, \omega_Z, l_Z], \text{ where}$$
$$n_X = [n_{X,1}, n_{X,2}, \dots, n_{X,r}], \quad n_Y = [n_{Y,1}, n_{Y,2}, \dots, n_{Y,r}], \quad n_Z = [n_{Z,1}, n_{Z,2}, \dots, n_{Z,r}].$$
$$w_X = [w_{X,1}, w_{X,2}, \dots, w_{X,r}], \quad w_Y = [w_{Y,1}, w_{Y,2}, \dots, w_{Y,r}], \quad w_Z = [w_{Z,1}, w_{Z,2}, \dots, w_{Z,r}],$$
$$l_X = [l_{X,1}, l_{X,2}, \dots, l_{X,r}], \quad l_Y = [l_{Y,1}, l_{Y,2}, \dots, l_{Y,r}], \quad l_Z = [l_{Z,1}, l_{Z,2}, \dots, l_{Z,r}].$$

A large verity of activation functions, we have chosen Log-Sigmoid  $q(s) = (1 + e^{-s})^{-1}$  in the presented study and applied to Equation (2), then we have:

$$[\hat{X}(s), \hat{Y}(s), \hat{Z}(s)] = \begin{bmatrix} \sum_{r=1}^{m} \frac{n_{X,r}}{1+e^{-(w_{X,r}s+l_{X,r})}}, \frac{n_{Y,r}}{1+e^{-(w_{Y,r}s+l_{Y,r})}}, \\ \sum_{r=1}^{m} \frac{n_{Z,r}}{1+e^{-(w_{Z,r}s+l_{Z,r})}}, \end{bmatrix}, \\ [\hat{X}'(s), \hat{Y}'(s), \hat{Z}'(s)] = \begin{bmatrix} \sum_{r=1}^{m} \frac{n_{X,r}w_{X,r}e^{-(w_{X,r}s+i_{X,r})}}{(1+e^{-(w_{X,r}s+l_{X,r})})^{2}}, \sum_{r=1}^{m} \frac{n_{Y,r}w_{Y,r}e^{-(w_{Y,r}s+i_{Y,r})}}{(1+e^{-(w_{Y,r}s+l_{Y,r})})^{2}}, \\ \sum_{r=1}^{m} \frac{n_{Z,r}w_{Z,r}e^{-(w_{Z,r}s+i_{Z,r})}}{(1+e^{-(w_{Z,r}s+l_{Z,r})})^{2}} \end{bmatrix}.$$
(3)

The appropriate arrangement of networks in Equation (3) can be exploited to solve differential systems as represented in (1) with the availability of suitable *W*. The ANNs parameters are optimized with combined strength of global and local search methodologies of GAs and IPA, i.e., ANN-GA-IPA.

GA is a famous global search procedure of optimization work to solve both linear and nonlinear systems. GA is typically used to control the results of the precise population for solving various steep/complicated models based on optimal training. In recent years, GA is implemented in the shotgun metabolomics [29], wellhead back pressure control system [30], bearing fault diagnosis of induction motors [31], energy efficient clustered wireless sensor networks [32], beam deflection monitoring systems [33], adjustment problem of sensor acquisition frequency [34], image processing optimization tasks [35], and torque adjustment for the ankle push-off in the walking bipedal robots [36]. The optimization performance in terms of efficiency, accuracy, and viability of GAs is further enhanced by introducing the concept of hybridization with efficient local search.

*Interior point algorithm* is a one of the local search optimization approaches generally applied to solve both types of constrained/unconstrained models in optimization tasks. It is a well-organized algorithm used to compute the results competently. Recently, IPA is a reliable treatment of the economic load dispatch problem [37], dynamic adjustments of step sizes and tolerances [38], active noise control systems [39], convex quadratic programming [40], and optimization of models representing the dynamics of heartbeat [41].

#### 2.2. Application ANN-GA-IPA to Environment and Economic System

In this section, the application procedure of ANN-GA-IPA is presented for the environmental and economic nonlinear model in terms of fitness function developments, formulation of pseudo code, and workflow of the procedural steps.

An objective function is presented as:

$$E = \sum_{r=i}^{4} E_r \tag{4}$$

$$E_1 = \frac{1}{N} \sum_{r=1}^{N} \left[ \hat{T}'_r + a \hat{F}_r \hat{T}_r + b \right]^2,$$
(5)

$$E_2 = \frac{1}{N} \sum_{r=1}^{N} \left[ \hat{F}'_r - d\hat{T}_r - c\hat{M}_r \hat{F}_r \right]^2,$$
(6)

$$E_{3} = \frac{1}{N} \sum_{r=1}^{N} \left[ \hat{M}_{r}' + d\hat{T}_{r} - b\hat{F}_{r}\hat{M}_{r} + \delta \right]^{2},$$
(7)

$$E_4 = \frac{1}{3} \Big[ \left( \hat{T}_0 - r_1 \right)^2 + \left( \hat{F}_0 - r_2 \right)^2 + \left( \hat{M}_0 - r_3 \right)^2 \Big], \tag{8}$$

where  $\hat{X}_r = X(s_r)$ ,  $\hat{Y}_r = Y(s_r)$ ,  $\hat{Z}_r = Z(s_r)$ , Nh = 1, and  $s_r = hr$ ,  $\hat{X}_r$ ,  $\hat{Y}_r$  and  $\hat{Z}_r$  indicate the proposed outcomes of ANN-GA-IPA for execution cost of control standards and new

technical diagnostics, elimination costs of emergencies values, and the competence of the system of industrial elements, respectively, i.e., indicator of environmental and economic systems. Accordingly, Equations (5)–(7) signify objective functions using indicators of nonlinear environmental and economic models while Equation (8) represents an objective function based on the initial conditions.

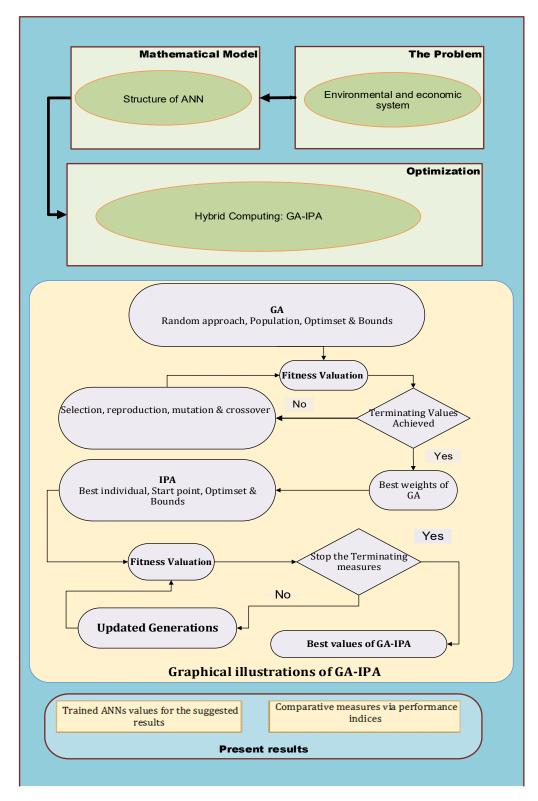
The optimization of the ANN model-based fitness function (5)–(8) conducted initially with GAs for the global search and performance of GAs by mean of efficiency in computational time is further enhanced by the procedure of IPA-based rapid local search. In other words, heuristic GA-IPA is implemented for the global search (exploration, GA) and effectiveness to exploit a solution (IPA, intensification), for finding the decision variables of ANN models of the environmental and economic system. The pseudo code for implementation of GA-IPA is narrated in Algorithm 1, while the procedural steps involved in implementation of ANN-GA-IPA are shown in Figure 1 for solving the environmental and economic systems.

**Algorithm 1.** Pseudo code for optimization for the environmental and economic nonlinear model by ANN-GA-IPA.

Start GA

**Inputs:** To measure the chromosomes of the same network element as: W = [n, w, l]Population: Set of chromosome is given as:  $W_X = [n_X, \omega_X, l_X], W_Y = [n_Y, \omega_Y, l_Y] \text{ and } W_Z = [n_Z, \omega_Z, l_Z].$ Output: Global weight values are W<sub>GB</sub> Initialization: To adjust the chromosomes selection, adjust the W<sub>GB</sub>. Fit Estimation: Modify the values of FIT (*E*) using the population (P) for systems 4 to 8 **Stopping standards:** Terminate if [Iterations = 60],  $[E = 10^{-06}]$ , [TolCon =  $10^{-10}$ ], [StallLimit = 120], [TolFun  $= 10^{-10}$ ] & [PopSize = 200]. Go to storage **Ranking:** For the FIT (*E*), rank *W*<sub>GB</sub> in population. Storage: Store W<sub>GB</sub>, time, iterations, *E* & count of function for the GA. **GA** process Ends **IPA Starts Inputs:** *W*<sub>GB</sub> is the Start point: **Output:** The best GAIPA weights are represented as  $W_{\text{GIPA}}$ **Initialize:** Iterations, Assignments &  $W_{GB}$ . **Terminating Standards:** Stop, if  $[E = 10^{-12}]$ , [MaxFunEvals = 100,000], [TolX =  $10^{-12}$ ], [TolFun  $= 10^{-12}$ ] & [Iterations = 500]. FIT approximation: Compute FIT & W<sub>GIPA</sub> using Equations (4) to (8). Amendments: Regulate 'fmincon' for the values of IPA, E to improve the 'W' for Equations (4)–(8). Accumulate: Transform WGIPA, function counts, time FIT, iterations for the IPA present runs.

IPA End



**Figure 1.** Design procedure of ANN-GA-IPA to solve the nonlinear environmental and economic model.

#### 2.3. Performance Measures

The statistical measures using the mean absolute deviation (MAD), semi interquartile range (S.I.R), variance account for (VAF), and Theil's inequality coefficient (TIC) using

$$[MAD_X, MAD_Y, MAD_Z] = \left[\sum_{r=1}^n |X_r - \hat{X}_r|, \sum_{r=1}^n |Y_r - \hat{Y}_r|, \sum_{r=1}^n |Z_r - \hat{Z}_r|\right],$$
(9)

$$\begin{cases} S.I.R = -\frac{1}{2} \times (q_1 - q_3), \\ q_1 = 1st \text{ quartile } \& q_3 = 3rd \text{ quartile,} \end{cases}$$
(10)

$$[\text{TIC}_{X}, \text{TIC}_{Y}, \text{TIC}_{Z}] = \begin{bmatrix} \frac{\sqrt{\frac{1}{n}\sum_{r=1}^{n} (X_{r} - \hat{X}_{r})^{2}}}{\left(\sqrt{\frac{1}{n}\sum_{r=1}^{n} \hat{X}_{r}^{2} + \sqrt{\frac{1}{n}\sum_{r=1}^{n} \hat{X}_{r}^{2}}\right)}, \\ \frac{\sqrt{\frac{1}{n}\sum_{r=1}^{n} (Y_{r} - \hat{Y}_{r})^{2}}}{\left(\sqrt{\frac{1}{n}\sum_{r=1}^{n} \hat{Y}_{r}^{2} + \sqrt{\frac{1}{n}\sum_{r=1}^{n} \hat{Y}_{r}^{2}}\right)}, \\ \frac{\sqrt{\frac{1}{n}\sum_{r=1}^{n} (Z_{r} - \hat{Z}_{r})^{2}}}{\left(\sqrt{\frac{1}{n}\sum_{r=1}^{n} \hat{Z}_{r}^{2} + \sqrt{\frac{1}{n}\sum_{r=1}^{n} \hat{Z}_{r}^{2}}\right)}, \end{bmatrix}, \qquad (11)$$

$$[\text{VAF}_{X}, \text{VAF}_{Y}, \text{VAF}_{Z}] = \begin{bmatrix} \left(1 - \frac{\text{var}(X_{r} - \hat{X}_{r})}{\text{var}(X_{r})}\right) * 100, \\ \left(1 - \frac{\text{var}(Y_{r} - \hat{Y}_{r})}{\text{var}(Z_{r})}\right) * 100, \\ \left(1 - \frac{\text{var}(Z_{r} - \hat{Z}_{r})}{\text{var}(Z_{r})}\right) * 100, \\ \left(1 - \frac{\text{var}(Z_{r} - \hat{Z}_{r})}{\text{var}(Z_{r})}\right) * 100, \\ \end{bmatrix}$$

$$[\text{EVAF}_{X}, \text{EVAF}_{Y}, \text{EVAF}_{Z}] = [|100 - \text{VAF}_{X}, 100 - \text{VAF}_{Y}, 100 - \text{VAF}_{Z}|].$$

where  $\hat{X}$ ,  $\hat{Y}$ , and  $\hat{Z}$  signify the approximate results.

#### 3. Results of Simulations

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In this section, the environmental and economic nonlinear model presented in the system (1) is numerically performed by using the computational ANN-GA-IPA. The obtained numerical outcomes of the environmental and economic nonlinear model are compared with the Adams result. The plots of AE, convergence analysis, and performance measures through different operatives are also presented. The simplified form of the environmental and economic nonlinear model using appropriate parameters is given as:

$$\begin{cases} \frac{dX}{ds} = 0.2X(s)(10 - X(s)) - 0.3Y(s) + 0.4Z(s), \quad X(0) = 2, \\ \frac{dY}{ds} = 0.2X(s)(10 - X(s)) + 0.3Y(s)(5 - Y(s)) + 0.3Z(s), \quad Y(0) = 4, \\ \frac{dZ}{ds} = 0.4X(s) - 0.3Y(s), \quad Z(0) = 3. \end{cases}$$
(13)

An objective function for the environmental and economic nonlinear model given in Equation (13) is written as:

$$E = \frac{1}{N} \sum_{r=1}^{N} \begin{pmatrix} \left[ \hat{X}'_r - 0.2 \hat{X}_r (10 - \hat{X}_r) + 0.3 \hat{Y}_r - 0.4 \hat{Z}_r \right]^2 + \\ \left[ \hat{Y}'_r - 0.2 \hat{X}_r (10 - \hat{X}_r) - 0.3 \hat{Y}_r (5 - \hat{Y}_r) - 0.3 \hat{Z}_r \right]^2 \\ + \left[ \hat{Z}'_r - 0.4 X_r + 0.3 \hat{Y}_r \right]^2 \\ + \frac{1}{3} \left[ \left( \hat{X}_0 - 2 \right)^2 + \left( \hat{Y}_0 - 4 \right)^2 + \left( \hat{Z}_0 - 3 \right)^2 \right].$$
(14)

The environmental and economic nonlinear model given in Equation (1) is applied to optimize the computational GAIPA for the ANN parameters using 30 number of variables

with the step size of 0.05. The best weight vectors are derived for solving the environmental and economic nonlinear model in the below equations are given as:

$$\begin{split} \hat{X}(s) &= \frac{2.4095}{1+e^{-(1.6650s-0.4688)}} - \frac{2.3755}{1+e^{-(2.0171s-2.8447)}} - \frac{-2.4751}{1+e^{-(0.2022s-2.0837)}} - \frac{-0.2371}{1+e^{-(0.2022s-2.0837)}} - \frac{-0.2371}{1+e^{-(0.2022s-2.0837)}} - \frac{-0.2371}{1+e^{-(0.2022s-0.7601)}} - \frac{-0.2371}{1+e^{-(0.2420s-0.7601)}} + \end{split}$$
(15)  
$$\begin{aligned} &\frac{2.4625}{1+e^{-(1.5802s-1.2805)}} - \frac{-0.7465}{1+e^{-(1.3418s-1.1636)}} + (15) \\ &\frac{-0.7051}{1+e^{-(0.0306s-1.0296)}} + \frac{1.9644}{1+e^{-(2.4220s-1.8985)}}, \end{aligned}$$
$$\hat{Y}(s) &= \frac{3.1478}{1+e^{-(2.7157s-0.0170)}} - \frac{1.4241}{1+e^{-(2.4220s-1.8985)}}, \\ &\frac{-0.7051}{1+e^{-(0.9108s-0.9828)}} - \frac{2.3612}{1+e^{-(2.2731s-0.1898)}} - \frac{2.3612}{1+e^{-(2.2731s-0.1898)}} - \frac{2.3612}{1+e^{-(2.2731s-0.1898)}}, \\ &\frac{1.7418}{1+e^{-(0.92288+1.0432)}} - \frac{0.0728}{1+e^{-(-0.0701s-0.7089)}} - \frac{0.4869}{1+e^{-(-0.2474s-0.4771)}} + \frac{1.1565}{1+e^{-(-0.0288s-1.0433)}} - \frac{0.0778}{1+e^{-(0.0385s-0.6263)}} - \frac{1.0697}{1+e^{-(0.0385s-0.6263)}} - \frac{0.2788}{1+e^{-(0.0383s-0.62263)}} - \frac{0.2788}{1+e^{-(0.0383s+0.6428)}} - \frac{0.2788}{1+e^{-(0.0383s+0.6228)}} - \frac{0.2788}{1+e^{-(0.0383s+0.6228)}} - \frac{0.177}{1+e^{-(0.033s+0.6628)}} - \frac{0.3777}{1+e^{-(0.020s+1.1699)}} + \frac{0.3777}{1+e^{-(0.030s+0.42977)}}, \end{split}$$

where,  $\hat{X}$ ,  $\hat{Y}$ , and  $\hat{Z}$  are the approximate results of Equation (13) for the environmental and economic system (1) by ANN-GA-IPA using the best weights of ANNs in the first equation of set (2). Figures 2–4 demonstrate the best weights performance, comparison of the results, and AE values to solve the nonlinear environmental and economic system using the computational performance of ANN-GA-IPA.

We conducted the implementation of the proposed integrated metaheuristic of ANN-GA-IPA by variants of parameters, i.e.,  $K_i$  for i = 1, 2, ..., 8, and initial conditions, i.e.,  $C_i$  for i = 1, 2, 3 with almost similar objective function as shown in Equation (14) and accuracy of the results are found in the similar range/levels of precision as that of the problem of environmental and economic systems presented in (13). Therefore, to avoid the redundant representation of illustrations, we confined in this study to present the results of ANN-GA-IPA for problem (13) for single and multiple autonomous execution of ANN-GA-IPA for effective remarks on accuracy, convergence, stability, and robustness.

The best weight values for the environmental and economic nonlinear model have been established in Figure 1a-c for 10 neurons and 30 variables. These weight vectors plots are demonstrated in the above Equations (15)–(17). The comparison of the result's performance for the control standards and new technical diagnostics, elimination costs of emergencies values, and the competence of the system of industrial elements based on the environmental and economic nonlinear model is provided in Figure 1d-f. The best and mean outcomes are derived using the ANN-GA-IPA based on the nonlinear environmental and economic system. The precise performance of the computational ANN-GA-IPA is observed for each class of the nonlinear environmental and economic model. The AE plots are derived in Figure 4a-c for each class of the environmental and economic nonlinear model. It is stated that the AE based on mean and best results is found in good trials. One can find that the best AE of the X(s), Y(s), and Z(s) based on the nonlinear environmental and economic model lie around 10-06-10-08, 10-04-10-06, and 10-05-10-08. The AE mean values for *X*(*s*), *Y*(*s*), and *Z*(*s*) lie 10-04-10-05, 10-04-10-06, and 10-05-10-06. This very good range of AE enhances the worth of the computational ANN-GA-IPA. The performance through EVAF, TIC, and MAD operators is observed in Figure 4. One can observe that the EVAF values for the control standards and new technical diagnostics, elimination costs of emergencies values, and the competence of the system of industrial elements based on the environmental and economic nonlinear model lie 10-04-10-05, 10-10-10-13, and 10-07-10-08.

Best Mean Exact Values Values 0.4 Inputs 1 L 0 0.2 0.6 0.8 (a) Best weights for the class X(s)(**d**) Comparison for the class X(s)7.5 Best 7 Mean Exact 6.5 Values Values 5.5 3.5 ∟ 0 Neurons 0.2 0.4 0.6 0.8 8 1 9 10 Inputs 2 (**b**) Best weights for the class Y(s)(e) Comparison for the class Y(s)– – Best 2.95 Mean Exact Values 2.9 Values 2.85 2.8 2.75 2.7 <mark>|</mark> 0 0.2 0.4 0.6 0.8 9 1 10 Inputs (c) Best weights for the class Z(s)(**f**) Comparison for the class Z(s)

The MAD values for these classes lie around 10-04-10-05. The TIC values for these classes lie around 10-08-10-09, 10-09-10-10, and 10-07-10-09.

Figure 2. Best values of the weights vectors and results comparison to solve the environmental and economic nonlinear model.

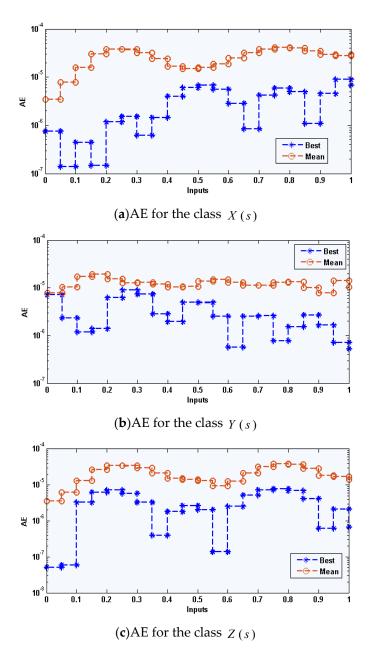
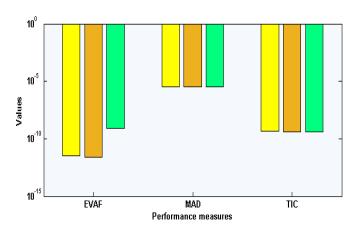


Figure 3. AE values for each class of the environmental and economic nonlinear model.



**Figure 4.** Performance indices through MAD, TIC, and EVAF for solving the environmental and economic nonlinear model.

The graphical representations of the statistical operators along with the performances of histograms/boxplot are illustrated in Figures 5–7 for solving each class of the environmental and economic nonlinear model. The convergence plots through the TIC, MAD, and EVAF operators for solving each category of the environmental and economic nonlinear model. One can see that the TIC performances lie around 10-08-10-09, 10-08-10-10, and 10-09-10-10. The MAD performances lie around 10-04-10-05, 10-04-10-06, and 10-05-10-06. Likewise, the EVAF performances lie around 10-09-10-11, 10-10-12, and 10-07-10-09. The achieved best performances via ANN-GA-IPA are calculated appropriately for the operators TIC, MAD, and EVAF.

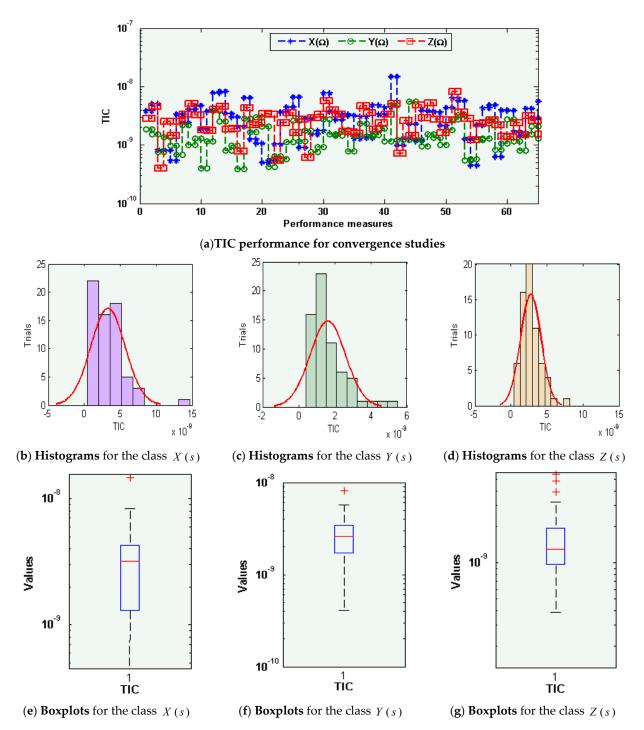


Figure 5. TIC operator performances based on ANN-GA-IPA to solve the environmental and economic nonlinear model.

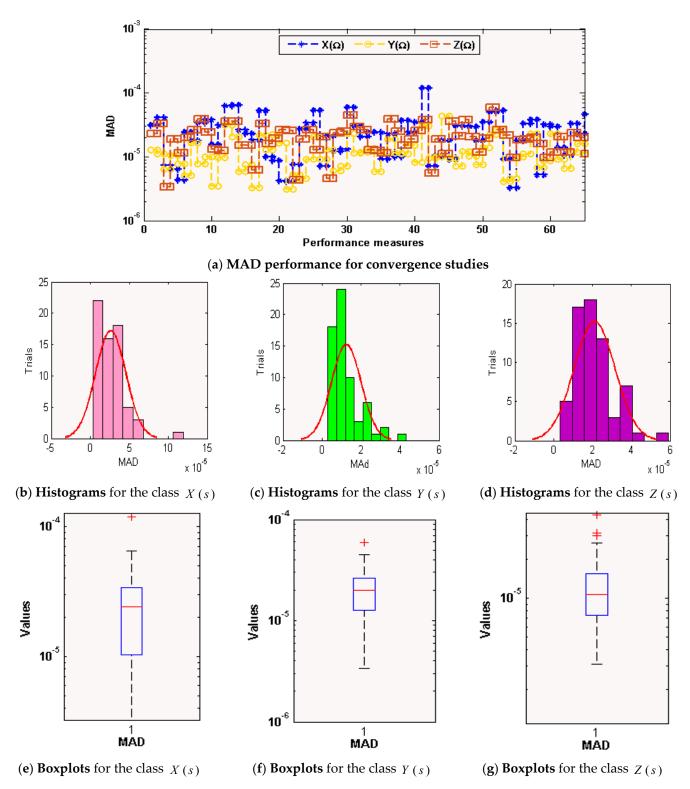


Figure 6. MAD operator performances based on ANN-GA-IPA to solve the environmental and economic nonlinear model.

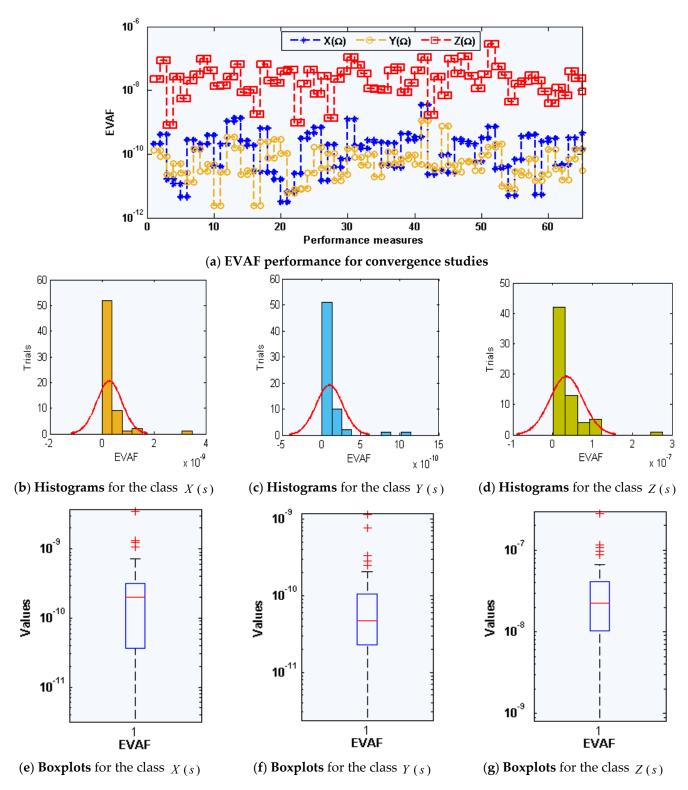


Figure 7. EVAF operator performances based on ANN-GA-IPA to solve the environmental and economic nonlinear model.

For accuracy performance, statistical studies are provided in Tables 2–4 to solve the environmental and economic nonlinear model using the statistical operators based on standard deviation (SD), median (MED), S.I.R, minimum (Min), and maximum (Max). The Min values show the best performances lie around 10-07-10-09, while the Max values specify the worst result lie 10-04-10-05, the MED, Mean, S.I.R, and STD performances lie 10-05-10-06 for each class of the environmental and economic nonlinear model. One can

realize performance worth through ANN-GA-IPA based on these statistical operator values lie around, in good measures, to solve the environmental and economic nonlinear model.

| Table 2. Statistical presentations of the environmental and economic nonlinear model for 2 | X(s). |
|--|-------|
|  |       |

|      | X(s)                   |                         |                         |                         |                         |                         |
|------|------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| S    | MIN                    | MAX                     | MED                     | MEAN                    | S.I.R                   | STD                     |
| 0    | $7.0743 	imes 10^{-8}$ | $4.7294 	imes 10^{-5}$  | $1.1257 \times 10^{-6}$ | $3.4587 	imes 10^{-6}$  | $1.4585 	imes 10^{-6}$  | $7.1764 \times 10^{-6}$ |
| 0.05 | $4.3791 	imes 10^{-9}$ | $6.2691 	imes 10^{-5}$  | $4.2221 \times 10^{-6}$ | $7.8281 	imes 10^{-6}$  | $2.7543 	imes 10^{-6}$  | $1.1642 	imes 10^{-5}$  |
| 0.1  | $4.3980	imes10^{-7}$   | $8.6225 \times 10^{-5}$ | $1.3970 	imes 10^{-5}$  | $1.5741\times10^{-5}$   | $5.5506	imes10^{-6}$    | $1.3619	imes10^{-5}$    |
| 0.15 | $1.4628	imes10^{-7}$   | $8.6543 	imes 10^{-5}$  | $3.1742 \times 10^{-5}$ | $2.9933 	imes 10^{-5}$  | $1.4389\times10^{-5}$   | $1.9541 \times 10^{-5}$ |
| 0.2  | $4.2069 	imes 10^{-7}$ | $1.2099\times10^{-4}$   | $3.8358 \times 10^{-5}$ | $3.7281 \times 10^{-5}$ | $2.2491 	imes 10^{-5}$  | $2.9381 	imes 10^{-5}$  |
| 0.25 | $1.0399 	imes 10^{-7}$ | $1.6466\times 10^{-4}$  | $3.4508	imes10^{-5}$    | $3.7597 	imes 10^{-5}$  | $2.5150 	imes 10^{-5}$  | $3.3362 	imes 10^{-5}$  |
| 0.3  | $5.8136 	imes 10^{-7}$ | $1.8272	imes10^{-4}$    | $2.6808 \times 10^{-5}$ | $3.2238 \times 10^{-5}$ | $2.0967 \times 10^{-5}$ | $3.1584 \times 10^{-5}$ |
| 0.35 | $2.0488 	imes 10^{-7}$ | $1.7620 	imes 10^{-4}$  | $1.7084 \times 10^{-5}$ | $2.4104\times10^{-5}$   | $1.4178\times 10^{-5}$  | $2.7355 \times 10^{-5}$ |
| 0.4  | $4.0904	imes10^{-7}$   | $1.5131	imes10^{-4}$    | $9.1530 	imes 10^{-6}$  | $1.6722 	imes 10^{-5}$  | $8.2770 	imes 10^{-6}$  | $2.4136	imes10^{-5}$    |
| 0.45 | $2.4855 	imes 10^{-7}$ | $1.1703	imes10^{-4}$    | $8.3726 \times 10^{-6}$ | $1.4900 	imes 10^{-5}$  | $4.5145	imes10^{-6}$    | $2.1680 	imes 10^{-5}$  |
| 0.5  | $6.3360 	imes 10^{-7}$ | $1.2621	imes10^{-4}$    | $8.7141 	imes 10^{-6}$  | $1.5906 \times 10^{-5}$ | $5.5567 	imes 10^{-6}$  | $2.0310 	imes 10^{-5}$  |
| 0.55 | $8.4288	imes10^{-8}$   | $1.2090	imes10^{-4}$    | $1.3416 \times 10^{-5}$ | $1.8517	imes10^{-5}$    | $6.4564	imes10^{-6}$    | $1.9342 	imes 10^{-5}$  |
| 0.6  | $8.3379 	imes 10^{-7}$ | $9.8065 	imes 10^{-5}$  | $2.1939 	imes 10^{-5}$  | $2.4509 \times 10^{-5}$ | $6.9207 	imes 10^{-6}$  | $1.8620 \times 10^{-5}$ |
| 0.65 | $2.2875 	imes 10^{-7}$ | $9.6707 	imes 10^{-5}$  | $3.2586 \times 10^{-5}$ | $3.1577 \times 10^{-5}$ | $1.5737 	imes 10^{-5}$  | $2.1481 	imes 10^{-5}$  |
| 0.7  | $1.8333	imes10^{-6}$   | $1.1287	imes10^{-4}$    | $3.9546 \times 10^{-5}$ | $3.7769 	imes 10^{-5}$  | $1.9937 	imes 10^{-5}$  | $2.6668 	imes 10^{-5}$  |
| 0.75 | $4.3106 	imes 10^{-7}$ | $1.2949\times10^{-4}$   | $4.2578 \times 10^{-5}$ | $4.0906 \times 10^{-5}$ | $2.3594 	imes 10^{-5}$  | $3.1227 \times 10^{-5}$ |
| 0.8  | $7.7704 	imes 10^{-8}$ | $1.5822 	imes 10^{-4}$  | $3.9862 \times 10^{-5}$ | $3.9923 	imes 10^{-5}$  | $2.2828\times 10^{-5}$  | $3.2959 \times 10^{-5}$ |
| 0.85 | $1.7169 	imes 10^{-7}$ | $1.9908	imes10^{-4}$    | $3.3272 \times 10^{-5}$ | $3.5134	imes10^{-5}$    | $1.9972 	imes 10^{-5}$  | $3.2436 	imes 10^{-5}$  |
| 0.9  | $8.7346 	imes 10^{-7}$ | $2.2773 	imes 10^{-4}$  | $2.2861 \times 10^{-5}$ | $2.9667 	imes 10^{-5}$  | $1.3509 \times 10^{-5}$ | $3.2102 \times 10^{-5}$ |
| 0.95 | $9.5136 	imes 10^{-7}$ | $2.3451	imes10^{-4}$    | $2.0569 \times 10^{-5}$ | $2.7424 \times 10^{-5}$ | $1.1005 	imes 10^{-5}$  | $3.3018	imes10^{-5}$    |
| 1    | $9.5870 	imes 10^{-7}$ | $2.1077\times10^{-4}$   | $2.3423\times10^{-5}$   | $2.8972 \times 10^{-5}$ | $1.1057\times10^{-5}$   | $3.0340\times10^{-5}$   |

**Table 3.** Statistical presentations of the environmental and economic nonlinear model for Y(s).

| _    | Y(s)                    |                         |                         |                         |                         |                         |
|------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| S    | MIN                     | MAX                     | MED                     | MEAN                    | S.I.R                   | STD                     |
| 0    | $9.1378 	imes 10^{-9}$  | $1.0984\times 10^{-4}$  | $2.792 \times 10^{-6}$  | $3.4587 	imes 10^{-6}$  | $3.6245 	imes 10^{-6}$  | $1.5545 \times 10^{-5}$ |
| 0.05 | $2.4578 	imes 10^{-7}$  | $9.9356 	imes 10^{-5}$  | $5.3448	imes10^{-6}$    | $7.8281 	imes 10^{-6}$  | $5.2596 	imes 10^{-6}$  | $1.4870 	imes 10^{-5}$  |
| 0.1  | $9.3748 	imes 10^{-7}$  | $8.3141 	imes 10^{-5}$  | $1.5007 \times 10^{-5}$ | $1.5741 \times 10^{-5}$ | $8.8015	imes10^{-6}$    | $1.5840 \times 10^{-5}$ |
| 0.15 | $2.0260 \times 10^{-7}$ | $6.8842 	imes 10^{-5}$  | $1.7456 \times 10^{-5}$ | $2.9933 	imes 10^{-5}$  | $1.2079 	imes 10^{-5}$  | $1.4222 \times 10^{-5}$ |
| 0.2  | $1.8344	imes10^{-7}$    | $5.8221 \times 10^{-5}$ | $1.2474 	imes 10^{-5}$  | $3.7281 \times 10^{-5}$ | $7.1484	imes10^{-6}$    | $1.3280 \times 10^{-5}$ |
| 0.25 | $1.6102 \times 10^{-7}$ | $6.3211 \times 10^{-5}$ | $8.8643 	imes 10^{-6}$  | $3.7597 \times 10^{-5}$ | $5.8968 	imes 10^{-6}$  | $1.2155 \times 10^{-5}$ |
| 0.3  | $3.7975 	imes 10^{-7}$  | $5.1710 \times 10^{-5}$ | $9.2806 \times 10^{-6}$ | $3.2238 \times 10^{-5}$ | $6.8494	imes10^{-6}$    | $1.1762 \times 10^{-5}$ |
| 0.35 | $3.2068 	imes 10^{-8}$  | $7.5259 \times 10^{-5}$ | $7.3542 \times 10^{-6}$ | $2.4104 	imes 10^{-5}$  | $6.8017	imes10^{-6}$    | $1.3281 \times 10^{-5}$ |
| 0.4  | $2.7219 \times 10^{-7}$ | $9.0351 \times 10^{-5}$ | $7.9833 \times 10^{-6}$ | $1.6722 \times 10^{-5}$ | $5.9056 \times 10^{-6}$ | $1.2623 \times 10^{-5}$ |
| 0.45 | $1.4999 	imes 10^{-7}$  | $7.8272 \times 10^{-5}$ | $7.8305 \times 10^{-6}$ | $1.4900 	imes 10^{-5}$  | $5.3639 	imes 10^{-6}$  | $1.1559 \times 10^{-5}$ |
| 0.5  | $8.1631	imes10^{-8}$    | $4.4175 	imes 10^{-5}$  | $1.1492 	imes 10^{-5}$  | $1.5906 	imes 10^{-5}$  | $7.7188	imes10^{-6}$    | $1.1044 	imes 10^{-5}$  |
| 0.55 | $6.7348 	imes 10^{-7}$  | $4.4434 	imes 10^{-5}$  | $1.2222 \times 10^{-5}$ | $1.8517 \times 10^{-5}$ | $8.0944 	imes 10^{-6}$  | $1.1129 \times 10^{-5}$ |
| 0.6  | $5.6913 	imes 10^{-7}$  | $4.5399 \times 10^{-5}$ | $9.5161 \times 10^{-6}$ | $2.4509 \times 10^{-5}$ | $8.2024 	imes 10^{-6}$  | $1.1023 \times 10^{-5}$ |
| 0.65 | $4.0292 \times 10^{-7}$ | $5.9049 	imes 10^{-5}$  | $9.1083	imes10^{-6}$    | $3.1577 \times 10^{-5}$ | $5.4083	imes10^{-6}$    | $9.9166 	imes 10^{-6}$  |
| 0.7  | $9.9417 	imes 10^{-9}$  | $5.7471 \times 10^{-5}$ | $8.2148 	imes 10^{-6}$  | $3.7769 \times 10^{-5}$ | $5.4840 	imes 10^{-6}$  | $1.1546 \times 10^{-5}$ |
| 0.75 | $2.6077 	imes 10^{-7}$  | $5.8973 \times 10^{-5}$ | $9.4506 \times 10^{-6}$ | $4.0906 \times 10^{-5}$ | $6.2179 	imes 10^{-6}$  | $1.3187 \times 10^{-5}$ |
| 0.8  | $1.8827	imes10^{-7}$    | $5.3973 	imes 10^{-5}$  | $9.7485 	imes 10^{-6}$  | $3.9923 \times 10^{-5}$ | $6.0827	imes10^{-6}$    | $1.1980	imes10^{-5}$    |
| 0.85 | $3.9154 	imes 10^{-7}$  | $4.1456 \times 10^{-5}$ | $7.5585 \times 10^{-6}$ | $3.5134 \times 10^{-5}$ | $4.7085 	imes 10^{-6}$  | $8.5556 \times 10^{-6}$ |
| 0.9  | $5.4210 	imes 10^{-7}$  | $6.6373 	imes 10^{-5}$  | $5.8519\times10^{-6}$   | $2.9667 	imes 10^{-5}$  | $3.3879 	imes 10^{-6}$  | $9.4999 	imes 10^{-6}$  |
| 0.95 | $4.0074	imes10^{-7}$    | $6.4268 	imes 10^{-5}$  | $1.0509 	imes 10^{-5}$  | $2.7424 \times 10^{-5}$ | $5.0057 	imes 10^{-6}$  | $1.2343 \times 10^{-5}$ |
| 1    | $4.5080 	imes 10^{-7}$  | $3.3751\times10^{-5}$   | $8.2858 	imes 10^{-6}$  | $2.8972 \times 10^{-5}$ | $4.1627\times 10^{-6}$  | $8.0614\times10^{-6}$   |

|      | $\mathbf{Z}(s)$         |                         |                         |                         |                         |                         |
|------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| S    | MIN                     | MAX                     | MED                     | MEAN                    | S.I.R                   | STD                     |
| 0    | $1.3315 \times 10^{-9}$ | $3.1457 \times 10^{-5}$ | $1.6205 \times 10^{-6}$ | $3.4587 	imes 10^{-6}$  | $2.1651 	imes 10^{-6}$  | $5.1288 \times 10^{-6}$ |
| 0.05 | $5.3875	imes10^{-8}$    | $2.3622 \times 10^{-5}$ | $4.3838	imes10^{-6}$    | $7.8281 	imes 10^{-6}$  | $4.2351	imes10^{-6}$    | $5.5084	imes10^{-6}$    |
| 0.1  | $7.6305 	imes 10^{-10}$ | $4.2407 	imes 10^{-5}$  | $1.1246 \times 10^{-5}$ | $1.5741 \times 10^{-5}$ | $4.8690 	imes 10^{-6}$  | $9.0944 	imes 10^{-6}$  |
| 0.15 | $2.5612 	imes 10^{-7}$  | $7.2251 	imes 10^{-5}$  | $2.5533 \times 10^{-5}$ | $2.9933 	imes 10^{-5}$  | $1.1121\times 10^{-5}$  | $1.6681\times 10^{-5}$  |
| 0.2  | $6.2359 	imes 10^{-7}$  | $1.0789 	imes 10^{-4}$  | $3.4484	imes10^{-5}$    | $3.7281 \times 10^{-5}$ | $1.6162\times10^{-5}$   | $2.2699 \times 10^{-5}$ |
| 0.25 | $2.9525 	imes 10^{-6}$  | $1.2191	imes10^{-4}$    | $3.3155 	imes 10^{-5}$  | $3.7597 	imes 10^{-5}$  | $1.7878 	imes 10^{-5}$  | $2.4080 \times 10^{-5}$ |
| 0.3  | $4.4801	imes10^{-7}$    | $1.1325	imes10^{-4}$    | $2.4715 	imes 10^{-5}$  | $3.2238 \times 10^{-5}$ | $1.4273 	imes 10^{-5}$  | $2.2405 	imes 10^{-5}$  |
| 0.35 | $7.2504 	imes 10^{-9}$  | $8.6395 	imes 10^{-5}$  | $1.5573 \times 10^{-5}$ | $2.4104 \times 10^{-5}$ | $1.0933 \times 10^{-5}$ | $1.9125 	imes 10^{-5}$  |
| 0.4  | $3.9256 	imes 10^{-8}$  | $7.9942 	imes 10^{-5}$  | $8.9671 	imes 10^{-6}$  | $1.6722 \times 10^{-5}$ | $9.3565 	imes 10^{-6}$  | $1.6111\times10^{-5}$   |
| 0.45 | $1.6522 	imes 10^{-8}$  | $6.5359 	imes 10^{-5}$  | $9.6346 	imes 10^{-6}$  | $1.4900 	imes 10^{-5}$  | $7.2259 	imes 10^{-6}$  | $1.3588	imes10^{-5}$    |
| 0.5  | $1.9922 	imes 10^{-7}$  | $6.3306 \times 10^{-5}$ | $8.7388 	imes 10^{-6}$  | $1.5906 \times 10^{-5}$ | $7.2795 	imes 10^{-6}$  | $1.2883 	imes 10^{-5}$  |
| 0.55 | $1.2826 	imes 10^{-7}$  | $8.4377 	imes 10^{-5}$  | $5.2377 \times 10^{-6}$ | $1.8517\times10^{-5}$   | $4.5867 	imes 10^{-6}$  | $1.3000 	imes 10^{-5}$  |
| 0.6  | $5.2687	imes10^{-7}$    | $7.7516 	imes 10^{-5}$  | $1.2001 	imes 10^{-5}$  | $2.4509\times10^{-5}$   | $5.6850 	imes 10^{-6}$  | $1.0936	imes10^{-5}$    |
| 0.65 | $2.4716 	imes 10^{-8}$  | $5.3962 \times 10^{-5}$ | $2.0504 \times 10^{-5}$ | $3.1577 \times 10^{-5}$ | $8.7259 	imes 10^{-6}$  | $1.3498	imes10^{-5}$    |
| 0.7  | $7.1897	imes10^{-7}$    | $7.8951 	imes 10^{-5}$  | $2.9520 \times 10^{-5}$ | $3.7769 	imes 10^{-5}$  | $1.1816\times 10^{-5}$  | $1.9020	imes10^{-5}$    |
| 0.75 | $1.5220	imes10^{-6}$    | $1.1358	imes10^{-4}$    | $3.4899 	imes 10^{-5}$  | $4.0906 	imes 10^{-5}$  | $1.1617	imes10^{-5}$    | $2.2560 	imes 10^{-5}$  |
| 0.8  | $1.4351	imes10^{-6}$    | $1.4452 	imes 10^{-4}$  | $3.4365 	imes 10^{-5}$  | $3.9923 	imes 10^{-5}$  | $1.2464 	imes 10^{-5}$  | $2.4768 \times 10^{-5}$ |
| 0.85 | $2.2068 	imes 10^{-7}$  | $1.3911	imes10^{-4}$    | $2.7204 \times 10^{-5}$ | $3.5134	imes10^{-5}$    | $1.2996 	imes 10^{-5}$  | $2.2994 	imes 10^{-5}$  |
| 0.9  | $4.4985 	imes 10^{-7}$  | $8.8317	imes10^{-5}$    | $1.6165\times 10^{-5}$  | $2.9667\times10^{-5}$   | $7.1411	imes10^{-6}$    | $1.6808	imes10^{-5}$    |
| 0.95 | $3.6249 	imes 10^{-7}$  | $6.9728 	imes 10^{-5}$  | $1.4044	imes10^{-5}$    | $2.7424 	imes 10^{-5}$  | $8.1852	imes10^{-6}$    | $1.5198\times10^{-5}$   |
| 1    | $9.8192	imes10^{-8}$    | $5.7973 	imes 10^{-5}$  | $1.1354\times 10^{-5}$  | $2.8972	imes10^{-5}$    | $6.7349\times10^{-6}$   | $1.2871 \times 10^{-5}$ |

**Table 4.** Statistical presentations of the environmental and economic nonlinear model for Z(s).

The global best performances of MAD, EVAF, and TIC operators for 65 trials based on the proposed computational ANN-GA-IPA are given in Table 5 to solve the environmental and economic nonlinear model. The global MED performances based on TIC, MAD, and EVAF gages lie 10-05-10-06, 10-08-10-09, and 10-10-10-11, while the S.I.R global performances of TIC, MAD, and EVAF lie 10-05-10-06, 10-09-10-10, and 10-08-10-11 to solve the environmental and economic nonlinear model. These optimal close results established the global presentations indicate the accurateness, correctness, and exactness of the proposed computational ANN-GA-IPA.

Table 5. Global measures performances of the environmental and economic nonlinear model.

| (G-TIC)  |   | (G-MAD)  |     | (G-EVAF)                 |                          |   |
|--|---|--|-----|--------------------------|--------------------------|---|
| Ω  | MED   | S.I.R  | MED | S.I.R                    | MED                      | S.I.R   |
| $\begin{array}{c} X(\Omega) \\ Y(\Omega) \\ Z(\Omega) \end{array}$ | $\begin{array}{c} 2.40785 \times 10^{-5} \\ 1.06193 \times 10^{-6} \\ 1.98223 \times 10^{-5} \end{array}$ | $\begin{array}{c} 1.17053\times10^{-5}\\ 3.97226\times10^{-6}\\ 6.73821\times10^{-6}\end{array}$ |     | $4.92191 	imes 10^{-10}$ | $4.67588 	imes 10^{-11}$ | $\begin{array}{c} 1.40658\times 10^{-10}\\ 4.10883\times 10^{-11}\\ 1.51931\times 10^{-8}\end{array}$ |

### 4. Conclusions

The purpose of this study is to treat the environmental and economic nonlinear model numerically using the ANNs strength together with the capability of global as well as local search schemes, i.e., ANN-GA-IPA. An objective function is designed on the basis of the environmental and economic nonlinear model and its boundary conditions. The optimization of the objective function based on the environmental and economic nonlinear model is performed using the ANN-GA-IPA strength. The proposed results are compared with the Adams solutions to check the correctness of the ANN-GA-IPA for solving the environmental and economic nonlinear model. The values of the AE are calculated in good measures to solve each category of the environmental and economic nonlinear model. Furthermore, the statistical operators based on MAD, TIC, and EVAF performances have been calculated accurately to solve each category of the environmental and economic

nonlinear model. The assessments through statistics performances for 65 independent executions using ANN-GA-IPA for the MED, Min, S.I.R, Max, STD, and mean operators authenticate correctness and worth of the designed computational ANN-GA-IPA. The global performances via statistical processes in terms of S.I.R and MED have been efficiently applied to each category of the environmental and economic nonlinear model.

In the future, the designed computational ANN-GA-IPA is capable to solve the biological nonlinear systems, fluid dynamic systems, and singular higher order systems.

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#### Nomenclature

| GAs             | Genetic Algorithms  |
|-----------------|---|
| ANNs            | Artificial neural networks                                      |
| IPS             | Interior-point scheme   |
| FFANNs          | Feed-forward ANNs   |
| ASA             | Active-set algorithm  |
| ANN-GA-ASA      | Hybrid of ANNs, GAs, and ASA                                    |
| MAD             | Mean absolute deviation   |
| VAF             | Variance account for  |
| S.I.R           | Semi-interquartile range  |
| Max             | Maximum   |
| Χ               | New technical diagnostics                                       |
| K <sub>i</sub>  | <i>i</i> th coefficient of environmental and economic indicator |
| $C_1, C_2, C_3$ | Values of Initial conditions                                    |
| $W_X$           | X components of W   |
| $W_Z$           | Z components of W   |
| $w_X, w_Y, w_Z$ | Weighting factor of X, Y, and Z                                 |
| Ε               | Fitness function  |
| IPA             | Interior point algorithm  |
| SQP             | Sequential quadratic programming                                |
| ANN-GA-IPA      | Hybrid of ANNs, GAs, and IPA                                    |
| FFANN-GASQP     | Hybrid of ANNs, GA, and SQP                                     |
| PSO             | Particle swarm optimization                                     |
| ANN-PSO-IPS     | Hybrid of ANNs, PSO, and IPS                                    |
| TIC             | Theil's inequality coefficient                                  |
| EVAF            | Error in VAF  |
| STD             | Standard Deviation  |
| Min             | Minimum   |
| Y               | elimination costs of emergencies values                         |
| Ζ               | the competence of the system of industrial elements             |
| W               | Known weights of ANNs or Chromosome of GAs                      |

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