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Research article

A novel proposed class of estimators under ranked set sampling: Simulation and diverse applications

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ABSTRACT

This study presents a novel enhanced exponential class of estimators for population mean under RSS by employing data on an auxiliary variable. The suggested estimators' mean square error (MSE) is calculated approximately at order one. The efficiency conditions that make the suggested enhanced exponential class of estimators superior to the traditional estimators are found. A simulation study using hypothetically drawn normal and exponential populations evaluates the execution of the suggested estimators. The findings demonstrate that the suggested estimators outperform their traditional equivalents. In addition, real data examples are examined to show how the proposed estimators can be implemented in various real life problems.

1. Introduction

The most frequent sampling method used in the establishing of statistical techniques is simple random sampling (SRS). It is unlikely to obtain a representative sample of the population by employing SRS if the sample size is insufficient. Numerous other sampling techniques like stratified sampling, systematic sampling, and cluster sampling, have been suggested in the literature as solutions to this issue. To increase the structure and reduce the possibility of an unrepresentative sample, these designs take into account prior knowledge of the underlying population's structure. In order to obtain the appropriate precision in drawing conclusions, [1] created ranked set sampling (RSS) that was intended to bring down the number of measured observations needed. It differs from the sampling methods previously discussed in that it does not need any prior knowledge about the structure of the population. This method of sampling is helpful in situations whenever ranking of the units of the sample without using their accurate amounts is simpler and less expensive than getting their accurate values.

The estimation methods consisting of RSS are often more efficacious than the estimation methods consisting of SRS counterparts. In the RSS literature, this problem has been researched for a number of common issues. The population mean computation is one of them and a very well known research topic among the survey researchers. Numerous enhanced and modified ratio, product, log, and regression type RSS based estimation methods of population mean based on supplementary information are available in the

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literature. Ref. [2] suggested RSS based traditional ratio estimator of population mean. Ref. [3] investigated RSS based traditional regression estimator of population mean. Ref. [4] examined RSS based ratio estimator for mean estimation. Ref. [5] investigated a generalised family of mean estimators consisting of RSS. Ref. [6] tested the efficiency of few RSS based ratio cum product category of estimators. Ref. [7] developed a generalized ratio cum product category of exponential estimator using RSS. Ref. [8] proposed a RSS based generalized class of estimators for mean estimation. Ref. [9] presented a RSS based exponential estimator of population mean. Refs. [10] and [11] proposed various RSS based improved classes of mean estimators. Ref. [12] examined a RSS based elevated generalized class of exponential ratio category of estimators. Similar to the above studies, many contributors have provided various modified and improved estimators for population mean using RSS, including Refs. [13–23].

A substantial work for the population mean estimation consisting of RSS has been done in the literature using ratio, log, and product categories of estimators. But, a very few works is available for the population mean estimation taking exponential estimators. The goal of this manuscript is to offer a new enhanced exponential class of estimators consisting of supplementary information under RSS, and to compare the proposed and existing estimators theoretically as well as empirically using real and artificial populations.

The RSS methodology, notations used, and a brief summary of the existing estimators along with their MSE expressions are provided in the next section. Section 3 proposes a generalized exponential class of estimators with its characteristics. Section 4 provides a performance comparison of the proposed and commonly used estimators. Simulation experiment is done in Section 5 to strengthen the theoretical results, while some real data sets are analyzed in Section 6. Section 7 conclude this research article.

2. Existing literature

To identify a ranked set sample based on the set with m size, one must initially choose m simple random samples of m size from the population. The next step is to order each sample of m size in the order of growing magnitude. The procedure of ranking in this stage is carried out utilizing a cheap approach that doesn't involve actual quantification of sample units, such as eye evaluation or human judgement. For real quantification, the sample unit from the i^{th} sample having judgement ranking i, $(i = 1, 2, ..., m)$ is chosen. To get a ranked set sample with $n = rm$ size, the full procedure may be performed r counts (cycles). It is essential to mention that the set of m size should be maintained low to allow for the requisite informal rankings. Let $Z_{[i]l}$ represent the l^{th} cycle's i^{th} judgement order statistic. The final sample is represented by the notation $Z_{[i]l} : i = 1, 2, ..., m; l = 1, 2, ..., r$. The accuracies and inaccuracies in the ranking procedure are shown by the usage of round and square brackets, respectively, in the subscripts of the variables.

Let's suppose for the purposes of obtaining the characteristics of numerous well known classes of estimators that $\delta_0 = (\bar{y}_{[n]} - \bar{Y})/\bar{Y}$, $\delta_1 = (\bar{z}_{(n)} - \bar{Z})/\bar{Z}$ implies that $E(\delta_0) = E(\delta_1) = 0$ and $\bar{z}_{(n)} = \sum_{i=1}^{m} Z_{(i)}/mr$ and $\bar{y}_{[n]} = \sum_{i=1}^{m} Y_{[i]}/mr$ are, respectively, the sample means of the variables Z and Y in RSS.

$$E(\delta_0^2) = \theta C_y^2 - W_{y_{[i]}}^2 \tag{1}$$

$$E(\delta_1^2) = \theta C_z^2 - W_{z_{(i)}}^2 \tag{2}$$

$$E(\delta_0, \delta_1) = \theta \rho_{zy} C_z C_y - W_{zy_{[i]}} \tag{3}$$

where $\theta = (mr)^{-1}$, $C_y = S_y/\bar{Y}$, $W_{y_{[i]}}^2 = \sum_{i=1}^{m}(\mu_{y_{[i]}} - \bar{Y})^2/m^2 r \bar{Y}^2$, $C_z = S_z/\bar{Z}$, $W_{z_{(i)}}^2 = \sum_{i=1}^{m}(\mu_{z_{(i)}} - \bar{Z})^2/m^2 r \bar{Z}^2$, $W_{zy_{[i]}} = \sum_{i=1}^{m}(\mu_{z_{(i)}} - \bar{Z})(\mu_{y_{[i]}} - \bar{Y})/m^2 r \bar{Z}\bar{Y}$, $\mu_{y_{[i]}} = E(Y_{[i]})$, and $\mu_{z_{(i)}} = E(Z_{(i)})$. $(\bar{Y}, \bar{Z})$ and (C_y, C_z) are the population's mean and coefficient of variation of variables y and z, respectively.

Further, in this section, we go through the estimators available in literature. The variance of the RSS based conventional mean estimator $t_1 = \bar{y}_{[n]}$ is:

$$V(t_1) = \bar{Y}^2(\theta C_y^2 - W_{y_{[i]}}^2)$$

The ratio estimator performs best when y and z have a strong positive correlation given the regression line (y on z) following a straight line from origin. Taking this advantage into consideration, [2] investigated the conventional RSS based ratio estimator as

$$t_2 = \bar{y}_{[n]} \frac{\bar{Z}}{\bar{z}_{(n)}}$$

The $MSE(t_2)$ is presented below as:

$$MSE(t_2) = \bar{Y}^2 \left\{ (\theta C_y^2 - W_{y_{[i]}}^2) + (\theta C_z^2 - W_{z_{(i)}}^2) - 2(\theta \rho_{zy} C_z C_y - W_{zy_{[i]}}) \right\}$$

When the regression between y and z is linear but the line of regression goes except the origin. In these circumstances, the ratio estimator's effectiveness is extremely poor. The superior choice is the regression estimator in such circumstances. Taking this fact into consideration, [3] employed the conventional RSS based regression estimator as

$$t_3 = \bar{y}_{[n]} + b(\bar{Z} - \bar{z}_{(n)})$$

where b is a constant known as the regression coefficient.

The optimum MSE for the optimum value of $b_{(opt)} = R(\theta\rho_{zy}C_zC_y - W_{zy_{[i]}})/(\theta C_y^2 - W^2_{y_{[i]}})$ is given by

$$minMSE(t_3) = \bar{Y}^2\left\{(\theta C_y^2 - W^2_{y_{[i]}}) - \frac{(\theta\rho_{zy}C_zC_y - W_{zy_{[i]}})^2}{(\theta C_z^2 - W^2_{z_{(i)}})}\right\}$$

Ref. [4] took inspiration from [2] and examined the ratio estimator under RSS examined by [24] as

$$t_4 = \omega\bar{y}_{[n]}\frac{\bar{Z}}{\bar{z}_{(n)}}$$

where ω is a constant.

The optimum MSE of the estimator t_4 for the optimum value of $\omega_{(opt)} = \{1 + (\theta\rho_{zy}C_zC_y - W_{zy_{[i]}})\}/\{1 + (\theta C_z^2 - W^2_{z_{(i)}})\}$ is presented as follows:

$$MSE(t_4)_{min} = \bar{Y}^2\{(\omega-1)^2 + (\theta C_z^2 - W^2_{z_{(i)}}) + \omega^2(\theta C_y^2 - W^2_{y_{[i]}}) - 2\omega(\theta\rho_{zy}C_zC_y - W_{zy_{[i]}})\}$$

Motivated by the works of [25], [26], [27], and [28], [6] examined the following RSS based estimators as

$$t_5 = \bar{y}_{[n]}\left(\frac{\bar{Z} + C_z}{\bar{z}_{(n)} + C_z}\right)$$
$$t_6 = \bar{y}_{[n]}\left(\frac{\bar{Z} + \beta_2(z)}{\bar{z}_{(n)} + \beta_2(z)}\right)$$
$$t_7 = \bar{y}_{[n]}\left(\frac{\bar{Z}C_z + \beta_2(z)}{\bar{z}_{(n)}C_z + \beta_2(z)}\right)$$
$$t_8 = \bar{y}_{[n]}\left(\frac{\bar{z}_{(n)}C_z + \beta_2(z)}{\bar{Z}C_z + \beta_2(z)}\right)$$
$$t_9 = \bar{y}_{[n]}\left\{\phi\left(\frac{\bar{Z}C_z + \beta_2(z)}{\bar{z}_{(n)}C_z + \beta_2(z)}\right) + (1-\phi)\left(\frac{\bar{z}_{(n)}C_z + \beta_2(z)}{\bar{Z}C_z + \beta_2(z)}\right)\right\}$$

where ϕ is a constant.

The MSE of the above estimators are presented as

$$MSE(t_l) = \bar{Y}^2\left\{(\theta C_y^2 - W^2_{y_{[i]}}) + \gamma_l^2(\theta C_z^2 - W^2_{z_{(i)}}) - 2\gamma_l(\theta\rho_{zy}C_zC_y - W_{zy_{[i]}})\right\},\ l = 5,6,7$$
$$MSE(t_8) = \bar{Y}^2\left\{(\theta C_y^2 - W^2_{y_{[i]}}) + \gamma_8^2(\theta C_z^2 - W^2_{z_{(i)}}) + 2\gamma_8(\theta\rho_{zy}C_zC_y - W_{zy_{[i]}})\right\}$$
$$MSE(t_9) = \bar{Y}^2\{(\theta C_y^2 - W^2_{y_{[i]}}) + (1-2\phi)^2d_3^2(\theta C_z^2 - W^2_{z_{(i)}}) + 2(1-2\phi)d_3(\theta\rho_{zy}C_zC_y - W_{zy_{[i]}})\}$$

The optimum MSE of the estimator t_9 at $\phi_{(opt)} = (d_3 + k)/2d_3 = \phi_0$ is presented as

$$MSE(t_9)_{min} = \bar{Y}^2\begin{bmatrix}(\theta C_y^2 - W^2_{y_{[i]}}) + (1-2\phi_0)^2d_3^2(\theta C_z^2 - W^2_{z_{(i)}}) \\ +2(1-2\phi_0)d_3(\theta\rho_{zy}C_zC_y - W_{zy_{[i]}})\end{bmatrix}$$

where $\gamma_5 = \{\bar{Z}/(\bar{Z} + C_z)\}$, $\gamma_6 = \{\bar{Z}/(\bar{Z} + \beta_2(z))\}$, $\gamma_7 = \gamma_8 = d_3 = \{\bar{Z}C_z/(\bar{Z}C_z + \beta_2(z))\}$.

Following [29], [7] adapted the RSS based exponential ratio and product estimators as

$$t_{10} = \bar{y}_{[n]}\exp\left(\frac{\bar{Z} - \bar{z}_{(n)}}{\bar{Z} + \bar{z}_{(n)}}\right)$$
$$t_{11} = \bar{y}_{[n]}\exp\left(\frac{\bar{z}_{(n)} - \bar{Z}}{\bar{z}_{(n)} + \bar{Z}}\right)$$

The MSE of these estimators is presented as

$$MSE(t_{10}) = \bar{Y}^2\left[(\theta C_y^2 - W^2_{y_{[i]}}) + \frac{1}{4}(\theta C_z^2 - W^2_{z_{(i)}}) - (\theta\rho_{zy}C_zC_y - W_{zy_{[i]}})\right]$$
$$MSE(t_{11}) = \bar{Y}^2\left[(\theta C_y^2 - W^2_{y_{[i]}}) + \frac{1}{4}(\theta C_z^2 - W^2_{z_{(i)}}) + (\theta\rho_{zy}C_zC_y - W_{zy_{[i]}})\right]$$

Further, [7] suggested a RSS based generalized ratio-cum-product exponential estimator as

$$t_{12} = \bar{y}_{[n]}\exp\left\{\frac{\left(\frac{\bar{Z}}{\bar{z}_{(n)}}\right)^{\alpha} - 1}{\left(\frac{\bar{Z}}{\bar{z}_{(n)}}\right)^{\alpha} + 1}\right\}$$

Table 1
List of sub class of the proposed estimator T.

Estimators	u	v
$T_{(1)} = k_1 \bar{y}_{[n]} \left(\frac{\bar{Z}}{\bar{z}_{(n)}}\right)^{k_2} \exp\left(\frac{\bar{Z}-\bar{z}_{(n)}}{\bar{Z}+\bar{z}_{(n)}}\right)$	1	0
$T_{(2)} = k_1 \bar{y}_{[n]} \left(\frac{\bar{Z}}{\bar{z}_{(n)}}\right)^{k_2} \exp\left\{\frac{(\bar{Z}+C_z)-(\bar{z}_{(n)}+C_z)}{(\bar{Z}+C_z)+(\bar{z}_{(n)}+C_z)}\right\}$	1	C_z
$T_{(3)} = k_1 \bar{y}_{[n]} \left(\frac{\bar{Z}}{\bar{z}_{(n)}}\right)^{k_2} \exp\left\{\frac{(\beta_2(z)\bar{Z}+C_z)-(\beta_2(z)\bar{z}_{(n)}+C_z)}{(\beta_2(z)\bar{Z}+C_z)+(\beta_2(z)\bar{z}_{(n)}+C_z)}\right\}$	$\beta_2(z)$	C_z
$T_{(4)} = k_1 \bar{y}_{[n]} \left(\frac{\bar{Z}}{\bar{z}_{(n)}}\right)^{k_2} \exp\left\{\frac{(C_z\bar{Z}+\beta_2(z))-(C_z\bar{z}_{(n)}+\beta_2(z))}{(C_z\bar{Z}+\beta_2(z))+(C_z\bar{z}_{(n)}+\beta_2(z))}\right\}$	C_z	$\beta_2(z)$
$T_{(5)} = k_1 \bar{y}_{[n]} \left(\frac{\bar{Z}}{\bar{z}_{(n)}}\right)^{k_2} \exp\left\{\frac{(\bar{Z}+\rho_{zy})-(\bar{z}_{(n)}+\rho_{zy})}{(\bar{Z}+\rho_{zy})+(\bar{z}_{(n)}+\rho_{zy})}\right\}$	1	ρ_{zy}
$T_{(6)} = k_1 \bar{y}_{[n]} \left(\frac{\bar{Z}}{\bar{z}_{(n)}}\right)^{k_2} \exp\left\{\frac{(\bar{Z}+\beta_2(z))-(\bar{z}_{(n)}+\beta_2(z))}{(\bar{Z}+\beta_2(z))+(\bar{z}_{(n)}+\beta_2(z))}\right\}$	1	$\beta_2(z)$
$T_{(7)} = k_1 \bar{y}_{[n]} \left(\frac{\bar{Z}}{\bar{z}_{(n)}}\right)^{k_2} \exp\left\{\frac{(C_z\bar{Z}+\rho_{zy})-(C_z\bar{z}_{(n)}+\rho_{zy})}{(C_z\bar{Z}+\rho_{zy})+(C_z\bar{z}_{(n)}+\rho_{zy})}\right\}$	C_z	ρ_{zy}
$T_{(8)} = k_1 \bar{y}_{[n]} \left(\frac{\bar{Z}}{\bar{z}_{(n)}}\right)^{k_2} \exp\left\{\frac{(\beta_2(z)\bar{Z}+\rho_{zy})-(\beta_2(z)\bar{z}_{(n)}+\rho_{zy})}{(\beta_2(z)\bar{Z}+\rho_{zy})+(\beta_2(z)\bar{z}_{(n)}+\rho_{zy})}\right\}$	$\beta_2(z)$	ρ_{zy}

where α is a constant to minimize the MSE of the estimator t_{12}. The optimum MSE of the estimator t_{12} at $\alpha_{(opt)} = 2(\theta\rho_{zy}C_zC_y - W_{zy_{[i]}})/(\theta C_z^2 - W_{z_{(i)}}^2)$ is presented as

$$MSE(t_{12})_{min} = \bar{Y}^2\left[(\theta C_y^2 - W_{y_{[i]}}^2) - \frac{(\theta\rho_{zy}C_zC_y - W_{zy_{[i]}})^2}{(\theta C_z^2 - W_{z_{(i)}}^2)}\right]$$

Ref. [8] took inspiration from [30] and suggested a RSS based generalized class of estimators as

$$t_{13} = \Delta\left(\frac{a\bar{Z}+b}{a\bar{z}_{(n)}+b}\right)^p + (1-\Delta)\left(\frac{a\bar{z}_{(n)}+b}{a\bar{Z}+b}\right)$$

where Δ is a constant and p is a real constant to develop numerous estimators. The optimum MSE of the estimator t_{13} at $\Delta_{(opt)} = -(\theta\rho_{zy}C_zC_y - W_{zy_{[i]}})/(\theta C_z^2 - W_{z_{(i)}}^2)$ is presented as

$$MSE(t_{13})_{min} = \bar{Y}^2\left[(\theta C_y^2 - W_{y_{[i]}}^2) - \frac{(\theta\rho_{zy}C_zC_y - W_{zy_{[i]}})^2}{(\theta C_z^2 - W_{z_{(i)}}^2)}\right]$$

Following [31], [9] suggested a RSS based exponential estimator as follows

$$t_{14} = \bar{y}_{[n]}\exp\left[\frac{\bar{Z}}{\{\bar{Z}+\phi(\bar{z}_{(n)}-\bar{Z})\}} - 1\right]$$

where ϕ is an optimizing scalar to be used to minimize the MSE expression. The optimum MSE of the estimator t_{14} at $\phi_{(opt)} = (\theta\rho_{zy}C_zC_y - W_{zy_{[i]}})/(\theta C_z^2 - W_{z_{(i)}}^2)$ is presented as

$$MSE(t_{14})_{min} = \bar{Y}^2\left[(\theta C_y^2 - W_{y_{[i]}}^2) - \frac{(\theta\rho_{zy}C_zC_y - W_{zy_{[i]}})^2}{(\theta C_z^2 - W_{z_{(i)}}^2)}\right]$$

3. Proposed class of estimators

Inspired by the works discussed in the last Section, we propose a novel enhanced exponential class of estimators for $\bar{Y}$ in RSS as follows:

$$T = k_1\bar{y}_{[n]}\left(\frac{\bar{Z}}{\bar{z}_{(n)}}\right)^{k_2}\exp\left\{\frac{(u\bar{Z}+v)-(u\bar{z}_{(n)}+v)}{(u\bar{Z}+v)+(u\bar{z}_{(n)}+v)}\right\}$$

where k_j, $j = 1, 2$ are constants to optimize the MSE, while u and v are real values or some known population parameters of the supplementary variables like coefficient of variation C_z, coefficient of kurtosis $\beta_2(z)$, standard deviation S_z, etc. Few sub class of the proposed estimator are given in Table 1. We write the suggested estimator T utilizing the notations provided in (1)-(3) as

$$T = k_1\bar{Y}(1+\delta_0)(1+\delta_1)^{-k_2}\exp\{-\lambda\delta_1(1+\lambda\delta_1)^{-1}\}$$

where $\lambda = 2\{u\bar{Z}/(u\bar{Z}+v)\}$.

Adapting Taylor's series expansion and neglecting the term with power more than 2, we get

$$T = k_1\bar{Y}(1+\delta_0)\left\{1 - k_2\delta_1 + \frac{k_2(k_2+1)}{2!}\delta_1^2 - \dots\right\}\left\{1 - \lambda\delta_1\left(1 - \lambda\delta_1 + \frac{3}{2}\lambda^2\delta_1^2\right)\right\}$$

After simplifying the above equation, we get

$$T-\bar{Y}=\bar{Y}\left[k_1\left\{1+\delta_0-k_2\delta_1-\lambda\delta_1-k_2\delta_0\delta_1-\lambda\delta_0\delta_1+\frac{k_2(k_2+1)}{2}\delta_1^2+k_2\lambda\delta_1^2+\frac{3}{2}\lambda^2\delta_1^2\right\}-1\right] \tag{4}$$

The bias of the estimator T approximated to the 1^{st} degree can be determined after applying the expectation on both sides of (4) as

$$Bias(T)=\bar{Y}\left[k_1\left\{\begin{array}{l}1-(k_2+\lambda)(\theta\rho_{zy}C_zC_y-W_{zy_{[i]}})\\ +\left\{\frac{k_2(k_2+1)}{2}+k_2\lambda+\frac{3}{2}\lambda^2\right\}(\theta C_z^2-W^2_{z_{(i)}})\end{array}\right\}-1\right]$$

The MSE of the estimator T to 1^{st} degree of approximation can be obtained by squaring and taking expectation on both sides of (4) as

$$MSE(T)=\bar{Y}^2\left(\begin{array}{l}1+k_1^2\left[\begin{array}{l}1+(\theta C_y^2-W^2_{y_{[i]}})+\{k_2(2k_2+1)+4\lambda^2+4k_2\lambda\}(\theta C_z^2-W^2_{z_{(i)}})\\ -4(k_2+\lambda)(\theta\rho_{zy}C_zC_y-W_{zy_{[i]}})\end{array}\right]\\ -2k_2\left[\begin{array}{l}1+\left\{\frac{k_2(k_2+1)}{2}+k_2\lambda+\frac{3}{2}\lambda^2\right\}(\theta C_z^2-W^2_{z_{(i)}})\\ -(k_2+\lambda)(\theta\rho_{zy}C_zC_y-W_{zy_{[i]}})\end{array}\right]\end{array}\right)$$

Further, the $MSE(T)$ may be written as

$$MSE(T)=\bar{Y}^2\left(1+k_1^2Z_1-2k_1Z_2\right)$$

where

$$Z_1= 1+(\theta C_y^2-W^2_{y_{[i]}})+\{k_2(2k_2+1)+4\lambda^2+4k_2\lambda\}(\theta C_z^2-W^2_{z_{(i)}})-4(k_2+\lambda)(\theta\rho_{zy}C_zC_y-W_{zy_{[i]}})\,,$$
$$Z_2= 1+\left\{\frac{k_2(k_2+1)}{2}+k_2\lambda+\frac{3}{2}\lambda^2\right\}(\theta C_z^2-W^2_{z_{(i)}})-(k_2+\lambda)(\theta\rho_{zy}C_zC_y-W_{zy_{[i]}})\,.$$

Minimizing the $MSE(T)$ against k_1 provides $k_{1(opt)}$ as

$$k_{1(opt)}=\frac{Z_2}{Z_1}$$

Putting the values of $k_{1(opt)}$ in the $MSE(T)$, we obtain

$$MSE(T)_{min}=\bar{Y}^2\left(1-\frac{Z_2^2}{Z_1}\right)$$

Note: The optimization of k_1 and k_2 simultaneously is very typical. Therefore, putting $k_1=1$ in the estimator T and minimize the $MSE(T)$ against k_2 provides the optimum value of k_2 as

$$k_{2(opt)}=-\lambda+\frac{(\theta\rho_{zy}C_zC_y-W_{zy_{[i]}})}{(\theta C_z^2-W^2_{z_{(i)}})}$$

4. Algebraic comparisons

The execution of the proposed estimators is done by the algebraic comparison of the MSEs of the proposed estimators and the commonly used estimators. The proposed estimators outperform the commonly used estimators under the below algebraic conditions.

$$MSE(T)<MSE(t_1)$$
$$\frac{Z_2^2}{Z_1}>1-(\theta C_y^2-W^2_{y_{[i]}})$$
$$MSE(T)<MSE(t_2)$$
$$\frac{Z_2^2}{Z_1}>1-(\theta C_y^2-W^2_{y_{[i]}})-(\theta C_z^2-W^2_{z_{(i)}})+2(\theta\rho_{zy}C_zC_y-W_{zy_{[i]}})$$
$$MSE(T)<MSE(t_3)$$
$$\frac{Z_2^2}{Z_1}>1-(\theta C_y^2-W^2_{y_{[i]}})+\frac{(\theta\rho_{zy}C_zC_y-W_{zy_{[i]}})^2}{(\theta C_z^2-W^2_{z_{(i)}})}$$
$$MSE(T)<MSE(t_4)$$
$$\frac{Z_2^2}{Z_1}>\left\{1-(\omega-1)^2-(\theta C_z^2-W^2_{z_{(i)}})-\omega^2(\theta C_y^2-W^2_{y_{[i]}})+2\omega(\theta\rho_{zy}C_zC_y-W_{zy_{[i]}})\right\}$$
$$MSE(T)<MSE(t_l)$$

$$\frac{Z_2^2}{Z_1} > 1 - (\theta C_y^2 - W_{y_{[i]}}^2) - \delta_i^2(\theta C_z^2 - W_{z_{(i)}}^2) + 2\delta_i(\theta \rho_{zy} C_z C_y - W_{zy_{[i]}})$$

$MSE(T) < MSE(t_8)$

$$\frac{Z_2^2}{Z_1} > 1 - (\theta C_y^2 - W_{y_{[i]}}^2) - \delta_4^2(\theta C_z^2 - W_{z_{(i)}}^2) - 2\delta_4(\theta \rho_{zy} C_z C_y - W_{zy_{[i]}})$$

$MSE(T) < MSE(t_9)$

$$\frac{Z_2^2}{Z_1} > 1 - (\theta C_y^2 - W_{y_{[i]}}^2) - (1 - 2\phi_0)^2 t_3^2(\theta C_z^2 - W_{z_{(i)}}^2) - 2(1 - 2\phi_0)t_3(\theta \rho_{zy} C_z C_y - W_{zy_{[i]}})$$

$MSE(T) < MSE(t_{10})$

$$\frac{Z_2^2}{Z_1} > 1 - (\theta C_y^2 - W_{y_{[i]}}^2) - \frac{1}{4}(\theta C_z^2 - W_{z_{(i)}}^2) + (\theta \rho_{zy} C_z C_y - W_{zy_{[i]}})$$

$MSE(T) < MSE(t_{11})$

$$\frac{Z_2^2}{Z_1} > 1 - (\theta C_y^2 - W_{y_{[i]}}^2) - \frac{1}{4}(\theta C_z^2 - W_{z_{(i)}}^2) - (\theta \rho_{zy} C_z C_y - W_{zy_{[i]}})$$

$MSE(T) < MSE(t_{12})$

$$\frac{Z_2^2}{Z_1} > 1 - (\theta C_y^2 - W_{y_{[i]}}^2) + \frac{(\theta \rho_{zy} C_z C_y - W_{zy_{[i]}})^2}{(\theta C_z^2 - W_{z_{(i)}}^2)}$$

$MSE(T) < MSE(t_{13})$

$$\frac{Z_2^2}{Z_1} > 1 - (\theta C_y^2 - W_{y_{[i]}}^2) + \frac{(\theta \rho_{zy} C_z C_y - W_{zy_{[i]}})^2}{(\theta C_z^2 - W_{z_{(i)}}^2)}$$

$MSE(T) < MSE(t_{14})$

$$\frac{Z_2^2}{Z_1} > 1 - (\theta C_y^2 - W_{y_{[i]}}^2) + \frac{(\theta \rho_{zy} C_z C_y - W_{zy_{[i]}})^2}{(\theta C_z^2 - W_{z_{(i)}}^2)}$$

5. Simulation study

To check the validity of the theoretical conclusions, a simulation experiment is carried out utilising artificially created populations. The artificial populations are generated by using the models given by [32] and recently used by [11,20–23]. The steps to perform the simulation study are described below:

Step 1. Generate a normal population and an exponential population both of $N = 600$ size. The observations on Z and Y variables are created by the models $Y = 9.8 + \sqrt{(1 - \rho_{zy}^2)}\, Y^* + \rho_{zy}\left(S_y/S_z\right) Z^*$ and $Z = 9.2 + Z^*$ with $X^* \sim N(23, 33)$ and $Y^* \sim N(9, 17)$ for normal population, while $X^* \sim Exp(0.06)$ and $Y^* \sim Exp(0.09)$ for exponential population.
Step 2. Quantify $n = 12$ ranked set samples with number of cycles $r = 4$ and set size $m = 3$ from the populations drawn in Step 1 by adopting RSS method.
Step 3. Obtain the essential statistics.
Step 4. Rerun the steps 1-3, 20,000 counts and obtain MSE and PRE using the expressions given in (5) and (6).

$$MSE(T_i) = \frac{\frac{1}{20{,}000}\sum_{i=1}^{20{,}000}(t_1 - \bar{Y})^2}{\frac{1}{20{,}000}\sum_{i=1}^{20{,}000}(T_i - \bar{Y})^2} \tag{5}$$

$$PRE = \frac{MSE(t_1)}{MSE(T_i)} \times 100 \tag{6}$$

It is worthwhile to note that various correlation coefficient $\rho_{zy} = 0.3, 0.5, 0.7, 0.9$ values are taken to study the characteristics of the proposed estimators. The simulation results for every population are listed in Tables 2–3 in form of MSE and PRE for several values of ρ_{zy}.

The important simulation findings are given in the following points:

- The simulation outcomes of Table 2 obtained using the normal population demonstrate that the members $T_{(1)}, T_{(2)}, \ldots, T_{(8)}$ of the proposed estimators T attain the lesser MSE and higher PRE for each value of ρ_{zy} and perform better than the competing estimators such as unbiased estimator t_1, traditional ratio estimator t_2, traditional regression estimator t_3, [4] estimator t_4, [6] estimator t_5, t_6,...,t_9, [7] estimators t_{10}, t_{11}, t_{12}, [8] estimator t_{13} and [9] estimator t_{14}.

Table 2
Simulation findings using artificially created normal population.

ρ_{zy} estimators	0.3		0.5		0.7		0.9	
	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE
t_1	80.42	100.00	80.42	100.00	80.42	100.00	80.42	100.00
t_2	113.18	71.05	99.20	81.06	89.04	90.31	81.74	98.38
t_3	72.41	111.05	70.44	114.16	67.63	118.90	64.01	125.64
t_4	106.15	75.76	92.73	86.72	83.05	96.83	76.24	105.48
t_5	109.93	73.15	97.20	82.73	87.67	91.73	80.61	99.77
t_6	105.26	76.40	93.86	85.68	85.20	94.39	78.61	102.29
t_7	104.22	77.16	92.74	86.71	84.22	95.48	77.85	103.30
t_8	130.19	61.77	130.60	61.57	133.07	60.43	138.80	57.94
t_9	73.28	109.75	71.35	112.71	68.60	117.23	65.04	123.65
t_{10}	84.91	94.71	79.73	100.86	75.64	106.31	72.10	111.54
t_{11}	99.71	80.65	101.26	79.41	103.38	77.79	106.72	75.35
t_{12}	72.41	111.05	70.44	114.16	67.63	118.90	64.01	125.64
t_{13}	72.41	111.05	70.44	114.16	67.63	118.90	64.01	125.64
t_{14}	72.41	111.05	70.44	114.16	67.63	118.90	64.01	125.64
$\mathbf{T_{(1)}}$	**65.03**	**123.66**	**63.19**	**127.27**	**60.69**	**132.50**	**57.43**	**140.01**
$\mathbf{T_{(2)}}$	**65.11**	**123.50**	**63.27**	**127.10**	**60.78**	**132.32**	**57.53**	**139.77**
$\mathbf{T_{(3)}}$	**65.07**	**123.59**	**63.22**	**127.20**	**60.73**	**132.42**	**57.48**	**139.91**
$\mathbf{T_{(4)}}$	**65.27**	**123.21**	**63.46**	**126.72**	**61.00**	**131.84**	**57.78**	**139.17**
$\mathbf{T_{(5)}}$	**65.07**	**123.59**	**63.23**	**127.18**	**60.74**	**132.40**	**57.49**	**139.87**
$\mathbf{T_{(6)}}$	**65.24**	**123.27**	**63.41**	**126.82**	**60.93**	**131.98**	**57.71**	**139.34**
$\mathbf{T_{(7)}}$	**65.08**	**123.58**	**63.24**	**127.16**	**60.75**	**132.37**	**57.51**	**139.84**
$\mathbf{T_{(8)}}$	**65.05**	**123.63**	**63.20**	**127.23**	**60.71**	**132.46**	**57.46**	**139.95**

The bold values in the Table indicate those with the least MSE and highest PRE.

Table 3
Simulation findings for artificially created exponential population.

ρ_{zy} estimators	0.3		0.5		0.7		0.9	
	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE
t_1	20.18	100.00	20.18	100.00	20.18	100.00	20.18	100.00
t_2	25.72	78.43	23.10	87.32	21.06	95.81	19.51	103.43
t_3	18.06	111.68	17.37	116.13	16.55	121.92	15.58	129.44
t_4	25.09	80.41	22.53	89.55	20.53	98.25	19.03	105.99
t_5	25.45	79.27	22.92	88.03	20.92	96.44	19.38	104.12
t_6	24.07	83.81	22.00	91.68	20.28	99.49	18.79	107.38
t_7	22.99	87.75	21.20	95.16	19.68	102.52	18.31	110.15
t_8	27.74	72.74	28.96	69.66	30.40	66.36	32.32	62.43
t_9	18.29	110.33	17.60	114.65	16.78	120.23	15.83	127.46
t_{10}	20.80	96.99	19.68	102.49	18.72	107.76	17.79	113.37
t_{11}	23.85	84.59	24.58	82.08	25.42	79.37	26.65	75.71
t_{12}	18.06	111.68	17.37	116.13	16.55	121.92	15.58	129.44
t_{13}	18.06	111.68	17.37	116.13	16.55	121.92	15.58	129.44
t_{14}	18.06	111.68	17.37	116.13	16.55	121.92	15.58	129.44
$\mathbf{T_{(1)}}$	**17.44**	**115.67**	**16.77**	**120.32**	**15.97**	**126.33**	**15.03**	**134.24**
$\mathbf{T_{(2)}}$	**17.44**	**115.65**	**16.77**	**120.30**	**15.97**	**126.31**	**15.03**	**134.20**
$\mathbf{T_{(3)}}$	**17.44**	**115.66**	**16.77**	**120.31**	**15.97**	**126.32**	**15.03**	**134.23**
$\mathbf{T_{(4)}}$	**17.47**	**115.48**	**16.80**	**120.07**	**16.01**	**126.01**	**15.08**	**133.78**
$\mathbf{T_{(5)}}$	**17.44**	**115.65**	**16.77**	**120.30**	**15.97**	**126.30**	**15.03**	**134.20**
$\mathbf{T_{(6)}}$	**17.46**	**115.57**	**16.78**	**120.19**	**15.99**	**126.17**	**15.06**	**133.98**
$\mathbf{T_{(7)}}$	**17.45**	**115.63**	**16.77**	**120.27**	**15.98**	**126.27**	**15.04**	**134.15**
$\mathbf{T_{(8)}}$	**17.44**	**115.66**	**16.77**	**120.31**	**15.97**	**126.32**	**15.03**	**134.23**

The bold values in the Table indicate those with the least MSE and highest PRE.

- The simulation outcomes of Table 3 obtained using the exponential population demonstrate that the members $T_{(1)}, T_{(2)},\ldots,T_{(8)}$ of the proposed estimators T attain the lesser MSE and higher PRE for each value of ρ_{zy} and perform better than the competing estimators such as unbiased estimator t_1, traditional ratio estimator t_2, traditional regression estimator t_3, [4] estimator t_4, [6] estimators t_5, t_6,…,t_9, [7] estimators t_{10}, t_{11}, t_{12}, [8] estimator t_{13} and [9] estimator t_{14}.
- The outcomes of Tables 2–3 demonstrate that when the values of the correlation coefficient rises, the MSE and PRE of the members of the suggested estimators reduce and grow, respectively.

Table 4
Parameters of real data sets.

Parameters	Data sets			
	1	2	3	4
$\bar{Y}$	4514.89	966.95	5182.63	36.65
$\bar{Z}$	4591.07	26441.72	285.00	14604.49
S_z	6315.21	45402.78	270.53	34064.88
S_y	6099.14	2389.77	1835.65	116.80
ρ_{zy}	0.95	0.71	0.91	0.22
N	69	204	80	124
n	12	12	12	12
m	3	3	3	3
r	4	4	4	4

6. Applications

The applications of the proposed estimators are presented through four different real data sets which are discussed below.

Data set 1: The first data set is chosen from [33], page no. 652 having amount of fish caught by marine recreational fisherman during 1995 (study variable Y) and the amount of fish caught by marine recreational fisherman during 1993 (auxiliary variable Z).

Data set 2: The second data set is chosen from [34], where the study variable Y is chosen as the apple production level and the auxiliary variable Z is chosen as the apple trees' quantity in 204 villages of the region of Black sea in Turkey in 1999.

Data set 3: The third data set is taken from [35], page no. 228, where the study variable is taken as the output for 80 factories in a particular area, while auxiliary variable Z is considered as the numbers of working persons for 80 factories in that area.

Data set 4: The fourth data set is taken from [36], page no. 662-665, based on 1983 population in millions (as study variable Y) and import (in millions of U.S. dollar) in 1983 (auxiliary variable Z).

Table 4 provides the data sets' descriptive statistics.

Using the descriptive statistics of data sets 1-4 reported in Table 4, the MSE and PRE of various estimators are tabulated utilizing the below mentioned mathematical expressions:

$$PRE = \frac{MSE(t_1)}{MSE(T_i)} \times 100$$

The findings of real data 1-4 reported in Table 5 show that the members $T_{(1)}$, $T_{(2)}$,...,$T_{(8)}$ of the proposed estimator T attain the least MSE and highest PRE and perform better than the competing estimators such as unbiased estimator t_1, traditional ratio estimator t_2, traditional regression estimator t_3, [4] estimator t_4, [6] estimator t_5, $t_6, ..., t_9$, [7] estimators t_{10}, t_{11}, t_{12}, [8] estimator t_{13} and [9] estimator t_{14}.

7. Conclusion

In this research article, a novel enhanced exponential class of estimators have been proposed for population mean estimation utilizing RSS. The mathematical expression of the MSE of the proposed estimators is found out to the approximation of degree one. The algebraic comparison has been performed to provide efficiency conditions under which the suggested estimator is found to be superior than the existing estimators. The performance of the suggested estimators has been assessed using a broad simulation study based on hypothetically drawn normal and exponential populations. Moreover, the applicability of the proposed estimators has been shown by examining several actual data sets, and the findings have been discussed. Based on the simulation and real data findings, the following observations have been made: **(i).** From the findings of Table 2 and Table 3 consisting of a simulation study using artificially made populations, the sub class $T_{(i)},\ i = 1, 2, ..., 8$ of the suggested estimators T have been found to be rewarding as compared to the reviewed estimators. **(ii).** From the results of Table 2 and Table 3, it has been noted that the MSE decreases as ρ_{zy} increase and vice versa according to PRE. **(iii).** From the numerical findings of Table 4 consisting of real data, the sub class $T_{(i)},\ i = 1, 2, ..., 8$ of the suggested estimators T perform superior than the commonly used estimators.

Thus, the suggested class of estimators provides a convincing argument for its merit.

CRediT authorship contribution statement

Najwan Alsadat: Conceived and designed the experiments; Performed the experiments; Wrote the paper.
M. Yusuf: Performed the experiments; Analyzed and interpreted the data.
Oluwafemi Samson Balogun: Performed the experiments; Contributed reagents, materials, analysis tools or data.
Mahmoud Abd El Raouf: Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data.
Hanan Alohali: Conceived and designed the experiments; Contributed reagents, materials, analysis tools or data.

Table 5
Numerical findings using real data sets.

ρ_{zy} estimators	Data set 1		Data set 2		Data set 3		Data set 4	
	MSE	PRE	MSE	PRE	MSE	PRE	MSE	PRE
t_1	3061362.00	100.00	465868.00	100.00	258894.00	100.00	1122.86	100.00
t_2	275050.70	1113.01	101008.60	461.21	878911.00	29.45	934.97	120.09
t_3	262430.10	1166.54	35175.32	1324.41	44829.60	577.50	921.68	121.82
t_4	274814.10	1113.97	71428.94	652.21	873649.90	29.63	628.07	178.77
t_5	274930.90	1113.50	101021.90	461.15	870588.80	29.73	934.95	120.09
t_6	274711.70	1114.39	101077.10	460.90	858296.10	30.16	934.89	120.10
t_7	274803.80	1114.01	101048.50	461.03	857210.50	30.20	934.93	120.10
t_8	12214929.00	25.06	1150070.00	40.50	3381529.00	7.65	1946.18	57.69
t_9	262445.70	1166.47	95709.95	486.74	44880.70	576.84	996.51	112.67
t_{10}	871344.70	351.33	243499.90	191.32	95591.87	270.83	949.45	118.26
t_{11}	6845104.00	44.72	768112.80	60.65	1368817.00	18.91	1455.21	77.16
t_{12}	262430.10	1166.54	35175.32	1324.41	44829.60	577.50	921.68	121.82
t_{13}	262430.10	1166.54	35175.32	1324.41	44829.60	577.50	921.68	121.82
t_{14}	262430.10	1166.54	35175.32	1324.41	44829.60	577.50	921.68	121.82
$\mathbf{T}_{(1)}$	**256333.40**	**1194.28**	**32178.61**	**1447.75**	**42429.46**	**610.17**	**534.14**	**210.21**
$\mathbf{T}_{(2)}$	**256341.40**	**1194.25**	**32178.27**	**1447.77**	**42458.45**	**609.75**	**534.15**	**210.21**
$\mathbf{T}_{(3)}$	**256335.50**	**1194.28**	**32178.57**	**1447.75**	**42441.75**	**609.99**	**534.14**	**210.21**
$\mathbf{T}_{(4)}$	**256350.00**	**1194.21**	**32177.60**	**1447.80**	**42504.59**	**609.09**	**534.15**	**210.21**
$\mathbf{T}_{(5)}$	**256339.00**	**1194.26**	**32178.47**	**1447.76**	**42457.41**	**609.77**	**534.14**	**210.21**
$\mathbf{T}_{(6)}$	**256356.20**	**1194.18**	**32176.88**	**1447.83**	**42500.86**	**609.14**	**534.17**	**210.20**
$\mathbf{T}_{(7)}$	**256337.50**	**1194.27**	**32178.53**	**1447.76**	**42458.89**	**609.75**	**534.14**	**210.21**
$\mathbf{T}_{(8)}$	**256334.80**	**1194.28**	**32178.59**	**1447.75**	**42441.31**	**610.00**	**534.14**	**210.21**

The bold values in the Table indicate those with the least MSE and highest PRE.

Declaration of competing interest

There is no conflict of interest regarding the submitted paper.

Data availability

Data included in article/supplementary material/referenced in article.

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