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“It gives you that sense of hope”: An exploration of technology use to mediate student engagement with mathematics

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A R T I C L E I N F O

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A B S T R A C T

Despite the predicted need for a more mathematically capable workforce, the proportion of students undertaking advanced mathematics courses in Australia and other comparable countries has stagnated or fallen, in part due to a lack of student engagement with mathematics in school. In society in general, technology use is commonplace, leading some educators to speculate that technology use for the teaching and learning of mathematics can improve student engagement. In this paper, using multiple case studies, we examine how teachers (n = 10), recognised by their peers as exemplary users of technology, take advantage of technological affordances to optimise student engagement with mathematics. Data was collected from three participant groups: Teachers, Leaders (n = 10), and student focus groups (n = 6). We examine both student and teacher perspectives, through the lens of the *Framework for Engagement with Mathematics (FEM),* to tease out the ways in which exemplary teachers use technology to enhance pedagogical relationships with students and their pedagogical repertoires. We find that the teachers and students reported evidence of all elements of the FEM, but to differing degrees. In particular, we identified that teachers used technological tools to enhance teacher awareness of individual student learning needs and to promote student-centred pedagogies leading to greater student engagement with mathematics. We contend that a greater awareness of the nuanced pedagogical affordances of a range of technological tools could lead teachers toward practices that enhance student engagement with mathematics, leading to an increase in students wishing to extend their mathematical knowledge beyond the compulsory school years.

1. Introduction

The quote “It gives you that sense of hope” is from a Year 11 mathematics student, drawn from a focus group conversation about his teacher, technology, and mathematics education. The focus group consisted of five senior secondary school students, who spoke about the transformational influence of their mathematics teacher and his use of technology. The teacher used the technology in ways that, when combined with his classroom teaching practices, facilitated high student engagement. This resulted in students who transitioned from being disengaged, with low confidence and low attainment in mathematics, to students who were engaged, more confident, and comfortable enough to seek help in front of their peers.

Low levels of student engagement with mathematics continue to be a challenge for educators internationally (Everingham et al., 2017). Students who disengage with mathematics often opt out of the study of mathematics beyond the compulsory years, resulting in more limited life and career opportunities in fields that require mathematics skills (Baroody et al., 2016). As technology has now become ubiquitous in classrooms around the world it is still regarded by many as a potential remedy for increasing student engagement with mathematics that will, in turn, increase academic outcomes and student enrolments in mathematics-related courses beyond the compulsory school years. Such expectations are reflected widely in literature (e.g. Beavis et al., 2015; Calder and Campbell, 2016; Pierce and Ball, 2009) and although studies indicate that the use of digital technologies does appear to improve student engagement (Attard, 2018; Bray and Tangney, 2015; Hilton, 2018; Ingram et al., 2016), few studies provide an in-depth and detailed view of how engagement is influenced by technology-related teaching practices. Moreover, the rapid pace of technology development and the exponential growth in its use within educational settings and students' home lives has likely reduced the novelty aspect of their use in classrooms, which may have previously played a role in influencing engagement (Attard, 2015). Therefore, we argue that research interrogating the subtle and nuanced ways in which technology-related teacher practices promote engagement is critical if educators are to maximise the potential

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benefits resulting from teacher and student access of affordances offered by current and emerging technologies.

This paper draws on a set of 10 case studies to explore engaging elements of pedagogical relationships and practices that emerge from the exemplary use of technology in mathematics classrooms ranging from pre-school to upper secondary education. The Framework for Engagement with Mathematics (FEM), as introduced by Attard (2014) is employed as an analytical lens to explore data from semi-structured teacher interviews, classroom observations and student focus groups. We use this framework to help us understand the subtleties involved in technology-related practices and the ways in which these practices can mediate student engagement with mathematics.

1.1. Technology and mathematics

A major benefit of contemporary technologies such as tablets is the wide range of affordances they offer beyond simply increasing the mobility of both the device and the learner (Bray and Tangney, 2017; Fabian et al., 2016). Mobile devices can provide access to unlimited information via the internet, and all content areas of the mathematics curriculum through software applications (apps) that provide opportunities for teachers to design and implement a range of activities from low level, mathematics-focussed fluency building games to more complex tasks that require students to analyse, evaluate, and create. Contemporary technologies also allow learners to interact with mathematics in more practical, dynamic and contextualised ways through visualisation, modelling, and manipulation (Bray and Tangney, 2017; Olive and Makar, 2010).

Unlike traditional textbooks that continue to feature heavily in mathematics classrooms, mobile devices provide opportunities for new types of teaching and learning interactions through the use of generic, productivity apps such as *OneNote* or *Evernote*, social media and learning management systems (LMS) (*Canvas, Echo, Google Classroom).* They also provide new ways of capturing student work samples, providing instruction and feedback, and fostering communication that extends beyond the classroom space (Attard, 2018; Calder and Campbell, 2016; Hilton, 2018; Ingram et al., 2016). For example, screen-casting apps such as *ShowMe* allow users to synchronously record audio and written responses. Other apps such as *Explain Everything* and *EduCreations* allow users to combine media such as video, still photography, audio, and drawing to create products that illustrate students' application and understanding of mathematical concepts. This has resulted in new opportunities for students to adapt from simply being consumers, to being producers: a significant shift from traditional mathematics classroom practices (Boaler, 2015).

Existing technology-related frameworks describe the knowledge teachers require to use technology effectively (TPACK), or they describe the varying levels of use in terms of redefining learning tasks (SAMR, TiM) (Florida Center for Instructional Technology, 2018; Koehler and Mishra, 2009; Puentedura, 2010). Similarly, there are frameworks that consider the levels of teacher integration of technology into existing practices (Niess et al., 2009) and the role of technology relating to practice. While Guerrero (2010) briefly mentions student engagement as an element of the management component of TPACK, there are no frameworks that specifically include student engagement. Research that has explored the ability for technology to improve engagement with mathematics often does so in a shallow manner. The construct of engagement often varies in terms of definition and the depth with which it is treated. For example, engagement is referred to as students having fun through games and outdoor learning, increased time on task, and increased focus (Fabian et al., 2016; Wijers et al., 2010). We argue that if engagement is one of the reasons technology is introduced into classrooms, we need to better understand how mathematics teachers can use technology to improve the quality of the engagement. It is hoped that improved engagement will result in more positive attitudes towards mathematics, improved academic achievement and lowered attrition rates.

1.2. Engagement and mathematics

Engagement occurs as a result of participation within an educational context when knowledge and learning are valued and applied by the student. In terms of engagement with mathematics in the context of this paper, we consider that engagement happens as a result of students who are procedurally engaged, actively participating in tasks and doing mathematics with the perspective that mathematics education is a pursuit that is worthwhile, valuable and useful both in and out of the classroom. We draw on the seminal review conducted by Fredricks et al. (2004) to further define engagement as a multidimensional construct operating at behavioural, cognitive and emotional levels resulting in a deeper student relationship with mathematics. Viewing engagement as the combination of behaviour, emotion and cognition is arguably more valuable than exploring the components individually, as in reality these elements are highly interrelated (Fredricks et al., 2004). Building on Fredricks et al.'s work, the Fair Go Project (Fair Go Team NSW Department of Education and Training, 2006) adapted this conceptualisation by changing emotional to affective engagement, and behavioural to operative engagement (Munns, 2007). Munns argued the change to affective engagement offered “a clearer pedagogical focus for teachers” (p.305), and the change from behavioural to operative “provided a stronger pedagogical and outcome focus for both teachers and students” (p.305).

This perspective informs the definition of engagement applied in this paper: the coming together of affective, cognitive, and operative dimensions that results in students valuing and enjoying school mathematics, and connecting schools mathematics to their own lives (Attard, 2013; Fair Go Team NSW Department of Education and Training, 2006; Munns and Martin, 2005). This definition incorporates the student's thoughts that are made evident through words and actions in relation to his or her effort towards learning and relational behaviours as they are observed in the mathematics classroom (Attard, 2014).

The Framework for Engagement with Mathematics (FEM) (Table 1) (Attard, 2014) draws on research and literature pertaining to student engagement and mathematics pedagogy to provide a tool that was originally intended to assist in planning for engaging mathematics experiences. The framework emerged from a longitudinal study (spanning three school years) of the factors that influence student engagement during the middle years of schooling (Grades 5 to 8) and has been used as a framework to assist in analyzing qualitative data based on classroom observations, interviews and focus group discussions in mathematics classrooms to determine how the use of digital technologies assist in increasing (or decreasing) student engagement (Attard, 2018; Hilton, 2018). The FEM considers student voice as a critical element in relation to influences on engagement with mathematics, hence student voice features heavily in the data informing this paper.

The FEM describes the influences on student engagement as consisting of two separate elements: pedagogical relationships and pedagogical repertoires. While considered separate, the two elements are also interrelated. Pedagogical relationships refer to the interpersonal educational relationships between teachers and students that serve to enhance engagement with mathematics. Pedagogical repertoires relate to the day-to-day teaching practices that are selected by the teacher. The development of positive pedagogical relationships is considered as the foundation for substantive student engagement. It is challenging to engage students without the establishment of such relationships, regardless of the quality of pedagogical repertoires. For example, if a teacher is unaware of individual student needs (TA) that result from continuous interactions (CI), or the teacher does not consider students' pre-existing knowledge (PK), it would be difficult to plan lessons that provide an appropriate challenge (CT) or are relevant to the lives of students (RT) (Attard, 2014).

Table 1. The Framework for engagement with mathematics (Attard, 2014).

Aspect	Code	Element
Pedagogical Relationships		In an engaging mathematics classroom, positive pedagogical relationships exist where these elements occur:
	TA	Teacher Awareness: the teacher is aware of each student's mathematical abilities and learning needs
	PK	Pre-existing Knowledge: students' backgrounds and pre-existing knowledge are acknowledged and contribute to the learning of others
	CI	Continuous Interaction: interaction amongst students and between teacher and students is continuous
	CF	Constructive Feedback: feedback to students is constructive, purposeful and timely
	PCK	Pedagogical Content Knowledge: the teacher models enthusiasm and an enjoyment of mathematics and has a strong Pedagogical Content Knowledge
Pedagogical Repertoires		Pedagogical repertoires include the following aspects:
	CT	Challenging Tasks: tasks are positive, provide opportunity for all students to achieve a level of success and are challenging for all
	RT	Relevant Tasks: the relevance of the mathematics curriculum is explicitly linked to students' lives outside the classroom and empowers students with the capacity to transform and reform their lives
	PC	Provision of Choice: students are provided an element of choice
	VT	Variety of Tasks: mathematics lessons regularly include a variety of tasks that cater to the diverse needs of learners
	SC	Substantive Conversation: there is substantive conversation about mathematical concepts and their applications to life
	ST	Student-centred Technology: Technology is embedded and used to enhance mathematical understanding through a student-centred approach to learning
Students are engaged with mathematics when:		
• Mathematics is a subject they enjoy learning • They value mathematics learning and see its relevance in their current and future lives, and • They see connections between the mathematics learned at school and the mathematics used beyond the classroom		

Once positive pedagogical repertoires are established, the implementation of engaging pedagogical repertoires can be considered. The FEM details six different elements that incorporate ideas presented in more traditionally recognised frameworks and constructs such as pedagogical content knowledge (PCK) (Shulman, 1986), TPACK (Koehler and Mishra, 2009), and Hill, Ball, and Schilling's Mathematical Knowledge for Teaching (MKT) (2008). The elements also address generic practices that have been found to directly influence student engagement with mathematics such as the provision of choice (PC) and the provision of a variety of tasks (VT).

Although the FEM contains a technology-specific element within its list of engaging pedagogical repertoires, in this paper the entire framework is used as a lens through which we explore and analyse technology-enhanced mathematics lessons and teacher practices. We investigate the pedagogical relationships that either inform or are developed by using digital technology, the pedagogical repertoires that incorporate technologies and we interrogate if and how the technology mediates engagement through the individual elements of the FEM.

1.3. What does research already tell us about technology, engagement, and the FEM?

According to Hoyles (2016), a major challenge for increasing student engagement with mathematics is to make mathematics more visible, suggesting that one way to do this is through the use of digital technology; "All too often mathematics is a black box that is kept closed, either as there is no reason to try to open it, or it is deemed as too complicated to even try" (p. 227). While student engagement continues to be a dominant reason for using technology in today's classrooms, research relating specifically to the influence of technology on student engagement in mathematics is still limited. Two Australian studies exploring the use of iPads in primary mathematics classrooms have reported some levels of improved engagement (Attard, 2015; Attard and Curry, 2012). The FEM was used to analyse the findings of three separate studies. When comparing the data from the three studies it was found that there were great variances in the levels of improvement in student engagement, and this was due to the use of a range of apps (with a variety of affordances), variations in teacher confidence and experience, and a great diversity in the ways in which the devices were used. Rather than the device or software making a difference to student engagement, it is suggested that it is the ways they are used, the purposes for their use and the "pedagogical practices that embed their use that determine how engaging they are" (Attard, 2018, p. 63).

A longitudinal study conducted in Australia by Hilton (2018) also used the FEM alongside a quantitative survey to explore increases in engagement during the first two years of a three-year phased program of iPad integration in a primary school. The FEM was specifically used to code data collected from the teachers and students. Hilton's findings indicated that there were high levels of student engagement resulting from iPad use, and in particular, increased engagement in students who may not otherwise have been engaged. The ability for students to learn via the use of multimodalities "and to use multiple modes when creating their own products is a strongly engaging factor" (p.156). Hilton also found that students were highly engaged through the use of apps that utilise a drill and practice approach. In addition, the use of iPads improved the engagement of students with a diverse range of learning needs.

2. Methodology

The research question that informs this paper is: In what ways does technology-related practice promote student engagement with mathematics?

This paper draws upon a broader study that was situated within an interpretive, social constructivism worldview. Within this view it is assumed that reality is socially constructed and that there is no single, observable reality (Corbin and Strauss, 2008; Creswell, 2013; Merriam, 2009). Within a social constructivism perspective, the individual seeks an understanding of the world by building subjective meanings of his or her experiences. These meanings vary, leading the researchers to explore several, complex views rather than reducing meanings to a smaller number of categories or ideas. Within a social constructivism view, the goal of research is to "rely as much as possible on the participants' views of the situation" (Creswell, 2013, p. 24). Such a view sits well with the aim of this study to investigate technology-mediated engagement from teachers' and students' perspectives. This view acknowledges that meanings are negotiated socially via interactions and through historical and cultural norms that exist for students and their teachers (Creswell, 2013).

A qualitative multiple case study approach was utilised to provide a set of in-depth studies drawn from a range of contexts. This approach allowed us to focus on the interactions, relationships and practices within each bounded case prior to conducting a cross-case comparison across all 10 cases (Cohen et al., 2018). The broader research explored the effective use of digital technology in 10 mathematics classrooms from pre-school to Year 12 (the final year of schooling in Australia). Ten case studies were conducted within eight Australian schools. Each case consisted of a classroom teacher, one member of the school leadership team and a focus group of five to six students (in cases A to G). Students in Cases H, I and J did not participate in focus groups due to their age. Cases were identified through a process of purposive sampling. The case studies were conducted in a combination of pre-school, primary and secondary schools from a mixture of public, private and Catholic systems in a range of socio-economic and geographic areas.

2.1. Participants

As the purpose of the broader study was to explore effective technology-related practices, case study teachers were identified through professional associations, professional teaching networks, and referrals, as teachers who are considered by their colleagues to be effective and innovative users of technology within mathematics classrooms. The 10 teachers came from a range of schools in terms of system, geography and socio-economic status. For this study we identified school leaders as those who had a formal leadership role within each of the case study schools. In some cases, the leaders were School Principals, in other cases, they were Deputy Principals, designated Technology Leaders, or Heads of Mathematics.

Students participating in focus groups were selected by their teachers, from the pool of students who had returned consent forms, as a representative sample of the case study teachers' students. Where possible students were chosen to represent a mixture of gender, ability, and attitudes towards mathematics.

Summary details relating to the participants' schools are included in Table 2 below:

Six of the case studies were drawn from individual teachers and schools. Two sets of case studies incorporated two teachers from the same school. This was not intentional on behalf of the researchers. Rather, the teachers involved had all been identified as exemplary users of technology and thus were included in the research. Likewise, the gender spread amongst the case study teachers unintentionally resulted in a group of all male secondary teachers and all female early childhood and primary teachers. Given that the majority of primary school teachers in Australia are female, the group was considered to represent the broader teaching population.

2.2. Data collection

Approval to conduct this research was provided by the Western Sydney University Human Research Ethics Committee (Approval No. H12456). In each of the case study sites data was collected from three participant groups: the case study teacher, a school leader, and students. Data collected from the case study teacher included classroom observations, lesson plans, and interviews. Students participated in a focus group discussion, and a nominated school leader participated in an interview. The multiple sources of data align with recommendations by Yin (2008) to provide a rich picture of each case and a detailed analysis of the case study teacher's classroom practices and student perceptions. For the purpose of this paper the data from the case study teachers, their students, and classroom observations will be used.

Semi-structured interviews were selected as a way of garnering in-depth information from teachers and school leaders in each of the cases. The open-ended nature of the interviews allowed each interviewee the opportunity to respond to the same set of prompts to increase the comparability of results while also allowing opportunities to delve deeper into participant responses and match any further questions to individuals and circumstances (Cohen et al., 2018). The interviews provided an opportunity for the researchers to explore the teachers' practices regarding the use of technology.

Interview prompts used in the teacher interviews were as follows:
Question set 1

- We have observed 2 (or 3) of your lessons where you have used technology for teaching mathematics.
- Can you explain your thinking when planning these lesson(s)?
- Did the lesson(s) proceed according to plan?
- On reflection, will you change anything when teaching these lesson(s) in future?

Question set 2

- In general, what do you consider to be the benefits of using technology for teaching mathematics?
- Are there particular mathematics topics that are better suited for teaching with technology? Which ones? Why?
- Are there particular mathematics topics which you teach without technology? Which ones? Why?
- In general, how do you decide when to use technology for teaching mathematics? And which technology to use?

Student focus groups prompts:

- We would like to know about how you like to learn mathematics.
- Do you like mathematics? Why? Why not?
- What are the most effective ways for you to learn mathematics?
- What types of technology have you used in the mathematics classroom?
- How does the technology help you to learn mathematics?
- Do you think you should be able to use more technology when learning mathematics?

Semi-structured observations were conducted to provide rich contextual information about the verbal, non-verbal, and physical

Table 2. Case Study details.

Case	*Teacher	Grade	**Experience	School Type	Location	***ICSEA (2018)	Students (2018)	Focus Group Students (Grade)
A	Adam	9–12	Mid	Catholic	Metropolitan	1076	1021	3 (9)
B	Ben	7–10	Early Career	Public	Regional	1080	1155	5 (7)
C	Cameron	9–12	Mid-Career	Catholic	Remote	1022	561	3 (9)
D	David	10–12	Mid-Career	Private	Metropolitan	1131	1327	6 (10–12)
E	Emma	6	Early Career	Private	Metropolitan	1184	1658	5 (6)
F	Fiona	3	Early Career	Public	Metropolitan	1084	478	6 (3)
G	Grace	3	Mid-Career					6 (3)
H	Helen	1/2	Early Career	Public	Metropolitan	989	356	N/A
I	Ivy	Special Unit	Mid-Career					N/A
J	Jane	Pre	Late-Career	Public	Metropolitan	1028	322	N/A

* Psuedonyms.
** Early Career = 0–5 years; Mid-career – 5–10 years; Late career = 10 + years.
*** Index of Community Socio-Educational Advantage (for more information see https://myschool.edu.au/media/1067/guide_to_understanding_icsea_values.pdf).

interactions between teachers, students, devices, and mathematical content (Cohen et al., 2018). Observations were also used during the teacher interviews and student focus groups and this provided a context for the participants and researchers from which to base some of the conversations. Field notes were also taken during the observations. The observation schedule focused on the following areas:

- Teacher roles (eg. Lecturing, interactive, coaching, moderating discussion)
- Student groupings (eg. Individual, pairs, small groups, whole class)
- Technologies used by teacher and students (eg. Tablet computers, laptops, software)
- Mathematics content and processes covered (eg. Problem solving, fluency building)
- Evidence of student engagement (eg. Cognitive, operative, or affective)

2.3. Data analysis

Data drawn from interviews and focus group discussions were audio recorded and transcribed verbatim. Observations were video-recorded. Data analysis was conducted in alignment with the research question posed in this paper. To do this all relevant data from interviews and focus group discussions from each of the case studies were collated to provide collective responses to the research question (Cohen et al., 2018; Saldana, 2016). Data from the interviews and focus groups were initially independently coded and categorised into broad themes. Field notes and observations were used to support further analysis. One of the dominant themes that emerged from the original coding was that of engagement, which led to a second coding of data within that theme, against the elements listed in the FEM (Attard, 2014) using *NVivo* software. The findings from the cross-case analysis relating to engagement are now presented.

3. Results

Items coded against the FEM were split almost evenly across the two main sections of the framework: pedagogical repertoires and pedagogical relationships. Each of the framework elements within the two sections had alignment with the data, indicating that technology use by these teachers promotes student engagement. Some items aligned with more than one element and were therefore assigned multiple codes. We now present the results, where we weave the data with our discussions. First, we address data pertaining to pedagogical relationships.

3.1. Pedagogical relationships

The development and maintenance of positive pedagogical relationships is considered the foundation for student engagement with mathematics. Without this foundation a teacher's repertoires may fail to engage students, regardless of their quality (Attard, 2014). Although there was alignment of data with each of the five elements in the pedagogical relationships section of the FEM (Table 3), the alignment was not evenly distributed. Data related to the ways in which technology facilitated the teachers' awareness of individual students' learning needs (TA) attracted a significantly higher number of references than the other elements.

As stated earlier, a number of items were coded against more than one element of the FEM, indicating the inter-related nature of the elements.

3.1.1. Teacher awareness (TA)

The data revealed that one of the most significant contributing elements of engagement resulting from technology use was the ease with which the technology allowed teachers to understand and respond to individual student learning needs (TA) as evidenced in the number of coding references (36%). This was facilitated in a range of ways. For example, in Case J, Jane used iPad apps that required the students to interact with mathematics in collaboration with the teacher or teacher's aide. This allowed the teachers to monitor and respond to student misconceptions or achievement. Jane explained the benefit of working this way with such young children (4-year-olds):

Table 3. Representative quotes, pedagogical relationships.

Pedagogical Relationships			
Element	Coding References	No. of Cases	Representative Quote
Teacher Awareness (TA)	24	9	The technology allows students to seamlessly move between them (levels of work) and also, surreptitiously move between it. So, they are not announcing to the class they are one level or another, they can just work on what they are comfortable with. (Ben, Case B)
Pre-existing Knowledge (PK)	7	5	But everything they do in the game they have to answer a maths question, and in the back end I've got control over the content. So, if they're doing length and measurement or probability that week or in two weeks, I can just do questions on that. Or I can say, we need to do some revision on this topic, and off they go. (Ivy, Case I)
Continuous Interaction (CI)	15	6	I like the fact that if you're really struggling on a single question or a section and you can't get it from the videos you can email him, and he makes a personalised video for you. (Student, Case D)
Constructive Feedback (CF)	8	4	I'll make a video explaining exactly what you've got to work on what you did wrong what you did right. (David, Case D)
Pedagogical Content Knowledge (PCK)	12	6	It's more like okay, well, what technology can support. What's the best way of teaching this concept? What's the best way of getting the students engaged with this this concept? If it does have a technology answer to that question, then I'm definitely going to be leaning towards it (Adam, Case A)

> They're familiar with it and they like to do it. It's that whole - it's that beautiful sense of agency. “I'm doing this and I'm learning it myself. If I'm getting right, I'm moving up to the next level and I'm doing” - so, I think there's - you can get - it's such a powerful tool. Because they can see. They can see their learning.
>
> (Jane, Case J)

Understanding her students’ abilities in mathematics as well as their abilities to use the technology was a priority for Fiona (Case F). Fiona is a Year 3 teacher, who at the time of data collection was in her first year of teaching. Fiona was conscious of designing tasks that challenged her students mathematically as well challenging their technology-related skills:

> I think knowing my kids, like I know that they are capable of using – the other day I introduced like a formula on a spreadsheet and, for

> example knowing my kids I know that that is something that they are capable of using, rather than – if, for example, they weren't really at that level I would do more games and things like that.
>
> (Fiona, Case F)

In Ben's classroom, students were offered three different levels of activities within each lesson to ensure each student's learning needs were being met. This differentiated learning evolved from Ben's concern that he was not able to address individual needs with a 'one size fits all' approach. Using a combination of *Canvas* and *OneNote* allowed Ben to provide differentiated tasks and keep track of student work which was uploaded to the learning management systems. This, according to Ben, has resulted in students becoming more confident in mathematics; a critical requirement if they are to develop positive attitudes that lead to the continued study of mathematics (Boaler, 2015). Ben talked about the success of this strategy:

> I have had a number of students say to me, certainly in the older classes, Year 9 and Year 10, "that is first maths exam I have ever passed in my life". And they have just stayed in foundation level the entire time. And they come out of the class with a sense of achievement and accomplishment because they have actually achieved something as opposed to getting half way through class, getting one or two right and then being thrown into the harder problems, getting confused and ultimately frustrated. So, I have found the student engagement has been much, much higher at all levels, up to Years 9 and 10.
>
> (Ben, Case B)

Although Ben could have provided different levels of tasks without the use of technology, locating them within a management system meant that students were able to select a level of task privately without fear of embarrassment. While the teachers intentionally used technology to track and respond to student needs, there was also evidence that students were aware their individual needs were being met. In Case F, students as young as Year 3 (approximately 8 years-old) talked about this:

> In the classroom the main technology we use is our iPads and the teacher gives questions on the board and then we all have to answer them on – like on *Explain Everything* or *SeeSaw*. So, she can interact, and we have these special groups in the classroom.
>
> (Student, Case F)

> And the reason that the teacher puts in questions is so when you play it the teacher knows and: "Oh, this person is having trouble at multiplication – next time I will try and work with this person to get better at it."
>
> (Student, Case F)

Senior secondary students also spoke about the benefits of technology enabling their teachers to address individual needs. In David's classroom (Case D), learning is differentiated through a broad range of tasks that are located in *OneNote.* Each student also has an individual *OneNote* file that provides David with access to their work. This structure lets David's students progress through the work in different ways through different trajectories, according to individual abilities. All of David's students begin at the same point, and his intention is that by the end of a unit of work/topic, they all achieve the set outcome. Several of David's students spoke about this element of their learning and the following comment typifies the sentiments of the group:

> Now there's videos and things that [teacher] has and we work at our own pace. I have for an entire lesson sat on a four-minute video re-watching it because I didn't understand it and there have been other lessons where I've gone through three videos and then just gone through three entire sections because I just understand what I'm learning.
>
> (Student, Case D)

The fact that students indicated an awareness that their needs are being met is significant in terms of improving or maintaining their engagement with mathematics. In the case of this study, it appears the use of tools such as learning management systems allowed the teachers' intentions and resulting actions to be more transparent to students, promoting positive pedagogical relationships and, according to the teachers, improving engagement.

3.1.2. Pre-existing knowledge (PK)

The acknowledgement and contribution of students' backgrounds and pre-existing knowledge (PK) attracted the least number of coding references ($n=7$). This may be due to the challenge in observing how pre-existing knowledge contributes to the learning of others and the close link between this element and the teacher awareness of student needs (TA) element. However, it is arguable that a teacher's understanding of pre-existing knowledge does contribute to the learning of others as a result of how it assists in determining teaching and learning activities that are subsequently undertaken by the whole group.

There are specific affordances offered by the use of technology that allowed the teachers in this study to understand their students' pre-existing knowledge and consequently influencing their planning. In six of the case studies, the teachers used technology as a portal to store student work samples allowing them continued and easy access for planning, assessment, and reporting. Learning management systems such as *Google Classroom, SeeSaw, OneNote,* and *Canvas,* and apps such as *Padlet* also allowed teachers to store work samples recorded using a range of media such as video, still photography, and audio recordings. In some cases, student work remained accessible from one year to the next, allowing teachers insight into their students' knowledge from the start of the school year.

Technology allowed the teachers opportunities to plan activities based upon their students' pre-existing knowledge. This was evident in Case H, where Helen used the *Plickers* app to conduct pre-assessments. *Plickers* replicates the use of a classroom response system that uses individual student codes printed on card rather than personal devices or clicker devices. As Helen's school did not have a BYOD program and had limited access to student devices at the time of data collection, *Plicker* allowed her to gather data on her students' pre-existing knowledge using multiple choice questions, an interactive whiteboard and her personal mobile phone. Helen talked about how she uses this data to inform teaching, gather assessment data, and share student work with parents. Although this strategy is easily replicated using traditional pen and paper assessment, technology provides instant results, is time saving, and allows children to be more operatively and affectively engaged in the process. The time saved in gathering and reporting results allows more lesson time and removes the negative attitudes often associated with mathematics and assessment (Henschel and Roick, 2017).

3.1.3. Continuous interaction (CI) and constructive feedback (CF)

The elements of continuous interaction (CI) and constructive feedback (CF) are grouped in this section due to the way technology enabled teachers, through opportunities for multi-directional communication within and outside the classroom, to provide feedback that was prompt, personalised and engaging, within those interactions. CI was more evident in data derived from student focus groups and teacher interviews in the secondary school case studies. However, observations in the primary classrooms revealed high levels of verbal interaction that appeared to be facilitated due to the nature of the technology-enhanced tasks observed. For example, Emma introduced a task that was embedded in a novel context, and it was that context, combined with the challenge of the task, that sparked high levels of teacher-student and student-student interactions.

Many of the tasks observed in the primary and early years case studies were collaborative, which naturally promoted higher levels of interaction. In contrast, the secondary tasks observed were all individual, yet the use of technology appeared to promote discussion. For example, in Adam's Year 9 classroom (Case A), a flipped learning approach was utilised and students were required to complete tasks independently but, with the help of flexible furniture, they were encouraged to form organic groups during class time. One student explained how this works:

> I enjoy when the teacher just teaches us for the first 15 minutes and then they give us a task and then we complete the task in groups. Most tasks we can get it from different perspectives. When we work in a group, one person might have this opinion, but another person might have a different opinion from a different angle. That's really good.
>
> (Student, Case A)

Self-paced learning through the use of a flipped learning approach (Cronhjort et al., 2018; Huang et al., 2018) in Cases A, B, and D provided teachers with more time for face to face interactions in the classroom. In David's classroom (Case D), lessons did not have a formal, traditional structure, and students worked at their own pace. Any class or group instruction, intervention or other assistance occurred at the point of need rather than at the start or completion of each lesson, and students were observed to engage in continuous interactions with each other and with David throughout the lesson. The nature of the interactions in this classroom had a positive impact on his students' engagement, and this comment is representative of the focus group sentiment:

> I think the fact that he's not up there teaching us all the time, using the opportunity to walk around and interact with us and help us out, I think that's something really important that most teachers don't usually get around to because they're always up the front and [unclear] I would not ask questions and ask for help but he's always coming around and asking how are you doing, you feel more obliged to actually speak up and be like, okay, yeah I don't know what's going on and you also get to have a pretty good relationship with him.
>
> (Student, Case D)

David's use of technology also enhanced the quality, timing and personalisation of feedback to students:

> In terms of like formative assessment there's a number of different ways that I do it. One of them is here's your diagnostic test, hand it back to me, I'll make a video explaining exactly what you've got to work on, what you did wrong, what you did right. I'll email it to you or maybe I'll email it to your parents so they can see where you're at. That's been, that's really, if I couldn't do anything else in technology, I'd probably choose that as my one, that's made a huge, huge difference. Students are getting individualised feedback.
>
> (David, Case D)

David and Cameron (Case C), both regularly used technology to promote interactions with students and their parents. Where Cameron used email, David used a combination of email and individually tailored videos, developing and extending the pedagogical relationships between teacher, student, and parents.

3.1.4. Pedagogical content knowledge (PCK)

The majority of teachers in this study appeared to give careful consideration to how technology could support students' learning and engagement, demonstrating PCK via the technology-related pedagogical choices they made. This was more common amongst the secondary teachers (9 out of 12 coding references). They each spoke about how they would not use technology if it did not enhance conceptual understanding, and the focus on mathematics rather than technology was clearly emphasised by Adam in this quote:

> With the younger years, I might use technology to engage the students. So, we might use say the *Desmos* marble slides, for instance. So, that's fun and engaging for the students. But it's still centred around the concepts that we need them to learn. So, I don't think oh, here's a cool tool, let's do the lesson on that.
>
> (Adam, Case A)

It appeared that there were two distinct uses of technology across the case studies. One use was based on organisational structures in the form of learning management systems, rather than apps that specifically focused on mathematics. However, these structures did appear to promote the development of pedagogical relationships which in turn improved student access to the mathematics. According to the teachers, this, alongside the use of mathematics-focused technology use such as *GeoGebra, Excel,* or *Desmos,* which were often embedded within the learning management systems, appeared to engage the students in these case studies.

An additional engaging element linked to teachers' PCK was the use of carefully selected videos from sources such as YouTube or self-made videos in the secondary case studies. PCK was also an important factor in considering the mathematical skills that could be enhanced through the use of technology. For example, Ben (Case B) discussed how he makes decisions of technology use:

> I see the use of technology in a few different ways. One is visualisation, you can use a lot of static props and hand things out and sometimes that's great to get hands on experience, and I still like to use dice and things like that in the classroom and there are some static props that I use because children like hands on stuff as well. But the ability to use things like *GeoGebra* but that's just one of the several that I use, you can actually very dynamically change things and they can visualise something very quickly.

In the primary classrooms, the choices of technology use included a range of generic productivity apps such as *Explain Everything* and *Padlet*, apps that allowed either teachers or students to design multiple choice questions such as *Plicker* and *Kahoot,* and game-based apps such as *Prodigy,* which allow the teacher to control the content and level of the mathematics involved, drawing on the teachers' PCK and resulting in high affective, cognitive and operative engagement. The following two quotes from Fiona's students are representative of the general attitudes towards the use of the *Prodigy* app:

> Well we were playing a game called Prodigy and it helps a lot in maths 'cause it is like a wizard game where to attack you need to work out a maths problem and then – but if you get it wrong you only have – you can only try twice and then it will show you the answer then you have got to remember it again the next time when it asks that question.
>
> (Student, Case F)

> I like working in maths and technology. And like [student] said, ...it is quite a challenge because they ask you very hard questions and the higher the levels get the higher – they give levels and the higher they get the more harder the questions are.
>
> (Student, Case F)

There was one case where apps that targeted specific mathematical content were observed to be used, and that was in Case J, the pre-school classroom. It was also noted that in almost all of the primary and preschool observations, concrete materials were used in conjunction with technology, which appeared to contribute to the operative engagement of the students.

Overall, data from this group of exemplary teachers and their students indicated their use of technology promoted the development of

Table 4. Representative quotes, pedagogical repertoires.

Pedagogical Repertoires			
Element	Coding References	No. of Cases	Representative Quote/Observation
Challenging Tasks (CT)	9	5	Driverless Car Lesson (Table 5) (Case E)
Relevant Tasks (RT)	12	6	...the end task was that they had to plan a trip around Australia so that was using maps and things like that and they had to use like – I gave them a map and they had to plot where they were going to stop and things like major landmarks and um figure out distances and plan them. (Fiona, Case F)
Provision of Choice (PC)	8	5	Now the way that I try to sequence that instead now is that if I provide them with enough resources and differentiated resources a student who can learn much more quickly than that can sort of head off, learn their content really, really quickly (David, Case D)
Variety of Tasks (VT)	13	9	At the moment our Year 10s are doing some, they're designing like a geometric sculpture. So, they're sort of using things like *3D Paint, Inventor, Tinkercad,* whatever. It's incredible actually because it's at a stage now where we're saying to students just, you need to design something in 3D and in order for you to do well in this assignment we wouldn't want to see it hand drawn, we'd want to see a graphical representation of that.
Substantive Conversation (SC)	8	4	Students are self-directed. Some ask teacher questions, some ask peers questions (Observation, Case D)
Student-Centred Technology (ST)	30	10	I like YouTube videos. YouTube videos, they step-by-step explain the task. Sometimes if you don't understand it from a teacher's point of view, the equation or question, someone on YouTube might do the same question and they'll have a different angle to it. That's really good. (Student, Case A)

Table 5. Driverless car activity (Case E).

'Driverless Car Prototype'

Congratulations! You are applying for a job to be a software engineer at Tesla. Your role will be coding the routes that the Tesla cars have to drive. To apply for this job, you need to provide a prototype to prove your coding and mathematical ability!

To prepare for this job interview your group will need to:

- Choose a real-life experience driving experience e.g driving from school to (insert name of local café)
- Design and Draw a map to scale of your area that includes the roads
- Design the code that Sphero will need to go
- Take a time lapse of Sphero completing the challenge
- Make any adjustments to your code
- Design another route using the same map

Things to think about and discuss in your group (record your answers in *OneNote*)

1) What would be the best 'scale' to use?
2) What is the total area that your map represents?
3) Why is it important for the map to be accurate?
4) What is the difference in speed that Sphero is going compared to an electric car?
5) Why is prototyping important?
6) How would this code for Sphero be converted into real time/real km?

Sharing with the interview panel:

- Upload a screenshot of your code to the Padlet link
- Turn your time lapse into a QR code and upload to the Padlet link

Reflection on Task:

This document contained a hyperlink to a *Google Form* that required students to respond to reflection prompts.

> positive pedagogical relationships. This quote from Grace synthesises the findings: "I mean, you have got to know your kids, you have to know your content, and know the resources to kind of bring it all together".
>
> (Grace, Case G).

3.2. Pedagogical repertoires

Engaging pedagogical repertoires are often enabled by quality pedagogical relationships, and the two are closely intertwined. In this study, there was evidence that the affordances offered by the various technologies accessed, and the ways this group of teachers utilised them, resulted in student engagement. Table 4 provides a snapshot of the number of coding references and a representative quote for each of the individual elements of pedagogical repertoires drawn from the data. As with pedagogical relationships, several items were coded against multiple elements, resulting in the discussion below weaving some of the elements together.

3.2.1. Challenging and relevant tasks (CT and RT)

The use of technology appeared to promote the design of challenging tasks for several of the case study teachers. For example, in Case A, the whole school planned teaching and learning from a problem-based learning approach. In Adam's classroom, this meant that each of his topics were introduced through a challenging problem that was introduced via the *Canvas* learning platform. Similarly, the school in Case E had a strong focus on inquiry-based learning, and all teachers were required to incorporate this approach across all subject areas, including mathematics. An example of such a task was observed during this study and is detailed in Table 5.

The Driverless Car Prototype lesson appeared to be highly engaging to students and this was evidenced by the amount of interaction that occurred, the length of time students appeared to be focused on the task (1 h), and their observed enthusiasm. The task was challenging as it required students to apply several mathematical concepts in order to address the criteria and the questions posed. It was open-ended and contextualised, and allowed students to access a range of technologies. This task aligns with the *redefinition* level of Puentedura's SAMR model of technology integration (2010) as it would have been inconceivable without the inclusion of technology.

In the younger classrooms, tasks that were observed as challenging and relevant were less complex and were often less open-ended. An example of the tasks observed was the use of *Kahoot,* as described earlier in this paper. Two teachers, Grace and Fiona, were observed to use this app at the end of their lessons to re-focus the students and gather assessment data. In each of these classrooms, students were visibly excited and all actively participated, indicating high levels of affective, operative, and cognitive engagement. Other examples of short challenging and relevant tasks included game-based iPad apps such as *Prodigy.* This game was discussed by teachers and students alike, in three of the case studies. The quote below is drawn from Ivy, who taught in a special unit for students with autism and other significant learning disabilities, unable to attend mainstream classrooms:

> I found *Prodigy* incredibly engaging, and so do the rest of the school, because it's a game. So, it's that role-play game. I think one of my autistic guys described it best. He said, "it's basically just Pokémon meets Undertale", which was his favourite game at the time, a role-play game.
>
> (Ivy, Case I)

Two common elements amongst all of the items coded at RT and CT is that they linked to students' interests and were mathematically challenging, sometimes opening up opportunities for students to access mathematical concepts beyond their year level. Some of the tasks above simply required students to be consumers, where others required them to become producers. Although typically considered to be low-level use of technology, the apps that required students to consume are still considered important contributors to student engagement due to their capability of providing instant feedback, tracking student progress, and allowing the teachers to determine the topic and level of content for students to work with.

3.2.2. Provision of choice and variety of tasks (PC & VT)

As with several of the other elements of pedagogical repertoires, choice was embedded in a variety of ways across the case studies. For example, in Cases B and D, students were provided with choice within every lesson due to the provision of tasks at different levels. In Case E, students were often given the choice to use technology, and which technology to use. For example, if they were responding to a task, they could record their responses directly onto their individual iPads, or they could respond in their books. This also occurred in Fiona's classroom (Case F), where students were also given the option to record their responses using video:

> …rather than just always having to write things down, there's different ways of presenting ideas and information. I like to get them to take videos or take voice recordings because sometimes it is not always about writing it down, and more about finding out what they know. Because [for] some of my kids it takes them forever to write things down in a book. It is just a different way to cater for all the needs in the classroom and being able to get them to show me their knowledge without literacy skills getting in the way and also just a different way of doing it. I just like to – I like things to be different all the time and not get bored.
>
> (Fiona, Case F)

Fiona's comment highlights how some tasks contain both PC and VT elements of pedagogical repertoires. In this study, there was evidence of choice without variety, and this was more common in the secondary classrooms or in lessons where the choice was linked to task responses, as in Fiona's example above. Further evidence of choice was embedded with the task design, and is illustrated in the Driverless Car Prototype task from Emma's classroom (Case E), where students could choose the location to situate their maps, linking to other elements such as RT and ST.

3.2.3. Substantive conversation (SC)

It was evident that substantive conversation (SC), which attracted eight coding references, resulted from a combination of teacher dialogue, technology-enriched tasks, and a range of other pedagogical decisions relating to the physical structure of classrooms. For example, in Ben's classroom, students accessed tasks via a learning management system but actually engaged in the task using a vertical whiteboarding system (Forrester et al., 2017) where students work on whiteboards to share and record solutions. This strategy promoted collaboration and mathematical reasoning amongst the students. Once work was completed, students would photograph their whiteboards and upload the work samples to *OneNote.*

Similarly, in many of the primary classrooms, students used a combination of technologies and manipulatives. Many of the tasks that promoted SC incorporated mathematical problem solving and investigation. For example, in Grace's classroom we observed a lesson that incorporated iPads and BeeBots (robotics and specifically designed floor mats). In the pre-school classroom (Case J), the use of mathematics iPad apps combined with close teacher supervision the use of videos (student-made and externally sourced), resulted in substantive conversations about mathematical concepts.

3.2.4. Student-centred technology (ST)

Although the FEM was not originally intended as a tool to use specifically with technology as a focus, this element was considered important to retain in this analysis as it brings together many of the other elements included in the pedagogical relationships and the repertoires sections. As is evidenced with the high number of items coded against this element ($n=30$), the majority of technology use observed across the 10 case studies was considered to be student-centred (ST). The ways in which the teachers developed ST that enhanced teaching and learning varied. For example, an ST approach resulted in classrooms where a flipped learning approach was adopted and enabled through the use of learning management systems. This approach allowed students to access materials before, during, and after lessons. An added benefit of this approach, as articulated earlier in this paper, was that often parents also had access to the learning resources, assessment results and student work samples. Emma commented on this in relation to her use of *OneNote*:

> And it just means that the learning is accessible at home, parents can see what we are doing and stuff isn't lost. OneNote is a resource is that they can then take with them and continue to grow with that.
>
> (Emma, Case E)

Two of the secondary teachers who used learning management systems also embedded differentiation as part of their routine pedagogical practices. This strategy had a significant effect on students and their confidence with mathematics. Not only was their learning more self-paced, it was tailored to their learning needs (TA), resulting in more opportunities for all students to achieve some academic success. The following excerpt of an exchange between the researcher and student from David's classroom highlights one high performing student's perceptions of working in this way:

> *Student:* Yeah, and just sit there and be like, okay, I have nothing else to do. So I could go at my own pace, and with the *GeoGebra* and stuff like that there's a few of them that have unlimited questions so you can just keep going on and on and on.
> *Researcher:* So you love doing unlimited questions?
> *Student:* Yeah. So I kind of just - I get to do as many as I need to keep going and I also get slowed down by other people or they can go back and watch again.
> *Researcher*: Can you go forward; not just repeating questions but is there more for you to do to extend yourself?
> *Student*: Yeah, yeah. So usually he has a topic in the *OneNote* and there will be all the videos to be done for the week. There will be a few - five videos sort of thing.
> *Researcher:* So, you just go crazy?
> *Student:* Yeah. So I can go do that and other people will be behind but there's always stuff for me to do. So yeah, I guess that's something I struggle with in other classes. But you can always do that, which is good. (Student, Case D)

In summary, the range of pedagogical repertoires observed across the 10 case studies indicated that the affordances of the technologies available to each of the teachers were utilised in ways that promoted student engagement. However, it is important to note that the technologies did not stand alone. They were embedded in teacher practices that were informed by the positive pedagogical relationships (also strengthened by technology use), and, reciprocally, those repertoires assisted in strengthening those positive relationships.

4. Conclusion

> I never really liked maths. Just growing up I always found it really hard and I could never think in that way. Just numbers, it just didn't really work with me as such. But in the past year I have been able to understand it a bit more just through this way of teaching.
>
> (Year 11 student, Case D)

The quote above provides evidence that the student's teacher and his technology-related practices have made a significant difference to his experience of mathematics. The purpose of this paper was to explore the ways in which technology-related practices promote student engagement with mathematics. The results of this study illustrate that, in each of the 10 case studies, technology-related practices promoted student engagement by supporting the development of positive pedagogical relationships and the design of engaging pedagogical repertoires. A fine-grained analysis of data drawn from the case studies indicated alignment with each element of the Framework for Engagement with Mathematics (Attard, 2014).

Analysis of data from teacher interviews, observations, and student focus groups indicated that the technology-related practices of this group of teachers appeared to address some of the well-documented issues known for causing disengagement. That is, a lack of curriculum relevance to students' lives, a divide between school mathematics and mathematics in the real world, affective issues concerning student confidence, and a move away from traditional textbook based lessons (Boaler, 2009). This was the result of technology-related practices that promoted positive pedagogical relationships through strategies that included variations of a flipped learning approach, the use of personalised videos, timely feedback, and personalised learning pathways. The pedagogical repertoires observed ranged from simple fluency building activities through apps designed specifically for mathematics, to contextualised tasks that required students to draw on a range of digital technologies in order to create a new product. A critical contributing factor to the engagement of students was the ways in which the technology enabled learning to extend beyond the classroom in ways that extended opportunities for communication, collaboration, and exploration.

In this paper we have presented data from the classrooms of teachers who are regarded by their peers as effective users of technology. Gaining insight into their technology-related practices allows us to better understand how the nuances of pedagogical decision making can have significant influence on student engagement. For example, the ways in which learning management systems are utilised can range from the use of the system as a static portal for teaching resources and student work such as that demonstrated in Case A, through to being a multi-directional system for communication between the teacher, students and parents through text and video as in Case D. On the surface, many of the practices discussed in this paper may appear to be standard practice, and in some cases, may not appear to be innovative. By interrogating these practices through the lens of the FEM we can see that regardless of the amount of access or the range of devices or software applications used, technology can be used to promote student engagement with mathematics.

This paper has added to existing knowledge of how technology-related practices mediate student engagement with mathematics, fostering in students, as articulated in the quote in the paper's title, a sense of hope. A deeper knowledge of how technology use alongside sound practice can improve student engagement with mathematics, laying the foundations for improved academic outcomes, and an understanding of the subtle effects of technology use through the FEM, may be of assistance to other researchers of mathematics education and practitioners alike. Continued research as technology continues to evolve is critical if teachers are to effectively maximise the affordances offered by contemporary technologies.

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Author contribution statement

C. Attard, K. Holmes: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

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