

Geometric figure–ground cues override standard depth from accretion-deletion

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Accretion-deletion is widely considered a decisive cue to surface depth ordering, with the accreting or deleting surface interpreted as behind an adjoining surface. However, Froyen, Feldman, and Singh (2013) have shown that when accretion-deletion occurs on both sides of a contour, accreting-deleting regions can also be perceived as in front and as self-occluding due to rotation in three dimensions. In this study we ask whether geometric figure–ground cues can override the traditional “depth from accretion-deletion” interpretation even when accretion-deletion takes place only on one side of a contour. We used two tasks: a relative-depth task (front/back), and a motion-classification task (translation/rotation). We conducted two experiments, in which texture in only one set of alternating regions was moving; the other set was static. Contrary to the traditional interpretation of accretion-deletion, the moving convex and symmetric regions were perceived as figural and rotating in three dimensions in roughly half of the trials. In the second experiment, giving different motion directions to the moving regions (thereby weakening motion-based grouping) further weakened the traditional accretion-deletion interpretation. Our results show that the

standard “depth from accretion-deletion” interpretation is overridden by static geometric cues to figure–ground. Overall, the results demonstrate a rich interaction between accretion-deletion, figure–ground, and structure from motion that is not captured by existing models of depth from motion.

Introduction

A crucial task that the visual system has to perform is to estimate three-dimensional (3-D) layout—i.e., relative depth ordering of surfaces—as well as the 3-D shape of these surfaces, from two-dimensional (2-D) retinal images. Figure–ground organization requires determining which regions own which contours in an image, and assigning “figure” and “ground” status to those regions accordingly. The region that has the figure status is shaped and bounded by this contour, while the ground region is perceptually unbounded and continues amodally behind the figural region (see, e.g., Nakayama, He, & Shimojo, 1995). The visual system

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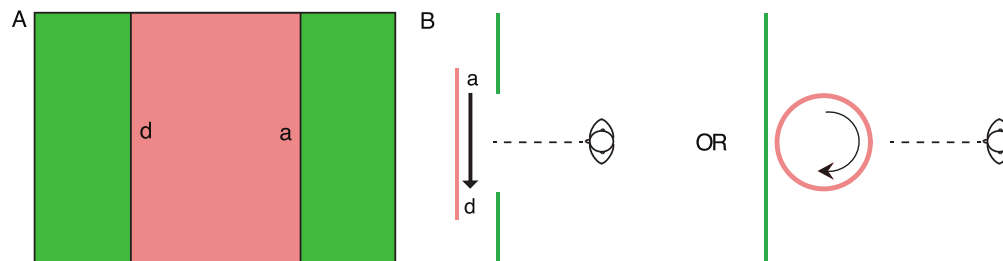


Figure 1. (A) The frontal projection of an accreting surface. The location of texture accretion is marked (a), and the location of texture deletion (d). The static surface is depicted in green, and the moving surface is depicted in red. (B) Overhead views of the two possible 3-D arrangements with different depth-order assignments that are both consistent with the frontal view of the accreting and deleting surface.

exploits numerous cues in order to assign figure and ground. An important class of figure–ground cue comprises shape factors, i.e., geometric properties of region or boundary shape that influence figural status. Many different geometric cues that tend to promote figural status have been proposed, such as symmetry (Kanizsa & Gerbino, 1976), convexity (Metzger, 1936/2006; Kanizsa & Gerbino, 1976; Burge, Peterson, & Palmer, 2005; Burge, Fowlkes, & Banks, 2010), parallelism (Metzger, 1936/2006; Morinaga, 1941), axiality and part salience (Hoffman & Singh, 1997; Froyen, Feldman, & Singh, 2010), and many others (for a review, see Wagemans et al., 2012).

Besides these static cues, there are also dynamic cues to figure–ground assignment where motion provides information about depth ordering. One such cue is accretion-deletion of textured regions (Kaplan, 1969; Mutch & Thompson, 1985; Thompson, Mutch, & Berzins, 1985). When a translating texture deletes at or accretes from a boundary, it tends to be perceived as disappearing or appearing from behind an occluding surface on the other side of the boundary. This in turn generates a vivid sense of figure and ground. This accretion-deletion of textured surfaces is often described as a decisive visual cue that can unambiguously assign depth order to surfaces (Gibson, Kaplan, Reynolds, & Wheeler, 1969; Kaplan, 1969; Mutch &

Thompson, 1985; Thompson et al., 1985; Niyogi, 1995; Howard & Rogers, 2002; Hegdé, Albright, & Stoner, 2004), and it has been used as such in computational models of depth from motion (Yonas, Craton, & Thompson, 1987; Berzhanskaya, Grossberg, & Mingolla, 2007; Beck, Ognibeni, & Neumann, 2008; Raudies & Neumann, 2010; Barnes & Mingolla, 2013; Layton & Yazdanbakhsh, 2015). Previous researchers have quantified the strength of this cue in terms of its ability to resolve the ambiguity of direction of rotation in orthogonally projected spheres, as when it overrides another important depth cue, motion parallax (Ono, Rogers, Ohmi, & Ono, 1988).

Contrary to the conventional view of accretion-deletion as a decisive cue to ground status (i.e., as a more distant surface), there exist stimuli in which the accreting/deleting side is interpreted as being clearly in front (i.e., closer to the observer). Specifically, the accretion-deletion of a surface can also be explained as arising from self-occlusion due to a 3-D object rotating in depth (Figure 1).¹ A striking example of this interpretation can be seen in the displays recently presented by Froyen et al. (2013). In their stimuli, accretion-deletion was introduced on both sides of a border. This created a bistable figure–ground stimulus where either the dark or the light regions were perceived as in front and rotating in depth (Figure 2).

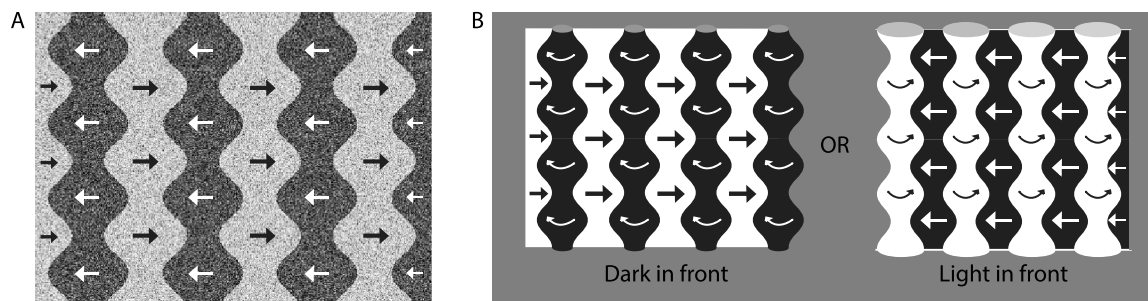


Figure 2. Display setup and phenomenology. (A) The displays were created by adding motion in one direction to odd regions and in the other direction to even regions in classical figure–ground displays. (B) This could yield one of two percepts, depending on which region was perceived as figural. The black regions were perceived as rotating in front of a white background which was seen as sliding behind them, or vice versa.

When geometric figure–ground cues (e.g., convexity) were introduced to the regions, this ambiguity was resolved such that the regions that were perceived as figural (e.g., convex) were also perceived as 3-D volumes rotating in depth (even though the constant-velocity texture motion is inconsistent with 3-D structure from motion). This possibility has occasionally been noted in passing by previous researchers (Kaplan, 1969; Yonas et al., 1987; Royden, Baker, & Allman, 1988) but has not been incorporated into standard accounts of accretion-deletion.

Recently, other studies have focused on the ambiguity caused by accretion-deletion. Kromrey, Bart, and Hegdé (2011) showed that accretion-deletion needs additional information about the occlusion border in order to unambiguously assign depth order. In their stimulus, an enclosed region containing translating random-dot texture was surrounded by random-dot texture. They observed that when the surrounding-region texture was flickering, the central region was seen as in front (in a paradoxical “moonwalk” motion) even though the texture in the central region was accreting and deleting. According to the authors, only when the delineation of the border between the center and the surround regions was made easier by segmentation cues (such as making the surrounding region static or increasing the luminance contrast between the two regions) was the interpretation that is consistent with the traditional account of accretion-deletion (i.e., seeing the translating texture as farther away) favored.

The studies summarized in the foregoing indicate that the shape of the border interacts with the accretion-deletion cue. Froyen et al. (2013), in particular, suggest that this interaction might have serious implications for accretion-deletion as a cue to depth. However, in the stimuli they used, accretion-deletion was present on both sides of each boundary, resulting in a fully ambiguous situation where accretion-deletion was unable to convey any information favoring one set of regions or the other. Moreover, it could be argued that the percept of rotating columns was obtained *only* because accretion-deletion was present in both sets of regions. (Since both sets of regions cannot be perceived as being in the back, one of them is “pushed” to the front, with the accretion-deletion attributed to self-occlusion due to rotation in 3-D.) In this article, by contrast, we introduce accretion-deletion in only one set of regions, keeping the other stationary. This allows us to study the inherent contribution of accretion-deletion to ordinal depth information for surfaces and its interaction with (static) geometric cues to figure and ground. An account of this interaction and the depth-order inversion that it causes might require us to rethink accretion-deletion as a reliable cue to relative depth.

In our experiments, we examined the interaction between two geometric cues to figure–ground and

accretion-deletion. We used multiple-region figure–ground stimuli similar to the one used by Froyen et al. (2013; see Figure 2). In our crucial experimental conditions, only one set of regions (either the odd or the even) contained accreting/deleting texture. According to standard accounts of accretion-deletion, this should lead to an unambiguous depth ordering with the regions containing accretion-deletion perceived as being in the back. We also introduced symmetry and/or convexity to one set of regions in order to examine the interaction between these geometric cues and accretion-deletion. In the “cue competition” condition, geometric cues (i.e., symmetry, convexity) were introduced on regions that contained accreting/deleting texture, whereas the other set of regions was static. In the “cue cooperation” condition, geometric cues were introduced on the static regions so that both cues suggested the same set of regions as figural. In this way we tested whether accretion-deletion is able to decisively assign depth-order and examined its interaction with the shape of the border where the texture is being accreted or deleted.

It is important to note that the terms “cue cooperation” and “cue competition” involve a concession to the traditional way of thinking about accretion-deletion, which presumes that accretion-deletion is a cue to ground status. However, this presumption is exactly what we question. In order to minimize confusion, we begin by describing our displays in this traditional way. However, this traditional presumption will be discussed and challenged in the General discussion, and it will then become clear that these terms are really an oversimplification of the complex interactions of cues at work in our displays.

In the following experiments, we used two different experimental tasks. The first task required subjects to indicate whether they saw a target region, indicated by arrows, in front relative to its adjacent region—which is a classical question to measure figure–ground. The other task required subjects to indicate whether they saw a rotational or a translational motion in the target region containing moving texture. In this way, we also aimed to understand the relationship between perceived relative depth and perceived 3-D shape of a region.

Experiment 1

In Experiment 1, the interaction between geometric figure–ground cues and accretion-deletion was examined by combining them in various conditions. In the “cue competition” condition, the two cues were introduced to the same region such that while geometric cues (convexity and symmetry in this case) were suggesting figural status in a particular region,

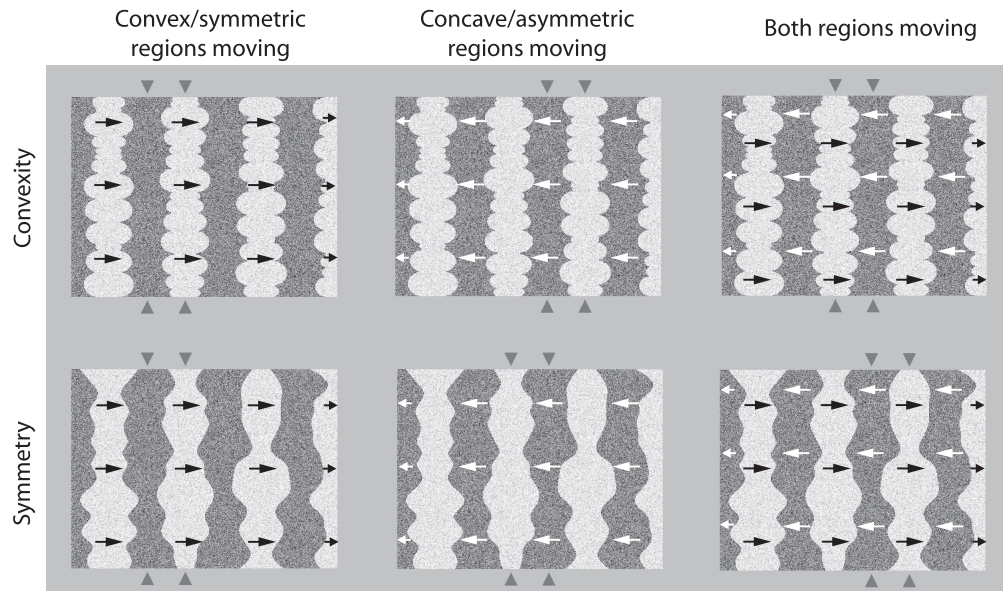


Figure 3. The six stimulus types used in the figure-ground task of Experiment 1. See demo movies of stimuli at <http://rucss.rutgers.edu/~manish/demos/RotatingColumns/RotColDemos.html> (arrows are not shown in the demo movies).

accretion-deletion (in its standard interpretation) was suggesting the opposite. Similarly, in the “cue cooperation” condition the two cues were introduced to different regions. There were also conditions designed to replicate the results of Froyen et al. (2013), where accretion-deletion cues were present in every region (i.e., all regions contained moving texture). Subjects performed the two different tasks on the same set of stimuli. Subjects’ responses of depth ordering (figure-ground task) and their interpretation of the motion (rotational or translational) were examined.

Method

Participants

Thirteen Rutgers University students who were unaware of the purpose of the experiment participated in the study. All had normal or corrected-to-normal visual acuity. Nine were paid for their participation, whereas the other four participated for course credit.

Stimuli

The stimulus for this experiment consisted of eight alternating black and white vertical regions. The stimuli were 7.3° high and 9.7° wide. Either the odd or the even regions were given geometric figure-ground cues (i.e., made symmetric or piecewise convex). In the weak-geometric-cue condition, the regions were given the symmetry cue, whereas in the strong-geometric-cue condition, the regions were given the convexity cue as

well as symmetry (Figure 3). The symmetric displays used here have previously been shown to yield a weak figural bias (Froyen et al., 2013). Here we use the term *convex* to refer to a boundary which is part-wise convex—i.e., can be segmented into parts, each of which is convex—with negative minima of curvature serving as the part boundaries (Hoffman & Singh, 1997). As noted previously, convex regions were also symmetric so that they could be interpreted as surfaces of revolution.²

As in the study by Froyen et al. (2013), the convex regions were created by using a series of half circles with random radii as a boundary and then mirroring it on the other side of the region. Symmetric contours were created by using B-spline functions with 20 control points. The control points for the symmetric contours were set so that the sum of signed curvature was kept at zero along each boundary. The area of each region was the same. The contours of each region were individually created such that no two regions were the same in terms of the geometry of their bounding contours.

On half of the trials, the odd regions were dark and the even regions were light colored; on the other half it was reversed (i.e., counterbalanced and crossed with other factors). The phase of the stimuli (e.g., whether the rightmost part of the display starts with a convex/symmetric or a nonconvex/asymmetric region) was also counterbalanced and crossed with other factors by mirroring the displays about their vertical middle axis.

To these stimuli, textural motion was added as a moving random-dot texture. For the dark regions, the

dot texture was sampled from a beta distribution, which has a probability density function as follows:

$$\text{Beta}(\alpha, \beta) : p(x|\alpha, \beta) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha, \beta)},$$

where $0 \leq x \leq 1$ and $\alpha, \beta > 0$ are shape parameters and $B(\alpha, \beta)$ is the beta function used as a normalization constant, with parameters $\alpha = 6, \beta = 2$ that resulted in a dark texture with sparsely scattered light pixels. The light regions contained random-dot texture sampled from a beta distribution with parameters $\alpha = 2, \beta = 6$, which resulted in a light texture with sparsely scattered dark pixels. The size of a single pixel was 1.47×1.47 arcmin. The texture could move either to the right or to the left, and it was implemented as follows. For the rightward motion, in each frame t the texture columns $[2, N]$ were taken from texture columns $[1, N - 1]$ in frame $t - 1$, and pixel luminance values in the first column in frame t were resampled from the same beta distribution. The implementation was the same for the leftward motion. This procedure was repeated at a rate of 40 frames/s, resulting in a motion with a speed of $0.98^\circ/\text{s}$.

We created three different types of stimuli, in terms of which regions contained moving texture. In one type, all regions had textural motion, with the odd and even regions moving in opposite directions (see the third column in Figure 3). In the second type of display, the regions where the geometric cues are introduced (the convex and/or symmetric regions) were made static and the other set of regions had consistent motion either to the left or to the right (see the second column in Figure 3). This is the “cue cooperation” condition, where (according to the standard interpretation of accretion-deletion) the two cues suggest the same relative depth interpretation. The third type of display was the opposite of the second type, where the nonconvex and asymmetric regions contained static texture while the convex/symmetric regions had consistent motion either to the left or to the right (see the first column in Figure 3). This is the “cue competition” condition, where (again, according to the standard interpretation) the relative depth interpretations that the two cues suggest conflict each other. The direction of motion was counterbalanced and crossed with other factors for all displays.

Design and procedure

Subjects sat 85 cm from a 21-in. CRT monitor (144 Hz, 1024×768 pixels) connected to a Windows XP PC. The experiment was presented using Psychtoolbox in MATLAB (Brainard, 1997; Kleiner et al., 2007). The experiment involved two tasks. One was the “figure-ground task,” in which subjects were asked which of the two indicated adjoining regions was in front. The

other task was the “rotation task,” in which subjects were asked whether they saw a rotation or translation in the indicated region. In both tasks, each trial started with 800 ms of premask, followed by 800 ms of the premask with a fixation cross added to it. The mask was created by randomly generating frames of figure-ground displays with unbiased contours (in terms of geometric cues to figure-ground) and then overlaying them on top of each other. The premask was used in order to exert more careful stimulus control by diminishing any potential visual persistence of the previous stimuli.

After the mask, the experimental display with moving textures was shown for 3 s. In the last two of these three seconds, two regions (for the figure-ground task) or a single region (for the rotation task) was indicated by triangle-shaped arrows that appeared at the top and bottom of the target region (5 pixels away from the display; see Figures 3 and 4). For the figure-ground task, the two (adjacent) target regions were chosen from four central regions. This limited the number of locations that the arrows could appear to three. These three locations can be seen in the upper row of Figure 3. For the rotation task, the question (whether rotation or translation is perceived) was asked regarding a single region. Since the arrow would have to appear on a region that has textural motion, there were only three different regions where the arrow could appear. At the beginning of each trial, the exact location of the arrows was randomly determined for that trial.

After 3 s (1 s without arrows and 2 s with arrows) the subjects were presented with a postmask for a minimum of 800 ms. The postmask was identical to the premask and was used in order to avoid any potential visual persistence of the stimulus after the termination of the display. Once this postmask was presented, the subjects were asked the experimental question with respect to their current task (either front/back or rotation/translation). These experimental questions were forced-choice questions. The subjects were instructed to respond with their first percept of depth order, so that any figure-ground reversals that might occur later would not influence the results. The subjects responded using the keyboard.

For the figure-ground task, subjects performed 96 experimental trials split into two blocks—i.e., 2 (geometric cues: convexity/symmetry) \times 3 (motion: in convex/symmetrical region; in nonconvex/asymmetric regions; in both regions) \times 2 (luminance: dark/bright) \times 2 (phase) \times 2 (direction of motion: right/left) \times 2 (repetition). For the rotation task, subjects performed 128 experimental trials split into two blocks. The experimental conditions were the same for the motion-interpretation task, except that the motion condition included four levels (rather than the three levels seen in

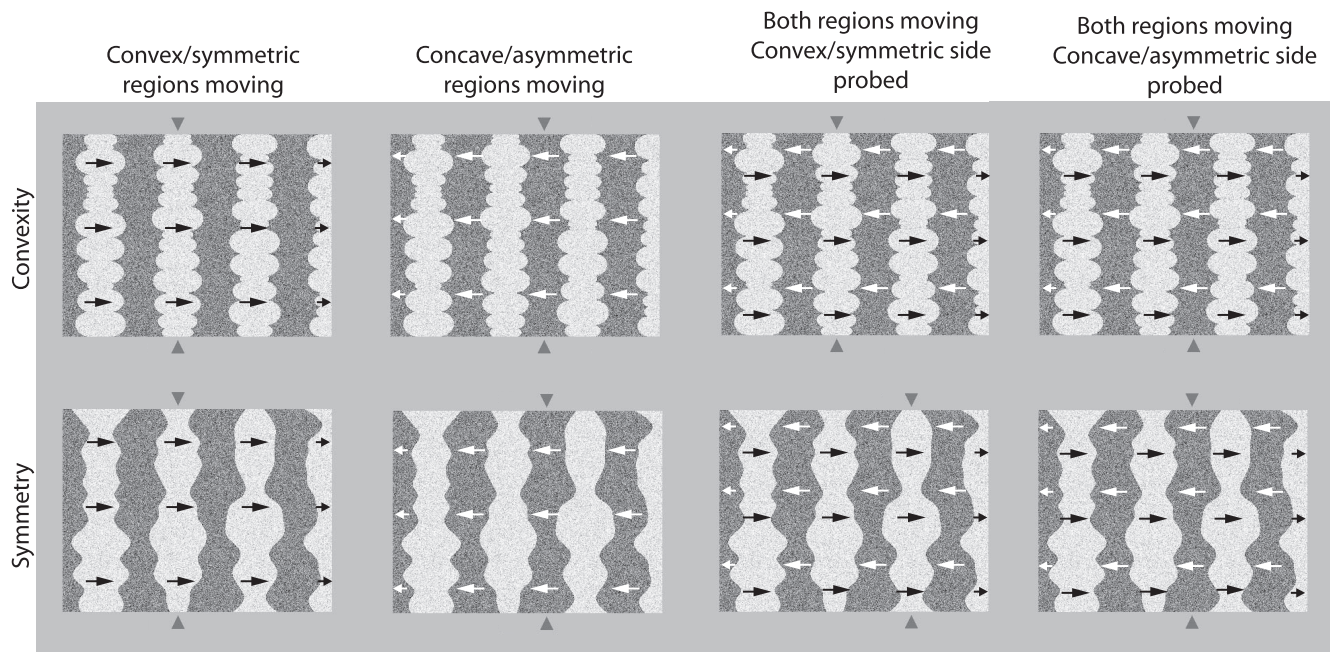


Figure 4. The eight stimulus types used in the rotation task of Experiment 1. See demo movies of stimuli at <http://rucss.rutgers.edu/~manish/demos/RotatingColumns/RotColDemos.html> (arrows are not shown in the demo movies).

Figure 3). The reason for this extra level is that for the condition where all regions on the display have motion, either the convex/symmetric or the nonconvex/asymmetric region could be probed (the last two columns in Figure 4). Including the two tasks, there were a total of 224 experimental trials. All conditions were counterbalanced for each subject, and trials were randomized for each subject separately. The order in which the subjects received the two tasks was also counterbalanced across subjects. Before the experimental trials began, 16 practice trials were run in order to acquaint the subjects with the displays and the tasks. It took approximately 50 min for subjects to complete the experiment.

Results

Figure 5 shows the results obtained from both tasks. When only one set of regions was moving, the results are plotted as the proportion of times subjects reported seeing the moving regions as in front for the figure-ground task and as rotating for the rotation task. When all regions had motion, the results are plotted as the proportion of times subjects reported seeing the indicated (i.e., marked with arrows) region as in front or as rotating, depending on the task. The top row in Figure 5 shows the results for the figure-ground task, and the bottom row is for the rotation task. The graphs in the left column are for the conditions where only one set of regions contained moving texture, whereas the

other regions were static. These conditions correspond to the first two columns depicted in Figures 3 and 4. The graphs in the right column of Figure 5 are for the conditions where all regions contained moving texture, with the odd and even regions moving in opposite directions. These conditions correspond to the right-most column in Figure 3 and the last two columns of Figure 4. The graphs in the left column (moving texture in only one set of regions) depict the results in the crucial conditions regarding our research questions. The graphs in the right column can be considered a replication of the results of Froyen et al. (2013).

We performed *t*-test analyses (for both task responses) in order to see whether the proportions shown in Figure 5 were significantly different from 0.5, i.e., chance level. The proportions that were significantly different from chance level are shown with their corresponding *p* values via asterisks in Figure 5. (For all the significant differences obtained, $t_{\max} = 30.99$, $t_{\min} = 3.3$, $df = 12$, $p < 0.05$. The maximum and minimum values reported in absolute values.) As seen from the top left graph of Figure 5, when only the texture of the regions that had the geometric figure-ground cues was moving, the proportion of times the moving regions were seen as figural did not differ from chance level (red bars in the top left graph of Figure 5), whereas when the texture of the regions that did not include the geometric cues was moving, the proportion was significantly lower than chance level (turquoise bars in the top left graph of Figure 5). In the condition in which both sets of

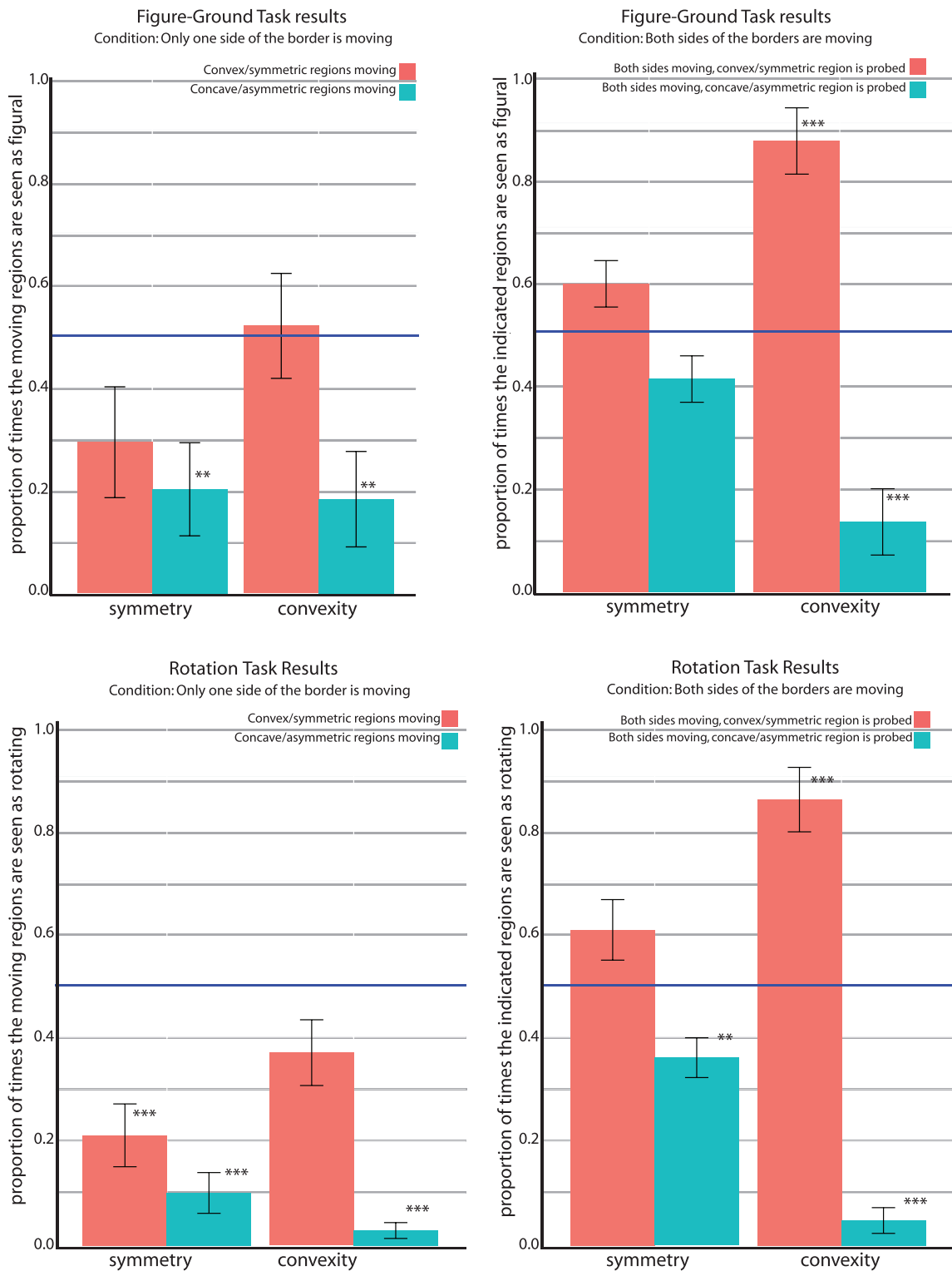


Figure 5. Results of Experiment 1. Error bars represent ± 1 standard error as computed between subjects. The blue line shows the chance level, i.e., where the proportion equals 0.5. Note that for presentation purposes, double the number of bars are present for the “both sides moving condition,” where the blue bars are calculated as $1 - p$ (red bars). The stars (*) represent the proportions that are significantly different than chance level. The number of stars indicate the significance level: *** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$.

regions had textural motion, the proportion of times the convex regions were seen as figural was significantly higher than chance level (red bar on the right side of the top right graph in Figure 5), whereas adding textural motion to both sets of regions in the symmetry condition resulted in a proportion that is around chance level (red bar on the left side of the top right graph in Figure 5). When the proportions in the bottom left graph of Figure 5 are examined, it is seen that subjects interpreted the textural motion mostly as translation, except in the condition where only part-wise convex regions had textural motion (the red bar on the right side of the bottom left graph of the Figure 5). In this condition, the proportion of times the moving part-wise convex regions were perceived as rotating was around chance level. When both sets of regions had textural motion, the motion in the part-wise convex regions was perceived as rotation almost all of the time (red bar on the right side of the bottom right graph of Figure 5), whereas the motion in the symmetric regions was interpreted as rotation (approximately) only half of the time (the red bar on the right side of the bottom right graph of Figure 5). The motion on the nonconvex and the asymmetric regions was mostly interpreted as translation (the turquoise bars on the bottom right graph of Figure 5).

In what follows, the results were analyzed for the geometric cue, the location of motion, and color factors. Other counterbalancing factors were found not to yield any significant main effects or interactions.

Figure-ground task

A multilevel logistic-regression analysis was done for the figure-ground-task responses. A likelihood-ratio test showed that including main effects for geometric cue and location of motion was a significant improvement over an unconditional-means model (i.e., containing only an intercept)—comparing models that include location of motion to the unconditional means model: $LR = 553.3$, $df = 7$, $p < 0.001$; comparing models that include location of motion and geometric cue to the one that only includes location of motion: $LR = 104.2$, $df = 5$, $p < 0.001$. The addition of the color factor (i.e., whether the target region is dark or light) to the model that includes the two main effects of location of motion and geometric cue was also a significant improvement over the model, $LR = 21.18$, $df = 6$, $p < 0.01$. The interaction between location of motion and geometric cue was also found to be a significant addition, $LR = 35.51$, $df = 15$, $p < 0.01$, yielding our final model. As expected, the regions that contained the geometric cues were more likely to be seen as figural when the geometric cue was convexity, $M = 0.52$, $SE = 0.05$, compared to when it was just symmetry, $M = 0.36$, $SE = 0.06$. Apart from the obvious fact that in one

condition we have two geometric cues (convexity and symmetry) whereas in the other there is only one (symmetry), it is also known that the symmetric displays here result in a rather weak figural bias (Froyen et al., 2013). Tukey pair-wise comparisons, done between the three different conditions of the location of motion, revealed the following effects. The proportion of times the convex/symmetric regions were seen as figural in the condition where regions on both sides of a boundary had motion (i.e., column 3 in Figure 3) was significantly higher than the proportion of times the moving regions were seen as figural in the condition where only convex/symmetric regions were moving (i.e., column 1 in Figure 3), $p < 0.05$, and also in the condition where only nonconvex/asymmetric regions were moving (i.e., column 2 in Figure 3), $p < 0.001$. The interaction between geometric cue and location of motion is seen when the effect of location of motion in the convexity condition is compared to its effect in the symmetry condition. In the condition in which nonconvex regions had moving texture, the proportion of times the moving regions were seen as figural was significantly lower than chance level (the rightmost, turquoise bar on the top left graph of Figure 5); however, when the part-wise convex regions had the moving texture, the proportion increase dramatically to chance level (the red bar on the right side of the top left graph of Figure 5). When both sides had moving texture, the part-wise convex regions were seen as figural almost all the time (the red bar on the right side of the top right graph of Figure 5). This strong effect of motion location was not observed in the symmetry condition (i.e., on the left side of the top left graph of Figure 5, it is seen that the difference between the red and the turquoise bars is relatively small, and it is seen from the red bar on the left side of the top right graph of Figure 5 that the proportion only goes up to chance level even when both sides had moving texture). Thus, whether accretion-deletion is introduced onto the convex/symmetric region or onto the nonconvex/asymmetric region makes a big difference when the geometric cue is convexity, but little difference when it is symmetry. Further investigation of the main effect of color shows that there is no structured effect of color on figure-ground responses; while some subjects have a bias towards dark regions ($n = 9$), others have a bias towards light regions ($n = 4$).

Rotation task

The same multilevel logistic-regression analysis was also done for the rotation-task responses. While a likelihood-ratio test showed that the main effect of location of motion was a significant improvement over the unconditional-means model, $LR = 580.25$, $df = 12$, $p < 0.001$, the main effect of geometric cue was no

significant expansion beyond location of motion, $LR = 7.7$, $df = 6$. However, the interaction between the two factors was a significant expansion, $LR = 178.93$, $df = 30$, $p < 0.001$. (Adding other factors was not found to yield significant expansions of this final model.) Tukey pair-wise comparisons were done between the four different conditions of location of motion. The order of the four conditions, in terms of the proportion of times the moving/indicated region was perceived as rotating, is as follows (bottom row in Figure 5): The highest proportion was obtained in the condition where both sides of the border were moving while the convex/symmetric region was indicated. The “cue competition” condition (i.e., only convex/symmetric region moving) followed it as the second. In the third position was the condition where both sides were moving and the nonconvex/asymmetric region was indicated. The lowest proportion was obtained in the “cue cooperation” condition, where only the nonconvex/asymmetric region was moving. All the pair-wise comparisons were significantly different from each other at $p < 0.001$, except the comparison between the “cue competition” condition and the condition where both regions were moving while the nonconvex/asymmetric region was indicated. The interaction between the location of motion and the geometric cue can be seen when the effect of location of motion is examined in different conditions of the geometric cue. For example, if the bottom left graph is examined, it can be seen that when symmetry was used as the geometric cue, the location of motion did not significantly influence the responses of subjects. However, when convexity was used, this difference became significant (i.e., the difference in proportion between the “cue competition” condition and the “cue cooperation” condition when the geometric cue is convexity), $p < 0.001$.

Between-tasks comparison

The response patterns obtained from the figure-ground task were fairly similar to those obtained from the rotation task. A regression analysis was done for each individual subject, where the predictor variable was the proportion of trials on which the moving/indicated region was seen as in front for each experimental condition and the predicted variable was the proportion of the times the moving/indicated region was seen as rotating for each experimental condition. As a result, each subject had eight data points on a scatter plot of the predictor and the predicted values. The regression analysis showed that for 11 subjects (out of 13) the proportion of figural responses on the figure-ground task was a significant predictor of the proportion of rotation percepts on the rotation task. For those 11 subjects, a significant regression result was

found: $R_{\max}^2 = 0.92$, $R_{\min}^2 = 0.60$, $F(1, 9)_{\max} = 69.43$, $F(1, 9)_{\min} = 8.96$, $p < 0.05$.

Discussion

The crucial condition in our experiment was the case where one side of each border was static while the other side had accreting/deleting texture. When the graph corresponding to this condition (left column of Figure 5) is examined, it can be seen that even in this type of display, taken as unambiguous by traditional accounts of accretion-deletion, the shape of the border exerts a strong influence on the figure-ground interpretation. As seen from the second red bar in the top left graph of Figure 5, convexity cancels out the effect of accretion-deletion (in its traditional sense) when the two cues are made to compete against each other. The percentage of times the accreting/deleting regions were seen in front was near 50% in this “cue competition” condition, whereas, according to the traditional accretion-deletion accounts, it should be at 0%.

It is also observed that both geometric cues combine with accretion-deletion to various degrees. The symmetry cue causes the regions that contain accretion-deletion to be perceived as figural on a certain proportion of the trials. In the conditions where both sides of a border had motion (the graphs in the right column of Figure 5), the responses of subjects were very similar to those obtained by Froyen et al. (2013). The subjects were more biased to see 3-D rotation in depth when both sides of a border had motion. This was expected, since introducing accretion-deletion to both sides of a border creates more ambiguity in the display. Geometric cues resolve this ambiguity, and that is why the effect of geometric cues is increased in these conditions. The high correlation between the response patterns obtained from the two tasks indicates that perception of 3-D rotation in these displays is connected to the figure-ground interpretation. This suggests that if the moving side was interpreted as figural, it is highly probable that it would also be perceived as a 3-D column rotating in depth (and vice versa).

Experiment 2

In Experiment 1, in the conditions where motion was introduced within one set of regions, the regions containing textural motion were all translating in the same direction. This common motion is likely to have resulted in a grouping effect, which may have biased subjects to see these moving regions as being grouped in the background and amodally completed behind the

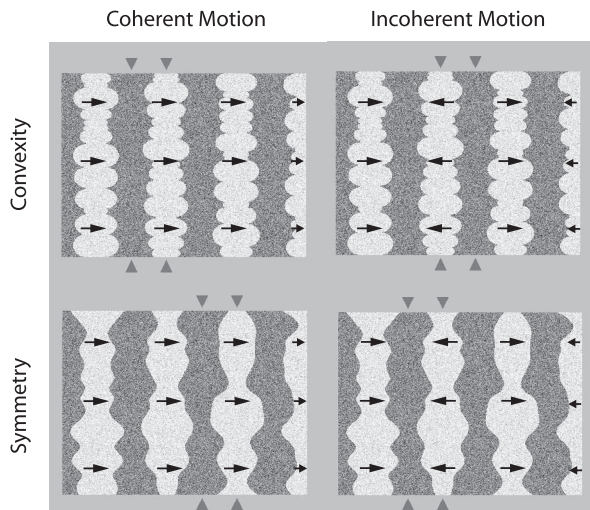


Figure 6. The stimuli used in the different conditions of Experiment 2. See demo movies of sample stimuli at <http://ruccs.rutgers.edu/~manish/demos/RotatingColumns/RotColDemos.html> (arrows are not shown in the demo movies).

other set of the regions. Specifically, in the “cue competition” condition, in which nonconvex regions were static and part-wise convex regions had accreting/deleting texture, the textural motion in those part-wise convex regions was moving coherently in the same direction and with the same speed. This common motion shared between the part-wise convex regions results in a bias to perceptually group these moving regions into one large sheet, which as a result leads to a bias to perceive the moving regions translating behind the static nonconvex regions. Such a bias would favor a “translating behind” interpretation of accretion-deletion and therefore would compete against the effect of geometric cues to figure-ground on moving regions. In order to examine the interaction of static geometric cues with accretion-deletion cues in the absence of any other cues to depth (such as this global motion-coherence cue), textural motion across regions was made incoherent in Experiment 2: Only the “cue competition” condition was used. In other words, the geometric and accretion-deletion cues were introduced to the same set of regions (either dark or light). On some trials, the textural motions in different regions were made incoherent by alternating the direction of motion in the moving regions such that there would be no motion-based grouping.

Method

Participants

Eight Rutgers University students, unaware of the purpose of the experiment, participated in this exper-

iment. All subjects had normal or corrected-to-normal visual ability. Subjects were paid for their participation.

Stimuli and procedure

The stimuli were generated in exactly the same manner as in Experiment 1, except that only the “cue competition” condition was used (in which only the symmetric/convex regions contained moving texture). To this we also added a version with incoherent motion across different regions (see Figure 6). In the coherent-motion condition, all the convex/symmetric regions were moving in the same direction (as in Experiment 1). All the nonconvex/asymmetric regions were kept static.

The experimental procedure was exactly the same as in Experiment 1. In order to minimize influences due to the particular shape of the contours, we used two different contours for each geometric cue. For each task, subjects performed 128 experimental trials split into two blocks—i.e., 2 (geometric cues: convexity/symmetry) \times 2 (motion: incoherent/coherent) \times 2 (luminance: dark/bright) \times 2 (phase) \times 2 (direction of motion: right/left) \times 2 (shape: two different contours for each geometric cue) \times 2 (repetition). Including the two tasks, there were a total of 256 experimental trials. It took just under 1 hr for each subject to complete the experiment.

Results

Figure 7 shows the results plotted as the proportion of times subjects reported seeing the moving regions in front (for the figure-ground task) or as rotating (for the rotation task). Responses were analyzed for the two essential factors, i.e., the geometric cue and motion type (coherent vs. incoherent). Except for the geometric cue, motion type, and color factors, other factors were found not to yield any main nor interaction effect.

We performed a *t*-test analysis to see whether the proportions reported on Figure 7 are significantly different from 0.5, i.e., chance level. The proportions that were significantly different from chance level are shown with their corresponding *p* values using asterisks in Figure 7. (Among all the significant differences obtained, $t_{\max} = 9.94$, $t_{\min} = 3.08$, $df = 12$, $p < 0.05$).³ Consistent with the results of Experiment 1, the proportion of seeing accreting/deleting regions in front was significantly lower than chance level in the symmetry condition, whereas when the boundaries were part-wise convex, the accreting/deleting regions were perceived in front approximately half of the time (left graph in Figure 7). When the motion was incoherent, the proportion of times the accreting/deleting regions were perceived as in front was slightly higher than the proportion obtained in the coherent-

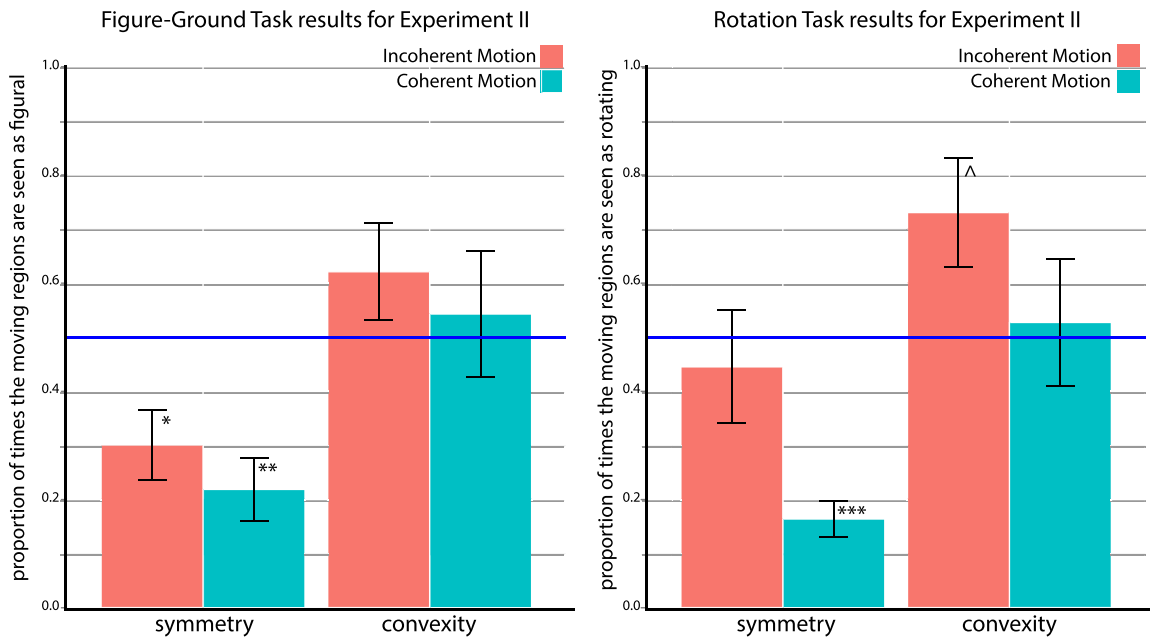


Figure 7. Results of Experiment 2. Error bars represent ± 1 standard error as computed between subjects. The blue line shows the chance level, i.e., where the proportion equals 0.5. The stars (*) represent the proportions that are significantly different than chance level. The number of stars indicate the significance level: *** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$; $\Delta p = 0.0569$.

motion condition (compare red bars to turquoise bars in the left graph of Figure 7). The effect of incoherent motion was much more apparent in the rotation-task results (compare red bars to turquoise bars in the right graph of Figure 7). Even in the symmetry condition, subjects perceived the accreting/deleting region as rotating almost half of the time. In the convexity condition, the proportion was almost significantly higher than chance level.

Figure-ground task

To analyze the responses of the subjects, a multilevel logistic regression was performed. A likelihood-ratio test showed that including main effects for geometric cue and motion type was a significant improvement over the baseline model including only the main effect of color—comparing models that include color and motion type to the one that just includes color: $LR = 15.83$, $df = 4$, $p < 0.01$; comparing models that include all three main effects to the one that includes motion type and color: $LR = 193.78$, $df = 5$, $p < 0.001$. There was no significant interaction between these main factors. As can be seen from the graph on the left in Figure 7, subjects were again more likely to see the moving regions as figural when they were convex and symmetric, $M = 0.58$, $SE = 0.1$, compared to the cases when these regions were just symmetric, $M = 0.26$, $SE = 0.06$. We found that the proportion of times people saw the moving region as figural was significantly higher

when the motion was incoherent, $M = 0.46$, $SE = 0.07$, compared to when it was coherent, $M = 0.38$, $SE = 0.08$. This is consistent with our prediction that motion grouping acts as an additional cue for relative depth perception. Regarding the color factor, subjects in general were more biased toward seeing the light regions as figural compared to dark regions. However, when individual responses were examined, there was not any systematic effect of color. Among eight subjects, six had this light-color bias and two had a dark-color bias (different from what was observed in Experiment 1, where nine subjects exhibited a dark-color bias and four exhibited a light-color bias). Thus, although individual subjects show a color bias, its direction is not systematic across subjects.

Rotation task

For the rotation-task responses, the multilevel logistic regression yielded the same significant main effects, which shows that including main effects for geometric cue and motion type was a significant improvement over the baseline model including only color—comparing models that include color and motion type to the one that includes just color: $LR = 102.52$, $df = 4$, $p < 0.001$; comparing models that include all three main effects to the one that includes motion type and color: $LR = 247.58$, $df = 5$, $p < 0.001$. In the same way, subjects were more likely to see the moving regions as rotating when they were convex and

symmetric, $M = 0.63$, $SE = 0.1$, compared to when they were just symmetric, $M = 0.3$, $SE = 0.06$. Subjects were also more likely to see the moving regions as rotating when the motion was incoherent, $M = 0.59$, $SE = 0.08$, rather than coherent, $M = 0.35$, $SE = 0.06$. The pattern of biases regarding the color factor was similar to the one obtained for the figure–ground task. Among eight subjects, six had a light-color bias and two had a dark-color bias.

Discussion

The responses obtained from the coherent-motion condition of the figure–ground task were similar to those obtained from Experiment 1. The subjects were more likely to see the moving regions as figural when the motion was incoherent across regions, compared to when it was coherent. This confirms that making the motion coherent (as was done in Experiment 1) introduces an additional factor that contradicts the geometric cues. It can also be seen that incoherent motion had a greater influence on the rotation task than on the figure–ground task. Subjects' judgments on the rotation task were more sensitive to the motion-coherence manipulation compared to their judgments on figure–ground assignment. A regression analysis of the responses obtained from the two tasks could not be done for this experiment because the number of experimental conditions used was four, which is very low for a regression analysis. However, as can be seen by comparing the two plots in Figure 7, the patterns of results are very similar for the two tasks. Overall, the significant difference observed between the responses of the coherent and incoherent conditions shows that eliminating the grouping effect by making the motion incoherent considerably alters subjects' responses. This incoherent-motion condition provides a cleaner and fairer comparison of the relative influence of the geometric cue and accretion-deletion, since motion coherence across regions is an additional global cue, whereas accretion-deletion and geometric cues are mostly local.

General discussion

Traditionally, accretion-deletion has been considered a decisive cue to depth order, such that it unambiguously assigns figure and ground status to image regions in dynamic 2-D images (Gibson et al., 1969; Kaplan, 1969; Mutch & Thompson, 1985; Thompson et al., 1985; Niyogi, 1995; Howard & Rogers, 2002; Hegdé et al., 2004). However, our results show that there is an inherent ambiguity about the relative depth informa-

tion conveyed by the accreting/deleting regions. Froyen et al. (2013) have shown that when an accretion-deletion cue is introduced on both sides of a border, an ambiguity about depth order is created. Their results show that this ambiguity can be resolved by introducing geometric figure–ground cues. However, a similar ambiguity was also observed in the “cue competition” condition of our study, where only one side of each border had an accreting-deleting texture, whereas the other side was static. Such a stimulus would be considered unambiguous in terms of traditional accounts of accretion-deletion, and the moving texture would be predicted to be consistently perceived as further away. Therefore, our results suggest that the relative depth ambiguity observed by Froyen et al. (2013) is not merely due to having accreting/deleting textures on both sides of a border, but is also dependent upon the way our visual system explains the accretion or deletion of the texture at the border, which can occur because of being occluded by the adjacent region or because of self-occlusion due to rotation in depth. Here, we showed that introducing static geometric cues to one set of regions and making the motion in those regions incoherent strongly modulates the way accretion-deletion of texture is explained by our visual system, and hence modulates the perception of relative depth and layered surface structure. Moreover, the perception of 3-D columns rotating in depth is so strong that it is observed in spite of the fact that the dot texture's motion has constant velocity, which is technically inconsistent with 3-D rotation.

This relation between perceived relative depth and perceived 3-D rotation is also supported by the fact that the responses obtained from the rotation task were fairly consistent with the responses obtained from the figure–ground task. Such similarities between the response patterns of the two different tasks suggest that the two judgments are strongly connected. In classical accretion-deletion stimuli seen in the literature, the static region is seen in front and interpreted as the region that occludes the disappearing texture. However, in our stimuli, when geometric cues favor figural status to the moving region, the visual system is confronted with evidence that the translating texture is in front and therefore infers that the region is a 2-D projection of a 3-D rotating column. Hence, the accreting and deleting texture is explained by dynamic self-occlusion due to rotation.

In both Experiments 1 and 2, the motion manipulations (changing the location of motion or altering the coherence of the textural motion) had a greater influence on the responses in the rotation task than in the figure–ground task. This can be observed on all the bar graphs presented. The response differences between the different motion conditions (the differences between the red and turquoise bars) were much higher in

the rotation task than in the figure–ground task. This suggests that the rotation-task responses were more sensitive to motion manipulations. We would argue that the question asked for the rotation task (i.e., whether the subjects saw a rotational motion in the moving region) may be considered a more reliable and indirect method for measuring figure–ground perception in these displays, rather than directly asking about figural status.

Individual differences were also observed among the participants, especially in the “cue competition” condition. This can also be seen from the large standard-error bars in the top left graph in Figure 5. Hildreth and Royden (2011) have shown that there are individual differences in the way people combine accretion-deletion and binocular disparity cues in a depth-order task. While some subjects give more weight to the accretion-deletion cue, other subjects give more weight to binocular disparity in their depth-order judgments. Consistent with that study, our results also suggest that there are individual differences in the way people use the accretion-deletion cue for depth-order interpretation.

Rotation-in-depth interpretations of accreting/deleting regions have been mentioned in the literature before (Kaplan, 1969; Yonas et al., 1987; Royden et al., 1988; Froyen et al., 2013), but not enough attention has been given to their implications for accretion-deletion as a cue to relative depth. In this article, we have shown that accreting/deleting textures can be easily interpreted as in front—not only in inherently ambiguous conditions, such as when accreting/deleting textures are present on both sides, but also in conditions in which accretion-deletion is traditionally considered unambiguous. These results raise serious concerns regarding the conventional view of accretion-deletion. They indicate that even though accretion-deletion provides some sort of ordinal depth information for surfaces, it is actually not a cue to depth as traditionally understood, because it is consistent with both “in front” and “in back” interpretations. In conditions where the accretion-deletion of a textured region is explained by self-occlusion due to rotation in depth, accretion-deletion is no longer serving as a cue to ground status. This makes the ordinal depth information provided by accretion-deletion highly dependent upon how the accreting/deleting texture is accounted for. For example, when the geometry of the border favors the “in front” interpretation of an accreting/deleting region, accretion-deletion would be neither competing nor cooperating with those factors for ordinal depth interpretation. Therefore, the situation is more complex than simple “cue competition” or “cue cooperation” and can be described more accurately as an interaction between accretion-deletion and geometric

cues to figure–ground. These conclusions suggest that the assumption that accretion-deletion indicates “behind” needs to be reconsidered or perhaps rejected.

The implications of our results also extend to motion as a cue, not only to depth but also to 3-D shape. In our stimuli, subjects perceived columns rotating in depth, even though the dot texture’s motion was linear, which is technically inconsistent with 3-D rotation (e.g., Ullman, 1979). Moreover, our results show that this rotation perception also depends on the geometry of the contour. This indicates that, contrary to the general view of motion cues as the strongest cues to depth and shape, the static contour geometry plays at least as important a role as the motion cues when it comes to determining percepts of relative depth and 3-D structure. This interaction between static geometric cues and motion cues points to a gap in the literature that has not received enough attention, and therefore calls for newer computational models that can account for this rich interaction.

Conclusion

In two experiments, we investigated whether the accretion-deletion cue is able to decisively assign figure–ground and examined its interaction with the geometry of the border at which the texture is being accreted or deleted. In Experiment 1 we showed that even in unambiguous accretion-deletion stimuli (i.e., unambiguous according to the traditional accounts of accretion-deletion), the geometric cue of convexity with symmetry can cancel out the effect of accretion-deletion, which also resulted in the regions that have accreting/deleting texture being seen as rotating columns in depth. In Experiment 2 we looked at the effect of motion grouping on figure–ground and rotational-motion interpretations. When the motion was made incoherent by alternating the direction of motion in convex regions, the influence of the geometric cues on the accretion-deletion cue increased. The symmetry cue alone did not override the effect of accretion-deletion, but it also combined with accretion-deletion for relative depth judgments. Here we see that cues based on static geometry (convexity and symmetry) override the motion cue for depth based on accretion-deletion (in its traditional sense).

Our results require us to reconsider the conventional view that accretion-deletion decisively dictates ground status. Exactly what information accretion-deletion conveys depends, rather, on the geometry of the boundary, along with other global factors such as the coherence of the motion. For example, in our visual stimuli, it seems that once the accretion and deletion of the texture is explained by the visual

system as a self-occlusion due to rotation, the accretion-deletion no longer functions as the figure-ground cue that indicates the occluded surface. A more general account is needed that can incorporate that kind of interaction of accretion-deletion with the geometric properties of the border. Another important point is how all of these findings connect with structure from motion. In our stimuli, the perception of 3-D columns rotating in depth is observed even though the dot texture's motion is linear, which is technically inconsistent with 3-D rotation of rigid objects (e.g., Ullman, 1979). In order to understand the relationship between figure-ground perception and perception of 3-D columns rotating in depth, more studies should be performed. Future studies might include running experiments where the velocity profile of the moving texture is manipulated and experiments with large numbers of trials within individual subjects. It is also important to study how accretion-deletion and geometric cues interact to give the perception of figure-ground segregation. For that, a new study can be done by applying gradual changes to both geometric and accretion-deletion cues.

Keywords: accretion-deletion, perceptual organization, figure-ground, depth perception, structure from motion

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Footnotes

¹ Note that the two interpretations shown in Figure 1 are meant to highlight the two qualitatively distinct depth-order possibilities: accreting/deleting surface (rotating) in the front or (sliding) in the back. Of course, quantitative variations on these are possible as well, e.g., in the precise curvatures of these surfaces.

² Throughout this article, when we refer to a region as convex, we mean that the region is convex and symmetric. It is worth noting, however, that in the study by Froyen et al. (2013) the percept of rotation was also obtained—paradoxically—when the contours

were asymmetric and therefore grossly inconsistent with 3-D rotation.

³ The maximum and minimum refer to maximum and minimum absolute values.

References

- Barnes, T., & Mingolla, E. (2013). A neural model of visual figure and ground in dynamically deforming shapes. *Neural Networks*, *37*, 141–164.
- Beck, C., Ognibeni, T., & Neumann, H. (2008). Object segmentation from motion discontinuities and temporal occlusion—A biologically inspired model. *PLoS One*, *3*(11), e3807.
- Berzhanskaya, J., Grossberg, S., & Mingolla, E. (2007). Laminar cortical dynamics of visual form and motion interactions during coherent object motion perception. *Spatial Vision*, *20*(4), 337–395.
- Brainard, D. (1997). The Psychophysics Toolbox. *Spatial Vision*, *10*(4), 433–436.
- Burge, J., Fowlkes, C. C., & Banks, M. S. (2010). Natural scene statistics predict how the figure-ground cue of convexity affects human depth perception. *The Journal of Neuroscience*, *30*(21), 7269–7280.
- Burge, J., Peterson, M. A., & Palmer, S. E. (2005). Ordinal configural cues combine with metric disparity in depth perception. *Journal of Vision*, *5*(6):5, 534–542, doi:10.1167/5.6.5. [PubMed] [Article]
- Froyen, V., Feldman, J., & Singh, M. (2010). A Bayesian framework for figure-ground interpretation. In J. Lafferty, C. K. I. Williams, J. Shawe-Taylor, R. S. Zemel, & A. Culotta (Eds.), *Advances in neural information processing systems: Vol. 3*, (pp. 631–639). Vancouver, Canada: Curran Associates.
- Froyen, V., Feldman, J., & Singh, M. (2013). Rotating columns: Relating structure-from-motion, accretion-deletion, and figure/ground. *Journal of Vision*, *13*(10):6, 1–12, doi:10.1167/13.10.6. [PubMed] [Article]
- Gibson, J., Kaplan, G., Reynolds, H., & Wheeler, K. (1969). The change from visible to invisible: A study of optical transitions. *Perception & Psychophysics*, *5*, 113–116.
- Hegd , J. Albright, T., & Stoner, G. (2004). Second-order motion conveys depth-order information. *Journal of Vision*, *4*(10):1, 838–842, doi:10.1167/4.10.1. [PubMed] [Article]
- Hildreth, E., & Royden, C. (2011). Integrating multiple

- cues to depth order at object boundaries. *Attention, Perception, & Psychophysics*, 73(7), 2218–2235.
- Hoffman, D., & Singh, M. (1997). Saliency of visual parts. *Cognition*, 63(1), 29–78.
- Howard, I., & Rogers, B. (2002). *Seeing in depth: Vol. 2. Depth perception I*. Toronto, Canada: Porteous.
- Kanizsa, G., & Gerbino, W. (1976). Convexity and symmetry in figure-ground organization. In M. Henle (Ed.), *Vision and artifact* (pp. 25–32). New York: Springer.
- Kaplan, G. (1969). Kinetic disruption of optical texture: The perception of depth at an edge. *Attention, Perception, & Psychophysics*, 6(4), 193–198.
- Kleiner, M., Brainard, D. H., & Pelli, D. G. (2007). What's new in Psychtoolbox-3? *Perception*, 36, ECVF Supplement 14.
- Kromrey, S., Bart, E., & Hegdé, J. (2011). What the “moonwalk” illusion reveals about the perception of relative depth from motion. *PLoS One*, 6(6), e20951, doi:10.1371/journal.pone.0020951.
- Layton, O. W., & Yazdanbakhsh, A. (2015). A neural model of border-ownership from kinetic occlusion. *Vision Research*, 106, 64–80.
- Metzger, F. (2006). *Laws of seeing* (L. Spillmann, S. Lehar, M. Stromeyer, & M. Wertheimer, Trans.). Cambridge, MA: MIT Press (Original work published 1936)
- Morinaga, S. (1941). Beobachtungen über Grundlagen und Wirkungen anschaulich gleichmäßiger Breite [Translation: Observations about fundamentals and effects describing equal width]. *Archiv für die gesamte Psychologie*, 110, 309–348.
- Mutch, K., & Thompson, W. (1985). Analysis of accretion and deletion at boundaries in dynamic scenes. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 2, 133–138.
- Nakayama, K., He, Z., & Shimojo, S. (1995). Visual surface representation: A critical link between lower-level and higher level vision. In S. Kosslyn & D. Osherson (Eds.), *Invitation to cognitive science* (pp. 1–70). Cambridge, MA: MIT Press.
- Niyogi, S. (1995). *Detecting kinetic occlusion*. Cambridge, MA: MIT Press.
- Ono, H., Rogers, B., Ohmi, M., & Ono, M. (1988). Dynamic occlusion and motion parallax in depth perception. *Perception*, 17(2), 255–266.
- Raudies, F., & Neumann, H. (2010). A neural model of the temporal dynamics of figure-ground segregation in motion perception. *Neural Networks*, 23(2), 160–176.
- Royden, C. S., Baker, J. F., & Allman, J. (1988). Perception of depth elicited by occluded and shearing motions of random dots. *Perception*, 17, 289–296.
- Thompson, W., Mutch, K., & Berzins, V. (1985). Dynamical occlusion analysis in optical flow fields. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 4, 374–383.
- Ullman, S. (1979). The interpretation of structure from motion. *Proceedings of the Royal Society of London*, 203, 405–426.
- Wagemans, J., Elder, J. H., Kubovy, M., Palmer, S. E., Peterson, M. A., Singh, M., & von der Heydt, R. (2012). A century of gestalt psychology in visual perception: 1. Perceptual grouping and figure-ground organization. *Psychological Bulletin*, 138(6), 1172–1217.
- Yonas, A., Craton, L., & Thompson, W. (1987). Relative motion: Kinetic information for the order of depth at an edge. *Attention, Perception, & Psychophysics*, 41(1), 53–59.