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Competition and dual users in complex contagion processes

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We study the competition of two spreading entities, for example innovations, in complex contagion processes in complex networks. We develop an analytical framework and examine the role of dual users, i.e. agents using both technologies. Searching for the spreading transition of the new innovation and the extinction transition of a preexisting one, we identify different phases depending on network mean degree, prevalence of preexisting technology, and thresholds of the contagion process. Competition with the preexisting technology effectively suppresses the spread of the new innovation, but it also allows for phases of coexistence. The existence of dual users largely modifies the transient dynamics creating new phases that promote the spread of a new innovation and extinction of a preexisting one. It enables the global spread of the new innovation even if the old one has the first-mover advantage.

The spread of rumors, fads, innovations or new technologies in a global scale occurs more frequently than before due to the intensive use of new information and communications technologies^{1,2}. In order to model and understand the mechanisms of spreading phenomena^{3–11}, quantitative approaches to contagion processes have become important. A pioneer work about diffusion of collective behavior⁴ and its variant adapted for the spread on complex networks⁵ proposed a model of “complex contagion”^{12–15} incorporating a threshold mechanism. This is often called the threshold model in which each agent requires a sufficiently large fraction of neighbors that have already adopted a spreading new technology to adopt it. Therefore, neighbors affect collectively (group interaction) the probability of adoption rather than independently as in the “simple contagion” of well known epidemic models¹⁶. Research on spreading by complex contagion sheds light on fundamental aspects of collective phenomena, non-linearity in diffusion, cascading dynamics, and first order phase transitions in complex networks. It has focused on aspects such as seed effect¹⁷, clustering¹⁸, modularity^{6,19–21}, multiplexity^{22,23}, multi-stage contagion²⁴, and interaction with simple contagion²⁵ or coordination processes²⁶.

Models of complex contagion typically assume that there exists no preexisting technology that competes with a new one, so that the new technology spreads in a system of susceptible agents^{2,5,6}. However, it is common that a preexisting technology is used by some fraction of the agents in the system. Therefore, the spread of a new competing technology involves the decline, coexistence or extinction of the actual predominant technology^{27–29}. Competition between ideas or products^{30–33} is an essential ingredient in the modeling of contagion process. While there exist several studies of simple contagion with cooperative epidemics^{34–36}, competing epidemics^{37,38}, and interacting epidemics on multi-layer networks^{39–41}, complex contagion with interactions among multiple spreading entities has not been, so far, addressed in detail.

A most clear example of competition in technology spreading is that of new social networking services in top of a preexisting technology^{27,28}. A new service competes with preexisting ones in the market share. The initial technology can take first-mover advantage such as preemption, technological leadership, and switching cost to other ones, and thus the spread of the new technology is effectively hindered. Therefore, one might expect that when an existing product occupies a dominant position in the market, the global spread of a newly launched product of similar characteristics is hardly successful. However, it is not rarely observed that a late technology successfully spreads into population, such as the success of new online social networking services²⁸. These observations call for an understanding of the competition dynamics in complex contagion processes. In this paper we address this question considering the spreading of a new technology competing with a preexistent one. An important fact in this context is that some agents use multiple technologies at the same time. For instance, it is reported that 52% of people use multiple different social networking services because different neighbors use different software²⁸. We thus explore the role of the agents using at the same time the old and new technologies. The consideration of these “dual users” is also motivated by analogy with bilingual agents on problems of language

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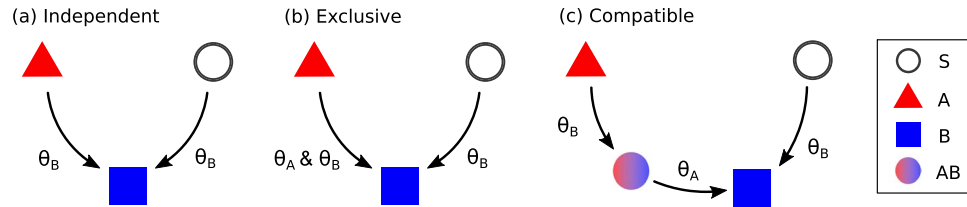


Figure 1. Schematic illustration of (a) independent, (b) exclusive, and (c) compatible models. Open circle, triangle, square, and filled circle respectively represent susceptible S , A , B , and dual user AB .

competition^{42–44} where they modify in essential ways the competition dynamics. Within a rich phenomenology of spreading and extinction transitions, as well as phases of coexistence, we find that the existence of dual users facilitates the spread of a new technology and the decline of the preexisting one. Dual users act as catalysts of extinction transitions and coexistence phases. Therefore, the conflict between the first-mover advantage and success of a late mover can be resolved by introducing “dual users” which promote the spread of a late mover.

Model

Here we consider a model of complex contagion of a new product B with threshold mechanism^{4,5} on a network with N nodes in which a preexisting technology A is prevalent. Initially, nodes can be either susceptible (S) or in the A state in which A has been adopted. We assume an initial fraction ρ_A of nodes in state A and an initial fraction ρ_S of nodes are in state S . To model the early adoption of a new technology B , we also select a small initial fraction ρ_B of nodes in state B as seeds to initiate the spreading of B . We consider three different models for the mechanism of spreading of B : independent [Fig. 1(a)], exclusive [Fig. 1(b)], and compatible [Fig. 1(c)] models. (i) In the independent model, the preexisting technology A and the new technology B are independent and do not interact with each other. If a fraction of neighbors of node i in B state is larger than a threshold θ_B ($k_B^i/k^i \geq \theta_B$) where k_j^i is the number of neighbors of i in j state and k^i is the total number of neighbors of node i (degree of node i), node i changes to state B regardless of the initial state of node i , either susceptible S or A . Therefore, the independent model is essentially the same as the original threshold model for the spreading of a single technology^{4,5}. (ii) In the exclusive model, A and B compete and additional requirements exist for a user of A to change its state adopting B . To be specific, if $k_B^i/k^i \geq \theta_B$, node i in state S adopts B , as the independent model. However, if node i is in state A , it changes to state B only if the fraction of B neighbors of i is larger than θ_B and at the same time the fraction of A neighbors is smaller than θ_A ($k_B^i/k^i \geq \theta_B$ and $k_A^i/k^i < \theta_A$). (iii) In the compatible model, we introduce dual users in an intermediate state AB in the transition from A to B modeling agents that use both technologies A and B . Nodes in the susceptible state S change to state B as in the independent or exclusive model. However, a node i in state A becomes a dual user AB by adopting B when $(k_B^i + k_{AB}^i)/k^i \geq \theta_B$. A dual user i (AB state) changes into B state by discarding A , when the fraction of neighbors in A is less than the threshold for A ($k_A^i/k^i < \theta_A$). In all three models, dynamics proceeds from the initial state until a final frozen configuration is reached which depends on parameters such as threshold parameters θ_A, θ_B , initial fraction ρ_A, ρ_B , and network structure as well.

Theory

The general question addressed is about the conditions for the spreading of B , but in particular we ask about the possible coexistence of A and B and the role of dual users in the spreading of B . To compute the final fraction of nodes in each state we provide a general theory of complex contagion in which each node can be in n states. The initial fraction of nodes with state x is ρ_x where $x \in \{1, 2, \dots, n\}$. The final average fraction of nodes in state x , R_x , can be expressed as

$$R_x = \sum_{y=1}^n [\rho_y f(y, x) - \rho_x f(x, y)], \quad (1)$$

where $f(x, y)$ is the transition probability from state x to y . On locally tree-like networks in the limit $N \rightarrow \infty$, the transition probability from x to y can be computed as^{5,17,45},

$$f(x, y) = \sum_{\vec{k}} P(k) T_0(\vec{k}) \prod_{w=1}^n q_w(x)^{k_w} \eta(x, y) \quad (2)$$

where $\eta(x, y)$ is a threshold function taking values 0 or 1 for the transition from state x to y defined by the adoption rule for a pair of states (x, y) , k_x represents the degree of a node in a given state x , k is the degree of a node $k = \sum_{x \in \{1, 2, \dots, n\}} k_x$, $P(k)$ is the degree distribution for nodes, the vector of degree for each state is $\vec{k} = \{k_1, k_2, \dots, k_n\}$, and multinomial distribution of them is $T_0(\vec{k}) = \frac{\prod_w k_w!}{k!}$. Here, $q_x(y)$ is the probability that a node, connected to a node in state y , is in state x .

In order to obtain $f(x, y)$, we first calculate $q_x(y)$ by introducing the transition probability $g(x, y, z)$ from state x to y for a node connected to a node in state z :

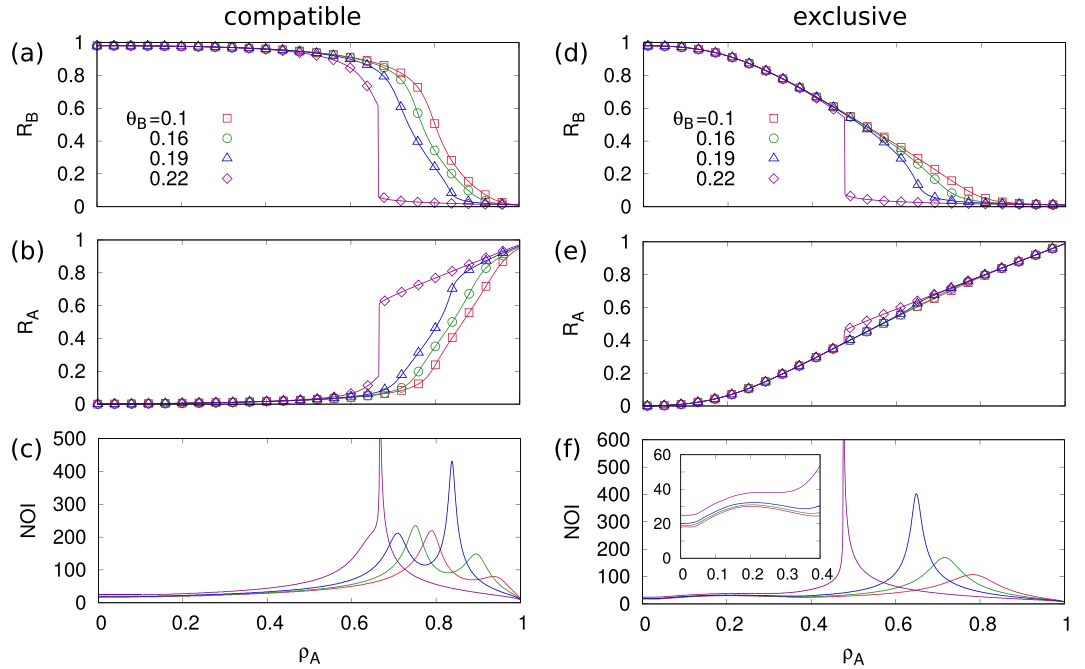


Figure 2. Theory (line) and simulation results (symbol) for the final fraction of A, B and NOI for (a–c) compatible and (d–f) exclusive with respect to ρ_A on ER networks with $N = 10^5$, $\langle k \rangle = 4$, $\theta_A = 0.2$, and $\rho_B = 0.01$ are shown with $\theta_B = 0.1$ (\square), 0.16 (\circ), 0.19 (\triangle), and 0.22 (\diamond). The local peak of NOI accurately indicates the transition of the spread of R_B and the extinction of R_A .

$$q_x(y) = \sum_{w=1}^n [\rho_w g(w, x, y) - \rho_x g(x, w, y)]. \tag{3}$$

Next, we define the excess degree of a node for a given state x as κ_x . Considering $g(x, y, z)$ where a node is connected to a node in state z , we can define the vector of excess degree for each state as $\vec{\kappa}_z = \{k_1, k_2, \dots, k_z - 1, \dots, k_n\}$. Assuming locally-tree like structures, we can compute $g(x, y, z)$ using the self-consistency equations^{5,17}:

$$g(x, y, z) = \sum_{\vec{\kappa}_z} \frac{kP(k)}{\langle k \rangle} T_1(\vec{\kappa}_z) \prod_{w=1}^n q_w(x)^{\kappa_w} \eta(x, y, z), \tag{4}$$

where $\eta(x, y, z)$ is a threshold function for the transition from state x to y for a node connected to a node in state z and $T_1(\vec{\kappa}_z) = \frac{\prod_w \kappa_w!}{(k-1)!}$. The differences among possible threshold contagion models are encoded in the threshold functions $\eta(x, y)$ and $\eta(x, y, z)$ that account for different transition rules. Details of this theoretical framework for our independent, exclusive, and compatible models are given in Methods. Our approximation is exact in a tree structure and produces very good agreement with numerical simulations for sparse tree-like graphs.

Results

In a threshold spreading model of a single technology ($n=2, x=S$ or $x=A$), there exists a single well known transition^{4,5} from a non-adopting phase to a spreading phase in which, by a global cascade mechanism, a large fraction of agents adopt the technology $x=A$ from an initial seed. In our set-up with A initially spread, and an initial seed of B , we search for transitions for the spreading of B , as well as for the extinction of A . When these transitions do not coincide there will be a phase of coexistence of A and B . Different phases and transitions can be identified calculating from Eqs. (1–4) the final fraction of nodes R_A and R_B in states A and B respectively. We define four possible phases. (i) Phase **S**: extinction of A and no spreading of B , (ii) Phase **A**: survival of A and no spreading of B , (iii) Phase **B**: extinction of A and spreading of B , and (iv) Coexistence phase **A + B**: survival of A and spread of B .

An example of how these phases can be identified for the compatible and exclusive model is given in Fig. 2 showing R_A and R_B for particular parameter values in an Erdős-Rényi (ER) network with $\langle k \rangle = 4$. The values chosen for θ_A and θ_B are in the range ($\theta < 1/4$) in which there is a spreading phase in the independent model [the details in the Supplementary Information (SI)]. Extremely good agreement is found between theory and simulation. The transition between different phases can be accurately identified by a local maximum in the number of iteration (NOI) of the recursion equations (Eq. 4), reflecting critical slowing down at the transition point. As the initial fraction of nodes in state A , ρ_A , decreases we can observe two transitions in R_A and R_B : a transition to extinction of A and a transition to spreading of B . For the compatible model, the transitions observed for R_A and R_B coincide when $\theta_B = 0.22$ (single peak of NOI), but they appear at different values of ρ_A when $\theta_B < 0.2$

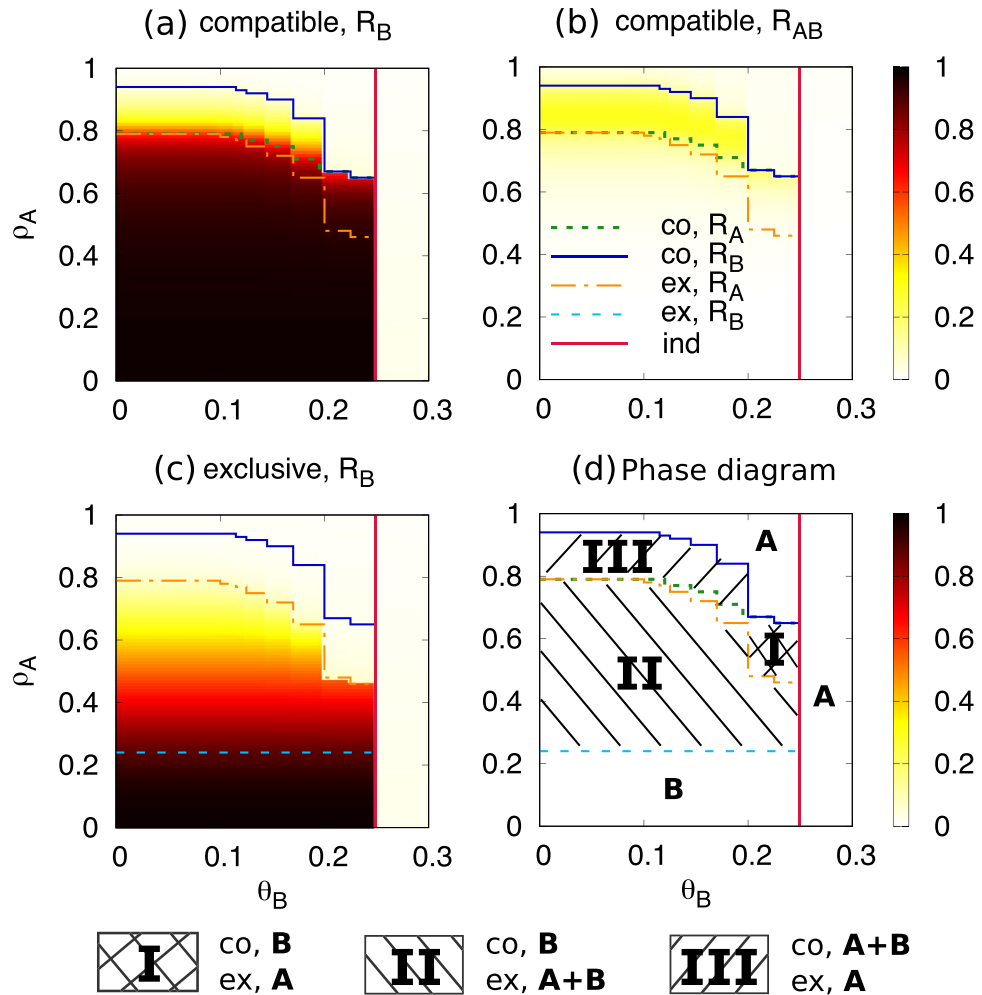


Figure 3. (a) R_B , (b) R_{AB} for the compatible model, (c) R_B for the exclusive model, and (d) phase diagram in the $\rho_A - \theta_B$ plane, with $\langle k \rangle = 4$, $\theta_A = 0.2$, $\rho_A = 0.6$, and $\rho_B = 0.01$. Transition lines are indicated as: (i) Compatible model: Extinction of A (co, R_A), spreading of B (co, R_B), (ii) Exclusive model: Extinction of A (ex, R_A), spreading of B (ex, R_B), and (iii) Transition for the independent model (ind).

allowing for a coexistence phase **A + B**. Note also that these transitions are discontinuous for $\theta > 0.2$ ($\theta_B = 0.22$), but become continuous for $\theta_B < 0.2$. For the exclusive model, a coexistence phase (**A + B**) appears consistently for all θ_B . The spreading transition for R_B is also discontinuous here for $\theta_B > 0.2$ and continuous for $\theta_B < 0.2$.

Proceeding along these lines we can construct from our analytical solution phase diagrams as a function of different parameters: Fig. 3 shows a plot density of R_B for the exclusive and compatible models in the plane $\theta_B - \rho_A$ with different transition lines (see Methods for their calculation). Varying θ_B , the transition lines in the phase diagram is discontinuous rather than a continuous curve because the threshold of our model is based on the ratio to the degree. Specifically, we can observe sharp transitions at the point where $\theta_B = n/k$ with n being an integer. Note that such discontinuity is also found in the prototypical threshold model⁵. Phase **A** always exists for $\theta_B > 1/4$ larger than the critical value of the independent model ($\theta_c = 1/4$) since θ_A is less than $1/4$. For $\theta_B < 1/4$ phase **A** occurs also in both models when ρ_A is above the critical value for spreading of B in the compatible model. On the other hand, also for $\theta_B < 1/4$, phase **B** occurs in both models when ρ_A is below the transition line for spreading of B in the exclusive model. In between these two regimes we find three domains of parameters in which results are different for the two models. These domains give evidence of the important role of dual users AB in favoring the spread of B. In region **I** we find phase **A** in the exclusive model and phase **B** in the compatible model: This is an extreme effect of dual users leading to the extinction of A and spreading of B. In region **II** we find phase **A + B** in the exclusive model and phase **B** in the compatible model: Dual users destroy coexistence in favor of full spreading of B. In region **III** we find phase **A** in the exclusive model and phase **A + B** in the compatible model: Dual users spread B allowing for coexistence. In this coexistence regime, there is a large proportion R_{AB} of dual users in the final state, while in regions **I** and **II** dual users play an important transient role in the spreading of B, but essentially disappear in the final state. In summary, dual users behave as an intermediate step in the transition from A to B, mitigating the first-mover advantages of A.

We next look at network effects that confirm the important role of dual users in the spreading of B (Fig. 4). We consider a point in region **II** of Fig. 3 and examine the effect of varying the average degree $\langle k \rangle$ of the ER network.

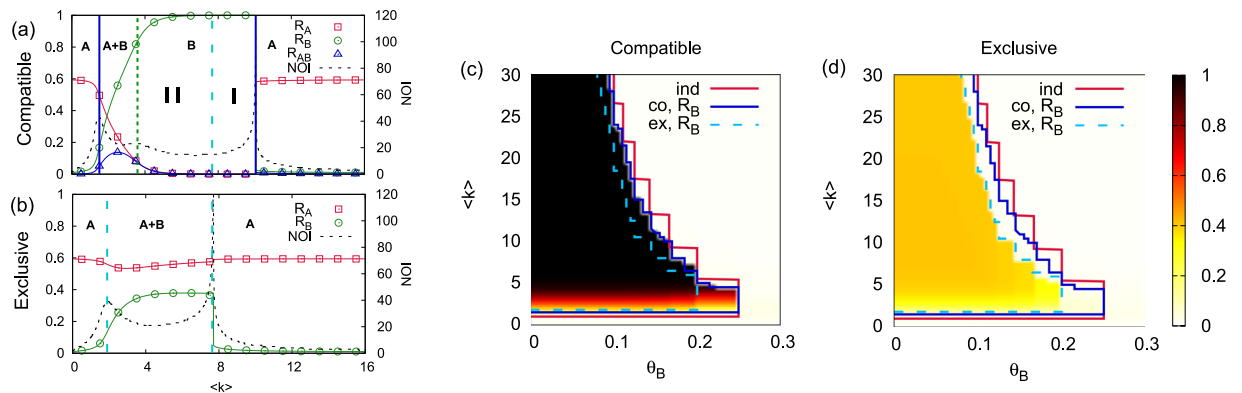


Figure 4. Theory (line) and numerical results (symbol) for the final fraction of A , B , AB nodes, and NOI for (a) compatible and (b) exclusive models on ER networks ($N = 10^5$) as a function of $\langle k \rangle$ with $\theta_A = 0.2$, $\theta_B = 0.16$, $\rho_A = 0.6$, and $\rho_B = 0.01$. The phases (A, B, and A + B) and transitions between them are identified by the peak of NOI. R_B for (c) compatible and (d) exclusive models in the $\langle k \rangle - \theta_B$ plane, with $\theta_A = 0.2$, $\rho_A = 0.6$, and $\rho_B = 0.01$. Transition lines are indicated as in Fig. 3.

The compatible model shows three transitions when increasing $\langle k \rangle$. Since networks do not form a globally connected component below the percolation threshold ($\langle k \rangle_{pc} = 1$), spreading of B does not occur when $\langle k \rangle < 1$. A first continuous spreading transition for B leads from phase A to the coexistence phase A + B. A second continuous extinction transition for A occurs around $\langle k \rangle \approx 4$ leading from phase A + B to phase B. A final transition from B to A occurs for the high connectivity regime ($\langle k \rangle \gtrsim 10$) where initial fraction of B does not exceed the threshold θ_B , so that the spread of B disappears abruptly. For the exclusive model, phase B between A + B and A no longer exists and $R_A > 0.5$ for all values of $\langle k \rangle$, indicating a strong preemptive advantage of A . Comparing the two models we find again regions I and II with the same characteristics than in Fig. 3 where dual users play a dominant role in the spreading of B . Again this transient dynamics affects with only a significant number of dual users in the final state (R_{AB}) in the coexistence phase A + B of the compatible model.

Dependence of our results on θ_B is seen in Fig. 4(c,d) where results for R_B are shown in the $\langle k \rangle - \theta_B$ plane. The range of parameters in which B spreads in the exclusive model is smaller than in the independent model due to the hindering effect of a preexisting technology A , but this effect is smaller for the compatible model. More importantly, when B spreads, the final fraction R_B is much larger in the compatible model than in the exclusive model. Note that phase B in Fig. 4(a) is within the R_B transition line of the compatible model [Fig. 4(c)], while phase A + B in Fig. 4(b) is within the R_B transition line of the exclusive model [Fig. 4(d)]. Results for the full range of values of $\langle k \rangle$, ρ_B , θ_B and θ_A are further shown in the SI. We find that our main conclusions are consistently valid for a broad range of parameters.

Discussion

In summary, we have discussed models of complex contagion for the spreading of a new innovation B competing with an established one A . We obtain perfect agreement between theory and simulation. Competition, as compared with the independent model, is shown to have a hindering effect on the spreading of the new technology. However, competition can create a gap between the extinction and spreading transitions that allows for a coexistence phase. It can also change the nature of the spreading transition from first to second order. Moreover, the existence of dual users is found to be a key mechanism to promote the spreading of B in a system dominated by a A , allowing to overcome the hindering effect of tightly clustered of agents using A . Dual users create new phases in which they either lead to extinction of A in favor of spreading of B , or destroy coexistence in favor of spreading of B or create coexistence by B spreading. The role of dual users provides a plausible explanation to reconcile two seemingly contradictory factors: the first-mover advantage meaning that the prevalence of A potentially blocks the spread of B , and the success of late-mover frequently observed in reality. Practically, our study suggests a strategic way of spreading a new technology in a competitive environment by offering it as a good compatible alternative instead of promoting changing to the new technology. Further studies may be needed to examine the role of dual-users in simple contagion models, opinion dynamics, and coevolutionary dynamics, to name a few.

Methods

Theory of compatible model. We implement the developed general theory of complex contagion into the compatible model. The final fraction of state A , B , AB , and S are defined as R_A , R_B , R_{AB} , and R_S , respectively. We further define the initial fraction of A , B , AB , and S as ρ_A , ρ_B , ρ_{AB} and ρ_S . These variables are normalized as $R_A + R_B + R_{AB} + R_S = 1$ and $\rho_A + \rho_B + \rho_{AB} + \rho_S = 1$. Note that $\rho_{AB} = 0$ since the coexisting state cannot exist initially. Then the main purpose of the theory is to calculate the final fraction of nodes in each state for a given initial condition. Given the initial fraction of each state, the final fraction of each state for the compatible model is given by

$$R_A = \rho_A [1 - f(A, AB) - f(A, B)], \quad (5)$$

$$R_B = \rho_B + \rho_S f(S, B) + \rho_A f(A, B), \quad (6)$$

$$R_{AB} = \rho_A f(A, AB), \quad (7)$$

where $f(x, y)$ represents the transition probability from state x to y . Then, the transition probability $f(x, y)$ on a locally-tree like network can be calculated as

$$f(x, y) = \sum_{\vec{k}} P(k) T_0(\vec{k}) q_A(x)^{k_A} q_B(x)^{k_B} q_{AB}(x)^{k_{AB}} q_S(x)^{k_S} \eta(x, y). \quad (8)$$

Here, k_x represents degree of a node for a given state x , k is the degree of a node $\vec{k} = \sum_{x \in \{A, B, AB, S\}} k_x$, $P(k)$ is the degree distribution of underlying networks, the vector of degree for each state is $\vec{k} = (k_S, k_A, k_B, k_{AB})$, and multinomial distribution of them is $T_0(\vec{k}) = \frac{k!}{k_S! k_A! k_B! k_{AB}!}$. $q_x(y)$ is the probability that a node, connected to a node in state y , is in state x . And, threshold $\eta(x, y)$ is given by the rule of adoption for a pair of state (x, y) . Specifically $\eta(S, B) = H(\frac{k_B + k_{AB}}{k} - \theta_B)$, $\eta(A, B) = H(\frac{k_B}{k} - \theta_B)H(\theta_A - \frac{k_A}{k})$, and $\eta(A, AB) = H(\frac{k_B}{k} - \theta_B)H(\frac{k_A}{k} - \theta_A)$ where $H(x)$ is the Heaviside step function, $H(x) = 0$ if $x < 0$ and $H(x) = 1$ if $x \geq 0$.

A set of variables $q_x(y)$ can be obtained by introducing the transition probability $g(x, y, z)$ from state x to y for a node connected to a node in state z :

$$q_A(z) = \rho_A [1 - g(A, AB, z) - g(A, B, z)], \quad (9)$$

$$q_B(z) = \rho_B + \rho_S g(S, B, z) + \rho_A g(A, B, z), \quad (10)$$

$$q_{AB}(z) = \rho_A g(A, AB, z), \quad (11)$$

where $z \in \{S, A\}$. Furthermore, $g(x, y, z)$ can be calculated by the self-consistency equations with the locally-tree like structures

$$g(x, y, z) = \sum_{\vec{\kappa}_z} \frac{kP(\vec{\kappa}_z)}{(\vec{\kappa}_z)} T_1(\vec{\kappa}_z) q_A(x)^{\kappa_A} q_B(x)^{\kappa_B} q_{AB}(x)^{\kappa_{AB}} q_S(x)^{\kappa_S} \eta(x, y, z). \quad (12)$$

where κ_i is the excess degree of state i , and $\eta(x, y, z)$ is a threshold function for the transition from state x to y for a node connected to a node in state z . $\eta(x, y, z)$ for our compatible model is given by $\eta(S, B, S) = \eta(S, B, A) = H(\frac{k_B + k_{AB}}{k} - \theta_B)$, $\eta(A, B, S) = H(\frac{k_B}{k} - \theta_B)H(\theta_A - \frac{k_A}{k})$, $\eta(A, B, A) = H(\frac{k_B}{k} - \theta_B)H(\theta_A - \frac{k_A + 1}{k})$, $\eta(A, AB, S) = H(\frac{k_B}{k} - \theta_B)H(\theta_A - \frac{k_A}{k})$, and $\eta(A, AB, A) = H(\frac{k_B}{k} - \theta_B)H(\frac{k_A + 1}{k} - \theta_A)$. Note that $\eta(A, B, A)$ is different from $\eta(A, B, S)$ in $H(\theta_A - \frac{k_A + 1}{k})$ since in $\eta(A, B, A)$ an additional neighbor with A state should be considered.

Theory of exclusive model. For the exclusive model, since coexisting AB state is not allowed, the final fraction of each state is given by

$$R_A = \rho_A [1 - f(A, B)], \quad (13)$$

$$R_B = \rho_B + \rho_S f(S, B) + \rho_A f(A, B). \quad (14)$$

Similarly, the variables $q_x(y)$ are

$$q_A(z) = \rho_A [1 - g(A, B, z)], \quad (15)$$

$$q_B(z) = \rho_B + \rho_S g(S, B, z) + \rho_A g(A, B, z), \quad (16)$$

where $z \in \{S, A\}$. The transition probability $f(x, y)$ and $g(x, y, z)$ can be computed by using the same equations for compatible model (Eqs. 8 and 12) but with different $\eta(x, y)$ and $\eta(x, y, z)$. To be specific, for the exclusive model, $\eta(S, B) = H(\frac{k_B}{k} - \theta_B)$, $\eta(A, B) = H(\frac{k_B}{k} - \theta_B)H(\theta_A - \frac{k_A}{k})$, $\eta(S, B, S) = \eta(S, B, A) = H(\frac{k_B}{k} - \theta_B)$, $\eta(A, B, S) = H(\frac{k_B}{k} - \theta_B)H(\theta_A - \frac{k_A}{k})$, and $\eta(A, B, A) = H(\frac{k_B}{k} - \theta_B)H(\theta_A - \frac{k_A + 1}{k})$.

Theory of independent model. For the independent model, nodes in A state do not interact with nodes in B state and thus the final fraction of nodes is given by

$$R_A = \rho_A [1 - f(A, B)], \quad (17)$$

$$R_B = \rho_B + \rho_S f(S, B) + \rho_A f(A, B). \quad (18)$$

And $q_x(y)$ are

$$q_A(z) = \rho_A [1 - g(A, B, z)], \quad (19)$$

$$q_B(z) = \rho_B + \rho_S g(S, B, z) + \rho_A g(A, B, z), \quad (20)$$

where $z \in \{S, A\}$. The thresholds for the independent model are simply given by $\eta(S, B) = \eta(A, B) = H(\frac{k_B}{k} - \theta_B)$ and $\eta(S, B, S) = \eta(S, B, A) = \eta(A, B, S) = \eta(A, B, A) = H(\frac{k_B}{k} - \theta_B)$.

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Author Contributions

All authors conceived the study, B.M. performed the research, all authors analyzed the data, and wrote and reviewed the paper.

Additional Information

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