Since January 2020 Elsevier has created a COVID-19 resource centre with free information in English and Mandarin on the novel coronavirus COVID-19. The COVID-19 resource centre is hosted on Elsevier Connect, the company's public news and information website.

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A fractal fractional order vaccination model of COVID-19 pandemic using Adam's moulton analysis

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ABSTRACT

The pandemic caused by coronaviruses (SARS-COV-2) is a zoonotic disease targeting the respiratory tract of active humans. Few mild symptoms of fever and tiredness get cured without any medicinal aid, whereas some severe symptoms of dry cough with breathing illness led to perceived risk of secondary transmission. This paper studies the effectiveness of vaccination in Covid-19 pandemic disease by modelling three compartments susceptible, vaccinated and infected (SVI) of Atangana Baleanu of Caputo (ABC) type derivatives in non-integer order. The disease dynamics is analysed and its stability is performed. Numerical approximation is derived using Adam's Moulton method and simulated to forecast the results for controllability of pandemic spread.

1. Introduction

Pandemic diseases are pondered to be a great threat to mankind deliberately . Recent Covid-19(SARS-COV-2) pandemic is a serious global health issue since its emergence from December 2019. This outbreak disrupts many countries' financial and health system abysmally [1,2]. Socio-economic and psychological challenges were faced by the people worldwide due to the contagiousness of COVID-19 [3]. Many studies were made about the pathogen, its infectious nature on human cells affecting the immune system. Symptoms of Covid-19 contagion range from mild asymptotic fever, cough to complicated airway disorder. The disease causing pathogen spreads through droplets of infected runny nose. The rapid spread of the virus transmission between population across the globe urged the health care system to improve the facilities to barge in the transmission globally. [4]. Several new virulent variants of coronavirus namely Delta, stealth Omicron were mutated recently. Mathematical modelling of the epidemic infections in terms of non-linear system help us to understand the disease spread and in process suggests to control its propagation. The pathogen transmission dynamics, population flow, issues arising in controlling the spread were studied through epidemiological models effectively. COVID-19 is a deadliest infectious disease the world ever had. From December 2019–April 2022, totally 514,952,802 infected persons with 20.3 million estimated fatalities around the globe [5,6]. India had its first infective case at the end of January 2020. All necessary preventive steps were implemented by The Government and Health Ministry to empower the innocent public about the right knowledge of the viral contagiousness, communal transmission, respiratory and self hygiene. Control measures are essential to mitigate the hike spread of contagious outbreaks. Home care, physical disinfection, uncrowdedness and self protection helps to unnerve the transmission chain, while inoculation combats for a fierce cure, neglecting complications. Vaccination is a simple, safe and effective way of immunising people against harmful diseases before the susceptible population come into contact with infective persons. It uses human body's natural defences to build resistance against specific infections and make the immune force stronger [7]. In India, vaccination programmes were started on 16 January 2021. India has administered more than 1.89

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billion vaccine doses on the whole till April 2022. In India, 91% of the adult population (above 12 years of age) has received at least single shot, and 77% with 2 shots were vaccinated [8].

Mathematical models have been earnestly involved in COVID-19 pandemic research to provide us deep knowledge about this specific disease with various effective control techniques [9–14]. Recently modellers and analysts acknowledge epidemic models using fractional derivatives. Qualitative, numerical, optimisation and significant contribution have been made by researchers in investigating fractional differential equations [15–21] . In recent scenario, the calculus of non-integer differential order is the most effective branch of mathematics which deals with kernels involving memory, past events and hereditary properties [22–24]. The concept of fractional calculus was introduced after 1695 as a simply academic generalisation of integer order derivatives and integrals. The interesting point of applying this calculus is the suitable variety of operators in use to derive better outcomes. The fractional derivative is a definite integral which geometrically accumulates the whole or vast neighbourhood of the function globally [25–27]. Many fractional order operators with power law, exponential and non-singular kernels which suitably fitted to complex systems were applied in various fields [28–30]. One of the important definitions which have been recently magnified is the ABC derivative with two parameters convened by Atangana, Baleanu and Caputo [31] in 2016, which exhibits the wide-spread kernel function detailed in [30,31]. Mittag Leffler function is the mostly used kernel in recent fractional order systems. The recent advancement of fractional order differintegrals combined with fractal dimension were introduced by Abdon Atangana [32]. The stable existence of the fractal–fractional system and its unique feature of results were explained by Atangana et al. in [33]. Researchers have keen interest working on several fields using fractal–fractional derivatives for its more efficient results than ordinary calculus and fractional derivatives [34–44] in modelling various real world problems listed as magnetohydro- dynamical fluid flow [35–37], control dynamics in banking [38], material change effects [39], problem on chemical kinetic hires [40], prototypic nature of an excited system [41] and various disease conceptual behaviour in [42–47],etc.

India reported 4.31 crores of positive infections with 5.24 lakhs of fatalities with 99% recovery rate as on 1st week of May 2022. Recently, several predictive models have been analysed on COVID-19 pandemic outbreak. Kamal Shah et al. developed a COVID-19 fractal model and analysed the effectual reactions in Pakistan region. [43], Ali.Z. Rabiei et al. Dynamics of SIR mathematical model for COVID-19 outbreak [44],Hasib Khan, et al. discussed SEIRQ compartmental model with quarantine control in [46], Epidemic forecasting model with environmental concentration on Covid-19 in India was presented by Mansour A. Abdulwassa et al. in [47]. This manuscript present a novel epidemic conceptual model of fractal ABC derivatives for COVID-19 with effective vaccination control strategy. This paper analyses SVI model with defensive inoculation activities protecting from new complicated infections, infection ratio of susceptible persons, post-vaccinated infections since the implementation of effective vaccine distribution amidst continuous spread of COVID-19 ailment.

A non-integer order pandemic model with the use of ABC fractal derivatives that encapsulates vaccination control measure to reduce the complicate virulence is built in this article. The significant points been discussed here are

(i) The combative speedy vaccination rollout to all public.

(ii) Impact of effectual vaccination in reducing complications.

(iii) Safeguarding the human resources by lessening contact rate with the rapid infections.

This study is synopsised as follows, Section 2 - Narrates basic necessary definitions and results on ABC fractal–fractional derivatives, Section 3 - Model formulation with parameters explained, Section 4 - Analysis of the designed model, Section 5 - Numerical algorithm for the model, Section 6 - Results and discussion and Section 7 - Conclusion.

2. Basic definitions and results

This section provides some necessary definitions of fractal–fractional operators required for framing and analysing the mathematical commensurate model. The fractal–fractional operators defined and named by the respective mathematicians who introduced them. In particular, we recall the Mittag-Leffler power law kernel in view of Atangana Baleanu (AB) operator in Caputo's style.

Definition 2.1. Let F = C [0,T], $F' = (0,T) \subset$ R, C [0,T]= (F,R) be space of continuous functions . Define T: [0,T] → R with norm,

$$\|(S,V,I)\| = \max_{t\in[0,T]} \{\|S(t) + V(t) + I(t)\|\} \text{ where S,V,I} \in \text{C[0,T]}.$$

Definition 2.2. The fractal–fractional derivative of function f(t) in Atangana Baleanu Caputo (ABC) sense is given by,

$$\text{ABC } D_{0+}^{\eta,p}\text{f(t)} = \frac{M(\eta)}{1-\eta}\int_0^t \frac{d}{dy^p}\text{f(y)}K_\eta\left(\frac{-\eta}{1-\eta}(t-y)^\eta\right)\ \text{dy}. \tag{1}$$

The order of derivation, $\eta > 0 \leq 1$ with fractal dimension p≤1 and $M(\eta) = (1-\eta) + \frac{\eta}{\Gamma\eta}$ is the normalisation operator takes only real positive values satisfying $M(\eta) = 1$ for $\eta = 0$ and 1,

For $\eta \in [0,1]$. This type of multipliers supports balance in modelling complex problems while integrating over the non-integer order η

Here K_η is the Mittag-Leffler function generalisation of exponential function given by

$$K_\eta(f) = \sum_{k=0}^{\infty} \frac{f^k}{K\eta + 1} \tag{2}$$

The corresponding integral with unit fractal dimension p is given by,

$$\text{AB } I_{f(t)} = \frac{1-\eta}{M(\eta)}f(t) + \frac{\eta p}{M(\eta)\Gamma\eta}\int_0^t (t-y)^{\eta-1}\text{f(y)}\ \text{dy}. \tag{3}$$

3. Formulation of fractal–fractional vaccination model for Covid-19

In this section we formulate a Covid-19 transmission model using new group of vaccinated population. Healthy unvaccinated susceptible persons who are exposed to infections is framed in this system. The new vaccinated class contains all the healthy public who received vaccine shots and been immunised. Both unvaccinated and vaccinated get infections due to heavy virulent features of the illness. It is a most challenging task to trace asymptomatic infections persists among people. This study investigates the Covid-19 dynamics with these conditions involved by framing in an epidemic model. Let us formulate the vaccination model on Covid-19 by denoting the total population counted to be N, which is splitted into three subclasses Susceptibles S(t), Vaccinated population V(t) and the contagious group I(t) with N(t) = S(t)+V (t) + I (t). The most significant feature of this study is to interrogate about the vaccination effectiveness in controlling the pandemic complications.

The commensurate model [48] explained with the above assumptions are designed in terms of fractal fractional order ABC derivatives. ABC operators treasures the inbuilt memory of complex biological phenomena. Memory and inheritance are the sufficient tools of the human immunity system. Hence ABC order fractal–fractional model explores the crossover behaviour dynamics of Covid-19 transference very realistically. The novel vaccination model for Covid-19 is formulated as follows,

$$\left.\begin{aligned} \text{ABC } D_{0+}^{\eta,1}\mathrm{S}(t) &= (1-\mathrm{v})\,\mathrm{a} - \frac{\beta \mathrm{SI}}{N} - \mu \mathrm{S} \\ \text{ABC } D_{0+}^{\eta,1}\mathrm{V}(t) &= \mathrm{va} - \frac{\alpha \mathrm{VI}}{N} - \mu \mathrm{V} + \gamma \mathrm{I} \\ \text{ABC } D_{0+}^{\eta,1}\mathrm{I}(t) &= \frac{\beta \mathrm{SI}}{N} + \frac{\alpha \mathrm{VI}}{N} - \mu \mathrm{I} - \gamma \mathrm{I} - \phi \mathrm{I} \end{aligned}\right\} \tag{4}$$

where $0 < \eta < 1$, ABC D_{0+}^{η} is the ABC derivative of order η, with the initial conditions S(0) ≥ 0, V(0) ≥ 0, I(0)≥ 0.

i. Susceptible population $S\ (t)$: Unvaccinated susceptible people become infected if exposed to infected people.
ii. Vaccinated population $V(t)$: Vaccinated people, susceptible to get infected due to vaccine breakthroughs.
iii. Infected population I(t): Infected persons both from unvaccinated and vaccinated susceptibles.

Parameters in the model briefed:

i. Total population under study be assumed as N(t).
ii. The birth rate of susceptible population $S(t)$ were assumed as 'a'
iii. The parameter 'v' notify the net- vaccination ratio with the overall probability lies between $0 < \mathrm{v} \leq 1$. The unvaccinated susceptibles acquires immunity at the rate of 'va' from $S(t)$ move on to $V(t)$.
iv. Incidence rate of susceptibles $\frac{\beta}{N}$ becomes contagious entering I(t).
v. Vaccinations breakthrough infections occur at the rate of $\frac{\alpha}{N}$ added to infected group I(t) .
vi. Occurrence of natural death among the people denoted by μ.
vii. Severity of the disease leads to mortality of infected persons denoted by ϕ neglected from I(t).
viii. Immunity built by exposed infection and recovery assumed as γ, moves to $V(t)$.

4. Analysis of the Covid-19 vaccination model:

Stability analysis of the proposed SVI model (4) is measured deliberately in this section. The qualitative analysis of the framed model is presented through positivity, boundedness and stability about its equilibrium points. The feasibility of the solution existence region depicts the ultimate range through norms positively.

Feasible neighbourhood: This theorem ensures the positive and feasible well-posedness of the bio-epidemiological model (4) for the initial values (S_0, V_0, I_0) defined on [0,1).

Theorem 4.1. *The feasible region of the solution is given by*

$$\|(S,V,I)\| = \max_{t\in[0,T]}\{\|S(t)+V(t)+I(t)\|, (S,V,I)\in R^3\}\ \textit{where S,V,I} \in \textit{C[0,T]}.$$

The condition for control of pandemic disease is $S_0 < \frac{\alpha V+\beta S}{\gamma+\phi+\mu} N_0$, where $\frac{\alpha V+\beta S}{\gamma+\phi+\mu} N_0$ is the disease severity rate.

Proof. The feasible region of the solution is derived as follows.

Let $N = S + V + I$ where N is the total population considered.

The sum of all equations in (1) we get,

$$\begin{aligned} \lim_{t\to\infty} \text{ABC } D_{0+}^{\eta}(\mathrm{S}+\mathrm{V}+\mathrm{I}) &\leq \mathrm{a} - \mu(\mathrm{S}+\mathrm{V}+\mathrm{I}) - \phi \mathrm{I} \\ &\leq \frac{a}{\mu}. \end{aligned}$$

Hence the system (1) is positively invariant with $\|(S,V,I)\| \in$ C [0,T].

The infected population of (1) shows that, ABC $D_{0+}^{\eta}\mathrm{I}(t) \leq \left(\frac{\beta \mathrm{S}+\alpha \mathrm{V}-N_0(\gamma+\phi+\mu)}{N_0}\right)\mathrm{I}(t)$ which implies the initial susceptible population $S_0 < \frac{\alpha V+\beta S}{\gamma+\phi+\mu} N_0$.

Hence the infected population will be decreasing as

ABC $D_{0+}^{\eta}\mathrm{I}(t) < 0$, whereas infections will increase if ABC $D_{0+}^{\eta}\mathrm{I}(t) > 0$, resulting in pandemic invasively.

Stability around equilibrium neighbourhoods

Theorem 4.2. *The contagion free equilibrium (CFE) of (1) is given by*

(i) $CFE = \left(\frac{\alpha V+\beta S}{\gamma+\phi+\mu} N_0, 0, 0\right)$.

(ii) The point CFE is asymptotically stable if the vaccine efficiency should be $> 1 - \frac{1}{R_0}$, *where* R_0 *is the reproduction number.*

Proof. To estimate the disease free state, let us equate system (1) to zero as follows, ABC $D_{0+}^{\eta,1}\mathrm{S}(t) = \mathrm{ABC}\ D_{0+}^{\eta,1}\mathrm{V}(t) = \mathrm{ABC}\ D_{0+}^{\eta,1}\mathrm{I}(t) = 0$.

The infected population of (1) implies $\left(\frac{\beta S+\alpha V-N_0(\gamma+\phi+\mu)}{N_0}\right)\mathrm{I}(t) = 0$

Then the susceptible population $S_0 = \frac{\alpha V+\beta S}{\gamma+\phi+\mu} N_0$, reveals that the spread of the pandemic will be controlled as there does not persists new infections and vanishing death rate, with the contagion free point, $CFE = \left(\frac{\alpha V+\beta S}{\gamma+\phi+\mu} N_0, 0, 0\right)$.

4.1. Basic reproduction ratio

The basic reproduction number is calculated from the Jacobian matrix . The threshold value $R_0 = \beta S + \alpha \mathcal{V} - N_0(\mu+\gamma+\phi)$ is the contagion metric of secondary infections transmitted.

By naming the equations of model (4) as P(t), Q(t), R(t), we accomplish,

$$\begin{aligned}
\mathrm{ABC}\ D_{0+}^{\eta,1}\mathrm{S}(t) &= (1-\mathrm{v})\,\mathrm{a} - \frac{\beta \mathrm{SI}}{N} - \mu \mathrm{S} &&= \mathrm{P}(t)\\
\mathrm{ABC}\ D_{0+}^{\eta,1}\mathrm{V}(t) &= \mathrm{va} - \frac{\alpha \mathrm{VI}}{N} - \mu \mathrm{V} + \gamma \mathrm{I} &&= \mathrm{Q}(t)\\
\mathrm{ABC} D_{0+}^{\eta,1}\mathrm{I}(t) &= \frac{\beta \mathrm{SI}}{N} + \frac{\alpha \mathrm{VI}}{N} - \mu \mathrm{I} - \gamma \mathrm{I} - \phi \mathrm{I} &&= \mathrm{R}(t)
\end{aligned} \tag{5}$$

Jacobian of the above system (5), $\mathrm{J} = \begin{pmatrix} \frac{\partial P}{\partial S} & \frac{\partial P}{\partial V} & \frac{\partial P}{\partial I} \\ \frac{\partial Q}{\partial S} & \frac{\partial Q}{\partial V} & \frac{\partial Q}{\partial I} \\ \frac{\partial R}{\partial S} & \frac{\partial R}{\partial V} & \frac{\partial R}{\partial I} \end{pmatrix}$

Equating the value of the determinant, $|J - \lambda I| = 0 \Rightarrow$

$$\begin{vmatrix} \frac{-\beta \mathrm{I}}{N} - \mu - \lambda & 0 & \frac{-\beta \mathrm{S}}{N} \\ 0 & -\frac{\alpha \mathrm{I}}{N} - \mu - \lambda & -\frac{\alpha \mathrm{V}}{N} + \gamma \\ \frac{\beta \mathrm{I}}{N} & \frac{\alpha \mathrm{I}}{N} & \frac{\beta \mathrm{S}}{N} + \frac{\alpha \mathrm{V}}{N} - \mu - \gamma - \phi - \lambda \end{vmatrix} = 0$$

$$\lambda_1 = -\left(\frac{\beta \mathrm{I}}{N} + \mu\right), \lambda_2 = -\left(\frac{\alpha \mathrm{I}}{N} + \mu\right), \lambda_3 = -\left(\mu + \gamma + \phi - \frac{\beta \mathrm{S}}{N} - \frac{\alpha \mathrm{V}}{N}\right).$$

The eigen values of the above matrix are all negative.

And $\lambda_3 = \frac{\beta S}{N} + \frac{\alpha V}{N} - \mu - \gamma - \phi$ which shows the ratio of immunised individual has to be effectively greater than $\left(1 - \frac{1}{\frac{\beta S}{N} + \frac{\alpha V}{N} - \mu - \gamma - \phi}\right)$, provided $\frac{(\beta S+\alpha V)}{N} - (\mu+\gamma+\phi) < 1$.

Hence the disease spread will be controlled from further extension with increasing efficient level of vaccinated population V(t).

5. Adams–Moulton numerical scheme:

This part of numerical derivation explores an apt approximation result for the designed model in Section 3. As a progress of solving fractal terms of SVI model, many analytical and semi-analytical numerical equations were found in literature [49]. In this study motivated from [50], we use Adams Moulton interpolation for the commensurate equations of same integral order.

The numerical algorithm of Atangana Baleanu Caputo fractal–fractional derivative is given,

Step 1:

We make use of Adams Moulton rule defined on $[t_j, t_{j+1}]$ for any function f, we have

$$\begin{aligned}
\mathrm{ABC}\ D_{0+}^{\eta,1}\left[\mathrm{f}\left(t_{j+1}\right)\right] &= \frac{1-\eta}{M(\eta)}\left[\frac{\mathrm{f}\left(t_{j+1}\right) - \mathrm{f}\left(t_j\right)}{2}\right] \\
&+ \frac{\eta}{M(\eta)\Gamma\eta}\sum_{i=0}^{\infty}(i+1)^{1-\eta} - (i)^{1-\eta}\left[\frac{\mathrm{f}\left(t_{i+1}\right) - \mathrm{f}\left(t_i\right)}{2}\right]
\end{aligned} \tag{6}$$

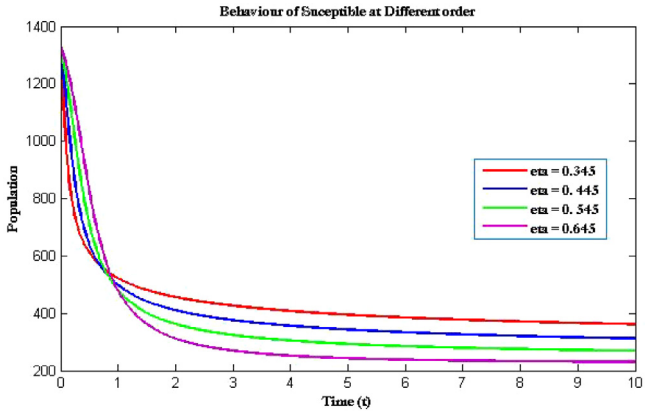

Fig. 1a. The plot optimises the susceptible population for various order of fractional differentiation η.

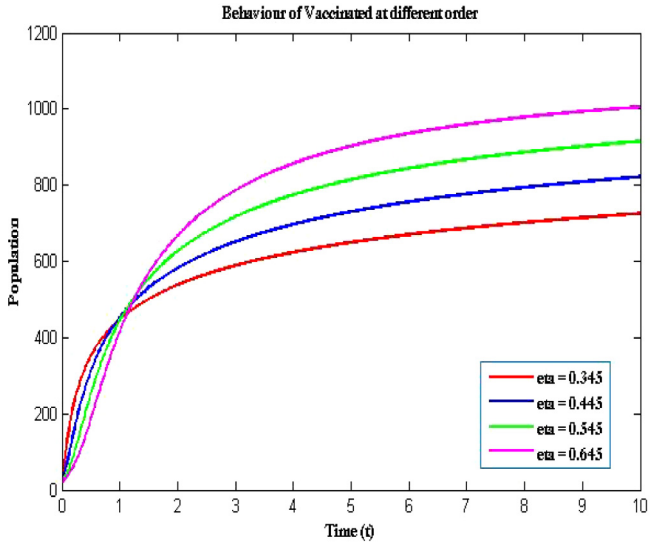

Fig. 1b. The plot shows the convergence of vaccinated population for various order of differentiation η.

Step 2:

The above Eq. (6) can be deformed in our model (4) for $[t_j, t_{j+1}], as$ the following convergence solution,

$$S(t_{j+1}) - S(t_j) = \mathrm{S}(0) + \frac{1-\eta}{M(\eta)}\left[(1-\mathrm{v})\,a - \frac{\beta}{N}\left(\frac{\mathrm{S}(t_{j+1}) - \mathrm{S}(t_j)}{2}\right)\left(\frac{\mathrm{I}(t_{j+1}) - \mathrm{I}(t_j)}{2}\right) - \mu\left(\frac{\mathrm{S}(t_{j+1}) - \mathrm{S}(t_j)}{2}\right)\right] + \frac{\eta}{M(\eta)\,\Gamma\eta}\sum_{i=0}^{\infty}(i+1)^{1-\eta} - (i)^{1-\eta}\left[(1-\mathrm{v})\,a - \frac{\beta}{N}\left(\frac{\mathrm{S}(t_{i+1}) - \mathrm{S}(t_i)}{2}\right)\left(\frac{\mathrm{I}(t_{i+1}) - \mathrm{I}(t_i)}{2}\right) - \mu\left(\frac{\mathrm{S}(t_{i+1}) - \mathrm{S}(t_i)}{2}\right)\right] \tag{7}$$

$$\mathrm{V}(t_{j+1}) - \mathrm{V}(t_j) = \mathrm{V}(0) + \frac{1-\eta}{M(\eta)}\left[\mathrm{v}a - \frac{\alpha}{N}\left(\frac{\mathrm{V}(t_{j+1}) - \mathrm{V}(t_j)}{2}\right)\left(\frac{\mathrm{I}(t_{j+1}) - \mathrm{I}(t_j)}{2}\right) - \mu\left(\frac{\mathrm{V}(t_{j+1}) - \mathrm{V}(t_j)}{2}\right) + \gamma\left(\frac{\mathrm{I}(t_{j+1}) - \mathrm{I}(t_j)}{2}\right)\right] + \frac{\eta}{M(\eta)\Gamma\eta}\sum_{i=0}^{\infty}(i+1)^{1-\eta} - (i)^{1-\eta}\left[\mathrm{v}a - \frac{\alpha}{N}\left(\frac{\mathrm{V}(t_{j+1}) - \mathrm{V}(t_j)}{2}\right)\left(\frac{\mathrm{I}(t_{i+1}) - \mathrm{I}(t_i)}{2}\right) - \mu\left(\frac{\mathrm{V}(t_{i+1}) - \mathrm{V}(t_i)}{2}\right) + \gamma\left(\frac{\mathrm{I}(t_{i+1}) - \mathrm{I}(t_i)}{2}\right)\right] \tag{8}$$

$$\mathrm{I}(t_{j+1}) - \mathrm{I}(t_j) = \mathrm{I}(0) +$$

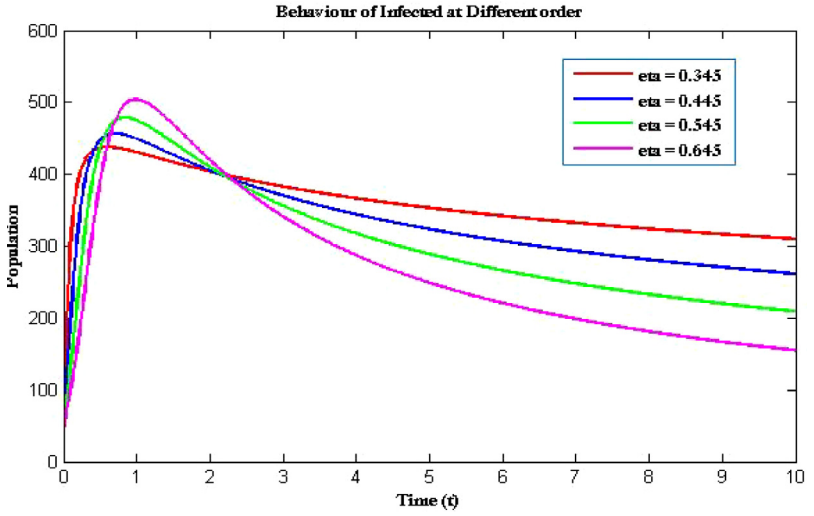

Fig. 1c. This plot shows the flow of infected population for various order of differentiation η.

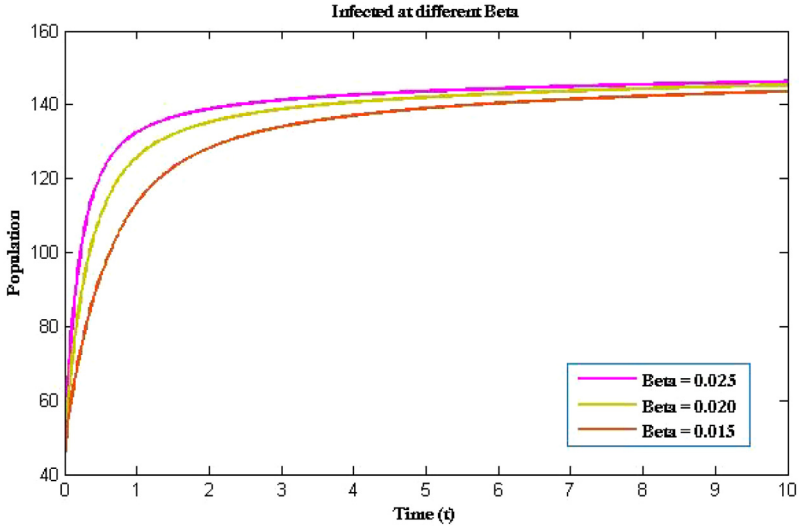

Fig. 2a. The plot depicting a gradual decline in reducing contacts with exposed infectives.

$$\frac{1-\eta}{M(\eta)}\left[\frac{\beta}{N}\left(\frac{S(t_{j+1})-S(t_j)}{2}\right)\left(\frac{I(t_{j+1})-I(t_j)}{2}\right)+\frac{\alpha}{N}\left(\frac{V(t_{j+1})-V(t_j)}{2}\right)\left(\frac{I(t_{j+1})-I(t_j)}{2}\right)\right.$$
$$\left.-\mu\left(\frac{I(t_{j+1})-I(t_j)}{2}\right)-\phi\left(\frac{I(t_{j+1})-I(t_j)}{2}\right)-\gamma\left(\frac{I(t_{j+1})-I(t_j)}{2}\right)\right]$$
$$+\frac{\eta}{M(\eta)\,\Gamma\eta}\sum_{i=0}^{\infty}(i+1)^{1-\eta}-(i)^{1-\eta}\left[\left[\frac{\beta}{N}\left(\frac{S(t_{i+1})-S(t_i)}{2}\right)\left(\frac{I(t_{i+1})-I(t_i)}{2}\right)+\frac{\alpha}{N}\left(\frac{V(t_{j+1})-V(t_j)}{2}\right)\left(\frac{I(t_{i+1})-I(t_i)}{2}\right)\right.\right.$$
$$\left.\left.-\mu\left(\frac{I(t_{i+1})-I(t_i)}{2}\right)-\phi\left(\frac{I(t_{i+1})-I(t_i)}{2}\right)-\gamma\left(\frac{I(t_{i+1})-I(t_i)}{2}\right)\right]\right] \tag{9}$$

The above numerical equations (7)–(9) will yield the most desired approximation to our model (4).

6. Results and discussion

In this section, numerical simulations were graphed for the Covid-19 model using the pandemic reported parameters in India from the websource [5]. An efficient approach of fractal dimensional fractional order differentiation were performed and found to

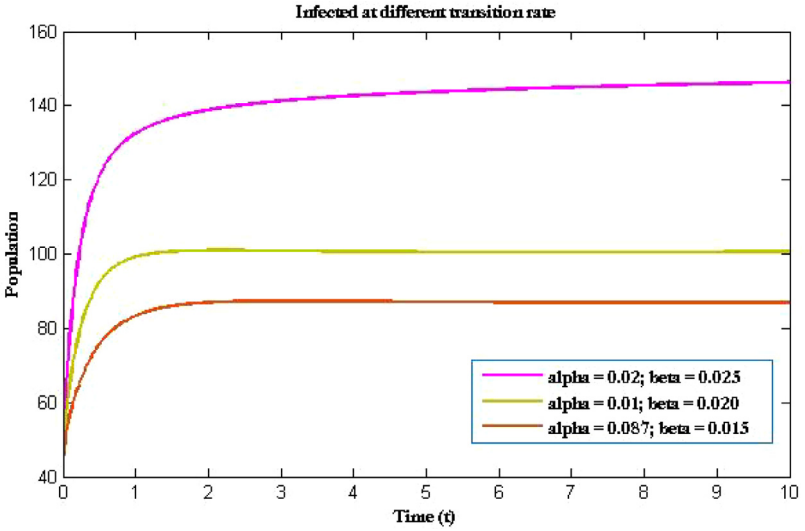

Fig. 2b. The plot explaining the vital flow of infected people for minimalised tracing rates.

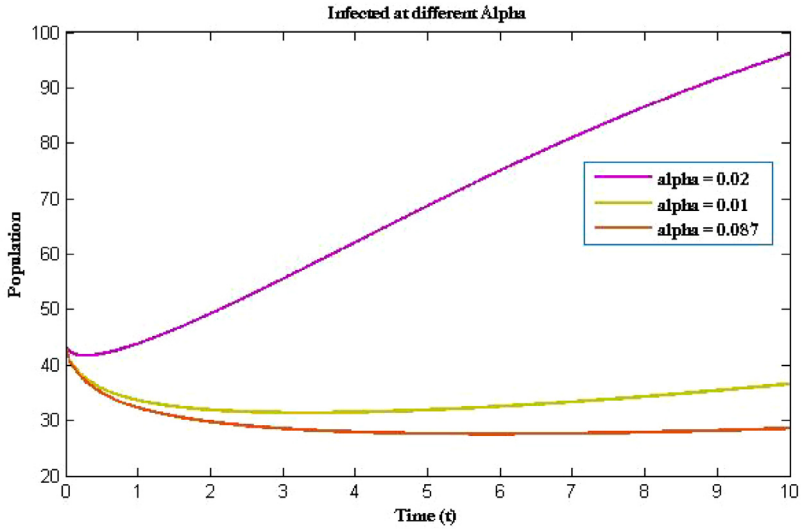

Fig. 2c. Plot confirming the control forecast by reduced vaccine breakthrough illness.

be more effectual for most least order. To illustrate our stability results, let us derive some numerical simulations by 3 different strategies in MATLAB by input values for the parameters involved.

i. For different fractional order η:

The numerical input for the parameters of the Covid-19 model involves, N=1401.23 millions, a = 0.0699, $\beta = 0.03, \alpha = 0.02$, $\mu = 0.002$, $\phi = 0.0001$, $\gamma = 0.99, \mathrm{v} = 0.63$. The simulated results for these parameters with $S_0 = 1339.81, V_0 = 17.66, \mathrm{I}_0 = 43.76$. are graphed below. The fractal–fractional order numerical simulations exhibited in Figs. 1a–1c confirmed the better approximate convergency of solution occurs at the minimal [15–20] arbitrary order $\eta = 0.345$.

The simulated results depict that the fractional order with fractal dimension 1, converges to a desired accuracy for lowest order of differentiation η. The least order decimal value iteration provides convenient results in all the three subclasses S(t), V(t), I(t) through Figs. 1a–1c.

ii. Minimised contact rates:

By minimising the contact rates of unvaccinated and vaccinated with exposed infectives, $\beta = 0.025$, 0.020, 0.015 and $\alpha = 0.02$, 0.01, 0.087 we get the variational flow of infected population I(t). The reduced numerical derivation for most small arbitrary order

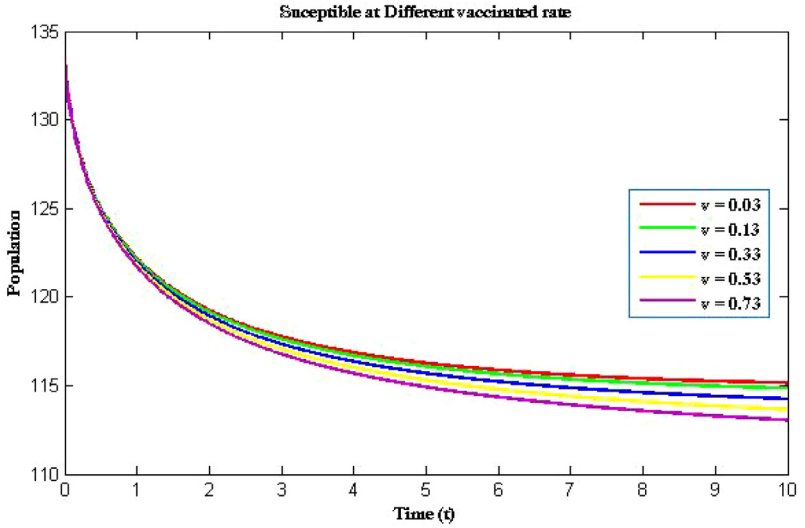

Fig. 3a. The plot of maximising the vaccination ratio results in declining susceptibles for fixed $\eta = 0.645$.

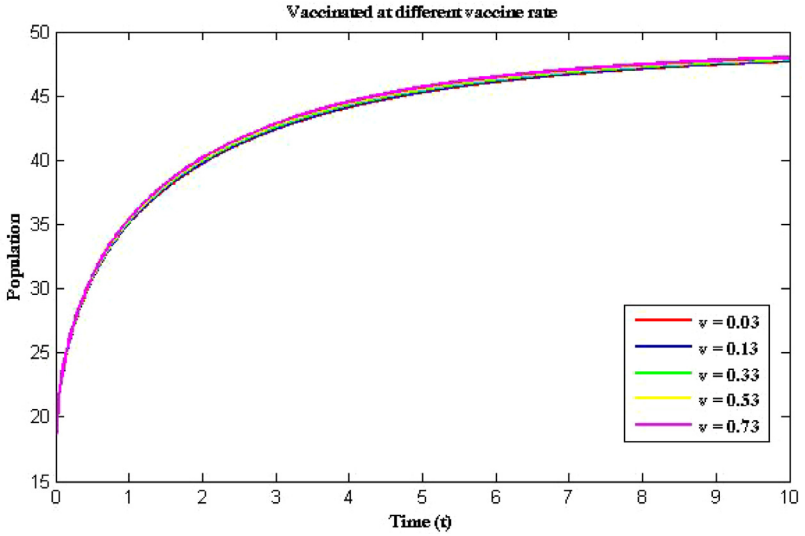

Fig. 3b. The plot depicting peak in vaccinated people with gradually increased vaccination rate for fixed $\eta = 0.645$.

$\eta = 0.645$ shown in different curves of Figs. 2a–2c, ensures an arise in highly inoculated population $V(t)$ due to the decaying contact with infectious. Hence by decreasing the contact rate (β, α) of susceptibles and vaccinated people with infectious ones, the flow of curves predict the control of disease at the expected time. Vaccination rollout is one of the noteworthy control measure in concern of this contagious transmission.

iii. Enlarged vaccination rates:

The numerical interpretation of Fig. 3 suggests the efficient increase in vaccination ratio reduces the vaccinated population against the disease. The impact of vaccine effectiveness in eradicating reproduction of new infections is shown in Figs. 3a and 3b

The effective control steps discussed above aid us with a better understanding of disease transmission. In view of, applied control measures the compartments behave with hike in healthy vaccinated and a gradual trough in infected and unvaccinated susceptibles shown in Fig. 4.

7. Conclusion

The COVID-19 fractal–fractional order SVI model with uni-control measure of vaccination explores the behaviour of three groups susceptibles, vaccinated and infected people in an ailment spread region. The proposed model was examined in terms of non-linear

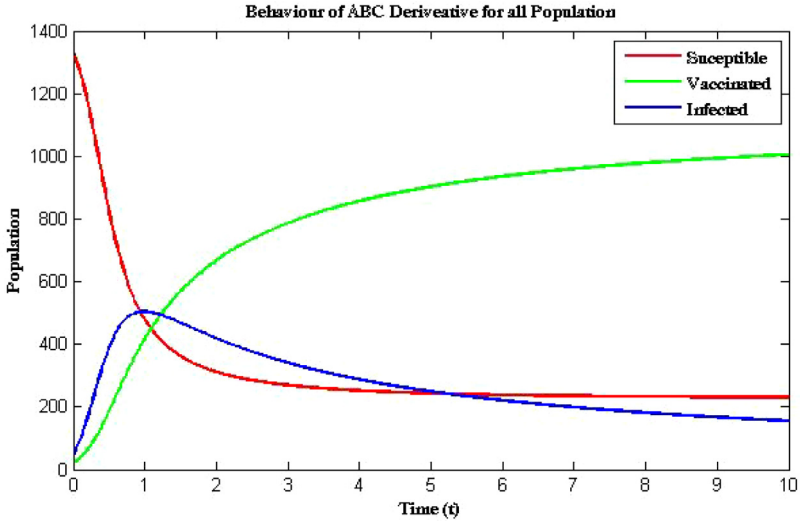

Fig. 4. Convergence behaviour of the pandemic conceptual model.

ABC type fractal–fractional order derivatives. The system is analysed qualitatively for contagion free equilibrium point CFE. The contagion free state CFE is locally asymptotic stable adhering close neighbourhood iff $R_0 < 1$, whereas the disease continues extinctly if $R_0 > 1$, in spite of effective vaccination . Transmission of COVID-19 ailment in India since the implementation of vaccination has been investigated for real data recorded from Our World in data (OWID) web-link. Rapid vaccinations were rolled out for the whole of our nation very expertly. The productive immunity of vaccine shots in triggering the memristive defence force of human cells is well-established in terms of Mittag-Leffler memory of ABC fractal kernel function [44,45]. The effect of mass vaccination shown graphically conjectures the decline in infections' spread effectively in due course are studied with different views and suggested below,

i. Different order of iterations $\eta = 0.345$, 0.445, 0.545, 0.645:

By varying the arbitrary order of differintegrals, the most appropriate results of the formulated non-linear model is achieved.

ii. Reduced contact rates β, α:

Lower-arbitrary order iterations of fractional differintegration for contagious system provide pertinent results as expected [40–51]. Hence fractional order graphs are simulated for the most minimal order $\eta = 0.645$.

Breaking the rate of susceptibles become contagious β, there is a sudden decline of susceptible population $S(t)$ to the ailment free state.

Declining the rate of vaccine breakthrough infections α, gradual hike in vaccinated population $V(t)$ is attained.

iii. Improvised vaccination:

Maximal vaccination results in immuned pinnacle of V(t).

Increased vaccinations 'v' results in decay of Susceptible population $S(t)$ due to movement of susceptible to vaccinated group $V(t)$ and minimalised infections.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Availability of data and materials

Not applicable.

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