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REGULAR PAPER

Transportation problem with interval-valued intuitionistic fuzzy sets: impact of a new ranking

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Abstract

To address uncertainty and hesitation of a real-life problem, interval-valued intuitionistic fuzzy sets (IVIFSs) have received increasing interest among researchers and industrialists. In this paper, we present an advanced illustration of IVIFSs using physical distancing during COVID-19 to understand the deep concept of IVIFSs. Due to special feature of an IVIFSs, it finds a better decision of a real-life problem having uncertainty and hesitation. Here some important arithmetic operations between two IVIFSs are also stated. Ranking of IVIFSs is a valuable tool and it is not easy to rank due to its ill-defined membership and non-membership degrees, and same difficulties arise in a wide variety of real-life problems. To tackle these difficulties, we introduce a new ranking function of IVIFSs, and it follows well to the law of trichotomy. And for its superiority, we compare it with some existing ranking functions by taking a suitable example. Furthermore, its applicability are tested on the basis of an IVIFSs. Further, it is very interesting to note that some unpredicted factors such as road condition, diesel prices, traffic condition and weather condition affect to the cost of transportation, and therefore, decision makers encounter uncertainty and hesitation to estimate cost of transportation. To resolve such issues, we consider transportation problem with IVIFSs parameters, and for its solution, a simple computational method is developed and illustrated.

Keywords Law of trichotomy · Intuitionistic fuzzy sets · Interval-valued intuitionistic fuzzy sets · Transportation problem · Uncertainty

1 Introduction

At present, the role of fuzzy optimization techniques in engineering and management applications has attracted massive attention because of their high accuracy, efficiency and adaptability that provides high-quality realistic results. Fuzzy optimization techniques have been highly explored in health, engineering and industrial sectors. Initially, the concept of mathematical logic was initiated by a greatest philosopher Aristotle. And his law of excluded middle became main tools for proving mathematical assertions. Later Cantor invented the set theory and this theory is presented by characteristic function that uses 0 and 1 only. Many conventional methods of the real-life problems based on fixed data are available in the literature, but due to increasing complexity, the problem based on fixed data cannot present to the situation properly.

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The idea of fuzzy sets (FS) was invented by Zadeh [1] which is an important tool to present the uncertainty and has been used by researchers [2,3], etc. in engineering and management sectors. Further, it is observed that the FS does not deal to the situation of uncertainty and hesitation both of a real-life problem.

Atanassov [4] has extended a fuzzy sets to intuitionistic fuzzy sets (IFS) by incorporating an additional degree, called non-membership degree. IFS is a very realistic and recent tool to deal the problem having uncertainty and hesitation both. Recently, several researchers [5–8] have used it in many sectors of engineering and management. Further, it is observed that a single membership degree and non-membership degree does not state properly to the situation of uncertainty and hesitation in the real-life problems due to ill-defined membership and non-membership degrees, and hence, we admit a kind of further uncertainty.

To enhance the capability of handling uncertainty and hesitation of an IFS, Atanassov and Gargov [9] invented an interval-valued intuitionistic fuzzy sets which is a generalization of IFS in which membership and non-membership

Table 1 Comparison of ranking methods

S.no.	Methods	Ranking	Ranking criteria
1.	Nayagam and Sivaraman [13]	$A_1 < A_2$	Based on new score and accuracy of IVIFSs
2.	Lee ranking method [14]	$A_1 < A_2$	Based on score and accuracy of IVIFSs
3.	Bharati and Singh [46]	$A_1 < A_2$	Based on value and ambiguity index of IVIFNs
4.	Proposed ranking	$A_1 < A_2$	Based on extended Yager's function of IVIFSs

degrees are intervals rather than fixed real number. Current research work has been focusing on operations of IVIFSs and some other interesting properties of IVIFSs [10–12]. Due to increasing complexity of many real-life optimization problem, it is often a challenge for the decision maker to provide the values of parameters in a precise way. Therefore, several research works have been carried out in this direction ranking of FS and IFS. Among several generalizations of FS, the notions of IVIFSs are an interesting and very useful tool in modeling and making decision of real-life problems under uncertainty and hesitation. Many ranking methods of FS and IFS are available and widely implemented in engineering, health and management sectors. And during the study, it is found that a very limited methods are presented in the literature [13,14], and therefore, it is very necessary to make a ranking method for IVIFSs. In the present paper, we introduced a new method of ranking of interval-valued intuitionistic fuzzy sets and compared it with some existing methods Table 1 based on an example.

Transportation problem is well-known optimization technique because of its simplicity and minimum transportation cost. In addition, it exhibits strong performance in real-life optimization problems. Initially, the basic structure of transportation problem is presented by Hitchcock [15] that is described well with linear programming problem. The main objective of a transportation problem is to transport products from a set of supply points to a set of demand points under minimum costs or maximum profits. There are three methods: Northwest corner method, least cost method and Vogel's approximation method (VAM) [16] are often used to determine initial basic solution (IBFS) of TP. VAM is a most common method that used to calculate the IBFS of a TP. The drawback of this method is to allocate items to the dummy cells of TP table. Several researchers [17,18] have modified VAM method of TP. In classical transportation problem, the costs of transportation were taken as fixed real numbers, but it is very interesting to note that the cost of transportation depends on various uncertain factors like fluctuation in diesel price, road condition, weather condition, etc. Therefore, in this situation the cost of single-objective transportation problem (SOTP) cannot be predicted exactly, but it can be estimated by developing a suitable model. Various researchers have estimated the cost of SOTP using FS, and some of them are: [19–21]. In these papers, only membership degree is used in the calculation to get optimal decisions. But in reality, the nature of real-life transportation problem includes hesitation as well which is not tackled by ordinary fuzzy sets.

A real-life transportation problem cannot be restricted to single-objective. Therefore, a multiobjective transportation problem (MOTP) became an important optimization technique. And several researches have been carried out on MOTP such as [12,22–27]. An IFS is expressed by a membership function and a non-membership function, and therefore, it a better tool than FS to deal the problem involving hesitation and uncertainty both. Recently, many research papers focusing on intuitionistic fuzzy transportation problems [28–41] have been published. Recently, Bharati and Malhotra [42] have presented a solution method of two-stage transportation problem (TSTP) using IFS. Liu [43] and Bharati [8] have studied fractional objective transportation problem (FOTP). Further, in ordinary IFS, the degrees of membership and non-membership take the values in the unit interval [0, 1]. In reality, however, we often encounter the situation that the degrees itself is frequently ill-defined as in the statement that the membership and non-membership degrees are "high," "low," "near 0.6," "middle," "not high," "very low," etc. To explain this fact, Atanassov and Gargov IVIFS in which membership and non-membership degrees are subsets of [0, 1] rather than a point in [0, 1]. Methods based on fuzzy and intuitionistic fuzzy sets can be improved by assigning these parameters as IVIFSs.

In this paper, our efforts is to develop an iterative method of an interval-valued intuitionistic fuzzy transportation problem (IVIFTPP). In IVIFTPP, the cost of the transportation problem is represented by triangular interval-valued intuitionistic fuzzy numbers which includes a triangle membership function and a triangle non-membership function, and it would be capable to tackle uncertainty and hesitation. For the optimal solutions of IVIFTPP, a new technique of ranking is adapted and it will be very simple computational viewpoints. The proposed iterative method of IVIFTPP would be attracted massive attention because of their high accuracy, efficiency and adaptability that searches high-quality realistic solutions than the existing methods of FS and IFS (Figs. 1 and 2).

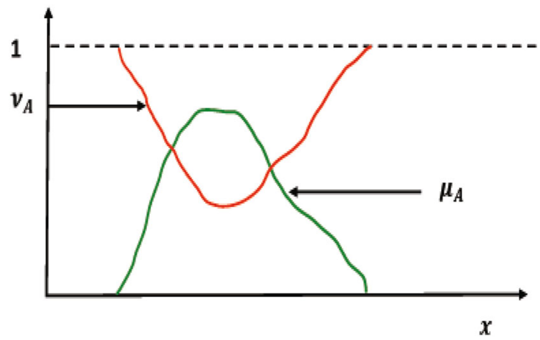

Fig. 1 Intuitionistic fuzzy set

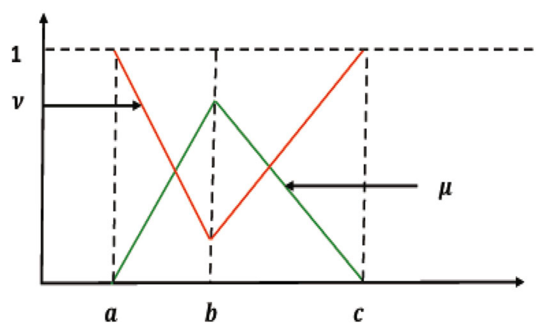

Fig. 2 Triangular intuitionistic fuzzy number

2 Preliminaries

Definition 1 (Atanassov [4]). Let X be an universal set. An intuitionistic fuzzy set A in X is a set of form $\tilde{A} = \{(x, \mu_A(x), \nu_A(x))\}$, where $\mu_A(x) : X \to [0, 1]$ and $\nu_A(x) : X \to [0, 1]$ define the degree of membership and degree of non-membership of the element $x \in X$, respectively, and for every $x \in X$, $0 \leq \mu_A(x)+\nu_A(x) \leq 1$. The value of $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called the degree of non-determinacy (or uncertainty) of the element $x \in X$ to the intuitionistic fuzzy set A. In IFS, if $\pi_A(x) = 0$, then an IFS becomes a FS and it takes the form $A = \{(x, \mu_A(x), 1 - \mu_A(x))\}$.

Definition 2 An intuitionistic fuzzy sets $A = \{(a, b, c), [\mu, \nu]\}$ where $a, b, c \in \mathbb{R}$ such that $a \leq b \leq c$. Then A is called a triangular intuitionistic fuzzy number if its membership and non-membership functions are of the form:

$$\mu_A(x) = \begin{cases} \mu, \ x = b \\ 0, \ x \geq c, x \leq a \\ \frac{x-a}{b-a}\mu, a < x < b \\ \frac{c-x}{c-b}\mu, b < x < c \end{cases} \tag{1}$$

$$\nu_A(x) = \begin{cases} \nu, \ x = b \\ 1, \ x \geq c, x \leq a \\ 1 - \frac{(1-\nu)(x-a)}{b-a}, a < x < b \\ \nu - \frac{(1-\nu)(x-b)}{c-b}, b < x < c \end{cases} \tag{2}$$

This study presents two main contributions: The first contribution of this study is to deal with the formulation of a new ranking function of interval-valued intuitionistic fuzzy numbers based on Yager's approach. Furthermore, it is felt that today is highly competitive market, the pressure on organizations to find better ways to create and deliver value to customers becomes stronger. The second contribution of this study is to deal with the formulation of a kind of transportation problems, known as interval-valued intuitionistic transportation problem that provide a powerful framework to meet this challenge.

2.1 Interval-valued intuitionistic fuzzy sets

The uncertainty and hesitation occur in every real-life problem, and therefore, it is very necessary to explain it. Now, suppose X represents set of 100 peoples of a village, and if we ask about the number of people who follow physical distancing during COVID-19 pandemic, the natural answer that we get are [30, 40], [35, 40], etc., and the number of people who do not follow physical distancing are [5, 10], [6, 8], etc. In the same manner, let $X = \{x_1, x_2, \ldots, x_N\}$ be the set of N people in a village. And let the number of people who follow physical distancing during COVID-19 pandemic be $[m_1(x), m_2(x)]$ and number of people who do not follow be $[n_1(x), n_2(x)]$.
Then $[m_1(x), m_2(x)] + [n_1(x), n_2(x)] \leq N$

$\Rightarrow \frac{[m_1(x), m_2(x)] + [n_1(x), n_2(x)]}{N} \leq 1$, since$N > 0$; hence, the following inequalities make sense
$\Rightarrow [\frac{[m_1(x), m_2(x)]}{N}] + [\frac{[n_1(x), n_2(x)]}{N}] \leq 1$
$\Rightarrow [\frac{m_1(x)}{N}, \frac{m_2(x)}{N}] + [\frac{n_1(x)}{N}, \frac{n_2(x)}{N}] \leq 1$

Therefore, $\{x \in X : [\frac{m_1(x)}{N}, \frac{m_2(x)}{N}], [\frac{n_1(x)}{N}, \frac{n_2(x)}{N}]\}$ is an interval-valued intuitionistic fuzzy set. Now, we shall represent the formal definition of interval-valued intuitionistic fuzzy sets.

Definition 3 (Atanassov and Gargov [9]). Let X be an universal set. An interval-valued intuitionistic fuzzy A in X is expressed as $A = \{(x, [\mu_A^-(x), \mu_A^+(x)], [\nu_A^-(x), \nu_A^+(x)]) : x \in X\}$, where $\mu_A^-(x) : X \to [0, 1]$, $\mu_A^+(x) : X \to [0, 1]$ define the lower and upper degrees of memberships, and $\nu_A^- : X \to [0, 1]$, $\nu_A^- : X \to [0, 1]$ define lower and upper degrees of non-memberships of the element $x \in X$. And for every $x \in X$, $0 \leq \mu_A^+(x) + \nu_A^+(x) \leq 1$.

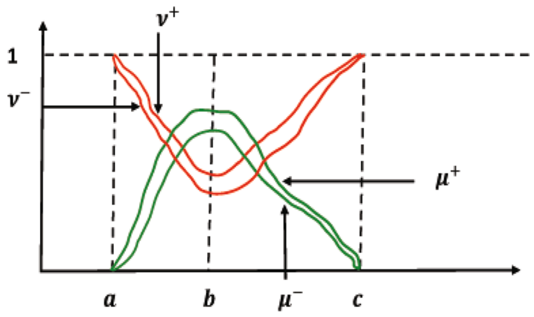

Fig. 3 Interval-valued intuitionistic fuzzy sets

The graphical representation of IVIFS is given in Fig. 3.

Definition 4 An interval-valued intuitionistic fuzzy number is expressed as:
$A = \{(a, b, c) : [\mu^-, \mu^+], [\nu^-, \nu^+])\}$, where $\mu^- : X \rightarrow [0, 1]$, $\mu^+ : X \rightarrow [0, 1]$ define the lower and upper degrees of memberships, and $\nu^- : X \rightarrow [0, 1]$, $\nu^+ : X \rightarrow [0, 1]$ define lower and upper degrees of non-memberships, and these are:

$$\mu_A^-(x) = \begin{cases} \mu^- \frac{(x-a)}{(b-a)}, & a < x < b \\ \mu^-, & x = b \\ \mu^- \frac{(c-x)}{(c-b)}, & b < x < c \end{cases} \tag{3}$$

$$\mu_A^+(x) = \begin{cases} \mu^+ \frac{(x-a)}{(b-a)}, & a < x < b \\ \mu^+, & x = b \\ \mu^+ \frac{(c-x)}{(c-b)}, & b < x < c \end{cases} \tag{4}$$

$$\nu_A^-(x) = \begin{cases} 1 - (1 - \nu^-) \frac{(x-a)}{(b-a)}, & a < x < b \\ \nu^-, & x = b \\ \nu^- + (1 - \nu^-) \frac{(x-b)}{(c-b)}, & b < x < c \end{cases} \tag{5}$$

$$\nu_A^+(x) = \begin{cases} 1 - (1 - \nu^+) \frac{(x-a)}{(b-a)}, & a < x < b \\ \nu^+, & x = b \\ \nu^+ + (1 - \nu^+) \frac{(x-b)}{(c-b)}, & b < x < c \end{cases} \tag{6}$$

2.2 Arithmetic operations

After IFS, IVIFS became a very popular tool in decision making due its special features. Li [44] proposed representation theorem of IVIF and defined operations between IVIFS. In this paper, we present arithmetic operations for triangular interval-valued intuitionistic fuzzy numbers. For this, let $A = \{(a_1, b_1, c_1), [\mu_A^-, \mu_A^+], [\nu_A^-, \nu_A^+])\}$ and $B = \{(a_2, b_2, c_2), [\mu_B^-, \mu_B^+], [\nu_B^-, \nu_B^+])\}$ be two triangular interval-valued intuitionistic fuzzy numbers, then

$$\begin{aligned} A \oplus B = \{&(a_1 + a_2, b_1 + b_2, c_1 + c_2), \\ &[\min(\mu_A^-, \mu_B^-) - \min(\mu_A^+, \mu_B^+)], \\ &[\max(\nu_A^-, \nu_B^-) - \max(\nu_A^+, \nu_B^+)]\} \end{aligned} \tag{7}$$

$$\begin{aligned} A \ominus B = \{&(a_1 - c_2, b_1 - b_2, c_1 - a_2), \\ &[\min(\mu_A^-, \mu_B^-) - \min(\mu_A^+, \mu_B^+)], \\ &[\max(\nu_A^-, \nu_B^-) - \max(\nu_A^+, \nu_B^+)]\} \end{aligned} \tag{8}$$

$$A \odot B = \begin{cases} \langle (a_1 a_2, b_1 b_2, c_1 c_2); \left[\min\{\mu_A^-, \mu_B^-\}, \min\{\mu_A^+, \mu_B^+\}\right], \\ \left[\max\{\nu_A^-, \nu_B^-\}, \max\{\nu_A^+, \nu_B^+\}\right] \rangle \text{ if } a_1, a_2 \in \mathbb{R}^+ \\ \langle (a_1 c_2, b_1 b_2, c_1 a_2); \left[\min\{\mu_A^-, \mu_B^-\}, \min\{\mu_A^+, \mu_B^+\}\right], \\ \left[\max\{\nu_A^-, \nu_B^-\}, \max\{\nu_A^+, \nu_B^+\}\right] \rangle \text{ if } a_1 < 0 \text{ and } a_2 > 0 \\ \langle (c_1 c_2, b_1 b_2, a_1 a_2); \left[\min\{\mu_A^-, \mu_B^-\}, \min\{\mu_A^+, \mu_B^+\}\right], \\ \left[\max\{\nu_A^-, \nu_B^-\}, \max\{\nu_A^+, \nu_B^+\}\right] \rangle \text{ if } c_1 < 0 \text{ and } c_2 > 0 \end{cases} \tag{9}$$

$$A \oslash B = \begin{cases} \left\langle \left(\frac{a_1}{c_2}, \frac{b_1}{b_2}, \frac{c_1}{a_2}\right); \left[\min\{\mu_A^-, \mu_B^-\}, \min\{\mu_A^+, \mu_B^+\}\right], \\ \left[\max\{\nu_A^-, \nu_B^-\}, \max\{\nu_A^+, \nu_B^+\}\right] \right\rangle \text{ if } c_1, c_2 \in \mathbb{R}^+ \\ \left\langle \left(\frac{c_1}{c_2}, \frac{b_1}{b_2}, \frac{a_1}{a_2}\right); \left[\min\{\mu_A^-, \mu_B^-\}, \min\{\mu_A^+, \mu_B^+\}\right], \\ \left[\max\{\nu_A^-, \nu_B^-\}, \max\{\nu_A^+, \nu_B^+\}\right] \right\rangle \text{ if } c_1 < 0 \text{ and } c_2 > 0 \\ \left\langle \left(\frac{c_1}{a_2}, \frac{b_1}{b_2}, \frac{a_1}{c_2}\right); \left[\min\{\mu_A^-, \mu_B^-\}, \min\{\mu_A^+, \mu_B^+\}\right], \\ \left[\max\{\nu_A^-, \nu_B^-\}, \max\{\nu_A^+, \nu_B^+\}\right] \right\rangle \text{ if } c_1 < 0 \text{ and } c_2 < 0 \end{cases} \tag{10}$$

$$k\dot{A} = \begin{cases} \langle (ka, kb, kc), [\mu_A^-, \mu_A^+], [\nu_A^-, \nu_A^+] \rangle \text{ if } k > 0 \\ \langle (kc, kb, ka), [\mu_A^-, \mu_A^+], [\nu_A^-, \nu_A^+] \rangle \text{ if } k < 0 \end{cases} \tag{11}$$

$$A^{-1} = \left\{ \left(\frac{1}{c}, \frac{1}{b}, \frac{1}{a} \right) [\mu_A^-, \mu_A^+], [\nu_A^-, \nu_A^+] \right\} \text{ if } a > 0 \tag{12}$$

(i). *Triangular intuitionistic fuzzy number:*
For a triangular intuitionistic fuzzy number, $\mu_A^- = \mu_A^+ = \mu_A$ and $\nu_A^- = \nu_A^+ = \nu_A$. Relations from (7) to (12) become: For this, let $A = \{(a_1, b_1, c_1); \{\mu_A, \nu_A\}\}$ and $B = \{(a_2, b_2, c_2); \{\mu_B, \nu_B\}\}$ be two triangular intuitionistic fuzzy numbers, then

$$A \oplus B = \{(a_1 + a_2, b_1 + b_2, c_1 + c_2); \{\min(\mu_A, \mu_B), \max(\nu_A, \nu_B)\}\} \tag{13}$$

$$A \ominus B = \{(a_1 - c_2, b_1 - b_2, c_1 - a_2); \{\min(\mu_A, \mu_B), \max(\nu_A, \nu_B)\}\} \tag{14}$$

$$A \odot B = \begin{cases} \langle(a_1a_2, b_1b_2, c_1c_2); \{\min(\mu_A, \mu_B), \max(\nu_A, \nu_B)\}\rangle \text{ if } a_1, a_2 \in \mathbb{R}^+ \\ \langle(a_1c_2, b_1b_2, c_1a_2); \{\min(\mu_A, \mu_B), \max(\nu_A, \nu_B)\}\rangle \text{ if } a_1 < 0 \text{ and } a_2 > 0 \\ \langle(c_1c_2, b_1b_2, a_1a_2); \{\min(\mu_A, \mu_B), \max(\nu_A, \nu_B)\}\rangle \text{ if } c_1 < 0 \text{ and } c_2 > 0 \end{cases} \tag{15}$$

$$A \oslash B = \begin{cases} \left\langle\left(\frac{a_1}{c_2}, \frac{b_1}{b_2}, \frac{c_1}{a_2}\right); \{\min(\mu_A, \mu_B), \max(\nu_A, \nu_B)\}\right\rangle \text{ if } c_1, c_2 \in \mathbb{R}^+ \\ \left\langle\left(\frac{c_1}{c_2}, \frac{b_1}{b_2}, \frac{a_1}{a_2}\right); \{\min(\mu_A, \mu_B), \max(\nu_A, \nu_B)\}\right\rangle \text{ if } c_1 < 0 \text{ and } c_2 > 0 \\ \left\langle\left(\frac{c_1}{a_2}, \frac{b_1}{b_2}, \frac{a_1}{c_2}\right); \{\min(\mu_A, \mu_B), \max(\nu_A, \nu_B)\}\right\rangle \text{ if } c_1 < 0 \text{ and } c_2 < 0 \end{cases} \tag{16}$$

$$k\dot{A} = \begin{cases} \langle(ka, kb, kc), \{\mu_A, \nu_A\}\rangle \text{ if } k > 0 \\ \langle(kc, kb, ka), \{\mu_A, \nu_A\}\rangle \text{ if } k < 0 \end{cases} \tag{17}$$

$$A^{-1} = \left\{\left(\frac{1}{c}, \frac{1}{b}, \frac{1}{a}\right); \{\mu_A, \nu_A\}\right\} \text{ if } a > 0 \tag{18}$$

(ii). *Triangular fuzzy number:*
For fuzzy number, $\mu_A^- = \mu_A^+ = 1$ and $\nu_A^- = \nu_A^+ = 0$. Relations from (7) to (12) become:
For this, let $A = \{(a_1, b_1, c_1); \{1, 0\}\}$ and $B = \{(a_2, b_2, c_2); \{\mu_B, \nu_B\}\}$ be two triangular intuitionistic fuzzy numbers, then

$$A \oplus B = \{(a_1 + a_2, b_1 + b_2, c_1 + c_2)\}\} \tag{19}$$

$$A \ominus B = \{(a_1 - c_2, b_1 - b_2, c_1 - a_2)\} \tag{20}$$

$$A \odot B = \begin{cases} \langle(a_1a_2, b_1b_2, c_1c_2) & \text{if } a_1, a_2 \in \mathbb{R}^+ \\ \langle(a_1c_2, b_1b_2, c_1a_2) & \text{if } a_1 < 0 \text{ and } a_2 > 0 \\ \langle(c_1c_2, b_1b_2, a_1a_2) & \text{if } c_1 < 0 \text{ and } c_2 > 0 \end{cases} \tag{21}$$

$$A \oslash B = \begin{cases} \left\langle\left(\frac{a_1}{c_2}, \frac{b_1}{b_2}, \frac{c_1}{a_2}\right)\right. & \text{if } c_1, c_2 \in \mathbb{R}^+ \\ \left\langle\left(\frac{c_1}{c_2}, \frac{b_1}{b_2}, \frac{a_1}{a_2}\right)\right. & \text{if } c_1 < 0 \text{ and } c_2 > 0 \\ \left\langle\left(\frac{c_1}{a_2}, \frac{b_1}{b_2}, \frac{a_1}{c_2}\right)\right. & \text{if } c_1 < 0 \text{ and } c_2 < 0 \end{cases} \tag{22}$$

$$k\dot{A} = \begin{cases} \langle(ka, kb, kc)\rangle \text{ if } k > 0 \\ \langle(kc, kb, ka) \text{ if } k < 0 \end{cases} \tag{23}$$

$$A^{-1} = \left\{\left(\frac{1}{c}, \frac{1}{b}, \frac{1}{a}\right)\right\} \text{ if } a > 0 \tag{24}$$

2.3 (α, β)-level sets

Let $\alpha \in \mathbb{R}$ such that $\alpha \in (0, 1)$. Then for $A = \{(a, b, c) : [\mu^-, \mu^+], [\nu^-, \nu^+])\}$ the $\alpha-$ cut of that TIVIFN is defined as:

$$\mu_A^-(x) \geq \alpha$$
$$\Rightarrow \mu^- \tfrac{(x-a)}{(b-a)} \geq \alpha$$
$$\Rightarrow \tfrac{(x-a)}{(b-a)} \geq \tfrac{\alpha}{\mu^-}$$
$$\Rightarrow (x-a) \geq \tfrac{\alpha}{\mu^-}(b-a)$$
$$\Rightarrow x \geq a + \tfrac{\alpha}{\mu^-}(b-a)$$

Now, $\mu_A^-(x) \geq \alpha$

$$\Rightarrow \mu^- \tfrac{(c-x)}{(c-b)} \geq \alpha$$
$$\Rightarrow \tfrac{(c-x)}{(c-b)} \geq \tfrac{\alpha}{\mu^-}$$
$$\Rightarrow (c-x) \geq \tfrac{\alpha}{\mu^-}(c-b)$$
$$\Rightarrow x \leq c - \tfrac{\alpha}{\mu^-}(c-b)$$
$$\Rightarrow x \in \left[a + \tfrac{\alpha}{\mu^-}(b-a), c - \tfrac{\alpha}{\mu^-}(c-b)\right]$$

Hence, we get

$$\left[a + \frac{\alpha}{\mu^-}(b-a), c - \frac{\alpha}{\mu^-}(c-b)\right] \tag{25}$$

Similarly,

$$\mu_A^+(x) \geq \alpha$$
$$\Rightarrow \mu^+ \tfrac{(x-a)}{(b-a)} \geq \alpha$$
$$\Rightarrow \tfrac{(x-a)}{(b-a)} \geq \tfrac{\alpha}{\mu^+}$$
$$\Rightarrow (x-a) \geq \tfrac{\alpha}{\mu^+}(b-a)$$
$$\Rightarrow x \geq a + \tfrac{\alpha}{\mu^+}(b-a)$$

Now, $\mu_A^+(x) \geq \alpha$

$$\Rightarrow \mu^+ \tfrac{(c-x)}{(c-b)} \geq \alpha$$
$$\Rightarrow \tfrac{(c-x)}{(c-b)} \geq \tfrac{\alpha}{\mu^+}$$

$\Rightarrow (c-x) \geq \frac{\alpha}{\mu^+}(c-b)$
$\Rightarrow x \leq c - \frac{\alpha}{\mu^+}(c-b)$

$$\Rightarrow x \in \left[a + \frac{\alpha}{\mu^+}(b-a), c - \frac{\alpha}{\mu^+}(c-b)\right]$$

Further, $\nu_A^-(x) \leq 1-\alpha$

$\Rightarrow 1-(1-\nu^-)\frac{(x-a)}{(b-a)} \leq 1-\alpha$
$\Rightarrow (1-\nu^-)\frac{(x-a)}{(b-a)} \geq \alpha$
$\Rightarrow \frac{(x-a)}{(b-a)} \geq \frac{\alpha}{(1-\nu^-)}$
$\Rightarrow (x-a) \geq (b-a)\frac{\alpha}{(1-\nu^-)}$
$\Rightarrow x \geq a + (b-a)\frac{\alpha}{(1-\nu^-)}$
$\nu_A^-(x) \leq 1-\alpha$
$\Rightarrow \nu^- + (1-\nu^-)\frac{(x-b)}{(c-b)} \leq 1-\alpha$
$\Rightarrow (1-\nu^-)\frac{(x-b)}{(c-b)} \leq 1-\nu^- -\alpha$
$\Rightarrow \frac{(x-b)}{(c-b)} \leq 1 - \frac{\alpha}{1-\nu^-}$
$\Rightarrow (x-b) \leq c-b-\frac{\alpha}{1-\nu^-}(c-b)$
$\Rightarrow x \leq c - \frac{\alpha}{1-\nu^-}(c-b)$

$$\Rightarrow x \in \left[a + (b-a)\frac{\alpha}{(1-\nu^-)}, c - \frac{\alpha}{1-\nu^-}(c-b)\right]$$

Similarly,

$$x \in \left[a + (b-a)\frac{\alpha}{(1-\nu^+)}, c - \frac{\alpha}{1-\nu^+}(c-b)\right].$$

3 A new ranking

In this section, we extend Yager's function [45] to help in the ranking of interval-valued intuitionistic fuzzy numbers. This function is the integral of the mean of the level sets associated with lower and upper memberships, and similarly with lower non-membership and upper non-memberships. We also verified some properties of the introduced functions. The merit of this function is that it does not require convexity, nor does it require normality of the interval-valued intuitionistic fuzzy sets ranked. Let $A_\alpha^{l\mu}$ and $A_\alpha^{u\mu}$ be level sets corresponding to lower and upper membership functions, respectively.
Similarly, let $A_\beta^{l\nu}$ and $A_\beta^{u\nu}$ be level sets corresponding to lower and upper non-membership functions, respectively.

Let $m(A_\alpha^{l\mu})$ and $m(A_\alpha^{u\mu})$ be means of level sets of lower and upper memberships, respectively.

Similarly, let $m(A_\beta^{l\nu})$ and $m(A_\beta^{u\nu})$ be means of level sets of lower and upper non-memberships, respectively. Then,

$$\begin{aligned} f_l^\mu(A) &= \int_0^{\max\ \mu} m(A_\alpha^\mu)\mathrm{d}\alpha \\ &= \frac{1}{2}\int_0^{\mu^-} a + \frac{\alpha}{\mu^-}(b-a) + c - \frac{\alpha}{\mu^-}(c-b)\mathrm{d}\alpha \\ &= \frac{1}{4}(a+2b+c)\mu - \\ f_l^\mu(A) &= \frac{1}{4}(a+2b+c)\mu^- \end{aligned}$$

Similarly for upper membership:

$$\begin{aligned} f_u^\mu(A) &= \int_0^{\max\ \mu} m(A_\alpha^\mu)\mathrm{d}\alpha \\ &= \frac{1}{2}\int_0^{\mu^+} a + \frac{\alpha}{\mu^+}(b-a) + c - \frac{\alpha}{\mu^+}(c-b)\mathrm{d}\alpha \\ &= \frac{1}{4}(a+2b+c)\mu^+ \\ f_u^\mu(A) &= \frac{1}{4}(a+2b+c)\mu^+ \\ F^\mu(A_\alpha^\mu) &= \sigma f_l^\mu(A) + (1-\sigma)f_u^\mu(A), \sigma \in [0,1] \end{aligned}$$

Since average represents a good choice, we take $\sigma = 0.5$

$$F^\mu(A_\alpha^\mu) = \frac{1}{8}(a+2b+c)(\mu^- + \mu^+) \tag{26}$$

In the same manner, we can proceed for non-memberships

$$\begin{aligned} f_l^\nu(A) &= \int_{\nu-}^1 m(A_\beta^\nu)\mathrm{d}\beta, \\ &= \int_{\nu-}^1 a + \frac{1-\beta}{1-\nu^-}(b-a) + b \\ &\quad + \left(1 - \frac{\beta-\nu^-}{1-\nu^-}\right)(c-b)\mathrm{d}\beta \\ &= \frac{1}{4}(a+2b+c)(1-\nu^-) \\ f_l^\nu(A) &= \frac{1}{4}(a+2b+c)(1-\nu^-) \\ &\times f_u^\nu(A) = \int_{\nu+}^1 m(A_\beta^\nu)\mathrm{d}\beta, \\ &= \int_{\nu+}^1 a + \frac{1-\beta}{1-\nu^+}(b-a) + b \\ &\quad + \left(1 - \frac{\beta-\nu^+}{1-\nu^+}\right)(c-b)\mathrm{d}\beta \\ &= \frac{1}{4}(a+2b+c)(1-\nu^+) \\ f_u^\nu(A) &= \frac{1}{4}(a+2b+c)(1-\nu^+). \end{aligned}$$

$$F^{\nu}(A^{\nu}_{\alpha}) = \sigma f^{\nu}_{l}(A) + (1-\sigma) f^{\nu}_{u}(A), \sigma \in [0,1]$$

Since average represents a good choice, we take $\sigma = 0.5$

$$F^{\nu}(A^{\nu}_{\alpha}) = \frac{1}{8}(a+2b+c)(2-\nu^{-}-\nu^{+}) \tag{27}$$

3.1 Properties of F^{μ} and F^{ν}

Property 1 Let $A = (a)$ be a crisp number then $F^{\mu}(a) = a$

Proof For a crisp number, max $\mu = \mu^{-} = \mu^{+} = 1$, max $\nu = \nu^{-} = \nu^{+} = 0$,

$$A^{\mu}_{\alpha} = a$$
$$F^{\mu}(A) = \int_0^{\max\ \mu} m(A^{\mu}_{\alpha}) d\alpha = \int_0^1 a \mathrm{d}\alpha = a$$

So, $F^{\mu}(a) = a$ and $F^{\nu}(a) = 0$. □

Property 2 If A is an ordinary subset of $\mathbb{R}$, then $F^{\mu}(A) = m(A)$ (F^{ν} does not make sense).

Proof Let $A = [a, b]$ be a subset of $\mathbb{R}$, then $m(A) = \frac{a+b}{2}$.

$$F^{\mu}(A) = \int_0^{\max\ \mu} m(A^{\mu}_{\alpha}) \mathrm{d}\alpha = \int_0^1 \frac{a+b}{2} \mathrm{d}\alpha$$
$$= \frac{a+b}{2} = \text{mean of } A.$$

□

Property 3 If $A = (p : \mu(p) = q)$. Then $F^{\mu}(A) = pq$.

Proof Clearly $A^{\mu}_{\alpha} = p$ and max $\mu = q$. Then by definition $F^{\mu}(A) = \int_0^{\max\ \mu} m(A^{\mu}_{\alpha})\mathrm{d}\alpha = \int_0^q p\mathrm{d}\alpha = pq$. □

Property 4 Let A be interval-valued intuitionistic fuzzy number and a be a crisp number such that $a \geq 1$. Then $F^{\mu}\left(\frac{A}{a}\right) = \frac{F^{\mu}(A)}{a}$.

Proof Let $B = \frac{A}{a}$ and $B = \frac{([\mu^{-}(x),\mu^{+}(x)],[\nu^{-}(x),\nu^{+}(x)])}{\frac{x}{a}}$.
If $z \in A$, then $\frac{x}{a} \in A^{\mu}_{\alpha}$ and $\frac{x}{a} \in B^{\nu}_{\alpha}$.
Therefore, $F^{\mu}(\frac{A}{a}) = \int_0^1 \frac{1}{a} A^{\mu}_{\alpha} \mathrm{d}\alpha = \frac{1}{a}\int_0^1 A^{\mu}_{\alpha}\mathrm{d}\alpha = \frac{F^{\mu}(A)}{a}$ (Fig. 4).
According to the law of trichotomy, every $x, y \in \mathbb{R}$ either $x < y$ or $x > y$ or $x = y$. It is pointed out that the law of trichotomy holds in classical logic, and it does not hold in fuzzy logic. In this paper, we introduce a new function $R : \aleph(\mathbb{R}) \to \mathbb{R}$ that assigns each interval-valued intuitionistic fuzzy number to a real number. The ranking function which is based on (26) and (27) is defined by

$$R(A) = \eta F^{\mu}(A^{\mu}_{\alpha}) + (1-\eta) F^{\nu}(A^{\nu}_{\beta}),$$
$$\eta \in [0,1], \alpha + \beta < 1. \tag{28}$$

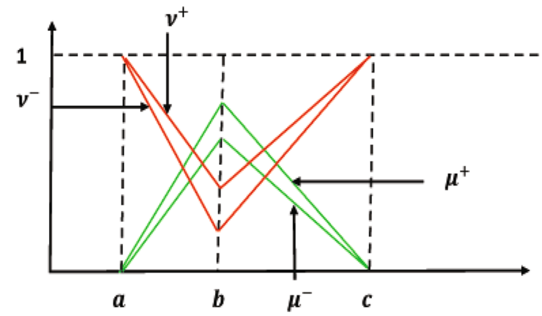

Fig. 4 Interval-valued intuitionistic triangular fuzzy number

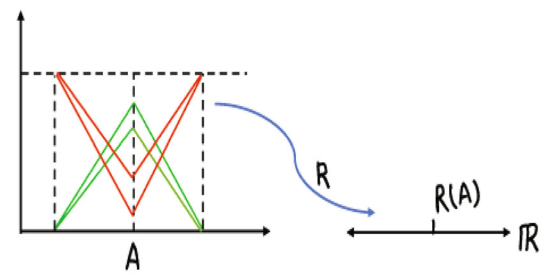

Fig. 5 Ranking function of interval-valued intuitionistic fuzzy sets

Figure 5 shows the ranking function of a collection of interval-valued intuitionistic fuzzy sets of real numbers.

$\aleph(\mathbb{R})$ denotes the collections of all interval-valued intuitionistic fuzzy numbers on $\mathbb{R}$, Therefore, ranking of interval-valued intuitionistic fuzzy sets is redefined as in the following manner

$$R(A) = \eta F^{\mu}(A^{\mu}_{\alpha}) + (1-\eta) F^{\nu}(A^{\nu}_{\beta}), \eta \in [0,1]$$

$\eta = 0.5$ represents best compromise choice, and thus, we take the same. □

Lemma *Let A and B be two interval-valued intuitionistic fuzzy numbers, and $R(A) = \frac{(a+2b+c)(\mu^{-}+\mu^{+}+2-\nu^{-}-\nu^{+})}{16}$. Then exactly one of the following is true:*

(i) *If $R(A_1) < R(A_2)$, then $A_1 < A_2$.*
(ii) *If $R(A_1) > R(A_2)$, then $A_1 > A_2$.*
(iii) *If $R(A_1) = R(A_2)$, then $A_1 = A_2$.*

Some remarks on proposed ranking function:

Remark 1 If A is an intuitionistic fuzzy number, then

$$R(A) = \frac{(a+2b+c)(\mu+\mu+2-\nu-\nu)}{16}$$
$$R(A) = \frac{(a+2b+c)(2\mu+2-2\nu)}{16}$$
$$R(A) = \frac{(a+2b+c)(\mu+1-\nu)}{8}.$$

Remark 2 If A is a triangular fuzzy number, then $R(A) = \frac{(a+2b+c)\mu}{4}$.

To see this, let A be a triangular fuzzy number. We can express A in interval-valued intuitionistic fuzzy sense as $A = \{(a, b, c), [\mu, \mu], [1 - \mu, 1 - \mu]\}$. For fuzzy set, $\mu + \nu = 1$ or $\nu = 1 - \mu$.
Then

$$R(A) = \frac{(a+2b+c)(\mu+\mu+2-(1-\mu)-(1-\mu))}{16}$$
$$R(A) = \frac{(a+2b+c)(\mu+\mu+2-1+\mu)-1+\mu))}{16}$$
$$R(A) = \frac{(a+2b+c)(4\mu)}{16}$$
$$R(A) = \frac{(a+2b+c)\mu}{4}$$

which is a very famous ranking of fuzzy numbers that have been utilized by several researchers.

Remark 3 If $A = a$ is a fixed real number, then $R(A) = a$. To see this, let A be a fixed real number. We can express any fixed real number in interval-valued intuitionistic fuzzy sense as $A = \{(a, a, a), [1, 1], [0, 0]\}$. Then

$$R(A) = \frac{(a+2a+a)(1+1+2-0-0)}{16}$$
$$R(A) = \frac{(16a)}{16}.$$
$$R(A) = a.$$

4 Comparison

In this section, the proposed ranking function that is defined by $R(A) = \frac{(a+2b+c)(\mu^{-}+\mu^{+}+2-\nu^{-}-\nu^{+})}{16}$ with some existing ranking function of interval-valued intuitionistic fuzzy sets. Nayagam and Sivaraman [13] presented an ranking function to rank IVIF sets, and that is defined as
Let $A = \{[a, b], [c, d]\}$ be an interval-valued intuitionistic fuzzy sets, then the ranking of A is defined as $LG(A) = \frac{\text{membership degree}+\delta\text{hesitancy degree}}{2}$, $\delta \in [0, 1]$. After simplification, we get

$$LG(A) = \frac{(a+b)(1-\delta)+\delta(2-(c+d))}{2}, \delta \in [0, 1]. \quad (29)$$

Lee [14] introduced the concept of novel score and deviation of interval-valued intuitionistic fuzzy sets. Further, he proposed a ranking methodology to rank a collection of interval-valued intuitionistic fuzzy sets based on novel score and deviation and the method are:

Let $A = \{[a, b], [c, d]\}$ be an interval-valued intuitionistic fuzzy sets then novel score $S(A)$ and deviation $D(A)$ are

$$S(A) = \frac{2+a+b-c-d}{3-a-b-c-d} \quad (30)$$
$$D(A) = b+d-a-c \quad (31)$$

For the comparison, we take two interval-valued intuitionistic fuzzy subsets of real numbers $A_1 = \{(1, 2, 3), [0.1, 0.2], [0.3, 0.5]\}$ and $A_2 = \{(1, 4, 7), [0.1, 0.2], [0.2, 0.3]\}$.
Proposed ranking function:

$$R(A) = \frac{(a+2b+c)(2+\mu^{-}+\mu^{+}-(\nu^{-}+\nu^{+}))}{16}$$
$$R(A_1) = \frac{(1+2(2)+3)(2+0.1+0.2-(0.3+0.5))}{16}$$
$$R(A_1) = \frac{(8)(1.5))}{16}$$
$$R(A_1) = 0.75$$

and

$$R(A_2) = \frac{(1+2(4)+7)(2+0.1+0.2-(0.2+0.3))}{16}$$
$$R(A_2) = \frac{(16)(1.8))}{16}$$
$$R(A_2) = 1.8$$

Clearly, $R(A_1) < R(A_2)$.
Therefore, $A_1 < A_2$.
Nayagam and Sivaraman ordering (29):

$$LG(A_1) = \frac{(0.1+0.2)(1-\delta)+\delta(2-(0.3+0.5))}{2}$$
$$LG(A_1) = \frac{0.3(1-\delta)+\delta(1.2)}{2}$$
$$LG(A_2) = \frac{(0.1+0.2)(1-\delta)+\delta(2-(0.2+0.3))}{2}$$
$$LG(A_2) = \frac{0.3(1-\delta)+\delta(1.5)}{2}$$

It is very clear that $LG(A_1) < LG(A_2)$ for every $\delta \in [0, 1]$.
Therefore, $A_1 < A_2$.
Using Lee ranking (30), we get

$$S(A_1) = \frac{2+0.1+0.2-0.3-0.5}{3-0.1-0.2-0.3-0.5} = \frac{3.1}{1.2} = 0.78$$
$$S(A_2) = \frac{2+0.1+0.2-0.2-0.3}{3-0.1-0.2-0.2-0.3} = \frac{1.5}{1.2} = 0.81$$

and

$$D(A_1) = 0.2+0.5-0.1-0.3 = 0.3$$
$$D(A_2) = 0.2+0.3-0.1-0.2 = 0.2$$

Since $S(A_1) < S(A_2)$, $A_1 < A_2$.

Finally we conclude that the proposed ranking function agrees with Nayagam and Sivaraman ranking function and Lee ranking, and the main difference between proposed function and existing function is: In the proposed ranking based on Yager's function, where as in Nayagam and Sivaraman and Lee function based on score and accuracy of interval-valued intuitionistic fuzzy sets, Bharati and Singh ranking is based on value and ambiguity indices of interval-valued intuitionistic fuzzy sets.

5 Interval-valued intuitionistic fuzzy transportation problem

An IVIFTPP is a very special case of interval-valued intuitionistic fuzzy linear programming problem (IVIFLPP), and IVIFLPP is solved by using simplex method. Simplex method provides a very weak initial basic solution to IVIFTPP, and it takes large time of computation. Therefore, for the basic feasible solutions of IVIFTPP, we may use one of the three methods: interval-valued intuitionistic fuzzy northwest corner method (IVIFNWCM), interval-valued intuitionistic fuzzy least cost method (IVIFLCM) and interval-valued intuitionistic fuzzy Vogel's approximation method (IVIFVAM). And its interval-valued intuitionistic fuzzy optimal solution is obtained by using interval-valued intuitionistic fuzzy u–v method. In this paper, we consider an IVIFTPP with m supplies and n demands.

Let $c = \{(c_1^{ij}, c_2^{ij}, c_3^{ij}), [\mu_{c_l^{ij}}, \mu_{c_u^{ij}}], [\nu_{c_l^{ij}}, \nu_{c_u^{ij}}]\}$ be a interval-valued intuitionistic fuzzy numbers representing to the cost of transportation to send one unit of thing from i^{th} place to j^{th} place. Let $a_i = \{(a_1^{ij}, a_2^{ij}, a_3^{ij}), [\mu_{a_l^{ij}}, \mu_{a_u^{ij}}], [\nu_{a_l^{ij}}, \nu_{a_u^{ij}}]\}$, and $b_j = \{(b_1^{ij}, b_2^{ij}, b_3^{ij}), [\mu_{b_l^{ij}}, \mu_{b_u^{ij}}], [\nu_{b_l^{ij}}, \nu_{b_u^{ij}}]\}$ represent supplies and demands, respectively. Then IVIFTP is presented as:

$$\begin{aligned}
\text{Minimize}\quad & \tilde{z} = \sum_{i=1}^{m}\sum_{j=1}^{n}\{(c_1^{ij}, c_2^{ij}, c_3^{ij}), [\mu_{c_l^{ij}}, \mu_{c_u^{ij}}], [\nu_{c_l^{ij}}, \nu_{c_u^{ij}}]\}x_{ij} \\
\text{S.t.}\quad & \sum_{j=1}^{n} x_{ij} \approx \{(a_1^i, a_2^i, a_3^i), [\mu_{a_l^i}, \mu_{a_u^i}], [\nu_{a_l^i}, \nu_{a_u^i}]\}, \\
& i = 1, 2, \ldots, m \\
& \sum_{i=1}^{m} x_{ij} \approx \{(b_1^j, b_2^j, b_3^j), [\mu_{b_l^j}, \mu_{b_u^j}], [\nu_{b_l^j}, \nu_{b_u^j}]\} \\
& x_{ij} \geq 0, i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n.
\end{aligned} \tag{32}$$

Table 2 Uncertain transportation

Destinations→ Sources↓	D_1	D_2	D_3	a^i
S_1	$\widetilde{c^{11}}$	$\widetilde{c^{12}}$	$\widetilde{c^{13}}$	$\widetilde{a^1}$
S_2	$\widetilde{c^{21}}$	$\widetilde{c^{22}}$	$\widetilde{c^{23}}$	$\widetilde{a^2}$
S_3	$\widetilde{c^{31}}$	$\widetilde{c^{32}}$	$\widetilde{c^{33}}$	$\widetilde{a^3}$
$\widetilde{b_j}$	$\widetilde{b_1}$	$\widetilde{b_2}$	$\widetilde{b_3}$	

Hitchcock [15] invented the basic transportation problem with fixed parameters, and it was modeled by standard linear programming without uncertainty and hesitation. After Zadeh's fuzzy sets, several transportation models have appeared in the literature in which uncertainty was a main problem to deal. In this paper, we tackle uncertainty and hesitant of transportation problem that are coming from all directions. All uncertainty and hesitation are dealt well using interval-valued intuitionistic fuzzy sets. Therefore, we focus on interval-valued intuitionistic fuzzy transportation problem (Table 2), and in sort, we call it IVIFTP problem. Here, we can classify interval-valued intuitionistic fuzzy transportation problem into four types that are discussed below:

5.1 Interval-valued intuitionistic fuzzy transportation problem of type 1

A transportation problem where costs are interval-valued intuitionistic fuzzy numbers, demands and supplied are real numbers is called IVIFTP of type 1. This type of transportation problem occurs because the cost of the transportation depends on various uncontrollable factors such as weather condition, road condition and traffic. Mathematically, a IVIFTP of type 1 is represented in the following way:

$$\begin{aligned}
\text{Minimize}\quad & \tilde{z} = \sum_{i=1}^{m}\sum_{j=1}^{n}\{(c_1^{ij}, c_2^{ij}, c_3^{ij}), [\mu_{c_l^{ij}}, \mu_{c_u^{ij}}], [\nu_{c_l^{ij}}, \nu_{c_u^{ij}}]\}x_{ij} \\
\text{S.t.}\quad & \sum_{j=1}^{n} x_{ij} = a_i, i = 1, 2, \ldots, m, \\
& \sum_{i=1}^{m} x_{ij} = b_j, j = 1, 2, \ldots, n, \\
& x_{ij} \geq 0, i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n.
\end{aligned} \tag{33}$$

5.2 Interval-valued intuitionistic fuzzy transportation problem of type 2

A transportation problem is that in which the demands and supplies are represented by interval-valued intuitionistic fuzzy numbers and cost of transportation is treated as fixed real numbers and that type of transportation problem is called IVIFTP of type 2.

Let $a_i = \{(a_1^{ij}, a_2^{ij}, a_3^{ij}), [\mu_{a_l^{ij}}, \mu_{a_u^{ij}}], [\nu_{a_l^{ij}}, \nu_{a_u^{ij}}]\}$, $b_j = \{(b_1^{ij}, b_2^{ij}, b_3^{ij}), [\mu_{b_l^{ij}}, \mu_{b_u^{ij}}], [\nu_{b_l^{ij}}, \nu_{b_u^{ij}}]\}$ and c_{ij} be the cost that spend to transport a product x_{ij} from the i^{th} origin to the j^{th} destination. The reason that appeal IVIFTP of type 2 is due to various uncontrollable factors such as storage capacity and public demand.

$$\begin{aligned}
\text{Minimize} \quad & \tilde{z} = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij} \\
\text{S.t.} \quad & \sum_{j=1}^{n} x_{ij} = \{(a_1^i, a_2^i, a_3^i), [\mu_{a_l^i}, \mu_{a_u^i}], [\nu_{a_l^i}, \nu_{a_u^i}]\}, \\
& i = 1, 2, \ldots, m, \\
& \sum_{i=1}^{m} x_{ij} = \{(b_1^j, b_2^j, b_3^j), [\mu_{b_l^j}, \mu_{b_u^j}], [\nu_{b_l^j}, \nu_{b_u^j}]\}, \\
& j = 1, 2, \ldots, n, \\
& x_{ij} \geq 0, i = 1, 2, \ldots, m; \; j = 1, 2, \ldots, n.
\end{aligned} \tag{34}$$

5.3 Interval-valued intuitionistic fuzzy transportation problem of type 3

A transportation problem is that in which all costs of transportation, demands and supplies are interval-valued intuitionistic fuzzy numbers. Let $\{(a_1^i, a_2^i, a_3^i), [\mu_{a_l^i}, \mu_{a_u^i}], [\nu_{a_l^i}, \nu_{a_u^i}]\}$, $i = 1, 2, \ldots, m$ be the quantity available at i^{th} origin, $\{(b_1^j, b_2^j, b_3^j), [\mu_{b_l^j}, \mu_{b_u^j}], [\nu_{b_l^j}, \nu_{b_u^j}]\}$, $j = 1, 2, \ldots, n$ be the quantity needed at j^{th} destination and $\{(c_1^{ij}, c_2^{ij}, c_3^{ij}), [\mu_{c_l^{ij}}, \mu_{c_u^{ij}}], [\nu_{c_l^{ij}}, \nu_{c_u^{ij}}]\}$ be the transportation cost require to send x_{ij} from i^{th} origin to j^{th} destination. The cost of the transportation as in type 1 and type 2.

$$\begin{aligned}
\text{Minimize} \quad & \tilde{z} = \sum_{i=1}^{m}\sum_{j=1}^{n} \{(c_1^{ij}, c_2^{ij}, c_3^{ij}), [\mu_{c_l^{ij}}, \mu_{c_u^{ij}}], [\nu_{c_l^{ij}}, \nu_{c_u^{ij}}]\} x_{ij} \\
\text{S.t.} \quad & \sum_{j=1}^{n} x_{ij} \approx \{(a_1^i, a_2^i, a_3^i), [\mu_{a_l^i}, \mu_{a_u^i}], [\nu_{a_l^i}, \nu_{a_u^i}]\}, \\
& i = 1, 2, \ldots, m \\
& \sum_{i=1}^{m} x_{ij} \approx \{(b_1^j, b_2^j, b_3^j), [\mu_{b_l^j}, \mu_{b_u^j}], [\nu_{b_l^j}, \nu_{b_u^j}]\} \\
& \tilde{x}_{ij} \geq \tilde{0}, i = 1, 2, \ldots, m; \; j = 1, 2, \ldots, n.
\end{aligned} \tag{35}$$

5.4 Interval-valued intuitionistic fuzzy transportation problem of type 4

A transportation problem is that in which all the cost of transportation, demands and supplies are interval-valued intuitionistic fuzzy numbers, and decision variables are interval-valued intuitionistic fuzzy numbers as well. This type of transportation is called a fully interval-valued intuitionistic fuzzy transportation problem a fully IVIFTPP. Recently, Kumar and Hussain [47] have studied fully intuitionistic fuzzy transportation problems. For the mathematical formulation, let $\{(a_1^i, a_2^i, a_3^i), [\mu_{a_l^i}, \mu_{a_u^i}], [\nu_{a_l^i}, \nu_{a_u^i}]\}$, $i = 1, 2, \ldots, m$ be the quantity available at the i^{th} origin, $\{(b_1^j, b_2^j, b_3^j), [\mu_{b_l^j}, \mu_{b_u^j}], [\nu_{b_l^j}, \nu_{b_u^j}]\}$ be the quantity needed at the j^{th} destination and $\{(c_1^{ij}, c_2^{ij}, c_3^{ij}), [\mu_{c_l^{ij}}, \mu_{c_u^{ij}}], [\nu_{c_l^{ij}}, \nu_{c_u^{ij}}]\}$ be the cost to transport $\tilde{x}_{ij}$ from the i^{th} origin to the j^{th} destination.

$$\begin{aligned}
\text{Minimize} \quad & \tilde{z} = \sum_{i=1}^{m}\sum_{j=1}^{n} \{(c_1^{ij}, c_2^{ij}, c_3^{ij}), [\mu_{c_l^{ij}}, \mu_{c_u^{ij}}], \\
& [\nu_{c_l^{ij}}, \nu_{c_u^{ij}}]\}\{(x_1^{ij}, x_2^{ij}, x_3^{ij}), [\mu_{x_l^{ij}}, \mu_{x_u^{ij}}], [\nu_{x_l^{ij}}, \nu_{x_u^{ij}}]\} \\
\text{S.t.} \quad & \sum_{j=1}^{n} \tilde{x}_{ij} \approx \{(a_1^i, a_2^i, a_3^i), [\mu_{a_l^i}, \mu_{a_u^i}], [\nu_{a_l^i}, \nu_{a_u^i}]\}, \\
& i = 1, 2, \ldots, m \\
& \sum_{i=1}^{m} \tilde{x}_{ij} \approx \{(b_1^j, b_2^j, b_3^j), [\mu_{b_l^j}, \mu_{b_u^j}], [\nu_{b_l^j}, \nu_{b_u^j}]\} \\
& \{(x_1^{ij}, x_2^{ij}, x_3^{ij}), [\mu_{x_l^{ij}}, \mu_{x_u^{ij}}], [\nu_{x_l^{ij}}, \nu_{x_u^{ij}}]\} \geq \tilde{0}, \\
& i = 1, 2, \ldots, m; \; j = 1, 2, \ldots, n.
\end{aligned} \tag{36}$$

5.5 Balanced interval-valued intuitionistic fuzzy transportation problem

An IVIFTPP in which sum of all demands is equal to the sum of all supplies and all these are done after taking ranking is called balanced IVIFTPP, mathematically if

$$\sum_{i=1}^{m} R(\tilde{a}^i) = \sum_{j=1}^{n} R(\tilde{b}^j). \tag{37}$$

Otherwise, it is called unbalanced interval-valued intuitionistic transportation problem.

5.6 Interval-valued intuitionistic fuzzy optimal solution

A basis feasible solution that minimizes to the cost of transportation or that maximizes the profit of transportation

Table 3 Uncertain transportation

Destinations→ / Sources↓	D_1	D_2	D_3	a_i
S_1	$R(\widetilde{c_{11}})$	$R(\widetilde{c_{12}})$	$R(\widetilde{c_{13}})$	$R(\widetilde{a_1})$
S_2	$R(\widetilde{c_{21}})$	$R(\widetilde{c_{22}})$	$R(\widetilde{c_{23}})$	$R(\widetilde{a_2})$
S_3	$R(\widetilde{c_{31}})$	$R(\widetilde{c_{32}})$	$R(\widetilde{c_{33}})$	$R(\widetilde{a_3})$
$R(\widetilde{b_j})$	$R(\widetilde{b_1})$	$R(\widetilde{b_2})$	$R(\widetilde{b_3})$	

problem is called an interval-valued intuitionistic fuzzy optimal solution (IVIFOS).

6 Computational method

The steps of the computational method are given below:

Step 1: In this step, the cost of transportation is expressed as triangular interval-valued intuitionistic fuzzy numbers.
Step 2: Write IVIFTPP in tabular form as in below, where in the table all the parameters are represented by triangular interval-valued triangular intuitionistic fuzzy numbers.

$$\begin{aligned}\widetilde{c^{ij}} &= \{(c_1^{ij}, c_2^{ij}, c_3^{ij}), [\mu_{c_l^{ij}}, \mu_{c_u^{ij}}], [\nu_{c_l^{ij}}, \nu_{c_u^{ij}}]\},\\ \widetilde{a^i} &= \{(a_1^i, a_2^i, a_3^i), [\mu_{a_l^i}, \mu_{a_u^i}], [\nu_{a_l^i}, \nu_{a_u^i}]\},\\ &\quad i = 1, 2, \ldots, m,\\ \widetilde{b^j} &= \{(b_1^j, b_2^j, b_3^j), [\mu_{b_l^j}, \mu_{b_u^j}], [\nu_{b_l^j}, \nu_{b_u^j}]\},\\ &\quad j = 1, 2, \ldots, n.\end{aligned}$$

Step 3: Using proposed ordering, we transform to the interval-valued intuitionistic fuzzy transportation problem (8) into its crisp form (Table 3), we get
Step 4: Now check whether it is balanced or not.

If $(\sum_{i=1}^{m} R(\tilde{a}^i) = \sum_{j=1}^{n} R(\tilde{b}^j))$, then TP is balanced.
If $(\sum_{i=1}^{m} R(\tilde{a}^i) \neq \sum_{j=1}^{n} R(\tilde{b}^j))$, then TP is unbalanced.

Step 5: If the given TP is balanced, then go to step *5* otherwise make it balanced by adding dummy rows or columns as required.
Step 6: In this step, we search the initial basic feasible solutions of the crisp transportation problem by using one of the following methods and methods are given below:

6.1 Interval-valued intuitionistic fuzzy least cost method

Step 1: In this step, the cost of transportation is expressed as interval-valued intuitionistic fuzzy numbers, particularly triangular interval-valued intuitionistic fuzzy numbers.
Step 2: Search a smallest $\tilde{c}^{ij}$ in the IVIFS cost matrix of the transportation problem. Suppose it be $\tilde{c}^{ij}$. Allocate $\widetilde{x_{ij}} = \min(\tilde{a}^i, \tilde{b}^j)$ in the cell (i, j).
Step 3: If $\widetilde{x_{ij}} = \tilde{a}^i$ cross off the i^{th} row of transportation table and decrease $\tilde{b}^j$ by $\tilde{a}^i$. Go to next step.

If $\widetilde{x_{ij}} = \tilde{b}^j$ cross off the j^{th} column of the transportation table and decrease $\tilde{a}^i$ by $\tilde{b}^j$. Go to next step.
If $\widetilde{x_{ij}} = \tilde{a}^i = \tilde{b}^j$ cross off either the i_{th} row or j_{th} column, but not both.

Step 4: Repeat steps *1* and *2* for the resulting reduced transportation table until all the requirements are satisfied.

6.2 Interval-valued intuitionistic fuzzy northwest corner method

Step 1: In this step, the cost of transportation is expressed as interval-valued intuitionistic fuzzy numbers, particularly triangular interval-valued intuitionistic fuzzy numbers.
Step 2: Select a northwest (upper left-hand) corner cell of the IVIF transportation table and allocate as much as possible so that either the capacity of the first row is exhausted or the destination requirement of the first column is satisfied, i.e., $\widetilde{x_{11}} = \min(\tilde{a}^1, \tilde{b}^1)$.
Step 3: If $\tilde{b}^1 > \tilde{a}^1$, then we move down vertically to the second row and make second allocation of magnitude $\widetilde{x_{21}} = \min(\tilde{a}^2, \tilde{b}^1) - \widetilde{x_{11}})$ in cell (2, 1).

If $R\tilde{b}^1 < \tilde{a}^1$, we move right horizontally to the second column and make the second allocation of magnitude $\widetilde{x_{12}} = \min(\tilde{a}^1) - \widetilde{x_{11}}, \tilde{b}^2)$ in cell (2, 1).
If $\tilde{b}^1 = \tilde{a}^1$, there is a tie for the second allocation. One can make the second allocation of magnitude.
$\widetilde{x_{12}} = \min(\tilde{a}^1 - \tilde{a}^1, \tilde{b}^1) = 0$ in the cell (1, 2), or $\widetilde{x_{21}} = \min(\tilde{a}^2, \tilde{b}^1 - \tilde{b}^1) = 0$ in the cell (2, 1).

Step 4: Repeat step 1 and step 2 moving down towards the lower/right corner of the transportation table until all the requirement satisfied.

6.3 Interval-valued intuitionistic fuzzy Vogel's approximation method

[16] Vogel's approximation method (VAM) is the most common method used to search initial basic feasible solution of TP. The demerit of this method is that VAM usually assigns items to the dummy cells before other cell in the table. Further, it was modified by several researchers such as: [17,18].

Step 1: In this step, the cost of transportation is expressed as interval-valued intuitionistic fuzzy numbers, particularly triangular interval-valued intuitionistic fuzzy numbers.
Step 2: For each row and column of the IVIF transportation table, identify the smallest and next smallest IVIFS cost with the help of the proposed ranking. And then calculate the penalty $p_i, p_j, i = 1, 2, \cdots m; j = 1, 2, \cdots n$ between them for each row and column.
Step 3: Identify the largest penalty $p_i, p_j, i = 1, 2, \cdots m$; $j = 1, 2, \cdots n$ among all the rows and columns. If a tie occurs, then choose any arbitrary cell. Let the greatest penalty occur corresponding to $kth, 1 \leq k \leq m$ row and let $\tilde{c}^{kj}$) be the smallest cost in the k_{th} row. Allocate the minimum of $\tilde{a}^i$ and $\tilde{b}^j$ or $x_{kj} = \min(\tilde{a}^i, \tilde{b}^j)$ in the $(k, j)^{th}$ cell, and cross either the k^{th} row and j^{th} column in the usual manner.
Step 4: Update the column and row penalties for the reduced transportation table and go to step 2. Repeat the procedure until all the requirements are satisfied.
Step 5: In this step, dual variables $\tilde{u}^i$ and $\tilde{v}^j$ corresponding to the i^{th} row and j^{th} column are defined, respectively, such that $\tilde{u}^i + \tilde{v}^j = \tilde{c}^{ij}$ for each basic cell (i, j).
Step 6: Define $\tilde{Z}^{ij} = \tilde{u}^i + \tilde{v}^j$ for all non-basic variables. Calculate $\tilde{Z}^{ij} - \tilde{c}^{ij}$, there are two cases occurred:

i. $\tilde{Z}^{ij} - \tilde{c}^{ij} \leq 0$, for all (i, j); then, current solution is optimal solution to the interval-valued intuitionistic fuzzy transportation problem and stop the process.
ii. $\tilde{Z}^{ij} - \tilde{c}^{ij} > 0$, for at least one (i, j). Go to next step.

Step 7: Assign quantity τ in the cell (i, j) for which $\tilde{Z}^{ij} - \tilde{c}^{ij}$ is most positive and make a loop as follows:
Step 8: Start from $\tau-$ cell and move alternatively horizontally and vertically to the nearest basic cell with the restriction that end point of the loop must not lie in any non-basic cell except $\tau-$ cell. In this way, return to τ cell to complete loop.
Step 9: Move along loop of $\tau-$ cell. Add and subtract τ successively to/from the allocations in the cell lying at the turning points of the loop. Take the value of τ to be minimum of x_{ij} from which τ subtracted.
Step 10: Inserting the value of τ in the above step, the next basic feasible solution is obtained which improves the interval-valued intuitionistic cost. While inserting value of τ, a cell assumes 0 value. This cell becomes non-basic. This gives us the improved basic feasible solutions.
Step 11: The optimal value of the objective function is calculated by $\tilde{Z} = \tilde{c}^{ij} * \widetilde{X^0}$.

7 Illustration

In this section, numerical example of [34] is taken to verify the proposed computational method of the interval-valued intuitionistic fuzzy transportation problem. Here, cost of transportation is represented by triangular interval-valued intuitionistic fuzzy numbers. It is very interesting to see that cost obtained from proposed approach is minimum that existing.

Step 1: The cost of transportation varies due to various uncertain situation like weather condition, traffic condition, petroleum price, etc. The value of cost cannot deal the situation properly; to address this situation, we express parameter by TIVIFNs (Table 4).
Step 2: Identify smallest element and next smallest element in each row and each column (Tables 5, 6).

$R(c_{11}) < R(c_{13}) < R(c_{22}) \Rightarrow c_{11} < c_{13} < c_{22}$, smallest cost and next cost are: c_{11}, c_{13}
$R(c_{21}) < R(c_{22}) < R(c_{23}) \Rightarrow c_{21} < c_{22} < c_{23}$, smallest cost and next cost are: c_{21}, c_{22}
$R(c_{31}) < R(c_{32}) < R(c_{33}) \Rightarrow c_{31} < c_{32} < c_{33}$, smallest cost and next cost are: c_{31}, c_{32}

Similarly for column

$R(c_{31}) < R(c_{11}) < R(c_{21}) \Rightarrow c_{31} < c_{11} < c_{21}$, smallest cost and next cost are: c_{31}, c_{11}
$R(c_{22}) < R(c_{12}) < R(c_{32}) \Rightarrow c_{22} < c_{12} < c_{32}$, smallest cost and next cost are: c_{22}, c_{12}
$R(c_{13}) < R(c_{23}) < R(c_{33}) \Rightarrow c_{13} < c_{23} < c_{33}$, smallest cost and next cost are: c_{13}, c_{23}

Step 3: Row penalties:
First row rp_1 :

$$\{(4, 6, 16), [0.3, 0.4], [0.03, 0.07]\} \ominus, \{(1, 4, 9), [0.1, 0.5], [0.01, 0.03]\} = \{(-5, 2, 15), [0.1, 0.4], [0.03, 0.07]\}$$

Second row rp_2 :

$$\{(5, 10, 15), [0.2, 0.5], [0.01, 0.04]\} \ominus \{(4, 5, 7), [0.3, 0.4], [0.01, 0.02]\} = \{(-2, 5, 11), [0.2, 0.4], [0.01, 0.04]\}$$

Table 4 Interval-valued intuitionistic fuzzy transportation problem

	D_1	D_2	D_3	a_i
S_1	{(1, 4, 9); [0.1, 0.5], [0.01, 0.03]}	{(3, 13, 14), [0.2, 0.4]; [0.02, 0.04]}	{(4, 6, 16); [0.3, 0.4], [0.03, 0.07]}	7
S_2	{(4, 5, 7); [0.3, 0.4], [0.01, 0.02]}	{(5, 10, 15); [0.2, 0.5], [0.01, 0.04]}	{(7, 16, 24); [0.3, 0.5], [0.02, 0.03]}	15
S_3	{(1, 3, 6); [0.4, 0.5], [0.01, 0.02]}	{(5, 13, 21); [0.3, 0.4], [0.03, 0.04]}	{(8, 18, 27); [0.4, 0.5], [0.05, 0.05]}	10
b_j	8	6	18	

Table 5 Interval-valued intuitionistic fuzzy transportation problem cont. . . .

	D_1	D_2	D_3	a_i
S_1	**(1, 4, 9), [0.1, 0.5], [0.01, 0.03]**	(3, 13, 14), [0.2, 0.4], [0.02, 0.04]	**(4, 6, 16), [0.3, 0.4], [0.03, 0.07]**	7
S_2	**(4, 5, 7), [0.3, 0.4], [0.01, 0.02]**	**(5, 10, 15), [0.2, 0.5], [0.01, 0.04]**	(7, 16, 24), [0.3, 0.5], [0.02, 0.03]	15
S_3	**(1, 3, 6), [0.4, 0.5], [0.01, 0.02]**	**(5, 13, 21), [0.3, 0.4], [0.03, 0.04]**	(8, 18, 27), [0.4, 0.5], [0.05, 0.05]	10
b_j	8	6	18	

The selected elements are represented by bold

Table 6 Interval-valued intuitionistic fuzzy transportation problem cont. . . .

	D_1	D_2	D_3	a_i
S_1	**(1, 4, 9), [0.1, 0.5], [0.01, 0.03]**	**(3, 13, 14), [0.2, 0.4], [0.02, 0.04]**	**(4, 6, 16), [0.3, 0.4], [0.03, 0.07]**	7
S_2	(4, 5, 7), [0.3, 0.4], [0.01, 0.02]	(5, 10, 15), [0.2, 0.5], [0.01, 0.04]	**(7, 16, 24), [0.3, 0.5], [0.02, 0.03]**	15
S_3	**(1, 3, 6), [0.4, 0.5], [0.01, 0.02]**	**(5, 13, 21), [0.3, 0.4], [0.03, 0.04]**	(8, 18, 27), [0.4, 0.5], [0.05, 0.05]	10
b_j	8	6	18	

The selected elements are represented by bold

Table 7 Basic feasible solutions

	D_1	D_2	D_3	a_i
S_1	(1, 4, 9), [0.1, 0.5], [0.01, 0.03]	(3, 13, 14), [0.2, 0.4], [0.02, 0.04]	(4, 6, 16), [0.3, 0.4], [0.03, 0.07] **(7)**	7
S_2	(4, 5, 7), [0.3, 0.4], [0.01, 0.02]	(5, 10, 15), [0.2, 0.5], [0.01, 0.04] **(6)**	(7, 16, 24), [0.3, 0.5], [0.02, 0.03] **(9)**	15
S_3	(1, 3, 6), [0.4, 0.5], [0.01, 0.02] **(8)**	(5, 13, 21), [0.3, 0.4], [0.03, 0.04]	(8, 18, 27), [0.4, 0.5], [0.05, 0.05] **(2)**	10
b_j	8	6	18	

The selected elements are represented by bold

Third row rp_3 :

$$\{(5, 13, 21), [0.3, 0.4], [0.03, 0.04]\} \\ \ominus\{(1, 3, 6), [0.4, 0.5], [0.01, 0.02]\} \\ = \{(-1, 10, 20), [0.3, 0.4], [0.03, 0.04]\}$$

Column penalties:
First column cp_1 :

$$\{(1, 4, 9), [0.1, 0.5], [0.01, 0.03]\} \\ \ominus, \{(1, 3, 6), [0.4, 0.5], [0.01, 0.02]\} \\ = \{(-5, 1, 8), [0.1, 0.5], [0.01, 0.03]\}$$

Second column cp_2 :

$$\{(3, 13, 14), [0.2, 0.4], [0.02, 0.04]\} \\ \ominus\{(5, 10, 15), [0.2, 0.5], [0.01, 0.04]\} \\ = \{(-12, 3, 9), [0.2, 0.4], [0.02, 0.04]\}$$

Third column cp_3 :

$$\{(7, 16, 24), [0.3, 0.5], [0.02, 0.03]\} \\ \ominus\{(4, 6, 16), [0.3, 0.4], [0.03, 0.07]\} \\ = \{(-9, 10, 20), [0.3, 0.4], [0.03, 0.07]\}$$

Step 4: Choose row or column having larger penalty. Third has maximum penalty, and third row is selected for allocation. Continuing in the same ways, we get basic feasible solutions (Table 7).

$x_{13} = 7, x_{22} = 6, x_{23} = 9, x_{31} = 8, x_{33} = 2$. Now we shall use interval-valued intuitionistic fuzzy $\widetilde{u_i} - \widetilde{v_j}$ method to get interval-valued intuitionistic fuzzy optimal solutions,

Here, we calculate interval-valued intuitionistic fuzzy net evaluations (i.e., $z_{ij} - c_{ij}$) for all non-basic cell.

Step 5:

Put $\widetilde{u_1} = 0$ (Table 8), we get

$$\widetilde{u_2} = \{(-9, 10, 20); [0.3, 0.4][0.03, 0.07]\}$$
$$\widetilde{u_3} = \{(-8, 12, 23); [0.3, 0.4][0.05, 0.07]\}$$
$$\widetilde{v_1} = \{(-22, 9, 14); [0.3, 0.4][0.05, 0.07]\}$$
$$\widetilde{v_2} = \{(-15, 0, 24); [0.2, 0.4][0.03, 0.07]\}$$
$$6\widetilde{v_3} = \{(4, 6, 16); [0.3, 0.4][0.03, 0.07]\}$$

Net evaluations corresponding to all non-basic cells

$$\begin{aligned}\widetilde{z_{11}} - \widetilde{c_{11}} &= \widetilde{u_1} + \widetilde{v_1} - \widetilde{c_{11}}\\ &= 0 + \{(-22, -9, 14); [0.3, 0.4][0.05, 0.07]\}\\ &\quad -\{(1, 4, 9), [0.1, 0.5], [0.01, 0.03]\}\\ &= \{(-31, -13, 13), [0.1, 0.5], [0.01, 0.03]\} < 0\\ &\quad \therefore \widetilde{z_{11}} - \widetilde{c_{11}} < 0\end{aligned}$$

$$\begin{aligned}\widetilde{z_{12}} - \widetilde{c_{12}} &= \widetilde{u_1} + \widetilde{v_2} - \widetilde{c_{12}}\\ &= 0 + \{(-15, 0, 24); [0.2, 0.4][0.03, 0.07]\}\\ &\quad -\{(3, 13, 14), [0.2, 0.4], [0.02, 0.04]\}\\ &= \{(-29, -13, 21), [0.2, 0.4], [0.03, 0.07]\} < 0\\ &\quad \therefore \widetilde{z_{12}} - \widetilde{c_{12}} < 0\end{aligned}$$

$$\begin{aligned}\widetilde{z_{21}} - \widetilde{c_{21}} &= \widetilde{u_2} + \widetilde{v_1} - \widetilde{c_{21}}\\ &= \{(-9, 10, 20); [0.3, 0.4][0.03, 0.07]\}\\ &\quad +\{(-22, -9, 14); [0.3, 0.4][0.05, 0.07]\}\\ &\quad -\{(4, 5, 7), [0.3, 0.4], [0.01, 0.02]\}\\ &= \{(-31, -1, 13); [0.3, 0.4], [0.03, 0.07]\}\\ &\quad -\{(4, 5, 7); [0.3, 0.4], [0.01, 0.02]\}\\ &= \{(-38, -6, 4); [0.3, 0.4], [0.03, 0.07]\} < 0\\ &\quad \therefore \widetilde{z_{21}} - \widetilde{c_{21}} < 0\end{aligned}$$

$$\begin{aligned}\widetilde{z_{32}} - \widetilde{c_{32}} &= \widetilde{u_3} + \widetilde{v_2} - \widetilde{c_{32}}\\ &= \{(-8, 12, 23); [0.3, 0.4][0.05, 0.07]\}\\ &\quad +\{(-15, 0, 24); [0.2, 0.4][0.03, 0.07]\}\\ &\quad -\{(5, 13, 21); [0.3, 0.4], [0.03, 0.04]\}\\ &= \{(-23, 12, 47); [0.3, 0.4], [0.03, 0.07]\}\\ &\quad -\{(5, 13, 21); [0.3, 0.4], [0.03, 0.04]\}\\ &= \{(-44, -1, 42); [0.3, 0.4], [0.03, 0.07]\} < 0\\ &\quad \therefore \widetilde{z_{32}} - \widetilde{c_{32}} < 0\end{aligned}$$

Step 6: Finally, we get for all non-basic cells: $\widetilde{z_{ij}} - \widetilde{c_{ij}} < 0$ Stop the process here. Therefore, we skip steps 7-9 and we shall go last step (i.e, step 11).

Table 8 Interval-valued intuitionistic fuzzy $\widetilde{u} - \widetilde{v}$ method

	$\widetilde{v_1}\{(-22, 9, 14); [0.3, 0.4][0.05, 0.07]\}$	$\widetilde{v_2}\{(-15, 0, 24); [0.2, 0.4][0.03, 0.07]\}$	$\widetilde{v_3}\{(4, 6, 16); [0.3, 0.4][0.03, 0.07]\}$
$\widetilde{u_1}0$	{(1, 4, 9); [0.1, 0.5], [0.01, 0.03]}	{(3, 13, 14), [0.2, 0.4]; [0.02, 0.04]}	{(4, 6, 16); [0.3, 0.4], [0.03, 0.07]}
$\widetilde{u_2}\{(-9, 10, 20); [0.3, 0.4], [0.03, 0.07]\}$	{(4, 5, 7); [0.3, 0.4], [0.01, 0.02]}	{(5, 10, 15); [0.2, 0.5], [0.01, 0.04]}	{(7, 16, 24); [0.3, 0.5], [0.02, 0.03]}
$\widetilde{u_3}\{(-8, 12, 23); [0.3, 0.4][0.05, 0.07]\}$	{(1, 3, 6); [0.4, 0.5], [0.01, 0.02]}	{(5, 13, 21); [0.3, 0.4], [0.03, 0.04]}	{(8, 18, 27); [0.4, 0.5], [0.05, 0.05]}

Table 9 Comparison with existing method

S.no.	Researchers	Fuzzy values
1.	Kumar and Hussain [47]	{(137, 292, 502); (12, 292, 961)}
2.	Proposed method	{(145, 306, 520); [0.2, 0.4], [0.03, 0.07]}

Step 11: Therefore, $x_{13} = 7, x_{22} = 6, x_{23} = 9, x_{31} = 8, x_{33} = 2$ are optimal solutions and minimum cost is :

$$\begin{aligned} &7 * \{(4, 6, 16); [0.3, 0.4], [0.03, 0.07]\} \\ &\quad +6 * \{(5, 10, 15), [0.2, 0.5], [0.01, 0.04]\} \\ &\quad +9 * \{(7, 16, 24); [0.3, 0.5], [0.02, 0.03]\} \\ &\quad +8 * \{(1, 3, 6); [0.4, 0.5], [0.01, 0.02]\} \\ &\quad +2 * \{(8, 18, 27), [0.4, 0.5], [0.05, 0.05]\} \\ &\quad = \{(145, 306, 520); [0.2, 0.4], [0.03, 0.07]\} \end{aligned}$$

Interval-valued intuitionistic fuzzy cost is: $z^0 = \{(145, 306, 520); [0.2, 0.4], [0.03, 0.07]\}$.

8 Conclusions

One of the very interesting problems of decision science is the ranking of interval-valued intuitionistic fuzzy sets. Also, it is very hard to develop a ranking function to rank interval-valued intuitionistic fuzzy sets. In the present paper, we use the law of trichotomy to order interval intuitionistic fuzzy sets, and for this, we introduce a transformation that is called ranking function. The proposed ranking function depends on both value of variable and interval-valued intuitionistic fuzzy degrees, and this is a beauty of proposed ranking function. Also, it distinguishes proposed ranking to existing ranking function. In the present paper, a computational methodology which is based on ranking function is developed and applied to an interval-valued intuitionistic fuzzy transportation problem to get a compromise solution. Further, it is very interesting to note that the proposed computation method predicts a minimum transportation cost as compared to the existing approach (see Tables 9 and 10), and interval-valued intuitionistic fuzzy degree of transportation cost is given in Fig. 6. To check the performance and superiority of the proposed ranking function, an illustrative example is presented (see Table 1). Also we compare our ranking function with existing ranking function and it shows that presented ordering function follows to the existing ranking function.

Due to the appearance of interval-valued intuitionistic fuzzy sets in real-life problems such as decision making, multiattribute decision making with incomplete weight, fuzzy forecasting, risk analysis, etc., clustering and artificial intelligence, the proposed method will be high performance and efficient.

Table 10 Solution approach of a transportation problem

S.no.	Researchers	Problem	Approach
1.	Das et al. [23]	MOTP	INs
2.	Li and Lai [12]	MOTP	FSs
3.	Zangiabadi and Maleki [24]	MOTP	FSs
4.	Wahed and Lee [25]	MOTP	FSs
5.	Liu and Kao [21]	SOTP	FSs
6.	Wahed [26]	MOTP	FSs
7.	Hussain and Kumar [30]	SOTP	IFSs
8.	Singh and Yadav [31]	SOTP	TIFNs
9.	Ebrahimnejad and Verdegay [32]	SOTP	IFNs
10.	Mahmoodirad et al. [33]	SOTP	IFNs
11.	Kumar [34]	SOTP	IFNs
12.	Roy [35,36]	SOTP	IFNs
13.	Kumar [37]	SOTP	TIFNs
14.	Jana [38]	SOTP	IFNs
15.	Singh and Yadav [39]	SOTP	IFNs
16.	Ebrahimnejad and Verdegay [40]	SOTP	IFNs
17.	Kour et al. [41]	SOTP	IFNs
18.	Liu [43]	FOTP	FNs
19.	Bharati [8]	FOTP	IFNs
20.	Bharati and Malhotra [42]	TSTP	IFNs
21.	Proposed method	SOTP	IVIFNs

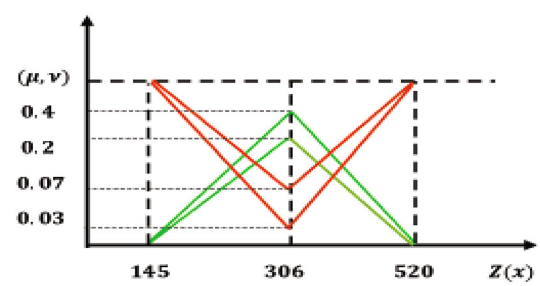

Fig. 6 Interval-valued intuitionistic fuzzy sets total cost

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Human and animal rights This article does not contain any studies with human or animal subjects performed by any of the authors.

Informed consent Informed consent was not required as no human or animals were involved.

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