

# Time-frequency BSS of biosignals

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Time–frequency (TF) representations are very important tools to understand and explain circumstances, where the frequency content of non-stationary signals varies in time. A variety of biosignals such as speech, electrocardiogram (ECG), electroencephalogram (EEG), and electromyogram (EMG) show some form of non-stationarity. Considering Priestley’s evolutionary (time-dependent) spectral theory for analysis of non-stationary signals, the authors defined a TF representation called evolutionary Slepian transform (EST). The evolutionary spectral theory generalises the definition of spectra while avoiding some of the shortcomings of bilinear TF methods. The performance of the EST in the representation of biosignals for the blind source separation (BSS) problem to extract information from a mixture of sources is studied. For example, in the case of EEG recordings, as electrodes are placed along the scalp, what is actually observed from EEG data at each electrode is a mixture of all the active sources. Separation of these sources from a mixture of observations is crucial for the analysis of recordings. In this study, they show that the EST can be used efficiently in the TF-based BSS problem of biosignals.

**1. Introduction:** Many practical signals such as speech, electrocardiogram (ECG), electroencephalogram (EEG), electromyogram (EMG) as well as signals arising from observations of many dynamic systems show non-stationarity. Time–frequency (TF) representations are very useful to understand and explain circumstances, where the frequency content of non-stationary signals varies with time. Most of the existing TF representations are based on the Wigner–Ville distribution (WVD) [1] or a smoothed version of the WVD [2]. A well known example for this is the spectrogram. The spectrogram, as the square modulus of the short time Fourier transform (STFT), is the WVD smoothed in time and frequency by the ambiguity function of the window used in the STFT [3].

A more recently developed method using the concepts of the STFT and continuous wavelet transform (CWT) is called S-transform (ST). The ST uses a Fourier kernel in the signal decomposition and preserves the phase information [4]. An improvement on the ST was proposed [5, 6] which is called the modified ST (MST). The MST is a signal-dependent version of the standard ST with an improved TF resolution. Another TF representation for spectral representation of non-stationary signals while avoiding some of the shortcomings of bilinear TF distributions (TFDs) [7, 8] is called the evolutionary spectrum (ES) or Priestley’s time-dependent spectrum. The evolutionary periodogram can be used to estimate the ES by allowing non-stationary signals to be modelled as a sum of complex sinusoids with time-varying complex amplitudes [9]. In [10], the discrete evolutionary transform (DET) was proposed for the computation of a kernel and the corresponding ES.

An important application for the spectral representation of non-stationary signals is in the blind source separation (BSS) problem [11–13]. The BSS problem can be defined as recovering  $n$  unknown sources from  $m$  observations (mixtures) of them [14]. In general, each sensor receives a linear mixture of source signals and the BSS methods recover all individual sources from the mixture or at least separate a particular source. For example, in the case of electroencephalography recordings, voltage fluctuations resulting from ionic current within the neurones of the brain are measured non-invasively. As electrodes are placed along the scalp, what we actually observe from EEG data is a mixture of all the active sources. Since the electrical signals must travel through human tissue to reach the electrodes, each measured signal can be assumed to be a linear mixture of source signals [15]. In addition,

scalp-recorded EEG signals include non-brain sources such as electrooculographic and electromyographic activities. The BSS methods are very useful for extracting these sources from the EEG data [16–27]. The unmixing matrix’s inverse can also be used to provide a spatial illustration of each BSS-extracted signal’s associated scalp location [21]. An example for the over-determined case, i.e. the number of observations are greater than the number of sources, a method based on second-order statistics and joint diagonalisation of a set of covariance matrices can be found in [25]. Other examples on spatial TFDs as a generalisation of bilinear TFDs, in the case of non-stationary signals, are in [26, 28]. Priestley’s ES was used also for array processing in semi-homogeneous random fields and for the direction of arrival estimation [29, 30].

In prior studies, we examined the signal reconstruction from the irregular samples and non-stationary signal representation problem using the evolutionary spectral techniques and the discrete prolate spheroidal sequences (DPSSs) [31–33]. Also known as Slepian sequences, the DPSS derive from the TF concentration problem and are defined to be the sequences with maximum spectral concentration for a given duration and bandwidth. In this Letter, we evaluate the performance of the evolutionary Slepian transform (EST) by comparing the EST with MST and smoothed Wigner Ville distribution (SWVD). We chose the ST for its being an adaptive form of the STFT and CWT and the WVD was chosen for its being a high-resolution TF representation. To be more precise, we used the improved versions as the MST and SWVD. We present an application of the EST in spectral representation of non-stationary signals for BSS problem. Experimental results demonstrate the efficiency of EST-based representation in the BSS problem. The Letter is organised as follows. In the next section, we review the ES and provide the fundamental equations of signal representation. In Section 3, we explain the EST and review properties of Slepian sequences. We briefly review the BSS problem and related formulation in Section 4. In Section 5, we present experimental methods and results.

**2. Review of the ES:** Introduced by Priestley, the evolutionary spectral theory describes the local power frequency distribution at each instant of time as the set of bandpass filter output powers. These powers are computed by averaging the squared output samples in time [7]. As a special case of Priestley’s ES, the Wold–Cramér ES considers a discrete-time non-stationary

signal  $x[n]$  as the output of a linear time-varying system with the impulse response  $h[n, m]$ , driven by a stationary white noise [8] as

$$x[n] = \sum_{m=-\infty}^n h[n, m]\varepsilon[m], \quad (1)$$

here  $x[n]$  is the output for the input  $\{\varepsilon[m]\}$  which is a stationary, zero-mean, unit-variance, white noise process. The representation in (1) is known as the Wold–Cramér decomposition [7]. The white noise process  $\{\varepsilon[m]\}$  can be expressed as a sum of sinusoids with random amplitudes and phases. Accordingly, the non-stationary process  $\{x[n]\}$  can be expressed as

$$x[n] = \int_{-\pi}^{\pi} H(n, \omega) e^{j\omega n} dZ(\omega), \quad (2)$$

where

$$H(n, \omega) = \sum_{m=-\infty}^n h[n, m] e^{-j\omega(n-m)}, \quad (3)$$

for  $Z(\omega)$  being a process with orthogonal increments. The variance of  $x[n]$  provides the power distribution of the non-stationary process  $\{x[n]\}$  at each time  $n$ , as a function of the frequency parameter  $\omega$  and the Wold–Cramér ES is defined as  $S(n, \omega) = |H(n, \omega)|^2$ . This definition was also proposed in [8] as a special case of Priestley’s ES if one restricts the function  $H(n, \omega)$  to the class of oscillatory functions that are slowly varying in time. In [9], a similar condition was applied to model the component  $x[n]$  for a particular frequency of interest  $\omega_0$  as

$$x[n] = x_0[n] + y_{\omega_0}[n] = A(n, \omega_0) e^{j\omega_0 n} + y_{\omega_0}[n], \quad (4)$$

where  $A(n, \omega_0)$  represents time-varying complex amplitude as  $A(n, \omega_0) = H(n, \omega_0) dZ(\omega_0)$  and  $y_{\omega_0}[n]$  being a zero-mean modelling error. It can be derived that the variance of  $A(n, \omega_0)$  is

$$E\{|A(n, \omega_0)|^2\} = S(n, \omega_0) \frac{d\omega_0}{2\pi}, \quad (5)$$

and repeating this process for all frequencies  $\omega$ , an estimate of the time-dependent spectral density  $S(n, \omega)$  can be obtained. The detailed description for the estimation of  $S(n, \omega)$  can be found in [9]. In [10], using the Gabor or Malvar representations with the Wold–Cramér representation, DET was defined to represent a non-stationary signal and its spectrum. In the DET, an evolutionary kernel can be obtained and the ES is the magnitude square of the evolutionary kernel [10] for the signal

$$x[n] = \sum_{k=0}^{K-1} X(n, \omega_k) e^{j\omega_k n}, \quad (6)$$

where  $\omega_k = 2\pi k/K$ ,  $0 \leq n \leq N-1$  and  $X(n, \omega_k)$  is called the evolutionary kernel [10]. In this case, associating with the sinusoidal representation in (1)

$$X(n, \omega_k) = \sum_{\ell=0}^{N-1} x(\ell) W_k(n, \ell) e^{-j\omega_k \ell}, \quad (7)$$

is an inverse discrete transformation that provides the evolutionary kernel,  $X(n, \omega_k)$  in terms of the signal.  $W_k(n, \ell)$  is in general a time- and frequency-dependent windows [10]. Here, the ES is defined as  $S_{\text{DET}}(n, \omega_k) = |X(n, \omega_k)|^2$ . A similar representation for the kernel was obtained in [9] by expressing the time-varying window as a set of orthogonal functions.

**3. Slepian sequences and the EST:** In the following section, we give a brief review of Slepian sequences and then the EST.

3.1. Slepian sequences: The discrete form of prolate spheroidal wave functions (PSWFs) [34] can be used efficiently for signal representation [31] and called DPSSs. DPSS resulted from the work of Slepian about the problem of concentrating a signal jointly in temporal and spectral domains [33]. These sequences are also known as Slepian sequences. PSWFs have been used in many applications such as analysis of non-stationary and nonlinear time series [35], communication theory [36] and their mathematical properties and computation are presented in [37]. The PSWFs are real-valued, finite support functions with maximum energy concentration in a given bandwidth. Given a sinc-function-based integral equation, the PSWFs  $\phi_n(\cdot)$  are the eigenfunctions as

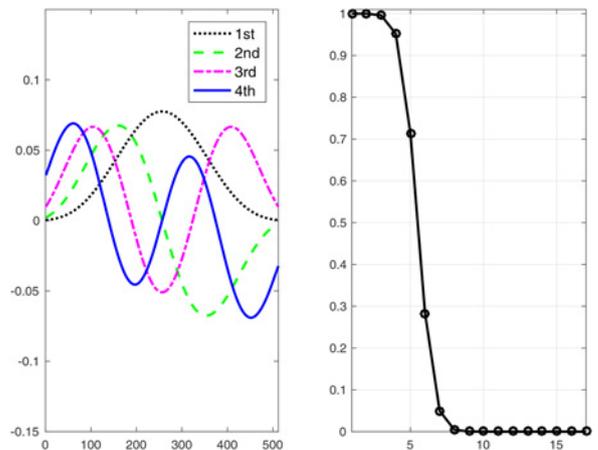
$$\phi_n(t) = \frac{1}{\lambda_n} \int_{-T}^T \phi_n(x) S(t-x) dx \quad (8)$$

where  $S(\cdot)$  is the sinc function. Since the sinc function is orthogonal, the PSWFs are also orthogonal and bases for finite energy signals such as the sinc function is. The discrete forms of the PSWF are characterised by the time-bandwidth product  $N\Omega$ , where  $N$  is their length and  $\Omega$  is the normalised bandwidth. Like their continuous-time equivalent (PSWF), the DPSSs are obtained by solving the following eigenvalue problem:

$$\lambda_k \phi_k(m) = \sum_{n=0}^{N-1} \frac{\sin(2\pi\Omega(n-m))}{\pi(n-m)} \phi_k(n). \quad (9)$$

Using the notation above, given  $N$  and  $0 < \Omega < 1/2$ , the DPSS is defined as a collection of  $N$  real valued, strictly bandlimited discrete-time sequences  $\mathbf{f}_{N,\Omega} = [\phi_{N,\Omega}^{(1)}, \phi_{N,\Omega}^{(2)}, \dots, \phi_{N,\Omega}^{(N)}]$  with their corresponding eigenvalues  $1 > \lambda^{(1)} > \lambda^{(2)} \dots \lambda^{(N)} > 0$ . The second Slepian sequence is orthogonal to the first Slepian sequence. The third Slepian sequence is orthogonal to both the first and second Slepian sequences. Continuing in this way, the Slepian sequences form an orthogonal set of bandlimited sequences and the DPSSs are also orthonormal.

There are  $2N\Omega - 1$  Slepian sequences with energy concentration ratios approximately equal to one, and for the rest the concentration ratios begin to approach zero (see Fig. 1). For a given integer  $K \leq N$ , we can get  $N \times K$  matrix formed by taking the first  $K$  columns of  $\mathbf{f}_{N,\Omega}$ . When  $K \approx 2N\Omega$ , it is a highly efficient basis that captures most of the signal energy [36, 37].



**Fig. 1** Left: the first four Slepian sequences for chosen  $N=512$  and  $N\Omega=3.5$  and right: energy concentrations, i.e. eigenvalues

3.2. Evolutionary ST: Stationary random signals can be approximated by the superpositions of random harmonic oscillations, i.e. superposition of sinusoids of all possible frequencies with randomly varying amplitudes and phases [38]. For non-stationary signals, to obtain a similar representation, non-stationary signals are considered as the output of a linear time-varying system with a stationary white noise input and the Wold–Cramér representation can be used [7]. In general, any discrete-time signal  $x[n]$  can be represented in terms of an orthogonal basis  $\{\phi_k[n]\}$  as

$$x[n] = \sum_{k=0}^{K-1} d_k \phi_k[n], \quad 0 \leq n \leq N-1, \quad (10)$$

$$d_k = \sum_{n=0}^{N-1} x[n] \phi_k^*[n], \quad 0 \leq k \leq K-1.$$

We showed in [31] that  $x[n]$  can be written as follows:

$$x[n] = \sum_{k=0}^{K-1} \underbrace{[d_k \phi_k[n] e^{-j\omega_k n}]}_{X(n, \omega_k)} e^{j\omega_k n}, \quad (11)$$

where  $\omega_k = 2\pi(k/N)$ . Then, the evolutionary kernel  $X(n, \omega_k)$  can be obtained in terms of  $x[n]$  by replacing the  $d_k$  coefficients with their definition in (18)

$$X(n, \omega_k) = d_k \phi_k[n] e^{-j\omega_k n} = \sum_{m=0}^{N-1} x[m] W_k(n, m) e^{-j\omega_k m}, \quad (12)$$

where we obtain the TF-dependent window as

$$W_k(n, m) = \phi_k[n] \phi_k^*[m] e^{-j\omega_k(n-m)}. \quad (13)$$

To obtain the evolutionary kernel, specifically the window  $W_k(n, m)$ , we considered DPSS  $\{\phi_k[n]\}$  as the bases of the representation. Accordingly, by taking the magnitude square  $|X(n, \omega_k)|^2$ , we obtain the ES  $S_{EST}(n, \omega_k) = |X(n, \omega_k)|^2$ .

In many practical applications, the exact bandwidth of the signal is known, so choosing the appropriate time-bandwidth product  $N\Omega$  to cover all the frequencies existing in the signal is straightforward. Therefore, having enough knowledge in the spectral characteristics of the signals, a precise representation can be obtained in the joint TF domain using the EST. Otherwise, we can use some bandwidth estimation techniques such as [39–43].

#### 4. Review of The BSS method:

4.1. Problem formulation: BSS covers a wide range of applications and has been a topic of great interest in diverse fields such as digital communications, pattern recognition, biomedical engineering and financial data analysis, among others. In general, the available BSS methods use the following data model for each signal received at each sensor [25]:

$$x[n] = Cs[n] + \mu[n], \quad (14)$$

such that

- $x[n] = [x_1[n], \dots, x_p[n]]^T$  is a  $p$  vector of observations;
- $s[n] = [s_1[n], \dots, s_q[n]]^T$  is a  $q$  vector of unknown sources;
- $C$  is a  $p \times q$  mixing or array matrix; and
- $\mu[n]$  is a zero mean and  $\sigma^2$  is a variance white noise vector.

The objective is to obtain an estimate  $\hat{C}$  of  $C$  and obtain sources as

$$\hat{s}[n] = \hat{C}^\# x[n] \simeq Gs[n] + \hat{C}^\# \mu[n] \quad (15)$$

where  $\#$  represents pseudoinverse and  $G$  is a matrix with only one non-zero entry per row and column [25]. In particular, the approaches using TF signal representations for BSS involve the following steps [44]:

- Estimation of the spatial TF spectra.
- Estimation of whitening matrix and noise variance.
- Joint diagonalisation of the noise compensated and whitened spatial TF spectra matrices.

More details on the BSS algorithm above can be found in [26, 44, 45].

4.2. Spatial evolutionary transform and BSS: In the TF approach for the BSS problem, using the data model received at each sensor, the cross-power spectral estimate can be written as [30]

$$\hat{S}_{xx}(n, \omega) = C \hat{S}_{ss}(n, \omega) C^H + \sigma^2 b[n]^H b[n] I. \quad (16)$$

In this Letter, in the equation above,  $\hat{S}_{xx}(n, \omega)$  is the evolutionary spatial Slepian estimate. Representing  $W$  as the  $p \times q$  whitening matrix

$$\tilde{S}_{xx}(n, \omega) = W(\hat{S}_{xx}(n, \omega) - \sigma^2 I) W^H \quad (17)$$

and letting  $U = WC$ , whitened and noise compensated matrix is

$$\tilde{S}_{xx}(n, \omega) = U \hat{S}_{ss}(n, \omega) U^H \quad (18)$$

where  $U$  is unitary and diagonalises  $\tilde{S}_{xx}(n, \omega)$  for any  $(n, \omega)$  [44–46]. The unitary matrix can be estimated from the eigenvectors of any  $\tilde{S}_{xx}(n, \omega)$  with distinct eigenvalues and the mixing matrix is obtained using  $C = W^\# U$ . The source signals are then estimated as in (15) [44].

**5. Methods:** Using the TF representation-based BSS algorithm given in [44], we simulated an overdetermined case (i.e.  $n \leq m$ ). The overdetermined case can be an example for speech signal processing, biomedical signal processing or telecommunications applications, where there are typically more sensors than the number of sources. Since the available EEG data recordings lack ground truth, we used the ground truth data provided by Independent Component Analysis Laboratory [17, 47], which are simulations of typical biosignals such as EEG (see Fig. 2) representing eyeblink, muscle movement of limbs and heart etc., to test the performance of the EST for the BSS. In our

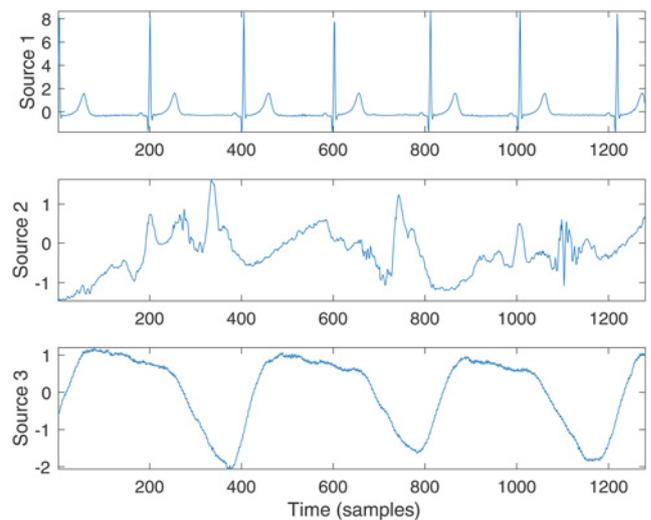


Fig. 2 Ground truth simulated biosignals for EEG data

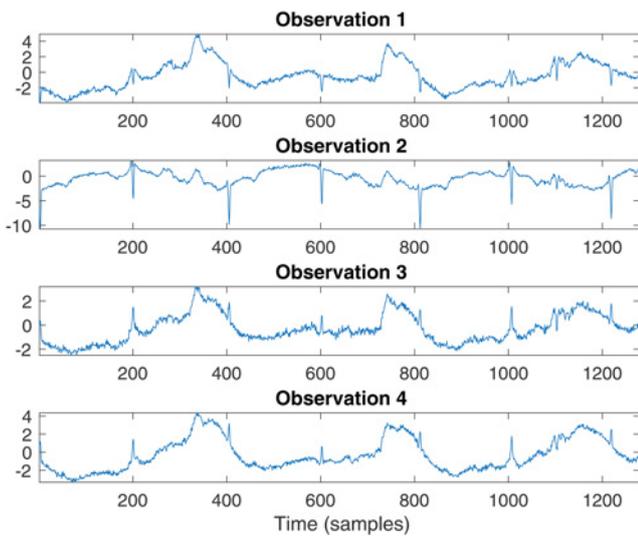


Fig. 3 Observations of simulated EEG data for  $n = 3$  and  $m = 4$

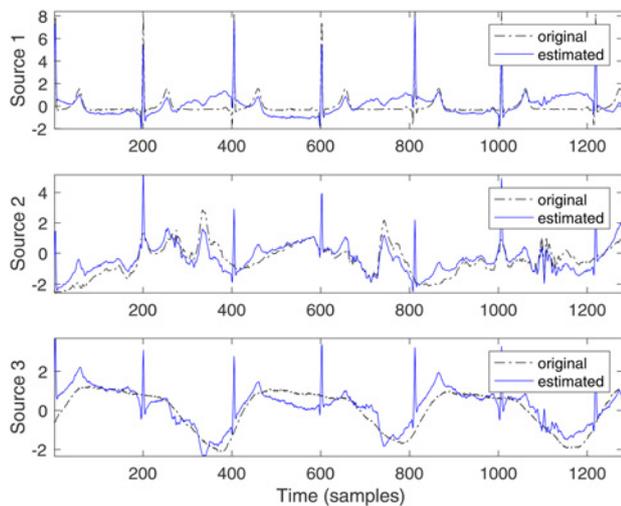


Fig. 4 Separated sources using the MST-based BSS

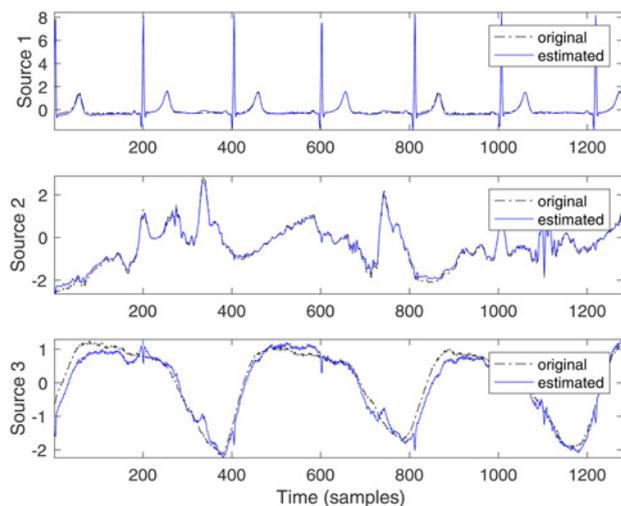


Fig. 5 Separated sources using the EST-based BSS

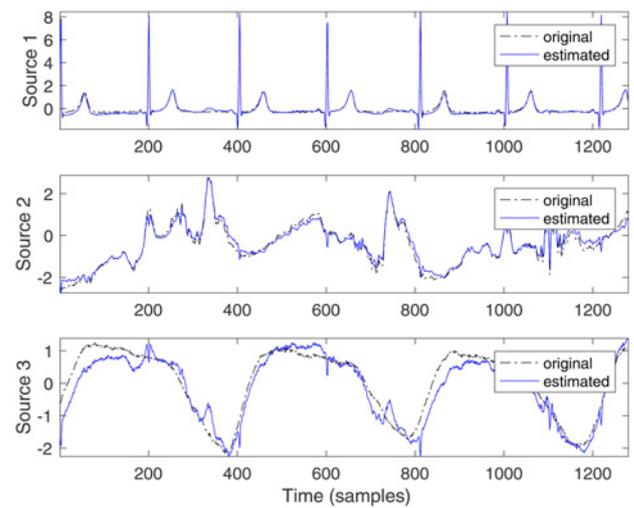


Fig. 6 Separated sources using the SWVD-based BSS

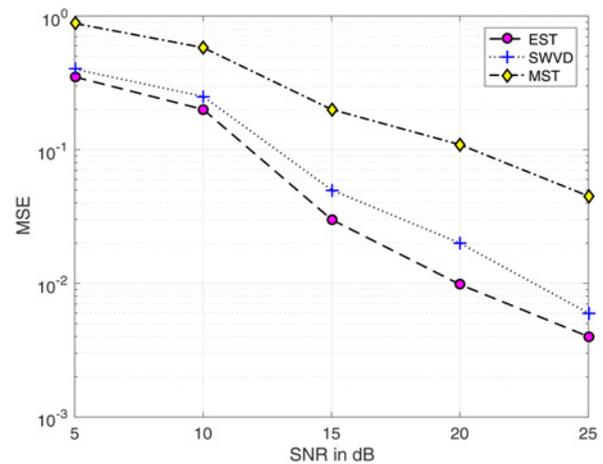


Fig. 7 MSE versus SNR for the Source 3 of EEG data

experiment, we chose a random matrix as the mixing matrix for generating noisy [signal-to-noise ratio (SNR)=20 dB] observations (see Fig. 3). The separation of three sources and estimation from observations, using the MST, EST and SWVD are presented in Figs. 4–6. To give an evaluation of the performance of source separation in terms of SNR and mean-squared error (MSE), we chose ground truth Source 3 of the EEG data and compared it with the separated Source 3 for various SNRs using the MST, EST and SWVD-based BSS methods (see Fig. 7).

**6. Conclusions:** In the BSS problem, we tested simulated EEG signals as source signals to be separated from the noisy observations obtained via a random mixing matrix. Comparing the results of estimated sources, we observed that EST provides the best performance in separation of individual simulated EEG signals. The EST is shown to be an efficient approach for separation of signals (biosignals and/or non-biosignals) from (electrophysiological) recordings. As our future work, we will expand our experiments to larger data sets for the BSS and also explore methods that can enable us to determine the number of sources in the observation mixtures.

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