



Research article

Unifying casualty distributions within and across conflicts

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ABSTRACT

The distribution of whole war sizes and the distribution of event sizes within individual wars, can both be well approximated by power laws where size is measured by the number of fatalities. However the power-law exponent value for whole wars has a substantially smaller magnitude – and hence a flatter distribution – than for individual wars. We provide detailed numerical evidence that confirms that these numerically different power-law exponent values are interrelated in a simple way by the effect of aggregating fatalities from individual events within wars to whole wars. We offer intuition for this finding and hence strengthen the case for a unified description and understanding of human conflict across scales.

1. Introduction

The distribution of whole war sizes is well approximated by a power-law (Cederman, 2003; González-Val, 2016; Clauset, 2018; Braumoeller, 2019), where size is measured as the total fatalities. This remarkable empirical finding is sometimes known as ‘Richardson’s Law’ in honor of its discovery more than half a century ago by one of the greatest scientists of the 20th century: Lewis Fry Richardson (Richardson, 1948, 1960; Gleditsch, 2020). Rather surprisingly, it turns out that approximate power laws also describe the distributions of event sizes within individual wars (Bohorquez et al., 2009; Johnson et al., 2013; Spagat et al., 2018) and terrorism (Clauset et al., 2007; Spagat et al., 2018) where size is now measured by fatalities per event. We stress that in both cases, the power laws are approximate not perfect, and we refer to Bohorquez et al. (2009) for a discussion of generative mechanisms that explain how these approximate power laws might arise and, importantly, the mechanistic origins of deviations from these power law distributions.

Here we focus on the relationship between these power-law exponents. Specifically we provide intuition that explains, and detailed numerical results that confirm, that the exponent values for whole wars and within wars are consistent with each other: namely, a power-law exponent value between 1.5 and 1.7 emerges for whole wars as a result of aggregating fatalities from events within wars that have a power-law

exponent near 2.5. These results in Figs. 1–3, which go beyond Johnson Restrepo et al. (2020), involve simulating individual wars with randomly selected power-law exponents distributed around 2.5, and with randomly selected event counts that are consistent with the distribution of event counts for empirical wars (see Fig. 3). These findings hence help solidify a unified understanding of human conflict across scales, from within individual wars to across wars. Interestingly, this emerging synthesis of micro and macro elements falls firmly within the spirit of Richardson’s life work, including his research on weather and fractals (Gleditsch, 2020).

2. Results

Cederman (2003), Clauset (2018) and most recently Braumoeller (2019), built on Richardson’s seminal work by showing that the distribution of severities for entire wars is an approximate power-law with an estimated exponent range $\alpha \sim 1.5$ –1.7, with the precise value depending on the date range and war types included. The goodness-of-fit values from these studies are fairly high ($p \sim 0.5$) which helps support the claim of an approximate power-law distribution. As noted above, we found earlier (Bohorquez et al. (2009)) that event sizes (i.e. fatalities) within each *individual war* i show an approximate power-law distribution which is spread broadly around an exponent value of $\beta_i \sim 2.5$, again with reasonably high p values. The statistical power-law testing proce-

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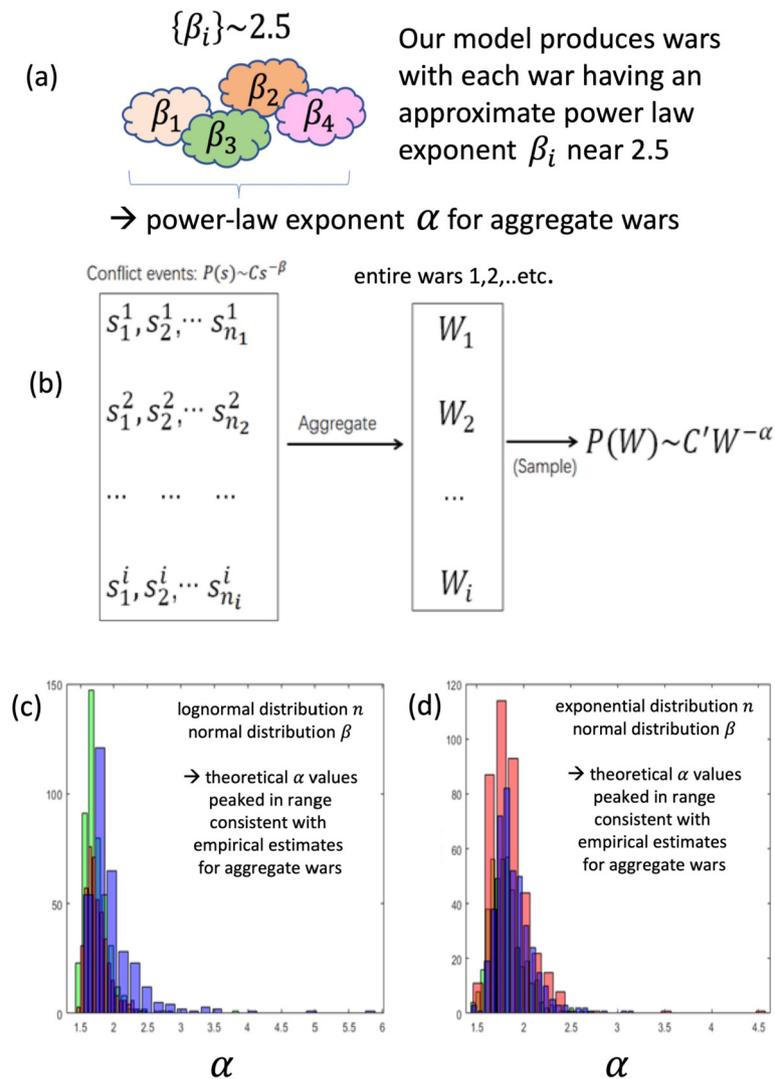


Fig. 1. (a) Schematic of the simulation procedure, shown in more detail in (b). To generate a number of wars consistent with our model, we generate events from power-law distributions with exponents distributed around 2.5 (see text). The aggregate size of each war is calculated, yielding $\{W_i\}$. This set of $\{W_i\}$ values is then used to generate the distribution for whole wars. (c) and (d) show that the resulting power-law exponents for the aggregate size of whole wars, are in the range observed empirically and that this finding is robust. Three examples are shown in each case, corresponding to three different choices of the mean number of events per war. The distribution of α values tends to be peaked in a range consistent with the empirical values for entire wars (i.e. $\alpha \sim 1.5-1.7$) and with similarly high goodness-of-fit values p ($p \sim 0.5$).

ture is the same in both cases: in the latter case of an individual war i , the process involves taking the histogram of the severity (i.e. fatalities) of individual events within this war and performing the standard power-law test to get the best-fit power law exponent (which we will refer to as β_i). In the former case of entire wars, the process involves taking the histogram of the severity for each entire war W_1, W_2, \dots etc. and then getting the best-fit power law exponent (which we will refer to as α).

To show the connection between the power-law exponents for events within wars and the power-law exponents for aggregated totals across whole wars, we start by simulating a set of events for an individual war i (see Fig. 1 1(b)). The number of events in each simulated war is drawn randomly from a lognormal distribution for one set of results, and from an exponential distribution for the other set of results. As shown in Fig. 3, the density for a lognormal distribution of the number of events per simulated war is indeed close to the empirical one, and far closer than for an exponential distribution. Specifically, we generate this number of events within a given war i by randomly sampling from a power-law distribution with power-law exponent β_i , where the value of β_i itself is picked randomly from a normal distribution of β_i

values whose mean value β lies in the vicinity of 2.5 and whose distribution has a spread (i.e. standard deviation) given by δ . For the trivial example of a spread $\delta = 0$ and mean $\beta = 2.5$ the power-law exponent $\beta_i = 2.5$ for all i . We carry out this procedure repeatedly to generate a number of different simulated wars $\{i\}$. The aggregated casualty total for each war i is given by W_i which is the sum of the individual events 1, 2, ... etc. within that war, i.e. $W_i = s_1^i + s_2^i + \dots$. This is shown schematically in Figs. 1(a) and (b). This entire process provides us with a set of values $\{W_i\}$ corresponding to the total fatalities in our simulated model wars $i = 1, 2, \dots$ which represents our model's predicted record for all human wars. We made sure to check that our conclusions are robust to the number of simulated wars. For the results shown, we simulated 100 wars but our conclusions are not sensitive to the number of wars as long as this number isn't too small (e.g. 10 or less). Whether we choose 100 or 150 etc. does not change our conclusions.

Fig. 1 displays our main findings. Fig. 2 provides further detail and illustrates the robustness of the results as we range across different choices of the mean β in the range 1.5 to 4.0 (horizontal axis), and for different choices of the standard deviation of the normal distribution of power-law exponents β_i (i.e. their spread δ around the mean β), as well

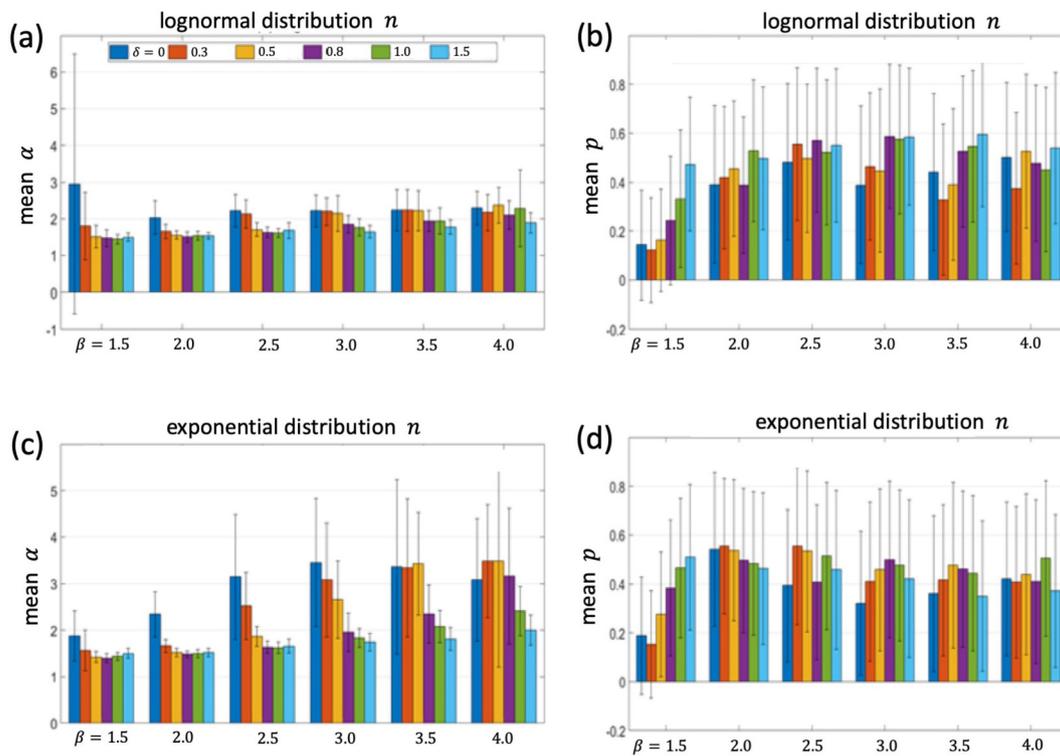


Fig. 2. Robustness of our main result for best-fit power-law exponent values across whole wars (α) from Fig. 1, and goodness-of-fit p , for different values of the mean individual war exponent (β , horizontal axis) and six values of its spread (δ , shown as six different colors). (a) and (b) correspond to a lognormal distribution for the number of events per war n , which is justified empirically (see Fig. 3). (c) and (d) show comparative results for an exponential distribution for n . The error bar in each case indicates the standard deviation. The W_{\min} values tend to be of order of magnitude $\sim 10^3$, akin to empirical conflict findings.

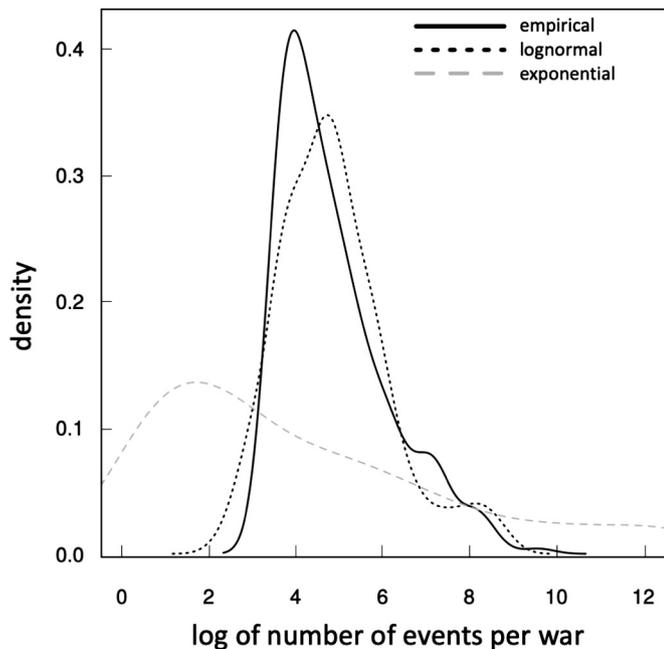


Fig. 3. Density distribution for the log of the number of events per war, from the empirical data in the GED database used in Spagat et al. (2018). Also shown are the best-fit lognormal and exponential distributions. The lognormal is the better fit to the empirical data.

as for different choices of the distribution of the numbers of events n_i for each simulated model war i (mean μ). Figs. 1(c)(d) and Fig. 2 show that the resulting power-law exponents across entire wars (i.e. α) are typically distributed with a peak in the same range $\alpha \sim 1.5-1.7$ as the

reported empirical values in the literature (e.g. see Clauset (2018)) as long as δ is not too small. This is particularly true for the lognormal distribution of the number of events per war (Fig. 3(a) and (b)), which is itself the closest to the empirical data for the number of events per war (Fig. 3). Fig. 2 also shows that the goodness-of-fit values are typically distributed around $p \sim 0.5$ as observed for the empirical data.

Importantly, Figs. 1(c)(d) and 2 confirm that the α distributions are more in line with the empirical findings for whole wars (i.e. power-law exponents in the range $\alpha \sim 1.5-1.7$ and $p \sim 0.5$) when event counts are drawn from lognormal distributions, as compared to when event counts are drawn from exponential distributions. This is exactly consistent with Fig. 3, which shows that the empirical event counts do indeed follow closely a lognormal distribution, and do so much better than an exponential distribution. Hence our simulations are empirically grounded, which lends further weight to our conclusions.

We offer the following intuitive explanation for our main finding that aggregating data from approximate power-law distributions with exponents $\{\beta_i\} \sim 2.5$ (i.e. events within individual wars) yields an approximate power-law distribution with exponent $\alpha \sim 1.5-1.7$ (totals for all wars). Some individual wars will by chance have β_i values well below 2.5, which means that they will each tend to have a relatively high ratio of very large events compared to small events. Moreover, the distribution of total event counts for individual conflicts is similar to a lognormal distribution (see Fig. 3) and hence has a somewhat extended upper tail. Putting these two facts together, there will be a non-negligible number of wars that will have both a large percentage of very large events because of their low β_i 's, and a large number of events overall. When these fatalities are aggregated across the war, this whole war will become very large – and these very large wars will together help fill the right hand tail of the distribution of whole war sizes. This stretches the distribution for whole wars to the right and hence generates a low estimated value of α , i.e. lower than the β_i 's. This explanation is strengthened by the following observation: the fact that the explana-

tion relies on the existence of a wide enough spread of $\{\beta_i\}$ around 2.5, is consistent with the simulations in Fig. 2 showing that when the distribution of the $\{\beta_i\}$ is highly concentrated near 2.5 then estimated values of α tend to exceed 2.0 and hence the distribution of whole war sizes comes out too steep compared to empirical reality (i.e. α too large).

This finding unifies the empirical power-law results for event sizes within individual wars with the empirical power-law result over whole wars, capturing how the scaling coefficient changes as we move from the intra-conflict level to the inter-conflict level. This means that looking at individual violent events within a single war is not the same as looking at individual wars within a collection of many wars, despite the fact that both phenomena can be captured reasonably well by power laws. The verbal message from this observation may seem straightforward to a historian but it is interesting to see it emerge from realistically simulated data while giving us additional insights into how this difference can be quantified. In particular, we see how compiling aggregate data across wars has the impact of lowering the value of the best-fit exponent. As a by-product of our study, we also showed that in the hypothetical case that all wars follow power laws with exponents $\{\beta_i\}$ equal to 2.5, then the power law for whole wars would also probably be somewhere near 2.5.

3. Discussion

We have provided detailed numerical evidence that the approximate power law obtained for whole war sizes (i.e. exponent $\alpha \sim 1.5-1.7$) arises naturally from the aggregation of casualties from individual wars in which the events have an approximate power law with exponents $\{\beta_i\} \sim 2.5$. This arises from the fact that each new war has a reasonable chance to be characterized by both a relatively high ratio of large events to small events *and* also by a large total number of events. This gives rise to enough very large wars in terms of total casualties (i.e. large W) that their effect is to flatten out the power-law and hence reduce the exponent well below 2.5. Our findings strengthen the case for a unified understanding of human conflict from microscopic to macroscopic scales.

Declarations

Author contribution statement

M. Spagat, S. van Weezel, D. J. Restrepo, M. Zheng, N. F. Johnson: Conceived and designed the experiments; Performed the experiments;

Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

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Competing interest statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

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