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Research article

Biased proportional hazard regression estimator in the existence of collinearity

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ABSTRACT

This paper proposed a new biased proportional hazard regression (PHR) estimator which is the combination of elastic net proportional hazard regression (ENPHR) and principal components proportional hazard regression (PCPHR) estimator. Comparison of proposed estimator with ENPHR, PCPHR, ridge PHR, lasso PHR, $r-k$ class PHR and maximum likelihood (ML) estimators is done in terms of scalar mean square error (MSE). Simulation study is conducted to examine the performance of each estimator. Furthermore, the developed estimator is utilized to analyze the infant mortality in Delhi, India.

1. Introduction

Survival Analysis is described as the group of techniques to examine the data in which dependent variable is defined as the time until the happening of an event of interest such as death in biological term or failure of a machinery system. In Survival analysis, proportional hazard regression (PHR) model is widely applicable. It is a regression model which model time data for the happening of an event and investigate the association between the survival time and explanatory variables.

Maximum likelihood (ML) technique is the widely applicable technique for the estimation of PHR model. Collinearity causes unstability in the ML estimates and increases the asymptotic covariance. One of the main reason of collinearity is the linear association among explanatory variables. In several practical situations, explanatory variables have dependency on each other, for example variables such as, birth order, mother age, antenatal care, tetatus toxoid and breastfeeding are frequently considered in the infant survival studies out of which birth order and mother has linear dependency on each other which is analyzed in this study. Collinearity is checked through condition number and If it is present in the dataset then ML estimates may not represent the actual effect of variables. Consequently, other estimators were developed.

In linear regression, Massy [15] proposed principal component regression (PCR) estimator, later Aguilera et al. [1], extended the same method in logistic regression. Lin et al. [12] utilized the principal component approach in PHR model. Hoerl and Kennard [7,8] introduced the estimator called ridge estimator in linear regression, Schafer et al. [23], Williams et al. [31] generalized the ridge estimator to logistic regression, Lukman et al. [13] generalized it for gamma regression model, Omer et al. [17] generalized it for zero-inflated Poisson regression model, Qasim et al. [21] generalized for Beta regression model and Xue et al. [32] spreaded this

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method of estimation in Cox PHR. Tibshirani [28] proposed lasso estimator in linear regression model and extended this approach in Cox PHR model in 1997, [29]. In linear regression model, Zou and Hastie [33] introduced a method called elastic net. Park and Hastie [20], utilized the approach in Cox PH model. In logistic regression, Ozkale and Arican [19] introduced an estimator called r-k class estimator that combined ridge estimator and principal component estimator, later Sirohi and Rai [25] generalized this technique in PHR model.

This research proposed an estimator that combines the elastic net proportional hazard and principal component proportional hazard regression model. Comparison of proposed estimator with PCPHR, ridge PHR, $r-k$ class PHR, lasso PHR, ENPHR and ML estimator is done through simulation. Proposed technique is then utilized to infer the infant's risk of death in Delhi.

2. Estimators in PHR

Cox PHR model [5], is defined as,

$$\gamma(t_i \mid x_i) = \gamma_0(t_i)exp(b'x_i)$$

where $\gamma(t_i \mid x_i)$ is the hazard of i^{th} individual at time $t_i, i = 1, 2, ..., n$, $x_i' = (x_{i1}, x_{i2}, ..., x_{iq})$ is the ith row of X which is a data matrix of order n x q with q known covariates vector, $b' = (b_1, b_2, ..., b_q)$ is the vector of regression coefficients and $\gamma_0(t_i)$ is the base line hazard function. Here, the research focused on the parametric form of proportional hazard model that means survival time follows some distribution. Rai and Sirohi [22], examined that the proportional hazard model performs better with the assumption of Weibull distribution. This study also has the same assumption, that means, $\gamma_0(t_i) = \theta t_i^{\theta-1}$, where θ is the parameter of Weibull distribution. The likelihood functional form of PHR model is defined as,

$$L(b,\theta) = \Pi_{i=1}^{n}(\gamma_0(t_i;\theta)e^{b'x_i})^{\delta_i} e^{-G_0(t_i;\theta)e^{b'x_i}} \tag{1}$$

where, δ_i represents the censoring indicator and having the values 0 (event has been occurred) and 1 (right censored). $G_0(t_i;\theta)$ represents the cumulative base line hazard function. Log-likelihood form of equation (1) is,

$$l(b,\theta) = \sum_{i=1}^{n}[\delta_i(log(\gamma_0(t_i)) + b'x_i) - G_0(t_i)e^{b'x_i}] \tag{2}$$

On Maximizing equation (2), we will get $\hat{b}$ (ML estimator) of b. Equation (2) represents a non-linear equation, therefore Newton-Raphson method is utilized to find the solution.

$$\hat{b}^{(m)} = \hat{b}^{(m-1)} + F(\hat{b}^{(m-1)})^{-1}U(\hat{b}^{(m-1)}) \tag{3}$$

In equation (3), first derivative of equation (2) is $U(\hat{b}^{(m-1)})$ and the negative of second derivative of equation (2) is $F(\hat{b}^{(m-1)})$, it is defined as,

$$F(\hat{b}^{(m-1)}) = X'V^{(m-1)}X \tag{4}$$

In equation (4), $V^{(m-1)}$ represents the diagonal matrix of order n x n with $v_{ii}^{(m-1)} = t_i^{\theta}e^{\hat{b}'^{(m-1)}x_i}$. Here estimation is based on the iterative process and iteration continued until the little changes in estimation. Ozkale and Arican [19] assumed the convergence criteria as, $\left\| b^{(m)} - b^{(m-1)} \right\| \leq 10^{-5}$, this study also follows the same criteria. In estimation collinearity is detected as an obstacle. Collinearity is the reason of unstable estimates and it increases the asymptotic covariance matrix $((X'\hat{V}X)^{-1})$.

To resolve collinearity, this research generalized the Zou and Hastie [33] approach in proportional hazard regression model. Log-likelihood function with penalty is defined as;

$$l_\alpha^k(b,\theta) = l(b,\theta) - k[\ \alpha|b|_1 + \frac{(1-\alpha)}{2}||b||^2] \tag{5}$$

In equation (5), $l_\alpha^k(b,\theta)$ is constricted log-likelihood, $||b|| = (\sum_{j=1}^{q} b_j^2)^{1/2}$ is the L2 norm and $|b|_1 = (\sum_{j=1}^{q}|b_j|)$ is the L1 norm, k and $0 \leq \alpha \leq 1$ are tuning parameters. Newton Raphson method is applied for the estimation of elastic net proportional hazard regression (ENPHR) estimator i.e. b_α^k.

$$\hat{b}_\alpha^{k(m)} = \hat{b}_\alpha^{k(m-1)} + F_\alpha^k(\hat{b}_\alpha^{k(m-1)})^{-1}U_\alpha^k(\hat{b}_\alpha^{k(m-1)}) \tag{6}$$

In equation (6), $U_\alpha^k(\hat{b}_\alpha^{k(m-1)})$ represents the first derivative of constricted log-likelihood and $F_\alpha^k(\hat{b}_\alpha^{k(m-1)})$ represent the negative matrix of second derivative,

$$U_\alpha^k(\hat{b}_\alpha^{k(m-1)}) = U(\hat{b}_\alpha^{k(m-1)}) - k[\alpha\ sgn(\hat{b}_\alpha^{k(m-1)}) + (1-\alpha)\hat{b}_\alpha^{k(m-1)}]$$

$$F_\alpha^k(\hat{b}_\alpha^{k(m-1)}) = F(\hat{b}_\alpha^{k(m-1)}) + k[\alpha\xi + (1-\alpha)I_q]$$

where, $\xi = diag(sgn'(\hat{b}_\alpha^{k(m-1)}))$ and $sgn'(.)$ is representing the derivative of $sgn(.)$ function. I_q is an identity matrix of order q x q. Generally, the choice of k depends on the idea of producing small increase in MSE. To estimate the tuning parameter k, there are many methods exist in literature for both linear and non-linear models [10,11]. In $r-k$ class PHR and ridge PHR model the choice of tuning parameter k has been $1/\hat{b}'\hat{b}$, $q/\hat{b}'\hat{b}$ and $(q+1)/\hat{b}'\hat{b}$ [32,25]. These three choices were investigated in this study.

3. $r-k-\alpha$ class PHR estimator

Smith and Marx [26] and Aguilera et al. [1] applied spectral decomposed information matrix $\psi = X'\hat{V}X$ to propose a reduced model. On using this approach, singular value decomposed information matrix ψ can be represented as $\psi = X'\hat{V}X = Z\Delta Z'$ where $Z = [Z_1 ... Z_q]$ is a q x q orthogonal matrix and $Z'\psi Z = P'\hat{V}P = \Delta = diag(\lambda_j)$ is a q x q diagonal matrix of the eigenvalues of $X'\hat{V}X$ ($\lambda_1 = \lambda_{max} \geq \lambda_2 \geq ... \geq \lambda_q = \lambda_{min}$). The principal components are defined as orthogonal linear spans with maximum variance of the columns of the matrix X denoted by $P_j = XZ_j, (j = 1, 2, ..., q)$, where Z_j are the eigen vectors of the information matrix ψ associated to their corresponding eigen values λ_j which are the variances of corresponding principal components. A reduced set of principal components is used to construct the PCPHR model, that reduces the problem of collinearity.

Aguilera et al. [1] applied logit transformation to represent the logistic regression model in the matrix form. Following this approach with principal components, Sirohi and Rai [25] expressed the PHR model as,

$$L = Xb = XZZ'b = P\mu \tag{7}$$

In equation (7) P and μ are partitioned as $P = [P_r \quad P_{q-r}]$ and $\mu = [\mu_r' \quad \mu_{q-r}']'$ where the principal components included in P_{q-r} $(r \leq q)$ will be deleted. Thus, Δ and Z are partitioned as

$$\begin{bmatrix} \Delta_r & 0 \\ 0 & \Delta_{q-r} \end{bmatrix}$$

and $Z = [Z_r \quad Z_{q-r}]$ where $P_r'\hat{V}P_r = Z_r'X'\hat{V}XZ_r = \Delta_r$ and $P_{q-r}'\hat{V}P_{q-r} = Z_{q-r}'X'\hat{V}XZ_{q-r} = \Delta_{q-r}$. On taking decomposition, logit-transformation is represented as, $L = P\mu = P_r\mu_r + P_{q-r}\mu_{q-r}$. To address collinearity through principal component regression with transition, the model $L = Xb = XZ_rZ_r'b + XZ_{q-r}Z_{q-r}'b = P_r\mu_r + P_{q-r}\mu_{q-r}$ is reduced to the model $L = P_r\mu_r$ where $\mu_r = Z_r'b$ and $P_r = XZ_r$. Reduced PHR model in the view of (1) is written as,

$$\gamma(t_i \mid p_{(r)i}) = \gamma_0(t_i) exp(\mu' p_{(r)i}), \ i = 1, 2, ..., n. \tag{8}$$

In equation (8), $p_{(r)i}' = [p_{i1}, p_{i2}, ..., p_{ir}]$ represent the ith row of P_r and $\mu_r' = [\mu_1, \mu_2, ..., \mu_r]$ is a parameter vector. The log-likelihood in respect of model (9) is,

$$l(\mu_r, \theta) = \sum_{i=1}^{n} [\delta_i(log(\gamma_0(t_i)) + \mu' p_{(r)i}) - G_0(t_i)e^{\mu' p_{(r)i}}] \tag{9}$$

On Maximizing equation (9), the ML estimator $\hat{\mu_r}$ of μ_r under equation (8) is,

$$\hat{\mu}_r^{(m)} = \hat{\mu}_r^{(m-1)} + F(\hat{\mu}_r^{(m-1)})^{-1} U(\hat{\mu}_r^{(m-1)}). \tag{10}$$

In equation (10), $U(\hat{\mu}_r^{(m-1)})$ represents first derivative of equation (9) using and $F(\hat{\mu}_r^{(m-1)})$ represents the negative of second derivative of equation (9), such as, $F(\hat{\mu}_r^{(m-1)})^{-1} = (P_r'V^{(m-1)}P_r)^{-1}$. Findings are converted into the originality as, $\hat{b}_r = Z_r\hat{\mu}_r = Z_rZ_r'\hat{b}$ and known as the PCPHR estimator.

Following the same technique, the reduced log likelihood function of ENPHR model is,

$$l_\alpha^k(\mu_r, \theta) = l(\mu_r, \theta) - k[\ \alpha|\mu_r|_1 + \frac{(1-\alpha)}{2}\|\mu\|_r{}^2] \tag{11}$$

On maximizing equation (11),

$$\hat{\mu}_{r\alpha}^{k(m)} = \hat{\mu}_{r\alpha}^{k(m-1)} + F_\alpha^k(\hat{\mu}_{r\alpha}^{k(m-1)})^{-1} U_\alpha^k(\hat{\mu}_{r\alpha}^{k(m-1)}) \tag{12}$$

where, $U_\alpha^k(\hat{\mu}_{r\alpha}^{k(m-1)})$ is the first derivative of equation (11) and $F_\alpha^k(\hat{\mu}_{r\alpha}^{k(m-1)})$ is the negative of second derivative of equation (11),

$$U_\alpha^k(\hat{\mu}_{r\alpha}^{k(m-1)}) = U(\hat{\mu}_{r\alpha}^{k(m-1)}) - k[\alpha\ sgn(\hat{\mu}_{r\alpha}^{k(m-1)}) + (1-\alpha)\hat{\mu}_{r\alpha}^{k(m-1)}]$$

$$F_\alpha^k(\hat{\mu}_{r\alpha}^{k(m-1)}) = F(\hat{\mu}_{r\alpha}^{k(m-1)}) + k[\alpha\xi + (1-\alpha)I_r]$$

Following Cessie and Houwelingen [3], equation (12) can be written as, $\hat{\mu}_{r\alpha}^{k(m)} = (P_r'\hat{V}P_r + k\alpha\xi + k(1-\alpha)I_r)^{-1}(P_r'\hat{V}P_r)\hat{\mu}_r^{(m)}$ and can be transformed in to the original parameters as, $\hat{b}_{r\alpha}^{k(m)} = Z_r(Z_r'X'\hat{V}XZ_r + k\alpha\xi + k(1-\alpha)I_r)^{-1}Z_r'X'\hat{V}XZ_rZ_r'\hat{b}^{(m)}$ which is called as approximated mth order $r-k-\alpha$ class estimator of the PHR model.

If $\alpha = 0$, $r-k-\alpha$ class estimator converts into $r-k$ class estimator and if $\alpha = 0$ with $r = q$ then $r-k-\alpha$ class estimator converts into ridge PHR and if $r = q$ with $\alpha = 1$ then $r-k-\alpha$ class estimator converts into lasso PHR. If $k = 0$, $r-k-\alpha$ class estimator converts into PCPHR estimator. If $r = q$, $r-k-\alpha$ class PHR estimator converts into ENPHR estimator. Therefore ridge PHR, lasso PHR, PCPHR, ENPHR and ML estimators are the particular cases of $r-k-\alpha$ class estimator.

4. MSE comparison

At final iteration ($\hat{b}^k_{\alpha r}$), asymptotic bias of $r-k-\alpha$ class PHR estimator is,

$$E(\hat{b}^k_{r\alpha}-b)=[Z_r(Z_r'X'\hat{V}XZ_r+k\alpha\xi+k(1-\alpha)I_r)^{-1}Z_r'X'\hat{V}XZ_rZ_r'-I_q]b \tag{13}$$

On using $I_q=Z_rZ_r'+Z_{q-r}Z_{q-r}'$ simplified form of equation (13) is,

$$\begin{aligned}E(\hat{b}^k_{r\alpha}-b)=[&-k(1-\alpha)Z_r(Z_r'X'\hat{V}XZ_r+k\alpha\xi+k(1-\alpha)I_r)^{-1}Z_r'\\&-k\alpha(Z_r'X'\hat{V}XZ_r+k\alpha\xi+k(1-\alpha)I_r)Z_r\xi Z_r'-Z_{q-r}Z_{q-r}']b\end{aligned} \tag{14}$$

asymptotic covariance is,

$$\begin{aligned}Cov(\hat{b}^k_{r\alpha})=&Z_r(Z_r'X'\hat{V}XZ_r+k\alpha\xi+k(1-\alpha)I_r)^{-1}Z_r'X'\hat{V}XZ_rZ_r'Cov(\hat{b})\\&Z_rZ_r'X'\hat{V}XZ_rZ_r'(Z_r'X'\hat{V}XZ_r+k\alpha\xi+k(1-\alpha)I_r)^{-1}\end{aligned} \tag{15}$$

On using equation (14) and equation (15), mean square error (MSE) of $r-k-\alpha$ class PHR estimator is written as,

$$\begin{aligned}MSE(b^k_{r\alpha})=&Z_r(Z_r'X'\hat{V}XZ_r+k\alpha\xi+k(1-\alpha)I_r)^{-1}Z_r'X'\hat{V}XZ_rZ_r'Cov(\hat{b})\\&Z_rZ_r'X'\hat{V}XZ_rZ_r'(Z_r'X'\hat{V}XZ_r+k\alpha\xi+k(1-\alpha)I_r)^{-1}+\\&[-k(1-\alpha)Z_r(Z_r'X'\hat{V}XZ_r+k\alpha\xi+k(1-\alpha)I_r)^{-1}Z_r'\\&-k\alpha(Z_r'X'\hat{V}XZ_r+k\alpha\xi+k(1-\alpha)I_r)Z_r\xi Z_r'-Z_{q-r}Z_{q-r}']bb'\\&[-k(1-\alpha)Z_r(Z_r'X'\hat{V}XZ_r+k\alpha\xi+k(1-\alpha)I_r)^{-1}Z_r'\\&-k\alpha(Z_r'X'\hat{V}XZ_r+k\alpha\xi+k(1-\alpha)I_r)Z_r\xi Z_r'-Z_{q-r}Z_{q-r}']\end{aligned} \tag{16}$$

$(X'\hat{V}X)^{-1}$ is expressed as, $Z\Delta^{-1}Z'=Z_r\Delta_r^{-1}Z_r'+Z_{q-r}\Delta_{q-r}^{-1}Z_{q-r}'$. Since $Z_rZ_{q-r}'=0$, Equation (16) is rewritten as,

$$\begin{aligned}MSE(\hat{b}^k_{r\alpha})=&Z_r(\Delta_r+k\alpha\xi+k(1-\alpha)I_r)^{-1}\Delta_r(\Delta_r+k\alpha\xi+k(1-\alpha)I_r)^{-1}Z_r'\\&+[-k(1-\alpha)Z_r(\Delta_r+k\alpha\xi+k(1-\alpha)I_r)^{-1}Z_r'\\&-k\alpha(\Delta_r+k\alpha\xi+k(1-\alpha)I_r)Z_r\xi Z_r'-Z_{q-r}Z_{q-r}']bb'\\&[-k(1-\alpha)Z_r(\Delta_r+k\alpha\xi+k(1-\alpha)I_r)^{-1}Z_r'\\&-k\alpha(\Delta_r+k\alpha\xi+k(1-\alpha)I_r)Z_r\xi Z_r'-Z_{q-r}Z_{q-r}']\end{aligned} \tag{17}$$

Asymptotic MSE of ENPHR estimator is computed by putting $r=q$ in equation (17).

$$\begin{aligned}MSE(\hat{b}^k_{\alpha})=&Z(\Delta+k\alpha\xi+k(1-\alpha)I_q)^{-1}\Delta(\Delta+k\alpha\xi+k(1-\alpha)I_q)^{-1}Z'\\&+[-k(1-\alpha)Z(\Delta+k\alpha\xi+k(1-\alpha)I_q)^{-1}Z'\\&-k\alpha(\Delta+k\alpha\xi+k(1-\alpha)I_q)Z\xi Z']bb'\\&[-k(1-\alpha)Z(\Delta+k\alpha\xi+k(1-\alpha)I_q)^{-1}Z'\\&-k\alpha(\Delta+k\alpha\xi+k(1-\alpha)I_q)Z\xi Z']\end{aligned}$$

Asymptotic MSE of r-k class estimator is computed by putting $\alpha=0$ in equation (17).

$$\begin{aligned}MSE(\hat{b}^k_{r\alpha})=&Z_r(\Delta_r+kI_r)^{-1}\Delta_r(\Delta_r+kI_r)^{-1}Z_r'+[-kZ_r(\Delta_r+kI_r)^{-1}Z_r'-Z_{q-r}Z_{q-r}']\\&bb'[-kZ_r(\Delta_r+kI_r)^{-1}Z_r'-Z_{q-r}Z_{q-r}']\end{aligned}$$

Asymptotic MSE of ridge PHR estimator is computed by putting $\alpha=0$ and $r=q$ in equation (17).

$$\begin{aligned}MSE(\hat{b}^k_{\alpha})=&Z(\Delta+kI_q)^{-1}\Delta(\Delta+kI_q)^{-1}Z'+[-kZ(\Delta+kI_q)^{-1}Z']\\&bb'[-kZ(\Delta+kI_q)^{-1}Z']\end{aligned}$$

Asymptotic MSE of lasso PHR estimator is computed by putting $r=q$ and $\alpha=1$ in equation (17).

$$\begin{aligned}MSE(\hat{b}^k_{\alpha})=&Z(\Delta+k\xi)^{-1}\Delta(\Delta+k\alpha\xi)^{-1}Z'+[-k(\Delta+k\xi)Z\xi Z']\\&bb'[-k(\Delta+k\xi)Z\xi Z']\end{aligned}$$

Asymptotic MSE of PCPHR estimator is computed by putting $k=0$ in equation (17).

$$MSE(\hat{b}_r)=Z_r(\Delta_r)^{-1}\Delta_r(\Delta_r)^{-1}Z_r'+[Z_{q-r}Z_{q-r}']bb'[Z_{q-r}Z_{q-r}']$$

Table 1
MSE for n = 50.

ρ	n = 50	$\hat{b}$	$\hat{b}_r$
0.6	q = 3	0.0592	0.0305
0.8		0.0942	0.0309
0.9		0.1650	0.0306
0.95		0.3066	0.0309
0.99		1.4380	0.0312
0.6	q = 6	0.1164	0.0914
0.8		0.1548	0.1113
0.9		0.2339	0.1414
0.95		0.3921	0.1854
0.99		1.6676	0.3519
0.6	q = 10	0.2202	0.1881
0.8		0.2630	0.2173
0.9		0.3579	0.2676
0.95		0.5472	0.3468
0.99		2.0564	0.6822

Asymptotic MSE of ML estimator is computed by putting $r = q$ and $k = 0$ in equation (17).

$$MSE(\hat{b}) = Z\Delta^{-1}Z'$$

5. Simulation study

Performance of different PHR estimator is compared through simulation study. A smaller MSE will be used as performance criteria. To complete this task, firstly we generate data. Here, survival function (sf) is defined as,

$$S(t_i) = exp(-G_0(t_i)e^{b'x_i})$$

and cumulative distribution function (cdf) is defined as,

$$F(t_i) = 1 - exp(H_0(t_i)e^{b'x_i})$$

It is assumed that the cdf (denoted by, M) follows uniform distribution between the range 0 and 1 i.e. $M \sim U(0,1)$, then $1 - M \sim U(0,1)$. In continuation the distribution of sf is also considered uniform in the range 0 to 1.

$$M = exp(-G_0(t_i)e^{b'x_i}) \sim U(0,1)$$

$$t = {G_0}^{-1}(-log(M)/e^{b'x_i})$$

Baseline is assumed to follow one parameter ($\theta = 0.5$) Weibull distribution then ${G_0}^{-1}(.) = t^{1/\theta}$, now survival time is defined as,

$$t = (-log(M)/e^{b'x_i})^{1/\theta} \tag{18}$$

Equation (18) is used to generate the survival times with different sizes of sample ($n = 50,\ 100,\ 200$ and 300). Distribution of censoring time is considered as uniform within the range 0 to 5, which provides approx 40% censoring rates. Five choices are considered for correlation coefficient ρ i.e. $0.99,\ 0.95,\ 0.9,\ 0.8$ and 0.6 (strong to modest). Total explanatory variables are taken as, $q = 3,\ 6$ and 10. Values of correlation coefficient are used to generate the two covariates and rest of the explanatory variables are generated through standard normal distribution. Initially the values of parameters are considered as, (0.250, 0.663, 0.450, 0.370, 0.730, −0.570, 0.350, 0.860, 0.930, −0.67) for $q = 10$, (0.250, 0.663, 0.450, −0.370, 0.730, −0.570) for $q = 6$ and (0.250, 0.663, 0.450) for $q = 3$. In the simulation procedure of $r - k$ class PHR model, Sirohi and Rai [25] decided the replication number 1000. In this research the procedure is also repeated 1000 times for each value of n, ρ and q. Convergence standard is considered, 10^{-5} that means iteration is terminated when $\left\| b^{(m)} - b^{(m-1)} \leq 10^{-5} \right\|$. Averaged MSE in case of various values of n, ρ and q are calculated for ML, ridge PHR, Lasso PHR, PCPHR, ENPHR, $r - k$ class and $r - k - \alpha$ class estimators. Tables 1–16 represents the results of simulation.

Ozkale and Arican [19] examined that the MSE of ridge logistic estimator is lower than the MSE of $r - k$ class, principal component and ML estimator when size of the sample is approximate 250 or more than that with severe collinearity and high number of explanatory variables. This study also sees the same behaviour with ridge estimator in PHR model. If the sample size is 50 and 100 then $r - k$ class estimator is performing better than ridge PHR, PCPHR and ML estimator for all categories of ρ and q and benefits are more high when $k = (q+1)/\hat{b}'\hat{b}$. For every sample size considered in the study and for all categories of ρ and q MSE of lasso PHR estimator is smaller than the MSE of PCPHR, ridge PHR, ENPHR, $r - k$ class and ML estimator which is lowest for $k = (q+1)/\hat{b}'\hat{b}$.

Proposed estimator ($r - k - \alpha$ class) has smaller MSE as compared to lasso PHR, PCPHR, ridge PHR, ENPHR, $r - k$ class and ML estimator. It will have minimum MSE for $\alpha = 1$ and $k = (q+1)/\hat{b}'\hat{b}$. Findings are consistent for each category of ρ, q and n.

Table 2
MSE for n = 50 and $k = 1/\hat{b}'\hat{b}$.

ρ		$\hat{b}_{\alpha}^{k}$					$\hat{b}_{r\alpha}^{k}$				
		$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.5$	$\alpha = 0.8$	$\alpha = 1$	$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.5$	$\alpha = 0.8$	$\alpha = 1$
0.6	q = 3	0.0229	0.0113	0.0076	0.0059	0.0051	0.0149	0.0072	0.0045	0.0033	0.0028
0.8		0.0246	0.0125	0.0086	0.0067	0.0058	0.0157	0.0082	0.0054	0.0041	0.0035
0.9		0.0302	0.0176	0.0126	0.0101	0.0090	0.0174	0.0107	0.0078	0.0063	0.0056
0.95		0.0382	0.0251	0.0186	0.0152	0.0136	0.0192	0.0131	0.0102	0.0086	0.0078
0.99		0.1069	0.0918	0.0799	0.0725	0.0689	0.0243	0.0209	0.0189	0.0177	0.0170
0.6	q = 6	0.0823	0.0474	0.0342	0.0277	0.0248	0.0579	0.0331	0.0232	0.0183	0.0161
0.8		0.0911	0.0529	0.0387	0.0317	0.0285	0.0642	0.0376	0.0269	0.0215	0.0191
0.9		0.1008	0.0603	0.0446	0.0367	0.0332	0.0715	0.0431	0.0315	0.0255	0.0228
0.95		0.1192	0.0793	0.0609	0.0510	0.0463	0.0847	0.0551	0.0424	0.0355	0.0323
0.99		0.2245	0.1944	0.1764	0.1670	0.1631	0.1280	0.0926	0.0766	0.0679	0.0638
0.6	q = 10	0.1669	0.1127	0.0880	0.0877	0.0688	0.1451	0.0991	0.0765	0.0642	0.0583
0.8		0.1798	0.1185	0.0919	0.0955	0.0714	0.1553	0.1035	0.0796	0.0667	0.0606
0.9		0.2041	0.1364	0.1067	0.1165	0.0834	0.1736	0.1178	0.0915	0.0771	0.0703
0.95		0.2360	0.1682	0.1356	0.1604	0.1090	0.1982	0.1393	0.1108	0.0948	0.0871
0.99		0.4068	0.3681	0.3532	0.4845	0.3528	0.2831	0.2255	0.1865	0.1638	0.1530

Table 3
MSE for n = 50 and $k = q/\hat{b}'\hat{b}$.

ρ		$\hat{b}_{\alpha}^{k}$					$\hat{b}_{r\alpha}^{k}$				
		$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.5$	$\alpha = 0.8$	$\alpha = 1$	$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.5$	$\alpha = 0.8$	$\alpha = 1$
0.6	q = 3	0.0111	0.0051	0.0032	0.0023	0.0019	0.0079	0.0030	0.0016	0.0011	0.0103
0.8		0.0113	0.0056	0.0036	0.0026	0.0022	0.0086	0.0037	0.0021	0.0015	0.0105
0.9		0.0137	0.0079	0.0055	0.0042	0.0037	0.0105	0.0056	0.0037	0.0028	0.0130
0.95		0.0167	0.0107	0.0076	0.0061	0.0053	0.0124	0.0076	0.0055	0.0044	0.0181
0.99		0.0379	0.0317	0.0278	0.0255	0.0245	0.0190	0.0159	0.0141	0.0129	0.0437
0.6	q = 6	0.0328	0.0156	0.0099	0.0073	0.0063	0.0232	0.0098	0.0057	0.0040	0.0033
0.8		0.0351	0.0177	0.0116	0.0088	0.0075	0.0253	0.0115	0.0069	0.0050	0.0041
0.9		0.0377	0.0199	0.0134	0.0103	0.0089	0.0281	0.0137	0.0086	0.0063	0.0054
0.95		0.0444	0.0254	0.0176	0.0138	0.0121	0.0339	0.0187	0.0128	0.0100	0.0087
0.99		0.0709	0.0515	0.0435	0.0401	0.0390	0.0576	0.0392	0.0314	0.0272	0.0253
0.6	q = 10	0.0641	0.0334	0.0223	0.0170	0.0147	0.0573	0.0269	0.0167	0.0122	0.0103
0.8		0.0634	0.0336	0.0225	0.0171	0.0147	0.0571	0.0275	0.0172	0.0125	0.0105
0.9		0.0684	0.0378	0.0258	0.0199	0.0173	0.0627	0.0318	0.0204	0.0152	0.0130
0.95		0.0775	0.0456	0.0327	0.0242	0.0233	0.0719	0.0397	0.0269	0.0208	0.0181
0.99		0.1116	0.0792	0.0669	0.0579	0.0630	0.1081	0.0719	0.0558	0.0437	0.0437

Table 4
MSE for n = 50 and $k = (q+1)/\hat{b}'\hat{b}$.

ρ		$\hat{b}_{\alpha}^{k}$					$\hat{b}_{r\alpha}^{k}$				
		$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.5$	$\alpha = 0.8$	$\alpha = 1$	$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.5$	$\alpha = 0.8$	$\alpha = 1$
0.6	q = 3	0.0089	0.0040	0.0024	0.0017	0.0014	0.0064	0.0023	0.0012	0.0008	0.0006
0.8		0.0089	0.0044	0.0027	0.0020	0.0016	0.0070	0.0028	0.0016	0.0011	0.0009
0.9		0.0110	0.0063	0.0043	0.0033	0.0028	0.0088	0.0045	0.0029	0.0022	0.0019
0.95		0.0134	0.0083	0.0059	0.0046	0.0041	0.0106	0.0063	0.0045	0.0036	0.0032
0.99		0.0289	0.0237	0.0206	0.0190	0.0182	0.0174	0.0145	0.0128	0.0117	0.0111
0.6	q = 6	0.0294	0.0138	0.0087	0.0063	0.0054	0.0208	0.0085	0.0049	0.0034	0.0028
0.8		0.0314	0.0157	0.0101	0.0076	0.0065	0.0227	0.0101	0.0060	0.0042	0.0035
0.9		0.0339	0.0177	0.0118	0.0090	0.0077	0.0253	0.0120	0.0075	0.0054	0.0046
0.95		0.0403	0.0226	0.0155	0.0121	0.0105	0.0306	0.0166	0.0113	0.0087	0.0076
0.99		0.0647	0.0458	0.0382	0.0349	0.0339	0.0531	0.0359	0.0286	0.0248	0.0230
0.6	q = 10	0.0602	0.0310	0.0206	0.0156	0.0135	0.0539	0.0248	0.0153	0.0111	0.0093
0.8		0.0595	0.0312	0.0207	0.0157	0.0135	0.0536	0.0254	0.0157	0.0113	0.0095
0.9		0.0643	0.0352	0.0238	0.0183	0.0159	0.0589	0.0295	0.0188	0.0139	0.0118
0.95		0.0731	0.0426	0.0304	0.0242	0.0214	0.0678	0.0249	0.0191	0.0191	0.0166
0.99		0.1061	0.0741	0.0620	0.0579	0.0578	0.1030	0.0525	0.0446	0.0446	0.0410

6. Application

This study included 264 infants of Delhi, India, out of which 230 infants were censored. Data is taken from the third phase of National Family Health Survey (NFHS III), India. Infant is defined as the child from zero to twelve months. Outcome variable is the

Table 5
MSE for n = 100.

ρ	n = 50	$\hat{b}$	$\hat{b}_r$
0.6	q = 3	0.0284	0.0143
0.8		0.0450	0.0143
0.9		0.0790	0.0142
0.95		0.1473	0.0143
0.99		0.6960	0.0141
0.6	q = 6	0.0521	0.0416
0.8		0.0692	0.0504
0.9		0.1050	0.0646
0.95		0.1769	0.0807
0.99		0.7527	0.1518
0.6	q = 10	0.0876	0.0768
0.8		0.1065	0.0903
0.9		0.1446	0.1104
0.95		0.2218	0.1440
0.99		0.8373	0.2821

Table 6
MSE for n = 100 and $k = 1/\hat{b}'\hat{b}$.

ρ		$\hat{b}^k_\alpha$					$\hat{b}^k_{r\alpha}$				
		α = 0	α = 0.2	α = 0.5	α = 0.8	α = 1	α = 0	α = 0.2	α = 0.5	α = 0.8	α = 1
0.6	q = 3	0.0134	0.0064	0.0043	0.0034	0.0029	0.0086	0.0043	0.0027	0.0020	0.0017
0.8		0.0153	0.0078	0.0055	0.0044	0.0039	0.0092	0.0051	0.0034	0.0026	0.0023
0.9		0.0170	0.0092	0.0065	0.0052	0.0046	0.0095	0.0057	0.0041	0.0032	0.0029
0.95		0.0219	0.0130	0.0093	0.0074	0.0066	0.0104	0.0072	0.0056	0.0047	0.0043
0.99		0.0507	0.0395	0.0314	0.0265	0.0242	0.0116	0.0095	0.0083	0.0077	0.0073
0.6	q = 6	0.0418	0.0239	0.0172	0.0140	0.0125	0.0297	0.0169	0.0118	0.0092	0.0081
0.8		0.0459	0.0255	0.0184	0.0150	0.0135	0.0325	0.0185	0.0130	0.0103	0.0091
0.9		0.0509	0.0285	0.0208	0.0170	0.0153	0.0364	0.0211	0.0151	0.0121	0.0107
0.95		0.0553	0.0325	0.0237	0.0194	0.0174	0.0401	0.0238	0.0173	0.0140	0.0125
0.99		0.0944	0.0731	0.0597	0.0519	0.0482	0.0565	0.0397	0.0318	0.0274	0.0254
0.6	q = 10	0.0709	0.0448	0.0338	0.0282	0.0256	0.0627	0.0406	0.0303	0.0248	0.0223
0.8		0.0788	0.0479	0.0359	0.0298	0.0271	0.0691	0.0435	0.0323	0.0264	0.0236
0.9		0.0868	0.0518	0.0388	0.0322	0.0293	0.0751	0.0468	0.0348	0.0285	0.0255
0.95		0.0965	0.0598	0.0449	0.0373	0.0337	0.0831	0.0535	0.0402	0.0331	0.0299
0.99		0.1409	0.1086	0.0903	0.0798	0.0748	0.1115	0.0798	0.0629	0.0535	0.0490

Table 7
MSE for n = 100 and $k = q/\hat{b}'\hat{b}$.

ρ		$\hat{b}^k_\alpha$					$\hat{b}^k_{r\alpha}$				
		α = 0	α = 0.2	α = 0.5	α = 0.8	α = 1	α = 0	α = 0.2	α = 0.5	α = 0.8	α = 1
0.6	q = 3	0.0071	0.0031	0.0019	0.0013	0.0011	0.0051	0.0019	0.0010	0.0007	0.0005
0.8		0.0076	0.0039	0.0025	0.0018	0.0015	0.0059	0.0025	0.0015	0.0010	0.0009
0.9		0.0079	0.0044	0.0029	0.0021	0.0018	0.0064	0.0031	0.0019	0.0014	0.0012
0.95		0.0096	0.0059	0.0041	0.0031	0.0027	0.0075	0.0044	0.0031	0.0024	0.0022
0.99		0.0184	0.0142	0.0113	0.0096	0.0088	0.0094	0.0072	0.0061	0.0055	0.0052
0.6	q = 6	0.0194	0.0087	0.0053	0.0038	0.0032	0.0139	0.0054	0.0029	0.0020	0.0016
0.8		0.0197	0.0093	0.0058	0.0042	0.0035	0.0145	0.0060	0.0034	0.0023	0.0019
0.9		0.0206	0.0103	0.0067	0.0049	0.0042	0.0157	0.0071	0.0042	0.0029	0.0024
0.95		0.0216	0.0112	0.0073	0.0054	0.0046	0.0171	0.0081	0.0051	0.0037	0.0031
0.99		0.0317	0.0208	0.0160	0.0135	0.0124	0.0265	0.0166	0.0124	0.0103	0.0093
0.6	q = 10	0.0304	0.0135	0.0082	0.0059	0.0049	0.0274	0.0106	0.0058	0.0039	0.0032
0.8		0.0310	0.0140	0.0086	0.0061	0.0051	0.0281	0.0112	0.0062	0.0042	0.0034
0.9		0.0314	0.0147	0.0091	0.0065	0.0055	0.0290	0.0120	0.0068	0.0046	0.0038
0.95		0.0331	0.0162	0.0103	0.0076	0.0064	0.0311	0.0141	0.0084	0.0060	0.0050
0.99		0.0430	0.0252	0.0183	0.0149	0.0133	0.0430	0.0242	0.0168	0.0132	0.0116

duration of the survival of infants. PHR model is used to evaluate the impact of birth order (Bord), mother age (Mage), antenatal care (ANC), tetanus toxoid (TT) and breastfeeding (BF) on the survival of infant. Several research have been taken to see the relationship of infant and child survival with different socio-economic, demographic, biological and environmental factors [9,4,2,6,16,27,24]. Kaplan Meier survival curve is utilized to see the pattern of infant survival with respect to different factors. Fig. 1 depicts that the

Table 8
MSE for n = 100 and $k = (q+1)/\hat{b}'\hat{b}$.

ρ		$\hat{b}^k_\alpha$					$\hat{b}^k_{r\alpha}$				
		α = 0	α = 0.2	α = 0.5	α = 0.8	α = 1	α = 0	α = 0.2	α = 0.5	α = 0.8	α = 1
0.6	q = 3	0.0058	0.0025	0.0014	0.0010	0.0008	0.0043	0.0015	0.0007	0.0005	0.0004
0.8		0.0062	0.0032	0.0019	0.0014	0.0011	0.0051	0.0020	0.0011	0.0008	0.0006
0.9		0.0064	0.0035	0.0022	0.0016	0.0014	0.0055	0.0025	0.0015	0.0011	0.0009
0.95		0.0078	0.0047	0.0032	0.0025	0.0021	0.0066	0.0038	0.0026	0.0020	0.0017
0.99		0.0142	0.0108	0.0086	0.0074	0.0068	0.0087	0.0066	0.0056	0.0050	0.0047
0.6	q = 6	0.0177	0.0078	0.0047	0.0033	0.0027	0.0126	0.0047	0.0025	0.0016	0.0013
0.8		0.0179	0.0083	0.0050	0.0036	0.0030	0.0132	0.0053	0.0029	0.0020	0.0016
0.9		0.0188	0.0093	0.0059	0.0043	0.0036	0.0143	0.0062	0.0036	0.0025	0.0021
0.95		0.0197	0.0100	0.0064	0.0047	0.0040	0.0156	0.0072	0.0044	0.0032	0.0027
0.99		0.0291	0.0186	0.0142	0.0119	0.0109	0.0245	0.0152	0.0113	0.0093	0.0084
0.6	q = 10	0.0287	0.0125	0.0075	0.0053	0.0044	0.0258	0.0098	0.0053	0.0035	0.0028
0.8		0.0292	0.0130	0.0079	0.0055	0.0046	0.0265	0.0103	0.0056	0.0037	0.0030
0.9		0.0297	0.0137	0.0084	0.0059	0.0049	0.0273	0.0111	0.0062	0.0042	0.0034
0.95		0.0313	0.0151	0.0095	0.0070	0.0059	0.0294	0.0131	0.0077	0.0054	0.0045
0.99		0.0411	0.0237	0.0171	0.0138	0.0123	0.0410	0.0228	0.0157	0.0123	0.0108

Table 9
MSE for n = 200.

ρ	n = 50	$\hat{b}$	$\hat{b}_r$
0.6	q = 3	0.0139	0.0069
0.8		0.0221	0.0068
0.9		0.0389	0.0068
0.95		0.0724	0.0069
0.99		0.3410	0.0068
0.6	q = 6	0.0246	0.0198
0.8		0.0329	0.0241
0.9		0.0500	0.0303
0.95		0.0843	0.0388
0.99		0.3600	0.0679
0.6	q = 10	0.0397	0.0352
0.8		0.0483	0.0411
0.9		0.0659	0.0512
0.95		0.1010	0.0662
0.99		0.3855	0.1317

Table 10
MSE for n = 200 and $k = 1/\hat{b}'\hat{b}$.

ρ		$\hat{b}^k_\alpha$					$\hat{b}^k_{r\alpha}$				
		α = 0	α = 0.2	α = 0.5	α = 0.8	α = 1	α = 0	α = 0.2	α = 0.5	α = 0.8	α = 1
0.6	q = 3	0.0086	0.0047	0.0034	0.0028	0.0025	0.0052	0.0032	0.0023	0.0018	0.0015
0.8		0.0099	0.0054	0.0040	0.0033	0.0029	0.0053	0.0034	0.0025	0.0020	0.0018
0.9		0.0109	0.0060	0.0045	0.0037	0.0033	0.0054	0.0037	0.0028	0.0023	0.0020
0.95		0.0129	0.0077	0.0058	0.0048	0.0043	0.0056	0.0041	0.0032	0.0027	0.0025
0.99		0.0274	0.0199	0.0151	0.0125	0.0113	0.0060	0.0050	0.0044	0.0041	0.0039
0.6	q = 6	0.0223	0.0139	0.0104	0.0088	0.0080	0.0159	0.0100	0.0073	0.0059	0.0053
0.8		0.0251	0.0147	0.0110	0.0092	0.0083	0.0177	0.0107	0.0078	0.0063	0.0056
0.9		0.0282	0.0159	0.0120	0.0100	0.0091	0.0199	0.0119	0.0086	0.0069	0.0062
0.95		0.0308	0.0175	0.0132	0.0110	0.0100	0.0219	0.0132	0.0096	0.0078	0.0069
0.99		0.0489	0.0347	0.0267	0.0223	0.0202	0.0301	0.0202	0.0157	0.0134	0.0124
0.6	q = 10	0.0344	0.0227	0.0174	0.0147	0.0135	0.0307	0.0209	0.0160	0.0133	0.0120
0.8		0.0387	0.0240	0.0183	0.0154	0.0141	0.0339	0.0222	0.0168	0.0139	0.0126
0.9		0.0437	0.0254	0.0192	0.0162	0.0148	0.0378	0.0239	0.0179	0.0148	0.0133
0.95		0.0494	0.0284	0.0215	0.0180	0.0164	0.0423	0.0271	0.0204	0.0169	0.0153
0.99		0.0689	0.0476	0.0369	0.0312	0.0285	0.0568	0.0413	0.0320	0.0270	0.0247

survival of infant is higher if infant is breastfeeding compared to the infant not breastfeeding. In the same manner we can see the pattern of other factors (Fig. 2–6). In this data birth order and mother age has moderate correlation (i.e. 0.64) to each other.

To examine collinearity, correlation between explanatory variable misleads some time. Hence Ozkale [18] approach is utilized to calculate the condition number which is 31.069 for the matrix $[(X^*)'(X^*)]$, here $X^* = V^{(1/2)}X$. Condition number is more than 30

Table 11
MSE for n = 200 and $k = q/\hat{b}'\hat{b}$.

ρ		$\hat{b}_{\alpha}^{k}$					$\hat{b}_{r\alpha}^{k}$				
		$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.5$	$\alpha = 0.8$	$\alpha = 1$	$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.5$	$\alpha = 0.8$	$\alpha = 1$
0.6	q = 3	0.0053	0.0027	0.0017	0.0013	0.0010	0.0038	0.0018	0.0010	0.0007	0.0006
0.8		0.0054	0.0030	0.0020	0.0015	0.0013	0.0041	0.0021	0.0013	0.0009	0.0007
0.9		0.0052	0.0033	0.0022	0.0017	0.0014	0.0043	0.0023	0.0015	0.0011	0.0009
0.95		0.0057	0.0040	0.0028	0.0022	0.0019	0.0046	0.0027	0.0019	0.0014	0.0013
0.99		0.0102	0.0080	0.0061	0.0050	0.0045	0.0052	0.0040	0.0033	0.0030	0.0028
0.6	q = 6	0.0126	0.0062	0.0039	0.0028	0.0023	0.0091	0.0039	0.0022	0.0015	0.0012
0.8		0.0125	0.0063	0.0040	0.0029	0.0024	0.0094	0.0041	0.0023	0.0016	0.0013
0.9		0.0126	0.0067	0.0044	0.0032	0.0027	0.0099	0.0044	0.0026	0.0018	0.0015
0.95		0.0127	0.0071	0.0047	0.0035	0.0030	0.0106	0.0049	0.0030	0.0021	0.0018
0.99		0.0165	0.0107	0.0079	0.0063	0.0056	0.0148	0.0089	0.0065	0.0053	0.0048
0.6	q = 10	0.0180	0.0081	0.0049	0.0035	0.0028	0.0163	0.0065	0.0035	0.0023	0.0018
0.8		0.0181	0.0083	0.0051	0.0035	0.0029	0.0165	0.0067	0.0036	0.0024	0.0019
0.9		0.0181	0.0087	0.0054	0.0038	0.0031	0.0169	0.0071	0.0039	0.0026	0.0021
0.95		0.0186	0.0094	0.0061	0.0044	0.0037	0.0178	0.0081	0.0047	0.0032	0.0026
0.99		0.0218	0.0126	0.0089	0.0071	0.0062	0.0227	0.0127	0.0086	0.0067	0.0058

Table 12
MSE for n = 200 and $k = (q+1)/\hat{b}'\hat{b}$.

ρ		$\hat{b}_{\alpha}^{k}$					$\hat{b}_{r\alpha}^{k}$				
		$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.5$	$\alpha = 0.8$	$\alpha = 1$	$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.5$	$\alpha = 0.8$	$\alpha = 1$
0.6	q = 3	0.0046	0.0023	0.0014	0.0009	0.0008	0.0034	0.0014	0.0008	0.0005	0.0004
0.8		0.0045	0.0026	0.0016	0.0012	0.0010	0.0037	0.0017	0.0010	0.0007	0.0005
0.9		0.0044	0.0027	0.0018	0.0013	0.0011	0.0039	0.0019	0.0012	0.0008	0.0007
0.95		0.0047	0.0033	0.0022	0.0017	0.0015	0.0043	0.0023	0.0015	0.0012	0.0010
0.99		0.0080	0.0062	0.0048	0.0039	0.0035	0.0050	0.0037	0.0030	0.0027	0.0025
0.6	q = 6	0.0118	0.0056	0.0034	0.0024	0.0019	0.0085	0.0035	0.0019	0.0012	0.0010
0.8		0.0116	0.0058	0.0035	0.0025	0.0020	0.0087	0.0036	0.0020	0.0013	0.0011
0.9		0.0117	0.0061	0.0039	0.0028	0.0023	0.0092	0.0040	0.0023	0.0015	0.0013
0.95		0.0118	0.0065	0.0042	0.0031	0.0026	0.0098	0.0044	0.0026	0.0019	0.0015
0.99		0.0153	0.0098	0.0071	0.0057	0.0050	0.0138	0.0082	0.0059	0.0048	0.0043
0.6	q = 10	0.0172	0.0076	0.0046	0.0031	0.0026	0.0156	0.0060	0.0032	0.0021	0.0016
0.8		0.0173	0.0078	0.0047	0.0032	0.0026	0.0158	0.0062	0.0033	0.0021	0.0017
0.9		0.0172	0.0082	0.0050	0.0035	0.0028	0.0161	0.0066	0.0036	0.0024	0.0019
0.95		0.0178	0.0089	0.0056	0.0041	0.0034	0.0170	0.0075	0.0043	0.0029	0.0024
0.99		0.0209	0.0119	0.0084	0.0066	0.0057	0.0217	0.0120	0.0081	0.0062	0.0054

Table 13
MSE for n = 300.

ρ	n = 50	$\hat{b}$	$\hat{b}_r$
0.6	q = 3	0.0092	0.0045
0.8		0.0147	0.0045
0.9		0.0257	0.0045
0.95		0.0481	0.0045
0.99		0.2270	0.0045
0.6	q = 6	0.0161	0.0130
0.8		0.0216	0.0159
0.9		0.0329	0.0201
0.95		0.0554	0.0263
0.99		0.2363	0.0497
0.6	q = 10	0.0257	0.0229
0.8		0.0313	0.0267
0.9		0.0427	0.0334
0.95		0.0656	0.0429
0.99		0.2497	0.0858

which means data is collinear, Mackinnon and Puterman [14]; Weissfeld and Sereika [30]; Ozkale [18]. Therefore, proposed method is applied to examine the impact of different factors on infant survival and compared with the existing methods in terms of MSE.

Table 17 represents the estimates of different methods on NFHS data. ML estimates reveal that there is a 0.821 (exp(-0.19711)) times low risk of infant death who breastfed additional one months than other. Risk of infant death is 0.957 times lower for the infants where mother age is one year more as compared to other. In terms of real units, if ANC, TT and family size increased by

Table 14
MSE for n = 300 and $k = 1/\hat{b}'\hat{b}$.

ρ		$\hat{b}^k_{\alpha}$					$\hat{b}^k_{r\alpha}$				
		$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.5$	$\alpha = 0.8$	$\alpha = 1$	$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.5$	$\alpha = 0.8$	$\alpha = 1$
0.6	q = 3	0.0064	0.0039	0.0030	0.0026	0.0023	0.0037	0.0026	0.0020	0.0017	0.0015
0.8		0.0078	0.0046	0.0036	0.0031	0.0028	0.0038	0.0028	0.0023	0.0019	0.0018
0.9		0.0087	0.0049	0.0039	0.0033	0.0030	0.0038	0.0029	0.0024	0.0021	0.0019
0.95		0.0101	0.0063	0.0050	0.0041	0.0037	0.0038	0.0030	0.0025	0.0022	0.0020
0.99		0.0183	0.0127	0.0095	0.0077	0.0069	0.0040	0.0035	0.0032	0.0029	0.0028
0.6	q = 6	0.0156	0.0106	0.0083	0.0072	0.0067	0.0111	0.0077	0.0060	0.0050	0.0046
0.8		0.0179	0.0111	0.0087	0.0075	0.0070	0.0126	0.0082	0.0063	0.0052	0.0047
0.9		0.0206	0.0121	0.0095	0.0081	0.0075	0.0143	0.0091	0.0069	0.0056	0.0051
0.95		0.0232	0.0133	0.0103	0.0088	0.0081	0.0160	0.0101	0.0076	0.0063	0.0057
0.99		0.0343	0.0230	0.0175	0.0146	0.0132	0.0223	0.0155	0.0121	0.0103	0.0095
0.6	q = 10	0.0231	0.0163	0.0128	0.0111	0.0103	0.0206	0.0150	0.0119	0.0102	0.0093
0.8		0.0263	0.0172	0.0135	0.0116	0.0108	0.0230	0.0160	0.0126	0.0108	0.0099
0.9		0.0305	0.0181	0.0141	0.0122	0.0113	0.0261	0.0172	0.0134	0.0113	0.0104
0.95		0.0348	0.0195	0.0151	0.0129	0.0119	0.0293	0.0189	0.0144	0.0121	0.0110
0.99		0.0459	0.0289	0.0219	0.0183	0.0167	0.0381	0.0265	0.0208	0.0176	0.0161

Table 15
MSE for n = 300 and $k = q/\hat{b}'\hat{b}$.

ρ		$\hat{b}^k_{\alpha}$					$\hat{b}^k_{r\alpha}$				
		$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.5$	$\alpha = 0.8$	$\alpha = 1$	$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.5$	$\alpha = 0.8$	$\alpha = 1$
0.6	q = 3	0.0044	0.0025	0.0018	0.0014	0.0012	0.0030	0.0017	0.0011	0.0008	0.0006
0.8		0.0044	0.0029	0.0021	0.0016	0.0014	0.0032	0.0020	0.0013	0.0009	0.0008
0.9		0.0042	0.0029	0.0021	0.0016	0.0014	0.0033	0.0021	0.0014	0.0011	0.0009
0.95		0.0045	0.0035	0.0026	0.0020	0.0017	0.0034	0.0023	0.0016	0.0013	0.0011
0.99		0.0066	0.0054	0.0041	0.0033	0.0030	0.0037	0.0029	0.0024	0.0021	0.0020
0.6	q = 6	0.0100	0.0055	0.0037	0.0027	0.0022	0.0072	0.0036	0.0021	0.0014	0.0012
0.8		0.0099	0.0056	0.0038	0.0028	0.0024	0.0075	0.0037	0.0022	0.0015	0.0012
0.9		0.0098	0.0058	0.0039	0.0029	0.0024	0.0079	0.0039	0.0023	0.0016	0.0013
0.95		0.0098	0.0061	0.0042	0.0032	0.0027	0.0084	0.0043	0.0027	0.0019	0.0016
0.99		0.0115	0.0080	0.0060	0.0049	0.0043	0.0112	0.0069	0.0050	0.0040	0.0036
0.6	q = 10	0.0138	0.0068	0.0043	0.0030	0.0025	0.0125	0.0055	0.0025	0.0020	0.0016
0.8		0.0140	0.0071	0.0046	0.0033	0.0027	0.0128	0.0059	0.0028	0.0022	0.0018
0.9		0.0138	0.0073	0.0048	0.0035	0.0029	0.0130	0.0061	0.0029	0.0024	0.0020
0.95		0.0137	0.0075	0.0050	0.0036	0.0030	0.0135	0.0064	0.0031	0.0026	0.0021
0.99		0.0149	0.0090	0.0064	0.0051	0.0044	0.0160	0.0091	0.0051	0.0045	0.0039

Table 16
MSE for n = 300 and $k = (q+1)/\hat{b}\hat{b}'$.

ρ		$\hat{b}^k_{\alpha}$					$\hat{b}^k_{r\alpha}$				
		$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.5$	$\alpha = 0.8$	$\alpha = 1$	$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.5$	$\alpha = 0.8$	$\alpha = 1$
0.6	q = 3	0.0038	0.0022	0.0015	0.0011	0.0009	0.0028	0.0015	0.0009	0.0006	0.0005
0.8		0.0038	0.0025	0.0017	0.0012	0.0010	0.0031	0.0017	0.0010	0.0007	0.0006
0.9		0.0035	0.0025	0.0017	0.0013	0.0011	0.0032	0.0018	0.0012	0.0008	0.0007
0.95		0.0037	0.0030	0.0021	0.0016	0.0014	0.0032	0.0020	0.0014	0.0011	0.0009
0.99		0.0052	0.0043	0.0033	0.0027	0.0024	0.0036	0.0027	0.0022	0.0019	0.0018
0.6	q = 6	0.0095	0.0051	0.0033	0.0023	0.0019	0.0068	0.0033	0.0018	0.0012	0.0010
0.8		0.0093	0.0052	0.0034	0.0025	0.0020	0.0070	0.0033	0.0019	0.0013	0.0010
0.9		0.0092	0.0053	0.0035	0.0025	0.0021	0.0074	0.0035	0.0020	0.0014	0.0011
0.95		0.0092	0.0056	0.0038	0.0028	0.0024	0.0079	0.0039	0.0024	0.0017	0.0014
0.99		0.0108	0.0074	0.0055	0.0044	0.0039	0.0105	0.0064	0.0046	0.0037	0.0032
0.6	q = 10	0.0133	0.0065	0.0040	0.0028	0.0023	0.0120	0.0052	0.0028	0.0018	0.0015
0.8		0.0134	0.0068	0.0043	0.0030	0.0025	0.0124	0.0055	0.0031	0.0020	0.0016
0.9		0.0132	0.0070	0.0045	0.0032	0.0026	0.0125	0.0057	0.0033	0.0022	0.0018
0.95		0.0132	0.0071	0.0046	0.0033	0.0028	0.0129	0.0060	0.0035	0.0024	0.0019
0.99		0.0144	0.0086	0.0061	0.0047	0.0041	0.0154	0.0086	0.0056	0.0042	0.0036

one unit, expected log of the relative hazard will decrease by 0.00335, 0.08488 and 0.00615 units respectively and these results are significant.

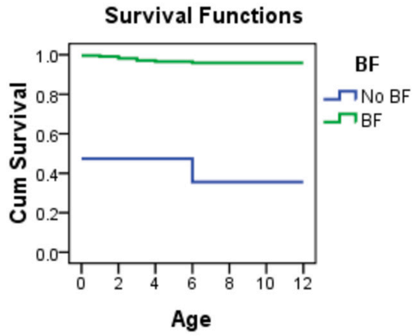

Fig. 1. Survival curves for Breastfeeding.

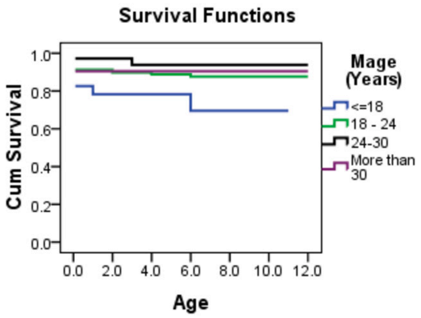

Fig. 2. Survival curves for Mother Age.

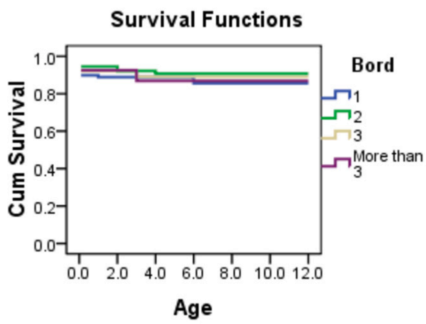

Fig. 3. Survival curves for Birth Order.

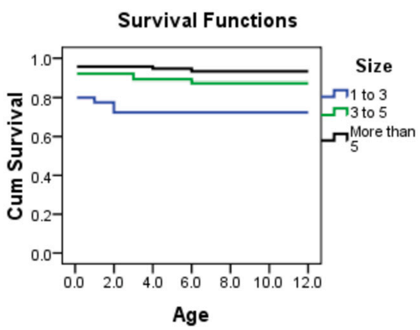

Fig. 4. Survival curves for Family Size.

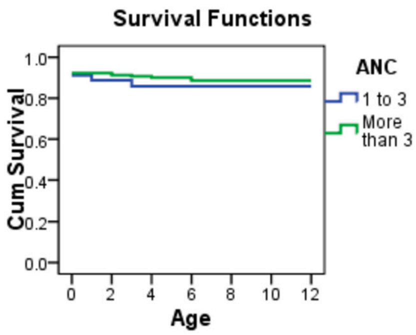

Fig. 5. Survival curves for Antenatal Care.

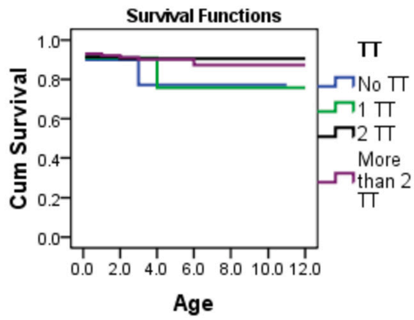

Fig. 6. Survival curves for Tetanus Toxoid.

Table 17
Estimated Values of parameters on NFHS data.

Category	$\hat{b}$	$\hat{b}_r$	$\hat{b}^k_\alpha$					$\hat{b}^k_{r\alpha}$				
$k = 1/\hat{b}'\hat{b}$			$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.5$	$\alpha = 0.8$	$\alpha = 1$	$\alpha = 0$	$\alpha = 0.2$	$\alpha = 0.5$	$\alpha = 0.8$	$\alpha = 1$
BF	-0.19710*	-0.16356*	-0.19306*	-0.19309*	-0.19305*	-0.19301*	-0.19299*	-0.16349*	-0.16322*	-0.16275*	-0.16229*	-0.16198*
Mage	-0.04332*	-0.02233*	-0.04333*	-0.04331*	-0.04328*	-0.04325*	-0.04323*	-0.02216*	-0.02216*	-0.02215*	-0.02211*	-0.02209*
Bord	0.02376*	0.01016	0.02376*	0.02357*	0.02330*	0.02302*	0.02284*	0.01020	0.01016	0.01015	0.01015	0.01015
Size	-0.00615*	-0.00413*	-0.00616*	-0.00617*	-0.00610*	-0.00612*	-0.00614*	-0.00414*	-0.00413*	-0.00414*	-0.00414*	-0.00415*
ANC	-0.00335*	-0.20977*	-0.00337*	-0.00340*	-0.00347*	-0.00353*	-0.00357*	-0.20974*	-0.20953*	-0.20918*	-0.20885*	-0.20863*
TT	-0.08488*	-0.03578*	-0.08469*	-0.08457*	-0.08425*	-0.08394*	-0.08373*	-0.03574*	-0.02695*	-0.03866*	-0.03129*	-0.03027*
$k = q/\hat{b}'\hat{b}$												
BF			-0.19688*	-0.19705*	-0.19713*	-0.19721*	-0.19725*	-0.16328*	-0.16127*	-0.15899*	-0.15656*	-0.15502*
Mage			-0.04240*	-0.04225*	-0.04209*	-0.04195*	-0.04187*	-0.02323*	-0.02336*	-0.02362*	-0.02398*	-0.02424*
Bord			0.02375*	0.02266*	0.02114*	0.01975*	0.01890*	0.01034	0.01025	0.01041	0.01074	0.01098
Size			-0.00603*	-0.00609*	-0.00624*	-0.00638*	-0.00647*	-0.00417*	-0.00418*	-0.00427*	-0.00442*	-0.00454*
ANC			-0.00347*	-0.00366*	-0.00400*	-0.00431*	-0.00451*	-0.20965*	-0.20817*	-0.20662*	-0.20509*	-0.20416*
TT			-0.08373*	-0.08305*	-0.08126*	-0.07956*	-0.07848*	-0.03564*	-0.04313*	-0.04975*	-0.04548*	-0.03857*
$k = (q+1)/\hat{b}'\hat{b}$												
BF			-0.19684*	-0.19704*	-0.19714*	-0.19722*	-0.19727*	-0.16332*	-0.16159*	-0.15828*	-0.15551*	-0.15376*
Mage			-0.04242*	-0.04223*	-0.04206*	-0.04190*	-0.04181*	-0.02322*	-0.02332*	-0.02373*	-0.02416*	-0.02447*
Bord			0.02374*	0.02248*	0.02074*	0.01917*	0.01822*	0.01031	0.01022	0.01051	0.01091	0.01121
Size			-0.00605*	-0.00612*	-0.00628*	-0.00644*	-0.00654*	-0.00417*	-0.00417*	-0.00431*	-0.00450*	-0.00464*
ANC			-0.00349*	-0.00371*	-0.00410*	-0.00445*	-0.00466*	-0.20967*	-0.20838*	-0.20617*	-0.20445*	-0.20342*
TT			-0.08354*	-0.08276*	-0.08070*	-0.07876*	-0.07752*	-0.03566*	-0.04220*	-0.05147*	-0.05757*	-0.06077*

Statistical significance: * at 5% level of significance.
BF: Breastfeeding.
Mage: Mother Age.
Bord: Birth Order.
Size: Family size.
ANC: Antenatal Care Visit.
TT: Tetanus Toxoid.

Table 18
MSE of different estimators on NFHS data.

Parameter	MSE $k = 1/\hat{b}'\hat{b}$	MSE $k = q/\hat{b}'\hat{b}$	MSE $k = (q+1)/\hat{b}'\hat{b}$
$\hat{b}$	0.008932		
$\hat{b}_r$	0.008599		
$\hat{b}_\alpha^k(\alpha = 0)$	0.008492	0.007853	0.007585
$\hat{b}_\alpha^k(\alpha = 0.2)$	0.007615	0.004732	0.004423
$\hat{b}_\alpha^k(\alpha = 0.5)$	0.006712	0.003942	0.003896
$\hat{b}_\alpha^k(\alpha = 0.8)$	0.006159	0.003935	0.003876
$\hat{b}_\alpha^k(\alpha = 1)$	0.005835	0.003921	0.003756
$\hat{b}_{r\alpha}^k(\alpha = 0)$	0.007924	0.006882	0.006762
$\hat{b}_{r\alpha}^k(\alpha = 0.2)$	0.006537	0.002493	0.002373
$\hat{b}_{r\alpha}^k(\alpha = 0.5)$	0.004373	0.002342	0.002127
$\hat{b}_{r\alpha}^k(\alpha = 0.8)$	0.003314	0.002273	0.002115
$\hat{b}_{r\alpha}^k(\alpha = 1)$	0.002962	0.002214	0.002105

Table 18 shows the MSE for each method. PCPHR estimator has lesser MSE as compared to ML, ridge, Elastic net and lasso estimator for various categories of k. MSE of $r-k-\alpha$ estimator is lower as compared to other all estimators and the value is lowest for $\alpha = 1$ and $k = (q+1)/\hat{b}'\hat{b}$.

7. Conclusion and discussion

This paper developed a new class of estimator i.e. $r-k-\alpha$ class which is the combination of elastic net proportional hazard model and principal component proportional hazard model. To examine the performance of $r-k-\alpha$ class estimator, simulation studies were showed and found that the developed estimator is more precised and accurate as compared to ML, ridge, lasso, elastic net, $r-k$ class and PCPHR estimator. This estimator gives the best results at $\alpha = 1$.

This method is further utilized to evaluate the effect of different variables on infant survival in Delhi, India. Third phase of NFHS, India is taken for data. In this dataset birth order and mother age are moderately correlated and it is examined that the collinearity is present. ML, ridge, lasso, elastic net estimates reveals that the effect of birth order is significant on infant mortality while it is not significant in the case of other estimates. PCPHR, $r-k$ class and $r-k-\alpha$ class estimators performs better than ML, ridge, lasso and elastic net in terms of MSE. Findings suggest that to overcome the problem of infant mortality, India should focus on breastfeeding, ANC visits and TT. Teenage pregnancy should be avoided and joint family system should be promoted to increase the size of family which is decreasing now days.

CRediT authorship contribution statement

Anu Sirohi: Conceptualization, Methodology, Writing – original draft. **Basim S.O. Alsaedi:** Formal analysis. **Marwan H. Ahelali:** Formal analysis, Methodology. **Mahesh Kumar Jayaswal:** Conceptualization, Validation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data is not available publically. The authors do not have permission to share the data.

References

[1] A.M. Aguilera, M. Escabias, M.J. Valderrama, Using principal components for estimating logistic regression with high-dimensional multicollinear data, Comput. Stat. Data Anal. 50 (2006) 1905–1924.
[2] J.C. Caldwell, Education as a factor in mortality decline: an examination of Nigerian data, Popul. Stud. 33 (3) (1979) 395–413.
[3] S.L.E. Cessie, J.C. Van Houwelingen, Ridge estimators in logistic regression, Appl. Stat. 41 (1) (1992) 191–201.
[4] R.K. Chandra, Immunological aspects of human milk, Nutr. Rev. 36 (9) (1978) 265–272.
[5] D.R. Cox, Regression models and life tables (with discussion), J. R. Stat. Soc. B 34 (1972) 187–220.
[6] J. Hobcraft, Womens education, child welfare and child survival: a review of the evidence, Health Transit. Rev. 3 (2) (1933) 159–173.
[7] A.E. Hoerl, R.W. Kennard, Ridge regression: biased estimation for nonorthogonal problems, Technometrics 12 (1970) 55–67.
[8] A.E. Hoerl, R.W. Kennard, Ridge regression: some simulations, Commun. Stat. 4 (2) (1970) 1105–1123.
[9] D.B. Jelliffe, E.F.P. Jelliffe, Human Milk in the Modern World, Oxford University Press, Oxford, 1978, pp. 84–96.
[10] B.M.G. Kibria, Performance of some new ridge regression estimators, Commun. Stat., Simul. Comput. 32 (2003) 419–435.
[11] B.M.G. Kibria, More than hundred (100) estimators for estimating the shrinkage parameter in a linear and generalized linear ridge regression models, J. Econom. Stat. 2 (2) (2022) 233–252.

[12] D. Lin, D. Banjevic, R.E. Jardine Beard, Using principal components in a proportional hazards model with applications in condition-based maintenance, J. Oper. Res. Soc. 57 (8) (2006) 910–919.
[13] A.F. Lukman, K. Ayinde, B.M.G. Kibria, E.T. Adewuyi, Modified ridge-type estimator for the gamma regression model, Commun. Stat., Simul. Comput. 51 (9) (2020) 5009–5023.
[14] M.J. Mackinnon, A.L. Puterman, Collinearity in generalized linear models, Commun. Stat., Theory Methods 18 (9) (1989) 3463–3472.
[15] W.F. Massy, Principal components regression in exploratory statistical research, J. Am. Stat. Assoc. 60 (309) (1965) 234–256.
[16] D.C. Nath, L.L. Donna, M.S. Steel, Analysis of Birth Interval in a Non Contracepting Indian Population, University of Washington, 1993.
[17] T. Omer, M. Kristofer, Pär Sjölander, B.M.G. Kibria, Improved estimators for the zero-inflated Poisson regression model in the presence of multicollinearity: simulation and applications of maternal death data, Commun. Stat. 7 (3) (2021) 394–412.
[18] M.R. Ozkale, The red indicator and corrected VIFs in generalized linear models, Commun. Stat., Simul. Comput. (2019) 1–27.
[19] M.R. Ozkale, E. Arican, A new biased estimator in logistic regression model, J. Theor. Appl. Stat. 50 (2) (2016) 233–253.
[20] M.Y. Park, T. Hastie, L1-regularization path algorithm for generalized linear models, J. R. Stat. Soc. B 69 (2007) 659–677.
[21] M. Qasim, K. Mansson, B.M.G. Kibria, On some beta ridge regression estimators: methods, simulation and application, J. Stat. Comput. Simul. 91 (9) (2021) 1699–1712.
[22] P.K. Rai, A. Sirohi, On the behavior of child mortality through different survival models in India, Int. J. Ecol. Econ. Stat. 41 (2) (2020) 19–33.
[23] R.L. Schafer, L.D. Roi, R.A. Wolfe, A ridge logistic estimator, Commun. Stat., Theory Methods 13 (1) (1984) 99–113.
[24] A. Sirohi, P.K. Rai, On the structure of infant mortality using accelerated failure time (AFT) model: a comparative study based on national family health survey (NFHS) data in India, Indian J. Public Health Res. Dev. 10 (2) (2019) 135–143.
[25] A. Sirohi, P.K. Rai, Some $r-k$ class proportional hazard regression models in the presence of collinearity: an evidence from Indian infant mortality, Commun. Stat., Simul. Comput. (2021).
[26] E.P. Smith, B.D. Marx, Ill-conditioned information matrices, generalized linear models and estimation of the effects of acid rain, Environmetrics 1 (1) (1990) 57–71.
[27] K.K. Singh, N. Pandey, A. Gautam, Effect of breastfeeding and maternal health care programme on infant mortality, Demogr. India 36 (2) (2007) 253–266.
[28] R. Tibshirani, Regression shrinkage and selection via the lasso, J. R. Stat. Soc. B 58 (1996) 267–288.
[29] R. Tibshirani, The lasso method for variable selection in the Cox model, Stat. Med. 16 (1997) 385–395.
[30] L.A. Weissfeld, S.M. Sereika, A multicollinearity diagnostic for generalized linear models, Commun. Stat., Theory Methods 20 (4) (1991) 1183–1198.
[31] U. Williams, B.M.G. Kibria, K. Mansson, Performance of some ridge regression estimators for the logistic regression model: an empirical comparison, Int. J. Stat. Reliab. Eng. 6 (1) (2019) 1–12.
[32] X. Xue, M.Y. Kim, E.S. Roy, Cox regression analysis in presence of collinearity: an application to assessment of health risks associated with occupational radiation exposure, Lifetime Data Anal. 12 (2007) 333–350.
[33] H. Zou, T. Hastie, Regularization and variable selection via the elastic net, J. R. Stat. Soc. B 67 (2) (2005) 301–320.