

Supplementary document

Proof of the theorems in Stability analysis subsection

Theorem 1. The equilibrium point $E_0 = (\frac{\Lambda}{\mu}, 0, 0, 0, 0, 0, 0)$ is locally asymptotically stable under some restriction on parameters.

Proof. The Jacobain matrix of the given system at non-trivial point $E_0 = (\frac{\Lambda}{\mu}, 0, 0, 0, 0, 0, 0)$ is obtained as follows:

$$J_{E_0} = \begin{pmatrix} -\mu & -\beta_1 & 0 & -\beta_2 & 0 & 0 & 0 \\ 0 & \beta_1 - \mu & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -a_1 & \beta_2 & 0 & 0 & 0 \\ 0 & 0 & \lambda_2 & -a_2 & 0 & 0 & \omega \\ 0 & 0 & \psi_1 & 0 & -(\gamma_1 + \mu) & 0 & 0 \\ 0 & 0 & 0 & \psi_2 & 0 & -(\mu + \gamma_2) & 0 \\ 0 & 0 & 0 & 0 & \gamma_1 & \gamma_2 & -(\omega + \mu) \end{pmatrix}$$

where

$$a_1 = (\lambda_2 + \mu + \psi_1), \quad a_2 = (\psi_2 + \mu + \sigma)$$

The characteristic equation $|J_{E_0} - ZI| = 0$, we have the following two eigen values $-\mu$ and $\beta_1 - \mu$, where other five eigenvalues are the roots of the following equations

$$M^5 + C_1 M^4 + C_2 M^3 + C_3 M^2 + C_4 M + C_5$$

where,

$$\begin{aligned} C_1 &= a_2 + a_1 + \gamma_2 + \gamma_1 + \omega + 3\mu, \\ C_2 &= (\omega + \mu)(a_2 + \mu + \gamma_2) + a_2(\mu + \gamma_2) + (a_2 + \gamma_2 + \omega + 2\mu)(a_1 + \gamma_1 + \mu) + a_1(\gamma_1 + \mu) - \lambda_2 \beta_2, \\ C_3 &= a_1(\gamma_1 + \mu)(a_2 + \omega + \gamma_2 + 2\mu) + a_2(\mu + \gamma_2)(\omega + \mu) + (a_1 + \gamma_1 + \mu)[(\omega + \mu)(a_2 + \mu + \gamma_2) + a_2(\mu + \gamma_2)] \\ &\quad - \psi_1 \gamma_1 \omega - \lambda_2 \beta_2(\gamma_1 + \gamma_2 + 3\mu), \\ C_4 &= a_2(\mu + \gamma_2)(\omega + \mu)(a_1 + \gamma_1 + \mu) + a_1(\gamma_1 + \mu)(a_2 + \mu + \gamma_2) - (a_1 + \gamma_1 + \mu)\gamma_1 \psi_1 \omega - \lambda_2 \beta_2(\gamma_1 + \mu)(\omega + \mu) \\ &\quad - (\gamma_1 + \omega + 2\mu)(\mu + \gamma_2) - \beta_2 \psi_1 \gamma_1 \omega \\ C_5 &= a_1 a_2(\mu + \gamma_2)(\omega + \mu)(\gamma_1 + \mu) - \psi_1 \gamma_1 \omega a_1(\gamma_1 + \mu) - \lambda_2 \beta_2(\gamma_1 + \mu)(\omega + \mu)(\mu + \gamma_2) - (\mu + \gamma_2)\beta_2 \psi_1 \gamma_1 \omega \end{aligned}$$

The equilibrium $E_0 = (\frac{\Lambda}{\mu}, 0, 0, 0, 0, 0, 0)$ is locally asymptotically stable is it satisfy the following Routh-Hurwitz criteria

$$\begin{aligned} C_i &> 0, \text{ for } i = 1, 2, 3, 4, 5 \\ C_2 C_1 - C_3^2 &> 0, \\ C_3(C_1 C_2 - C_3^2) - C_4 C_1^2 &> 0, \\ C_4(C_1 C_3 - C_2^2) - C_5 C_1^2 &> 0, \\ C_5(C_2 C_3 - C_4^2) &> 0 \end{aligned}$$

□

Theorem 2. The depression free equilibrium point $E_1 = (S^*, B^*, 0, 0, 0, 0, 0)$, where $S^* = \frac{\Lambda}{\beta_1}$ and $B^* = (\frac{\beta_1}{\mu} - 1) \frac{\Lambda}{\beta_1}$ is locally asymptotically stable under some restriction on parameters.

Proof. The jacobain matrix of the given system at non-trivial point $E_1 = (S^*, B^*, 0, 0, 0, 0, 0)$ is obtained by

$$J_{E_1} = \begin{pmatrix} -b_1 - \mu & b_2 & b_3 & b_4 & b_3 & b_3 & b_3 \\ b_1 & -b_2 - \mu & -b_3 & -b_3 & -b_3 & -b_3 & -b_3 \\ b_5 & 0 & b_6 & b_7 & 0 & 0 & 0 \\ 0 & 0 & b_8 & b_9 & 0 & 0 & \omega \\ 0 & 0 & \psi_1 & 0 & b_{10} & 0 & 0 \\ 0 & 0 & 0 & \psi_2 & 0 & b_{11} & 0 \\ 0 & 0 & 0 & 0 & \gamma_1 & \gamma_2 & -(\omega + \mu) \end{pmatrix}$$

where

$$b_1 = \frac{\beta_1 B^2}{(S+B)^2}; b_2 = \frac{-\beta_1 S^2}{(S+B)^2}, b_3 = \frac{\beta_1 BS}{(S+B)^2}, b_4 = \frac{\beta_1 BS}{(S+B)^2} + \frac{\beta_2 S}{N}, b_5 = \frac{\lambda_1 BP}{(S+B)^2} = 0, b_{11} = -(\mu + \gamma_2)$$

$$b_6 = -(\frac{\lambda_1 B}{(S+B)} + (\lambda_2 + \mu + \psi_1)), b_7 = \frac{-\beta_2 S}{(S+B)^2}, b_8 = \frac{\lambda_1 B}{(S+B)} + \lambda_2, b_9 = -(\psi_2 + \mu + \sigma), b_{10} = -(\gamma_1 + \mu),$$

The characteristic equation $|J_{E_1} - YI| = 0$, and the eigenvalues are the roots of the following equation

$$Y^7 + G_1 Y^6 + G_2 Y^5 + G_3 Y^4 + G_4 Y^3 + G_5 Y^2 + G_6 Y + G_7 = 0$$

where

$$\begin{aligned} G_1 &= b_1 + b_2 - b_6 - b_9 - b_{10} - b_{11} + 3\mu + \omega \\ G_2 &= b_6(b_9 - b_2 - b_1 + b_{10} + b_{11} - 3\mu - \omega) + b_9(b_6 - b_1 - b_2 + b_{10} + b_{11} - 3\mu - \omega) + b_{10}(b_9 + b_6 - b_1 - b_2 \\ &\quad + b_{11} - 3\mu - \omega) + b_{11}(b_9 + b_6 - b_1 - b_2 + b_{10} - 3\mu - \omega) + b_3 b_5 + b_7 b_8 + 2b_1 \mu + 2b_2 \mu + b_1 \omega + b_2 \omega + 2\mu \omega + 3\mu^2 \\ G_3 &= (b_1 + b_2 - 3b_6 - 3b_9 - 3b_{10} - 3b_{11})\mu^2 + \mu^2 \omega + \mu^3 + (b_1 b_6 b_9 + b_1 b_6 b_{10} + b_1 b_6 b_{11} + b_2 b_6 b_9 + b_2 b_6 b_{10} + b_2 b_6 b_{11} \\ &\quad + b_1 b_9 b_{10} + b_1 b_9 b_{11} + b_2 b_9 b_{10} + b_2 b_9 b_{11} + b_1 b_{10} b_{11} + b_2 b_{10} b_{11}) - (b_1 b_7 b_8 + b_2 b_7 b_8 + b_6 b_9 b_{10} + b_6 b_9 b_{11} \\ &\quad + b_6 b_{10} b_{11} + b_9 b_{10} b_{11}) + b_7 b_8 b_{10} + b_7 b_8 b_{11} + (b_3 b_5 b_9 + b_3 b_5 b_{10} + b_3 b_5 b_{11} - b_4 b_5 b_8) - b_3 b_5 \psi_1 - 2(b_1 b_6 \\ &\quad + b_2 b_6 + b_3 b_5 + b_1 b_9 + b_1 b_{10} + b_2 b_9 + b_1 b_{11} + b_2 b_{10} + b_2 b_{11})\mu + 3(b_6 b_9 + b_6 b_{10} + b_6 b_{11} + b_9 b_{10} + b_9 b_{11} + b_{10} b_{11})\mu \\ &\quad - (b_1 b_6 + b_2 b_6 + b_3 b_5 + b_1 b_9 + b_1 b_{10} + b_2 b_9 + b_1 b_{11} + b_2 b_{10} + b_2 b_{11})\omega + (b_6 b_9 + b_6 b_{10} + b_6 b_{11} + b_9 b_{10} \\ &\quad + b_9 b_{11} + b_{10} b_{11})\omega + b_1 \mu \omega + b_2 \mu \omega - 2(b_6 + b_9 + b_{10} + b_{11})\mu \omega - \gamma_2 \omega \psi_2 \\ G_4 &= -(b_9 + b_{10} + b_{11} + b_6)\mu^3 + (3b_6 b_9 - b_1 b_6 - b_2 b_6 - b_3 b_5 - b_1 b_9 - b_1 b_{10} - b_2 b_9 - b_1 b_{11} - b_2 b_{10} - b_2 b_{11} \\ &\quad - 3b_7 b_8 + 3b_6 b_{10} + 3b_6 b_{11} + 3b_9 b_{10} + 3b_9 b_{11} + 3b_{10} b_{11} - (b_6 + b_9 + b_{10} + b_{11})\omega)\mu^2 + (-b_1 b_6 - b_2 b_6 - b_3 b_5 \\ &\quad + -b_1 b_9 - b_1 b_{10} - b_2 b_9 - b_1 b_{11} - b_2 b_{10} - b_2 b_{11} + 2b_6 b_9 + 2b_6 b_{10} + 2b_6 b_{11} + 2b_9 b_{10} + 2b_9 b_{11} + 2b_{10} b_{11})\mu \omega \\ &\quad + (b_2 b_3 b_5 b_8 - b_2 b_4 b_5 b_8) + 2(b_1 b_6 b_9 + b_1 b_6 b_{10} + b_1 b_6 b_{11} + b_2 b_6 b_9 + b_2 b_6 b_{10} + b_2 b_6 b_{11} \\ &\quad + b_3 b_5 b_9 + b_3 b_5 b_{10} + b_3 b_5 b_{11} + b_1 b_9 b_{10} + b_1 b_9 b_{11} + b_2 b_9 b_{10} + b_2 b_9 b_{11} + b_2 b_{10} b_{11})\mu \\ &\quad - 3(b_6 b_9 b_{10} + b_6 b_9 b_{11} + b_6 b_{10} b_{11} + b_9 b_{10} b_{11})\mu \\ G_5 &= 3b_6 b_9 \mu^2 - b_9 \mu^3 - b_{10} \mu^3 - b_{11} \mu^3 - b_1 b_6 \mu^2 - b_2 b_6 \mu^2 - b_3 b_5 \mu^2 - b_1 b_9 \mu^2 - b_1 b_{10} \mu^2 - b_2 b_9 \mu^2 \\ &\quad - b_1 b_{11} \mu^2 - b_2 b_{10} \mu^2 - b_2 b_{11} \mu^2 - b_6 \mu^3 - 3b_7 b_8 \mu^2 + 3b_6 b_{10} \mu^2 + 3b_6 b_{11} \mu^2 + 3b_9 b_{10} \mu^2 + 3b_9 b_{11} \mu^2 \\ &\quad + 3b_{10} b_{11} \mu^2 - 2b_3 b_5 \mu \psi_1 + b_2 b_3 b_5 b_8 - b_2 b_4 b_5 b_8 - b_1 b_6 b_9 b_{10} + b_1 b_7 b_8 b_{10} - b_1 b_6 b_9 b_{11} + b_1 b_7 b_8 b_{11} \\ &\quad - b_2 b_6 b_9 b_{10} + b_2 b_7 b_8 b_{10} - b_3 b_5 b_9 b_{10} + b_4 b_5 b_8 b_{10} - b_1 b_6 b_{10} b_{11} - b_2 b_6 b_9 b_{11} + b_2 b_7 b_8 b_{11} - b_3 b_5 b_9 b_{11} \\ &\quad + b_4 b_5 b_8 b_{11} - b_2 b_6 b_{10} b_{11} - b_3 b_5 b_{10} b_{11} - b_1 b_9 b_{10} b_{11} - b_2 b_9 b_{10} b_{11} + b_6 b_9 b_{10} b_{11} - b_7 b_8 b_{10} b_{11} \\ &\quad + 2b_1 b_6 b_9 \mu - 2b_1 b_7 b_8 \mu + 2b_1 b_6 b_{10} \mu + 2b_2 b_6 b_9 \mu - 2b_2 b_7 b_8 \mu + 2b_3 b_5 b_9 \mu - 2b_4 b_5 b_8 \mu \\ &\quad + 2b_1 b_6 b_{11} \mu + 2b_2 b_6 b_{10} \mu + 2b_3 b_5 b_{10} \mu + 2b_2 b_6 b_{11} \mu + 2b_3 b_5 b_{11} \mu + 2b_1 b_9 b_{10} \mu + 2b_1 b_9 b_{11} \mu \\ &\quad + 2b_2 b_9 b_{10} \mu + 2b_1 b_{10} b_{11} \mu + 2b_2 b_9 b_{11} \mu + 2b_2 b_{10} b_{11} \mu - 3b_6 b_9 b_{10} \mu + 3b_7 b_8 b_{10} \mu - 3b_6 b_9 b_{11} \mu \\ &\quad + 3b_7 b_8 b_{11} \mu - 3b_6 b_{10} b_{11} \mu - 3b_9 b_{10} b_{11} \mu - b_3 b_5 b_8 \psi_2 + b_3 b_5 b_9 \psi_1 + b_3 b_5 b_{11} \psi_1 - b_3 b_5 \gamma_1 \psi_1 \end{aligned}$$

$$\begin{aligned}
G_6 &= \mu^3 (b_7 b_8 (b_{10} + b_{11}) - b_6 b_9 (b_{10} + b_{11}) - b_6 b_{10} b_{11} - b_9 b_{10} b_{11}) + \mu^2 (b_1 (b_7 b_8 (b_{10} + b_{11}) - b_6 b_9 (b_{10} + b_{11}) \\
&\quad - b_6 b_{10} b_{11} - b_9 b_{10} b_{11}) + b_2 (b_7 b_8 (b_{10} + b_{11}) - b_6 b_9 (b_{10} + b_{11}) - b_6 b_{10} b_{11} - b_9 b_{10} b_{11}) + b_3 b_5 (b_9 (\psi_1 + b_{10} \\
&\quad + b_{11}) - b_8 \psi_2 - b_{10} b_{11}) + b_4 b_5 (b_8 (b_{10} + b_{11}) - b_9 b_{10} b_{11}) + 3b_6 b_9 b_{10} b_{11} - 3b_7 b_8 b_{10} b_{11} - b_3 b_5 b_8 \psi_2 \\
&\quad + b_3 b_5 b_9 \psi_1 + b_3 b_5 b_{11} \psi_1) + \mu (b_2 (2b_6 b_9 b_{10} b_{11} - 2b_7 b_8 b_{10} b_{11} - b_3 b_5 (b_8 (1 + \psi_2) - b_9 (1 + \psi_1) - b_{11} (1 + \psi_1)) \\
&\quad + b_4 b_5 (2b_8 b_{10} b_{11} - b_8 (b_{10} + b_{11})) + 2b_1 (b_6 b_9 b_{10} b_{11} - b_7 b_8 b_{10} b_{11}) + b_3 b_5 (2b_8 b_{10} \psi_2 - 2b_9 b_{11} \psi_1 + b_9 \gamma_1 \psi_1 \\
&\quad - b_8 \gamma_2 \psi_2 + b_{11} \gamma_1 \psi_1) - b_4 b_5 (2b_8 b_{10} b_{11})) + b_2 b_3 b_5 (b_8 b_{10} b_{11} - b_8 (b_{10} + b_{11})) - b_2 b_4 b_5 (b_8 (b_{10} + b_{11}))) \\
G_7 &= (b_6 b_9 b_{10} b_{11} - b_7 b_8 b_{10} b_{11}) \mu^3 + [(b_1 b_6 b_9 b_{10} b_{11} - b_1 b_7 b_8 b_{10} b_{11} + b_2 b_6 b_9 b_{10} b_{11} - b_2 b_7 b_8 b_{10} b_{11} \\
&\quad + b_3 b_5 b_9 b_{10} b_{11} - b_4 b_5 b_8 b_{10} b_{11} + b_6 b_9 b_{10} b_{11} \omega - b_7 b_8 b_{10} b_{11} \omega + b_3 b_5 b_8 b_{10} \psi_2 - b_3 b_5 b_9 b_{11} \psi_1 - b_6 b_{10} \gamma_2 \omega \psi_2 \\
&\quad + b_7 b_{11} \gamma_1 \omega \psi_1) \mu^2 + [(b_2 b_3 b_5 b_8 b_{10} b_{11} - b_2 b_4 b_5 b_8 b_{10} b_{11} + b_2 b_3 b_5 b_8 b_{10} \omega - b_2 b_4 b_5 b_8 b_{10} \omega + b_1 b_6 b_9 b_{10} b_{11} \omega \\
&\quad - b_1 b_7 b_8 b_{10} b_{11} \omega + b_2 b_6 b_9 b_{10} b_{11} \omega - b_2 b_7 b_8 b_{10} b_{11} \omega + b_3 b_5 b_9 b_{10} b_{11} \omega - b_4 b_5 b_8 b_{10} b_{11} \omega + b_3 b_5 b_8 b_{10} \gamma_2 \psi_2 \\
&\quad - b_3 b_5 b_9 b_{11} \gamma_1 - b_2 b_3 b_5 b_{11} \gamma_1 \omega \psi_1 + b_2 b_4 b_5 b_{11} \gamma_1 \omega \psi_1 + b_3 b_5 b_8 b_{10} \mu \omega \psi_2 - b_3 b_5 b_9 b_{11} \mu \omega \psi_1 - b_1 b_6 b_{10} \gamma_2 \mu \omega \psi_2 \\
&\quad + b_1 b_7 b_{11} \gamma_1 \mu \omega \psi_1 - b_2 b_6 b_{10} \gamma_2 \mu \omega \psi_2 + b_2 b_7 b_{11} \gamma_1 \mu \omega \psi_1 - b_3 b_5 b_{10} \gamma_2 \mu \omega \psi_2 + b_4 b_5 b_{11} \gamma_1 \mu \omega \psi_1 - b_3 b_5 \gamma_1 \mu \omega \psi_1 \psi_2 \\
&\quad + b_3 b_5 \gamma_2 \mu \omega \psi_1 \psi_2) \mu
\end{aligned}$$

If the following Routh-Hurwitz criteria satisfied, then E_1 will be locally asymptotically stable.

$$-G_i > 0, \text{ for } i = 1, 2, 3, 4, 5, 6, 7$$

$$G_7 G_6 G_5 - G_5^2 > 0$$

$$(G_7 G_6 G_5 - G_5^2)(G_7 G_4 G_6 - G_6^2 - G_7^2 G_3) - G_6 G_5 (G_7 G_3 G_5 - G_4^2) > 0$$

$$G_7 (G_6 (G_5 (G_4 G_3 - G_2 G_5) - G_5 (G_3 G_2 - G_1 G_4)) + G_6^2 G_2) - G_5 G_6 G_3^2 + G_6 G_5 G_2 G_3 > 0$$

$$G_7 G_6 G_5 G_4 G_3 G_2 - G_5^2 G_6 G_4 G_3 - G_6 G_5 G_3^2 + G_6 G_5 G_4 G_2 - G_6 G_5 G_1 G_4 G_3 + G_6 G_4^2 G_2 > 0$$

□

Theorem 3. The saboteurs population free equilibrium point $E_2 = (S^*, 0, P^*, D^*, M^*, C^*, R^*)$, is locally asymptotically stable under some restriction on parameters.

Proof. The Jacobian matrix of the given system at non-trivial point $E_2 = (S^*, 0, P^*, D^*, M^*, C^*, R^*)$, is obtained as follows:

$$J_{E_2} = \begin{pmatrix} -(a_1 + \mu) & a_2 & a_3 & a_4 & a_3 & a_3 & a_3 \\ 0 & a_5 & 0 & 0 & 0 & 0 & 0 \\ a_1 & a_6 & -(a_3 + \mu) & -a_4 & -a_3 & -a_3 & -a_3 \\ 0 & a_7 & \lambda_2 & a_8 & 0 & 0 & \omega \\ 0 & 0 & \psi_1 & 0 & -(\gamma_1 + \mu) & 0 & 0 \\ 0 & 0 & 0 & \psi_2 & 0 & -(\mu + \gamma_2) & 0 \\ 0 & 0 & 0 & 0 & \gamma_1 & \gamma_2 & -(\omega + \mu) \end{pmatrix}$$

where

$$\begin{aligned}
N &= (S + P + D + M + C + R), a_1 = \frac{\beta_2 D N - \beta_2 D S}{N^2}, a_2 = \frac{-\beta_1 S}{N} + \frac{\beta_2 D S}{N^2}, a_4 = -\left(\frac{\beta_2 S N - \beta_2 D S}{N^2}\right) \\
a_3 &= \frac{\beta_2 D S}{N^2}, a_5 = \frac{\beta_1 S}{N}, a_6 = -\left(\frac{\beta_2 D S}{N^2} - \frac{\lambda_1 P}{N}\right), a_7 = \frac{\lambda_1 P}{N}, a_8 = -\left(\frac{\beta_2 D S}{N^2} - \frac{-\lambda_1 P}{N}\right)
\end{aligned}$$

The characteristic equation $|J_{E_2} - ZI| = 0$, we have the following eigen value

$$a_5 = \frac{\beta_1 S}{N}$$

And the other eigen values are obtained from the following equation

$$Z^6 + H_1 Z^5 + H_2 Z^4 + H_3 Z^3 + H_4 Z^2 + H_5 Z + H_6 = 0$$

where

$$\begin{aligned}
H_1 &= a_1 + a_3 - a_8 + \gamma_1 + \gamma_2 + 5\mu + \omega \\
H_2 &= a_1(\gamma_1 + \gamma_2 + 4\mu + \omega) + a_3(\gamma_1 + \gamma_2 + 4\mu + \omega + \psi_1) - (a_3 + a_1 + a_8)(\gamma_1 + \gamma_2 + \omega) - a_1a_8 - a_4\lambda_2 + \gamma_1\gamma_2 + 10\mu^2 \\
&\quad + 4\mu\omega + 4(\gamma_1 + \gamma_2)\mu \\
H_3 &= (6a_1 + 6a_3 - 10a_8 + 6\gamma_1 + 6\gamma_2 + 6\omega)\mu^2 + 10\mu^3 - (a_1 + a_3)a_8(\gamma_1 + \gamma_2) - a_3a_8\psi_1 + (a_4\gamma_1 + a_4\gamma_2 + 4a_4\lambda_2 + 3a_3\lambda_2)\mu \\
&\quad + (a_4\lambda_2 + 3a_1 + 3a_3)\omega + (3\gamma_1 + 3\gamma_2 + 3a_1 + 3a_3)\mu\omega + (1 + a_3)\gamma_1\psi_1 + (1 + a_3)\gamma_2\psi_1 + \gamma_1\gamma_2(\mu + \omega) - \gamma_2\omega\psi_2 \\
H_4 &= \mu^4(5 + 4\omega) + \mu^3(4a_1 + 4a_3 - 10a_8 + 4\gamma_1 + 4\gamma_2) + \mu^2(3\gamma_1\omega + 3\gamma_2\omega - 6a_1a_8 - 6a_3a_8 + 3a_1\gamma_1 + 3a_1\gamma_2 + 3a_3\gamma_1 + 3a_3\gamma_2) \\
&\quad + \mu^2(6a_4\lambda_2 + 3\gamma_1\gamma_2 - 6a_8\gamma_1 - 6a_8\gamma_2 - 6a_8\omega + 3a_3\psi_1) + \mu(3a_4\gamma_1\lambda_2 + 3a_4\gamma_2\lambda_2 + 2a_1\gamma_1\omega + 2a_1\gamma_2\omega + 2a_3\gamma_1\omega \\
&\quad + 2a_3\gamma_2\omega - 3a_8\gamma_1\omega - 3a_8\gamma_2\omega) + (2a_3\gamma_1\mu + 2a_3\gamma_2\mu - a_1\gamma_2\omega\psi_2 + a_4\gamma_1\omega\psi_1 + a_3\gamma_2\omega\psi_1 + a_4\gamma_2\lambda_2\omega - a_3\gamma_2\omega\psi_2) \\
&\quad + (3a_4\lambda_2\mu\omega + 3a_3\lambda_2\mu\psi_2 + 2\gamma_1\gamma_2\mu\omega + a_3\lambda_2\omega\psi_2 + 2a_3\mu\omega\psi_1 - \gamma_1\gamma_2\omega\psi_2 - 3\gamma_2\mu\omega\psi_2) \\
&\quad - (a_1a_8\gamma_1\gamma_2 + a_3a_8\gamma_1\gamma_2 + 3a_1a_8\gamma_1\mu + 3a_1a_8\gamma_2\mu + 3a_3a_8\gamma_1\mu + 3a_3a_8\gamma_2\mu + a_1a_8\gamma_1\omega + a_1a_8\gamma_2\omega + a_3a_8\gamma_1\omega + a_3a_8\gamma_2\omega) \\
&\quad + (a_4\gamma_1\gamma_2\lambda_2 + 2a_1\gamma_1\gamma_2\mu + 2a_3\gamma_1\gamma_2\mu - 3a_8\gamma_1\gamma_2\mu - 3a_1a_8\mu\omega - 3a_3a_8\mu\omega + a_1\gamma_1\gamma_2\omega + a_3\gamma_1\gamma_2\omega - a_8\gamma_1\gamma_2\omega - 3a_3a_8\mu\psi_1) \\
H_5 &= \mu^5 + (a_1 + a_3 - 5a_8 + \gamma_1 + \gamma_2 + \omega)\mu^4 + [\gamma_1\omega + \gamma_2\omega + a_1\gamma_1 + a_1\gamma_2 + a_3\gamma_1 + a_3\gamma_2 - 4a_1a_8 - 4a_3a_8 + 4a_4\lambda_2 \\
&\quad - 4a_8(\gamma_1 + \gamma_2) + a_1\omega + a_3\omega - 4a_8\omega + a_3\psi_1]\mu^3 + [a_1\gamma_1\gamma_2 + a_3\gamma_1\gamma_2 - 3(a_1a_8\gamma_1 + a_1a_8\gamma_2 + a_3a_8\gamma_1 + a_3a_8\gamma_2) \\
&\quad + 3a_4\lambda_2(\gamma_1 + \gamma_2) - 3(a_1a_8 + a_3a_8)(\gamma_1 + \gamma_2)]\mu^2 + [-2(a_1a_8 + a_3a_8)\gamma_1\gamma_2 + (a_1\gamma_1\gamma_2 + a_3\gamma_1\gamma_2 - 2a_8\gamma_1\gamma_2) \\
&\quad + 2a_3\gamma_1\lambda_2\psi_2 + 2a_3\gamma_2\lambda_2\psi_2 + a_4\gamma_1\gamma_2\lambda_2\omega - a_3a_8\gamma_2\omega\psi_1]\mu + [-2\gamma_1\gamma_2 + 2a_3\lambda_2\psi_2] \\
H_6 &= -a_8\mu^5 + (a_4\lambda_2 - a_1a_8 - a_3a_8 - a_8\gamma_1 - a_8\gamma_2 - a_8\omega)\mu^4 + (-a_1a_8\gamma_1 - a_1a_8\gamma_2 - a_3a_8\gamma_1 - a_3a_8\gamma_2 - a_8\gamma_1\gamma_2 - a_1a_8\omega \\
&\quad - a_3a_8\omega - a_3a_8\psi_1 + a_4\gamma_1\lambda_2 + a_4\gamma_2\lambda_2 + a_4\lambda_2\omega + a_3\lambda_2\psi_2 - \gamma_2\omega\psi_2)\mu^3 + (-a_1a_8\gamma_1\gamma_2 - a_3a_8\gamma_1\gamma_2 - a_1a_8\gamma_1\omega \\
&\quad - a_1a_8\gamma_2\omega - a_3a_8\gamma_1\omega - a_3a_8\gamma_2\omega - a_3a_8\gamma_1\psi_1 - a_3a_8\gamma_2\psi_1 + a_4\gamma_1\gamma_2\lambda_2 - a_8\gamma_1\gamma_2\omega - a_3\mu^2\omega\psi_1 + a_4\gamma_1\lambda_2\omega \\
&\quad + a_4\gamma_2\lambda_2\omega + a_3\gamma_1\lambda_2\psi_2 + a_3\gamma_2\lambda_2\psi_2 - a_1\gamma_2\omega\psi_2 + a_4\gamma_1\omega\psi_1 - a_3\gamma_2\omega\psi_2 + a_3\lambda_2\omega\psi_2 - \gamma_1\gamma_2\omega\psi_2)\mu^2 \\
&\quad + (-a_1a_8\gamma_1\gamma_2\omega - a_3a_8\gamma_1\gamma_2\omega - a_3a_8\gamma_1\gamma_2\psi_1 - a_3a_8\gamma_2\omega\psi_1 + a_4\gamma_1\gamma_2\lambda_2\omega + a_3\gamma_1\gamma_2\lambda_2\psi_2 - a_1\gamma_1\gamma_2\omega\psi_2 - a_3\gamma_1\gamma_2\omega\psi_2 \\
&\quad + a_4\gamma_1\gamma_2\omega\psi_1 + a_3\gamma_1\lambda_2\omega\psi_2 + a_3\gamma_1\omega\psi_1\psi_2 - a_3\gamma_2\omega\psi_1\psi_2)\mu
\end{aligned}$$

If the following conditions hold, E_2 will be locally asymptotically stable according to the Routh-Hurwitz criteria:

$$\begin{aligned}
&-H_i > 0 \text{ for } i = 1, 2, 3, 4, 5, 6 \\
&-(H_1H_2H_3) > H_3^2 + H_1^2H_4 \\
&-(H_1H_4 - H_5)(H_1H_2H_3 - H_3^2 - H_1^2H_4) > H_5(H_1H_2 - H_3)^2H_1H_5^2 > 0 \\
&-(H_1H_5^2 - H_2H_6)(H_1H_2H_3 - H_3^2 - H_1^2H_4) > H_6(H_1H_2 - H_3)^2H_1H_6^2 > 0
\end{aligned}$$

□

Theorem 4. The Endemic equilibrium point $E_3 = (S^{**}, B^{**}, P^{**}, D^{**}, M^{**}, C^{**}, R^{**})$, is locally asymptotically stable under some restriction on parameters.

Proof. The Jacobain matrix of the given system at non-trivial point $E_3 = (S^{**}, B^{**}, P^{**}, D^{**}, M^{**}, C^{**}, R^{**})$ is obtained as follows:

$$J_{E_3} = \begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_5 & a_6 & a_7 & a_7 & a_7 & a_7 & a_7 \\ a_8 & a_9 & a_{10} & a_{11} & a_{12} & a_{12} & a_{12} \\ a_{15} & a_{14} & a_{15} & a_{16} & a_{13} & a_{13} & a_{13} + \omega \\ 0 & 0 & \psi_1 & 0 & a_{17} & 0 & 0 \\ 0 & 0 & 0 & \psi_2 & 0 & a_{18} & 0 \\ 0 & 0 & 0 & 0 & \gamma_1 & \gamma_2 & -(\mu + \omega) \end{pmatrix}$$

where

$$\begin{aligned}
a_1 &= -\left(\frac{(\beta_1BN - \beta_1BS)}{N^2} + \frac{(\beta_2DN - \beta_2DS)}{N^2} + \mu\right), \quad a_2 = -\frac{(\beta_1SN - \beta_1BS)}{N^2} + \frac{\beta_2DS}{N^2}, \quad a_{17} = -(\gamma_1 + \mu), \\
a_3 &= \frac{(\beta_1BS - \beta_2DS)}{N^2}, \quad a_4 = \frac{\beta_1BS}{N^2} - \frac{(\beta_2SN - \beta_2DS)}{N^2}, \quad a_5 = \frac{\beta_1BN - \beta_1BS}{N^2}, \quad a_7 = -\frac{\beta_1BS}{N^2}, \quad a_{18} = -(\gamma_2 + \mu),
\end{aligned}$$

$$\begin{aligned}
a_6 &= \frac{(\beta_1 SN - \beta_1 BS)}{N^2} - \mu, \quad a_8 = \frac{(\beta_2 DN - \beta - 2DS)}{N^2} + \frac{\lambda_1 BP}{N^2}, \quad a_9 = \frac{(\lambda_1 PN - \lambda_1 BP)}{N^2} - \frac{\beta_2 DS}{N^2}, \\
a_{10} &= \frac{(\lambda_1 BN - \lambda_1 BP)}{N^2} - \frac{\beta_2 DS}{N^2} - (\lambda_2 + \mu + \psi_1), \quad a_{11} = \frac{(\beta_2 SN - \beta_2 DS)}{N^2} + \frac{\lambda_1 BP}{N^2}, \quad a_{13} = \frac{-\lambda_1 BP}{N^2}, \\
a_{12} &= \frac{(\lambda_1 BP - \beta_2 DS)}{N^2}, \quad a_{14} = \frac{(\lambda_1 PN - \lambda_1 BP)}{N^2}, \quad a_{15} = \frac{(\lambda_1 BN - \lambda_1 BP)}{N^2} + \lambda_2, \quad a_{16} = -(\psi_2 + \mu + \sigma) - \frac{\lambda_1 BP}{N^2}.
\end{aligned}$$

The characteristic equation $|J_{E_3} - XI| = 0$ and the eigen values are the roots of the following equations,

$$X^7 + K_1 X^6 + K_2 X^5 + K_3 X^4 + K_4 X^3 + K_5 X^2 + K_6 X + K_7 = 0.$$

The coefficient K_i 's can be found use any software with symbolic computation. If the following Routh-Hurwitz criteria holds, then the equilibrium point E_3 is locally asymptotically stable;

$$-K_i > 0, \text{ for } i = 1, 2, 3, 4, 5, 6, 7$$

$$K_7 K_6 K_5 - K_5^2 > 0,$$

$$(K_7 K_6 K_5 - K_5^2)(K_7 K_4 K_6 - K_6^2 - K_7^2 K_3) - K_6 K_5 (K_7 K_3 K_5 - K_4^2) > 0,$$

$$K_7 (K_6 (K_5 (K_4 K_3 - K_2 K_5) - K_5 (K_3 K_2 - K_1 K_4)) + K_6^2 K_2) - K_5 K_6 K_3^2 + K_6 K_5 K_2 K_3 > 0,$$

$$K_7 K_6 K_5 K_4 K_3 K_2 - K_5^2 K_6 K_4 K_3 - K_6 K_5 K_3^2 + K_6 K_5 K_4 K_2 - K_6 K_5 K_1 K_4 K_3 + K_6 K_4^2 K_2 > 0.$$

□