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Locating leakage in pipelines based on the adjoint equation of inversion modeling

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ABSTRACT

This paper presents an adjoint method for locating potential leakage in a single-phase fluid pipeline based on the analytic solution of inversion modeling. By studying the mechanism of pipeline leakage pressure, the adjoint equation based on the governing equation of transient flow is established in the single-liquid phase aspect using inverse adjoint theory and sensitivity analysis method. The inverse transient adjoint equation is primarily derived from the single linear fluid pipeline in the semi-infinite domain. The Laplace method is then used to obtain an analytical solution that determines the location of pipeline leakage. The experimental results indicate that the analytic solution can quickly and accurately judge the leakage location of the pipeline. Furthermore, it presents a new approach to engineering applications, such as gas-liquid two-phase flow complex pipe networks, etc.

1. Introduction

Leakages can occur in all types of pipelines due to a combination of factors. The leakage of the pipelines increases the risk of contaminant entry and further increases the projected expenditure of resources and costs. Pipeline leakage poses potential risks to public health and causes financial losses [1,2], while also burdening the environment by wasting energy [3–8]. These phenomena have gained worldwide attention.

Nowadays, there are many advanced and mature leak detection and location technologies for long straight pipelines (e.g., long oil pipelines, natural gas pipelines) [3–5,7,9], such as acoustic and infrared detection methods, negative pressure wave method, transient model method, distributed optical fiber method, etc. Among these technologies, the transient analysis method exhibits more advantages in leakage detection and pipeline location [10–16]. Transient analysis is a highly effective method for detecting and locating small leaks in pipelines due to its high sensitivity in flow conditions and pressure [17,18]. This method is more accurate in locating leaks because it employs mathematical modeling to determine the leak location [19,20]. Additionally, the method is also relatively fast compared to other detection methods as it is based on the analysis of transient pressure waves [21]. Transient analysis is non-intrusive, making it particularly useful for continuous processes, as it does not require pipeline shutdown or interruption during the detection process [22]. Moreover, it is cost-effective due to its minimal equipment and resource requirements [23].

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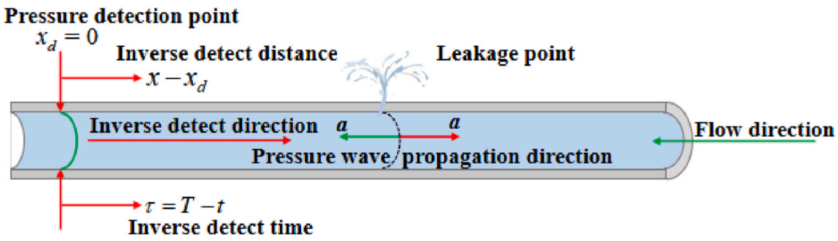

Fig. 1. Schematic diagram of leak location detection.

The inverse-transient analysis (ITA) overthrew the ordinary opinion of leaking detection research and was first proposed by Pudar and Liggett [24]. The "forward problem" is defined in the forward model, which assumes known demands and system characteristics, and then solves for resulting state quantities such as pressure and flow rate. In contrast, the inverse problem described in the inverse model assumes that the system state (pressures, flow, etc.) is known/measured, however some other parameters (pipe roughness, leakage rate, etc.) are unknown [25]. For instance, the state parameters of the pipe network are measured during transient leakage and compared to measurements corresponding to the same pipe network without leakage. Then, the inverse problem is calibrated to determine the leak size, location, etc. This is a typical ITA example. Recently, several researchers [26,27] have reported promising leak detection results using the ITA method.

In the ITA method, the goal function is to minimize the sum of squared differences between observed and simulated pressure sequences to solve the inverse problem. Over the past 20 years, Kapelan et al. [28–31] have conducted inverse transient analysis for leak detection analysis and roughness correction by combining prior information of parameters. Nash et al. [32] used a multiple linear regression method to effectively analyze serial pipe network leaks. Vitkovsky et al. [33–36] studied the inverse experimental detection and analysis of pipeline leaks on the basis of a constructed topology structure pipeline network and adjusted the forward model through the optimization algorithm, further using the weighted function pipeline unsteady friction model for numerical error correction. Kim et al. [37] used the impedance method for inverse transient analysis of branch pipe network systems with leaks and blockages. Dong et al. [38] proposed a genetic algorithm-based inverse transient location method for water supply pipe networks. To solve the inverse problem, methods such as Levenberge-Marquardt (LM), Shuffled Complex Evolution (SCE), Systematic Levenberge Marquardt (SLM), and Surrogate Assisted Genetic Algorithm (SAGA) have been developed. However, the accurate calibration of the transient solver and the optimization speed are inherently contradictory, as increasing accuracy sacrifices computing speed. A recent trend has been to optimize ITA algorithms to circumvent the accuracy/speed tradeoff all together so as to achieve the best solution state [6,18,39–44].

However, most of the existing inverse detection methods rely on separate detection signals, which are greatly affected by the sensor and the quality of detection. These methods are limited to signal analysis and processing and are subject to more severe conditions. Therefore, they cannot be applied to the prevalent pattern of leakage detection. To solve the leakage issue, we attempt to interpret the inner propagation mechanism with flow knowledge. In this regard, we propose to adopt the idea of inverse transient analysis and solve the pipeline flow field based on the adjoint method, which provides a novel approach for related research in this field.

The adjoint method is a highly effective technique used in various fields of science and engineering for solving optimization problems [45–47] and conducting sensitivity analysis [48–50]. Marchuk and Penenko [51–55] made significant contributions to the development of adjoint methods. Lions [56] applied the adjoint method to study the governing of partial differential equations, while other researchers have applied this method to identify and quantify sources of pollution in environmental systems. For example, Piasecki et al. [57] used the adjoint method to determine the sensitivity of pollutant solute concentrations in river pollution to distant sources. Neupauer et al. [49,58,59] applied the adjoint method to identify the location of groundwater pollutant sources and release times. Zhai et al. [60,61] used the adjoint method to achieve rapid identification of indoor pollution sources. Xue et al. [62] further improved the adjoint method for identifying multiple pollutant sources in open spaces. Jing et al. [63] used the adjoint equation method to inversely estimate the release amount of pollution sources in rivers. Kauker et al. [64] used the adjoint sensitivity analysis method to establish an information system for the potential diffusion of radioactive pollutants in the northern marine environment.

In summary, it has been observed that while the adjoint method has been successfully employed in pollution detection in water and air systems, it has not been utilized in fluid leakage detection in pipelines. Similar to the transport of contaminants, the mathematical model for leakage detection involves the advection-dispersion equation, while the propagation of pressure waves can be described by the relevant equation.

In this research, we apply the adjoint method to solve the inverse pressure wave transmission equation. The adjoint method replaces the governing equation with pressure and velocity as dependent variables is replaced by the adjoint equation with the adjoint state as dependent variables. The key to efficiently computing the adjoint solution of the related system is to compute the sensitivity. In the inverse model, the adjoint equation model is equivalent to the forward model for describing the physical processes. However, the flow of information in the pipeline is reversed, meaning that the adjoint state is transmitted inversely in time.

Furthermore, the paper utilizes the sensitivity analysis method to derive the adjoint equation, which describes the propagation variation of the pressure wave with time and displacement in reverse processes. This allows us to determine the exact location of the pressure mutation, which is transmitted through the pipeline. This location is likely the most probable location of the leakage in the pipeline.

2. The derivation of the adjoint equation of the single-phase flow governing equation

Fig. 1 illustrates the propagation of a negative pressure wave in both directions along the pipeline when leakage occurs. A pressure sensor is installed at the pressure detection point $(x_d = 0)$, and the location of the detection point can in principle be chosen arbitrarily to determine whether the pressure can be detected. For the pressure wave generated for any location, we can select several possible leakage locations and run a forward model for each of them. However, using a forward model to obtain such information can be computationally inefficient as it requires one simulation for each potential leakage location. In contrast, by using the inverse model, all necessary information about possible leakage locations can be obtained through a single simulation.

2.1. Forward model and solution

In this dissertation, the transient flow governing equation is used to describe pressure transport, disregarding the effects of friction, damping, and other factors. The equation is formulated as follows:

$$\begin{cases} \dfrac{\partial p}{\partial t} + \rho a^2 \dfrac{\partial v}{\partial x} = 0 \\ \dfrac{\partial v}{\partial t} + \dfrac{1}{\rho}\dfrac{\partial p}{\partial x} = 0 \end{cases} \quad t \geq 0, 0 \leq x < +\infty \tag{1}$$

Initial condition: $p(x,t) = 0$ at $t = 0$, $v(x,t) = 0$ at $t = 0$.

Boundary condition: $p(x,t) = p_0(t)$ at $x = 0$, $p(x,t) = 0$ at $x \to +\infty$, $v(x,t) = 0$ at $x = 0$.

Where p is pressure, Pa; v is liquid velocity, m/s; x is the spatial dimension, m; t is time, s; a is the velocity of pressure wave, m/s; ρ is the density of the liquid, kg/m^3.

Taking the Laplace transform of $p(x,t)$ and $v(x,t)$ with respect to $t \to s$:

$$\begin{aligned} P(x,s) &= \int_0^{+\infty} p(x,t)e^{-st}dt \\ V(x,s) &= \int_0^{+\infty} v(x,t)e^{-st}dt \end{aligned} \tag{2}$$

Using Laplace transforms, we can transform the coupled equation (2) above:

$$\begin{cases} \dfrac{dP(x,s)}{dx} + \rho s V(x,s) = 0 \\ \dfrac{dV(x,s)}{dx} + \dfrac{1}{\rho a^2} s P(x,s) = 0 \end{cases} \tag{3}$$

The homogeneous linear differential equation (3) with constant coefficients can be derived by simplification:

$$\begin{cases} \dfrac{d^2 P(x,s)}{dx^2} - \dfrac{s^2}{a^2} P(x,s) = 0 \\ \dfrac{d^2 V(x,s)}{dx^2} - \dfrac{s^2}{a^2} V(x,s) = 0 \end{cases} \tag{4}$$

Initial condition: $P(x,0) = 0, V(x,0) = 0$ at $s = 0$.

Boundary condition: $P(0,s) = P_0(s), P(+\infty,s) = 0, V(0,s) = 0$.

The general solution of the above equation (4) is as follows:

$$P(x,s) = A_1 e^{\frac{s}{a}x} + A_2 e^{-\frac{s}{a}x} \ (A_1, A_2, \text{belongs to the constant}) \tag{5}$$

Substituted initial conditions and boundary conditions into the solution of equation (5):

$$P(x,s) = P_0(s)e^{-\frac{s}{a}x} \tag{6}$$

In equation (6), $P_0(s) = \int_0^{+\infty} p_0(t)e^{-st}dt$, indicating the formula as a function of the Laplace transformation. Here, $p_0(t)$ denotes that the pressure wave is detected from the initial location $(x = 0)$. For the purpose of analyzing the propagation characteristics of the pressure wave in the pipe network, it is necessary to take the inverse Laplace transformation and deduce the corresponding original function $p(x,t)$. For the sake of confirmation, it is assumed that the amplitude of the pressure wave at $x = 0$ is p_0, which is the step function is imported from the initial location, namely:

$$p_0(t) = p_0 U(t) \tag{7}$$

Including: $U(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$.

By definition: $p_0(s) = \frac{p_0}{s}$.

Thus, by applying the delay theorem of the Laplace transform, it is possible to obtain the pressure instantaneous expression (8) of the pipe network at an arbitrary point, based on the above relational expression (7):

$$p(x,t) = p_0 U\left(t - \frac{x}{a}\right) \tag{8}$$

2.2. Inverse model

2.2.1. The derivation of the adjoint equation

To locate pipeline leaks, it is necessary to construct an inverse problem for the governing differential equations of pressure and velocity and use canonical transformation to convert it into a peak source finding problem of pressure fluctuations in the pipeline. This allows for analytical solutions to be carried out. In the problem of pipe network leakage location, we use adjoint theory as a method to obtain the inverse pressure wave transmission equation (inverse model). In the inverse model, the adjoint equation model is an equivalent description of the forward model that describes the physical process. The difference is that the flow of information is opposite (i.e., the adjoint state propagates backward in time).

The adjoint theory used in this paper is the sensitivity analysis method. The sensitivity analysis method replaces the governing equation with pressure and velocity (forward operator) as dependent variables is replaced by the adjoint equation with the adjoint state as dependent variables (adjoint operator).

To perform sensitivity analysis, a performance measure must be defined for quantifying the system parameters, which measures the marginal sensitivity (derivative) when the parameter values change. Even if the purpose is not sensitivity analysis, it is possible to derive adjoint equations using the sensitivity method. In this section, a performance measure is utilized to determine the adjoint equation for the single-phase flow pressure governing equation.

This objective function, L (the performance measure), for quantifying the state quantity of the system as follows:

$$L = \iint_{x,t} h(\alpha, p) dxdt \tag{9}$$

The state of the system is represented by a function $h(\alpha,p)$, the system parameters are represented by α (e.g., $\alpha = [v, D, a]$), the pressure is represented by p, and the integration is in the whole space-time domain.

By differentiating with respect to α_k, the marginal sensitivity of the system with respect to a parameter α_k can be obtained:

$$\frac{dL}{d\alpha_k} = \iint_{x,t} \left[\frac{\partial h(\alpha,p)}{\partial \alpha_k} + \frac{\partial h(\alpha_k,p)}{\partial p}\varphi\right] dxdt \tag{10}$$

Where $t \le t'$ represents the sensitivity of the state parameter. When x, t and other parameters in α are fixed, φ means that the system state parameter changes with the parameter α_k.

Since the state sensitivity is unknown, the transient governing equation φ is obtained by combining adjoint theory with the governing equation of single-phase flow pressure. Differentiating the couple of equation (1), and their boundary and initial conditions with respect to α_k gives:

$$\begin{cases} \dfrac{\partial}{\partial t}\left(\dfrac{\partial p}{\partial \alpha_k}\right) + \rho a^2 \dfrac{\partial}{\partial x}\left(\dfrac{\partial v}{\partial \alpha_k}\right) = 0 \\ \dfrac{\partial}{\partial t}\left(\dfrac{\partial v}{\alpha_k}\right) + \dfrac{1}{\rho}\dfrac{\partial}{\partial x}\left(\dfrac{\partial p}{\partial \alpha_k}\right) = 0 \end{cases} \tag{11}$$

$$\frac{\partial p(x,t)}{\partial \alpha_k} = 0 \text{ at } t = 0 \ , \ \frac{\partial v(x,t)}{\partial \alpha_k} = 0 \text{ at } t = 0$$

$$\frac{\partial p(x,t)}{\partial \alpha_k} = \frac{\partial p_0(t)}{\partial \alpha_k} \text{ at } x = 0 \ , \ \frac{\partial p(x,t)}{\partial \alpha_k} = 0 \text{ at } x \to +\infty$$

$$\frac{\partial v(x,t)}{\partial \alpha_k} = 0 \text{ at } x = 0 \ , \ \frac{\partial v(x,t)}{\partial \alpha_k} = 0 \text{ at } x \to +\infty$$

Substituted $\varphi = \partial p / \partial \alpha_k$, $\lambda = \partial v / \partial \alpha_k$ into the above equation (11):

$$\begin{cases} \dfrac{\partial}{\partial t}(\varphi) + \rho a^2 \dfrac{\partial}{\partial x}(\lambda) = 0 \\ \dfrac{\partial}{\partial t}(\lambda) + \dfrac{1}{\rho}\dfrac{\partial}{\partial x}(\varphi) = 0 \end{cases} \tag{12}$$

$\varphi(x,t) = 0$ at $t = 0$, $\lambda(x,t) = 0$ at $t = 0$.
$\varphi(x,t) = 0$ at $x = 0$, $\varphi(x,t) = 0$ as $x \to +\infty$.
$\lambda(x,t) = 0$ at $x = 0$, $\lambda(x,t) = 0$ at $x \to +\infty$.

Following that, in order to obtain the governing equations in a similar form according to the adjoint states φ^*, λ^* which are respectively just arbitrary functions. In the first step, we define η and ξ as the inner product, to be $\iint_{x,t}\eta\bar{\xi}\,dxdt$, with $\bar{\xi}$ denoting complex conjugate. In this paper, all functions are real, which means $\bar{\xi}=\xi$. Taking the inner product of φ^*, λ^* and each term on both sides of (12) gives:

$$\int_0^T\int_{x_1}^{x_2}\left[\varphi^*\left(\frac{\partial\varphi}{\partial t}+\rho a^2\frac{\partial\lambda}{\partial x}\right)\right]dxdt=0 \tag{13}$$

$$\int_0^T\int_{x_1}^{x_2}\left[\lambda^*\left(\frac{\partial\lambda}{\partial t}+\frac{1}{\rho}\frac{\partial\varphi}{\partial x}\right)\right]dxdt=0 \tag{14}$$

Integral interval changed: $x_1\le x\le x_2$ and $0\le t\le T$ (It will be shown later that the last time T corresponds to the detection time). By manipulating the equations term by term, a similar equation can be obtained with φ^*,λ^* as the state variable.

Taking the two terms of the first equation (13), we can obtain equation (15) and (16):

$$\int_0^T\int_{x_1}^{x_2}\varphi^*\frac{\partial\varphi}{\partial t}dxdt=\int_0^T\int_{x_1}^{x_2}\frac{\partial\varphi\varphi^*}{\partial t}dxdt-\int_0^T\int_{x_1}^{x_2}\varphi\frac{\partial\varphi^*}{\partial t}dxdt \tag{15}$$

$$\int_0^T\int_{x_1}^{x_2}\varphi^*\frac{\partial\lambda}{\partial x}dxdt=\int_0^T\int_{x_1}^{x_2}\frac{\partial\lambda\varphi^*}{\partial x}dxdt-\int_0^T\int_{x_1}^{x_2}\lambda\frac{\partial\varphi^*}{\partial x}dxdt \tag{16}$$

Taking the two terms of the second equation (14), we can obtain equations (17) and (18):

$$\int_0^T\int_{x_1}^{x_2}\lambda^*\frac{\partial\lambda}{\partial t}dxdt=\int_0^T\int_{x_1}^{x_2}\frac{\partial\lambda\lambda^*}{\partial t}dxdt-\int_0^T\int_{x_1}^{x_2}\lambda\frac{\partial\lambda^*}{\partial t}dxdt \tag{17}$$

$$\int_0^T\int_{x_1}^{x_2}\lambda^*\frac{\partial\varphi}{\partial x}dxdt=\int_0^T\int_{x_1}^{x_2}\frac{\partial\varphi\lambda^*}{\partial x}dxdt-\int_0^T\int_{x_1}^{x_2}\varphi\frac{\partial\lambda^*}{\partial x}dxdt \tag{18}$$

Substituting (15–18) into (13) and (14), rearranging each term gives:

$$\int_0^T\int_{x_1}^{x_2}-\varphi\left(\frac{\partial\varphi^*}{\partial t}+\frac{1}{\rho}\frac{\partial\lambda^*}{\partial x}\right)-\lambda\left(\frac{\partial\lambda^*}{\partial t}+\rho a^2\frac{\partial\varphi^*}{\partial x}\right)+\left[\frac{\partial\varphi\varphi^*}{\partial t}+\frac{\partial\lambda\lambda^*}{\partial t}+\rho a^2\frac{\partial\lambda\varphi^*}{\partial x}+\frac{1}{\rho}\frac{\partial\varphi\lambda^*}{\partial x}\right]dxdt=0 \tag{19}$$

Since the left-hand side of equation (19) is equal to zero, adding this to the right side of the marginal sensitivity equation (10) in the inverse model yields:

$$\begin{aligned}\frac{dL}{d\alpha_k}=&\int_0^T\int_{x_1}^{x_2}\left\{\frac{\partial h(\alpha,p)}{\partial\alpha_k}+\varphi\left[\frac{\partial h(\alpha,p)}{\partial p}-\frac{\partial\varphi^*}{\partial t}-\frac{1}{\rho}\frac{\partial\lambda^*}{\partial x}\right]-\right.\\ &\left.\lambda\left[\rho a^2\frac{\partial\varphi^*}{\partial x}+\frac{\partial\lambda^*}{\partial t}\right]+\left(\frac{\partial\varphi\varphi^*}{\partial t}+\frac{\partial\lambda\lambda^*}{\partial t}\right)+\left(\rho a^2\frac{\partial\lambda\varphi^*}{\partial x}+\frac{1}{\rho}\frac{\partial\varphi\lambda^*}{\partial x}\right)\right\}dxdt\end{aligned} \tag{20}$$

The final two terms of this equation (20) are assessed at the boundary condition. Therefore, these terms can be condensed as follows:

$$\int_0^T\int_{x_1}^{x_2}\left(\frac{\partial\varphi\varphi^*}{\partial t}+\frac{\partial\lambda\lambda^*}{\partial t}\right)dxdt=\int_{x_1}^{x_2}(\varphi\varphi^*+\lambda\lambda^*)\big|_T 0dx \tag{21}$$

And

$$\int_0^T\int_{x_1}^{x_2}\left(\rho a^2\frac{\partial\lambda\varphi^*}{\partial x}+\frac{1}{\rho}\frac{\partial\varphi\lambda^*}{\partial x}\right)dxdt=\int_0^T\left(\rho a^2\lambda\varphi^*+\frac{1}{\rho}\varphi\lambda^*\right)\big|_{x_2}x_1dt \tag{22}$$

During this exercise, the goal is to eliminate the unknown sensitivity of the state parameter φ from equations (21) and (22). The state sensitivity φ from equations (21) and (22) can be eliminated using the arbitrariness of the adjoint state φ^*. Taking these into consideration, the adjoint governing equation is as follows:

$$\begin{cases}\dfrac{\partial h(\alpha,p)}{\partial p}-\dfrac{\partial\varphi^*}{\partial t}-\dfrac{1}{\rho}\dfrac{\partial\lambda^*}{\partial x}=0\\[2ex] \dfrac{\partial\lambda^*}{\partial t}+\rho a^2\dfrac{\partial\varphi^*}{\partial x}=0\end{cases} \tag{23}$$

Furthermore φ^* and λ^* must satisfy the following requirements, respectively:

$$(\varphi\varphi^*+\lambda\lambda^*)\Big|\begin{matrix}T\\0\end{matrix}=0 \tag{24}$$

$$\left(\rho a^2\lambda\varphi^* + \frac{1}{\rho}\varphi\lambda^*\right)\Big|_{x_1}^{x_2} = 0 \tag{25}$$

Using these initial and boundary conditions with (24) and (25), these initial and boundary conditions corresponding to the adjoint equations are obtained. Substituting, into (24), which are left with $(\varphi\varphi^* + \lambda\lambda^*)|_{t=T} = 0$. We can obtain the initial conditions for the adjoint equations are $\varphi(x,T) = 0$, $\lambda(x,T) = 0$. For the boundary conditions, substituting the boundary conditions , into (25), which are left with $(\rho^2 a^2\lambda\varphi^* + 1/\rho\varphi\lambda^*)|_{x=0} = 0$. We can obtain the boundary conditions for the adjoint equations are $\varphi^*(0,t) = p_0(t)$, $\lambda^*(0,t) = 0$.

Thus, the undetermined term in the adjoint equation is only the function $h(\alpha,p)$. Among them, the pressure p in the function is the independent variable, and the variables in the governing equation (12) are also in terms of pressure and velocity. Therefore, the form of $h(\alpha,p)$ depends on the specific working conditions to be studied. The appropriate function, h, is

$$h(\alpha,p) = p(x,t)\delta(x-x')\delta(t-t') \tag{26}$$

Where $p(x,t)$ is the pressure wave propagation function, δ is the Dirac delta function, x' and t' are the location and time of the leak.

According to equation (9), L is the performance measure of pressure detection in the space-time domain. Integrating over the (x,t) domain after substituting (26) into (9) gives:

$$L = p(x',t') \tag{27}$$

The above Equation (27) represents the expression of the objective function L.Taking the Frechet derivative of (26) on both sides, we can obtain the following:

$$\frac{\partial h(\alpha,p)}{\partial p} = \delta(x-x')\delta(t-t') \tag{28}$$

We note that when solving the forward problem, the time T represents the upper bound in the time domain. If the pressure at (x,t) is a performance measure in the forward model, it is obvious that only the case when $t \le t'$ needs to be considered. This means that the upper bound in the time domain can be presumed to $T = t'$ in the forward problem. For the inverse problem, the propagation time of the inverse pressure $\tau = t' - t$. Taking $T = t'$, then the final form of adjoint equation (23) and its corresponding initial and boundary conditions for the pressure and velocity on the semi-infinite interval can be obtained according to equation (28) as follows:

$$\begin{cases} \dfrac{\partial\varphi^*}{\partial t} + \dfrac{1}{\rho}\dfrac{\partial\lambda^*}{\partial x} = \delta(x-x')\delta(t-T) \\ \dfrac{\partial\lambda^*}{\partial t} + \rho a^2\dfrac{\partial\varphi^*}{\partial x} = 0 \end{cases} \tag{29}$$

Initial condition: $\varphi^*(x,T) = 0$, $\lambda^*(x,T) = 0$.

Boundary condition: $\varphi^*(0,t) = p_0(t)$, $\lambda^*(0,t) = 0$.

2.2.2. The solution of the adjoint equation

Since the adjoint equation is a partial differential equation, the Laplace transform is often used to transform them into the ordinary differential equations that are easy to solve. To seek the solution to the adjoint equations above, a new time variable $\tau = T - t$ is defined and substituted into equation (29) for equivalent transformation:

$$\begin{cases} -\dfrac{\partial\varphi^*}{\partial\tau} + \dfrac{1}{\rho}\dfrac{\partial\lambda^*}{\partial x} = \delta(x-x')\delta(\tau) \\ -\dfrac{\partial\lambda^*}{\partial\tau} + \rho a^2\dfrac{\partial\varphi^*}{\partial x} = 0 \end{cases} \tag{30}$$

Initial condition: $\varphi^*(x,0) = 0$, $\lambda^*(x,0) = 0$.

Boundary condition: $\varphi^*(0,t) = p_0(t)$, $\lambda^*(0,t) = 0$.

Applying the Laplace transform to the left and right sides of equation (30) with respect to time $\tau(\tau\to s)$ gives:

$$\begin{cases} -s\psi + \varphi^*|_{\tau=0} + \dfrac{1}{\rho}\dfrac{d\lambda\!\!\!\!-}{dx} = g(x) \\ -s\lambda\!\!\!\!- + \lambda^*|_{\tau=0} + \rho a^2\dfrac{d\psi}{dx} = 0 \end{cases} \tag{31}$$

Initial condition: $\psi(x,0) = 0$, $\lambda\!\!\!\!-(x,0) = 0$.

Boundary condition: $\psi(0,s) = p_0(s) = \frac{p_0}{s}$, $\lambda\!\!\!\!-(0,s) = 0$.

Where ψ is the Laplace transform of φ^*, $g(x) = \delta(x - x')$. $\varphi^*|_{\tau=0}$ and $\lambda^*|_{\tau=0}$ are equal to zero, according to $\tau = 0$. Applying the Laplace transform to the left and right sides of equation (31) again with respect to $x(x\to r)$ gives:

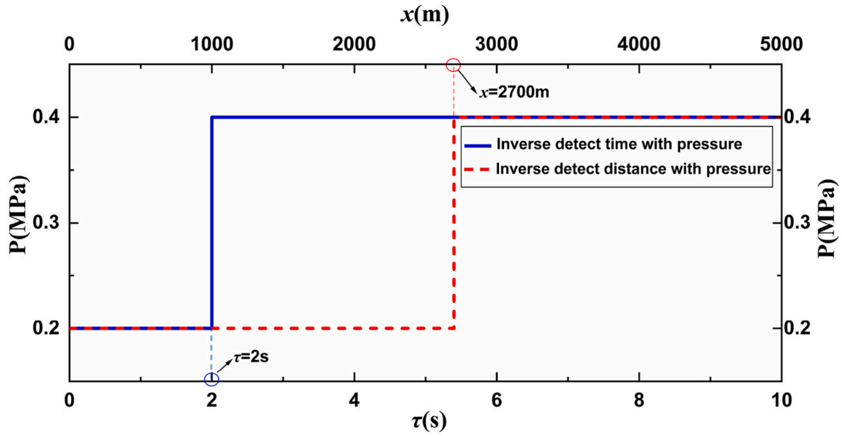

Fig. 2. Pressure changing with inverse propagated time and distance.

$$\begin{cases} -s\widehat{\psi}+\frac{1}{\rho}r\widehat{\lambda}-\frac{1}{\rho}\lambda|_{x=0}=\widehat{g(x)} \\ -s\widehat{\lambda}+\rho a^2 r\widehat{\psi}-\rho a^2\psi|_{x=0}=0 \end{cases} \tag{32}$$

Where $\widehat{\psi}$ is the Laplace transform of ψ, $\widehat{\lambda}$, $\widehat{g(r)}$ is the Laplace transform of λ and $g(x)$ respectively. Based on the boundary condition at $x=0$, equation (32) can be simplified as follows:

$$\begin{cases} -s\widehat{\psi}+\frac{1}{\rho}r\widehat{\lambda}=\widehat{g(x)} \\ -s\widehat{\lambda}+\rho a^2 r\widehat{\psi}-\rho a^2\frac{p_0}{s}=0 \end{cases} \tag{33}$$

Further, equation (33) can be simplified as follows:

$$\widehat{\psi}=\frac{p_0}{s}\frac{a^2 r}{a^2r^2-s^2}+\frac{s\widehat{g(r)}}{a^2r^2-s^2} \tag{34}$$

Taking the inverse Laplace transform for the above equation (34) with respect to $r(r\to x)$ as follows:

$$\psi=L^{-1}_{r\to x}\left[\frac{p_0}{s}\frac{a^2 r}{a^2r^2-s^2}+\frac{s\widehat{g(r)}}{a^2r^2-s^2}\right] \tag{35}$$

According to equation (35), the following expression for ψ is obtained by using the table of Laplace transform and the convolution theorem:

$$\psi=\frac{p_0}{s}\left(e^{\frac{s}{a}x}+e^{-\frac{s}{a}x}\right)+\frac{1}{2a}\int_0^x\left[e^{\frac{s}{a}(x-x'')}-e^{-\frac{s}{a}(x-x'')}\right]g(x'')dx'' \tag{36}$$

Taking the inverse Laplace transform for the above equation (36) with respect to s time, and using some algebraic operations and the shifting property, the solution of the adjoint equation can be obtained as below:

$$\varphi^*(x,\tau)=\frac{p_0}{2}\left[U\left(\tau+\frac{x}{a}\right)+U\left(\tau-\frac{x}{a}\right)\right]+\frac{1}{2a}\left[\delta\left(\tau+\frac{x}{a}\right)-\delta\left(\tau-\frac{x}{a}\right)\right] \tag{37}$$

For the negative pressure wave propagated at the detection point ($x_d=0$), this equation (37)describes the pressure change at time τ, and leakage location x before the pressure wave was detected at $x_d=0$.

To facilitate the analysis from the perspective of gradient change, it can be seen that equation (38) is composed of a linear superposition of a unit step function and a pulse function based on the mathematical characteristics. The partial derivatives of equation (39) with respect to time and displacement independent variables can be solved as follows:

$$\partial\varphi^*\Big/\partial\tau=\frac{p_0}{2}[\delta(\tau+x/a)-\delta(\tau-x/a)] \tag{38}$$

$$\partial\varphi^*\Big/\partial x=\frac{p_0}{2a}[\delta(\tau+x/a)-\delta(\tau-x/a)] \tag{39}$$

The analytical solution and partial derivative of the adjoint equation provide the theoretical basis for locating the leakage location in the pipe network.

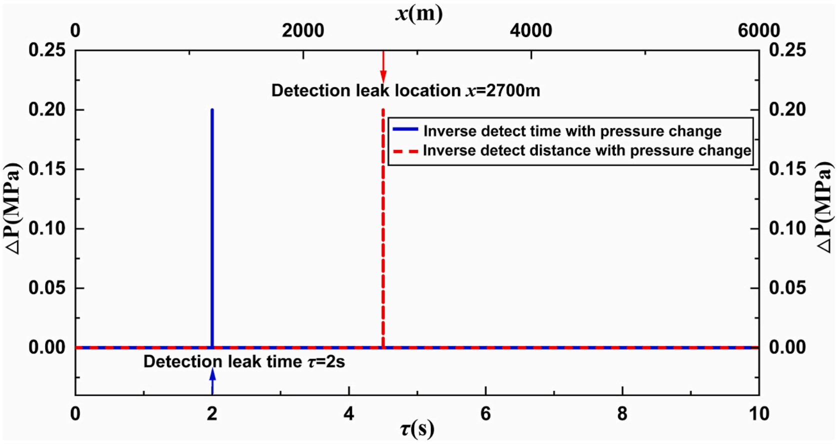

Fig. 3. The differential pressure changing with inverse propagated time and distance.

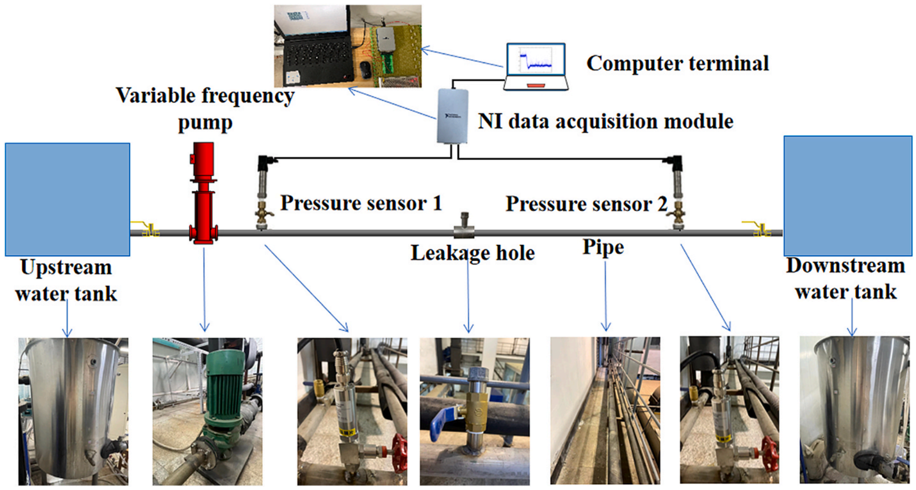

Fig. 4. The diagram of the pipeline experiment system.

3. The method and analysis of leakage detection with adjoint equation

Assuming that the negative pressure wave is detected at the pressure detection point at $x_d = 0$. The initial pressure of the pipeline is 0.4Mpa. Fig. 2 shows that the inverse propagated time from $x_d = 0$ to simulated leakage location $x = 2700$ m is $\tau = 2$ s, and the pressure signal exhibits a step change. The pressure wave velocity is calculated according to 1350 m/s, and it is evident that the leakage detection time is consistent with the location of the leakage. Similarly, Fig. 2 shows the pressure change curve of pressure wave in reverse propagation when the time of the simulated leakage process is $\tau = 5.36$. The mutation of pressure value from 0.2 MPa to 0.4 MPa at leakage location $x = 2700$ m.

Moreover, from the perspective of differential pressure, we could obtain the more apparent features directly, which describe the pressure mutation to identify leakage location.

In this process, the value of differential pressure hardly changes with the time variable until detected time at $\tau = 2$ s, as shown in Fig. 3. At this point, a pulse of differential pressure occurs, indicating the pressure change caused by leakage. Similarly, the differential pressure during inverse propagation hardly changes with the propagated distance until it reaches $x = 2700$ m, where a pulse of differential pressure occurs due to pipeline leakage.

4. Experimental validation of single-phase flow transient adjoint equation model

The experimental pipeline is composed of galvanized steel pipes, with the length of 100 m, the diameter of 40 mm, and the wall thickness of 3.5 mm. The leakage part is composed of valve, pipe and metal cap. Leakage part consists of valve, pipe and metal cap. The leak hole is located above the metal cap, 45 m from the inlet, and the diameter of the leak hole is adjustable (220 mm). The high-frequency dynamic pressure sensors are used to measure the pressure signal at a distance of 5 m from the head and end, with the

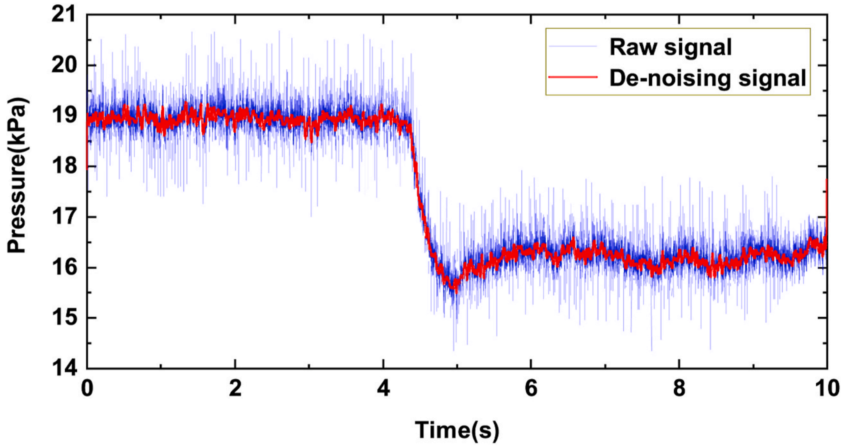

Fig. 5. Dynamic pressure changes at the monitoring point.

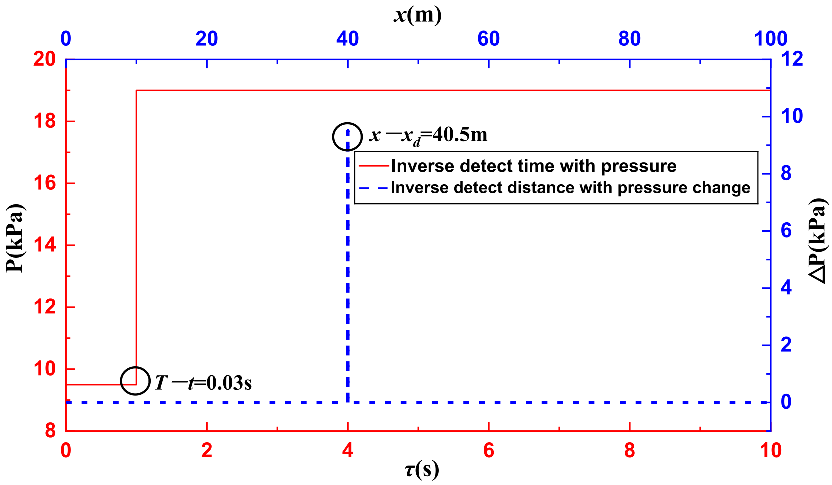

Fig. 6. Leakage detection and location based on the adjoint equation of inversion modeling.

resonant frequency ≥200 kHz. The Leak holes and sensors are connected by valves, and their positions can be interchanged as per the experimental requirements. Pressure signals are acquired by NI9205 at a frequency of 2000Hz. The measured temperature is 293k, the inlet pressure is 19 kPa, and the velocity of the negative pressure wave used is 1350 m/s. The leakage rate ranges from 0.1L/min to 0.5L/min. The experimental setup for the pipeline leakage is depicted in Fig. 4.

Fig. 5 depicts the variation of dynamic pressure in the pipeline when the leakage occurs at 40 m away from the detection point towards the leakage position. To de-noise the original signal, the wavelet threshold (WT) method is used. As shown in the figure, the average value of the initial pressure signal in the pipeline is approximately 19.0 kPa. Upon reaching a stable state, the flow and pressure in the pipeline experience a sudden change when the leakage hole is opened, resulting in a significant drop in the average pressure value, which stabilizes at approximately 16.0 kPa. Furthermore, there is a significant change in the amplitude of the pressure signal.

Combining the adjoint model of the derived pressure governing equation with experimental data, the solution of the equation was verified and analyzed. The pressure change over time is shown in Fig. 6. It is observed that the calculation result of the inverse adjoint model is exactly opposite to the experimental result, which is attributed to the inverse treatment of the time term of the adjoint equation. Furthermore, the inverse adjoint model neglects pipe resistance and has no effect on noise, resulting in a higher pressure drop compared to the experiments. Generally, the forward pressure propagation curve measured in the experiment agrees well with the back propagation curve obtained from the adjoint equation of single-phase flow.

In Fig. 6, the sudden change in differential pressure from 9.5 kPa to 19 kPa occurs at $x - x_d = 40.5$ m, indicating the presence of a leak in the pipeline with a propagation time of $\tau = 0.03$ s from the sensor to the leakage position. However, it should be noted that the frictional resistance is not considered in the model proposed in this study, and hence, the actual distance from the pressure detection point to the leakage point is $x - x_d = 40$ m.

To validate the accuracy of the proposed model, we adjusted the positions of the two pressure sensors and the leak point, and the

Table 1
Leakage localization results of different distance from the leakage point to the pipeline inlet.

Leakage distance x (m)	Detection point x_d (m)	Inverse detect distance $\|x-x_d\|$ (m)	AEIM in this paper $\|x-x_d\|$ (m)	Relative error (%)
5	45	40	40.5	1.25
	95	90	87.75	2.5
45	5	40	40.5	1.25
	95	50	49.41	1.18
95	45	50	49.41	1.18
	5	90	87.75	2.5

leak location results for the remaining two positions of the pressure sensors placed at 5 m, 45 m, and 95 m from the inlet are presented in Table 1. The adjoint equation of inversion modeling (AEIM) accurately calculates the leakage location with an error of less than 3%.

5. Conclusions

In this paper, the pressure transport governing equation of single-phase flow is firstly established. The pressure wave effect of the pipeline leakage on the flow field is used as the basis to construct the sensitivity function. The sensitivity function is used to derive the transient adjoint equation of the pipeline flow field by taking the derivative of the pressure. The adjoint equation is reverse processed in time and solved using Laplace transform and inverse transform to obtain the inverse problem solution model, which can be used to validate the high-dimensional numerical solutions of the inverse model.

The inverse-in-time propagated pressure can offer information about the leakage location of the pressure before it is detected. For each detection, the benefit of the inverse model is that it only needs to be solved once to acquire the leakage location and pressure information of the pipeline at a given time. This makes the inverse model less computationally intensive and more efficient than the forward model in simulation, as there are fewer simulation models to run. The results of the analysis of the plot of pressure and differential pressure changes with time and displacement are satisfactory, and this method can solve gas-liquid two-phase flow complex pipe networks.

In this paper, the inverse adjoint method is derived from the theoretical analysis. We do not consider the influence of the friction resistance of the pipeline in the derived inverse adjoint method. In future work, we will add the dynamic resistance term to the adjoint equation to make the inverse adjoint model more realistic and further verify the accuracy of the proposed method. Additionally, the method proposed in this paper is not applicable to pipelines with high pressure and temperature variations.

Declarations

Author contribution statement

Chang Chang: Performed the experiments; Analyzed and interpreted the data; Wrote the paper.

Xiangli Li & Wenbin Zhou: Conceived and designed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data.

Lin Duanmu: Contributed reagents, materials, analysis tools or data; Wrote the paper.

Hongwei Li: Analyzed and interpreted the data.

Data availability statement

Data will be made available on request.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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