Heliyon 8 (2022) e12122

Contents lists available at ScienceDirect

Heliyon

journal homepage: www.cell.com/heliyon

Review article

# Soliton: A dispersion-less solution with existence and its types

Geeta Arora [a], Richa Rani [a], Homan Emadifar [b,*]

[a] Department of Mathematics, Lovely Professional University, Phagwara, Punjab, India
[b] Department of Mathematics, Hamedan Branch, Islamic Azad University, Hamedan, Iran

ARTICLE INFO

*Keywords:*
Korteweg de Vries equation
sine-Gordon equation
Camassa-Holm equation
Nonlinear Schrodinger equation
Solitons
Properties of solitons

ABSTRACT

A solitary wave is the dispersion-less solution of nonlinear evolutionary equations that travels at a constant speed without dissipating its energy. The purpose of this article is to provide insight into the discovery and history of solitons. The different types of the solitons are discussed in brief that is helpful for the researchers. For the discussion of the nature of solitons, the solution behavior of the Korteweg de Vries equation (KdV), the sine-Gordon (SG), the Camassa-Holm (CH) equation, and the nonlinear Schrodinger (NLS) equation are considered. This article deals with the various applications of solitons in different fields such as biophysics, nonlinear optics, Bose-Einstein condensation, plasma physics, Josephson junction, etc. focusing on the properties of solitons based on their classification.

## 1. Introduction

Solitons are a special type of long-wave that are non-dispersive and travel in the form of packets with constant velocity. They are also called shallow-water waves with a permanent shape. A soliton has the special property that its shape remains unchanged when it collides with another soliton. This behaviour of solitons has attracted mathematicians, physicists, and engineers because of their robustness and applicability in physics applications. The soliton exists as a solution to nonlinear partial differential equations.

This paper is an effort to give quick exposure to the researchers in this area by providing information about the many types of soliton and their applications. The present work is elaborating the differential equations having the soliton solutions by giving an outline to the users regarding the soliton types that are contributing and hence enriching the discipline to explore new findings.

### *1.1. History*

A soliton, or solitary wave, is a type of self-reinforcing wave packet that continues to propagate at a constant speed and retains its original shape. Over the past three decades, the solitary wave phenomenon has been at the forefront of many advances in physics and mathematics. Numerous nonlinear evolutionary equations have been shown to have soliton solutions by numerical calculations and theoretical research. The theory of the soliton has close ties to contemporary physics and is used to provide explanations for a wide variety of physical problems at the cutting edge of this dynamic field.

The credit to the discovery of the concept of soliton goes to John Scott Russell [1], a Scottish naval architect, who in 1834 observed a "Great Translation Wave" in the shallow waters of the Great Britain Canal. He attempted to demonstrate a constant wave by constructing a channel so that the wave could travel a great distance with the channel. He placed the boat in the canal with a rope to which he tied the horses on either side. He found that the wave came to rest due to the obstruction of the wave propagation by the boat, but continued to move at a constant speed without losing its shape. He followed the wave for about 8 miles and found that the wave moves at a constant speed up to 2 miles without losing its shape. He continued his research and briefly described the properties of translational waves as follows:

(a) The waves can travel large distances with constant speed.
(b) The waves never merge, unlike normal waves.
(c) The speed of a wave depends on its size and its width depends on the depth of water.
(d) The higher waves travel faster than the smaller waves.
(e) The velocity of waves can be formulated by an equation which is as follows:

$$V=\sqrt{G(H+A)}$$

where $G$ is the acceleration due to gravity, $A$ is the amplitude of solitary waves; $H$ is the height of shallow water channel and $V$ is the velocity of travelling waves.

* Corresponding author.
*E-mail address:* homan_emadi@yahoo.com (H. Emadifar).

https://doi.org/10.1016/j.heliyon.2022.e12122
Received 16 June 2022; Received in revised form 16 November 2022; Accepted 28 November 2022
2405-8440/© 2022 The Author(s). Published by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).

The results of Russell were not appreciated by the mathematical society and also denied by a researcher named Array. In 1845, Array published his book "Tides and Waves", in which he presented a theory of long waves and focused on the speed of waves which depends on their height and amplitude. This theory indicated that solitary waves by Russell could not exist [2].

In the 1870s, two great physicists, Joseph Boussinesq and Lord Rayleigh, independently further illuminated Russell's observations in the form of a mathematical model [3]. Boussinesq and Rayleigh observed the velocity of a solitary wave and related its height to distance, discussing the properties of high and small waves.

In 1895 [4], the Dutch mathematician Diederik Korteweg, together with Gustav de Vries, formulated a nonlinear partial differential equation, which became famous as the KdV equation, recognizing soliton solutions to describe the shallow water waves. This equation mathematically proves solitary water waves and plays an important role in the development of soliton theory.

In 1955, Fermi, Pasta, and Ulam [5] studied a computer simulation of a one-dimensional nonlinear lattice to discuss its equilibrium state. They believed that the nonlinear interactions with respect to the normal modes of the linear system resulted in the energy of the system being uniformly distributed among all modes. But when they examined the KdV equation numerically, the results reversed this notion. The energy was again distributed unevenly among all modes, but the system returned to its initial position after some time. The problem later became known as the FPU problem.

To understand this recursion phenomenon, Zabusky and Martin Kruskal [6] studied the FPU problem again in 1965. They solved the KdV equation numerically in terms of a nonlinear grid. Further, they noted the surprising property that the interaction of two solitary waves of the KdV equation exhibits elastic behaviour. When two solitary waves collide, they reappear without changing their original shape, size, and velocity. These properties of elastic collision between two particles make them behave like stable particles. They called these solitary waves 'solitons' because of their particle-like behaviour like protons, electrons, photons, etc. This is how the soliton was invented.

When the soliton was discovered, there was no mathematical tool to solve the initial value problem of nonlinear integrable partial differential equations (PDEs). Later, Gardner, Kruskal, Miura, and Greene (GKMG) invented a technique for solving nonlinear PDEs known as inverse scattering (IST). A year later, another mathematical approach for dealing with nonlinear problems was developed by Lax [7]. In this concept, an integrable PDE was framed into a standard form called a Lax pair. Then, Zakharov and Shabat [8] generalized this as a linear matrix eigenvalue problem and solved the nonlinear Schrodinger equation (NLSE) using IST and obtained a soliton solution.

Another scheme, known as the AKNS scheme, was developed by Ablowitz, Kaup, Newell, and Segur [9], who identified solitons with nonlinear evolutionary equations. This scheme was first used to numerically solve the sine-Gordon equation, which was later used to solve several other nonlinear PDEs. There are several other methods such as the bilinear Hirota method and the Backlund transform that are commonly used to solve integrable nonlinear PDEs.

The paper is organized with the first section of introduction and history of soliton. In the second section, the types of solitons are discussed. In the third section, some applications of solitons in the field of science and engineering are briefly introduced. In the fourth section, some nonlinear evolutionary equations that help in the description of solitons are discussed. In the fifth section, concluding remarks on solitons are made.

## 2. Types of solitons

Solitons can be categorized in a number of different ways. Topological and nontopological solitons are two different types of solitons. By taking into account their profiles as permanent and time dependent, all solitons can be split into two groups regardless of their topological character. For instance, all breathers have internal dynamics even though they are static, but kink solitons have a permanent profile (in ideal systems). As a result, their shape changes over time. According to the characteristics of the nonlinear equations, this section deals with some common characteristics of the solitons.

### 2.1. Kink Soliton

Kink soliton is a one-dimensional solitary wave, which signifies a change in the solution value due to the transition from one state to another [10, 11]. They are also known as topological solitons because their velocity does not depend on the wave amplitude.

Topological solitons [12] are defined as a localized lumps of energy in a nonlinear system. They are stable particle-like objects with finite mass, have a smooth structure and appear like monopoles in the nonlinear classical field theory.

The collision properties of solitons are observed in both kinks and anti-kinks solutions. There are many evolutionary equations that yield a kink soliton, such as the KdV equation, the sine-Gordon equation, the Burger's equation, Ostrovsky equation [13], etc. Kink waves rise or fall from one asymptotic state to another and approach a constant level at infinity. The kink-type soliton has been presented by the sine-Gordon equation in section 4.

### 2.2. Breather

A breather is a nonlinear wave in which energy accumulates in an oscillatory and bounded manner. They oscillate in both time and space, but sometimes exhibit oscillations in space and can localize in time. Once a breather reaches its maximum amplitude, it decays symmetrically and eventually disappears. The sine-Gordon equation (SG) [14] and the nonlinear Schrodinger (NLS) equation [15] are examples of one-dimensional PDEs that contain breather-type soliton solutions. In Section 4, the soliton solution of the sine-Gordon equation admitting a breather type was presented.

### 2.3. Gap solitons

These are the solitons that occur in finite gaps in the domain of continuous systems. These types of solitons have been discussed by the NLS equations with periodic solutions observed experimentally in nonlinear optics and Bose-Einstein condensation [16]. Optical gap solitons [17], which exists in nonlinear optical media, are electromagnetic field structures.

The difference between a regular soliton and a gap soliton is due to the dispersion of the group velocity of the photonic band structure. The gap solitons that occur in NLS are presented in Section 4.

### 2.4. Envelope solitons

Envelope solitons are solitary wave solutions that occur in a dispersive nonlinear medium [18, 19, 20]. Envelope solitons can be divided into light and dark solitons. Bright solitons occur with a localized intensity peaking over a constant wave background, while dark solitons are described as a concavity in the continuous background. From the NLS equation bright soliton solutions are derived in the anomalous dispersion regime and dark soliton solutions are derived in the normal dispersion regime [21]. The dark solitons are more stable and less affected by background noise and interference compared to the light solitons. Apart from NLS equation the Chaffee–Infante equation [22] and Kaup–Kupershmidt equation [23] also plays an important role in bright and dark soliton. The envelope soliton resulting from the NLS equation has been presented in Section 4.

### 2.5. Solitary waves with discontinuous derivatives

There are solitary waves with discontinuous derivatives, which can be classified as peakons, cuspons, and compactons [24].

***Peakons*** are solitary waves whose peaks have a discontinuous first derivative [25, 26]. This type of solitary wave solution is smooth, except for a peak at one corner of its vertex. In particular, peakons maintain their velocity and shape after colliding with other peakons. The equation of Camassa-Holm (CH) and the integral equation of Degasperis Procesi (DP) have peakon-type solutions. The peakon solution for the equation CH is presented in Section 4.

***Cuspons*** are soliton solutions where the solutions have cusps at crests [27]. In some special cases, the solutions of the CH and DP equation are of cuspons type. The coupons solution for the DP equation has been presented in Section 4.

***Compactons*** are solitary waves that have a finite wavelength, are free of exponential tails, and have robust soliton-like solutions. They are special solitary waves that have the property of maintaining their shape and travelling at the same speed after colliding with other compactons. The nonlinear dispersive K (n,n) equation, which is a family of nonlinear KdV-like equations, yields a soliton solution of the compacton type presented in Section 4.

## 3. Applications of solitons

Solitons have a broader application perspective in various fields such as biophysics, field theory, plasma physics, fluid dynamics, photonic crystal fibers, optical fibers, condensed matter physics, Josephson junction and Bose-Einstein condensates, surface waves, etc. Some of the above applications are briefly discussed below:

### 3.1. Biophysics

Study of solitons is used in biophysics in the DNA lattice. When a protein comes near to soliton, some conformational changes occur, which cause intracellular communication. This communication of solitons on the DNA lattice is described by Feynman diagrams, which describe the survival of cellular life [28]. The solitary wave is also used to study various biophysical phenomena.

The Davydov soliton [29] is one such soliton that exists as a solution to an equation describing the energy distribution in hydrogen-bonded spines. The nonlinear dynamics of DNA molecules also reveal the presence of solitary waves [30], which arose in the process of splitting double-stranded DNA into single strands [31].

### 3.2. Field theory

Solitons appear in both classical and quantum field theory [10]. Topological solitons exist in field theory [12] in the form of kinks, monopoles, vortices, and skyrmions. In two-dimensional quantum field theory, the sine-Gordon equation has solutions for topological solitons that can be mapped onto the elementary excitations of an exactly solvable quantum field theory [32].

### 3.3. Plasma physics

The study of solitary waves is also related to the study of plasma physics, which contains charged particles in large numbers [33]. For example, the KdV equation reflects the change of charge from neutrality. Another equation describing solitons and solitary-wave solutions for the study of plasma physics is the KP equation, variants of KdV and the KP equation. In addition, the soliton in plasma is studied in various contexts, e.g., to discuss the interaction of solitons in collisionless plasma [6], in Langmuir wave collapse for plasma [34], in the study of soliton stability in plasma and hydrodynamics [35], and in ionic-acoustic solitons in plasma [36, 37] etc. Benjamin–Bona–Mahony (BBM) [38] is considered as an improvement of the KdV equation and used to describe the properties of the long surface gravity wave, acoustic-gravity waves in compressible fluids, hydromagnetic waves in a cold plasma, and acoustic waves in an harmonic crystals.

### 3.4. Fluid dynamics

Solitary waves are also among the characteristics of fluid dynamics. The "translational wave" described by Russell was a water wave [39] and Korteweg and de Vries described a shallow water wave by the KdV equation, which also occurs in a long-wavelength limit. Solitary waves also exist in deep water, as shown by the work of Vladimir Zakharov [40], who set up the NLSE (nonlinear Schrodinger equation) to study these waves. Solitary wave solutions have been constructed in many models of fluid dynamics. For example, tidal wells have been explained using dispersive shock waves, the theory of non-propagating surface--wave solitons [41], the small-amplitude gravity capillary wave as an envelope soliton [42], and the soliton mean-field theory in macroscopic flow hydrodynamics [43], etc.

### 3.5. Optical fiber

In optical fibers, light propagates as solitons. Information is sent through the optical fiber in the form of packets as solitons. Since solitons travel at a speed equal to that of light, they provide high-speed connectivity and a high-bandwidth network [44]. This property feature of optical solitons makes them useful for high-speed communication over an optical fiber [45]. There are applications in a variety of fields related to fiber optics, such as soliton photonic switches, which are used for optical switching by using the process of position shifting of the spatial soliton after collision [46]. In addition, trapping solitons in optical fibers can be used to develop optical logic gates [47]. In the study of optical solitons, the nonlinear Schrodinger equation is crucial [48]. The Fokas-Lenells (FL) equation [49, 50] has been derived as an alternate model equation of the Schrodinger equation for the higher-order terms, and it represents the propagation of short pulses in optical fibers. Complex perturbed Gerdjikov–Ivanov equation [51] describes the physical characterization of the optical soliton waves to mitigate internet bottlenecks with many different applications in the telecommunication industry. Additionally, telegraph equation [52, 53] has an important application electromagnetic waves in communication.

### 3.6. Josephson junctions

The Josephson junction [54] is a nonlinear oscillator consisting of two weakly coupled superconductors separated by a thin nonconducting layer for the passage of electrons. Solitary waves exist as the propagation of electromagnetic waves between two superconductors. These junctions are used in the fabrication of mechanical circuits, e.g., SQUIDs (Superconducting Quantum Interference Devices).

### 3.7. Bose-Einstein condensates (BEC)

In 1924, Bose and Einstein demonstrated the process of Bose-Einstein condensates. At a very low temperature, a finite fraction of particles in a dilute base gas can assume the same quantum state known as BEC. The macroscopic dynamics of BEC near temperature zero is modelled by the Gross-Pitaevskii equation [55, 56]. BECs were experimentally detected in 1995 by trapping atoms of dilute alkali vapours in a magnetic trap, which was then cooled to an extremely low temperature on the order of micro-Kelvins [57, 58]. Rogue waves in nonlinear Schrodinger models with variable coefficients are also an important application for Bose-Einstein condensates [59, 60].

## 4. Nonlinear evolutionary equations and their examples

Nonlinear evolutionary equations (NLEEs) describe nonlinear science in a dynamical way, i.e., in the two dimensions of space and time through the nonlinear systems. These equations are used to describe nonlinear phenomena in various fields of science such as physics, chemistry, biology and ecology, pattern formation, solitons and nonlinear dispersion, etc. They are considered a unique tool for describing and characterizing phenomena in science and engineering. NLEE are nonlinear partial differential equations whose solution exists in the form of solitons and has several important properties. For the discussion of the types of soliton solutions and their properties, three NLEEs are considered, including the well-known KdV equation, the nonlinear Schrodinger equation, the sine-Gordon equation, and the Camassa-Holm equation. A brief discussion is done on the type of solution and the physical behaviour using MATLAB 2014, which are briefly described below:

### *4.1. Korteweg de Vries (KdV) equation*

The KdV is a very simple model of the wave equation, which is hyperbolic in nature. It is a nonlinear equation that links dispersion and nonlinearity. It is the most important class of NLEEs with various applications in engineering and natural sciences. It was originally discovered by Lord Rayleigh in 1812; subsequently, it was mathematically introduced by Joseph Boussinesq in 1877 and rediscovered by Diedrik Korteweg and Gustav de Vries in 1905, who introduced this equation in modelling shallow water waves. The KdV equation plays an important role in the study of compressible fluids in fluid mechanics, in the description of the properties of electron plasmas, in the study of oceanic water waves, and in the study of mass transport problems associated with chemical compounds [61]. A simple generalization of the KdV equation is given as:

$$u_t + \alpha u u_x + u_{xxx} = 0, -\infty < x < \infty, 0 \le t < \infty$$

where $u$ is the wave amplitude, $x$ and $t$ are the space and time variables respectively and the subscripts represent differentiation with respect to the relevant variable. A travelling wave solution of permanent form occurs due to a balance between the dispersive term and the nonlinear term.

The nonlinear dispersive K (n,n) equation, which is a family of nonlinear KdV like equations give compactons type soliton solution is given as:

$$u_t + a(u^n)_x + (u^n)_{xxx} = 0, -\infty < x < \infty, 0 \le t < \infty$$

where a = 1 results in compact solitary travelling soliton. The compactons soliton solution obtained for the K (n,n) equation is presented in Figure 6 for the exact solution given as [61]:

$$u(x,t) = \sqrt{\cos(x-t)}, 0 < x < 1.5, 0 \le t \le 1.5.$$

### *4.2. Sine-Gordon equation*

The sine-Gordon equation (SG) is a nonlinear partial differential equation of hyperbolic nature that has soliton solutions. The structure of the soliton solutions is the same as that of the KdV equations. It was originally introduced by Edmond Bour in 1862 and rediscovered by Frenkel and Kontorova in 1939 while studying crystal dislocations [62]. The sine-Gordon equation is given as:

$$u_{tt} - u_{xx} + \sin(u) = 0, -\infty < x < \infty, 0 \le t < \infty$$

where $u$ is a field variable, $t$ represents the time and $x$ denotes the space coordinate in the direction of propagation. The SG equation admits the soliton solution as presented as in Figures 1 and 7.

The breather soliton solution obtained for the SG equation is presented in Figure 1 for the exact solution given as [62]:

$$u(x,t) = 4\, arctan\left(\frac{v sinh\left(\frac{x}{1-v^2}\right)}{cosh\left(\frac{vt}{1-v^2}\right)}\right), -20 < x < 20, 0 \le t \le 30.$$

The kink solitons that occurred for the SG equation is presented in Figure 7 for the exact solution given as [62]:

$$u(x,t) = 4\tan^{-1}\left(e^{(x-vt)/d}\right), -15 < x < 15, 0 \le t < 10 \text{ for } v = 0.5.$$

where $v$ represents the velocity of soliton with $d = \sqrt{1-v^2}$ is the Lorenz contraction factor.

This equation has a wide range of application in physics, not only in relativistic field theories but also in study of solid-state physics, nonlinear optics, shape waves, mechanical transmission lines and Josephson junction, Bloch wall motion of magnetic crystals and nonlinear dynamics of DNA etc.

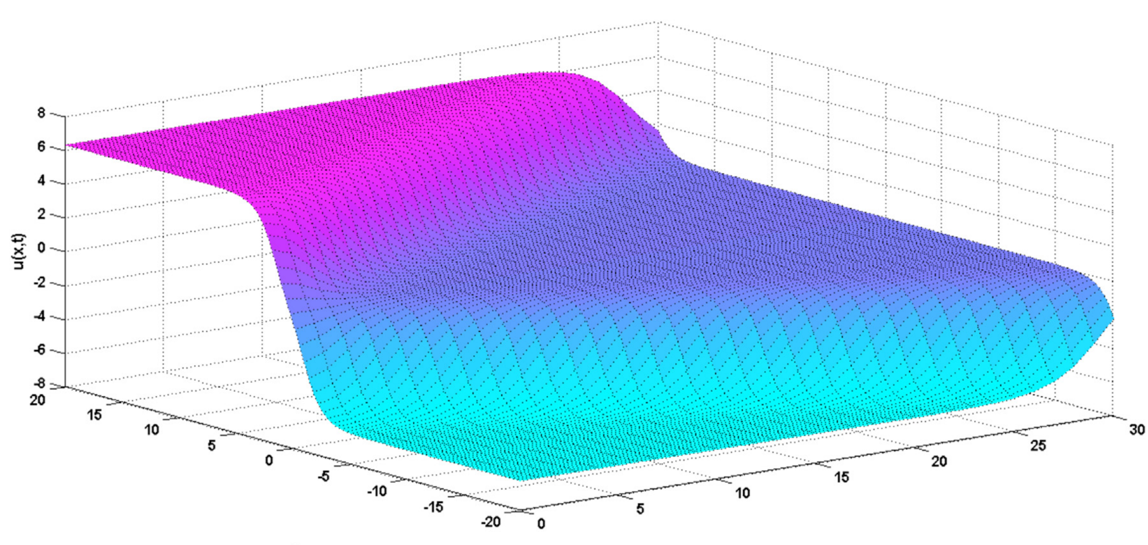


**Figure 1.** Breathers type soliton for SG equation.

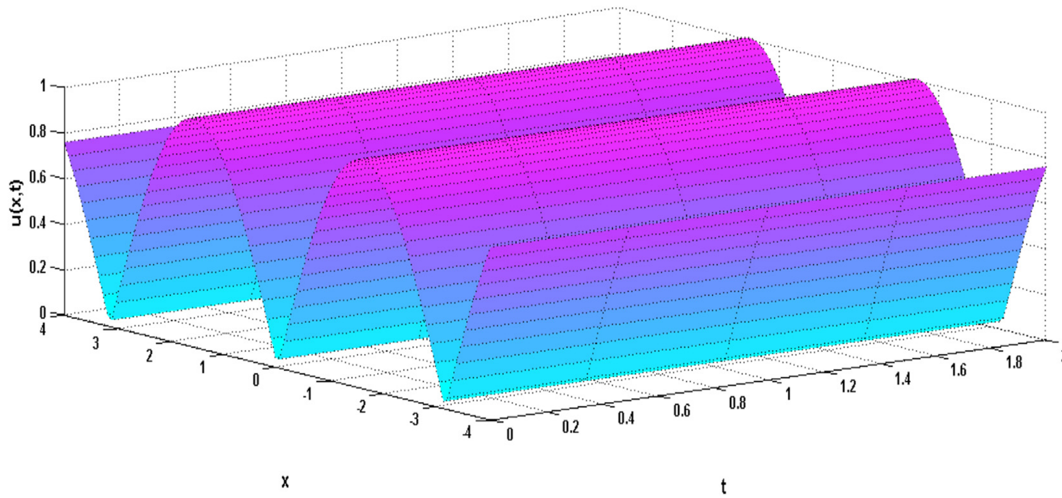


**Figure 2.** Gap solitons from NLS Equation.

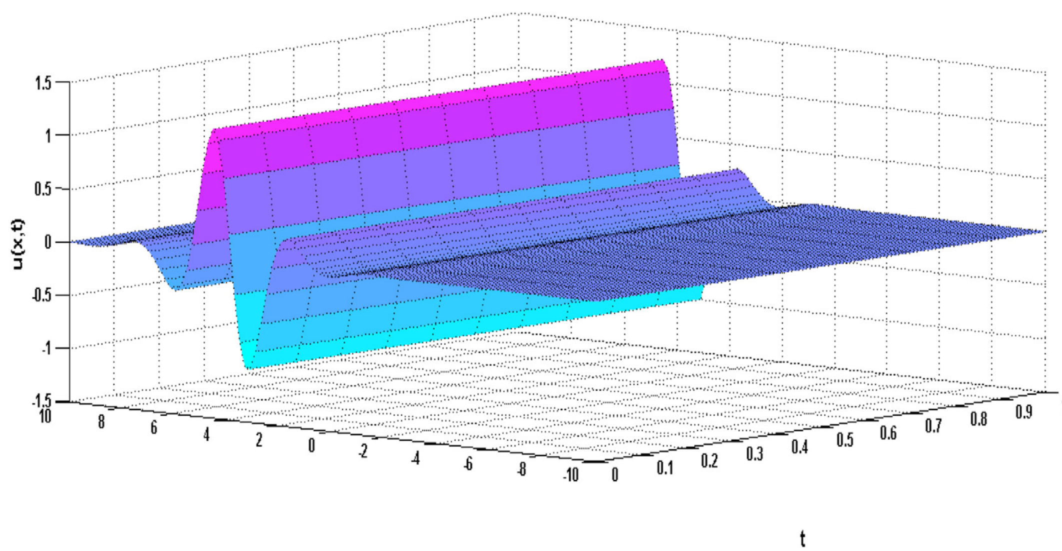


**Figure 3.** Envelope soliton of NLS equation.

### *4.3. Nonlinear Schrodinger equation (NLS)*

One of the most important dynamical models in nonlinear physics is nonlinear Schrodinger equation which presents the function of wave in nonlinear and dispersive motion and is given by:

$$iu_t = u_{xx} + g|u|^2u, -\infty < x < \infty, 0 \leq t < \infty$$

where u is the complex field function and g is a constant. The first function of wave i.e., dispersion effect makes the waveform spread and the second function causes the steepening of waveform due to its nonlinear effect. The NLS equation admits the soliton solution as presented as in Figures 2 and 3.

The gap solitons that occurred for NLS equation is presented in Figure 2 for the exact solution given as [63]:

$$u(x,t) = sin(x)e^{-i1.5t}, \ -4 < x < 4, 0 \leq t \leq 2.$$

The envelope soliton that occurred for NLS equation is presented in Figure 3 for the exact solution given as [63]:

$$u(x,t) = \sqrt{2}\cos(2x-3t)\text{sech}(x-4t), \ -10 < x < 10, 0 \leq t \leq 1.$$

NLS equation has localized solutions which have applications in many fields such as plasmas, electromagnetism and many other instability phenomena. It is also helpful in problem of optical pulse propagation in asymmetric, twin core optical fibers etc. Optical solitons, which are one of the most important solutions of nonlinear Schrodinger equation, are used in optical fiber communication [44, 48, 64, 65, 66, 67, 68, 69].

### *4.4. Camassa-Holm equation*

This equation is first introduced by Camassa and Holm [70] by the use of Hamiltonian method.

The Camassa-Holm equation of the form:

$$u_t + 2ku_x - u_{xxt} + 3uu_x = 2u_xu_{xx} + uu_{xxx}$$

where $u$ denotes the fluid velocity and the parameter $k$ is a constant related to the critical shallow water wave speed. This is an entirely integrable dispersive water wave equation for all $k$ and for $k = 0$, it has travelling solution of the form $ce^{-|x-a|}$ which are called peakons because they have a discontinuous first derivative at the weak peak. The Camassa-Holm (CH) equation has peakon type solutions.

The Peakon solution for CH equation is shown in Figure 4 with exact solution [71]:

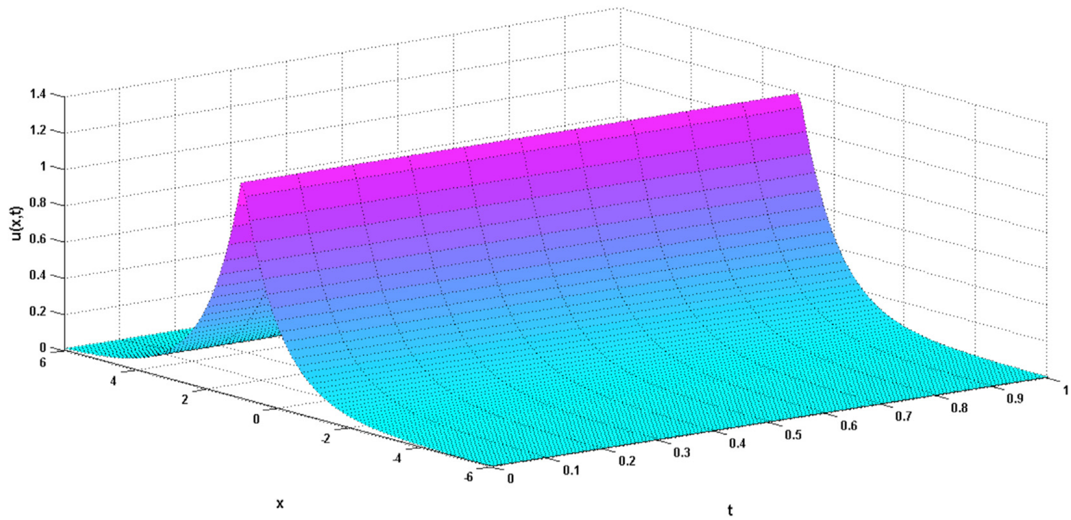


**Figure 4.** Peakon Soliton from CH equation.

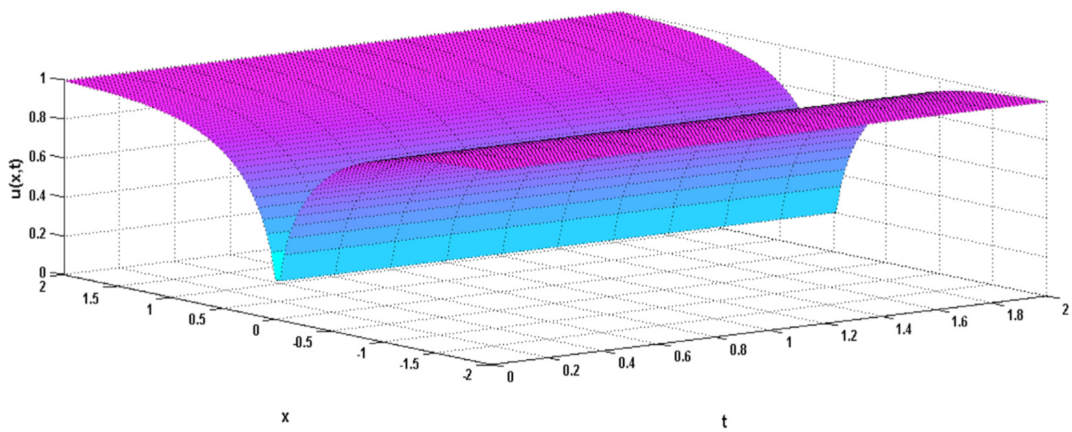


**Figure 5.** Cuspons Soliton from DP equation.

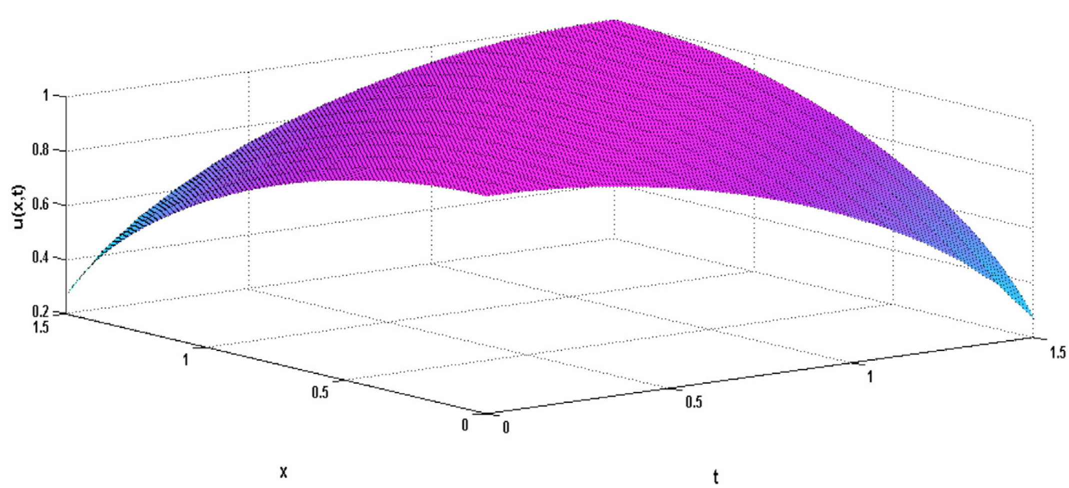


**Figure 6.** Compactons soliton from K (n,n) equation.

$$u(x,t)=\sqrt{\frac{3}{2}}e^{-|x-t|},\ -6<x<6, 0\le t\le 1.$$

The DP equation also describes shallow water nonlinear waves and its asymptotic accuracy resembles as that of CH equation:

The DP equation is given by:

$$m_t+m_xu+3mu_x=0, m=u-u_{xx}.$$

The cuspons solution for DP equation is shown in Figure 5 with exact solution [72]:

$$u(x,t)=\sqrt{1-e^{-2|x|}},\ -2<x<2, 0\le t\le 1.$$

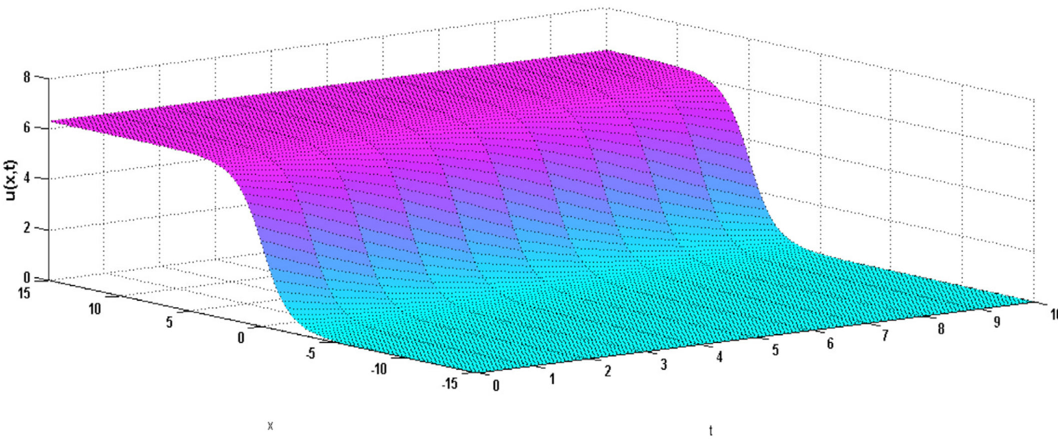


**Figure 7.** Kink Soliton for SG equation.

In addition to the above equations, there are many equations which have important applications in field of science and technology and yields soliton type solutions i.e. Kolmogorov–Petrovskii–Piskunov equation [73], generalized (2 + 1)-dimensional shallow water waves equation [74], human immunodeficiency virus (HIV)-1 infection of CD4+ T-cells fractional biomathematical model for constructing novel solitary wave solutions [75], Fokas–Lenells equation [49], Klein–Fock–Gordon equation [76] relates to Schrodinger equation, phi-four equation [77] which is a particular case of the Klein–Fock–Gordon equation, telegraph equation [52], Chaffee–Infante equation [22], Benjamin–Bona–Mahony (BBM) [38], Cahn–Allen equation [78], Klein–Gordon–Zakharov equation [36, 79], Kaup–Kupershmidt equation [23], Fisher-Kolmogorov-Petrovskii-Piskunov [80], Kadomtsev−Petviashvili equation [81], Ginzburg–Landau equation [82], Hirota–Satsuma–Shallow Water Wave Equation [83] (2 + 1)-dimensional Kadomtsev–Petviashvili–Benjamin–Bona–Mahony equation [84], cubic–quintic nonlinear Helmholtz model [85], Ostrovsky equation [13], Vakhnenko–Parkes equation which is reduced from the Ostrovsky equation [86], complex perturbed Gerdjikov–Ivanov (CPGI) equation [51], etc.

## 5. Conclusion

In recent decades, nonlinear equations have appeared in various forms to study the behavior of complex natural phenomena in different branches of science and technology. A system whose output is not proportional to its input is said to be nonlinear. Most nonlinear phenomena are modelled in terms of a nonlinear evolutionary equation (NLEE) due to linear and nonlinear effects. The solution of these NLEEs leads to solitary waves and periodic solutions, which plays an important role in the description of nonlinear physical phenomena.

In the present work, solitons have been discussed with a brief history of their existence, followed by the different types of soliton solutions. The application of soliton solutions in the various scientific and engineering fields has been discussed. The types of solitons for the known NLEES have also been discussed to arouse the interest of the readers.

From this article, one may have acquired a glimpse into the relationship between dispersion and nonlinearity in the differential equations, it is not yet explained how solitons keep their forms and velocities after colliding. This characteristic, which makes particles seem like they have their own independent reality, has profound mathematical significance and goes far beyond ordinary curiosity. A limitation is that not all nonlinear partial differential equations have soliton solutions, and this is acknowledged in the paper.

## Declarations

### *Author contribution statement*

All authors listed have significantly contributed to the development and the writing of this article.

### *Funding statement*

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

### *Data availability statement*

The authors are unable or have chosen not to specify which data has been used.

### *Declaration of interest's statement*

The authors declare no competing interests.

### *Additional information*

No additional information is available for this paper.

## Acknowledgements

Authors are thankful to the reviewers for the constructive comments to enhance the quality of the manuscript.

## References

[1] J.S. Russell, Report on waves, the report of the meeting of the British association for the advancement of science, Nature (1844) 311–390.
[2] R.L. Horne, A (very) brief introduction to soliton theory in a class of nonlinear PDEs, Russell J. Bertrand Russell Arch. (2002).
[3] M. Wadati, Introduction to solitons, Pramana – J. Phys. 57 (5–6) (2001) 841–847.
[4] D.J. Korteweg, G. de Vries, XLI. On the change of form of long waves advancing in a rectangular canal, and on a new type of long stationary waves, London, Edinburgh, Dublin Philos. Mag. J. Sci. 39 (240) (1895) 422–443.
[5] E. Fermi, J.R. Pasta, S.M. Ulam, Studies of non-linear prolems (technical Report), Collect. Work. E. Fermi 2 (May) (1965) 978–988.
[6] N.J. Zabuskal, M.D. Kruskal, Interaction of 'Solitons' in a collisionless plasma and the recurrence of initial states N, Phys. Rev. Lett. 15 (6) (1965) 240–243.
[7] P.D. Lax, Integrals of nonlinear equations of evolution and solitary waves, Commun. Pure Appl. Math. 21 (5) (1968) 467–490.
[8] A.B. Shabat, V.E. Zakharov, Exact theory of two-dimensional self-focussing and one-dimensional self-modeulation of waves in nonlinear media, Sov. Phys. Jept. 34 (1) (1972) 62.

[9] M.J. Ablowitz, D.J. Kaup, A.C. Newell, H. Segur, Inverse scattering transform-Fourier analysis for nonlinear problems, Stud. Appl. Math. 53 (4) (1974) 249–315.
[10] Y. Yousefi, K.K. Muminov, A Simple Classification of Solitons, pp. 1–18, [Online]. Available: http://arxiv.org/abs/1206.1294, 2012.
[11] P. Govind, Agrawal and Clifford Headley, "Kink solitons and optical shocks in dispersive nonlinear media, Phys. Rev. A 46 (3) (1992) 1573–1577.
[12] A.L. Fabian, R. Kohl, A. Biswas, Perturbation of topological solitons due to sine-Gordon equation and its type, Commun. Nonlinear Sci. Numer. Simulat. 14 (4) (2009) 1227–1244.
[13] M.M. Khater, Diverse solitary and Jacobian solutions in a continually laminated fluid with respect to shear flows through the Ostrovsky equation, Mod. Phys. Lett. B 35 (13) (2021), 2150220.
[14] S. Johnson, A. Biswas, Breather dynamics of the sine-Gordon equation, Commun. Theor. Phys. 59 (6) (2013) 664–670.
[15] M. Tajiri, Y. Watanabe, Breather solutions to the focusing nonlinear Schrödinger equation, Phys. Rev. E 57 (3) (1998) 3510–3519.
[16] R. Carretero, J. Frantzeskakis, Nonlinear waves in Bose-Einstein condensates: physical relevance and mathematical techniques, Nonlinearity 21 (7) (2008) 139.
[17] A.B. Aceves, Optical gap solitons: past, present, and future; theory and experiments, Chaos 584 (2000) (2011) 584–589.
[18] Y.S. Kivshar, Stable vector solitons composed of bright and dark pulses, Opt. Lett. 17 (19) (1992) 1322–1324.
[19] R. Radhakrishnan, M. Lakshmanan, Bright and dark soliton solutions to coupled nonlinear Schrodinger equations, J. Phys. Math. Gen. 28 (9) (1995) 2683–2692.
[20] A.R. Seadawy, D. Lu, Bright and dark solitary wave soliton solutions for the generalized higher order nonlinear Schrödinger equation and its stability, Results Phys. 7 (2017) 43–48.
[21] K. Hosseini, M. Mirzazadeh, D. Baleanu, S. Salahshour, L. Akinyemi, Optical solitons of a high-order nonlinear Schrödinger equation involving nonlinear dispersions and Kerr effect, Opt. Quant. Electron. 54 (3) (2022) 1–12.
[22] M.M.A. Khater, B. Ghanbari, On the solitary wave solutions and physical characterization of gas diffusion in a homogeneous medium via some efficient techniques, Eur. Phys. J. Plus 136 (4) (2021) 1–28.
[23] M.M.A. Khater, et al., Bright–dark soliton waves' dynamics in pseudo spherical surfaces through the nonlinear kaup–kupershmidt equation, Symmetry 13 (6) (2021) 1–20.
[24] L. Najera, M. Carrillo, M.A. Agüero, Non-classical solitons and the broken hydrogen bonds in DNA vibrational dynamics, Adv. Stud. Theor. Phys. 4 (9–12) (2010) 495–510.
[25] J. Chen, J. Chen, J. Zhou, Compacton, peakon and solitary wave solutions of the Osmosis K (3,2) equation, J. Sci. Res. Rep. 5 (4) (2015) 275–284.
[26] S.S. Behzadi, Numerical solution of fuzzy Camassa-Holm equation by using homotopy analysis methods, J. Appl. Anal. Comput. 1 (3) (2011) 315–323.
[27] J. Li, Y. Zhang, Exact loop solutions, cusp solutions, solitary wave solutions and periodic wave solutions for the special CH-DP equation, Nonlinear Anal. R. World Appl. 10 (4) (2009) 2502–2507.
[28] A.H.J. Wang, et al., Molecular structure of a left-Handed double helical DNA fragment at atomic resolution, Nature 282 (5740) (1979) 680–686.
[29] A.C. Scott, Davydov's soliton, Phys. D Nonlinear Phenom. 51 (1–3) (1991) 333–342.
[30] W. Alka, A. Goyal, C. Nagaraja Kumar, Nonlinear dynamics of DNA – Riccati generalized solitary wave solutions, Phys. Lett. Sect. A Gen. At. Solid State Phys. 375 (3) (2011) 480–483.
[31] M. Peyrard, Nonlinear dynamics and statistical physics of DNA, Nonlinearity 17 (2) (2004) R1.
[32] J.S. Song, Theory of magnetic monopoles and electric-magnetic duality: a prelude to S -duality, J. Undergrad. Sci 3 (1996) 47–55. Summer.
[33] H.G. Abdelwahed, E.K. El-Shewy, M.A.E. Abdelrahman, A.F. Alsarhana, On the physical nonlinear (n+1)-dimensional Schrödinger equation applications, Results Phys. 21 (2021), 103798.
[34] V.E. Zakharov, Collapse of Langmuir waves, Sov. Phys. JETP 35 (5) (1972) 908–914.
[35] E.A. Kuznetsov, A.M. Rubenchik, V.E. Zakharov, Soliton stability in plasmas and hydrodynamics, Phys. Rep. 142 (3) (1986) 103–165.
[36] M.M.A. Khater, A.E.S. Ahmed, Strong Langmuir turbulence dynamics through the trigonometric quintic and exponential b-spline schemes, AIMS Math. 6 (6) (2021) 5896–5908.
[37] M. Tajiri, M. Tuda, On large amplitude ion acoustic solitons in plasma with negative ions, J. Phys. Soc. Jpn. 54 (1) (1985) 19–22.
[38] M.M.A. Khater, T.A. Nofal, H. Abu-Zinadah, M.S.M. Lotayif, D. Lu, Novel computational and accurate numerical solutions of the modified Benjamin–Bona–Mahony (BBM) equation arising in the optical illusions field, Alex. Eng. J. 60 (1) (2021) 1797–1806.
[39] D.H. Peregrine, Water waves, nonlinear Schrödinger equations and their solutions, J. Aust. Math. Soc. Ser. B. Appl. Math. 25 (1) (1983) 16–43.
[40] V.E. Zakharov, Stability of periodic waves of finite amplitude on the surface of a deep fluid, J. Appl. Mech. Tech. Phys. 9 (2) (1968) 190–194.
[41] A. L, S. Putterman, Theory of non-propagating surface-wave solitons, J. Fluid Mech. 148 (1984) 443–449.
[42] M. Longuet-Higgins, Capillary-gravity waves of solitary type on deep water, J. Fluid Mech. 252 (1993) 703–711.
[43] M.D. Maiden, D.V. Anderson, N.A. Franco, G.A. El, M.A. Hoefer, Solitonic dispersive hydrodynamics: theory and observation, Phys. Rev. Lett. 120 (14) (2018) 1–8.
[44] M.Y. Wang, Optical solitons of the perturbed nonlinear Schrödinger equation in Kerr media, Optik 243 (2021), 167382.
[45] A. Hasegawa, An historical review of application of optical solitons for high speed communications, Chaos 10 (3) (2000) 475–485.
[46] T.-T. Shi, S. Chi, Nonlinear photonic switching by using the spatial soliton collision, Opt. Lett. 15 (20) (1990) 1123.
[47] M.N. Islam, Ultrafast all-optical logic gates based on soliton trapping in fibers, Opt. Lett. 14 (22) (1989) 1257–1259.
[48] G. Arora, R. Rani, H. Emadifar, Numerical solutions of nonlinear Schrodinger equation with applications in optical fiber communication, Optik 266 (2022), 169661.
[49] M.M.A. Khater, A.E.S. Ahmed, S.H. Alfalqi, J.F. Alzaidi, S. Elbendary, A.M. Alabdali, Computational and approximate solutions of complex nonlinear Fokas–Lenells equation arising in optical fiber, Results Phys. 25 (2021), 104322.
[50] M. Khater, Recent electronic communications; optical quasi–monochromatic soliton waves in fiber medium of the perturbed Fokas–Lenells equation, Opt. Quant. Electron. 59 (9) (2022) 1–12.
[51] M.M. Khater, Abundant wave solutions of the perturbed Gerdjikov–Ivanov equation in telecommunication industry, Mod. Phys. Lett. B 35 (26) (2021), 2150456.
[52] M.M.A. Khater, K.S. Nisar, M.S. Mohamed, Numerical investigation for the fractional nonlinear space-time telegraph equation via the trigonometric Quintic B-spline scheme, Math. Methods Appl. Sci. 44 (6) (2021) 4598–4606.
[53] M.M. Khater, D. Lu, Analytical versus numerical solutions of the nonlinear fractional time–space telegraph equation, Mod. Phys. Lett. B 35 (19) (2021), 2150324.
[54] L.V. Ginzburg, et al., Determination of the current–phase relation in Josephson junctions by means of an asymmetric two-junction SQUID, JETP Lett. 107 (1) (2018) 48–54.
[55] L. Ferrari, Approaching bose-einstein condensation, Eur. J. Phys. 32 (6) (2011) 1547–1557.
[56] J. Rogel-Salazar, The Gross-Pitaevskii equation and Bose-Einstein condensates, Eur. J. Phys. 34 (2) (2013) 247–257.
[57] J.S. He, E.G. Charalampidis, P.G. Kevrekidis, D.J. Frantzeskakis, Rogue waves in nonlinear Schrödinger models with variable coefficients: application to Bose-Einstein condensates, Phys. Lett. Sect. A Gen. At. Solid State Phys. 378 (5–6) (2014) 577–583.
[58] K.B. Davis, et al., Bose Einstein Condensation in a Gas of Sodium Atoms, 1995, pp. 3969–3972.
[59] M.H. Anderson, J.R. Ensher, M.R. Matthews, C.E. Wieman, E.A. Cornell, Observation of bose-einstein condensation in a dilute atomic vapor, Collect. Pap. Carl Wieman 269 (5221) (1995) 453–456.
[60] K. Abedi, V. Ahmadi, S. Gholmohammadi, E. Darabi, M.H. Yavari, Soliton solution of nonlinear Schrodinger equation with application to Bose-Einstein condensation using the FD method, Second Int. Conf. Adv. Optoelectron. Lasers 7009 (2008) 125–131.
[61] A.M. Wazwaz, Compactons dispersive structures for variants of the K(n,n) and the KP equations, Chaos Soliton Fractals 13 (5) (2002) 1053–1062.
[62] R.C. Mittal, R. Bhatia, Numerical solution of nonlinear sine-Gordon equation by modified cubic B-spline collocation method, Int. J. Partial Differ. Equ. 2014 (1) (2014) 1–8.
[63] G. Arora, V. Joshi, R.C. Mittal, Numerical simulation of nonlinear Schrödinger equation in one and two dimensions, Math. Model. Comput. Simul. 11 (4) (2019) 634–648.
[64] H.A. Alkhidhr, Closed-form solutions to the perturbed NLSE with Kerr law nonlinearity in optical fibers, Results Phys. 22 (2021) 103875.
[65] K. Hosseini, et al., A high-order nonlinear Schrödinger equation with the weak non-local nonlinearity and its optical solitons, Results Phys. 23 (2021), 104035.
[66] S. Malik, S. Kumar, K.S. Nisar, C. Ahamed Saleel, Different analytical approaches for finding novel optical solitons with generalized third-order nonlinear Schrödinger equation, Results Phys. 29 (2021), 104755.
[67] M.S. Osman, et al., On global behavior for complex soliton solutions of the perturbed nonlinear Schrödinger equation in nonlinear optical fibers, J. Ocean Eng. Sci. (xxxx) (2021).
[68] M.M.A. Khater, S. Anwar, K.U. Tariq, M.S. Mohamed, Some optical soliton solutions to the perturbed nonlinear Schrödinger equation by modified Khater method, AIP Adv. 11 (2) (2021), 025130.
[69] M.M.A. Khater, et al., Sub-10-fs-pulse propagation between analytical and numerical investigation, Results Phys. 25 (2021), 104133.
[70] R. Camassa, D.D. Holm, An integrable shallow water equation with peaked solitons, Phys. Rev. Lett. 71 (11) (1993) 1661–1664.
[71] G. Gui, Y. Liu, P.J. Olver, C. Qu, Wave-breaking and peakons for a modified Camassa-Holm equation, Commun. Math. Phys. 319 (3) (2013) 731–759.
[72] M. Tamsir, V.K. Srivastava, R. Jiwari, An algorithm based on exponential modified cubic B-spline differential quadrature method for nonlinear Burgers' equation, Appl. Math. Comput. 290 (2016) 111–124.
[73] M.M.A. Khater, M.S. Mohamed, R.A.M. Attia, On semi analytical and numerical simulations for a mathematical biological model; the time-fractional nonlinear Kolmogorov–Petrovskii–Piskunov (KPP) equation, Chaos, Solit. Fractals 144 (2021), 110676.
[74] Y. Chu, M.M.A. Khater, Y.S. Hamed, Diverse novel analytical and semi-analytical wave solutions of the generalized (2+1)-dimensional shallow water waves model, AIP Adv. 11 (1) (2021), 015223.
[75] M.M.A. Khater, A.E.S. Ahmed, M.A. El-Shorbagy, Abundant stable computational solutions of Atangana–Baleanu fractional nonlinear HIV-1 infection of CD4+ T-cells of immunodeficiency syndrome, Results Phys. 22 (2021), 103890.
[76] M.M. Mostafa, M.S. Mohamed, S.K. Elagan, Diverse accurate computational solutions of the nonlinear Klein–Fock–Gordon equation, Results Phys. 23 (2021), 104003.
[77] M.M.A. Khater, A.A. Mousa, M.A. El-Shorbagy, R.A.M. Attia, Analytical and semi-analytical solutions for Phi-four equation through three recent schemes, Results Phys. 22 (2021), 103954.

[78] M.M.A. Khater, A. Bekir, D. Lu, R.A.M. Attia, Analytical and semi-analytical solutions for time-fractional Cahn–Allen equation, Math. Methods Appl. Sci. 44 (3) (2021) 2682–2691.

[79] M.M.A. Khater, A.A. Mousa, M.A. El-Shorbagy, R.A.M. Attia, Abundant novel wave solutions of nonlinear Klein–Gordon–Zakharov (KGZ) model, Eur. Phys. J. Plus 136 (5) (2021) 1–11.

[80] M.M.A. Khater, A.M. Alabdali, Multiple novels and accurate traveling wave and numerical solutions of the (2+1) dimensional Fisher-Kolmogorov-petrovskii-piskunov equation, Mathematics 9 (12) (2021) 1–13.

[81] K. Hosseini, D. Baleanu, S. Rezapour, S. Salahshour, M. Mirzazadeh, M. Samavat, Multi-complexiton and positive multi-complexiton structures to a generalized B-type Kadomtsev−Petviashvili equation, J. Ocean Eng. Sci. (xxxx) (2022).

[82] K. Hosseini, M. Mirzazadeh, L. Akinyemi, D. Baleanu, S. Salahshour, Optical solitons to the Ginzburg – Landau equation including the parabolic nonlinearity, 2021, pp. 0–11.

[83] C. Yue, D. Lu, M.M.A. Khater, Abundant wave accurate analytical solutions of the fractional nonlinear hirota–satsuma–shallow water wave equation, Fluid 6 (7) (2021) 1–13.

[84] W. Li, L. Akinyemi, D. Lu, M.M.A. Khater, Abundant traveling wave and numerical solutions of weakly dispersive long waves model, Symmetry 13 (6) (2021) 1–15.

[85] M.M. Khater, Diverse bistable dark novel explicit wave solutions of cubic–quintic nonlinear Helmholtz model, Mod. Phys. Lett. B 35 (26) (2021), 2150441.

[86] M.M. Khater, Abundant breather and semi-analytical investigation: on high-frequency waves' dynamics in the relaxation medium, Mod. Phys. Lett. B 35 (22) (2021), 2150372.