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ORIGINAL ARTICLE

A construction of a conformal Chebyshev chaotic map based authentication protocol for healthcare telemedicine services

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Abstract

The outbreak of coronavirus has caused widespread global havoc, and the implementation of lockdown to contain the spread of the virus has caused increased levels of online healthcare services. Upgraded network technology gives birth to a new interface "telecare medicine information systems" in short TMIS. In this system, a user from a remote area and a server located at the hospital can establish a connection to share the necessary information between them. But, it is very clear that all the information is always being transmitted over a public channel. Chaotic map possesses a dynamic structure and it plays a very important role in the construction of a secure and efficient authentication protocols, but they are generally found vulnerable to identity-guess, password-guess, impersonation, and stolen smart-card. We have analyzed (Li et al. in Fut Gen Comput Syst 840:149–159, 2018; Madhusudhan and Nayak Chaitanya in A robust authentication scheme for telecare medical information systems, 2008; Zhang et al in Privacy protection for telecare medicine information systems using a chaotic map-based three-factor authenticated key agreement scheme, 2017; Dharminder and Gupta in Pratik security analysis and application of Chebyshev Chaotic map in the authentication protocols, 2019) and found that Bergamo's attack (IEEE Trans Circ Syst 52(7):1382–1393, 2005) cannot be resisted by the protocol. Although few of the protocols ensures efficient computations but they cannot ensure an anonymous and secure communication. Therefore, we have proposed a secure and efficient chaotic map based authentication protocol that can be used in telecare medicine information system. This protocol supports verified session keys with only two messages of exchange. Moreover, we have analysed the performance of proposed protocol with relevant protocols and it is being implemented in "Automated Validation of Internet Security Protocols and Applications" respectively.

Keywords Authentication · Security · Privacy · Key agreement · Chaotic mapping

Introduction

The adoption of advanced health care "Telemedicne Information System" and "Telematics" applications in health care needs an integrated strategy to the different social, financial, cultural and political impacts and hurdles of information and communication technologies. Both security and privacy [37–39,42] are two important attributes required to construct a secure authentication protocol. Security and privacy of "Telemedicne Information System" are the impressive components that are of great interest to the field of health care. Because the Internet is truly an open network with many potential security gaps, close consideration and measures must be required to ensure safe medical facilities and the safety of patient data. Both health care and treatment are two very important factors in the human's life (see data in Fig. 1). Upgraded technology in the field of online health care services such as variety of medical sensors, smart phones, and smart robotics helps the patients to facilitate the health care services in the remote areas. In these days, most of the doctors are employing robots and smart digital sensor in surgeries is an application of computer science in health care services [40,41]. There are other applications such as artificial intelligence and machine learning are used to detect the medical conditions of a patient. Nowadays, a patient possessing smart

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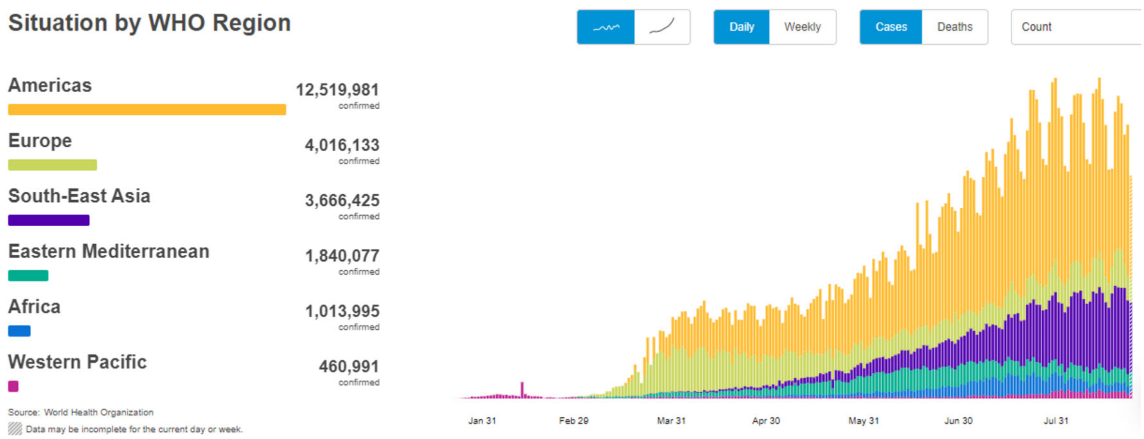


Source: World Health Organization

▨ Data may be incomplete for the current day or week.

Fig. 1 A world wide COVID-19 data recorded on August 25, 2020 by WHO

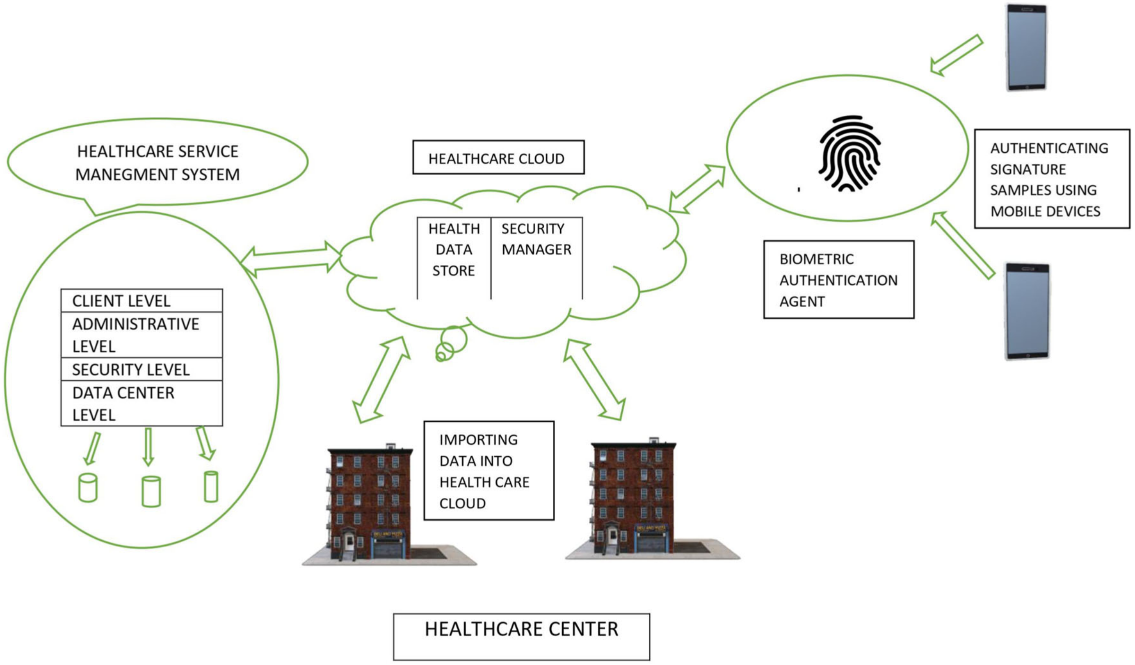

HEALTHCARE SERVICE MANEGMENT SYSTEM
CLIENT LEVEL
ADMINISTRATIVE LEVEL
SECURITY LEVEL
DATA CENTER LEVEL
HEALTHCARE CLOUD
HEALTH DATA STORE
SECURITY MANAGER
IMPORTING DATA INTO HEALTH CARE CLOUD
AUTHENTICATING SIGNATURE SAMPLES USING MOBILE DEVICES
BIOMETRIC AUTHENTICATION AGENT
HEALTHCARE CENTER

Fig. 2 A typical model of health-care services with respect to authentication protocol

sensors and mobile phones can enjoy benefits of health care services around the world (see Fig. 2).

Patients can be benefited with online health care services via their smart phones, i-pads, and other smart sensors, but their security and privacy are two very important components during communication on public channel. In 2012, Wu et al. [10] designed a secure and anonymous authentication protocol to benefit the patients at their home. In the same year, Wei et al. [9] analyses the security of the protocol [10] and it is found vulnerable to two-factor authentication. In order to eradicate the two-factor authentication defect, a fresh design is needed for two-factor authentication. In the same year, Zhu [12] discussed the security attributes such as password guessing in the protocol [9] and invented a password-guess resistant protocol, although he didn't seem to think about communicating anonymously. In 2012, Chen et al. [4] designed an efficient and secure lightweight authentication protocol that preserves an anonymous communication in health care telemedicine services.

In 2013, Lin et al. [7] observed that identity can traced in [4] using both dictionary and password guess along with stolen smart card information. He tried to remove most of the existing attacks and he invented an anonymous authentication protocol. In the same year, Cao and Zhai [3] discussed both security and privacy of [4] and they found that the protocol is vulnerable against both identity guess and password guess along with the information stored in the smart card. Three protocols discussed [3,7,12] are found insecure to input verification procedure due to which they cannot differentiate incorrect inputs with in short time interval. The anonymous communication is another important factor that is missing in [9,10,12,32] respectively. In 2013, Guo et al. [14] used the complex dynamic structure of chaotic maps to design a new secure authentication protocol, but Hao et al. [15] discussed the security of the protocol and he found that two important attributes traceability and anonymity are missing, and he tried to fill the gap with a new design [14]. In 2014, Jiang et al. [16] reviewed both security and privacy attributes in [15] and he found the protocol is vulnerable to stolen smart card attack.

In the year 2016, Li et al. [21] designed a secure and efficient chaotic map-based authentication protocol to secure the communication in health care services, but in the year 2018, Madhusudhan et al. [20] discussed the attacks in [21] such as password guess, and impersonations, and he tried to remove these attacks as discussed in [20]. In the year 2018, Jiang et al. [28] introduced a secure and efficient protocol to improve the telemedicine services in health care sector, but it is not much efficient and it requires to exchange three messages to establish secure and fresh session key. In the same year 2018, Wu et al. [29] introduced a secure and efficient authentication protocol based on RFID and Radhakrishnan et al. [19] also proposed a new design to secure the health care telemedicine services, but their protocols found susceptible to password guess, identity guess and also for stolen card information too. In the same year 2018, Zhang et al. [25] introduced a lightweight and secure authentication protocol for the mobile devices used in heath care telemedicine services, but it is also susceptible to identity guess, password guess and replay attacks. In 2018, Madhusudhan et al. [20] designed an efficient, and secure, and robust protocol for telecare services, but Dharminder et al. [35] discussed the security of the protocol [20] and they found it susceptible to identity guess, password guess, impersonations, and stolen smart card. In the same year 2020, Dharminder et al. [34] introduced a new design for authentication scheme based on RSA, but it uses the modulo operations that decreases the efficiency of the protocol due to costly modulo exponentiation.

In the Table 2, we have observed various security attributes achieved by the existing relevant chaotic map based authentication protocols used to secure TMIS system, where the symbol $\surd$ denotes "yes", and × denotes "not" respectively. In the Tables 1, 2 one can see that existing protocols in the TMIS environment suffers various vulnerabilities such as password-guess, identity-guess, impersonations, replaying of older messages, and stolen smart card attacks. In the proposed design, we have discussed two important components security and privacy in the form of security attributes such as identity guess, impersonations, password guess, anonymity, replaying of messages, and stolen smart card information in the protocols [1,20,25,35] respectively. In the design [35], we have analyzed that a user U_i selects ID_i, PW_i and calculates $A_i = h(ID_i||PW_i)$, then he sends $< ID_i, A_i >$ to the server. Next, the server chooses a random $n_i \in Z_p^*$, and does the computation $B_i = T_x(ID_i||n_i) \oplus A_i$, it sends $< h(.), B_i, n_i >$ to the corresponding user. Now, the user U_i calculates $B_i \oplus A_i = T_x(ID_i||n_i)$ then $D_i = h(T_x(ID_i||n_i)||PW_i||ID_i)$ and $N_i = n_i \oplus A_i$, and then store B_i, D_i, N_i in the corresponding smart card. We have observed that a user knows $n_i \in Z_p^*$, and $T_x(ID_i||n_i)$ can executes Bergamo's attack and computes x' as $T_{x'}(ID_i||n_i) = T_x(ID_i||n_i)$ and uses x' in the ongoing communication on the open channel.

Similarly, in the design [1], we have analyzed a vulnerability in the session key established during the communication. In the design [1], an adversary $\mathcal{A}$ obtains the information from earlier transmitted information M_1 and M_2. Moreover, $\mathcal{A}$ computes u' with knowledge of x, $T_u(x)$ satisfying $T_u(x) = T_{u'}(x)$. Finally, $\mathcal{A}$ guesses v' under the previous knowledge x, $T_v(x)$ satisfies $T_v(x) = T_{v'}(x)$ as:

$$u' = \frac{arccos(T_u(x)) + 2k\pi}{arccos(x)}$$
$$v' = \frac{arccos(T_v(x)) + 2k\pi}{arccos(x)}$$

Where k is a positive integer, finally $\mathcal{A}$ finds $T_{v'}T_{u'}(x) = T_vT_u(x) = Sk_u = Sk_s$ that plays the role of session key.

Similarly, in the design [25], we have analyzed a vulnerability in the session key established during the communication. In the design [25], an adversary $\mathcal{A}$ obtains the information from earlier transmitted messages $\{a_i, T_u(x), Nid_i\}$, $\{b_i, T_s(x), m\}$ and $\{Y_i, Nid_i, Z_i, h(.), V_i, x\}$ stored in the corresponding smart card. Furthers, $\mathcal{A}$ obtains u' under the knowledge of x, $T_u(x)$ satisfies $T_u(x) = T_{u'}(x)$. Furthers, $\mathcal{A}$ makes a guess s' under the knowledge of x, $T_s(x)$ satisfies $T_s(x) = T_{s'}(x)$.

$$u' = \frac{arccos(T_u(x)) + 2k\pi}{arccos(x)}$$
$$s' = \frac{arccos(T_s(x)) + 2k\pi}{arccos(x)}$$

Where k is a positive integer, then $\mathcal{A}$ computes $T_{s'}T_{u'}(x) = T_sT_u(x) = Sk_u = Sk_s$ that plays the role of session key during communication.

Similarly, in the design [20], we have analyzed a vulnerability in the session key established during the communication. In the design [20], an adversary $\mathcal{A}$ obtains the information from earlier transmitted messages $\{Cid_i, E_i, D_i, F_i\}$, $\{H_i, T_z(ID_i||n_i)\}$ and $\{B_i, h(.), C_i, n_i, D_i\}$ that is stored in the smart card. Furthers, $\mathcal{A}$ computes y' under the knowledge $ID_i, n_i, T_y(ID_i||n_i)$ satisfies $T_y(ID_i||n_i) = T_{y'}(ID_i||n_i)$. Furthers, $\mathcal{A}$ makes a guess z' under the knowledge ID_i, n_i, $T_z(ID_i||n_i)$ satisfies $T_z(ID_i||n_i) = T_{z'}(ID_i||n_i)$ as:

Table 1 A review on security attributes of authentication protocols based on password for TMIS

Security parameters\protocols	[3]	[4]	[11]	[7]	[30]	[24]	[13]	[31]
Anonymity	√	√	√	√	√	√	√	×
Password-guess	×	×	×	√	√	√	×	×
Session-key verification	√	×	×	×	×	√	×	×
Impersonations	√	×	√	√	√	√	×	√
Replay-messages	×	√	√	√	√	√	√	√
Key-agreement	√	√	√	√	√	√	√	√
Stolen-card	√	√	√	√	√	√	√	√
Password-change	√	√	×	×	×	×	×	√

Table 2 A review of security attributes of authentication protocols based on chaotic map for TMIS

Security parameters/protocols	[24]	[15]	[22]	[23]	[19]	[25]	[21]	[20]
Anonymity	√	√	×	√	×	×	×	×
Insider-security	√	√	√	√	√	√	√	√
Password-guess	√	√	√	√	×	×	×	×
Stolen-cards	√	√	√	√	×	√	√	×
User’s impersonations	√	√	√	√	√	√	×	×
Key-agreement	√	√	√	√	√	√	√	√
Server’s impersonations	√	√	√	√	√	√	×	×
Replay-messages	√	√	√	√	√	×	√	√
Key-agreement	√	√	√	√	√	√	√	√
Key-verification	√	×	√	×	√	√	×	√

$$y' = \frac{arccos(T_u(ID_i||n_i)) + 2k\pi}{arccos(ID_i||n_i)}$$

$$z' = \frac{arccos(T_z(ID_i||n_i)) + 2k\pi}{arccos(ID_i||n_i)}$$

Where k is a positive integer, then $\mathcal{A}$ computes $T_{s'}T_{u'}(ID_i||n_i) = T_sT_u(ID_i||n_i)$ and $Sk_i = Sk_j = h(h(ID_i)||T_z'T_y'(ID_i||n_i)||T_y'(ID_i||n_i))$ that plays the role of session key during communication.

To handle the issues in [1,20,25,35], we have an idea to compute $x = h(ID_i||s)$, where ID_i is the identity of i^{th} user and “s” is the long term secret key of the server, in this way a user possesses x that results from different ID_i concatenated with master key of the server to produce different secret keys x for each of the user. Now, the x will plays the role in place of master secret that is different for each of the user. Therefore, we have designed a new authentication protocol possessing both security and efficiency using the dynamic Chaos theory. The security of the presented scheme have been analyzed in random Oracle with this we also use the tool for authentication called “Automated Validation of Internet Security Protocols and Applications” respectively. Moreover, the presented protocol resists session key violation problem, that is proposed by Bergamo et al. [33] and establishes a session key with just two messages of exchange.

Table 3 Some of useful notation and symbol

Notation	Description
U_i	User-i
Sr_j	Server-j
$\mathcal{A}$	Adversary
Sc_i	Smart Card
ID_i	Identity for U_i
$TMIS$	Telecare Medicine Information System
Ps_i	Password for U_i
s	Secret Key for S_j
$h(\cdot)$	A secure hashing
bm_i	biometric imprint
$\oplus$	Bitwise XOR
$\|\|$	Concatenation of two strings

Preliminaries

In this section, we will discuss some of the basic notations, terminologies and basic properties of conformal Chaos maps used in the proposed protocol. A conformal map is an angle-preserving transformation that preserves local angles. A brief review of some of the useful notations are also given in Table 3.

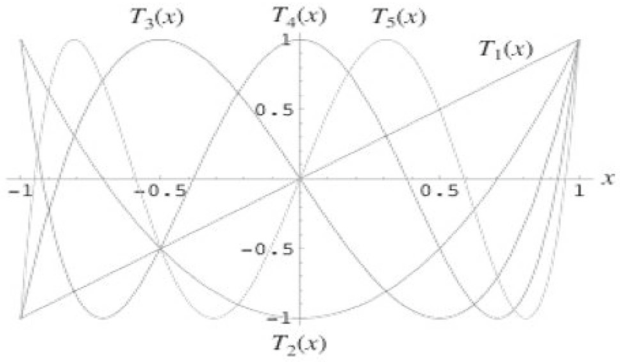


Fig. 3 A review of dynamic structure of Chebyshev polynomials

Chebyshev chaotic mapping

As seen in Fig. 3, chaotic maps have a complex dynamic structure and it is well known for its pseudo randomness. In this subsection, we have discussed some of basic definitions and dynamic properties [17].

Definition 1 A polynomial $T_\nu(y) : (-\infty, +\infty) \rightarrow [-1, +1]$ is introduced by Chebyshev in variable "y" and of positive degree ν, besides that $T_\nu(y) = cos(\nu(arccos(y)))$ and the recurrence $T_\nu(y)$ is described as $T_\nu(y) = (2yT_{\nu-1}(y) - T_{\nu-2}(y))$, whereas $y \in (-\infty, +\infty)$ and $T_0(y) = 1, T_1(y) = y$

Theorem 1 *The set of Chebyshev polynomials on* $R = (-\infty, +\infty)$ *satisfies semigroup property under composition of chaotic maps.*

Proof Chebyshev polynomials given in variable x is $T_\nu(y) : (-\infty, +\infty) \rightarrow [-1, +1]$, where $\nu > 0$ is an integer, and $T_\nu(y) = cos(\nu(arccos(y)))$ under recurrence relation $T_\nu(y) = (2yT_{\nu-1}(y) - T_{\nu-2}(y))$, furthers $y \in (-\infty, +\infty)$ and $T_0(y) = 1$, $T_1(y) = y$. Therefore, we get difference equation along with characteristics equation $\rho^2 - 2y\rho + 1 = 0$. Therefore, $\rho_1 = y + \sqrt{y^2 - 1}$ and $\rho_2 = y - \sqrt{y^2 - 1}$ are two characteristics roots of characteristics equation $\rho^2 - 2y\rho + 1 = 0$ along with $\rho_1 + \rho_2 = 2y$ and $\rho_1\rho_2 = 1$. Therefore,

$$\begin{aligned} T_n(y) &= \frac{\rho_1^n + \rho_2^n}{2} \\ &= \frac{(y + \sqrt{y^2 - 1})^n + (y - \sqrt{y^2 - 1})^n}{2} \\ &= \sum_{i=0}^{[\frac{n}{2}]} C_i^n y^{n-2i} (y^2 - 1)^i \end{aligned}$$

Where n, i are positive integers and $C_i^n = \frac{n!}{i!(n-i)!}$. Furthers, $T_{n_1}(T_{n_2}(y)) = \frac{\xi_1^{n_1} + \xi_2^{n_1}}{2}$ where n_1, n_2 are positive integers and $\xi_1 + \xi_2 = 2T_{n_2}(y)$, and $\xi_1\xi_2 = 1$. Therefore, $\xi_1 + \xi_2^{-1} = \rho_1^{n2} + \rho_1^{-n2}$ then $\xi_1 = \rho_1^{n2}$ or $\xi_1 = \rho_1^{-n2}$. Furthermore, $T_n(y) = \frac{\rho_1^n + \rho_2^n}{2}$ gives us $T_{n_1n_2}(y) = \frac{\rho_1^{n_1n_2} + \rho_2^{n_1n_2}}{2}$. Hence $T_{n_1n_2}(y) = T_{n_1}(T_{n_2}(y)) = T_{n_2}(T_{n_1}(y)) = T_{n_2n_1}(y)$. □

Definition 2 If it is asked to find u such that $T_u(x) = y$, where the values y and x are known to the adversary. Then this problem is know as Discrete Logarithm Problem (DLP).

Definition 3 Computational Diffie-Hellman Problem (CDHP) can be stated to find $T_{uv}(x)$, where the values x, $T_u(x)$ and $T_v(x)$ are known to the adversary.

Fuzzy extractor

A fuzzy extractor(E_f) [2] is an extraction mechanism that is used to extract a random uniform string from biometric imprints (bm_i). It consists of two algorithms $I(.)$ and $R(.)$. $I(.)$ is a probabilistic algorithm that produced two strings b_1, b_2 as output after taking bm_i an input parameter, where b_1 is private key and b_2 is a helper string. $R(.)$ is an algorithm that is used to regenerate the private key b_1 after taking noisy biometric parameter bm_i' and helper string b_2 as input, where $|bm_i - bm_i'| \leq \delta bm$.

Proposed authentication protocol under chaotic mapping

We have proposed a secure and efficient chaotic map based authentication that can be divided into four phases, (1) registration-phase, (2) login-phase, (3) authentication-phase and (4) password-update-phase.

Registration-phase

U_i registers to the concern server Sr_j via a secure channel as written in the following lines.

- U_i selects ID_i, Ps_i, and imprints his own biometric bm_i then compute $I(bm_i) = (b_1, b_2)$, $Ui_1 = h(ID_i||Ps_i||b_1)$ and transmits $\{ID_i, Ui_1\}$ to Sr_j.
- After getting the information $\{ID_i, Ui_1\}$, Sr_j chooses arbitrary $p_i \in Z_p^*$, then compute first $x = h(ID_i||s)$, where "s" is the private key of Sr_j. Now, it computes $Si_1 = T_s(ID_i)$, $Si_2 = T_x(ID||p_i)$. Then, it calculates the value $Si_3 = (Si_1||Si_2) \oplus Ui_1$ and store $h(.)$, Si_3, p_i.
- Sr_j delivers a hidden information $\{h(.), Si_3, p_i\}$, after stored it into the smart-card, to U_i via a private channel.
- Finally, U_i does the computations $Si_1||Si_2 = Si_3 \oplus Ui_1$, $Ui_2 = h(Si_1\|Si_2||Ps_i||ID_i||b_1)$, $P_i = p_i \oplus Ui_1$ and store $\{h(.), Si_3, Ui_2, P_i, b_2\}$ into the smart card.

Fig. 4 A description of registration phase via secure channel

User (U)	secure channel	Server (S)
U_i selects $ID_i,\ Ps_i$ and imprint bm_i Computes $I(bm_i) = (b_1,\ b_2)$ $Ui_1 = h(ID_i\|\|Ps_i\|\|b_1)$		
	$\xrightarrow{< ID_i,\ Ui_1 >}$	choose random $p_i \in Z_p^*$ Computes $x = h(ID_i\|\|s)$ $Si_1 = T_s(ID_i)$ $Si_2 = T_x(ID_i\|p_i)$ $Si_3 = (Si_1\|Si_2) \oplus Ui_1$
	$\xleftarrow{< h(.),\ Si_3,\ p_i >}$	
Compute $Si_1, Si_2 = Si_3 \oplus Ui_1$ $Ui_2 = h(Si_1\|Si_2\|\|Ps_i\|\|ID_i\|\|b_1)$ $P_i = p_i \oplus Ui_1$ Stores $P_i,\ Si_3,\ Ui_2,\ b_2$ in smart card		

User (U)	Public channel	Server (S)
U_i inputs ID_i and Ps_i imprint bm_i Smart card computes $I(bm_i) = (b_1, b_2)$ $Ui'_1 = h(ID_i\|\|Ps_i\|\|b_1)$ $Si'_1, Si'_2 = Si_3 \oplus Ui'_1$ verifies $Ui_2 = h(Si'_1\|Si'_2\|\|Ps_i\|\|ID_i\|\|b_1)$ choose y randomly $Ui_3 = T_y(ID_i\|\|p_i)$ $Ui_4 = T_y(ID_i)$ $Sk_i = h(T_y(Si_2)\|\|T_1) = h(T_yT_x(ID_i\|\|p_i)\|\|T_1)$ $Ui_5 = h(Ui_3\|\|Ui_4\|\|Sk_i\|\|T_1)$ $NID_i = (Ui_3\|\|Ui_5\|\|ID_i) \oplus T_y(Si_1) = (Ui_3\|\|Ui_5\|\|ID_i) \oplus T_yT_s(ID_i)$		
	$\xrightarrow{< Ui_4,\ NID_i,\ T_1 >}$	Check T_1 is valid Compute $T_s(Ui_4) = T_sT_y(ID_i)$ $T_sT_y(ID_i) \oplus NID_i = ID'_i, Ui'_3, Ui'_5$ $x' = h(ID'_i\|\|s)$ $Sk'_i = h(T_{x'}(Ui'_3)\|\|T_1) = h(T_{x'}T'_y(ID_i\|\|p_i)\|\|T_1)$ Verifies $Ui'_5 =?Ui_5$ $Ui'_5 = h(Ui'_3\|\|Ui_4\|\|Sk'_i\|\|T_1)$ Computes $Si_4 = h(Sk_i\|\|ID_i\|\|T_2)$
	$\xleftarrow{< Si_4,\ T_2 >}$	
checks T_2 Computes $Si'_4 = h(Sk_i\|\|ID_i\|\|T'_2)$ verifies $Si'_4 = Si_4$		

Fig. 5 A description of of login and authentication phases

Login phase

If U_i wants to login to Sr_j then:

- U_i firstly insert the card into card reader machine, then input $ID_i,\ Ps_i$ and imprints the biometric bm_i, then the card computes $I(bm_i) = (b_1,\ b_2)$ and $Ui'_1 = h(ID_i||Ps_i||b_1)$.
- Using Ui'_1 smart card executes the step $Si'_1,\ Si'_2 = Si_3 \oplus Ui'_1$ and computes $Ui'_2 = h(Si'_1\|Si'_2||Ps_i||ID_i||b_1)$ and proceeds for the legal verification $Ui'_2 =?Ui_2$.
- The choose $y \in Z_p^*$ arbitrarily and furthers proceeds to compute $Ui_3 = T_y(ID_i||p_i)$, $Ui_4 = T_y(ID_i)$, $Sk_i = h(T_y(Si_2)||T_1) = h(T_yT_x(ID_i||p_i)||T_1)$, $Ui_5 = h(Ui_3||Ui_4||Sk_i||T_1)$, $NID_i = (Ui_3||Ui_5||ID_i) \oplus T_y(Si_1) = (Ui_3||Ui_5||ID_i \oplus T_yT_s(ID_i)$ and then U_i transmits $< Ui_4,\ NID_i,\ T_1 >$ to Sr_j.

Authentication phase

Sr_j receives $< Ui_4,\ NID_i,\ T_1 >$ from corresponding U_i and executes the process of authentication:

– Sr_j verifies the time stamp T_1, and computes $T_s(Ui_4) = T_sT_y(ID_i)$, $T_sT_y(ID_i) \oplus NID_i = ID'_i, Ui'_3, Ui'_5$ then after getting ID'_i Sr_j computes $x' = h(ID'_i||s)$, $Sk'_i = h(T_{x'}(Ui'_3)||T_1) = h(T_{x'}T'_y(ID_i||p_i)||T_1)$ and then ver-

ifies Ui'_5 $=?Ui_5$ where $Ui'_5 = h(Ui'_3||Ui_4||Sk'_i||T_1)$ finally Sr_j computes $Si_4 = h(Sk_i||ID_i||T_2)$.
- Sr_j transmits the information $\{Si_4,\ T_2\}$ to U_i through public channel.
- After receiving the information $\{Si_4,\ T_2\}$ from Sr_j, then U_i confirms T_2 is valid or not, then he calculates $Si'_4 = h(Sk_i||ID_i||T_2)$, and proceeds the verification $Si'_4 = Si_4$, and establishes a session key $Sk_i = h(T_yT_x(ID_i||p_i)||T_1)$.

Password update phase

To update password U_i, executes the following steps:

- U_i inputs ID_i and Ps_i. Furthers, he imprints biometric, then computes $I(bm_i) = (b_1,\ b_2)$ and $Ui'_1 = h(ID_i||Ps_i||b_1)$. Using Ui'_1, the Sc_i obtains Si'_1, $Si'_2 = Si_3 \oplus Ui'_1$ and $Ui'_2 = h(Si'_1\|Si'_2||Ps_i||ID_i||b_1)$ and proceeds for the verification $Ui'_2 = Ui_2$.
- U_i inputs Ps_i^{new} and proceeds the computation $Ui_1^{new} = h(ID_i||Ps_i^{new}||b_1)$, $Ui_2^{new} = h(Si'_1||Si'_2||Ps_i^{new}||ID_i||b_1)$, $Si_3^{new} = Ui_1^{new} \oplus Si_1||Si_2$ and updates the values $Si_3,\ Ui_2$ with $Si_3^{new},\ Ui_2^{new}$.

Security analysis

Formal security analysis

At first, we have to define a framework $\mathcal{P}$ to verify the security of the presented protocol and then, under random oracle, we will implement the presented protocol.

Security-model Suppose the i^{th} instance of a user U_i is denoted by $M_i \in (U_i,\ Sr_j)$, and $\mathcal{A}$ be an attacker that governs the connection between U_i and Sr_j. An illustration of $\mathcal{A}$, is therefore stated as follows:

Extract: With the help of extract query, $\mathcal{A}$ could get the private key of a user U_i.

Send(m, M_i): With the help of send query, $\mathcal{A}$ could be able to send arbitrary message m, to random oracle then in response of m, random oracle have to reply with a computational output.

Hash(m): In this query $\mathcal{A}$ sends random massage m to $H(.)$, then oracle select $s \in Z_p^*$ randomly and reply with s, after storing it into hash list H_{ij} with m. Initially H_{ij} assumed to be empty.

Reveal(M_i): If an adversary $\mathcal{A}$ process a reveal query to oracle, then oracle permits $\mathcal{A}$, to know about session key Sk_i.

Corrupt(M_i): Corrupt query allows $\mathcal{A}$ to obtains the private key of M_i, by corrupting M_i.

Test(M_i): If oracle receives test query, oracle guesses a random bit $b \in \{0, 1\}$. Then two cases aries: (1) if b = 0, then oracle reply with arbitrary number, (2) if b = 1, then oracle reply with session key Sk_i.

Suppose a bit b is selected from corrupt query phase and $Succ(\mathcal{A})$ correctly estimates the value of b, then the advantage $Adv_{\mathcal{A},P}(k)$, against the protocol retained by adversary is specified as:

$$Adv_{\mathcal{A},P}(k) = |2.Pr[Succ(\mathcal{A})] - 1|$$

Mutual authentication is established by security analysis in Random Oracle for the suggested scheme.

Chaotic based assumption: Suppose $x \in Z_p^*$ is a secret key of Sr_j, p is a prime number with length n, then from generation algorithm $Gen(1^n) = p$

$\exists$ a negligible function $neg(n)$ such that:

$$\begin{aligned} &Pr[Gen(1^n) \rightarrow p, x, r, T_x(r) \leftarrow Z_p* \\ &: \mathcal{A}(1^n, p, T_x(r)) \rightarrow x] = neg(n) \end{aligned}$$

$\forall$ probabilistic polynomial time adversary $\mathcal{A}$.

Collision resistance attack(CRA) algorithm: If for $h(.)$, $\mathcal{A}$, then we have

$$Adv_{\mathcal{A}} = Pr[(c, c') \overset{R}{\leftarrow} Adv : c \neq c' and h(c) = h(c')]$$

Theorem 2 *Suppose $\mathcal{R}$ be a RO and if $\mathcal{A}$ breach the suggested mutual authentication protocol, then we can design an algorithm M_o, to solve CRA problem, together with $\mathcal{A}$.*

Proof At first, M_o detects $Ui_4,\ NID_i,\ H_i$ and then to solve the problem of CRA, M_o attempts to calculate X_i* and $r*$ and verifies $h(r^*||Ps_i||ID_i||H_i) =?Ui_2$ or $Ui_5 = ?h(Ui_3||Ui_4||Sk_u||T_1))$. M_o have access to $\mathcal{R},\ ,\ p,\ \omega,\ T$, $Gen(.)$, also M_o can reach to $\mathcal{A}$ through query. □

Hash query: When $\mathcal{A}$ asks hash query H_q, for ID_i, M_o first check H_{ij} list for ID_i. If M_o found it in the list H_{ij}, then it reply h_{i1}, otherwise M_o calculates $h_u = H_1(ID_i)$ and put it in the list along with ID_i, and sends h_u back to $\mathcal{A}$.

Extract: In extract query $\mathcal{A}$ sends query on ID_i. After getting extract query M_o checks $h_u \in \{H_i, Ui_4, NID_i\}$. If M_o fails to verify, then M_o ends the process. After this, M_o searches $ID_i \in H_{ij}$, if present, then it responds, else calculates $X_i = Ui_4$ and return $h(Si_2||Ps_i||ID_i||H_i) = Ui_2$ to $\mathcal{A}$.

Send-queries: In send query there are three phases discussed as below, first U_i request for login to Sr_j, then U_i sends a message $< (Ui_4,\ NID_i,\ T_1) >$ to Sr_j, and at last Sr_j responds (Si_4, T_2). We will describe this phase by a game between U_i and Sr_j respectively.

1. $\mathcal{A}$ start it with sending a query, in response of that Mo is supposed to reply a login message to $\mathcal{A}$.

2. $\mathcal{A}$ sends polynomial time send queries, to login into Sr_j then M_o computes corresponding to i^{th} query as $Ui_4 = T_y(ID_i)$ and $Ui_2 = h(Si_2||Ps_i||ID_i||p_i)$ responds to $\mathcal{A}$.
3. $\mathcal{A}$ submits $(Ui_2,\ U_i)$, then M_o verifies first whether $h_u \in H_{ij}$ or not. If it is present, then M_o returns a failure, else ends the query Q_{e_1}. Furthers, Mo computes $R_1 = T_y(ID_i||p_i)$, and $Sk = T_jT_y(ID_i||p_i)$ for arbitrary j, y and $E_i = h(T_y(ID_i||p_i)||Sk||T_1)$ and returns the output to $\mathcal{A}$.
4. $\mathcal{A}$ submits $((T_y(r_i), Ui_5), LS_j)$, then M_o computes $S_i = h(Id_i||r)$ and $Ui_2 = h(T_x(ID_i||p_i))||Ps_i||ID_i||p_i)$, where r is chosen arbitrarily, then check the equation $W_i =?h(T_x(r_i))||Ps_i||ID_i||p_i)$, if holds, then M_o does the computation $Sk = T_yT_j(ID_i||N_i)$, $Si_4 = h(Sk||T_2)$ responds (Si_4, T_2) to $\mathcal{A}$.
5. $\mathcal{A}$ submits $((Si_4, T_2), U_i)$, then M_o does computations discussed above and get V_2, then verify $Si_4* = ?h(Sk||T_2)$ and at last authenticates $\mathcal{A}$.

Analysis If $\mathcal{A}$ violates the authentication process between U_i and Sr_j, then without knowing about the private key, the authentication massage $M_1 = T_y(Si_2)$ could be forged by $\mathcal{A}$. $\mathcal{A}$ can send a duplicate massage Si_i*

If without any knowledge about privet key, $\mathcal{A}$ is able to forge $M_1 = T_y(r_i)$, then it can sends a duplicate Si_i*. If $M_1 = h(X) \in L_{ij}$, then failed to proceed P_{c_2}. Else, M_o solves the problem of CRA. As $H(ID_i) \notin L_{ij}$, $A = h(B) \notin (s_{i1}, s_{i2} \dots)$, and let the chance of success of M_o be α, and the chance of violating the protocol be β respectively. Then, if event P_c , P_{c_1}, P_{c_2} exist then hash query, extract query and send query are legal. So, to break CRA problem M_o takes the help of $\mathcal{A}$, if none of P_c , P_{c_1}, P_{c_2} happened. Thus,

$$Pr[\neg P_c \wedge \neg P_{c_1} \wedge \neg P_{c_2}] = \left(\frac{P_q}{H_q}\right)^{P_q+S_q} \left(\frac{H_q - E_q}{H_q}\right)$$

Therefore, M_o is successful with advantage

$$\alpha \geq \left(\beta - \frac{1}{2^k}\right)\left(\frac{P_q}{H_q}\right)^{P_q+s_q} \left(\frac{H_q - P_q}{H_q}\right)$$

So, we can observe that the algorithm M_o has the advantage, so if $\mathcal{A}$ can breach the scheme, then M_o can breach the suggested protocol, using subroutine $\mathcal{A}$.

Theorem 3 *The protocol is secured against chosen massage attack in RO model, if Chaotic discrete logarithm problem (CDLP) holds.*

Proof By contradiction, let us prove this, let us say that there is $\mathcal{A}$, who breaches proposed scheme against chosen massage attack, so we can model model an algorithm mo that violates the discrete logarithm presumption based on the Chaotic map (CDLP), which implies that $\mathcal{A}$ breaches the proposed system, then mo breaches the proposed system as well, which means $\mathcal{A}$ breaches the proposed scheme with non-negligible advantage.

Game 1: Suppose that $\mathcal{A}$ is playing a chosen massage attack game with the suggested scheme. At first, $\mathcal{A}$ possesses security parameter 1^n and map $\mathcal{T}$, then $\mathcal{A}$ use massage space $\mathcal{M}$, to send massages (M_0, M_1), to perform encryption queries Q_E. After getting massage, oracle selects arbitrary $m \in \{0, 1\}$ and applies $E_1 = \mathcal{T}_x(M_m)$ and selects $y \in \{0, 1\}^n$ computes $E_2 = y \oplus M_m$, then reply $E_m = (E_1, E_2)$. Finally, if $\mathcal{A}$ guesses a correct bit m', where $m = m'$ then $\mathcal{A}$ wins the game.
Game 2: In this phase Mo tries to breach the assumptions. Since CDLP says challenger can obtain $s, q \leftarrow Z_n*$ by running $Gen(.)$ algorithm and returns $(1^n, \mathcal{T}_y(s)\ )$ to Mo. Now Mo has to return s without having y.
Game 3: To guess y correctly, $\mathcal{A}$ has three options:

1. If $\mathcal{T}_z\mathcal{T}_x(r) = \mathcal{T}_{z'}\mathcal{T}_x(r)$ implies $z' = z$, $\mathcal{A}$ continues to oracle with a random z, then it calculates $C_1 = \mathcal{T}_z(r)$, else $RO(z) = y$, and reports $y' = y$. However, $\mathcal{A}$ have non negligible chance of conducting such a query, so Mo will win with a non-negligible gain. However, it has been concluded that Mo violates the assumption with a marginal chance that implies $\mathcal{A}$ has a neg(n) gain.
2. In this case $\mathcal{A}$ have no clue about y because there is no query about z following $\mathcal{T}_z\mathcal{T}_x(r) = \mathcal{T}_{z'}\mathcal{T}_x(r)$. Now since $C_2 = y \oplus M_b$ works as one time pad so the probability of winning of $\mathcal{A}$ is $Pr[\mathcal{A} : win] = \frac{1}{2}$.
3. So the probability of $\mathcal{A}$ for winning the game is $Pr[\mathcal{A} : win] = \frac{1}{2} + neg(n)$, that contradicts our assumption.

□

Informal security analysis

Anonymous

If for any protocol, it is impossible for any adversary, to find user's real identity then we says that protocol follows the anonymous property. In our protocol, user's identity is not used over public channel, instead we use $NID_i = (Ui_3||Ui_5||ID_i) \oplus T_y(Si_1)$. Adversary can intercept NID_i and Ui_4 but it is not possible for adversary to extract ID_i from NID_i because for that $\mathcal{A}$ needs to compute $T_s(Ui_4)$ that requires long term secret key s of server.

Password guessing attack

Password guessing attack is not possible on proposed scheme because $\mathcal{A}$ needs Si_3, Ui_2, ID_i, H_i simultaneously to implement it successfully. Adversary might get Si_3, Ui_2 by side channel attack or by power analysis attack but then he also need ID_i and H_i, which is not possible.

Privileged insider attack

Since in registration phase user U_i send ID_i, Ui_1 to the server Sr_j, where $Ui_1 = h(ID_i||Ps_i||H_i)$. So it is not possible for any insider to know the password and biometric of any user because these are protected by hash function $h(.)$. It makes proposed scheme secure against privileged insider attack.

Impersonation attack

If $\mathcal{A}$ wants to act like a legitimate user, he needs to send NID_i correctly to the server, where $NID_i = (Ui_3||Ui_5||ID_i) \oplus T_y(Si_1) = (Ui_3||Ui_5||ID_i \oplus T_yT_s(ID_i)$. To create NID_i, $\mathcal{A}$ needs Si_2), which is protected by Ui_1 and $Ui_1 = h(ID_i||Ps_i||H_i)$. So basically $\mathcal{A}$ needs ID_i, Ps_i and H_i to impersonate user.

Again if $\mathcal{A}$ wants to impersonate server then he need to create Si_4 correctly and for this $\mathcal{A}$ needs server's long term term secret key s. So above discussion suggest that proposed scheme is secure against impersonation attack.

Reply attack

Since we use different random number y for every session in proposed scheme where session key depends upon y, with this we also use time stamp, to avoid this type of attacks. For every session, the timestamp is uniquely chosen. The timestamp uniqueness property limits the duplication of log-in messages. This indicates that the proposed system is responsive to replay attacks.

Perfect forward secrecy

Even if long term secret key s is compromised, suggested scheme is secure. Because to create session key $\mathcal{A}$ needs to compute x and for this $\mathcal{A}$ need ID_i which is not possible because we do not send ID_i through public channel.

Man in the middle attack

If any adversary $\mathcal{A}$ wants to implement man in the middle attack then firstly $\mathcal{A}$, intercept the massage Ui_4, NID_i, T_1, where $Ui_4 = T_y(ID_i)$ and $NID_i = (Ui_3||Ui_5||ID_i) \oplus T_y(Si_1) = (Ui_3||Ui_5||ID_i \oplus T_yT_s(ID_i)$. Since ID_i of U_i is hidden in NID_i and it is not practically possible for any adversary to extract ID_i from public channel information therefore adversary fails to forge Ui_4. So proposed scheme is secure against man in the middle attack.

Stolen card attack

If adversary get access to the smart card of a user and extract P_i, Si_3 and Ui_2 from smart card. Even then he could not able to get any meaningful information, that helps $\mathcal{A}$ to breach the security of proposed protocol because all of these are secured by hash function. $\mathcal{A}$ needs ID_i, Ps_i and H_i simultaneously get useful information which is not possible.

Simulation and output using "Automated Validation of Internet Security Protocols and Applications"

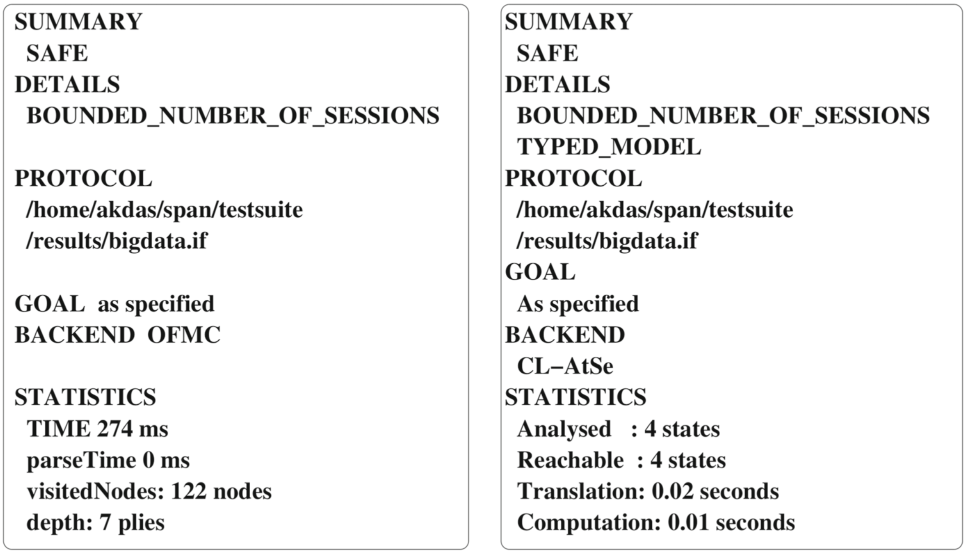
In this subsection, we simulate the scheme using "Automated Validation of Internet Security Protocols and Applications" in short AVISPA tool to analyse formal security (man in the middle attack, replay attack) [36]. We have provided essential illustration on the basic output in OFMC and ATSE modes in Fig. [6].

Performance analysis

In this section, we have analyzed the performance of the proposed protocol and the performance of the proposed protocol has been compared with the related chaotic map based authentication scheme in the Table 4, where the cost of various operations are $h_t \approx 0.0005s$, $s_t \approx 0.0087s$, $m_t \approx 0.06307s$, and $c_t \approx 0.02102s$ denote the time for hashing, message encryption under symmetric key, one ordinary multiplication in Z_p^* and chaotic based operation respectively.

We have analysed the performance of chaotic map based authentication protocols [1,15,16,20,22,24,25] with the proposed protocol. As we know our mobile phones has limited storage and random access memory, and internet connectivity is another problem, that all the telecare medicine services runs on limited bandwidth network that is why we need a secure and efficient authentication protocol. Both computation and communication efficiency are very important and these two costs of computations have been compared with existing protocols in the Table 4. The various operation cost estimated via executing an experiment on intel $Pentiums-4$ (1024 MB ram) processor as in [6,35] with this computation cost described in Fig. 7.

In addition, Liu et al. [22] runs with computation cost $4h_t + 2c_t$ at user side, $5h_t + 2c_t$ at server side, Jiang et al. [16], at user side runs with computation cost $2h_t + s_t + c_t$ at server side use $2h_t+2s_t+3c_t$, Hao et al. [15] at user side runs



```
SUMMARY
 SAFE
DETAILS
 BOUNDED_NUMBER_OF_SESSIONS

PROTOCOL
 /home/akdas/span/testsuite
 /results/bigdata.if

GOAL  as specified
BACKEND  OFMC

STATISTICS
 TIME 274 ms
 parseTime 0 ms
 visitedNodes: 122 nodes
 depth: 7 plies
```

```
SUMMARY
 SAFE
DETAILS
 BOUNDED_NUMBER_OF_SESSIONS
 TYPED_MODEL
PROTOCOL
 /home/akdas/span/testsuite
 /results/bigdata.if
GOAL
 As specified
BACKEND
 CL–AtSe
STATISTICS
 Analysed   : 4 states
 Reachable  : 4 states
 Translation: 0.02 seconds
 Computation: 0.01 seconds
```

Fig. 6 Illustration of output of HLPSL code in OFMC and ATSE modes

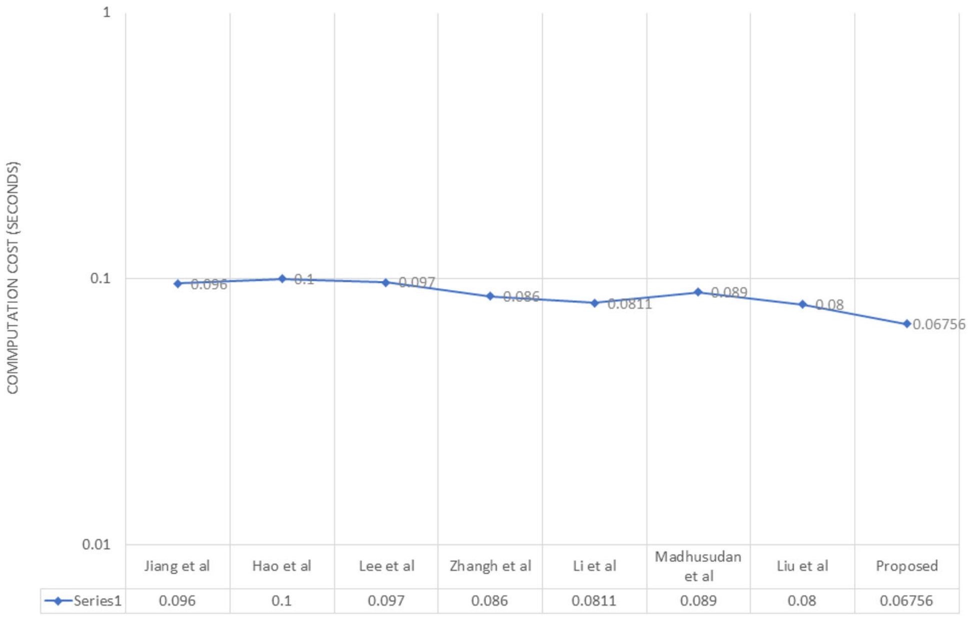


Fig. 7 Computation cost comparison

Table 4 A performance analysis of the proposed protocol with recent chaotic map-based authentication protocols

Schemes	User-computation	Server-computation	Messages
Liu et al.'s [22]	$4h_t + 2c_t$	$5h_t + 2c_t$	3
Jiang et al.'s [16]	$2h_t + s_t + c_t$	$2h_t + 2s_t + 3c_t$	2
Hao et al.'s [15]	$2c_t + 3h_t + 2s_t$	$2c_t + 3s_t + 2h_t$	2
Lee's [24]	$2c_t + 7h_t$	$2c_t + 8h_t$	2
Zhang et al.'s [25]	$6h_t + 2c_t$	$4h_t + 1c_t + 2s_t$	3
Madhusudhan et al's [20]	$7h_t + 2c_t$	$3h_t + 2c_t$	2
Li et al.'s [1]	$7h_t + 2c_t$	$7h_t + 2c_t$	2
Proposed	$4c_t + 4h_t$	$2c_t + 4h_t$	2

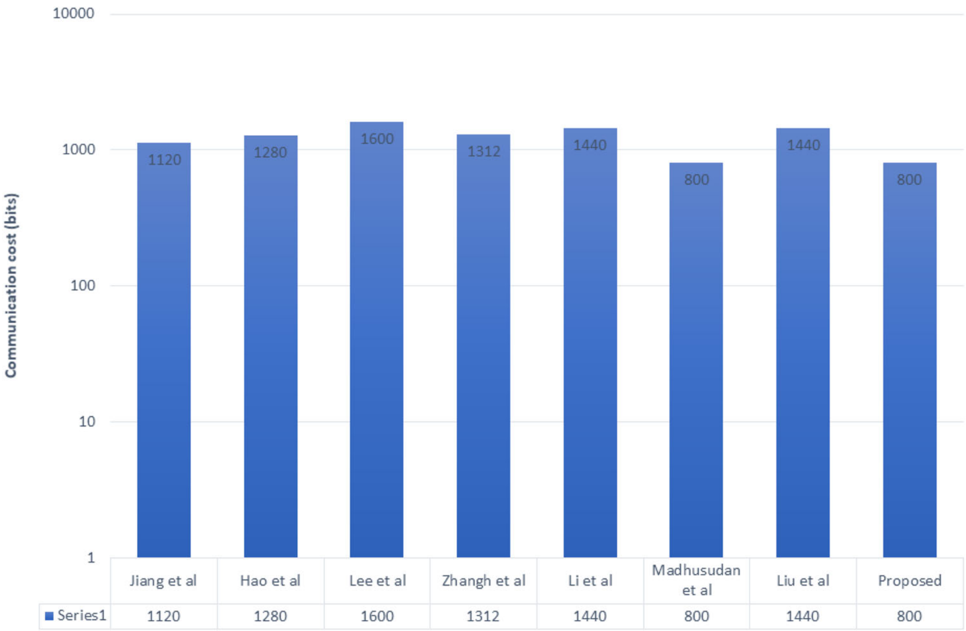


Fig. 8 Communication cost comparison

with computation cost $2c_t + 3h_t + 2s_t$, at sever computation cost is $2c_t + 3s_t + 2h_t$, Lee et al. [24], at the user side runs with computation cost $2c_t + 7h_t$, at the server side it takes $2c_t + 8h_t$, Zhang et al. [25] runs with computation cost $6h_t + 2c_t$ on the user side, and at server cost is $4h_t + 1c_t + 2s_t$, Madhusudhan et al. [20] at user side runs with computation cost $7h_t + 2c_t$, at server side $3h_t + 2c_t$, Li et al. [1] runs with computation cost $7h_t + 2c_t$ at the user side, and at server side cost is $7h_t + 2c_t$, whereas the suggested protocol runs with computation cost $4c_t + 4h_t$ at the user side, $2c_t + 4h_t$ at the server side respectively.

In this article, we have considered the cost of communication in the form of hashing, chaotic operation, and time-stamp as 160-bits, and symmetric encryption outputs standard 256 bits, whereas total cost of communication is given in the Fig. 8 and the cost of computation is shown in the Fig. 8.

Conclusion

This article provides a review on the security of recently proposed chaotic map based authentication protocol. The suggested design is free from most of the existing vulnerabilities such as password-guess, identity-guess, impersonations, replaying of messages, and stolen smart cards attacks and it also gives the idea how a poor verification results vulnerabilities. Furthermore, we have observed that the proposed design fulfills the requirement of session key verification in just two message exchange. In future, we can implement the protocol in vehicular communications, and digital rights management system etc.

Declarations

Conflict of interests All the authors have no conflict of interests.

Research involving human participants/animals Research does not involve any human participant and/or animal performed by any of the authors.

Informed consent All the authors have agreed to this submission.

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