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## ORIGINAL ARTICLE

# Agreement between total corneal astigmatism calculated by vector summation and total corneal astigmatism measured by ray tracing using Galilei double Scheimpflug analyzer

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### KEYWORDS

Total corneal astigmatism;  
Ray tracing;  
Vector summation

### Abstract

**Purpose:** To evaluate the agreement between total corneal astigmatism calculated by vector summation of anterior and posterior corneal astigmatism ( $TCA_{Vec}$ ) and total corneal astigmatism measured by ray tracing ( $TCA_{Ray}$ ).

**Methods:** This study enrolled a total of 204 right eyes of 204 normal subjects. The eyes were measured using a Galilei double Scheimpflug analyzer. The measured parameters included simulated keratometric astigmatism using the keratometric index, anterior corneal astigmatism using the corneal refractive index, posterior corneal astigmatism, and  $TCA_{Ray}$ .  $TCA_{Vec}$  was derived by vector summation of the astigmatism on the anterior and posterior corneal surfaces. The magnitudes and axes of  $TCA_{Vec}$  and  $TCA_{Ray}$  were compared. The Pearson correlation coefficient and Bland–Altman plots were used to assess the relationship and agreement between  $TCA_{Vec}$  and  $TCA_{Ray}$ , respectively.

**Results:** The mean  $TCA_{Vec}$  and  $TCA_{Ray}$  magnitudes were  $0.76 \pm 0.57\text{D}$  and  $1.00 \pm 0.78\text{D}$ , respectively ( $P < 0.001$ ). The mean axis orientations were  $85.12 \pm 30.26^\circ$  and  $89.67 \pm 36.76^\circ$ , respectively ( $P = 0.02$ ). Strong correlations were found between the  $TCA_{Vec}$  and  $TCA_{Ray}$  magnitudes ( $r = 0.96$ ,  $P < 0.001$ ). Moderate associations were observed between the  $TCA_{Vec}$  and  $TCA_{Ray}$  axes ( $r = 0.75$ ,  $P < 0.001$ ). Bland–Altman plots produced the 95% limits of agreement for the  $TCA_{Vec}$  and  $TCA_{Ray}$  magnitudes from  $-0.33$  to  $0.82\text{D}$ . The 95% limits of agreement between the  $TCA_{Vec}$  and  $TCA_{Ray}$  axes was  $-43.0$  to  $52.1^\circ$ .

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**Conclusion:** The magnitudes and axes of astigmatisms measured by the vector summation and ray tracing methods cannot be used interchangeably. There was a systematic error between the TCA<sub>Vec</sub> and TCA<sub>Ray</sub> magnitudes.

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## PALABRAS CLAVE

Astigmatismo corneal total;

Trazado de rayos;  
Suma de vectores

## Concordancia entre el astigmatismo corneal total calculado mediante sumación vectorial, y el astigmatismo corneal total medido mediante trazado de rayos utilizando un analizador Galilei de doble cámara Scheimpflug

### Resumen

**Objetivo:** Evaluar la concordancia entre el astigmatismo corneal total calculado mediante suma de los vectores del astigmatismo corneal anterior y posterior (TCA<sub>Vec</sub>), y el astigmatismo corneal total medido mediante trazado de rayos (TCA<sub>Ray</sub>).

**Métodos:** Este estudio incluyó a un total de 204 ojos derechos de 204 sujetos normales. Los ojos se midieron utilizando un analizador Galilei de doble cámara Scheimpflug. Los parámetros medidos incluyeron el astigmatismo queratométrico simulado utilizando el índice queratométrico, el astigmatismo corneal anterior utilizando el índice refractivo corneal, el astigmatismo corneal posterior, y TCA<sub>Ray</sub>. TCA<sub>Vec</sub> se calculó mediante la suma de los vectores del astigmatismo en las superficies corneales anterior y posterior. Se compararon las magnitudes y ejes de TCA<sub>Vec</sub> y TCA<sub>Ray</sub>. Se utilizaron el coeficiente de correlación de Pearson y los diagramas de Bland-Altman para valorar la relación y concordancia entre TCA<sub>Vec</sub> y TCA<sub>Ray</sub>, respectivamente.

**Resultados:** Los valores medios de las magnitudes TCA<sub>Vec</sub> y TCA<sub>Ray</sub> fueron  $0,76 \pm 0,57$  D y  $1 \pm 0,78$  D, respectivamente ( $P < 0,001$ ). Las orientaciones medias del eje fueron  $85,12 \pm 30,26$  grados y  $89,67 \pm 36,76$  grados, respectivamente ( $P = 0,02$ ). Se encontraron fuertes correlaciones entre las magnitudes de TCA<sub>Vec</sub> y TCA<sub>Ray</sub> ( $r = 0,96$ ,  $P < 0,001$ ). Se observaron asociaciones moderadas entre los ejes de TCA<sub>Vec</sub> y TCA<sub>Ray</sub> ( $r = 0,75$ ,  $P < 0,001$ ). Los diagramas de Bland-Altman establecieron un 95% de límite de acuerdo para las magnitudes de TCA<sub>Vec</sub> y TCA<sub>Ray</sub> de -0,33 a 0,82 D. Los límites de acuerdo del 95% entre los ejes de TCA<sub>Vec</sub> y TCA<sub>Ray</sub> fueron de -43 a 52,1 grados.

**Conclusión:** Las magnitudes y ejes de los astigmatismos medidos mediante suma de vectores y trazado de rayos no pueden intercambiarse. Se produjo un error sistemático entre las magnitudes de TCA<sub>Vec</sub> y TCA<sub>Ray</sub>.

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## Introduction

Toric intraocular lenses (IOLs) have been developed to reduce corneal astigmatism following cataract surgery.<sup>1</sup> The accurate measurement of corneal astigmatism is of the utmost importance for the prevention of astigmatism over- or undercorrection by toric IOLs. Conventional keratometers estimate the corneal refractive power and astigmatism from anterior corneal measurements and a fictitious keratometric index without taking into account the posterior corneal power.<sup>2</sup> However, neglecting the effect of posterior corneal astigmatism on anterior corneal astigmatism can lead to unacceptable toric IOL power calculation results. Several studies report that incorporating posterior astigmatism into the toric IOL calculation improves refractive outcomes.<sup>3–6</sup> Recently, the development of new technologies, such as slit-scanning devices, Scheimpflug devices, and optical coherence tomography, has made the quantitative

measurement of the posterior corneal curvature in a clinical setting possible.<sup>7,8</sup> The Galilei dual Scheimpflug system (Ziemer Ophthalmic System AG, Zurich, Switzerland) is a noninvasive diagnostic instrument designed for the analysis of the anterior eye segment characteristics with an excellent repeatability of corneal power measurements.<sup>9,10</sup> This system combines both technologies, including Placido imaging, which provides curvature data, and Scheimpflug imaging, which is optimal for precise elevation measurements. This enables the calculation of the total corneal astigmatism using ray tracing of the anterior and posterior corneal surfaces, taking into account the effect of corneal thickness on the corneal power.<sup>4,6,11–14</sup> Alternatively, total corneal astigmatism can be calculated from the astigmatism of each surface using vector analysis.<sup>5,15–17</sup> However, a confirmation of the agreement between these two total corneal astigmatisms is still lacking and the best method for achieving optimum astigmatic outcomes after toric IOL

implantation has not been determined. The purpose of this study was to evaluate the agreement among magnitudes and axis orientations of three corneal astigmatisms, including simulated keratometric astigmatism, calculated total corneal astigmatism using the vector summation of anterior and posterior corneal astigmatism, and Galilei-derived total corneal astigmatism using ray tracing of the anterior and posterior corneal surfaces. Previous studies found an improvement in the prediction of the refractive astigmatism error when the ray tracing method was used to calculate the cylinder power of toric IOLs.<sup>3,4,6</sup> Therefore, we considered astigmatism measured by ray tracing to be the gold standard and compared simulated keratometric astigmatism and total corneal astigmatism calculated by vector summation with this benchmark in the current study.

## Methods

This prospective observational case series enrolled 204 right eyes of 204 normal subjects. The only ocular problem was refractive error, and a history or diagnosis of any ocular pathology, such as dry eye, keratoconus, glaucoma, retinal disease, or any previous ocular surgery led to subject exclusion. The study was approved by the Institutional Review Board of the Ophthalmic Research Center, which is affiliated with the Shahid Beheshti University of Medical Sciences, Tehran, Iran, and followed the tenets of the Declaration of Helsinki. Informed consent was signed by all participants after the purpose of the study was explained.

Contact lens users were asked to discontinue the use of lenses for at least 2 weeks and 1-month before the examination for soft and gas-permeable contact lenses, respectively. A complete ocular examination, including slit-lamp biomicroscopy, manifest refraction, uncorrected and best-spectacle corrected visual acuity using the Snellen acuity chart, intraocular pressure measurement, and dilated funduscopic examination was performed. For measurements using the Galilei dual Scheimpflug analyzer (software version 5.2; Ziener Ophthalmic System AG), the participants were seated with their chin on a chinrest and forehead against the forehead strap while focusing on the target. The alignment of the scan center with the corneal apex was confirmed using an initial Scheimpflug image formed on the monitor together with a guide line. The measurements were confirmed under a quality-specification window. Good-quality dual Scheimpflug analyzer scans (green check mark displayed on the map) were included in the study. If the comments were marked yellow or red (i.e., not acceptable), then the examination was repeated or the patient's data were excluded.

The following measurements were obtained using the Galilei dual Scheimpflug analyzer: simulated keratometric astigmatism, anterior and posterior corneal astigmatism, total corneal astigmatism, and central corneal thickness. All measurements were made across the 1.0–4.0 mm central zone of the cornea. Simulated keratometric astigmatism (KA<sub>Sim</sub>) was calculated using the anterior corneal radii and a keratometric index of 1.3375. For the calculation of anterior corneal astigmatism, the anterior corneal radii and air (1.00) and corneal refractive (1.376) indices were

used. Posterior corneal astigmatism was calculated using the posterior corneal radii and refractive index of the cornea (1.376) and the aqueous humor (1.336). The Galilei-derived total corneal astigmatism was calculated by tracking the path of incident rays of light through the anterior corneal surface and the posterior corneal surface using a ray tracing method and the Snell law. The actual indices of the refractions of each medium (air=1.00, cornea=1.376, and aqueous=1.336) were used in the total corneal astigmatism calculation using the ray tracing method (TCA<sub>Ray</sub>).

The calculated total corneal astigmatism was obtained as the vector summation of anterior and posterior corneal astigmatism (TCA<sub>Vec</sub>).<sup>15</sup> Before vector analysis, the power of the flat and steep meridian of the anterior corneal surface was converted to the posterior corneal surface plane using the vergence formula: vergence power = 1.376/[(1.376/P) – d], where P is the power of anterior corneal surface in the flat or steep meridian and d is the average pachymetric values over the central 4-mm area. The astigmatism at the posterior corneal surface plane caused by the anterior surface was the difference between the vergence powers of the anterior flat and steep meridians. TCA<sub>Vec</sub> was then obtained by vector summation of the astigmatism at the posterior corneal surface plane created by the anterior corneal surface and the astigmatism from the posterior corneal surface. For vector summation, anterior astigmatism was defined using the positive magnitude and steep axis. Because the posterior corneal surface has a divergent optical effect, the posterior corneal astigmatism was also defined with a positive magnitude but using the flat axis for vector addition.

The astigmatism magnitudes measured by KA<sub>Sim</sub>, TCA<sub>Vec</sub>, and TCA<sub>Ray</sub> were compared using the power vector analysis according to Thibos et al.<sup>18</sup> Each astigmatism value was converted to rectangular vectors J0 and J45 using the following equations: J0 = -(C/2) × Cos (2α) and J45 = -(C/2) × Sin (2α), where J0 is the Jackson cross-cylinder power at axis 90° and 180°, J45 is the Jackson cross-cylinder power at axis 45° and 135°, C is the negative cylinder, and α is the axis of the flat meridian. In this representation, positive values of J0 indicate with-the-rule (WTR) astigmatism and negative values indicate against-the-rule (ATR) astigmatism. Power J45 represents oblique astigmatism.

Since the significance of the difference of astigmatism axes depends on the magnitude of astigmatism, the weighted axes difference between the KA<sub>Sim</sub> and TCA<sub>Ray</sub> axes, between the TCA<sub>Vec</sub>, and TCA<sub>Ray</sub> axes, and between TCA<sub>Vec</sub> and KA<sub>Sim</sub> axes was calculated using the following equations: DA = 2 × TCA<sub>Ray</sub> × sin (2Axis<sub>Sim</sub> – 2Axis<sub>Ray</sub>), DA = 2 × TCA<sub>Ray</sub> × sin (2Axis<sub>Vec</sub> – 2Axis<sub>Ray</sub>), and DA = 2 × KA<sub>Sim</sub> × sin (2Axis<sub>Vec</sub> – 2Axis<sub>Sim</sub>), respectively, where DA is the weighted axes difference, and Axis<sub>Sim</sub>, Axis<sub>Vec</sub>, and Axis<sub>Ray</sub> denote the axis of astigmatism measured by the simulated keratometry, vector summation, and ray tracing methods, respectively.<sup>19</sup> DA is a quality criterion based on the power-vector approach. Geometrically, DA is the length of the difference vector between both methods given that the astigmatism power found with the two methods are comparable. This approach makes it possible to compare axis values, even when the actual astigmatism powers are different.<sup>19</sup>

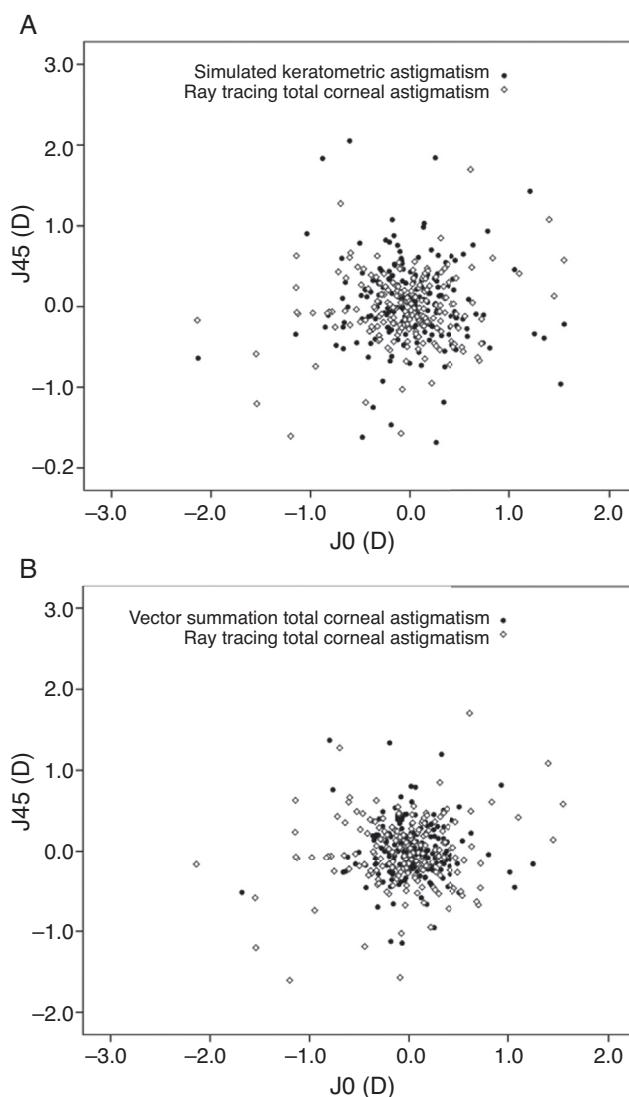
## Statistical analysis

Data were analyzed using SPSS statistical software version 21 (IBM Corp., Armonk, NY, USA). Values indicating means and standard deviations, ranges, frequencies and percentages were used to express the data. The normal distribution of continuous variables was confirmed using a Kolmogorov-Smirnov test and Q-Q plot. The magnitudes and orientations of three different astigmatisms were compared using repeated measure ANOVA and Friedman tests with the Bonferroni correction for multiple comparisons. The absolute mean difference of astigmatism magnitude between  $KA_{Sim}$  and  $TCA_{Ray}$  was compared with that between  $TCA_{Vec}$  and  $TCA_{Ray}$  using a paired *t*-test and Wilcoxon signed rank test. The MacNemar test was used to compare the percentage of eyes that had magnitudes and axis orientations of  $KA_{Sim}$  and  $TCA_{Vec}$  different from those of  $TCA_{Ray}$ . The Pearson correlation coefficient was calculated to assess the relationship between  $KA_{Sim}$  and  $TCA_{Ray}$  and between  $TCA_{Vec}$  and  $TCA_{Ray}$  with respect to the magnitudes and axis orientations. Bland-Altman plots and the 95% limits of agreement (95% LoA; mean difference  $\pm 1.96$  standard deviation) were applied to determine the agreement between  $KA_{Sim}$  and  $TCA_{Ray}$  and between  $TCA_{Vec}$  and  $TCA_{Ray}$  in terms of magnitudes and axis orientations.  $P < 0.05$  was considered a statistical significance. All  $P$ -values are two-sided.

## Results

A total of 204 right eyes of 204 normal subjects (88 male subjects) were analyzed. The mean participant age was  $29.9 \pm 5.9$  years (range, 20 to 54 years). The mean spherical equivalent refraction was  $-2.68 \pm 1.87$  D (range, -13.88 to +0.75 D). The mean simulated keratometry and keratometric astigmatism were  $43.74 \pm 1.14$  D (range, 41.09–45.98 D) and  $1.09 \pm 0.82$  D (0.06–4.45 D), respectively. The mean anterior corneal astigmatism magnitude using the corneal refractive index was  $1.22 \pm 0.91$  D (range, 0.06–4.94 D). Regarding the anterior corneal surface, 155 eyes (76.0%) had WTR astigmatism (the steep meridian was within 60–120°), 21 eyes (10.3%) had ATR astigmatism (the steep meridian was within 0–30° or 150–180°), and 28 eyes (13.7%) had oblique astigmatism. The mean posterior corneal power was  $-6.30 \pm 0.20$  D (range, -6.84 to -5.83 D), and the mean posterior corneal astigmatism magnitude was  $-0.31 \pm 0.14$  D (range, -0.82 to -0.08 D). Regarding the posterior corneal surface, 1 eye (0.5%) had WTR astigmatism (the steep meridian was within 0–30° or 150–180°), 198 eyes (97.1%) had ATR astigmatism (the steep meridian was within 60–120°), and 5 eyes (2.4%) had oblique astigmatism. The principle meridians of the anterior and posterior corneal astigmatism coincided in the same patient in 172 eyes (84.3%).

**Table 1** shows and compares the magnitudes, axis orientations,  $J_0$ , and  $J_{45}$  among  $KA_{Sim}$ ,  $TCA_{Vec}$ , and  $TCA_{Ray}$  in the entire study group. As demonstrated, the magnitudes of these three astigmatisms were significantly different.  $KA_{Sim}$  yielded the highest magnitude and  $TCA_{Vec}$  had the lowest magnitude. The results of multiple comparisons revealed that  $TCA_{Vec}$  produced significantly lower astigmatism magnitude than  $KA_{Sim}$  ( $P < 0.001$ ) and  $TCA_{Ray}$  ( $P < 0.001$ ). The  $KA_{Sim}$  magnitude was significantly higher than the  $TCA_{Ray}$



**Figure 1** Vector analysis of astigmatic magnitudes measured by the three methods, plotted as an astigmatic vector for each eye. (A) Vector analysis of astigmatic magnitude measured by keratometric astigmatism versus ray tracing. (B) Vector analysis of astigmatic magnitudes measured by vector summation of the anterior and posterior corneal astigmatism versus ray tracing ( $J_0$  = Jackson cross-cylinder power, axes at 90° and 180°;  $J_{45}$  = Jackson cross-cylinder power, axes at 45° and 135°).

magnitude ( $P < 0.001$ ). The absolute mean difference in the magnitudes of astigmatism between  $KA_{Sim}$  and  $TCA_{Ray}$  ( $0.16 \pm 0.09$  D, range, 0.0–0.45 D) was significantly lower than that between  $TCA_{Vec}$  and  $TCA_{Ray}$  ( $0.28 \pm 0.26$  D, range, 0.0–1.30 D;  $P < 0.001$ ). The astigmatic magnitudes of  $KA_{Sim}$  were within  $\pm 0.50$  D of the  $TCA_{Ray}$  magnitudes in all eyes (100%). The percentages of eyes with the magnitudes of  $TCA_{Vec}$  within  $\pm 0.50$  D and  $\pm 1.0$  D of the  $TCA_{Ray}$  magnitudes were 84.3% and 97.6%, respectively. There was no significant difference in the  $J_0$  and  $J_{45}$  values among the three methods of astigmatism measurement (**Table 1** and **Fig. 1**).

The axis orientations of astigmatisms measured by the three methods were significantly different (**Table 1**). The results of multiple comparisons revealed a significant

**Table 1** Comparisons of keratometric astigmatism (KA<sub>Sim</sub>), total corneal astigmatism calculated by vector summation (TCA<sub>Vec</sub>), and total corneal astigmatism measured by ray tracing (TCA<sub>Ray</sub>) in terms of magnitude, axis, and component of Jackson cross cylinder at 0 and 90° (J0) and at 45 and 135° (J45).

	KA <sub>Sim</sub>	TCA <sub>Vec</sub>	TCA <sub>Ray</sub>	P value
Magnitude (D)	1.09 ± 0.82 (0.06 to 4.45)	0.76 ± 0.57 (0.05 to 3.52)	1.00 ± 0.78 (0.06 to 4.29)	<0.001
Axis orientation (°)	85.55 ± 30.51 (1.0 to 176.0)	85.12 ± 30.26 (1.03 to 175.89)	89.67 ± 36.76 (2.0 to 177.0)	0.002
J0 (D)	0.020 ± 0.46 (-2.13 to 1.55)	0.024 ± 0.32 (-1.68 to 1.25)	-0.044 ± 0.47 (-2.14 to 1.55)	0.08
J45 (D)	0.037 ± 0.52 (-1.86 to 2.05)	0.021 ± 0.35 (-1.15 to 1.37)	-0.034 ± 0.42 (-1.61 to 1.70)	0.11

difference in axis orientations between KA<sub>Sim</sub> and TCA<sub>Ray</sub> ( $P=0.04$ ) and between TCA<sub>Vec</sub> and TCA<sub>Ray</sub> ( $P=0.02$ ). There was no significant difference between KA<sub>Sim</sub> and TCA<sub>Vec</sub> in terms of axis orientations ( $P>0.99$ ). The percentages of eyes with the KA<sub>Sim</sub> axis within ±5°, ±10°, and ±15° of the TCA<sub>Ray</sub> axis were 66.2%, 80.9%, and 88.2%, respectively. The percentages of eyes with the TCA<sub>Vec</sub> axis within ±5°, ±10°, and ±15° of the TCA<sub>Ray</sub> axis were 61.8%, 79.9%, and 87.8%, respectively. Weighted axes difference was  $-0.16 \pm 1.75$  D between KA<sub>Sim</sub> and TCA<sub>Ray</sub>,  $-0.17 \pm 1.74$  D between TCA<sub>Vec</sub> and TCA<sub>Ray</sub>, and  $-0.04 \pm 0.43$  D between KA<sub>Sim</sub> and TCA<sub>Vec</sub> ( $P<0.001$ ). This approach revealed that the length of the difference vector was longest between TCA<sub>Vec</sub> and TCA<sub>Ray</sub> and shortest between KA<sub>Sim</sub> and TCA<sub>Vec</sub>.

Strong statistically significant correlations were found between the KA<sub>Sim</sub> and TCA<sub>Ray</sub> magnitudes ( $r=0.98$ ,  $P<0.001$ ) and between the TCA<sub>Vec</sub> and TCA<sub>Ray</sub> magnitudes ( $r=0.96$ ,  $P<0.001$ ). Moderate significant associations were observed between the KA<sub>Sim</sub> and TCA<sub>Ray</sub> axes ( $r=0.77$ ,  $P<0.001$ ) and between the TCA<sub>Vec</sub> and TCA<sub>Ray</sub> axes ( $r=0.75$ ,  $P<0.001$ ) (Fig. 2).

Bland–Altman plots produced a 95% LoA for the KA<sub>Sim</sub> and TCA<sub>Ray</sub> magnitudes from  $-0.40$  to  $0.21$  D (mean difference =  $-0.10$  D). This range was wider for the TCA<sub>Vec</sub> and TCA<sub>Ray</sub> magnitudes (from  $-0.33$  to  $0.82$  D; mean difference =  $0.24$  D) (Fig. 3). The mean difference between the KA<sub>Sim</sub> and TCA<sub>Ray</sub> axes was  $4.1^\circ$  ( $\pm 95\%$  LoA:  $-42.2$  to  $50.4^\circ$ ), which was similar to that between the TCA<sub>Vec</sub> and TCA<sub>Ray</sub> axes ( $4.5^\circ$ ,  $\pm 95\%$  LoA:  $-43.0$  to  $52.1^\circ$ ) (Fig. 3).

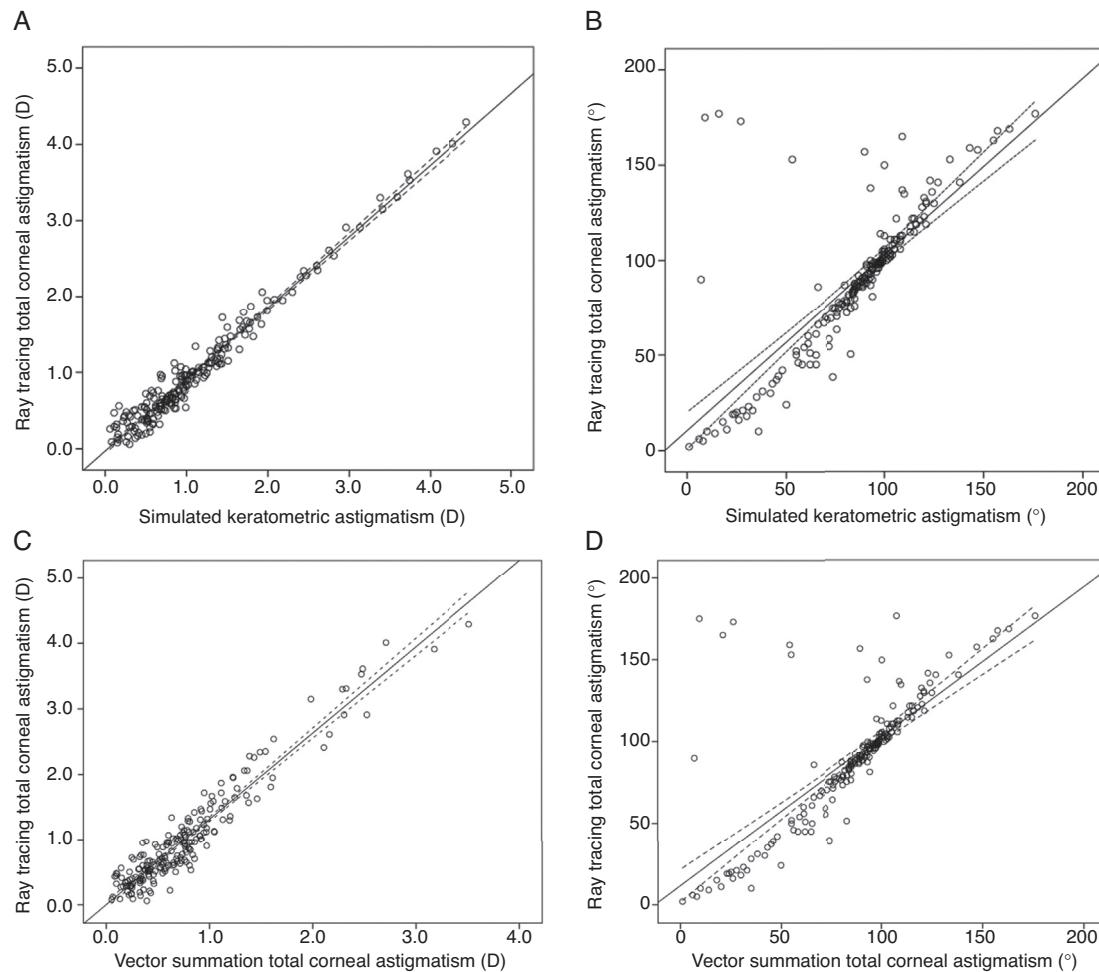
## Discussion

Since posterior corneal imaging became possible, it has received increasing clinical interest, particularly for the accurate correction of astigmatic errors by toric IOLs.<sup>3–6</sup> Three methods are used to determine the effect of posterior corneal astigmatism on anterior corneal astigmatism. Simulated keratometry measures the radius of the anterior corneal surface only and uses the keratometric index (1.3375 for most devices) to compensate for the posterior corneal surface and to convert anterior measurements into total corneal power and astigmatism without knowing the curvature of the posterior corneal surface.<sup>2</sup> The keratometric index is used on the basis of the premise that the anterior and posterior corneal curvatures have a constant and linear relationship.<sup>20,21</sup> However, this premise is not always true because the anterior-posterior curvature ratio varies from eye to eye.<sup>20,21</sup> Total corneal astigmatism can also be

calculated from the astigmatism of anterior and posterior surfaces using vector summation.<sup>5,15–17</sup> This method uses the paraxial approximation with the assumption of parallel rays approaching the posterior corneal surface.<sup>22</sup> Another method is the ray tracing of the anterior and posterior corneal surfaces.<sup>3,4,6,11–14</sup> Ray tracing propagates incoming parallel rays and uses the Snell law to refract these rays through the anterior and posterior corneal surfaces, taking into account the effect of corneal thickness on the corneal power. This approach, instead of assuming that parallel rays reach the posterior corneal surface, accounts for the refraction of rays by the anterior corneal surface.<sup>22</sup> To the best of our knowledge, no study has evaluated whether the astigmatism magnitudes and axis orientations measured by vector summation and ray tracing can be used interchangeably.

We evaluated different corneal astigmatisms from a dual Scheimpflug topographer because studies have shown that the system has excellent repeatability of corneal power measurements.<sup>9,10</sup> The advantage of the Galilei dual Scheimpflug analyzer is that KA<sub>Sim</sub>, anterior and posterior corneal astigmatism, and TCA<sub>Ray</sub> are measured simultaneously, thereby eliminating the potential disagreement among the measurements that may be caused by the realignment of the eye. The results of the current study show disagreement among KA<sub>Sim</sub>, TCA<sub>Vec</sub>, and TCA<sub>Ray</sub> in terms of magnitude and axis orientation. Compared with TCA<sub>Ray</sub>, KA<sub>Sim</sub> overestimated and TCA<sub>Vec</sub> underestimated the astigmatism magnitude. The agreement was narrower between the KA<sub>Sim</sub> and TCA<sub>Ray</sub> magnitudes than between the TCA<sub>Vec</sub> and TCA<sub>Ray</sub> magnitudes. Similar to our results, previous studies reported that the TCA<sub>Ray</sub> magnitude was significantly lower than the KA<sub>Sim</sub> magnitude, which is due to the contribution from the posterior corneal surface.<sup>12–14</sup> However, contradicting results have been reported. Zhang et al.<sup>4</sup> and Srivannaboon et al.<sup>11</sup> reported no significant difference between the KA<sub>Sim</sub> and TCA<sub>Ray</sub> magnitudes. This discrepancy is likely due to the participant's age, which was older in the study by Zhang et al.<sup>4</sup> (68.0 years) and Srivannaboon et al.<sup>11</sup> (66.14 years) than in our study (29.9 years). Anterior corneal astigmatism shifts toward ATR from WTR with increasing age, while the orientation of posterior astigmatism remains unchanged.<sup>17,23</sup> Thus, the compensating effects of the posterior corneal surface on anterior corneal astigmatism decreases with advancing age. This explains the discrepancy between our results and the results obtained by Zhang et al.<sup>4</sup> and Srivannaboon et al.<sup>11</sup>

The method we used to calculate total corneal astigmatism by vector summation was performed according to the approach adopted by Ho et al.<sup>15</sup> They reported that

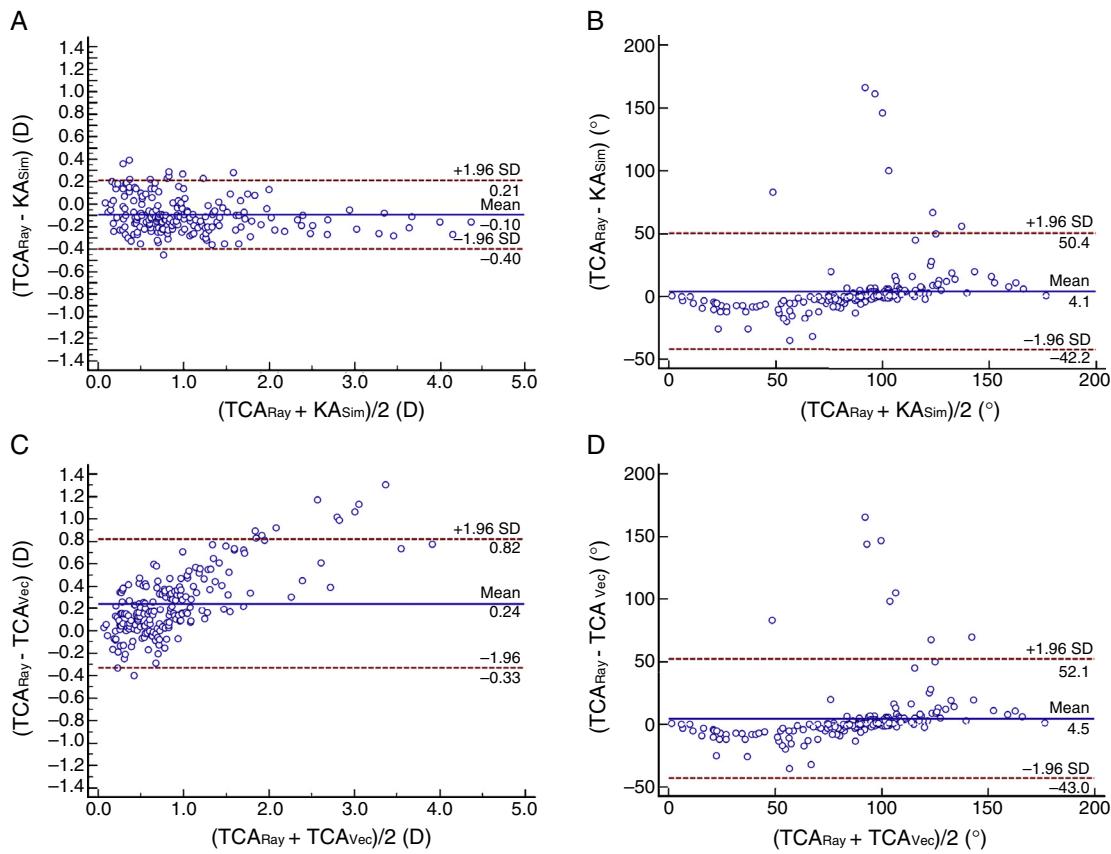


**Figure 2** Scattergrams illustrating the relationships between keratometric astigmatism ( $KA_{sim}$ ) and total corneal astigmatism measured by ray tracing ( $TCA_{Ray}$ ) and between total corneal astigmatism calculated by vector summation ( $TCA_{Vec}$ ) and  $TCA_{Ray}$  in terms of magnitudes and orientations. (A) The relationship between the magnitudes of  $KA_{sim}$  and  $TCA_{Ray}$ ; the regression formula was  $TCA_{Ray}$  magnitude =  $-0.03 + 0.94 \times KA_{sim}$  magnitude ( $r = 0.98$ ,  $P < 0.001$ ). (B) The relationship between the axis orientations of  $KA_{sim}$  and  $TCA_{Ray}$ ; the regression formula was  $TCA_{Ray}$  orientation =  $10.46 + 0.93 \times KA_{sim}$  orientation ( $r = 0.77$ ,  $P < 0.001$ ). (C) The relationship between the magnitudes of  $TCA_{Vec}$  and  $TCA_{Ray}$ ; the regression formula was  $TCA_{Ray}$  magnitude =  $0.003 + 1.32 \times TCA_{Vec}$  magnitude ( $r = 0.96$ ,  $P < 0.001$ ). (D) The relationship between the axis orientations of  $TCA_{Vec}$  and  $TCA_{Ray}$ ; the regression formula was  $TCA_{Ray}$  orientation =  $11.68 + 0.92 \times TCA_{Vec}$  orientation ( $r = 0.75$ ,  $P < 0.001$ ). Dotted lines indicate 95% confidence intervals for the regression line.

the vector summation of posterior corneal astigmatism to anterior corneal astigmatism resulted in a reduction in the astigmatism of the anterior corneal surface, which is consistent with our findings.<sup>15</sup> Similarly, Prisant et al.<sup>16</sup> observed that vector summation of Orbscan measurements of posterior and anterior corneal surface toricity resulted in an astigmatism reduction in overall topographical astigmatism compared to anterior topographical astigmatism.

Despite the identification of a significant difference among the three methods of astigmatism magnitude measurement, a strong association was found between the  $KA_{sim}$  and  $TCA_{Ray}$  magnitudes and between the  $TCA_{Vec}$  and  $TCA_{Ray}$  magnitudes in the present study. This finding indicates that the difference was caused by a systematic error and not by a random error. The presence of a fixed systematic bias may allow appropriate conversion equations to be derived such that the  $KA_{sim}$  and  $TCA_{Vec}$  magnitudes can be changed

to the  $TCA_{Ray}$  magnitude. The intercept and slope parameters reported in the current study can be used for this purpose. As previously mentioned, the systematic measurement difference between the  $KA_{sim}$  and  $TCA_{Ray}$  magnitudes mainly arises from neglecting the posterior corneal astigmatism in the  $KA_{sim}$  calculation. The potential explanation for the systematic measurement difference between the  $TCA_{Vec}$  and  $TCA_{Ray}$  magnitudes can be attributed to the assumption by which the posterior corneal astigmatism is calculated. Corneal astigmatism from the posterior corneal surface is calculated using the paraxial approximation with the assumption of parallel rays approaching the posterior corneal surface.<sup>12</sup> However, the rays propagating to the posterior surface have already been refracted by the anterior surface; therefore, the “effective” posterior power will be less than what is calculated using parallel incident rays and a paraxial formula.<sup>22</sup> Thus, the vector summation of



**Figure 3** Bland-Altman plots show the difference between two measurements divided by the mean of two measurements. (A) Difference in the magnitudes of keratometric astigmatism ( $KA_{Sim}$ ) and total corneal astigmatism measured by ray tracing ( $TCA_{Ray}$ ) against the mean magnitudes of  $KA_{Sim}$  and  $TCA_{Ray}$ . (B) Difference in the axes of  $KA_{Sim}$  and  $TCA_{Ray}$  against the mean axes of  $KA_{Sim}$  and  $TCA_{Ray}$ . (C) Difference in the magnitudes of total corneal astigmatism calculated by vector analysis ( $TCA_{Vec}$ ) and  $TCA_{Ray}$  against the mean magnitudes of  $TCA_{Vec}$  and  $TCA_{Ray}$ . (D) Difference in the axes of  $TCA_{Vec}$  and  $TCA_{Ray}$  against the mean axes of  $TCA_{Vec}$  and  $TCA_{Ray}$ . The solid lines represent mean differences and dotted lines are the upper and lower borders of the 95% limits of agreement.

anterior and posterior corneal astigmatism would underestimate total corneal astigmatism, which was the case in the current study. This may result in an underestimation of the cylinder power of the toric IOLs when using  $TCA_{Vec}$ . Ray tracing does not rely on paraxial optics and is the better method with which to calculate total corneal refractive power.

The  $KA_{Sim}$  and  $TCA_{Vec}$  axes were significantly different from the  $TCA_{Ray}$  axis. An axis difference of more than 10° was detected in 19.1% of eyes between  $KA_{Sim}$  and  $TCA_{Ray}$  and in 20.1% of eyes between  $TCA_{Vec}$  and  $TCA_{Ray}$ . These rates are very similar to those reported in a previous study by Koch et al.,<sup>12</sup> who found that the difference in the location of the steep meridian between  $KA_{Sim}$  and  $TCA_{Ray}$  was higher than 10° in 17.2% of eyes. The axis orientations of  $KA_{Sim}$  and  $TCA_{Vec}$  were comparable. In addition, weighted axes difference revealed that the length of the difference vector was shortest between  $KA_{Sim}$  and  $TCA_{Vec}$ . These findings are consistent with those obtained in previous studies, demonstrating that the axis location was similar between  $KA_{Sim}$  and  $TCA_{Vec}$ .<sup>15,16</sup> A potential explanation may be the small difference between the anterior and posterior principal meridians, which we observed in most eyes included in the current study. Because the anterior and posterior principal meridians were closely aligned, the modification

in axis induced by vector summation was not statistically significant.

The results of the current study indicate that compared to the ray tracing method, residual postoperative astigmatism can be as high as 0.82 D and axis misalignment can vary from -43.0 to 52.1° if the vector summation method is used for toric IOL calculation. However, the limitation of our study is that we did not validate the outcomes of toric IOL implantation calculated by  $TCA_{Vec}$  versus  $TCA_{Ray}$ . Therefore, we are not certain whether  $TCA_{Ray}$  is superior to  $TCA_{Vec}$  in regard to toric IOL calculation. Several studies have shown that toric IOL calculation using  $TCA_{Ray}$  provides a more accurate residual astigmatism prediction than calculation using  $KA_{Sim}$ . Savini and Næser<sup>3</sup> reported a mean 0.59-D cylinder overcorrection for WTR eyes and a mean 0.32-D undercorrection for ATR eyes when  $KA_{Sim}$  was used to select the cylinder power of the toric IOLs. When Pentacam-derived  $TCA_{Ray}$  was used, the error in refractive astigmatism was not statistically significant from zero.<sup>3</sup> Zhang et al.<sup>4</sup> reported that the prediction error of the vector mean astigmatism after toric IOL implantation was slightly smaller when the Galilei-derived  $TCA_{Ray}$  was used than when the  $KA_{Sim}$  was used. However, the difference did not reach a significant level.<sup>4</sup> They reported that using keratometric

measurements tended to overcorrect WTR astigmatism and undercorrect ATR astigmatism, whereas these biases were not observed with TCA<sub>Ray</sub>.<sup>4</sup> Using the Galilei TCA<sub>Ray</sub> values to calculate the cylinder power of toric IOLs, Koch et al.<sup>6</sup> found a significant WTR prediction error of 0.57 D in the WTR group but no significant WTR/ATR prediction errors in the ATR group. They concluded that the Galilei TCA<sub>Ray</sub> may still underestimate the posterior corneal astigmatism in WTR eyes.<sup>6</sup> Only one study testing the accuracy of toric IOL calculation using TCA<sub>Vec</sub> has been published. Eom et al.<sup>5</sup> reported that toric IOL calculation using TCA<sub>Vec</sub> provided a more accurate residual astigmatism prediction (0.30 D) than calculation using KA<sub>Sim</sub> (0.50 D). However, the results of the study by Eom et al.<sup>5</sup> cannot be compared with the results by Savini and Næser<sup>3</sup>, Zhang et al.<sup>4</sup> and Koch et al.<sup>6</sup> because the former authors estimated posterior corneal astigmatism from anterior corneal astigmatism using linear regression analysis parameters of the reference subjects.

In summary, the current study indicates a discrepancy among simulate keratometric astigmatism, total corneal astigmatism calculated by vector summation, and total corneal astigmatism measured by ray tracing. Although we did not validate the outcomes of toric IOL implantation in a clinical setting, our findings may be helpful for a more accurate correction of astigmatism using toric IOL implantation. Our results suggest that the direct measurement of total corneal astigmatism by Scheimpflug imaging and ray tracing may be the best option with which to calculate the astigmatism to be surgically corrected and that using vector summation of anterior and posterior corneal astigmatism may cause a significant error (undercorrection) in the cylinder power of toric IOLs. A study is warranted to assess which of the investigated methods (vector summation versus ray tracing) allows the best refractive outcomes in patients undergoing toric IOL implantation during cataract surgery.

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## Conflicts of interest

There are no conflicts of interest.

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