ASN-ASAS SYMPOSIUM: FUTURE OF DATA ANALYTICS IN NUTRITION: Mathematical modeling in ruminant nutrition: approaches and paradigms, extant models, and thoughts for upcoming predictive analytics[1,2]

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ABSTRACT: This paper outlines typical terminology for modeling and highlights key historical and forthcoming aspects of mathematical modeling. Mathematical models (**MM**) are mental conceptualizations, enclosed in a virtual domain, whose purpose is to translate real-life situations into mathematical formulations to describe existing patterns or forecast future behaviors in real-life situations. The appropriateness of the virtual representation of real-life situations through MM depends on the modeler's ability to synthesize essential concepts and associate their interrelationships with measured data. The development of MM paralleled the evolution of digital computing. The scientific community has only slightly accepted and used MM, in part because scientists are trained in experimental research and not systems thinking. The scientific advancements in ruminant production have been tangible but incipient because we are still learning how to connect experimental research data and concepts through MM, a process that is still obscure to many scientists. Our inability to ask the right questions and to define the boundaries of our problem when developing models might have limited the breadth and depth of MM in agriculture. Artificial intelligence (**AI**) has been developed in tandem with the need to analyze big data using high-performance computing. However, the emergence of AI, a computational technology that is data-intensive and requires less systems thinking of how things are interrelated, may further reduce the interest in mechanistic, conceptual MM. Artificial intelligence might provide, however, a paradigm shift in MM, including nutrition modeling, by creating novel opportunities to understand the underlying mechanisms when integrating large amounts of quantifiable data. Associating AI with mechanistic models may eventually lead to the development of hybrid mechanistic machine-learning modeling. Modelers must learn how to integrate powerful data-driven tools and knowledge-driven approaches into functional models that are sustainable and resilient. The successful future of MM might rely on the development of redesigned models that can integrate existing technological advancements in data analytics to take advantage of accumulated scientific knowledge. However, the next evolution may require the creation of novel technologies for data gathering and analyses and the rethinking of innovative MM concepts rather than spending resources in collecting futile data or amending old technologies.

Key words: artificial intelligence, computer program, deep learning, machine learning, mathematical modeling and simulation, prediction

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INTRODUCTION

Mathematical models (**MM**) are mental conceptualizations, enclosed in a virtual domain, whose purpose is to translate real-life situations into mathematical formulations (symbolically or numerically) to describe existing patterns or forecast future behaviors in the real-life situations (Figure 1). The development of MM is a cyclical process that occurs iteratively and continuously. More recently, their application in research is referred to as in silico experimentation (Tedeschi and Fox, 2018). Although Ludwig von Bertalanffy introduced the systems theory concept in the 1940s (von Bertalanffy, 1969), the acceptance and use of systems-oriented research by the scientific community have been difficult to attain and of limited reach.

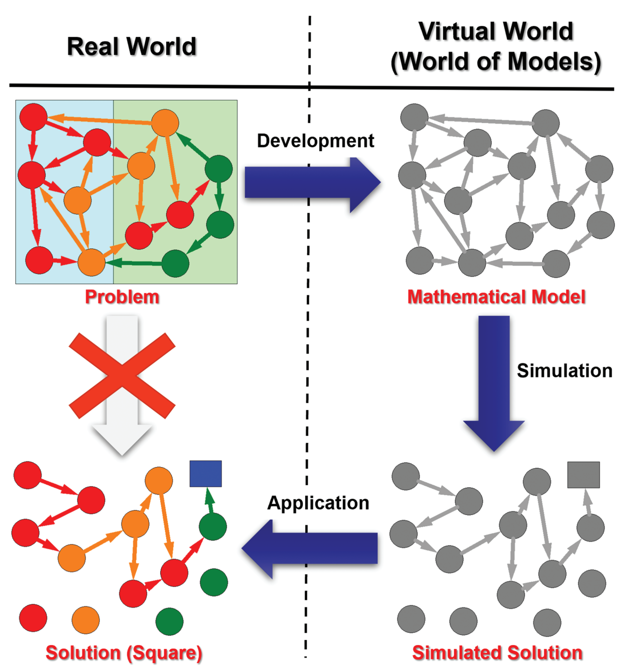

Figure 1. Illustration of the cooperation between the real world and virtual world (the world of models) to solve problems encountered in the real world. The large blue arrows (development, simulation, and application) represent the only possible route for solving the problem. The circles represent the variables of interest, the square represents the solution, the arrows between variables represent causal relationships, and the vertical dashed line represents the boundaries between real and virtual worlds. Colors represent different domains.

Scientists, in general, have been trained in experimental research and not systems thinking, and the concept of virtualization of reality has been confined to the design of controlled experimentation. The appropriateness of the virtual representation of real-life situations through mathematical modeling depends on the modeler's ability to synthesize essential concepts and associate their interrelationships with measured data. In this sense, MM often serve as decision-support systems (**DSS**), and even when a solution does not present itself in the virtual world, the model can ease the identification of possible solutions or expose the boundaries and gaps of the scientific knowledge, as shown in Figure 1. The user can obtain a feasible solution for the real-world problem by using other operational research tools such as optimization, or use the outputs of the model for meta-modeling purposes, or the creation of MM based on the outputs of other independent models. In general, the development of DSS has only been possible with the advancement of digital computing and data analysis, which enabled the first technological wave in mathematical modeling.

For about 50 yr, mathematical modeling has been used to develop DSS to assist with many aspects of livestock production in diverse environmental conditions. During the 1940s and 1950s, several important livestock-related experiments were planned and conducted by different, mostly university-associated and governmental organizations around the world. Together, their data and results formed the common base of our scientific knowledge. Experimental results were published in scientific papers (Leroy, 1954; Blaxter and Graham, 1955; Blaxter and Wainman, 1961), reports (National Research Council, 1944a, 1944b, 1945a, 1945b, 1945c, 1949), and books (Brody, 1945; Kleiber, 1961; Blaxter, 1962). The publication of these experiment results raised more questions, which prompted the formation and establishment of public, governmental research entities to investigate further the recent findings by the scientific community and to promote discoveries. The accumulation of data and knowledge compelled scientists to develop ways to combine and apply the new information being generated by these research entities with the old information of animal nutrition. For quite some time, the release of scientific publications (e.g., papers, reports, and extension bulletins) containing newly acquired information and recommendations in tabular form was enough. However, as the knowledge increased, its management and dissemination through static tabular forms were neither sufficient to contain the vast amount of information being accumulated nor quick enough to allow stakeholders to develop recommendations for production conditions outside those in which the data were generated. Computer models containing the knowledge in mathematical formulations (e.g., equations) were needed to solve the problem of the ever-growing body of data and knowledge being generated by the scientific community. Unfortunately, the development of computerized DSS did not become a reality until the mid-1960s, when the perception of the massive capability of such systems started to flourish for applications such as communications-driven, data-driven, document-driven, knowledge-driven, and model-driven DSS (Power, 2008). With the advancement of computing in the 1960s, mathematical modeling became feasible, and nutrition models have been developed since then (Tedeschi et al., 2014a).

The objectives of this paper are to illustrate the application of DSS in ruminant nutrition by characterizing different paradigms and approaches used in developing MM, briefly describe the evolution of different lines of thoughts in nutrition modeling, and exemplify the progression of an applied DSS in large- and small-ruminants nutrition, and to provide some initiatives to push forward the mathematical modeling field in animal science given recent advancements in predictive data analytics, a potential second technological wave in the evolution of mathematical modeling.

MATHEMATICAL MODELING APPROACHES AND PARADIGMS

Definitions

In this paper, *data-crunching* is the process involved in the management and preparation of large amounts of data and information (e.g., big data) for an analytical purpose; *data analytics* is the process of examining data sets to obtain relationships among variables and to draw conclusions from the information therein, and it is typically achieved with statistical tools; and *predictive analytics* is the process of making predictions and forecastings, typically achieved with modeling tools, about unknown future events. The following definitions and notations commonly used in system dynamics modeling (Forrester, 1961; Sterman, 2000) were adopted throughout this paper for clarification and standardization. Level, state, or stock variables accumulate values over time; they hold the contents from one time to another during simulation, serving as the memory of the system; and they can only be changed (increased or decreased) by rate or flow variables, which represents inflows or outflows, respectively, to and from the level (state or stock) variables. The rate (flow) variables have the same dimension as the level (state or stock) per unit of the time period. All other variables in the model are auxiliary and, from a reductionist perspective, they can be eliminated. They only help the modeler to visualize and build the model. Consequently, a MM can be collapsed to level and rate variables (and time in dynamic models). Endogenous variables are variables that affect and are affected by other variables in the model, whereas exogenous variables can affect but cannot be affected by variables in the model because they are outside of the model boundaries. The number of level (state or stock) variables in the model dictates its order. For instance, a MM with one independent level variable is deemed a first-order model, two independent level variables a second-order model, and so on. A MM is deemed linear when the rate (flow) variables are linear combinations of the level (state or stock) variables and any exogenous variables. The graphical representation of level vs. rate will always yield a straight line for linear models, whereas for nonlinear models it will yield curved lines. The graphical representation of levels over time, however, may depict a nonlinear behavior even for linear models.

Applications

Mathematical models, in general, have an important role in solving problems, especially in those conditions in which unforeseen variable relationships exist and stakeholders need to make decisions to improve production. Specific applications of MM include the improvement of animal performance, reduction of production cost, and reduction of excretion of nutrients by accounting for more of the variation in predicting requirements and feed utilization (Tedeschi et al., 2005). The public's lack of awareness and limited knowledge about MM are the main culprits of the negative perception of modeling and simulation, which has hindered their development and broader application (Tedeschi et al., 2015b). Mathematical models are not immune to failures, and unintended consequences arise when a model's limitations are misunderstood during the assessment of its appropriateness to solve a perceived problem. Despite their fallibility, MM are great tools for biological systems because they help us to identify areas in the scientific knowledge that have limited information and need additional research.

Approaches

Models can be categorized in many ways, depending on their scope and purpose (France and Thornley, 1984; Haefner, 1996; Meerschaert, 2007; Thornley and France, 2007). Such categorizations include *descriptive* vs. *prescriptive* (i.e., elucidative vs. predictive) when the modeling context is application; *static* (i.e., steady state) vs. *dynamic*, which can be further categorized as *discrete* vs. *continuous*, when the modeling context is time; *deterministic* vs. *stochastic* (i.e., probabilistic) when the modeling context is prediction (Guttorp, 1995); or *empirical* vs. *mechanistic* (i.e., theoretical or rational) when the modeling context is the nature of the model. The different approaches to developing an MM can be mixed (e.g., a deterministic, dynamic, mechanistic model). Within the predictive analytics context, Miller (2014) considered 3 general approaches: the *traditional* approach uses linear regressions to estimate parameters through fitting models to data (similar to the empirical category); the *data-adaptive* or *data-driven* approach searches through data to find useful predictors (similar to artificial intelligence—**AI**); and the *model-dependent* approach defines the model (similar to the mechanistic category) and uses it to generate data (e.g., meta-modeling), predictions, or recommendations. Others have proposed additional approaches to categorizing MM such as *teleonomic* vs. *teleologic* models and *functional* models (France and Kebreab, 2008; Tedeschi and Fox, 2018).

Categorizing the MM sets the stage for the tasks of model development, such as determining model boundaries, assumptions, and what type of data and data analytics are needed. However, unnecessary modeling complexity and nonessential categorization can easily overwhelm users or even knowledgeable modelers, entangling them in details, obscuring the bigger picture, and causing them to lose sight of the forest for the trees (Tedeschi and Fox, 2018). Figure 2 depicts critical components and steps of three major approaches for model development (empirical, mechanistic or knowledge-driven, and AI or data-driven).

Hybridization of these approaches is possible and may be employed more often in practice than has been recognized. The combination of models and methods usually works best in the predictive context (Miller, 2014). The empirical approach relies largely on the goodness of fit through statistical analyses and data selection, whereas the mechanistic approach (i.e., knowledge-driven) requires the conceptualization of hypotheses of what and how endogenous variables are interconnected (i.e., affect and are affected by other variables) and some data mining. The AI approach (i.e., data-driven) is at its core empirical, but recent development in this field (i.e., machine learning and deep learning) can be thought of as having some mechanistic elements. The AI approach relies almost exclusively on neural network analysis as the base for establishing the nodes (i.e., neurons) structure and layers. Figure 2 shows important steps in the model development:

1) *Data management* indicates the development of databases following pre-established criteria for data acceptance.
2) *Model conceptualization* indicates the logical arrangement of important variables towards a common purpose.
3) *Model coding* indicates the parameterization process of variables purely statistically or ideologically.
4) *Training and evaluation*, intrinsic processes in the AI approach, train the neural network formulation and establish the adequacy of its prediction. If the adequacy of the prediction is suboptimum, the algorithm seeks out additional resources to improve its predictability or alters the neural network formulation (layers) by itself.
5) *Model evaluation* indicates how well the MM precisely and accurately makes predictions given its purpose (Tedeschi, 2006).

Divergences

The separation between mechanistic vs. empirical is not always clear. At times, the difference has been contentious among researchers who have used it, improperly, to indicate the superiority of mechanistic over empirical models. For our purposes, the superiority of a model is related to its ability to satisfactorily perform (e.g., describe or predict) based on its intended purpose and development context (Tedeschi,

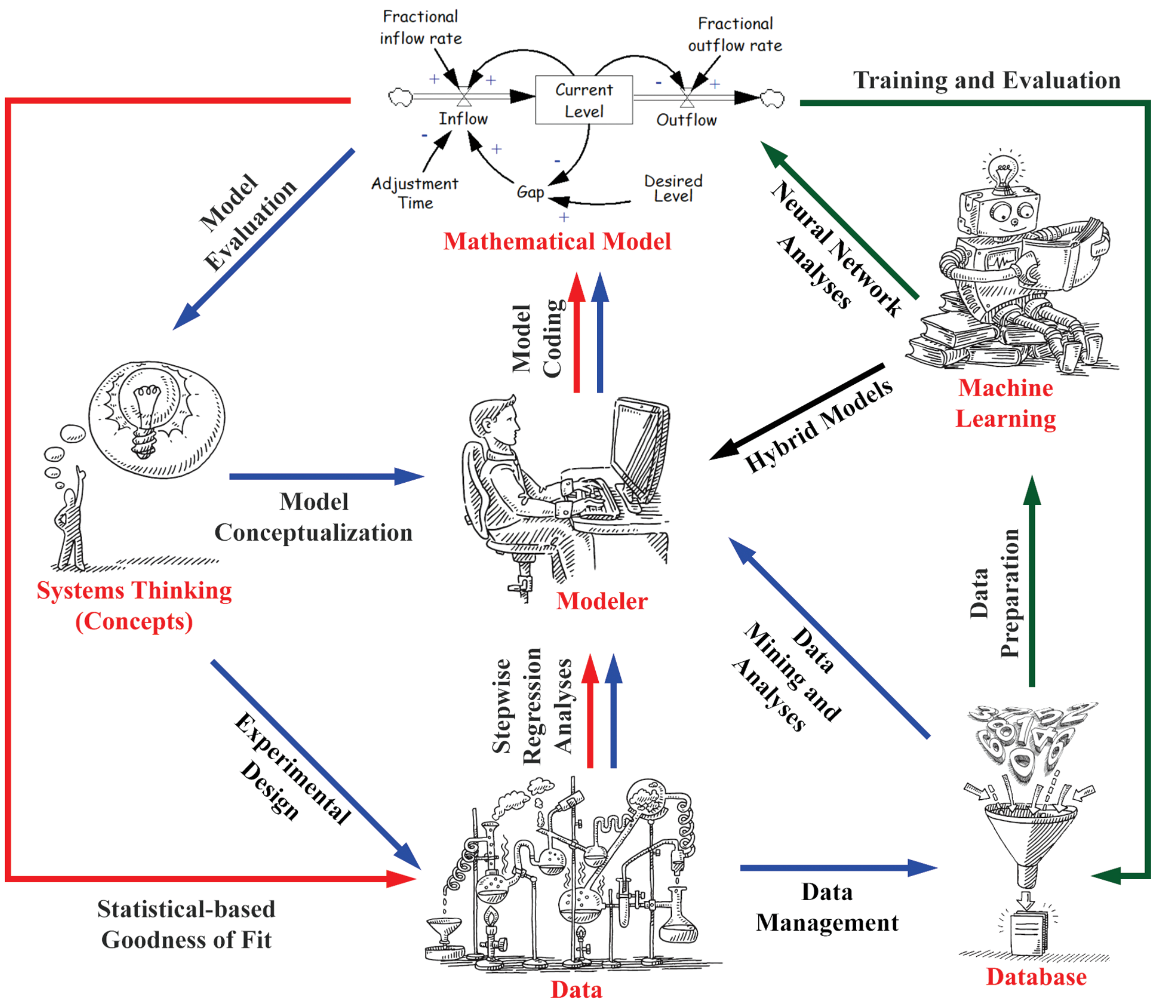

Figure 2. Illustration of pathways for the development of mathematical models using different approaches and paradigms: red is empirical, blue is mechanistic, and green is artificial intelligence.

2006). Similarly, *model validation* is not a valid statement in mathematical modeling because it has often been misused to prove the rightness and legitimacy of models and promote their acceptance and usability (Oreskes et al., 1994; Sterman, 2002). The misuse has even led to alternative terminology such as "evaludation" as an attempt to clarify the issue (Augusiak et al., 2014). The term *model evaluation* or *model testing* is preferred instead (Tedeschi, 2006).

A mechanistic model is usually represented as a model made of a nested (i.e., vertical) structure of entities (i.e., objects) that are localized at different strata (i.e., ranks). This nested structure implies that an object of a higher rank depends on the outcome of one or more objects from one or more lower or nested ranks. For instance, the response of cells (rank #1) to a given stimulus (i.e., change of status) will affect the response of an organ (rank #2) that is made up of these cells. In this case, cell organelles could be assigned to rank #0 and the animal body (a group of organs) to rank #3, and so forth. Mechanistic models can also be represented by a hierarchical representation of phenomena, but in a horizontal structure rather than a vertical one, in which the response of an object depends on the outcome of a previous object within the same rank. For instance, in ruminants, compartmental modeling (digesta passing through the rumen to the small intestine to the large intestine) states that what happens to the digesta in the large intestine, for instance, depends on what happened to it in the rumen before the large intestine can initiate its series of events (e.g., digestion and absorption). Within this context, MM that intrinsically rely on time are naturally categorized as mechanistic if each time step represents a change of status of level variables. Consequently, the nested/vertical structure relies on the necessary mechanisms employed or required by the parts to make the whole, whereas the hierarchical/horizontal structure conveys the sequential mechanisms that objects need to go through in order to reach an end: that is, the parts follow a supply chain process to yield the final product. Both types of models have intrinsic mechanisms that ordain the logic of the calculation. In contrast, the main premise in the relatively new discipline of systems biology modeling is that the sum of the parts is not necessarily equal to the whole. In other words, modeling the parts independently may not yield the outcome observed with the whole, which contrasts with the underlying principle of mechanistic modeling. In this case, a holistic viewpoint is necessary, and inverse problem modeling (**IPM**) is employed to develop the MM (Engl et al., 2009; Vargas-Villamil and Tedeschi, 2014; Guzzi et al., 2018).

Paradigms

The creation of MM can be accomplished with different paradigms. Some paradigms are more appropriate than others depending on the purpose and nature of the model, which is largely imposed by the degree of abstraction (global vs. individual). Models with global, or high, abstraction are less detailed-oriented and have a macro scale. Models with individual, or low, abstraction are more detailed-oriented (complex) and have a micro scale. Individual-abstraction models usually have a short time step and sometimes have multiple time scales, further complicating the computational process. The four commonly used types of paradigms are discrete-events modeling (**DEM**) (Fishman, 2001; Law, 2007), dynamic systems, agent-based (or individual-based) modeling (**ABM**) (Hellweger and Bucci, 2009; Crooks and Hailegiorgis, 2014), and system dynamics (or feedback-based systems) modeling (**SDM**) (Ford, 1999; Sterman, 2000; Morecroft, 2007). The DEM relies heavily on stochasticity to create time points (i.e., events) at which variables change their value or state rather than change continuously with time (Fishman, 2001). The ABM relies on self-governing, individual agents made of properties, behavioral rules, memory, and resources that allow each agent to independently make decisions upon the occurrence of an event (Macal and North, 2005), usually triggered by a probabilistic distribution and randomness generators. The SDM is concerned with the behavior of complex systems, and it relies on the theory of nonlinear dynamics and feedback processes in which the structure of the system (variable associations) gives rise to specific behavior over time (Tedeschi et al., 2011). Conceptually, SDM and IPM both determine the model's internal structure that is responsible for the behavior of the system. From a simplistic viewpoint, the goal of SDM and IPM is to build a model with the fewest number of variables that obey their causal relationships and that can accurately mirror the system's behavior. Early proponents and adopters of systems thinking have used SDM to develop DSS in agricultural sciences (Bawden, 1991; Yin and Struik, 2010; Tedeschi et al., 2013). The SDM is usually employed to solve high-abstraction problems and dynamic systems find their way with low-abstraction problems, but both are mainly for continuous-type problems. The DEM and ABM have a broader scope of abstraction but require discrete-type problems. Hybridization of paradigms for model development is also possible, and common examples include discrete-event dynamic modeling (Sandefur, 1991, 1993) and hybrid agent-based system dynamic modeling (Vincenot et al., 2011; Wallentin and Neuwirth, 2017; Kim et al., 2019).

EXTANT MATHEMATICAL MODELS IN RUMINANT PRODUCTION

Many MM for ruminants exist, and they differ significantly in numerous ways. Figure 3 depicts the chronological evolution of influential MM for nutrition (Tedeschi et al., 2014a; Tedeschi and Fox, 2018) and, more specifically, for producing grazing ruminants (Tedeschi et al., 2019) and their derivative works. Around the world, the most commonly used static and deterministic nutrition models are based on the National Research Council (NRC, 2000, 2001, 2007) in the United States, the Agricultural Research Council (ARC, 1965) and Agricultural and Food Research Council (AFRC, 1993) in the United Kingdom, the Institut National de la Recherche Agronomique (INRA, 1989) in France, the Commonwealth Scientific and Industrial Research Organization (CSIRO, 1990, 2007) in Australia, the Rostock Feed Evaluation System (Jentsch et al., 2003; Chudy, 2006) in Germany, the DVE/OEB [DarmVerteerbaar Eiwit (ileal digestible protein)/Onbestendig Eiwit Balans (rumen degradable protein balance)] system (Tamminga et al., 1994; Van Duinkerken et al., 2011) in the Netherlands, and the Nordic Feed Evaluation System [**NorFor**; Volden (2011)] in Scandinavia. Other nutrition models containing mechanistic or dynamic elements include the Cornell Net Carbohydrate and Protein System [**CNCPS**; Fox et al. (2004); Tylutki et al. (2008)], Ruminant (Herrero, 1997; Herrero et al., 2013), Molly (Baldwin, 1995), and Karoline (Danfær et al., 2006a, b). These nutrition models have been modified to account for specific production concerns of their eras by including novel or revised submodels, subsequently leading to many derivative models. For instance, the INRA (1989)

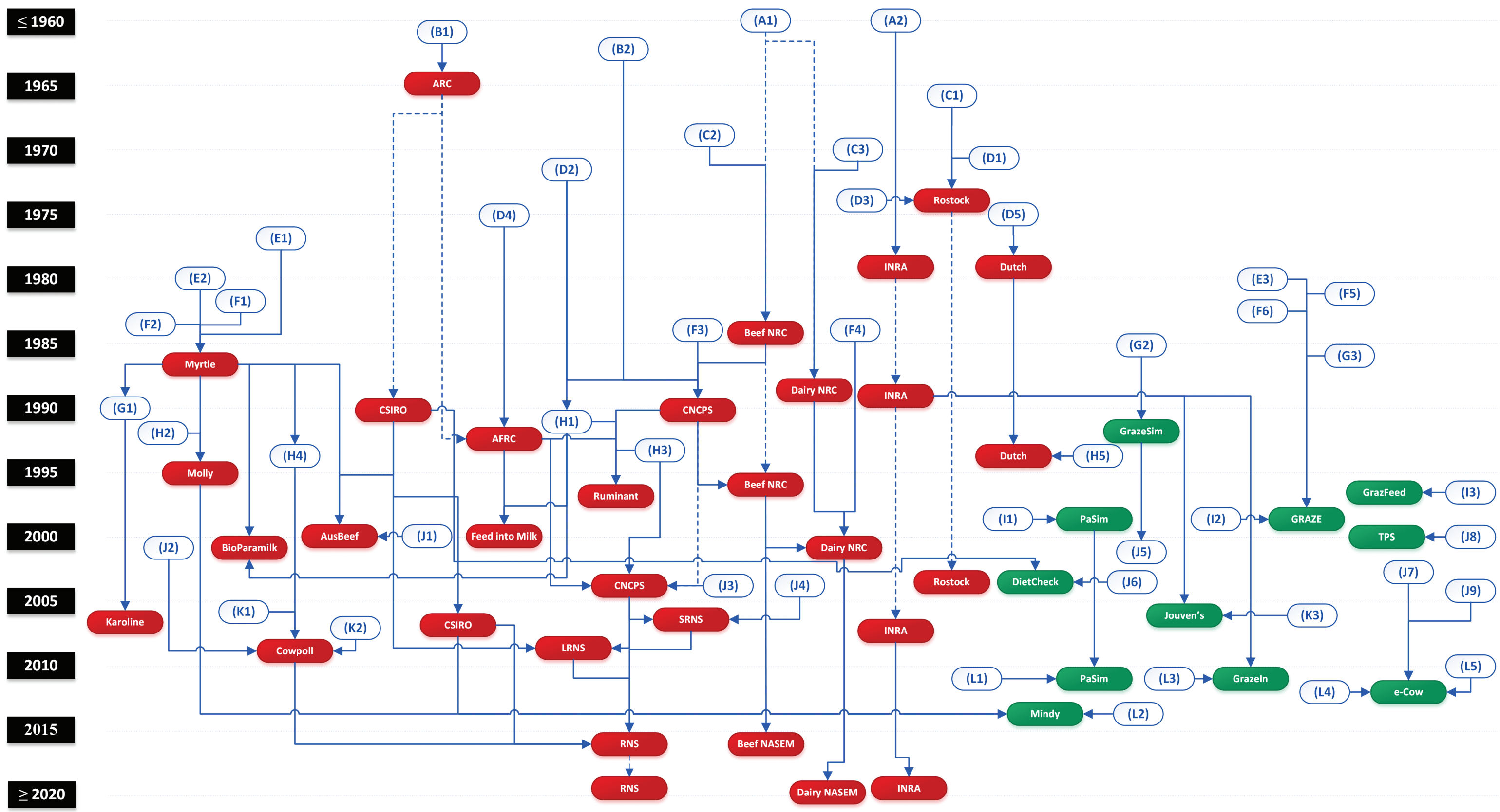

Figure 3. Chronological evolution of key mathematical models whose primary goal lies within ruminant nutrition only (red squircles) or pasture/grazing ruminants (green squircles) domains. Approximate year of publication or release is shown on the left. The solid line represents a direct relationship of influence, and the dashed line represents that at least one other version or edition was released in between the marks. The lack of lines connecting the same model does not imply the model has been phased out. AFRC = Agricultural and Food Research Council; ARC = Agricultural Research Council; CNCPS = Cornell Net Carbohydrate and Protein System; LRNS = Large Ruminant Nutrition System; CSIRO = Commonwealth Scientific and Industrial Research Organization; INRA = Institut National de la Recherche Agronomique; NASEM = National Academies of Sciences, Engineering, and Medicine; NRC = National Research Council; RNS = Ruminant Nutrition System; SRNS = Small Ruminant Nutrition System; TPS = Tropical Pasture Simulator. Key references (empty blue squircles) are (A1) NRC (1945a, b), (A2) Leroy (1954), (B1) (Blaxter, 1962), (B2) Van Soest (1963a) and Van Soest (1963b), (C1) Nehring et al. (1966), (C2) Lofgreen and Garrett (1968), (C3) Moe et al. (1970), (D1) Schiemann et al. (1971), (D2) Waldo et al. (1972), (D3) Hoffmann et al. (1974), (D4) Ministry of Agriculture, Fisheries and Food (1975), (D5) Van Es (1975), (E1) Baldwin et al. (1977), (E2) Baldwin et al. (1980), (E3) Loewer et al. (1980), (F1) France et al. (1982), (F2) Gill et al. (1984), (F3) Fox and Black (1984), (F4) Conrad et al. (1984), (F5) Loewer et al. (1981), (F6) Loewer et al. (1983), (G1) Danfær (1990), (G2) Mertens (1985; 1987), (G3) Bridges et al. (1986), (H1) Illius and Gordon (1991), (H2) France et al. (1992), (H3) Russell et al. (1992), Sniffen et al. (1992), and Fox et al. (1992), (H4) Dijkstra et al. (1992), Neal et al. (1992), and Dijkstra (1993), (H5) Tamminga et al. (1994), (I1) Riedo et al. (1998) based on the Hurley Pasture Model (Thornley, 1998), (I2) Loewer (1998), (I3) Freer et al. (1997), (J1) Nagorcka et al. (2000), (J2) Mills et al. (2001), (J3) Fox et al. (2004), (J4) Cannas et al. (2004) and Tedeschi et al. (2010), (J5) Vazquez and Smith (2001), (J6) Heard et al. (2004), (J7) Baudracco et al. (2010), (J8) Herrero et al. (2000b; 2000a), (J9) Vetharaniam et al. (2003), (K1) Bannink et al. (2006), (K2) Bannink et al. (2008), (K3) Jouven et al. (2006a, b), (L1) Graux et al. (2011), (L2) Gregorini et al. (2013a), (L3) Delagarde et al. (2011a; 2011b) and Faverdin et al. (2011), (L4) Baudracco et al. (2012), and (L5) Friggens et al. (2004). Adapted from Tedeschi and Fox (2018).

went through significant overhauls in 2007 (INRA, 2007) and 2018 (INRA, 2018) with the intent of revisiting the calculations of available dietary energy and protein by including digestive dynamics (ruminal degradation and passage rates) and microbial growth (Sauvant et al., 2014; Sauvant and Nozière, 2016). The Ruminant Nutrition System [**RNS**; Tedeschi and Fox (2018)], a CNCPS-based model, incorporated many additional submodels and revised equations as discussed below. Dumas et al. (2008) portrayed a historical perspective of how early ruminant nutrition knowledge led scientists to dwell on MM in the search for unanswered questions. Some review papers have compared and highlighted the modern state of agricultural system models (Jones et al., 2017). Others have contrasted the different ways nutrition models represent important elements in predicting the requirements and dietary supplies of energy and nutrients to improve ruminant production while providing a more contemporary perspective of mathematical modeling in the field of ruminant nutrition (Sørensen, 1998; Tedeschi et al., 2005; Tedeschi et al., 2014a; Tedeschi et al., 2015a) as well as some prerequisites to advance the utility of animal systems modeling (McNamara et al., 2016a).

Mathematical Nutrition Models

Ruminant production DSS became fully embodied and more evident after the 1960s (Figure 3), though many mathematical modeling efforts took place prior to 1925 (Dumas et al., 2008). In the United States, the first, and ultimately unsuccessful, request to study nutrient requirements of food animals, especially protein, was issued in 1910 by Henry P. Armsby (Christensen, 1932). The National Research Council (**NRC**) underwrote a second request in 1917. The resulting *Cooperative Experiments upon the Protein Requirements for Growth of Cattle* had several participating experimental stations across the country from 1918 to 1923 (Christensen, 1932) and culminated with the publications of two reports summarizing the experimental results (NRC, 1921, 1924). Several reports were released by the then-called National Academy of Sciences–National Research Council, including the first attempt to establish nutrient requirements of beef cattle (NRC, 1945a) and dairy cattle (NRC, 1945b). In 1974, a report on the *Research Needs in Animal Nutrition* was released (NRC, 1974) with the intent to address important issues for ruminant nutrition at that time, such as non-protein nitrogen utilization, better understanding of rumen fermentation, nutrient requirements of "exotic" breeds, and factors affecting feed intake and utilization, among many others. As discussed above, computer modeling was not even brought up during these early deliberations because experimental data were still being collected and digital computing was in its infancy with few practical applications (Power, 2008).

Today, precision feeding is possibly the most relevant application of nutrition models for the livestock industry. The primary reason is mid-1990s federal and state regulations that required feeding programs to be more protective of water and air quality by minimizing excess of nutrients in the environment. Consequently, precision feeding (a technical misnomer—from a modeling perspective it should be called *accurate feeding*) encompasses accurate diet balancing and formulation in unique production situations to deliver appropriate energy and nutrients that allow animals to express their genetic production potential. In the process of applying precision feeding, the minimization of excess nutrients (those that will not be absorbed and utilized by the animal) helps us to decrease nutrient excretion to the environment, especially nitrogen (Cerosaletti et al., 2004) and phosphorus (Vasconcelos et al., 2007).

In the United States, two major schools of thought have dominated the modeling efforts in ruminant nutrition. The first school was based on a more biochemical, process-based, fundamental-type model initiated in the late 1970s, including submodels for rumen function (Baldwin et al., 1977) and postabsorptive metabolism (Baldwin and Black, 1979). After a series of integration with existing United Kingdom models in the early 1980s, the first model of lactating dairy cows was developed in 1984 (France, 2013) and published in 1987 (Baldwin et al., 1987a; Baldwin et al., 1987b; Baldwin et al., 1987c). Molly, a dynamic, mechanistic model based on biochemical reactions in animal metabolism, became available in the 1990s (Baldwin, 1995). Molly's research and modeling efforts inspired new developments and improvements in many places around the world (Nagorcka et al., 2000; Hanigan, 2005; Gregorini et al., 2013b; McNamara and Shields, 2013; Gregorini et al., 2015; McNamara et al., 2016b). Concomitantly, the modeling efforts of the second school, a more functional-oriented, applied-type modeling approach that is based on the NRC recommendations, started in the late 1970s at Cornell University (Chalupa and Boston, 2003; Sniffen, 2006). Many papers have been published on the specific components of this second school's CNCPS model (Tedeschi and Fox, 2018).

National Research Council. As indicated above, the NRC's feed evaluation and nutrient requirements of ruminants started in the mid-1940s with the publications of the *Recommended Nutrient Allowances for Beef Cattle* (NRC, 1945a) and *Recommended Nutrient Allowances for Dairy Cattle* (NRC, 1945b). As scientific knowledge was acquired, the information contained in subsequent publications grew exponentially as did citations and number of pages to them (Figure 4). Multiple factors may have facilitated the growth in the size of the NRC publications. The rate of knowledge acquisition and the interest in the enhancement of these publications were so intense that the first 6 revisions happened quickly (on average, less than 7 yr apart) compared with more recent publication rates.

The first revision of the beef and dairy NRC publications was issued in 1950 (NRC, 1950a; 1950b). The second revisions of the dairy (NRC, 1956) and beef (NRC, 1958) publications were retitled to *Nutrient Requirements* instead of *Recommended Nutrient Allowances*. At that time, establishing protein requirements for cattle was critical for increasing production. They were expressed as concentrations in the diet because most recommendations were based on summaries of experiments using feeding trials in which performance and digestibilities were routinely measured as the concentration of protein in the diet was gradually increased. The third revisions occurred in 1963 for beef (NRC, 1963) and in 1966 for dairy (NRC, 1966). Subsequent revisions for nutrient requirements of beef and dairy cattle had significant modifications. In the 1960s, metabolism trials started to take place, and the research results led to the development of net energy systems for cattle, which were published in the fourth revisions of the beef NRC (1970) and dairy NRC (1971). In the 1970s, rumen microorganisms received increased scrutiny, and by the 1980s, the factorial method was used to compute protein requirements. For beef cattle, the fifth revision was released in 1976 (NRC, 1976). The sixth revision, released in 1984 (NRC, 1984), contained major changes in the energy requirements section and included the concepts of ruminal protein degradation and bypass.

For dairy cattle, the fifth *Nutrient Requirements* revision was issued in 1978 (NRC, 1978), with major modifications to the calculation of protein requirements based on the work of Swanson (1977), including unavailable feed protein and feed protein solubility. The sixth revision, released in 1989 (NRC, 1989), included the concept of ruminally undegraded protein and microbial crude protein as the main sources of metabolizable protein.

The seventh revisions of both the beef and dairy NRC publications saw a drastic increase in

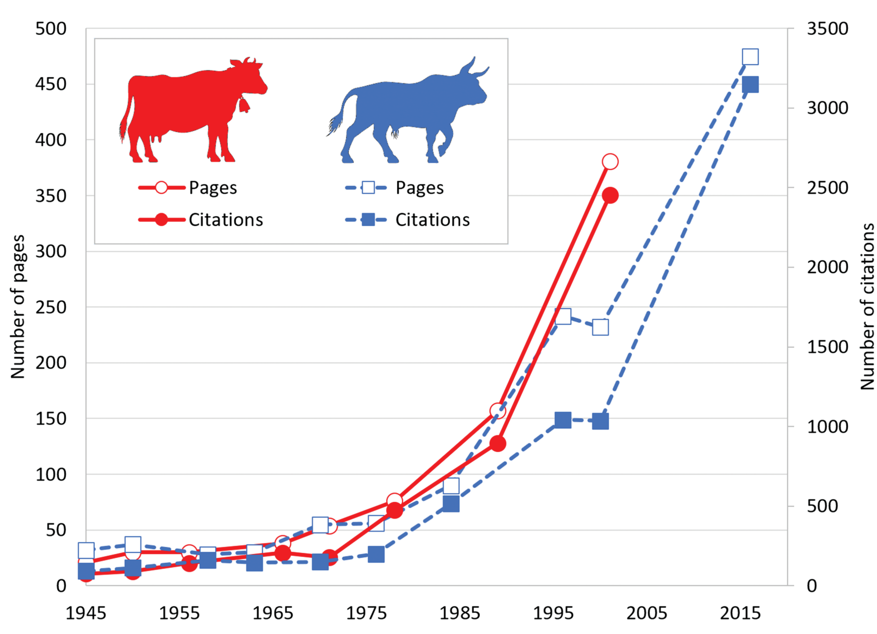

Figure 4. Indicators of knowledge progression of the National Research Council's *Nutrient Requirements for Beef and Dairy Cattle* throughout the years.

the numbers of pages and citations to the publications (Figure 4). Major modifications were proposed, motivated by the extensive data collection and analyses of accumulated experimental research enabled by more accessible digital computing. Along with the development of net energy systems for beef (NRC, 1970, 1976, 1984) and dairy (NRC, 1971, 1989) cattle and the mathematical description of the rumen fermentation (NRC, 1985, 1989), equations needed to initiate the prediction of requirements for each primary physiological function (maintenance, growth, pregnancy, lactation, rumen fermentation, intestinal digestion and absorption, and metabolism) allowed the development of more complex and mechanistic nutritional models. These models were released with the seventh revisions of the beef (NRC, 1996, 2000) and dairy (NRC, 2001) cattle publications and again with the eighth revision for beef cattle (NASEM, 2016) after the inclusion of additional advancements. The latest NRC publications include the concept of degradation kinetics for feed protein, to compute readily available, potentially available, and unavailable protein fractions. Because of the removal of so-called safety factors when formulating and balancing rations and the more accurate estimates of energy and nutrient requirements for diverse production conditions, these computations have informed DSS and reduced the cost per unit of production while reducing the excretion of excess nutrients, including N, P, and greenhouse gasses, to meet U.S. government regulations.

Cornell Net Carbohydrate and Protein System. The concepts of the CNCPS were initially published in 1992 (Fox et al., 1992; Russell et al., 1992; Sniffen et al., 1992; O'Connor et al., 1993), but the engine and calculation logic of the model were developed in the 1980s (Fox et al., 1990). At that time, a large portion of the requirement submodels of the CNCPS was based on the NRC publications. In 1996 this scenario was reversed, and the NRC (1996, 2000) adopted many concepts from the CNCPS modeling effort (Tedeschi and Fox, 2018) that have extended until the seventh revision for dairy (NRC, 2001) and the eighth revision for beef (NASEM, 2016) cattle. For the supply side, the CNCPS model was heavily based on Peter J. Van Soest's ideas about the fractionation of carbohydrate (Van Soest, 1967) and protein (Van Soest et al., 1981), which themselves rest on many concepts of the classification of carbohydrate and protein for ruminants dating back to the 1950s with the work of Lauri and Irja Paloheimo (Paloheimo and Paloheimo, 1949).

The CNCPS possesses the characteristics of a deterministic, static, and empirical model, with some mechanistic features, whose main objective is to function as an applied DSS. The modeling core of the CNCPS limits its usability as a fully mechanistic, dynamic model, though some continuous simulations can be achieved pending the adaptation of some elements (Reynoso-Campos et al., 2004; Tedeschi et al., 2004). CNCPS-based models utilize detailed fractionation of dietary carbohydrate and protein (Sniffen et al., 1992) and horizontal mechanistic elements (i.e., supply chain process) to compute total digestible nutrients. The mechanistic elements include ruminal fermentation of nutrients and production of volatile fatty acids and ruminal pH (Pitt et al., 1996), two pools of ruminal bacteria (Russell et al., 1992), and intestinal digestibility for undegraded feed. The animal requirements are essentially based on those recommended by the NRC (1996, 2000) and NASEM (2016) publications for beef cattle and the NRC (2001) publication for dairy cattle.

Tedeschi and Fox (2018) meticulously reviewed significant modifications and additional submodels implemented during the development of the RNS compared with the original 1990s CNCPS supply model (Fox et al., 2004; Tylutki et al., 2008), including 1) the adoption of urea-N used for anabolism rather than recycled ruminal N (Eisemann and Tedeschi, 2016), 2) a more mechanistic ruminal fiber degradation submodel based on GnG1 models (Vieira et al., 2008a, 2008b), 3) a revised microbial growth submodel to account for deficiency of ruminal N and branched-chain amino acids, 4) a revised volatile fatty acids and ruminal pH submodel, 5) a revised methane yield calculation, 6) a lipids and long-chain fatty acids submodel (Moate et al., 2004), 7) revised submodels of ruminal passage rates (Seo et al., 2006; Seo et al., 2007; Seo et al., 2009), 8) a revised fecal submodel with corrections proposed by Cannas et al. (2004), and 9) a slightly modified calculation logic for metabolizable energy from digestible energy and total digestible nutrients. Despite the enormous efforts in data collection, development and improvement of methodology, and meticulous use of cutting-edge statistical analyses, inconsistencies have been identified and recommendations have been proposed (Alderman et al., 2001a; Alderman et al., 2001b; Alderman et al., 2001c). Recently, others (Galyean and Tedeschi, 2014; Galyean et al., 2016; Tedeschi et al., 2017; Tedeschi, 2019) have brought to light additional flaws and limitations in the NRC- and CNCPS-based models. These include restrictions and problems associated with the fixed and long-standing 82% efficiency index of conversion of digestible energy to metabolizable energy, the conversion of metabolizable energy to net energies for maintenance and growth, the empirical prediction of ruminal bacteria growth, the contribution of microbial protein to metabolizable protein, the quantification of urea-N recycled in the rumen and truly used by the ruminal microbes for anabolism, the efficiency of use of metabolizable protein by the ruminant animal, the energy requirement for maintenance for grazing animals, the inconsistencies in predicting protein retained by growing cattle, and the energy required for animals under cold-stress conditions, among many others. Some of these inconsistencies were inherited because of limitations (often by design) in the methods employed to measure the required data (Tedeschi, 2019). Solutions to these limitations may require procedural changes to the methods and considerable quantities of new data.

Tedeschi et al. (2014a) summarized the evolution of six empirical and five mechanistic nutrition models, describing their key characteristics and highlighting their similarities and differences. These authors also performed a comparative prediction of milk production of dairy cows among four nutrition models. They developed a database of milk production from 37 published studies from six regions of the world, totaling 173 data points: 19 for Africa, 45 for Asia, 16 for Europe, 12 for Latin America, 44 for North America, and 37 for Oceania. Tedeschi et al. (2014a) indicated that these four nutrition models could not easily be compared, despite their similar assumptions and calculations, because the conceptual and structural foundations inherent to their intended purposes were too different. They concluded that not all nutrition models were suitable for predicting milk production of dairy cows and that simpler systems might be more resilient to variations in studies and production conditions around the world. Later, on another assessment of model predictability, Tedeschi et al. (2015a) reached a similar conclusion that the prediction of metabolizable protein required for lactation was uniform among nutrition models, but the metabolizable protein required for growth varied largely.

Integrated Mathematical Models

Whole-farm decision support systems (**WFDSS**) use a multiobjective modeling approach in which independent DSS are systematically and harmoniously integrated into a highly aggregated platform to simulate specific operations within the boundary of a farm, ranch, or basin. As shown in Figure 3, several WFDSS have been developed for ruminant production, including the Agricultural Production Systems Simulator (**APSIM**) (Moore et al., 2007), Australian Dairy Grazing Systems (**DairyMod**) (Johnson et al., 2008), DairyNZ Whole Farm Model, Discrete Event Simulation Environment (**DIESE**) (Martin-Clouaire and Clouaire, 2009), EcoMod (Johnson et al., 2008), Farm Assessment Tool (**FASSET**) (Berntsen et al., 2003), GRAZE (Loewer, 1998), GRAZPLAN (Donnelly et al., 1997; Moore et al., 1997), Great Plains Framework for Agricultural Resource Management (**GPFARM**) (Andales et al., 2003), Hurley Pasture Model (**HPM**) (Thornley, 1998), Integrated Farm System Model (**IFSM**) (Rotz et al., 1999; Rotz et al., 2005), LINCFARM, Pasture Simulation (**PaSim**) (Graux et al., 2011), PROGRASS, Sustainable Grazing Systems (**SGS**) (Johnson et al., 2003), and Whole Farm Model (**WFM**).

The literature of WFDSS aimed at modeling grazing ruminant animals is vast and slowly expanding. The interest in integrating scientific knowledge of animals, plants, and soil to understand the behavior of animal agricultural systems and to better manage and control them has led the scientific community to develop individual models and integrate them for a common goal: maximize productivity (per area or per animal) while minimizing the use of resources as an attempt to increase efficiency and profitability. In the United States, such DSS were promoted starting in the mid-1970s following the many NRC publications on nutrient requirements of cattle (Loewer, 1998). However, the modeling limitations of complex systems (e.g., WFDSS) such as overparameterization, inadequate parameter estimation, and simulation instability led to well-known chaotic behavior (Woodward, 1998). Furthermore, many of these individual models did not "speak the same language": they had different objectives and purposes, and their modeling approaches and paradigms were distinct enough that integrating them required their total re-engineering and re-programming. These inherent discrepancies have created inconsistencies and delays in the development of WFDSS for ruminant production, but the field has been moderately active in the last decade. Not until recently have some of these WFDSS been evaluated under different production scenarios. Bryant and Snow

(2008) reviewed nine pastoral simulation models (APSIM, EcoMod, FASSET, GRAZPLAN, GPFARM, HPM, IFSM, LINCFARM, and WFM) and concluded that there was a need to include the effect of pests and diseases on pasture production as well as improved animal performance predictions, including a more mechanistic model for voluntary feed intake and ruminal fermentation processes. Snow et al. (2014) summarized six of these models (APSIM, AgMod, DIESE, FASSET, GRAZPLAN, and IFSM) and compared their different approaches to model forage mixtures in the paddocks, animal–forage interactions, N transfers by the animal in the paddocks, management of the whole farm, and future prospects. They also provided ideas and solutions for the inherent limitations of these six models.

Environmental Aspects. Recently, the emission of greenhouse gases (**GHG**) from ruminant production operations (i.e., methane and nitrous oxide) became an important issue within the scientific community because of its perceived contribution to the global warming phenomenon (Tedeschi and Fox, 2018, Ch. 3). Currently, the net abatement potential of GHG from ruminant production systems can be obtained only through WFDSS and life-cycle assessments (Eckard et al., 2010). These results have led to the issuing of recommendations for effective reduction in the emission of GHG (Crosson et al., 2011). Del Prado et al. (2013) indicated that WFDSS are the appropriate scale for mitigating GHG emissions because the farm represents the unit at which management decisions are made. They analyzed different approaches to modeling GHG. Most of the reviews of WFDSS suitability for GHG assessment have discussed the strengths and drawbacks of WFDSS, but they lack model intercomparisons under different production systems. Tedeschi et al. (2014a) indicated that accurate prediction of milk production by dairy cows by mathematical nutrition models is a critical prerequisite to further development of systems that can effectively and correctly estimate the contribution of large ruminants to GHG emissions and their true share of the global warming event. The inaccuracies in predicting GHG become even more complicated and uncertain when the whole farm system is considered. Given the complex nature of WFDSS, Tedeschi et al. (2014b) recommended that simple nutrition models should be used with WFDSS to predict GHG emissions for the time being.

Sustainable Production. The ability to forecast social and economic aspects that prevent the broader use of WFDSS in decisions involving sustainability is limited. More integrated approaches are needed to combine MM from different fields within animal production to develop substantial programs of sustainable intensification (Garnett and Godfray, 2012; Tedeschi et al., 2015b). Liu et al. (2015) suggested that a "holistic approach to integrating various components of coupled human and natural systems across all dimensions is necessary to address complex interconnections and identify effective solutions to sustainability challenges." The development of integrated systems and cross-scale interactions of dynamic systems may facilitate social–ecological resilience, with a focus on our complex adaptive transformability, learning capacity, and ability to innovate (Folke, 2006). The SDM paradigm can combine accumulated scientific data with knowledge and strategic management to improve the animal industry by better assessing market opportunities with biological limitations and potentials of the agroindustry (Tedeschi et al., 2011) while accounting for the three pillars of sustainability: environmental, social, and economic aspects (Makkar, 2013; Makkar and Ankers, 2014; Tedeschi et al., 2015b).

Disease Outbreak. Another important, and more recent, application of integrated and dynamic DSS is in the control and management of disease outbreak. The development of mathematical epidemiological models simulating animal infectious diseases and providing solutions to minimize their life-threatening menace to animals and humans has advanced considerably in the United States (Harvey et al., 2007) and Europe (Lantier, 2014) in the last decade. Epidemiological DSS help us to understand the dynamics of spreading infectious diseases, such as foot-and-mouth disease, in susceptible populations (Webb et al., 2017). Lofgren et al. (2014) used real-time modeling and simulation tools to identify the spread of the 2014 outbreak of Ebola virus in West Africa and provide timely guidance for policymakers. Perry et al. (2013) believe that though the use of powerful MM of the distribution and dynamics of livestock disease have been increased in the last decade, incomplete understanding of the models' underlying assumptions may result in dangerous decisions that might create a false confidence of our understanding of the model predictions. Furthermore, many of these epidemiological DSS seek to aid understanding of the spreading dynamics of infectious diseases, not necessarily their prevention. The latter could be addressed by accounting for animal nutritional deficiencies as well as animal management malpractices if nutrition were incorporated in the DSS for epidemiological modeling.

Opportunities

Although integrated systems are required to develop more inclusive WFDSS to assist with sustainability, there are several limitations in modeling the dynamics of metabolism (McNamara, 2004), including lack of detailed and accurate data likely because of limitations in experimental focus and design (McNamara et al., 2016a). For instance, accurate nutrition and growth models could assist in the management of feedlot animals if the models accurately predicted body composition brought about by fat and protein deposition, two of the most influential variables in predicting animal requirements for growth. However, different genotypes have different rates of fat and protein deposition, and few MM accounts for them. Since the early 1980s, there have been considerable efforts in the understanding of growth of ruminants and the development of DSS to predict it (Loewer et al., 1980; Loewer et al., 1983; Bridges et al., 1986; Oltjen et al., 1986; Di Marco and Baldwin, 1989; Keele et al., 1992; Williams and Bennett, 1995; Kilpatrick and Steen, 1999; Oltjen et al., 2000; Hoch and Agabriel, 2004; Tedeschi et al., 2004). Because many factors inherent to the genetic makeup of the animal affect its composition of gain, the incorporation of nutrition with a genetic predisposition may likely advance the modeling and simulation of growth biology. Tedeschi (2015) provided a preliminary modeling approach to combine a nutrition and growth model with molecular breeding values obtained from commercial, single-nucleotide polymorphism panels. The author indicated that the molecular breeding values for the ribeye area were an important piece of genetic information for increasing the precision in predicting mature weight at a given body composition.

The future of mathematical modeling intrigues many researchers. Understanding it guides the investment of resources, including the time devoted to new learning experiences, towards the development of new techniques and the exploration of scientific frontiers. As depicted in Figure 3, the rise in the development of MM for ruminants occurred in 1985, and, as expected, a 10-yr delay was observed for pasture-related modeling. A collapse in the release of new MM for ruminant nutrition became evident after 2010. It is hard to distinguish when the period of great model development and idea-sharing within the modeling community ended and the period of development decline and reshuffling of ideas within the community, plagued by a lack of innovation in nutrition modeling, started.

The field of animal nutrition modeling seems to have been stagnant for quite some time. On the one hand, this apparent stagnation may indicate that the field has reached a certain level of maturity that adequately meets the expectations of producers and stakeholders, taking away any pressure for further development. On the other hand, this apparent stagnation might be the reflection of many deficiencies acting alone or in combination that are suppressing interest by the scientific community and limiting resources to further develop the field. Continuous and effective communication and knowledge-sharing with non-scientists stakeholders is vital to raising their awareness and appreciation for complex modeling. Historically, however, this communication, including clear instructions on the acquisition of inputs needed to operate complex modeling in practice (Newman et al., 2000), has not been properly executed for many reasons (Cartwright et al., 2016).

There are indications that computer-based modeling and simulation are, in general, important in the learning and teaching of sciences, as well as proposals to include modeling in **STEM** (science, technology, engineering, and mathematics) curricula (Feurzeig and Roberts, 1999). Systems thinking has been commended as a required discipline for the development of systems-oriented MM (Senge, 1990; Sherwood, 2002). Systems thinking has to do with how we perceive the connection among entities (i.e., objects and variables) within a defined boundary; in essence, it is how we see the forest for the trees. However, under specific circumstances, the shortage or decline of innovative modeling in agriculture and life sciences may be partially explained by academia's failure to properly introduce students to MM (or systems thinking for that matter) and the overloading of faculty, which decreases their time for critical thinking about the subject.

Another deficiency leading to this apparent stagnation is the lack of novel ideas and concepts to further challenge the status quo. Reduced funding at the state and federal levels may have also contributed to the ever-declining rate of scientific production in agriculture (Rouquette et al., 2009; Black, 2018). The lack of learning experiences, slow transfer of knowledge, and the shortage of resources may not be exclusive to agriculture, but they are certainly restraining its development.

On the bright side, novel developments may be on the horizon with the advancement of innovative technologies in data analytics, such as deep learning. We may be entering an era of growth like the one in the 1950s, when the development and application of digital computing gave the needed boost to mathematical modeling in agriculture. The integration of mathematical modeling and AI is likely to spur an avant-garde technological wave in predictive analytics, yielding hybrid knowledge- and data-driven models.

HYBRID KNOWLEDGE- AND DATA-DRIVEN MATHEMATICAL MODELING

The artificial neural network (**ANN**) technique has been around for some decades. It comprises many single, connected processors, called nodes, that are assembled to computationally mimic the perceived function of human brain neurons. Thousands of ANN neurons are interconnected among themselves and embedded in multiple layers of similar or different shapes (i.e., different neuron connection layouts). The ANN neurons of the first layer usually receive the inputs (e.g., values of independent variables), one input per neuron. When activated, each neuron sends a signal to another neuron in the next layer. This process happens subsequently throughout all layers until the ANN produces an overall output (e.g., a dependent variable).

The basic building block of an ANN is the adaptive linear element that consists of cascaded neurons (i.e., layers) that produce binary outputs (±1) depending on the pattern of inputs (Widrow and Lehr, 1990). Many different forms and architectures of the basic ANN technique exist, including supervised and unsupervised learning, back-propagation, deep learning, and reinforcement learning, among many others (LeCun et al., 2015; Schmidhuber, 2015). These variants have been developed since the 1960s to improve the reliability and stability of imagery and sound recognition, patterns of quantifiable data over time, and prediction of output given different combinatorial variables, among many other uses. The mathematics behind these ANN variants are sophisticated, complex, and expanding as novel techniques are developed by combining operational research tools (e.g., dynamic programming and Markov chain) to assist in the credit assignment for problems of different characteristics (Widrow and Lehr, 1990).

Artificial intelligence comprises a group of extremely powerful data analytics, including machine learning (**ML**) and deep learning (**DL**), that have benefited from the quick progress of ANN since the 1950s. A typical computer program uses inputs (i.e., raw data and independent variables) and hard code (i.e., logic and calculation rules) to produce outputs (i.e., dependent variables). In contrast, ML and DL use inputs and outputs to generate a set of rules (mostly statistical and optimization methods) that can sufficiently and accurately represent the data for detection and classification (LeCun et al., 2015; Chollet and Allaire, 2018).

Despite current applications of AI to solve problems in many different fields, including agriculture, and the tremendous technological advancement and refinements of AI, its role and utility in mathematical modeling are still unknown. Although some studies comparing ML and AI were improving the recognition of objects or increasing the predictability of models, other studies were identifying the limitations and shortcomings of this technology (NASEM, 2018). For instance, DL is a data-thirsty process that requires large data sets for training and evaluation processes (Figure 2) and, ideally, large variability within the data sets to cover as many combinatorial possibilities among variables as practicable (Kamilaris and Prenafeta-Boldú, 2018). Although the bootstrapping technique can partially alleviate the data shortage problem (Breiman, 1996), it may exclude natural variations and correlations among variables. The bootstrapping technique should be carefully used as it cannot substitute measured data. The second, and perhaps most serious, the drawback with the adoption of AI and its variants is the lack of transparency in the reasoning behind each prediction. Once an ANN layout is developed, almost nothing is known about the underlying mechanisms that produce the overall output (Knight, 2017). Indeed, DL methods are commonly called representation-learning methods with low to high degrees of abstraction as the number of layers increases (LeCun et al., 2015).

Unlike ML, DL has been shown to help solve multidimensional problems with intricate structures in several fields of science, including pharmaceutical, medical, physical, and psychological challenges (LeCun et al., 2015). The DL is a compelling data-crunching technique, but it may not be a genuine modeling approach because it is a black box whose workings we do not know or understand. DL alone incompletely fulfills the hierarchical learning steps of Ackoff's (1989) data–information–knowledge–wisdom (**DIKW**; Figure 5) pyramid that humans have been taught for centuries because it cannot provide insightful knowledge that leads to wisdom. The wisdom in the

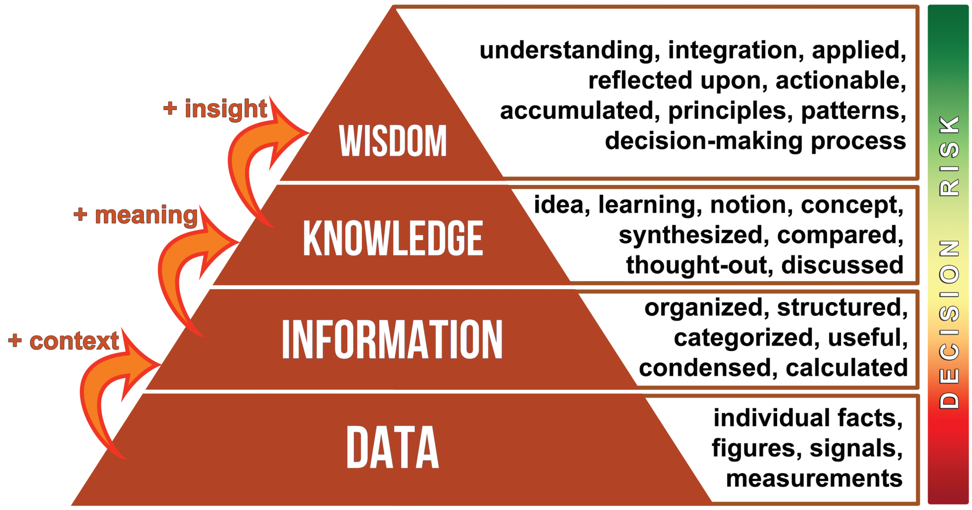

Figure 5. The data–information–knowledge–wisdom pyramid based on Ackoff (1989). Data have no value until they are processed into a useable form given a context. Information contains data that underwent some kind of organization and systematic analyses. Knowledge represents information that has been gained and put into use, generally by a human. Wisdom is the possession of knowledge used to make intelligent connections between different agents and patterns needed to understand the principles and underlying mechanisms that govern the behavior of the data. In the decision risk color scale, red indicates high risk and green indicates the low risk associated with decision-making processes.

DIKW hierarchy (Figure 5) adds value to knowledge through methodical judgments, an important characteristic that differentiates humans from machines (Ackoff, 1989). The question then becomes, can we move forward with DL and mechanistic mathematical modeling and, if so, how?

Despite being incipient, the applications of ML and DL in agriculture are already a reality (Kamilaris and Prenafeta-Boldú, 2018; Liakos et al., 2018). However, their integration with MM, more specifically mechanistic modeling, is embryonic. In cattle production, few studies in animal welfare (Dutta et al., 2015), genome-wide predictions (González-Recio et al., 2014) and breed classification (Santoni et al., 2015), genomics' expected progeny difference (Okut et al., 2013), anatomical biometrics for animal identification/recognition (Kumar et al., 2018), animal growth (Alonso et al., 2013; Alonso et al., 2015), and rumen functioning (Craninx et al., 2008; Dong and Zhao, 2014) have used AI technologies alone or in combination with other statistical methods. Craninx et al. (2008), for instance, compared the adequacy of ML to multilinear regression techniques for predicting ruminal volatile fatty acids production, measured by milk fatty acid composition, using data from 10 studies (n = 138 observations) of rumen cannulated dairy cows. They reported that no significant differences between the techniques based on the mean square error of prediction statistic. Kumar et al. (2018) used DL and muzzle biometrics (imagery) for registration, unique identification, and verification of cattle. This is an interesting application of DL ability to process images. The use of DL with animals' physical biometrics, to improve our ability to identify desired body characteristics and project growth patterns and carcass composition, has an enormous potential to identify optimum slaughter time of live cattle (Tedeschi, 2017).

The integration of knowledge- and data-driven modeling technologies, yielding hybrid artificial MM, seems plausible in the near future, after the fever of adopting new technology passes. Some fields have already partially addressed the possibility of incorporating ML with other modeling techniques. For instance, though it is not entirely clear how IPM can benefit from AI techniques, Vemuri (2003) might have shed some light on how ML can assist with broader usage of IPM. The supervised learning architecture is most commonly used in DL. However, unsupervised learning and reinforcement learning might be the way to combine DL and mechanistic MM because most human learning about the world's complexity is done in an unsupervised way, i.e., there is no pre-established relationship among variables, we learn them from inside-out. LeCun et al. (2015) indicated that AI is progressing by combining representation–learning methods (e.g., DL) with complex reasoning, perhaps including mechanistic modeling.

The data analytics field can be daunting to those with inadequate understanding. When combined with modeling approaches, data analytics may even frighten some potential users away from predictive analytics. Although there have been localized efforts (Xu and Rhee, 2014), our society must stimulate adequate training in AI technologies: their possibilities, drawbacks, and opportunities. There is no good in teaching how to properly collect data when principles in data analytics, and modeling for that matter, are absent.

CONCLUSION

Our inability to pose the right questions about the problem that needs to be solved and define its boundaries when developing models, as well as our intrinsic ambition to develop models to simulate systems rather than problems, might have limited the breadth and depth of mathematical modeling in agriculture and perhaps other fields of science. The emergence of data-intense computational technologies that require less systems-thinking about how things are interrelated may have helped disperse the interest in mechanistic, conceptual mathematical modeling. It also may have shifted the interest of, and attracted adopters to, statistics-oriented, data-intense, less-mechanistic modeling approaches such as AI. AI has its niche, but it cannot entirely replace mechanistic learning and systems-thinking approaches. Data-driven and knowledge-driven approaches must be merged into functional DSS that are sustainable and resilient by transferring fundamental knowledge while providing effective forecasting experiences. The premature adoption of AI or its derivations, likely sparked by the excitement of using cutting-edge technology, at the expense of knowledge-driven approaches may be obfuscating unintended consequences, such as the lack of learning and teaching practices, poor transfer of knowledge for training of future leaders and researchers, and the shortage of resources for experimental research. The future success of mathematical modeling relies on the development of redesigned models that can integrate existing technological advancements in data analytics to take advantage of accumulated scientific knowledge. However, reaching the next technological level requires the investment of resources in creating novel technologies for data gathering and analyses, confronting established assumptions, and rethinking and pioneering concepts rather than amending limited technologies or continuing to collect futile data (Tedeschi et al., 2017; Black, 2018).

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