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Paramagnetic resonance in spin-polarized disordered Bose-Einstein condensates

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We study the pseudo-spin density response of a disordered two-dimensional spin-polarized Bose gas to weak alternating magnetic field, assuming that one of the spin states of the doublet is macroscopically occupied and Bose-condensed while the occupation of the other state remains much smaller. We calculate spatial and temporal dispersions of spin susceptibility of the gas taking into account spin-flip processes due to the transverse-longitudinal splitting, considering microcavity exciton polaritons as a testbed. Further, we use the Bogoliubov theory of weakly-interacting gases and show that the time-dependent magnetic field power absorption exhibits double resonance structure corresponding to two particle spin states (contrast to paramagnetic resonance in regular spin-polarized electron gas). We analyze the widths of these resonances caused by scattering on the disorder and show that, in contrast with the ballistic regime, in the presence of impurities, the polariton scattering on them is twofold: scattering on the impurity potential directly and scattering on the spatially fluctuating condensate density caused by the disorder. As a result, the width of the resonance associated with the Bose-condensed spin state can be surprisingly narrow in comparison with the width of the resonance associated with the non-condensed state.

Conventional paramagnetic resonance also referred to as the electron spin resonance, is a phenomenon known from the physics of electrons in metals¹. After its discovery, this phenomenon was, in particular, used in the proposal of a quantum cyclotron², it was employed to improve the measurements of the electronic magnetic moment and the fine structure constant³, and it has been utilized in the calculations of the magnetic transition dipole moments⁴.

In this article, we propose a new type of the paramagnetic resonance applied to bosonic systems. It is crucial that the bosons should possess spin degree of freedom and they can be represented by, for instance, a cold atomic gas⁵ under the applied magnetic field. Such systems have attracted substantial interest recently⁶. Another alternative is exciton polaritons (EPs) in a semiconductor microcavity. We will consider the latter system and show that the paramagnetic resonance in bosonic gases possesses new features over against two-dimensional (2D) electronic systems.

Due to their hybrid half-light–half-matter nature, EPs demonstrate a number of peculiar properties, standing aside from other quasiparticles in solid-state. In particular, their small effective mass (10^{-4} – 10^{-5} of free electron mass) inherited from the photons together with strong particle–particle interaction taken from the excitons make EP systems suitable for observation of quantum collective phenomena at astonishingly high temperatures^{7,8}. Other significant effects have been reported, such as EP superfluidity⁹, the Josephson effect¹⁰, formation of vortices¹¹. Some of the theoretically predicted phenomena such as polariton self-trapping¹², polariton-mediated superconductivity¹³ are to be measured.

Beside fundamental importance, the strong coupling regime can be used in various optoelectronic applications¹⁴. A polariton laser should be mentioned here^{15–18} as a manifestation of BEC-based alternative light source. Coherently pumped microcavities also give us polariton neurons¹⁹ and polariton integrated circuits²⁰. Further,

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semiconductor microcavities under incoherent background pumping (for instance, electric current injection) can be used in optical routers^{21,22}, detectors of terahertz radiation^{23,24}, high-speed optical switches^{25,26} and more.

One of the most significant quantum properties governing the dynamics of EPs, is their spin degree of freedom (also referred to as polarization)²⁷. It opens a way to spin-optonics²⁸. On one hand, as opposed to classical optics, where nonlinear Kerr interaction is usually weak, spin-optonics is in a more favourable position thanks to advantageous relatively strong particle-particle interaction. On the other hand, as opposed to spintronics, using EPs can reduce the dramatic impact of the carrier spin relaxation and decoherence^{29–32}. Polariton spin dynamics has been extensively studied in literature^{33–36}, although many issues remain undiscovered.

Pseudospin susceptibility

Dynamics of EPs in a microcavity can be described by the spinor wave function, having two components related to two polariton spin states, $\hat{\psi}(\mathbf{r}, t) = (\psi_+(\mathbf{r}, t), \psi_-(\mathbf{r}, t))^T$. Our goal is to study the response of the polariton spin density, $S^l(\mathbf{r}, t) = \hat{\psi}^\dagger(\mathbf{r}, t)\sigma^l\hat{\psi}(\mathbf{r}, t)$, where σ^l are the Pauli matrices ($l = x, y, z$), to external space and time fluctuating magnetic field, $\mathbf{B}(\mathbf{r}, t) = (0, 0, B(\mathbf{r}, t))$, where $B(\mathbf{r}, t) = B_0 \cos(\mathbf{k}\mathbf{r} - \omega t)$. Let us assume that the magnitude of this field is low enough thus a linear response theory can be applied. In its framework, the spin susceptibility is defined as³⁷

$$S^i(\mathbf{r}, t) = \iint d\mathbf{r}' dt' \chi_{ij}(\mathbf{r}, \mathbf{r}'; t, t') B_j(\mathbf{r}', t'). \quad (1)$$

Utilising the EP interacting Hamiltonian in a special form³⁸,

$$\hat{H}_{\text{int}} = \frac{1}{2} U_0 (|\psi_+|^4 + |\psi_-|^4) + U_2 |\psi_+|^2 |\psi_-|^2,$$

where $U_2 = U_0 - 2U_1$, U_0 and U_1 are polariton-polariton interacting constants, we can write the Gross-Pitaevskii equation (GPE) for each of the spin components of the EP doublet:

$$i\dot{\psi}_\pm = (\hat{E}_p - \mu + u(\mathbf{r}) + U_0 |\psi_\pm|^2 + U_2 |\psi_\mp|^2 \pm \mathcal{F})\psi_\pm + \alpha p_\mp^2 \psi_\mp, \quad (2)$$

where $\hat{E}_p = \hat{\mathbf{p}}^2/2M$ is the operator of kinetic energy of EPs with mass M (we assume parabolic dispersion at not very high p for simplicity), μ is the chemical potential. The non-diagonal terms $\alpha p_\pm^2 = \alpha(p_x \pm ip_y)^2$ account for the TE-TM splitting of polariton states, mixing the '+' and '-' spinor components. An external magnetic perturbation is given here via the term $\mathcal{F}(\mathbf{r}, t) = \frac{1}{2} g_s \mu_B B(\mathbf{r}, t)$. Here g_s is an effective polariton g -factor, μ_B is the Bohr magneton, and we also assume that the perturbation is real for simplicity, $B^*(\mathbf{r}, t) = B(\mathbf{r}, t)$. Randomly fluctuating impurity potential is assumed to have zero mean value, $\langle u(\mathbf{r}) \rangle = 0$, and the following statistical properties:

$$\langle u(\mathbf{r})u(\mathbf{r}') \rangle = u_0^2 \delta_{\mathbf{r},\mathbf{r}'}, \quad \langle u(\mathbf{p})u(\mathbf{p}') \rangle = u_0^2 \delta_{\mathbf{p},-\mathbf{p}'}, \quad (3)$$

where $\langle \dots \rangle$ means the averaging over the impurities positions.

Usually, EP lifetime is restricted to 5–20 ps. However here we assume that the bosonic system is a closed quantum system, thus neglecting the particle losses and assuming relatively long lifetime of EPs^{39,40}. In the steady state (quasi-equilibrium) and in the absence of TE-TM splitting, the ground state of the EP condensate is sensitive to the sign of the interacting parameter, U_1 ^{27,38}. If $U_1 > 0$, the ground state is a composition of equally populated spin-up and spin-down components of EP spinor. If, instead, $U_1 < 0$, the ground state is characterized by nearly zero population of one of the circular component of the EP spinor and macroscopic population of the other one³⁸. We will consider this case ($U_1 < 0$). Under the action of external perturbation, $\mathcal{F}(\mathbf{r}, t)$, the TE-TM terms cause transitions of EPs from the condensed component (let it be ψ_+) to the other one (ψ_-), which was empty initially. We assume that the occupation of the condensed component ever remains much larger, $|\psi_+|^2 \gg |\psi_-|^2$. Then we can disregard the non-linear terms proportional to $U_0 |\psi_-|^2$ and $(U_0 - 2U_1) |\psi_-|^2$ in Eq. (2). After these agreements, the evolution equations read:

$$\begin{aligned} (i\partial_t - \hat{E}_p + \mu - U_0 |\psi_+|^2 - u(\mathbf{r}) - \mathcal{F})\psi_+ &= \alpha p_-^2 \psi_-, \\ (i\partial_t - \hat{E}_p + \mu - U_2 |\psi_+|^2 - u(\mathbf{r}) + \mathcal{F})\psi_- &= \alpha p_+^2 \psi_+. \end{aligned} \quad (4)$$

Considering here \mathcal{F} as a perturbation, we write:

$$\begin{pmatrix} \psi_+(\mathbf{r}, t) \\ \psi_-(\mathbf{r}, t) \end{pmatrix} \rightarrow \begin{pmatrix} \psi_0(\mathbf{r}) + \delta\psi_+(\mathbf{r}, t) \\ \delta\psi_-(\mathbf{r}, t) \end{pmatrix}, \quad (5)$$

where we have extracted the condensate fraction, $\psi_0(\mathbf{r})$, of ψ_+ state and denoted small corrections, $\delta\psi_\pm$, assuming $\delta\psi_\pm \sim \delta\psi_- \sim \mathcal{F}$. Substituting (5) into (4) and keeping only zero and first-order terms with respect to \mathcal{F} , we find that zero-order terms describe the ground state of EP condensate in the impurity potential (see Supplementary):

$$\left[\hat{E}_p - \mu + U_0 |\psi_0(\mathbf{r})|^2 + u(\mathbf{r}) \right] \psi_0(\mathbf{r}) = 0, \quad (6)$$

while the first-order terms contain information about EP dynamics due to external perturbations,

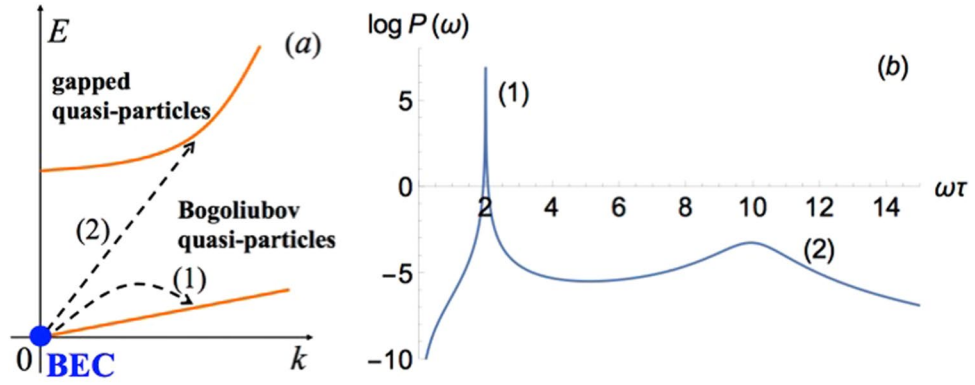


Figure 1. (a) Schematic of the quasi-particle spectrum of the system with two types of transitions: (1) and (2). Blue solid dot is the condensate of '+' polarized EPs. (b) Power absorption spectrum. The peaks (1) and (2) result from the transitions (1) and (2) from (a).

$$\begin{aligned} \hat{G}^{-1} \begin{pmatrix} \delta\psi_+ \\ \delta\psi_+^* \end{pmatrix} - \hat{K} \begin{pmatrix} \delta\psi_- \\ \delta\psi_-^* \end{pmatrix} &= \psi_0(\mathbf{r}) \mathcal{F}(\mathbf{r}, t) \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \\ \hat{\mathcal{G}}^{-1} \begin{pmatrix} \delta\psi_- \\ \delta\psi_-^* \end{pmatrix} - \hat{K}^* \begin{pmatrix} \delta\psi_+ \\ \delta\psi_+^* \end{pmatrix} &= 0, \hat{K} = \begin{pmatrix} \alpha p_-^2 & 0 \\ 0 & \alpha p_+^2 \end{pmatrix}, \end{aligned} \tag{7}$$

where the Green's functions, \hat{G} and $\hat{\mathcal{G}}$, are explicitly presented in Supplementary. The formal solution of the system (7) reads:

$$\begin{aligned} \begin{pmatrix} \delta\psi_+(\mathbf{r}, t) \\ \delta\psi_+^*(\mathbf{r}, t) \end{pmatrix} &= \iint d\mathbf{r}' dt' \hat{G}^R(\mathbf{r}, \mathbf{r}'; t - t') \left[\psi_0(\mathbf{r}') \mathcal{F}(\mathbf{r}', t') \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \hat{K} \begin{pmatrix} \delta\psi_-(\mathbf{r}') \\ \delta\psi_-^*(\mathbf{r}') \end{pmatrix} \right], \\ \begin{pmatrix} \delta\psi_-(\mathbf{r}, t) \\ \delta\psi_-^*(\mathbf{r}, t) \end{pmatrix} &= \iint d\mathbf{r}' dt' \hat{\mathcal{G}}^R(\mathbf{r}, \mathbf{r}'; t - t') \hat{K}^* \begin{pmatrix} \delta\psi_+(\mathbf{r}') \\ \delta\psi_+^*(\mathbf{r}') \end{pmatrix}, \end{aligned} \tag{8}$$

and now the components of the spin density can be expressed as:

$$\begin{aligned} S^x(\mathbf{r}, t) &\approx \langle \psi_0(\mathbf{r}) [\delta\psi_-(\mathbf{r}, t) + \delta\psi_-^*(\mathbf{r}, t)] \rangle, \\ S^y(\mathbf{r}, t) &\approx -i \langle \psi_0(\mathbf{r}) [\delta\psi_-(\mathbf{r}, t) - \delta\psi_-^*(\mathbf{r}, t)] \rangle, \\ S^z(\mathbf{r}, t) - \langle \psi_0^2(\mathbf{r}) \rangle &\approx \langle \psi_0(\mathbf{r}) [\delta\psi_+(\mathbf{r}, t) + \delta\psi_+^*(\mathbf{r}, t)] \rangle. \end{aligned} \tag{9}$$

Let us consider different regimes.

Ballistic regime

In an ideally pure sample where polariton-impurity scattering can be neglected, $\psi_0(r)$ is uniform in space, $\psi_0(\mathbf{r}) \equiv \psi_0 = \sqrt{n_c}$.

Then from Eq. (6) we get $\mu = U_0 n_c$, and

$$\hat{\mathcal{G}}^R(\varepsilon, \mathbf{p}) = \frac{\begin{pmatrix} \varepsilon + \mathcal{E}_p & 0 \\ 0 & -\varepsilon + \mathcal{E}_p \end{pmatrix}}{(\varepsilon + i\delta)^2 - \mathcal{E}_p^2}, \tag{10}$$

$$\hat{G}^R(\varepsilon, \mathbf{p}) = \frac{\begin{pmatrix} \varepsilon + E_p + U_0 n_c & -U_0 n_c \\ -U_0 n_c & -\varepsilon + E_p + U_0 n_c \end{pmatrix}}{(\varepsilon + i\delta)^2 - \varepsilon_p^2},$$

where $\varepsilon_p = \sqrt{E_p(E_p + 2U_0 n_c)} = sp\sqrt{1 + p^2 \xi^2}$ is a Bogoliubov quasiparticle spectrum, $\xi = 1/2Ms$ is a healing length, $s^2 = U_0 n_c/M$ is the excitations velocity and $\mathcal{E}_p = 2|U_{1c}| + E_p$ is a gapped dispersion branch of low-populated EP circular component³⁸, see Fig. 1a. Then the exact solutions of Eq. (7) read

$$\begin{pmatrix} \delta\psi_+(\mathbf{k}, \omega) \\ \delta\psi_+^*(\mathbf{k}, \omega) \end{pmatrix} = \sqrt{n_c} \hat{L}^{-1}(\mathbf{k}, \omega) \mathcal{F}(\mathbf{k}, \omega) \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad (11)$$

$$\begin{pmatrix} \delta\psi_-(\mathbf{k}, \omega) \\ \delta\psi_-^*(\mathbf{k}, \omega) \end{pmatrix} = \hat{\mathcal{G}}^R(\mathbf{k}, \omega) \hat{K}^* \begin{pmatrix} \delta\psi_+(\mathbf{k}, \omega) \\ \delta\psi_+^*(\mathbf{k}, \omega) \end{pmatrix},$$

with $\hat{L}^{-1} = (\hat{G}^{-1} - \alpha^2 p^4 \hat{\mathcal{G}})^{-1}$. Calculating this inverse matrix, we keep all the α -containing terms in the numerator and disregard their contribution to the denominator in determinant which appears in the matrix calculation, assuming that the TE-TM splitting is small and does not affect the dispersions, ε_k and \mathcal{E}_k . Then in the lowest order in α we obtain the transverse,

$$\chi_{xz}(\mathbf{k}, \omega) = \frac{\alpha n_c g_s A_+ + A_-}{2\mu_b^{-1} D_\varepsilon D_\varepsilon}, \quad (12)$$

$$\chi_{yz}(\mathbf{k}, \omega) = \frac{\alpha n_c g_s A_+ - A_-}{i2\mu_b^{-1} D_\varepsilon D_\varepsilon}, \quad (13)$$

where $A_+ = k_+^2(\omega + E_k)(\omega + \mathcal{E}_k)$, $A_- = k_-^2(\omega - E_k)(\omega - \mathcal{E}_k)$, $D_\varepsilon = (\omega + i\delta)^2 - \varepsilon_k^2$, $D_\varepsilon = (\omega + i\delta)^2 - \varepsilon_k^2$, and longitudinal,

$$\chi_{zz}(\mathbf{k}, \omega) = \frac{g_s \mu_b n_c E_k}{D_\varepsilon} \left[1 + \frac{(2M\alpha)^2 E_k \mathcal{E}_k}{D_\varepsilon} \right], \quad (14)$$

pseudo-spin susceptibilities. They experience resonance in the vicinity of the frequency of the collective (Bogoliubov) mode of the condensate, $\omega \approx \varepsilon_k$. Moreover, TE-TM splitting results in transitions of particles between the spin-polarized components of the EP doublet which results in emergence of an additional resonance at $\omega \approx \mathcal{E}_k$. It should be mentioned that both the transverse (12), (13) and longitudinal (14) susceptibilities diverge at frequencies corresponding to the exact resonance, $\omega = \varepsilon_k$ or $\omega = \mathcal{E}_k$ due to infinitely small scattering rates of '+' and '-' EPs.

Finite polariton-impurities scattering

Accounting for the scattering mechanisms results in the line broadening and finite values of susceptibilities (12)-(14) at resonances. The most significant contributions to EP non-radiative lifetime at low temperatures are given by the polariton-polariton⁴¹ and polariton-disorder scattering. We will analyze here the latter case. A naive approach, commonly used in literature, is to assume that the $i\delta$ terms in (12), (13) and (14) have finite value, associated with some phenomenological particle scattering time, $\delta \rightarrow 1/\tau$, where τ is independent of the momentum and energy. However, what will happen with the scattering time when EPs condense?

In the presence of a disorder caused by impurities, the ground state of the system is to be determined from Eq. (6). To solve this equation and find $\psi_0(\mathbf{r})$, we follow the approach suggested in ref. 42 (for 3D excitonic systems). In its framework, the impurity field, $u(\mathbf{r})$, produces a static fluctuation of the condensate density, $\psi_0(\mathbf{r})$, assumed to be weak enough thus it cannot destroy the condensate, $\psi_0(\mathbf{r}) = \sqrt{n_c} + \phi(\mathbf{r})$, where $|\phi(\mathbf{r})| \ll \sqrt{n_c}$. Further, linearization of Eq. (6) with respect to $\phi(\mathbf{r})$ gives:

$$\left[\hat{E}_p - \delta\mu + 2U_0 n_c + u(\mathbf{r}) \right] \frac{\phi(\mathbf{r})}{\sqrt{n_c}} = - (u(\mathbf{r}) - \delta\mu), \quad (15)$$

where $\delta\mu = \mu - U_0 n_c$ is a correction to the chemical potential. The formal solution of this equation reads:

$$\phi(\mathbf{r}) = \sqrt{n_c} \int d\mathbf{r}' g(\mathbf{r}, \mathbf{r}') (u(\mathbf{r}') - \delta\mu), \quad (16)$$

where

$$\left[-\hat{E}_p + \delta\mu - 2U_0 n_c - u(\mathbf{r}) \right] g(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}') \quad (17)$$

and $\delta\mu$ is determined by the condition $\langle \phi(\mathbf{r}) \rangle = 0$. In the lowest order of the perturbation theory, we use the Green's function, $g(\mathbf{r}, \mathbf{r}')$, taken at $u(\mathbf{r}) = 0$ and find the fluctuating part of the ground state wave function:

$$\phi(\mathbf{p}) = \sqrt{n_c} g(\mathbf{p}) u(\mathbf{p}), \quad g(\mathbf{p}) = - \frac{1}{2U_0 n_c} \frac{1}{1 + p^2 \xi^2} \quad (18)$$

and $\delta\mu = 0$. Now one can find the disorder-averaged Green's functions and EP-impurity scattering times. To do this, one needs to linearize the Green's functions (see Eq. (9) in Supplementary) with respect to $\phi(\mathbf{r})$ to get the matrix equations: $\hat{G}^R = \hat{G}_0^R + \hat{G}_0^R \hat{X} \hat{G}^R$ and $\hat{\mathcal{G}}^R = \hat{\mathcal{G}}_0^R + \hat{\mathcal{G}}_0^R \hat{\lambda} \hat{\mathcal{G}}^R$, where the bare (without disorder) functions, $\hat{G}_0^R, \hat{\mathcal{G}}_0^R$, are given by Eq. (10) and we denote

$$\hat{X}(\mathbf{r}) = u(\mathbf{r}) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + 2U_0\sqrt{n_c}\phi(\mathbf{r}) \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \quad (19)$$

$$\hat{\mathcal{X}}(\mathbf{r}) = [u(\mathbf{r}) + 2\sqrt{n_c}(U_0 - 2U_1)\phi(\mathbf{r})] \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (20)$$

These potentials describe the EP scattering on impurity field (terms $\sim u(\mathbf{r})$) and on the static fluctuations of the condensate density (terms $\sim \phi(\mathbf{r})$). Now we apply the standard Feynman diagram technique and find that in the lowest order of the Born approximation, the impurity self-energies take the standard form: $\hat{W}(\mathbf{r} - \mathbf{r}') = \langle \hat{X}(\mathbf{r})\hat{G}_0^R(\mathbf{r} - \mathbf{r}')\hat{X}(\mathbf{r}') \rangle$ and $\hat{\mathcal{W}}(\mathbf{r} - \mathbf{r}') = \langle \hat{\mathcal{X}}(\mathbf{r})\hat{\mathcal{G}}_0^R(\mathbf{r} - \mathbf{r}')\hat{\mathcal{X}}(\mathbf{r}') \rangle$. The Green's functions averaged over the disorder can be found from the matrix Dyson equations⁴³, $\langle \hat{G}^{-1} \rangle = \hat{G}_0^{-1} - \hat{W}$ and $\langle \hat{\mathcal{G}}^{-1} \rangle = \hat{\mathcal{G}}_0^{-1} - \hat{\mathcal{W}}$. At this point, the general consideration with the spectrum of the Bogliubov quasiparticles, $\varepsilon_k = sk\sqrt{1 + k^2\xi^2}$, and arbitrary k becomes a tricky issue. However, we can restrict our consideration to the most important analytical case of quasi-linear Bogliubov dispersion, $\varepsilon_k \approx sk$, under the condition $k\xi \ll 1$. Taking into account Eqs (19) and (20), we find:

$$\hat{\mathcal{W}}(\varepsilon) = u_0^2 \left(\frac{2U_1}{U_0} \right)^2 \int \frac{d\mathbf{p}}{(2\pi)^2} \hat{\mathcal{G}}_0^R(\mathbf{p}, \varepsilon), \quad (21)$$

$$\hat{W}(\varepsilon) = u_0^2 \int \frac{d\mathbf{p}}{(2\pi)^2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \hat{G}_0^R(\mathbf{p}, \varepsilon) \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

Substituting the bare Green's functions (10) into Eq. (21), averaging over the disorder and using the matrix equations $\langle \hat{G}^{-1} \rangle = \hat{G}_0^{-1} - \hat{W}$, $\langle \hat{\mathcal{G}}^{-1} \rangle = \hat{\mathcal{G}}_0^{-1} - \hat{\mathcal{W}}$, we can now find the impurity-mediated scattering times.

Results and Discussion

In our chosen limit, $k\xi \ll 1$, and at the mass shells $\varepsilon = sk$ for '+' polarized polaritons and $\varepsilon = \mathcal{E}_k$ for '-' polaritons, we find the polariton-impurity scattering rates:

$$\gamma_k^+ = \frac{1}{\tau}(k\xi)^3, \quad \gamma_k^- = \frac{1}{\tau} \left(\frac{2U_1}{U_0} \right)^2. \quad (22)$$

Here $1/\tau = Mu_0^2$ is the inverse scattering time in the normal (not condensed) state. As it is expected to be, '-' polaritons which are assumed to be in the normal state, have regular scattering lifetime ($2U_1/U_0 \sim 1^{44,45}$), whereas the scattering of polaritons in the condensed state turns out severely suppressed due to $(k\xi)^3 \ll 1$.

Scattering rates (22) together with the expressions for the longitudinal and transverse spin susceptibilities, (12)–(14), are the key results of this article. They determine the paramagnetic absorption line widths. From these expressions it is obvious that the response line width of the macroscopically occupied component of the polariton function ('+' in our case) is much less in comparison with the line width of the initially unoccupied, '-', component of the doublet, since $\gamma_k^+/\gamma_k^- \sim (k\xi)^3 \ll 1$. This fundamental result can be beneficial in experiments, checking whether one of the components is Bose-condensed or not.

The response of the system is conventionally described by the power absorption:

$$P_{k\omega} \sim -\omega B_0^2 \text{Im}\chi_{zz}(\mathbf{k}, \omega). \quad (23)$$

To explain qualitatively the structure of its spectrum, we consider the quantum transitions of the particles under external perturbation, shown in Fig. 1a. In usual electronic systems, the power absorption spectrum of the paramagnetic resonance is characterised by single resonance associated with the transitions between two spin-resolved electron levels. In contrast to this situation, in our bosonic system we have a double-peak structure of the resonance. This is due to the fact that effectively, our system has three levels. Indeed, as one can see from Fig. 1a, beside the condensate itself, there are two branches of excitations with energies ε_k and E_k in the system. The transitions from the BEC to these two branches results in the double resonance structure, see Fig. 1b. Thus the presence of the BEC is crucial for the considered effect.

The second important difference from the regular paramagnetic resonance is the requirement to use nonuniform alternating magnetic field instead of a homogeneous one. In other words, finite values of $k = |\mathbf{k}|$ are required (EPs in the BEC have zero momentum and in order to excite them one has to transfer the momentum from an external excitation). The third difference is absence of external uniform magnetic field since in our case the spin polarization occurs due to the strong exchange interaction between EPs.

We operate with two free parameters which can be determined by the experiment and the semiconductor sample: (i) the wave vector of the external perturbation, k , and (ii) impurity scattering time, τ . For (i), we have the following constraint: $k\xi \ll 1$. In order to fix (ii), we take $U_0 n_c \tau = 10$, since our theory is feasible if $U_0 n_c \tau \gg 1$. Taking into account that $|U_1| \approx 0.5U_0$, we have $2|U_1|n\tau = 10$. Since in usual GaAs samples $U_0 n_c \sim 0.05 \div 0.5$ meV or it can be smaller, and this parameter can be controlled by the number of particles in the condensate, n_c , we find $\tau \gg 10$ ps for $U_0 n_c \sim 0.05$ and $\tau \gg 1$ ps for $U_0 n_c \sim 0.5$, respectively. Using the dimensionless units of

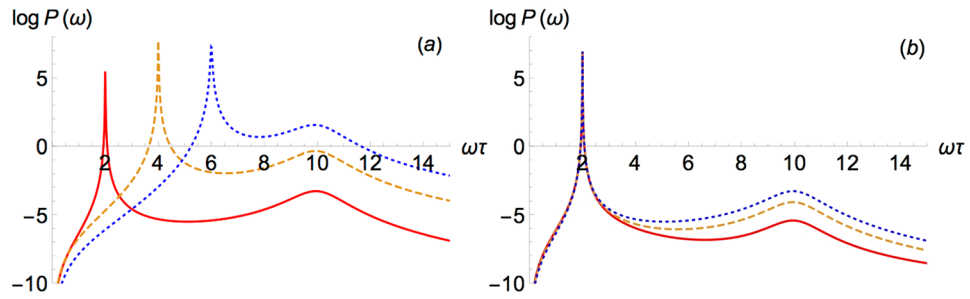


Figure 2. Power absorption spectrum for (a) various values of $k\xi$: 0.1 (red solid), 0.2 (yellow dashed) and 0.3 (blue dotted) and (b) various values of $M\alpha$: 0.1 (red solid), 0.2 (yellow dashed) and 0.3 (blue dotted curve).

TE-TM splitting, $M\alpha$, we plot the power absorption spectrum in Fig. 2 for different values of $k\xi$ (a) and $M\alpha$ (b). Here we estimate $M\alpha$ using⁴⁶ and GaAs alloys parameters⁴⁷, see Fig. 2. Clearly, both the positions of the resonances and their widths depend on (i) and (ii). It can be useful for experimental testing of our theory. The value k determines the position and width of the first resonance ($\omega \sim sk$), whereas α determines the height of the second resonance. In fact, the position of the second resonance is determined by the EP blueshift value, $2|U_1|n_c$. This value also gives an estimation of the characteristic magnetic field frequency, $\omega \sim 2|U_1|n_c$, required to observe the effect. Since in modern samples the lifetime can approach values $\tau \approx 180$ ps^{39,40} and we should satisfy $U_0n_c\tau \gg 1$, we find $U_0n_c \gg 0.004$ meV and thus $\omega \gg 0.7 \times 10^{10} \text{ s}^{-1} = 7$ GHz. We can also roughly estimate the magnitude of the external magnetic field such that it can be considered as a perturbation. One can find it from the relation, $U_0n \gg g\mu_B B_0$, thus $B_0 \ll U_0n/(g\mu_B) \approx 0.5$ T at $U_0n_c = 0.05$ meV and $g\mu_B = 0.11$ meV/T¹⁷. Let us also estimate the minimal magnetic field required for the observation of the effect. The time transfer from the condensate to excited modes of the system, which can be estimated as $\hbar/(g\mu_B B) \approx 6/B$ (ps · T) for $g\mu_B = 0.11$ meV/T, should be of the order of particle lifetime. For $\tau \approx 180$ ps we find $B_{(min)} \approx 34 \cdot 10^{-3}$ T. Therefore one has to find ways to realise experimentally large enough values of $B > B_{(min)}$ at $\omega > 7$ GHz or make samples with long enough EP lifetime.

If we assume a hypothetical situation, when instead of having only z component the initial perturbation has an in-plane component $\sim \hat{\sigma}_x B(\mathbf{r}, t)$, where $\hat{\sigma}_x$ is a Pauli matrix, then initially in the absence of spin-orbit coupling the transitions (2) in Fig. 1a would be allowed, whereas (1) would be banned. With the account of the spin-orbit interaction, one can make the transitions (1) allowed for the in-plane perturbation. Thus, in the case of the in-plane perturbation we can also expect the same behavior of the system manifesting a two-resonance profile similar to one shown in Fig. 1b.

One more important point to mention is the role of polariton-polariton scattering to the widths of peaks of the paramagnetic resonance. It can become significant in a particularly clean cavity, where impurity scattering is negligible. It is known that the particle-particle scattering rate in a 2D Bose gas calculated within the Bogliubov theory depends on the wave vector as k^3 . One can expect that the particle-particle scattering rate in the normal (not Bose-condensed) phase will behave as a square of its energy, $E_k^2 \sim k^4$ and it will be less than in the condensed phase. Thus we expect that in this situation, the width of the low-occupied component can become narrower than the macroscopically occupied component which is the opposite situation to what we have observed here. In order to give a conclusive answer, one should also consider the scattering between the condensed, ψ_+ , and non-condensed, ψ_- , EPs. This interesting question is beyond the scope of present article.

The second issue is the case $U_1 > 0$. In the case of equally populated circular components of the EP doublet, occurring at $U_1 > 0$, the Zeeman splitting becomes strongly suppressed by the particle-particle interaction up to some critical value of the constant magnetic field^{16,17,27}. Thus, the paramagnetic resonance may only occur if the magnitude of the alternating magnetic field exceeds some critical value. This question also deserves an extra consideration.

Finally, we believe that a similar physics might be observed in indirect exciton gases with spin-orbit Rashba or Dresselhaus interaction in the limit of large exchange interaction between the electron and hole within the exciton. Indeed, as it has been shown in ref. 48, the indirect exciton Hamiltonian has a form which exactly coincides with the EP Hamiltonian in the presence of the TE-TM splitting.

Conclusions

We have developed a microscopic theory of paramagnetic resonance in a spin-polarized polariton gas in a disordered microcavity. Pseudospin susceptibilities were calculated accounting for TE-TM splitting. We have shown that both longitudinal and transverse susceptibilities have a double resonance structure, responsible for different polariton spin states, and calculated the widths of the peaks of the paramagnetic resonance taking into account the polariton-impurity scattering. In contrast to ordinary disordered electronic systems, exciton polaritons in the presence of the BEC phase can scatter off both the impurity potential and impurity-stimulated fluctuations of the condensate density. We analyze those scattering processes and find that the polariton-impurity scattering rates are dramatically different for macroscopically, on one hand, and low occupied, on the other hand, components of the polariton doublet.

References

- Zavoisky, E. Spin-magnetic resonance in paramagnetics. *Fizicheskii Zhurnal* **9**, 211–245 (1945).
- Peil, S. & Gabrielse, G. Observing the quantum limit of an electron cyclotron: QND measurements of quantum jumps between Fock states. *Phys. Rev. Lett.* **83**(7), 1287–1290, doi:10.1103/PhysRevLett.83.1287 (1999).
- Odom, B., Hanneke, D., D'Urso, B. & Gabrielse, G. New measurement of the electron magnetic moment using a one-electron quantum cyclotron. *Phys. Rev. Lett.* **97**, 030801, doi:10.1103/PhysRevLett.97.030801 (2006).
- Nehrkorn, J., Schnegg, A., Hollmack, K. & Stoll, S. General magnetic transition dipole moments for electron paramagnetic resonance. *Phys. Rev. Lett.* **114**, 010801, doi:10.1103/PhysRevLett.114.010801 (2015).
- Saarikoski, H., Reimann, S. M., Harju, A. & Manninen, M. Vortices in quantum droplets: Analogies between boson and fermion systems. *Rev. Mod. Phys.* **82**, 2785–2834, doi:10.1103/RevModPhys.82.2785 (2010).
- Morsch, O. & Oberthaler, M. Dynamics of Bose-Einstein condensates in optical lattices. *Rev. Mod. Phys.* **78**, 179–215, doi:10.1103/RevModPhys.78.179 (2006).
- Kasprzak, J. *et al.* Bose-Einstein condensation of exciton polaritons. *Nature* **443**, 409–414, doi:10.1038/nature05131 (2006).
- Christopoulos, S. *et al.* Room-temperature polariton lasing in semiconductor microcavities. *Phys. Rev. Lett.* **98**, 126405, doi:10.1103/PhysRevLett.98.126405 (2007).
- Amo, A. *et al.* Collective fluid dynamics of a polariton condensate in a semiconductor microcavity. *Nature* **457**, 291–5, doi:10.1038/nature07640 (2009).
- Lagoudakis, K. G., Pietka, B., Wouters, M., Andre, R. & Deveaud-Pledran, B. Coherent oscillations in an exciton-polariton Josephson junction. *Phys. Rev. Lett.* **105**, 120403, doi:10.1103/PhysRevLett.105.120403 (2010).
- Lagoudakis, K. *et al.* Observation of half-quantum vortices in an exciton-polariton condensate. *Science* **13**, 974–976, doi:10.1126/science.1177980 (2009).
- Shelykh, I. A., Solnyshkov, D. D., Pavlovic, G. & Malpuech, G. Josephson effects in condensates of excitons and exciton polaritons. *Phys. Rev. B* **78**, 041302, doi:10.1103/PhysRevB.78.041302 (2008).
- Laussy, F. P., Kavokin, A. V. & Shelykh, I. A. Exciton-polariton mediated superconductivity. *Phys. Rev. Lett.* **104**, 106402, doi:10.1103/PhysRevLett.104.106402 (2010).
- Liew, T. C. H., Shelykh, I. A. & Malpuech, G. Polariton devices. *Physica E* **43**, 1543–1568, doi:10.1016/j.physe.2011.04.003 (2011).
- Imamoglu, A. & Ram, J. R. Quantum dynamics of exciton lasers. *Phys. Lett. A* **214**, 193–198, doi:10.1016/0375-9601(96)00175-2 (1996).
- Walker, P. *et al.* Suppression of Zeeman Splitting of the Energy Levels of Exciton-Polariton Condensates in Semiconductor Microcavities in an External Magnetic Field. *Phys. Rev. Lett.* **106**, 257401, doi:10.1103/PhysRevLett.106.257401 (2011).
- Schneider, C. *et al.* An electrically pumped polariton laser. *Nature (London)* **497**, 348–52, doi:10.1038/nature12036 (2013).
- Karpov, D. V. & Savenko, I. G. Operation of a semiconductor microcavity under electric excitation. *Appl. Phys. Lett.* **109**, 061110, doi:10.1063/1.4960797 (2016).
- Liew, T. C. H., Kavokin, A. V. & Shelykh, I. A. Optical circuits based on polariton neurons in semiconductor microcavities. *Phys. Rev. Lett.* **101**, 016402, doi:10.1103/PhysRevLett.101.016402 (2008).
- Liew, T. C. H. *et al.* Exciton-polariton integrated circuits. *Phys. Rev. B* **82**, 033302, doi:10.1103/PhysRevB.82.033302 (2010).
- Flayac, H. & Savenko, I. G. An exciton-polariton mediated all-optical router. *Appl. Phys. Lett.* **103**(20), 201105, doi:10.1063/1.4830007 (2013).
- Marsault, F. *et al.* Realization of an all optical exciton-polariton router. *Appl. Phys. Lett.* **107**(20), 201115, doi:10.1063/1.4936158 (2015).
- Kavokin, K. V. *et al.* Stimulated emission of terahertz radiation by exciton-polariton lasers. *Appl. Phys. Lett.* **97**, 201111, doi:10.1063/1.3519978 (2010).
- Savenko, I. G., Shelykh, I. A. & Kaliteevski, M. A. Nonlinear terahertz emission in semiconductor microcavities. *Phys. Rev. Lett.* **107**, 027401, doi:10.1103/PhysRevLett.107.027401 (2011).
- Amo, A. *et al.* Exciton-polariton spin switches. *Nat. Photon* **4**, 361–366, doi:10.1038/nphoton.2010.79 (2010).
- Wertz, E. *et al.* Spontaneous formation and optical manipulation of extended polariton condensates. *Nature Phys* **6**, 860–864, doi:10.1038/nphys1750 (2010).
- Shelykh, I. A., Kavokin, A. V. & Malpuech, G. Spin dynamics of exciton polaritons in microcavities. *Phys. Stat. Sol. (b)* **242**(11), 2271–2289, doi:10.1002/(ISSN)1521-3951 (2005).
- Shelykh, I. A. *et al.* Semiconductor microcavity as a spin-dependent optoelectronic device. *Phys. Rev. B* **70**, 035320, doi:10.1103/PhysRevB.70.035320 (2004).
- Glazov, M. M., Semina, M. A., Sherman, E. Ya. & Kavokin, A. V. Spin noise of exciton polaritons in microcavities. *Phys. Rev. B* **88**, 041309(R), doi:10.1103/PhysRevB.88.041309 (2013).
- Smirnov, D. S. & Glazov, M. M. Exciton spin noise in quantum wells. *Phys. Rev. B* **90**, 085303, doi:10.1103/PhysRevB.90.085303 (2014).
- Kavokin, A. V., Malpuech, G. & Glazov, M. Optical spin Hall effect. *Phys. Rev. Lett.* **95**, 136601, doi:10.1103/PhysRevLett.95.136601 (2005).
- Glazov, M. M. & Golub, L. E. Quantum and classical multiple-scattering effects in the spin dynamics of cavity polaritons. *Phys. Rev. B* **77**, 165341, doi:10.1103/PhysRevB.77.165341 (2008).
- Dufferwiel, S. *et al.* Spin textures of exciton-polaritons in a tunable microcavity with large TE-TM splitting. *Phys. Rev. Lett.* **115**, 246401, doi:10.1103/PhysRevLett.115.246401 (2015).
- Ohadi, H. *et al.* Spontaneous spin bifurcations and ferromagnetic phase transitions in a spinor exciton-polariton condensate. *Phys. Rev. X* **5**, 031002, doi:10.1103/PhysRevX.5.031002 (2015).
- Flayac, H., Solnyshkov, D. D. & Malpuech, G. Oblique half-solitons and their generation in exciton-polariton condensates. *Phys. Rev. B* **83**, 193305, doi:10.1103/PhysRevB.83.193305 (2011).
- Flayac, H., Tercas, H., Solnyshkov, D. D. & Malpuech, G. Superfluidity of spinor Bose-Einstein condensates. *Phys. Rev. B* **88**, 184503, doi:10.1103/PhysRevLett.112.066402 (2013).
- Poshakinskiy, A. V. & Tarasenko, S. A. Spatiotemporal spin fluctuations caused by spin-orbit-coupled Brownian motion. *Phys. Rev. B* **92**, 045308, doi:10.1103/PhysRevB.92.045308 (2015).
- Rubo, Yu. Chapter IV, Physics of Quantum Fluids (New Trends and Hot Topics in Atomic and Polariton Condensates). (Springer Series in solid-state sciences, Editors: Bramati A. & odugno, M. doi:10.1007/978-3-642-37569-9)
- Steger, M., Gautham, C., Snoko, D. W., Pfeiffer, L. & West, K. Slow reflection and two-photon generation of microcavity exciton-polaritons. *Optica* **2**(1), 1–5, doi:10.1364/OPTICA.2.000001 (2015).
- Nelsen, B. *et al.* Dissipationless flow and sharp threshold of a polariton condensate with long lifetime. *Phys. Rev. X* **3**, 041015, doi:10.1103/PhysRevX.3.041015 (2013).
- Kovalev, V. M., Savenko, I. G. & Iorsh, I. V. Ultrafast exciton-polariton scattering towards the Dirac points. *J. of Phys.: Cond. Mat.* **28**(10), 105301, doi:10.1088/0953-8984/28/10/105301 (2016).
- Gergel, V. A., Kazarinov, R. F. & Suris, R. A. Rarefied imperfect Bose gas in a field of randomly distributed fixed impurities. *Sov. Phys. JETP* **31**(2), 367–373 (1970).
- Kovalev, V. M. & Chaplik, A. V. Soon in. *J. Exp. Theor. Phys.* **122**, 499 (2016).

44. Shelykh, I. A., Rubo, Yu. G., Malpuech, G., Solnyshkov, D. D. & Kavokin, A. Polarization and propagation of polariton condensates. *Phys. Rev. Lett.* **97**, 066402, doi:[10.1103/PhysRevLett.97.066402](https://doi.org/10.1103/PhysRevLett.97.066402) (2006).
45. Renucci, P. *et al.* Microcavity polariton spin quantum beats without a magnetic field: A manifestation of Coulomb exchange in dense and polarized polariton systems. *Phys. Rev. B* **72**, 075317, doi:[10.1103/PhysRevB.72.075317](https://doi.org/10.1103/PhysRevB.72.075317) (2005).
46. Cilibrizzi, P. *et al.* Half-skyrmion spin textures in polariton microcavities. *Phys. Rev. B* **94**, 45315, doi:[10.1103/PhysRevB.94.045315](https://doi.org/10.1103/PhysRevB.94.045315) (2016).
47. Vurgaftmana, I., Meyer, J. R. & Ram-Mohan, L. R. Band parameters for III-V compound semiconductors and their alloys. *J. Appl. Phys. (Applied Physics Review)* **89**(11), 5815–5875, doi:[10.1063/1.1368156](https://doi.org/10.1063/1.1368156) (2001).
48. Durnev, M. V. & Glazov, M. M. Spin-dependent coherent transport of two-dimensional excitons. *Phys. Rev. B* **93**, 155409, doi:[10.1103/PhysRevB.93.155409](https://doi.org/10.1103/PhysRevB.93.155409) (2016).

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Author Contributions

V.M.K. and I.G.S. designed and performed research and wrote the paper.

Additional Information

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