Heliyon 7 (2021) e07243

Contents lists available at ScienceDirect

Heliyon

journal homepage: www.cell.com/heliyon

Research article

Power-laws in dog behavior may pave the way to predictive models: A pattern analysis study

Arunita Banerjee [a], Nandan Das [b], Rajib Dey [b,c], Shouvik Majumder [d], Piuli Shit [a], Ayan Banerjee [c,***], Nirmalya Ghosh [c,**], Anindita Bhadra [a,*]

[a] Behavior and Ecology Lab, Department of Biological Sciences, Indian Institute of Science Education and Research Kolkata, Mohanpur, Nadia, PIN 741246, West Bengal, India

[b] Tissue Optics and Microcirculation Imaging, School of Physics, National University of Ireland, Galway, Ireland

[c] Department of Physical Sciences, Indian Institute of Science Education and Research Kolkata, Mohanpur, Nadia, PIN 741246, West Bengal, India

[d] Department of Mathematical Sciences, Indian Institute of Science Education and Research Kolkata, Mohanpur, Nadia, PIN 741246, West Bengal, India

ARTICLE INFO

Keywords:
Power law
Dog behavior
Multifractality
Scale invariance
Fractal patterns

ABSTRACT

Apparently random events in nature often reveal hidden patterns when analyzed using diverse and robust statistical tools. Power law distributions, for example, project diverse natural phenomenon, ranging from earthquakes to heartbeat dynamics into a common platform of self-similarity. Animal behavior in specific contexts has been shown to follow power law distributions. However, the behavioral repertoire of a species in its entirety has never been analyzed for the existence of such underlying patterns. Here we show that the frequency-rank data of randomly sighted behaviors at the population level of free-ranging dogs follow a scale-invariant power law behavior. It suggests that irrespective of changes in location of sightings, seasonal variations and observer bias, datasets exhibit a conserved trend of scale invariance. The data also exhibits robust self-similarity patterns at different scales which we extract using multifractal detrended fluctuation analysis. We observe that the probability of consecutive occurrence of behaviors of adjacent ranks is much higher than behaviors widely separated in rank. The findings open up the possibility of designing predictive models of behavior from correlations existing in true time series of behavioral data and exploring the general behavioral repertoire of a species for the presence of syntax.

1. Introduction

Simon de Laplace, one of the pioneers of probability theory, propounded that events look random to us only because we are limited by our ability to grasp the numerous hidden factors that influence, and thereby affect such events [1]. An interesting commonality that has emerged across studies on many apparently disparate phenomena, from the distributions of species in specific habitats, city sizes, world populations, web-traffic, words in human languages to connections between nodes in social networks, is the presence of a power-law distribution [2]. Power-law distributions represent classic cases of order in chaos and are often considered as a signature for the presence of underlying mechanisms that lead to the observed data [3]. The analysis of power-law and scaling relationships in behavioral data can be used to identify the existence of universal principles within the seemingly arbitrary behavioral repertoire of a species. While specific behavioral categories like locomotion, foraging and vocalizations have been subjected to such analysis, to the best of our knowledge, the entire behavioral repertoire of a species has never been tested for such patterns.

The behavioral repertoire of a species might contain hidden patterns, which carry important information that is not revealed easily to an observer. In fact, power-law distributions have been observed in various behaviors in the context of movement, such as foraging patterns of animals [4, 5], free-flight of *Drosophila* [6], swarming behavior [7], and even human mobility [8]. These behavioral patterns can be approximated by Lévy flights [5, 7], and also display scale-free characteristics. While the scale-free nature of power-law distributions in the living world is quite ubiquitous, a scale invariant fractal-like nature has also been reported in

* Corresponding author.
** Corresponding author.
*** Corresponding author.
E-mail addresses: ayan@iiserkol.ac.in (A. Banerjee), nghosh@iiserkol.ac.in (N. Ghosh), abhadra@iiserkol.ac.in (A. Bhadra).

https://doi.org/10.1016/j.heliyon.2021.e07243
Received 28 January 2021; Received in revised form 21 April 2021; Accepted 3 June 2021

a few observations of behavior, such as sleep-wake cycle transitions [9], social behavior of wild chimpanzees [10], diving patterns in seabird foraging [11], etc. A study on Japanese macaques showed that the behavioral patterns of these primates display a fractal nature, and parasite load affects the level of complexity of their behavior [12].

The fractal nature of behavioral signatures can thus be used to understand the behavioral patterns of animals, as well as to identify anomalous behavior in species of interest. However, monofractal measures that are characterized by a single scaling exponent are rarely observed in natural phenomena, which rather exhibit more complex scaling behavior comprising of a multitude of scaling exponents corresponding to many constituent interwoven fractal subsets. This special class of complex self-affine processes – termed as multifractals [13] – have been observed and studied in a wide variety of natural phenomena, structures and processes, which include the physiological time series of heartbeats [14], the sun's magnetic field dynamics [15], turbulence phenomenon [16], swimming behavior in zooplanktons [17], and so on. The signatures of multifractality are more robust in cases where its origin is due to a broad probability distribution, so that it cannot be removed by random shuffling of the fluctuation series, as is the case for statistical fractals. Thus, in the case of the former, the quantification of multifractality provides potentially invaluable information on the underlying complex correlations and scaling behavior – which can enable entirely new perspectives in the analysis of any natural phenomena exhibiting self-similarity [18, 19].

Human languages have been known to demonstrate similar probabilistic distributions. Specifically, the frequency of occurrence of the words from a piece of text mostly follow a power-law distribution obeying the Zipf-Mandelbrot law [20]. Interestingly, this trend remains conserved and consistent across all known human languages and has been used to assign language-like property or lack thereof to other analogous datasets as diverse as inscriptions from ancient civilizations such as the Indus valley script [21], animal vocalizations like dolphin whistles [22] and bird songs [23], behavioral displays and sequences pertaining to courtship like the "push-up" displays of lizards [24] and even genetic distribution in an organism [25]. Animal communication systems are often a complex mix of visual, auditory, tactile and olfactory signals, and these modes of communication can be either used singly or in combination by individuals, depending on context. Behavioral displays are an important form of animal communication, which have been extensively studied in the context of mating and aggression in a diverse range of species [26, 27, 28]. While most mammals and birds are known to use vocalizations for communication – postures, gaits and other behaviors also play important roles, especially in social interactions. For example, ritualized aggression can be achieved through postures, thereby avoiding overt display of active aggression, and helping in the maintenance of social hierarchies [29, 30]. We wondered whether the repertoire of behaviors, both interactive and non-interactive, shown by individuals of a species, might be the equivalent of a corpus in the context of human languages, and thus, could be expected to follow the Zipf-Mandelbrot law. This was a curiosity-driven question, and not an attempt to prove that behavioral profiles of animals are akin to any spoken or written language. However, we assumed that if behavioral repertoires indeed showed language-like tendencies, this could suggest inherent similarities in communication systems of animals and could lead to further explorations for syntax and similar structures within the behavioral profiles of species.

In this study, we explore this paradigm in dogs - the first species to have been domesticated by humans and having a long history of co-evolution with our species [31]. Free-ranging dogs, which contribute to nearly 80% of the world's dog population, can provide interesting insights into the biology of dogs in general, and the evolution of the dog-human relationship in particular [32]. Many studies attempt to understand the communication between dogs and humans using vocalizations and gestures [33, 34, 35, 36]. Free-ranging dogs are social, with groups displaying interesting cooperation-conflict dynamics [37, 38] and individuals showing various degrees of socialization with humans [39]. In India, the free-ranging dogs are ubiquitous, experiencing a wide variety of interactions with humans, from very positive to very negative [36, 40]. They spend a large proportion of time in inactivity [41] and their activity patterns are not easy to predict.

Using random, population-level sampling of behavior, we investigated whether the free-ranging dog behavioral repertoire has any inherent language-like pattern and also tested for other properties like the Pareto Principle and Shannon Entropy which are likely to be found in such datasets. The Pareto Principle indicates that the majority of the observed data (80%) is likely to be composed of 20% of the type of observations, while the Shannon Entropy is a measure of information theory used to estimate the variability in the possible outcomes of such data. Accordingly, we hypothesized that the higher-ranking behaviors or rarely sighted behaviors will contribute in lesser proportion to the data but will provide more information about the system under study. The study is designed for a better understanding of the non-verbal communication system of these animals and to gauge its relevance in information theory and the possibility of building predictive models through probabilistic tools and parser evaluation of the observed data.

2. Results

We obtained data on 5669, 1308 and 506 random sightings of free-ranging dogs during three different sampling bouts in various parts of India (SupplementaryTable S2). The number of behaviors observed in these three data sets were 83, 36 and 29 respectively, summing up to a total of 93 unique behaviors in the combined data set. The frequency of observed behaviors plotted against their ranks showed power-law distributions of the nature $P(r) = r^{-\alpha}$, with the log-log plots having slopes of $\alpha \sim 1.73$ (Table 1; Figure 1a; 1b). Thus, though the behavioral repertoire showed a general power-law distribution, it did not follow the Zipf-Mandelbrot law, which requires the data to fit a slope of -1 in the log-log scale. Though the data did not seem to follow one of the prime signatures of linguistics, it did show agreement with the Pareto principle [42], with 80% of the cumulative proportion of the time activity budget being explained by 20% of the observed behaviors (on a normalized scale) (Supplementary FigureS1a). The Shannon entropy of the behavioral data scaled with the frequency of occurrence of the behaviors (Supplementary Figure S1b), suggesting that the least frequent behaviors provided the most information about the system, which is also true for languages [43].

Since power-laws are typically scale-invariant, we checked for scaling in our data using the three data sets and all their possible combinations (Supplementary Table S1). Indeed, our data showed scale-invariance, with the slopes being very similar (range: -1.717 to -1.822), in spite of the widely differing population sizes (Figure 1c; 1d). However, the scaling behavior was rather intriguing as the slope α was not uniform throughout the entire range of rank r (Figure 2a), with different values for αover three selected r –ranges: low ($r = 1 to 21$), intermediate ($r = 21$ *to* 56) and high ($r = 51$ *to* 93). This is in contrast to the value obtained for α (~ 1.81)while fitting over the entire range of the ranks for the combined data set. The corresponding representative behavioral fluctuation series $\xi(n)$numerically generated using a single power-law probability approximation of the frequency vs rank data was then subjected to MFDFA analysis. The derived generalized Hurst exponent $h(q)$exhibited a bi-fractal scaling behavior $\left[h(q) \sim \frac{1}{q} \; for \; (q(\alpha 1)) \; and \; h(q) \sim \frac{1}{(\alpha-1)} \; for \; (q \leq (\alpha - 1))\right]$(Supplementary FigureS2), which is expected for an uncorrelated random fluctuation series generated using a power-law probability distribution [7]. Next, we proceeded to generate a fluctuation series $\xi(n)$ numerically using multiple power-law probability distributions (see Methods) and once again subjected the series to MFDFA analysis. The observed wide range of large and small fluctuations in the detrended fluctuation series underscores the complex nature and the overall

Table 1. This table provides the sample size for each data set used in the analysis, along with the slope (α) and error for each, which suggests scale invariance.

Dataset	Sample size	Slope	Error
Data Set P	506	-1.74309	0.058762
Data Set C	1308	-1.73654	0.088568
Data Set P+C	1814	-1.74122	0.066323
Data Set A	5669	-1.71677	0.072575
Data Set A+C	6980	-1.75794	0.077475
Data Set A+P+C	7482	-1.82206	0.072396
Data Set P+A	6175	-1.76273	0.070593

randomness of the behavioral fluctuations (Figure 2b). The MFDFA-derived moment (q) dependence of the fluctuation functions $logF_q(s)$vs $\log s$ (Figure 2c) indicates the presence of multifractal scaling, as the slopes vary significantly for the entire range of q. The resulting continuous variations of $h(q)$ and $\tau(q)$ with varying q furnishes concrete evidence of multifractality (Figure 2d), with the variations of $h(q)$being more prominent for negative values of q as compared to positive values. This is in sharp contrast to that observed for the fluctuation series with single power-law probability (Supplementary Fig. S1c). These multifractal trends suggest that consecutive occurrence of behaviors of adjacent ranks (representing small fluctuations that are captured by the negative$q$) dominate the overall scaling behavior as compared to the behaviors that are widely separated in rank (representing large fluctuations that are captured by the positive$q$). The corresponding strength of multifractality is subsequently quantified via the width ($\Delta\beta$) of the singularity spectrum $f(\beta)$ (Figure 2e), with the reasonably large magnitude of $\Delta\beta(=1.12)$ clearly demonstrating strong multifractality in the behavioral data of free-ranging dogs.

3. Discussion

The implication of our findings in the context of the analysis of behavioral data is rather intriguing. While the data does not follow the Zipf-Mandelbrot law, commonly exhibited by human languages, it projects a strong power-law behavior which retains scale invariant trends irrespective of spacio-temporal changes and observer bias. The information obtained from the multifractal analysis of the behavioral repertoire of a species is indeed of a richer hue. The multifractal trends gleaned using the MFDFA analysis indicates dominant self-similarity among the closely occurring ranks as compared to the distantly placed ranks. In the present context, these results imply that for a single dog, the probability of consecutive occurrence of behaviors of adjacent rank is much higher than behaviors widely separated in rank. This inference is quite remarkable, given that the data pool is drawn from a large number of randomly sighted behaviors at the population level. From a purely probabilistic standpoint, this would demand that the probability of occurrence of a high-frequency behavior (high rank) would be very large immediately after a low frequency behavior (low rank). Specifically, a behavior such as sleeping (Rank 1, highly probable) should follow barking (Rank 18, less probable). However, this is rarely expected to occur in practice, which is also validated by our multifractal analysis.

The results from the Pareto-plot and Shannon Entropy of the data complement each other and indicate that the commonly observed behaviors, which form the bulk of the data are of the very generic kind and mostly include passive resting behaviors or maintenance activities like grooming and foraging, which are not species-specific. This result is in agreement with other species in which detailed time-activity budgets have been created, revealing the majority of time in the life of animals being spent in behavioral “states”, which provide less information about the species, and a small proportion of time being spent in more energy-intensive behavioral “events” which carry a high amount of information about the species, and about individual level variation within the species [44]. In the case of the dogs, the higher-ranking behaviors, which are rarely sighted, include play behaviors, mating related behaviors, interactions with humans like tail wagging, aggressive and affiliative interactions, etc and provide more species-specific information about the system. Such a distribution of observed data is also very characteristic of other power-law distributions exhibited by various analogous data sets like human languages, courtship displays and animal vocalizations.

Since the study is based on a random sampling method and deals with population-level data analysis, it represents a corpus where the entities

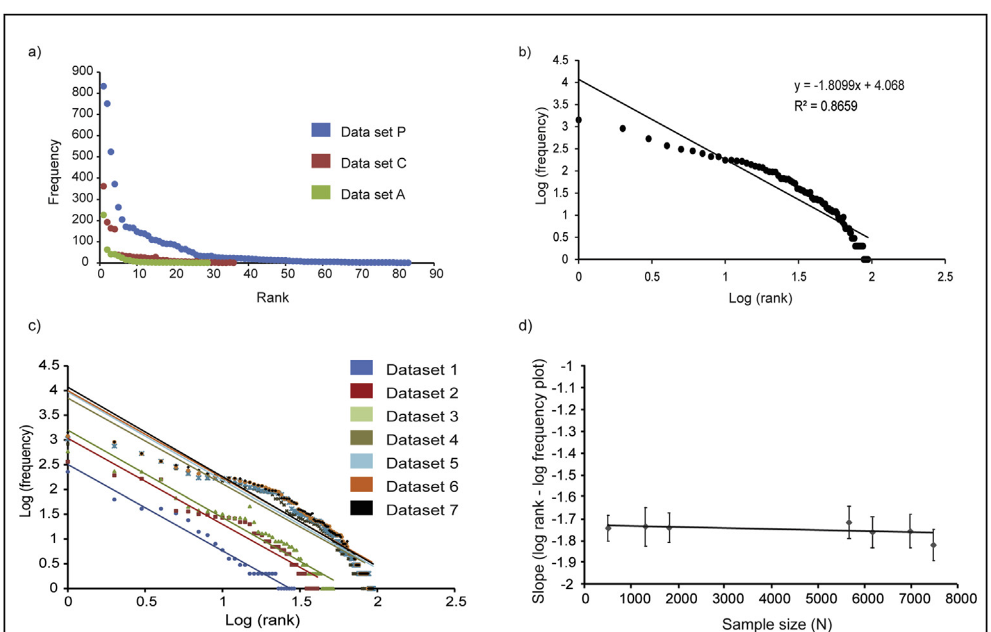

Figure 1. a) Scatterplots showing the frequency distributions of observed behaviors arranged according to rank in the three data sets (N = 506, 1308 and 5669 for P, C and A respectively). b) The black dots represent the frequency-rank distribution plotted on a log-log scale for all 7482 observations (data sets P+C+A), which yielded 93 unique ranks. The line represents the linear fit to the curve represented by the equation y = -1.8099x + 4.068, having a R^2 value of 0.8659. c) The frequency vs rank plots (on a log-log scale) of the three data sets and all their possible combinations, yielding 7 data sets. Each colour represents a data set and the straight line of the corresponding colour represents the linear fit to the data. d) A scatterplot showing the mean (dots) and standard errors (lines) of the slopes of the log-log plots as shown in “c”. The nearly straight line parallel to the x-axis fitting the data suggests scale invariance with change in sample size for this data.

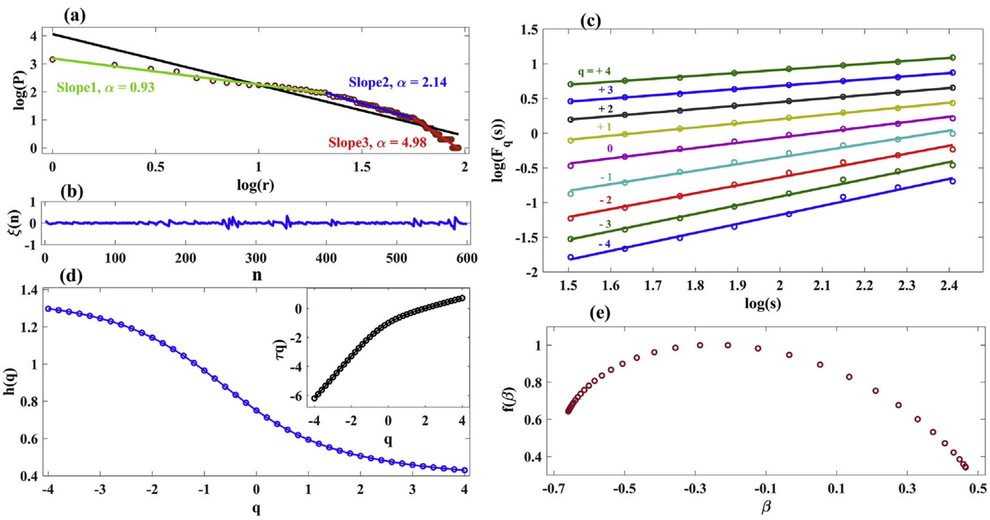

Figure 2. *Multifractal patterns in the statistically equivalent behavioral random fluctuation series generated using multiple power-law approximation of the Frequency vs behavioral rank data.* (a)Power-law fitting of the Frequency (P) vs rank (r) data of dog behavior (shown in natural logarithm scale). Fitting at three selected r -ranges (lower (red), intermediate (blue) and higher (green), respectively) yield different values for the scaling exponent (indicating multifractality). Overall fitting over the entire range of the rank with a single power-law ($\boldsymbol{\alpha} = 1.81$) is shown by black line. (b)–(e): Results of the MFDFA analysis on the fluctuation series $\xi(n)$ generated using *multiple power-law approximation* of frequency vs rank data. (b) The detrended (by least square polynomial fitting) fluctuation series representing the behavioral fluctuations $[\xi(n)]$. (c) The log-log plot of the moment (q = -4 to +4) dependent fluctuation function $F_q(s)$ vs s. Considerable variations in the slopes for the entire range of q values indicate the presence of multifractal scaling. (d) The variation of generalized Hurst exponent $h(q)$(derived from the slopes of $\log F_q(s)$ *vs* $\log(s)$) and $\tau(q)$(derived using Eq. (3), shown in inset) with varying q. Continuous variations of $h(q)$ and $\tau(q)$ with varying q confirm multifractality. (e) The corresponding multifractality is quantified via the singularity spectrum $f(\beta)$ (derived using Eq. 4) and its full width $\Delta\beta$ is noted.

are drawn out of randomized pools. So, it is noteworthy that, multifractal analysis of languages with word pools drawn from multiple corpora instead of a single corpus would probably lead to traits similar to the observations we report here. The statistical significance of such results is strongly indicative of successful extrapolation of this data into the construction of probability-based predictive models. The proposed multifractal analysis in combination with appropriate probabilistic models may in principle be able to predict a behavior at a later time when the behavior at a certain time is known. To achieve that goal, we intend to extend this study towards the true time series of behavioral data of free-ranging dogs to investigate the existence of possible long and short-range correlations and their effects on the resulting multifractal trends. The resulting short-range correlations can be exploited to predict immediate behavioral responses to certain stimuli, while long-range correlations might provide insights into personalities of individuals and nuances layering their behavior. Such information would be of great value in behavioral studies and prove to be most useful in mitigating dog-human conflict. Our method is also completely general in nature, and can, in principle, address similar behavioral questions in any species.

4. Methods

4.1. Sampling

The data was collected through instantaneous scan sampling of free-ranging dogs in urban and semi-urban habitats. The observer traversed along a pre-decided route, at random time points during the day, on different days. Whenever one or more dogs were sighted, the age class (adult/juvenile), sex, instantaneous behavior at the time of sighting for each dog were recorded along with the date, time and location of sighting [45]. The behaviors were noted following the ethogram compiled by the Dog Lab (Supplementary Table S1). Three data sets, collected at different times from different locations, were used for this analysis (Supplementary Table S2; Figure S1).

4.2. Data analysis

4.2.1. Zipf-Mandelbrot law in behavior

The frequency (number of occurrences) of each behavior was estimated from the data sets. For each data set, the behaviors were ranked according to their frequencies of occurrence, with the most frequent behavior being rank 1. The behavioral frequencies were plotted against their respective ranks, to check for a power law distribution. The frequency and rank were re-plotted on a log-log scale and the slope of the resulting trend line was considered to check for a fit to the Zipf-Mandelbrot law [42, 43]. The three data sets and all their possible combinations were used to check for scale invariance.

The behavioral ranks were normalized on a scale of 1–100 for each data set separately, and the cumulative time spent in each behavior was calculated, such that the total time was 1. The behaviors contributing to the first 80% of time spent were identified for each data set and the distributions were tested for adherence to the Pareto principle. The Shannon entropy was estimated for the data sets and plotted against the ranks of the behaviors to check if the frequency of occurrence influenced the Shannon entropy.

4.2.2. Generation of the statistically equivalent behavioral random fluctuation series from the frequency vs rank data

The collected behavioral data of free-ranging dogs were ranked according to their frequencies (P)of occurrence, with the most frequent behavior being ranked $r = 1$. The corresponding Frequency vs rank data (for the range$r = 1 to 93$)was first fitted to a single power-law distribution$P(r) \sim r^{-\alpha}$, yielding a value of$\alpha = 1.81$(shown in Figure 2a). A series of random numbers that statistically represent the random sequence of different behavioral events (with rank r) were generated through sampling of the random numbers by the resulting power-law distribution. The numerical ranks of the representative behaviors in the series were represented by an appropriately scaled (by the mean and the standard deviations of rank) random variable$\xi(n)$,with $n$representing any random sequence of behavioral event.

However, the actual frequency vs rank $(P(r))$behavioral data exhibited three different power-law coefficients α for three different range of ranks (Figure 2a). The corresponding series $\xi(n)$ was therefore obtained by sampling the random numbers with power-law distributions with three different values for $\alpha(0.93, 2.14, 4.98)$ and then by random shuffling of the generated sequences. This process of generation of the uncorrelated random fluctuation series using multiple power-law probability distribution is specifically applicable here because the behavioral data were not collected in any given sequence of time rather these were collected at random time points. Thus, in this scenario, the $\xi(n)$ series of random events synthesized using the aforementioned method represents the actual random behavioral fluctuation data in a statistical sense. The $\xi(n)$ fluctuation series generated using either a single or multiple power-law probability distribution were subsequently subjected to multifractal detrended fluctuation analysis.

4.2.3. Multifractal detrended fluctuations analysis

A statistical monofractal series is one whose variance exhibits a power-law scaling described by a single scaling exponent, namely, Hurst exponent, H $(0<H<1)$ [46]. A multifractal series, on the other hand, exhibits complex scaling behavior comprising of many interwoven fractal subsets characterized by different local Hurst exponents [13]. Multifractal detrended fluctuation analysis (MFDFA) is a generalized approach to characterize such complex multi-affine processes [47]. Using this approach, the fluctuation profile $\xi(n)$ (series of length N, $n = 1.....N$) is first divided into $N_s = \text{int}(N/s)$ segments m of equal length s. The local trends $(y_m(n))$ of each segment m are determined by polynomial fitting. The fitted trends are then subtracted from the profile to obtain the detrended fluctuations and the corresponding variance of a segment is subsequently obtained as

$$F^2(m,s) = \frac{1}{s}\sum_{n=1}^{s}\left[Y\{(m-1)s+n\} - y_m(n)\right]^2 \tag{1}$$

The variances are then averaged over all the segments to construct the moment (q) dependent fluctuation function

$$F_q(s) = \left\{\frac{1}{2N_s}\sum_{m=1}^{2N_s}\left[F^2(m,s)\right]^{\frac{q}{2}}\right\}^{1/q} \tag{2}$$

In order to quantify the scaling behavior, the fluctuation function is approximated to follow a power-law scaling $F_q(s) \sim s^{h(q)}$.The multifractality (if any) of the signal is subsequently characterized via the moment dependence of the generalized Hurst scaling exponent $h(q)$, the classical multifractal scaling exponent $\tau(q)$, and the singularity spectrum $f(\beta)$. These are related as

$$\tau(q) = qh(q) - 1 \tag{3}$$

$$\beta = \frac{d\tau}{dq},\ f(\beta) = q\beta - \tau(q) \tag{4}$$

whereβ is the singularity strength and the full width of $f(\beta)$,$\Delta\beta$ (taken at $f(\beta) = 0$) is a quantitative measure of the strength of multifractality. Note that $h(q=2)$ corresponds to the Hurst exponent $(H)$of an equivalent monofractal series.

Declarations

Author contribution statement

Arunita Banerjee: Performed the experiments; Analyzed and interpreted the data; Wrote the paper.

Nandan Das, Rajib Dey, Shouvik Majumder, Piuli Shit: Analyzed and interpreted the data.

Ayan Banerjee, Nirmalya Ghosh, Anindita Bhadra: Conceived and designed the experiments; Analyzed and interpreted the data; Wrote the paper.

Funding statement

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Data availability statement

Data included in article/supplementary material/referenced in article.

Declaration of interests statement

The authors declare no conflict of interest.

Additional information

Supplementary content related to this article has been published online at https://doi.org/10.1016/j.heliyon.2021.e07243.

Acknowledgements

The authors would like to thank Ms. Sreejani Sen Majumder and Dr. Ankita Chatterjee for their participation in this earlier work (Sen Majumder et al 2014a), for which AB was the corresponding author.

References

[1] J. Horne, The Ontology of Randomness. Encyclopedia of Information Science and Technology, fourth ed., 2018, pp. 1845–1855.
[2] A. Clauset, C.R. Shalizi, M.E. Newman, Power-law distributions in empirical data, SIAM Rev. 51 (4) (2009) 661–703.
[3] B.J. Cole, Fractal time in animal behavior: the movement activity of Drosophila, Anim. Behav. 50 (5) (1995) 1317–1324.
[4] D.W. Sims, E.J. Southall, N.E. Humphries, G.C. Hays, C.J.A. Bradshaw, J.W. Pitchford, A. James, M.Z. Ahmed, A.S. Brierley, M.A. Hindell, D. Morritt, M.K. Musyl, D. Righton, E.L.C. Shepard, V.J. Wearmouth, R.P. Wilson, M.J. Witt, J.D. Metcalfe, Scaling laws of marine predator search behavior, Nature 451 (7182) (2008) 1098–1102.
[5] G.M. Viswanathan, V. Afanasyev, S.V. Buldyrev, E.J. Murphy, P.A. Prince, H.E. Stanley, Lévy flight search patterns of wandering albatrosses, Nature 381 (6581) (1996) 413–415.
[6] A.M. Reynolds, M.A. Frye, Free-flight odor tracking in Drosophila is consistent with an optimal intermittent scale-free search, PloS One 2 (4) (2007) e354.
[7] G. Ariel, A. Rabani, S. Benisty, J.D. Partridge, R.M. Harshey, A. Be'er, Swarming bacteria migrate by Lévy walk, Nat. Commun. 6 (2015) 8396.
[8] D. Brockmann, L. Hufnagel, T. Geisel, The scaling laws of human travel, Nature 439 (7075) (2006) 462–465.
[9] C.-C. Lo, T. Chou, T. Penzel, T.E. Scammell, R.E. Strecker, H.E. Stanley, P.C. Ivanov, Common scale-invariant patterns of sleep–wake transitions across mammalian species, Proc. Natl. Acad. Sci. Unit. States Am. 101 (50) (2004) 17545–17548.
[10] C.L. Alados, M.A. Huffman, Fractal long-range correlations in behavioral sequences of wild chimpanzees: a non-invasive analytical tool for the evaluation of health, Ethology 106 (2) (2000) 105–116.
[11] A.J. MacIntosh, L. Pelletier, A. Chiaradia, A. Kato, Y. Ropert-Coudert, Temporal fractals in seabird foraging behavior: diving through the scales of time, Sci. Rep. 3 (2013) 1884.
[12] A.J.J. MacIntosh, C.L. Alados, M.A. Huffman, Fractal analysis of behavior in a wild primate: behavioral complexity in health and disease, J. R. Soc. Interface 8 (63) (2011) 1497–1509.
[13] H.E. Stanley, P. Meakin, Multifractal phenomena in physics and chemistry, Nature 335 (6189) (1988) 405.
[14] P.C. Ivanov, L.A.N. Amaral, A.L. Goldberger, S. Havlin, M.G. Rosenblum, Z.R. Struzik, H.E. Stanley, Multifractality in human heartbeat dynamics, Nature 399 (6735) (1999) 461.
[15] R.J. McAteer, C.A. Young, J. Ireland, P.T. Gallagher, The bursty nature of solar flare X-ray emission, Astrophys. J. 662 (1) (2007) 691.
[16] F. Argoul, A. Arneodo, G. Grasseau, Y. Gagne, E.J. Hopfinger, U. Frisch, Wavelet analysis of turbulence reveals the multifractal nature of the richardson cascade, Nature 338 (6210) (1989) 51.
[17] M. Uttieri, E. Zambianchi, J.R. Strickler, M.G. Mazzocchi, Fractal characterization of three-dimensional zooplankton swimming trajectories, Ecol. Model. 185 (1) (2005) 51–63.

[18] C. Song, S. Havlin, H.A. Makse, Self-similarity of complex networks, Nature 433 (7024) (2005) 392–395.
[19] D. Sornette, Critical Phenomena in Natural Sciences: Chaos, Fractals, Selforganization and Disorder: Concepts and Tools, Springer Science & Business Media, 2006.
[20] A. Mehri, M. Jamaati, Variation of zipf's exponent in one hundred live languages: a study of the holy bible translations, Phys. Lett. 381 (31) (2017) 2470–2477.
[21] N. Yadav, H. Joglekar, R.P.N. Rao, M.N. Vahia, R. Adhikari, I. Mahadevan, Statistical analysis of the Indus script using N-grams, PloS One 5 (3) (2010), e9506.
[22] B. McCOWAN, S.F. Hanser, L.R. Doyle, Quantitative tools for comparing animal communication systems: information theory applied to bottlenose dolphin whistle repertoires, Anim. Behav. 57 (2) (1999) 409–419.
[23] J.P. Hailman, M.S. Ficken, Combinatorial animal communication with computable syntax: chick-a-dee calling qualifies as "language" by structural linguistics, Anim. Behav. 34 (6) (1986) 1899–1901.
[24] Structural complexity in a lizard communication system: the Sceloporus graciosus "push-up" display on JSTOR. https://www.jstor.org/stable/1446717?casa_token=R0GU-hCIrxgAAAAA%3AFc-QGkIEWRWXMVSRkvkRHS8MPgDYbcx0x5PqA804nYMUqvoqGbet5vEANN95esUTETXOR8H4pL3lvzmi2qlX7gofjh79Zl4A0l8V0xK5uZ7teRESHZgugw&seq=1#metadata_info_tab_contents. (Accessed 22 March 2021).
[25] D.B. Searls, The language of genes, Nature 420 (6912) (2002) 211–217.
[26] A.D.M. Wilson, E.M. Whattam, R. Bennett, L. Visanuvimol, C. Lauzon, S.M. Bertram, Behavioral correlations across activity, mating, exploration, aggression, and antipredator contexts in the European house cricket, Acheta domesticus, Behav. Ecol. Sociobiol. 64 (5) (2010) 703–715.
[27] P. Stockley, A. Campbell, Female competition and aggression: interdisciplinary perspectives, Phil. Trans. Biol. Sci. 368 (1631) (2013) 20130073.
[28] J. Burgdorf, R.A. Kroes, J.R. Moskal, J.G. Pfaus, S.M. Brudzynski, J. Panksepp, Ultrasonic vocalizations of rats (Rattus norvegicus) during mating, play, and aggression: behavioral concomitants, relationship to reward, and self-administration of playback, J. Comp. Psychol. 122 (4) (2008) 357–367.
[29] T.E. Deiker, The Ritualization of Aggression in Cichlasoma Cutteri, 1971, p. 2045. LSU Historical Dissertations and Theses, https://digitalcommons.lsu.edu/gradschool_disstheses/2045.
[30] C.B. Jones, The functions of status in the mantled howler monkey, Alouatta palliata gray: intraspecific competition for group membership in a folivorous neotropical primate, Primates 21 (3) (1980) 389–405.
[31] S.E. Lea, B. Osthaus, In what sense are dogs special? Canine cognition in comparative context, Learn. Behav. 46 (4) (2018) 335–363.
[32] J. Serpell, The Domestic Dog, Cambridge University Press, 2016.
[33] D. Bhattacharjee, N.D. N, S. Gupta, S. Sau, R. Sarkar, A. Biswas, A. Banerjee, D. Babu, D. Mehta, A. Bhadra, Free-ranging dogs show age related plasticity in their ability to follow human pointing, PloS One 12 (7) (2017), e0180643.
[34] S. Yin, B. McCowan, Barking in domestic dogs: context specificity and individual identification, Anim. Behav. 68 (2) (2004) 343–355.
[35] A. Andics, A. Gábor, M. Gácsi, T. Faragó, D. Szabó, Á. Miklósi, Neural mechanisms for lexical processing in dogs, Science 353 (6303) (2016) 1030–1032.
[36] Frontiers, Free-ranging dogs understand human intentions and adjust their behavioral responses accordingly, ecology and evolution. https://www.frontiersin.org/articles/10.3389/fevo.2018.00232/full. (Accessed 23 January 2019).
[37] M. Paul, A. Bhadra, Selfish pups: weaning conflict and milk theft in free-ranging dogs, PloS One 12 (2) (2017), e0170590.
[38] M. Paul, A. Bhadra, The great Indian joint families of free-ranging dogs, PloS One 13 (5) (2018), e0197328.
[39] D. Bhattacharjee, S. Sau, J. Das, A. Bhadra, Free-ranging dogs prefer petting over food in repeated interactions with unfamiliar humans, J. Exp. Biol. 220 (24) (2017) 4654–4660.
[40] M. Paul, S.S. Majumder, S. Sau, A.K. Nandi, A. Bhadra, High early life mortality in free-ranging dogs is largely influenced by humans, Sci. Rep. 6 (2016) 19641.
[41] S.S. Majumder, A. Chatterjee, A. Bhadra, A dog's day with humans – time activity budget of free-ranging dogs in India, Curr. Sci. 106 (6) (2014) 5.
[42] M.E. Newman, Power laws, Pareto distributions and zipf's law, Contemp. Phys. 46 (5) (2005) 323–351.
[43] R.F. i Cancho, R.V. Solé, Least effort and the origins of scaling in human language, Proc. Natl. Acad. Sci. Unit. States Am. 100 (3) (2003) 788–791.
[44] R. Gadagkar, The Social Biology of Ropalidia Marginata: toward Understanding the Evolution of Eusociality, Harvard University Press, 2009.
[45] S.S. Majumder, A. Bhadra, A. Ghosh, S. Mitra, D. Bhattacharjee, J. Chatterjee, A.K. Nandi, A. Bhadra, To Be or not to Be social: foraging associations of free-ranging dogs in an urban ecosystem, Acta Ethol. 17 (1) (2014) 1–8.
[46] B.B. Mandelbrot, J.R. Wallis, Joseph Noah, Operational hydrology, Water Resour. Res. 4 (5) (1968) 909–918.
[47] J.W. Kantelhardt, S.A. Zschiegner, E. Koscielny-Bunde, S. Havlin, A. Bunde, H.E. Stanley, Multifractal detrended fluctuation analysis of nonstationary time series, Phys. Stat. Mech. Appl. 316 (1–4) (2002) 87–114.