THE JOURNAL OF PHYSICAL CHEMISTRY LETTERS
A JOURNAL OF THE AMERICAN CHEMICAL SOCIETY

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Letter

Colossal Power Extraction from Active Cyclic Brownian Information Engines

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Cite This: *J. Phys. Chem. Lett.* 2022, 13, 6912–6918

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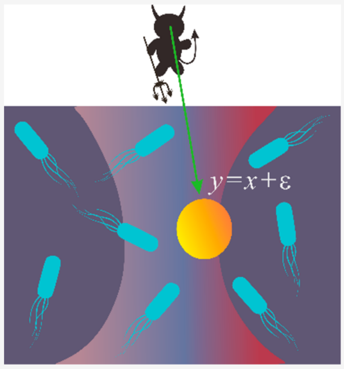
ABSTRACT: Brownian information engines can extract work from thermal fluctuations by utilizing information. To date, the studies on Brownian information engines consider the system in a thermal bath; however, many processes in nature occur in a nonequilibrium setting, such as the suspensions of self-propelled microorganisms or cellular environments called an active bath. Here, we introduce an archetypal model for a Maxwell-demon type cyclic Brownian information engine operating in a Gaussian correlated active bath capable of extracting more work than its thermal counterpart. We obtain a general integral fluctuation theorem for the active engine that includes additional mutual information gained from the active bath with a unique effective temperature. This effective description modifies the generalized second law and provides a new upper bound for the extracted work. Unlike the passive information engine operating in a thermal bath, the active information engine extracts colossal power that peaks at the finite cycle period. Our study provides fundamental insights into the design and functioning of synthetic and biological submicrometer motors in active baths under measurement and feedback control.



Information engines, a modern realization of thought experiments such as Maxwell's demon[1] and the Szilard engine,[2] are stochastic devices capable of extracting mechanical work from a single heat bath by exploiting the information acquired from measurements. Recent progress in information thermodynamics has provided the inevitable upper bound of the work that can be extracted from an information engine by generalizing the second law of thermodynamics:[3−10]

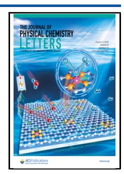
$$\langle -W\rangle \leq -\Delta F + k_{\mathrm{B}}T\langle \Delta I\rangle \quad (1)$$

where $\langle\cdots\rangle$ denotes ensemble average. According to eq 1, the average work extracted from an information engine $\langle -W\rangle$ operating in a thermal bath of temperature T is bounded by the associated free energy difference $-\Delta F$ and the average mutual information gain $\langle\Delta I\rangle$ between the system and feedback controller multiplied by $k_{\mathrm{B}}T$, where k_{B} is the Boltzmann constant.

Various models of information engines operating in thermal baths have been theoretically proposed[5,6,11−13] and experimentally verified in classical[14−24] and quantum[25−27] systems. Whether these models and, in particular, the laws of information thermodynamics also apply to information engines operating in athermal baths, such as swimming bacteria and active colloidal particles[28−39] or cellular environments,[40−43] remains to be explored.

Brownian particles in such active baths are subject to violent agitation due to the uncorrelated thermal fluctuations of the solvent molecules and the correlated fluctuations generated by the active components. Consequently, they are in a perpetual nonequilibrium state. Recent studies on nonfeedback-driven cyclic active heat engines operating between active baths of different activity (temperature) reveal that the active heat engines can extract work beyond the limit set by the Carnot bound.[44−48] However, because of the limitations in the existing experimental techniques, which require a long time to change the activity of the active bath, the active heat engines realized in the experiment operate in the quasistatic limit with the cycle period much longer than the thermal relaxation time.[44] Moreover, many physiochemical processes in nature occur far from equilibrium in the active bath and exchange energy and information.[40,49−51] For example, biological motors are essentially modeled as information engines that use the information on the fluctuations to extract energy from the noisy environment by rectifying random fluctuations.[50,52] Although the biological motors inside the living cells operate in the active environment, prior studies on the information-driven motors consider the system only in a thermal environment. Thus, a more feasible physical model for such processes would be an efficient finite cycle stochastic engine operating in the active bath of constant activity. Herein, we introduce an experimentally feasible cyclic information engine operating in an active bath capable of extracting more work than the acquired information.

Received: June 8, 2022
Accepted: July 19, 2022
Published: July 22, 2022



https://doi.org/10.1021/acs.jpclett.2c01736
J. Phys. Chem. Lett. 2022, 13, 6912–6918

The active information engine examined herein consists of a Brownian particle in a harmonic potential well that is subjected to the periodic measurement and feedback control under the influence of Gaussian colored noise, which is a typical model used for active baths.[28,35] We examine the performance of the active information engine as a function of the cycle period, measurement error, and strength and correlation time of the active noise. We find that the thermodynamic quantities such as work, heat, and mutual information of the active engines are greater than those of the passive engines operating in the thermal bath. The average extracted work per cycle in the steady state where $\Delta F = 0$ can exceed the bound in eq 1, but it is always bounded by the modified generalized second law $\langle -W \rangle \le k_B T_{\text{eff}} \langle \Delta I \rangle$, where $k_B T_{\text{eff}}$ is equivalent to the average effective energy of the particle in the active bath. The modified second law can also be derived from the generalized integral fluctuation theorem that we obtain for the cyclic active information engine as $\langle \exp[-(W/k_B T_{\text{eff}} + \Delta I)] \rangle = 1$.

One of the key challenges in designing efficient stochastic engines is maximizing the extracted work and power simultaneously.[50,53] We show that the extracted power of the active information engine is a maximum for a finite cycle period nearly equal to the thermal relaxation time of the particle where the extracted work is also near the maximum. Depending on the active noise parameters, this power can be orders of magnitude larger than those of passive information engines, which exhibit maximum power for ultrafast cycle periods where the extracted work vanishes.[16,19,22] For example, we find that for a strongly correlated active bath of strength $f_{\text{act}} \approx 2$ pN and correlation time $\tau_c \approx 25$ ms, the peak power is $\sim 10^4$ $k_B T/s$, which is ~ 50 times larger than its passive counterparts, indicating that the active engines with a finite cycle period can extract a colossal amount of power from the active bath. We also confirm our analytical results using numerical simulations.

Active Bath Model. We consider the one-dimensional motion of a Brownian particle in a harmonic potential, $V(x, \lambda) = (k/2)(x - \lambda)^2$, where x is the particle position, k the stiffness, and λ the center of the potential in an active bath of temperature T. The motion of the particle is described by the overdamped Langevin equation:

$$\gamma \frac{dx}{dt} = -k(x - \lambda) + \xi_{\text{th}}(t) + \xi_{\text{act}}(t) \tag{2}$$

The thermal noise $\xi_{\text{th}}(t)$ follows a Gaussian white noise with zero mean $\langle \xi_{\text{th}}(t) \rangle = 0$ and no correlation $\langle \xi_{\text{th}}(t)\xi_{\text{th}}(t') \rangle = 2\gamma k_B T \delta(t - t')$, where γ is the dissipation coefficient. The active noise $\xi_{\text{act}}(t)$ is characterized by an exponentially correlated Gaussian noise with a zero mean $\langle \xi_{\text{act}}(t) \rangle = 0$ and correlation of[35]

$$\langle \xi_{\text{act}}(t)\xi_{\text{act}}(t') \rangle = f_{\text{act}}^2 \exp(-|t - t'|/\tau_c) \tag{3}$$

Here, f_{act} is the strength and τ_c is the correlation time of the active noise. In the absence of active noise, the particle is in thermal equilibrium with a Gaussian distribution of $p(x) = (2\pi S)^{-1/2} \exp[-(x - \lambda)^2/2S]$, where $S = k_B T/k$ is the equilibrium variance in the thermal bath. The thermal relaxation time of a particle in the harmonic potential is $\tau_r = \gamma/k$.

In the presence of active noise, the probability distribution function (PDF) of the particle position at any time t still follows a Gaussian distribution but with a variance $S_{\text{act}}(t)$, which can be calculated by solving eq 2 (see the Supporting Information and refs 35 and 54):

$$S_{\text{act}}(t) = S(0)e^{-2t/\tau_r} + \left[S + \frac{f_{\text{act}}^2 \tau_r}{\gamma^2(1/\tau_r + 1/\tau_c)}\right][1 - e^{-2t/\tau_r}] - \frac{2f_{\text{act}}^2}{\gamma^2(1/\tau_r^2 - 1/\tau_c^2)}[e^{-t(1/\tau_r + 1/\tau_c)} - e^{-2t/\tau_r}] \tag{4}$$

where $S(0)$ is the initial variance of the particle position distribution at $t = 0$. Considering the long time limit $t \gg \tau_c$, the active noise correlation decays fully, and the particle reaches a nonequilibrium steady state. The generalized equipartition theorem can then be defined in the active bath as $\lim_{t\to\infty}(k/2)S_{\text{act}} = (k_B/2)(T + T_{\text{act}})$,[35] where

$$T_{\text{act}} = f_{\text{act}}^2/[k_B\gamma(1/\tau_r + 1/\tau_c)] \tag{5}$$

is the active temperature of the particle owing to the active noise source in the medium.

Active Information Engine. Each engine cycle of period τ includes three steps: particle position measurement, instantaneous shift of the potential center, and relaxation. Figure 1 shows schematics of the ith engine cycle operating in the active bath. Here, the information engine measures the particle position x with respect to the potential center λ_{i-1} as $y \equiv x + \varepsilon$. The error in the measurement $\varepsilon \equiv y - x$ is assumed to follow the Gaussian distribution $p(y|x) = (2\pi N)^{-1/2} \exp[-(y - x)^2/2N]$ of variance N. During the feedback step, the trap center is shifted instantaneously to the measurement outcome $\lambda_{i-1} \to \lambda_i = y$. In the relative frame of reference, the trap center is fixed at the origin while the particle is transported instantaneously to $-y$.[55] In the last step, the particle relaxes in the shifted potential for time τ before the next cycle begins. The particle dynamics during the relaxation follows the overdamped Langevin eq 2. Because the measurement and feedback control are instantaneous, the cycle period τ is the relaxation period. In the subsequent $(i + 1)$th cycle, the particle position is measured with respect to the shifted potential center λ_i (the origin is reset) and the same feedback protocol is repeated. Because the origin is reset, the particle dynamics in the shifted potential is independent of all previous measurement.[11]

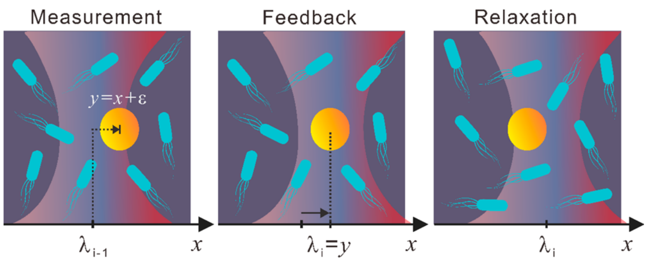


Figure 1. Illustration of the ith engine cycle of a Brownian information engine consisting of a colloidal particle in an optical trap operating in an active bath of swimming bacteria. The particle is initially located at x with respect to the optical trap center λ_{i-1}. During the measurement step, the information engine perceives the particle position x as $y = x + \varepsilon$. The error in the measurement ε follows a Gaussian white noise of variance N. During the feedback step, the trap center is shifted instantaneously to $\lambda_i = y$. During the relaxation step, the particle relaxes in the active bath for a duration τ with the fixed trap center λ_i until the next cycle begins.

https://doi.org/10.1021/acs.jpclett.2c01736
J. Phys. Chem. Lett. 2022, 13, 6912−6918

After many repetitions of the feedback cycles, the engine approaches a nonequilibrium steady state. Therefore, in the relative frame of reference, the PDF of the particle position in the steady state at the beginning of the relaxation (immediately after the feedback step) is the same as the error distribution $p(y|x)$ with variance N. The PDF of the particle position in the steady state after time τ (at the beginning of the next cycle) is given by $p(x) = (2\pi S^*(\tau))^{-1/2} \exp[-x^2/2S^*(\tau)]$. The steady-state variance $S^*(\tau)$ is obtained by substituting $S(0) = N$ and $t = \tau$ in eq 4.

In the absence of active noise ($f_{\mathrm{act}} = 0$), $S^*(\tau)$ reduces to the steady-state variance of a passive information engine operating in a thermal bath of temperature T as $S^*_{\mathrm{th}}(\tau) = S + (N - S)e^{-2\tau/\tau_r}$.[22] For ultrafast active and passive engines where $\tau \to 0$, the particle does not have time to relax immediately after feedback control; hence, the steady-state variance is equivalent to the variance of the measurement error, $S^*(\tau \to 0) \approx N$. Conversely, the steady-state variance for slower cycle active engines reduces to $S^*(\tau \to \infty) \approx S + f^2_{\mathrm{act}}\tau_r/[\gamma^2(1/\tau_r + 1/\tau_c)]$, which is greater than $S^*_{\mathrm{th}}(\tau \to \infty) \approx S$ of the passive engine. For a given cycle period τ, the departure of $S^*(\tau)$ from the thermal equilibrium variance S can be interpreted in terms of the effective temperature of the particle in the active bath under measurement and feedback control:

$$k_B T_{\mathrm{eff}}(\tau) = kS^*(\tau) \tag{6}$$

Equation 6 is the modified generalized equipartition theorem for the information engine operating in the active bath. It can be observed from eqs 4–6 that the effective temperature of the slower cycle active engines is equal to the effective temperature of the particle in the active bath, $T_{\mathrm{eff}}(\tau \to \infty) \approx (T + T_{\mathrm{act}})$.

Because $p(x)$ and $p(y|x)$ are Gaussian, the PDF of the measurement outcome $p(y) = \int p(x)\, p(y|x)\, dx$ is also Gaussian with variance $S^*(\tau) + N$. We can also obtain the conditional PDF immediately after the measurement $p(x|y)$ using Bayes' theorem, $p(x|y)p(y) = p(y|x)p(x)$.[22,56]

Thermodynamics of the Engine. In the overdamped limit, the kinetic energy of the particle can be ignored, so the change in total energy of the particle during the shift of the potential center is given by the change in potential energy $\Delta V(x)$. Therefore, the work performed on the particle during each shifting of the potential center is equal to the change in its potential energy plus the heat dissipated into the bath, following the thermodynamic first law.[22,57] However, because the potential is shifted instantaneously after the measurement, the particle has no time to move and dissipate energy. Therefore, the work done on the particle during the feedback step is equal to the change in potential energy $W \equiv \Delta V = (1/2)k[(x - y)^2 - x^2)]$. Because the potential center remains fixed during the relaxation step, no work is done on the particle, and only heat is dissipated. Hence, the average extracted work by the particle per cycle in the steady state is given by

$$\begin{aligned}\langle -W \rangle &= -\frac{k}{2}\int dx\, dy\, p(y|x)p(x)[(x - y)^2 - x^2] \\ &= \frac{k}{2}(S^*(\tau) - N)\end{aligned} \tag{7}$$

Equation 7 shows the average extracted work $\langle -W \rangle$ is always positive as long as $S^* > N$. The average extracted work per cycle for a passive engine operating in a thermal bath is given by $\langle -W \rangle_{\mathrm{th}} = (k/2)(S^*_{\mathrm{th}}(\tau) - N)$.[22] Because $S^*(\tau) \geq S^*_{\mathrm{th}}(\tau)$, the extracted work for the active engine with a finite cycle period ($\tau > 0$) is always greater than its thermal counterpart, $\langle -W \rangle > \langle -W \rangle_{\mathrm{th}}$. The maximum amount of extractable work is $\langle -W \rangle_{\max} = (k_B/2)(T + T_{\mathrm{act}})$, which is obtained for the error-free active engine ($N = 0$) with a slower cycle period ($\tau \to \infty$). Therefore, the error-free and quasistatic cycle active information engines are capable of extracting work equal to the total mean effective energy of the particle in the active bath. The average heat supplied to the system in the steady state during the relaxation step is equal to the average extracted work during the feedback, $\langle Q \rangle = \langle -W \rangle$.

We can also find the average mutual information gain for each measurement between the particle position x and the measurement outcome y as

$$\langle I \rangle = \int dx\, dy\, p(y|x)p(x)\ln\frac{p(y|x)}{p(y)} = \frac{1}{2}\ln\left(1 + \frac{S^*(\tau)}{N}\right) \tag{8}$$

Equation 8 shows that the average mutual information gain by the active engine is greater than that of the passive engine operating in a thermal bath, $\langle I \rangle \geq \langle I \rangle_{\mathrm{th}} = (1/2)\ln(1 + S^*_{\mathrm{th}}(\tau)/N)$.

Entropy Production. For the information engine operating in a thermal bath, the total entropy production (normalized by k_B) per cycle in steady state is given by $\langle \Delta S_{\mathrm{tot}} \rangle_{\mathrm{th}} = \langle \Delta S_{\mathrm{sys}} \rangle + \langle \Delta S_m \rangle + \langle \Delta I \rangle$, where ΔS_{sys} is the system entropy change, ΔS_m the entropy change of the medium, and ΔI the net information gain per cycle.[10,24] Equation 6, in a way, suggests the steady-state dynamics of the active information engine is equivalent to that of a passive engine operating in a medium with effective temperature T_{eff}, so the total entropy production $\langle \Delta S_{\mathrm{tot}} \rangle$ of the active information engine should have a similar form as $\langle \Delta S_{\mathrm{tot}} \rangle_{\mathrm{th}}$. Because the PDF of the particle position in the steady state at the beginning of the measurement $p(x, 0)$ and at the end of relaxation $p(x, \tau)$ are the same, there is no change in the average system entropy during each cycle, $\langle \Delta S_{\mathrm{sys}} \rangle \equiv \langle -\ln p(x, \tau) + \ln p(x, 0) \rangle = 0$. In addition, resetting the trap center erases the mutual information between x and y, thus $\langle \Delta I \rangle = \langle I \rangle$. The entropy change of the medium can be estimated as the average heat dissipation per cycle in the steady state divided by the effective temperature, $\langle \Delta S_m \rangle \equiv -\langle Q \rangle / k_B T_{\mathrm{eff}} = \langle W \rangle / k_B T_{\mathrm{eff}}$. Therefore, using the thermodynamic second law $\langle \Delta S_{\mathrm{tot}} \rangle = \langle W \rangle / k_B T_{\mathrm{eff}} + \langle I \rangle \geq 0$, we obtain the bound for the average extracted work of the cyclic information engine in the active bath:

$$\langle -W \rangle \leq k_B T_{\mathrm{eff}} \langle I \rangle \tag{9}$$

Integral Fluctuation Theorem. We can also modify the generalized integral fluctuation theorem,[6] $\langle e^{-(W-\Delta F)/k_B T - \Delta I)} \rangle = 1$, for the cyclic information engine operating in an active bath, where $\Delta F = 0$, as (see the Supporting Information)

$$\langle e^{-(W/k_B T_{\mathrm{eff}} + \Delta I)} \rangle = \int dx\, dy\, p(y|x)p(x) e^{-(W/k_B T_{\mathrm{eff}} + I)} = 1 \tag{10}$$

Note that applying the Jansen inequality to eq 10 yields the modified generalized second law in eq 9, which provides a general bound for the extracted work.

Numerical results. We validate the analytical results in eqs 7–10 via numerical simulations. To achieve this, we numerically solve eq 2 for the cyclic information engine using the Euler method with a time step of $\Delta t = 50\ \mu s$ and obtain the distributions for the particle position x and the measurement outcome y. The input parameters are $T = 293$ K, $\gamma = 6\pi\eta a \approx$

https://doi.org/10.1021/acs.jpclett.2c01736
J. Phys. Chem. Lett. 2022, 13, 6912–6918

18.8 nNm^{-1}s, and $S = (20\ \mathrm{nm})^2$. The stiffness of the harmonic potential is then $k \equiv k_BT/S \approx 10$ pN/μm, and the thermal relaxation time of the particle is $\tau_r = \gamma/k \approx 1.88$ ms. We study the performance of the information engine as a function of f_{act} and τ_c for a fixed distribution of the measurement error $N/S = 0.1$. In addition, we propose that an active information engine with these parameters can be realized in an experiment using the active optical feedback trap technique.[22,58]

Figure 2a shows a plot of the average extracted work $\langle -W/k_BT\rangle$ as a function of the rescaled cycle period τ/τ_r for a fixed value of $\tau_c \gg \tau_r$ and various values of f_{act}. Here, $\langle -W/k_BT\rangle$ is obtained by averaging $-W/k_BT = -(k/2k_BT)[(x-y)^2 - x^2]$ over more than 3.3×10^6 engine cycles. The numerical results (solid circles) agree well with the theoretical predictions of eq 7 (solid curves). We find that, for a given value of f_{act}, the extracted work increases with the cycle period and saturates when $\tau \gtrsim 5\tau_r$. For direct comparison, we also plot $\langle -W/k_BT\rangle$ for the passive engine operating in a thermal bath of temperature T (see the black data in Figure 2a). The extracted work for the active engine is always greater than that for the passive engine. The extracted work increases with f_{act}, and when $f_{act} \gg f_{th}$, where $f_{th} = \sqrt{k_BTk} \approx 0.2$ pN is the thermal strength of the particle in the harmonic potential, the active engine can extract enormous work from the correlated active bath by exploiting the information about the microstates of the system.

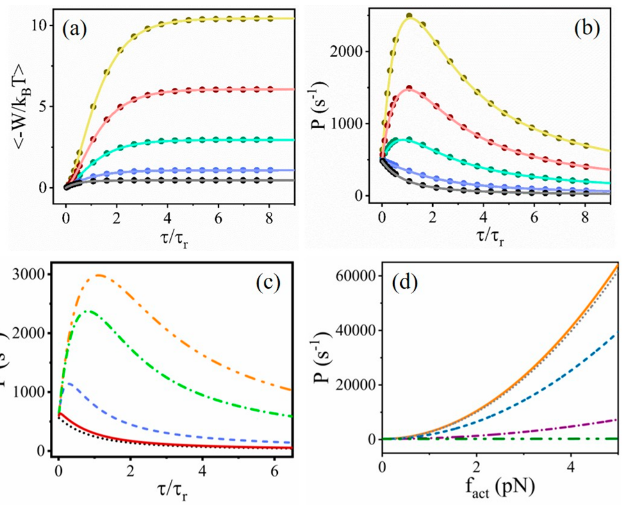


Figure 2. (a) Average extracted work per cycle $\langle -W/k_BT\rangle$ and (b) average extracted power $P \equiv \langle -W/k_BT\rangle/\tau$ of the information engine as a function of the rescaled cycle period τ/τ_r and under a fixed measurement error of $N/S = 0.1$ in a thermal bath (black), as well as in an active bath of fixed correlation time $\tau_c = 25$ ms and noise strength $f_{act} \approx 0.2$ pN (blue), 0.5 pN (olive green), 0.7 pN (burgundy), and 0.9 pN (dark yellow). The solid curves represent the theoretical plots of eq 7 in panel a and eq 7 divided by τ in panel b. (c) Theoretical plot of P vs τ/τ_r for active noise of fixed strength $f_{act} \approx 1$ pN and rescaled correlation time $\tau_c/\tau_r = 0.001$ (black dotted), 0.01 (burgundy solid), 0.1 (blue dashed), 1 (green dash−dotted), and 10 (orange dash−dot−dotted). (d) Theoretical plot of P vs f_{act} evaluated at $\tau/\tau_r = 1$ for $\tau_c/\tau_r = 0.0005$ (olive green dash−dot−dotted) 0.05 (purple dash−dotted), 0.5 (blue dashed), 5 (gray dotted), and 13 (orange solid).

Figure 2b shows a plot of the average extracted power $P \equiv \langle -W/k_BT\rangle/\tau$. For the passive information engines, P is maximum for ultrafast engines with a vanishing cycle period $\tau \to 0$ (see the black data in Figure 2b), for which the extracted work vanishes $\langle -W/k_BT\rangle \to 0$. The extracted power P for ultrafast active information engines is equal to that of ultrafast passive engines. However, P for the finite cycle ($\tau > 0$) active information engine is always greater than its passive counterpart. Interestingly, for $f_{act} > f_{th}$ (see the olive green, burgundy, and dark yellow data in Figure 2b), P for the active information engine increases with τ and reaches a maximum when the cycle period is almost equal to the thermal relaxation time $\tau \approx \tau_r$.

The peak power observed at the finite cycle period ($\tau \approx \tau_r$) is mainly due to the correlation time of the active noise (see Figures 2c and S4 and eqs S9−S14). For $\tau_c \ll \tau_r$, the active noise is equivalent to the Gaussian white noise. Here, P is a maximum for ultrafast engines ($\tau \to 0$) irrespective of the magnitude of f_{act} (Figures 2c and S4a). Also, for $f_{act} \lesssim f_{th}$, thermal noise is dominant and P still exhibits maximum for ultrafast engines regardless of τ_c (Figure S4b,c). We find that P peaks at the finite cycle period only when $f_{act} \gtrsim f_{th}$ and $\tau_c \gtrsim \tau_r$. The peak position shifts toward the higher values of τ and saturates to $\tau \approx 1.26\tau_r$ when $f_{act} \gg f_{th}$ and $\tau_c \gg \tau_r$ (Figure S4c). Figure 2d shows the dependence of the extracted power on the active noise parameters (τ_c and f_{act}) when the cycle period is equal to the thermal relaxation time of the particle $\tau = \tau_r$. In the white Gaussian regime of the active noise, $\tau_c \ll \tau_r$ (olive green curve), the extracted power of the active engine is similar to its passive counterpart. The extracted power increases with increase in τ_c and f_{act}, and for a given $f_{act} > 0$, it increases with τ_c and saturates when $\tau_c \gtrsim 5\tau_r$. Therefore, the finite-cycle active information engine can extract colossal power from the strongly correlated active bath with $\tau_c \gtrsim 5\tau_r$ and $f_{act} \gg f_{th}$.

Figure 3 shows the average mutual information $\langle I\rangle$ between x and y as a function of τ/τ_r under similar conditions as in Figure 2a ($\tau_c \gg \tau_r$ and varied f_{act}). Here, $\langle I\rangle$ is obtained by averaging $\ln[p(y|x)/p(y)]$. The numerical results (solid circles) agree well with the theoretical predictions of eq 8 (solid curves). We find that $\langle I\rangle$ increases with τ and saturates for slower engines $\tau \gg \tau_r$. The saturated value of $\langle I\rangle$ is greater than that of the passive engine. In addition, $\langle I\rangle$ increases with the strength of the active noise.

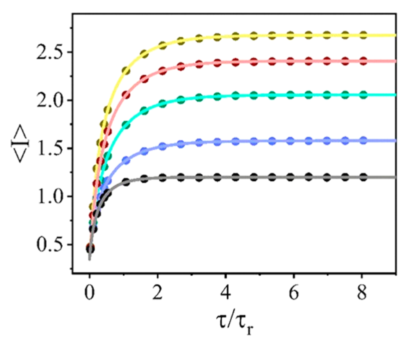


Figure 3. Average mutual information $\langle I\rangle$ per cycle vs τ/τ_r under conditions similar to those of Figure 2a. The solid curves are the theoretical plots of eq 8.

Next, we study the performance of the active engine as a function of τ_c for different values of f_{act}. The extracted work $\langle -W/k_BT\rangle$ and mutual information $\langle I\rangle$ increase with τ_c and saturate when $\tau_c \gg \tau_r$. For passive engines, the extracted work is always bounded by the mutual information, $\langle -W/k_BT\rangle \leq \langle I\rangle$, following the generalized second law in eq 1. For active

https://doi.org/10.1021/acs.jpclett.2c01736
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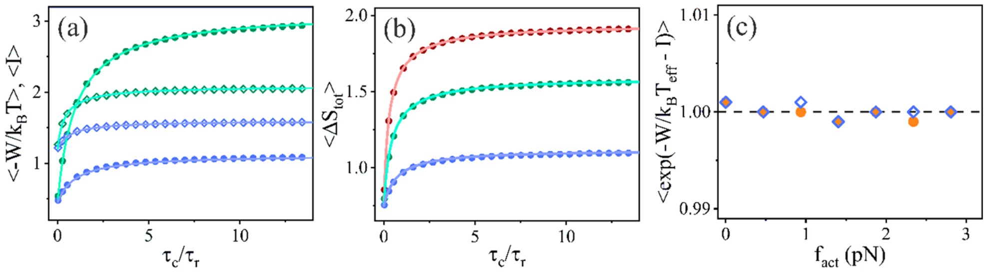

Figure 4. (a) Plot of the extracted work $\langle -W/k_BT\rangle$ (circles) and mutual information $\langle I\rangle$ (diamonds) per cycle as a function of τ_c/τ_r with f_{act} = 0.2 pN (blue) and 0.5 pN (olive green) for slower engines τ = 15 ms with a measurement error of N/S = 0.1. (b) Average total entropy production per cycle $\langle \Delta S_{tot}\rangle = \langle W\rangle/k_BT_{eff} + \langle I\rangle$ as a function of τ_c/τ_r with f_{act} = 0.2 pN (blue), 0.5 pN (cyan), and 0.7 pN (burgundy) for τ = 15 ms and N/S = 0.1. The solid curves show the theoretical predictions. (c) Plot of $\langle e^{-(W/k_BT_{eff}+I)}\rangle$ as a function of f_{act} for fixed values of τ_c = 25 ms and N/S = 0.1 for slower engines τ = 15 ms (orange circles) and faster engines τ = 0.5 ms (blue diamonds).

engines, this is true only for relatively smaller active noise strengths, and when $f_{act} \gg f_{th}$, the saturated value of $\langle -W/k_BT\rangle$ can exceed $\langle I\rangle$ (see the olive green data in Figures 4a and S2). However, it is shown that $\langle -W\rangle$ is always bounded by $k_BT_{eff}\langle I\rangle$. To further examine this phenomenon, we measure the total entropy production per cycle $\langle \Delta S_{tot}\rangle = \langle W\rangle/k_BT_{eff} + \langle I\rangle$ as a function of τ_c, as shown in Figure 4b. $\langle \Delta S_{tot}\rangle$ increases with τ_c and saturates when $\tau_c \gg \tau_r$. Moreover, we find that $\langle \Delta S_{tot}\rangle > 0$, thereby validating the modified generalized second law in eq 9 (Figures 4b and S3). Finally, we test the integral fluctuation theorem in eq 10. To achieve this, we evaluate $\langle e^{-(W/k_BT_{eff}+I)}\rangle$ as a function of f_{act} for $\tau_c \gg \tau_r$ and find it to be equal to unity irrespective of the cycle period (Figure 4c).

In conclusion, we introduced an exactly solvable model for a Maxwell-demon type cyclic information engine operating in a Gaussian correlated active bath. We found that the active engine can extract maximum work and power simultaneously; the extracted power peaks at the finite cycle period where the extracted work becomes nearly saturated. In particular, we showed that the finite cycle active engine is capable of extracting colossal power from the strongly correlated active bath by exploiting positional information concerning the state of the system. We derived the total entropy production for the cyclic active information engine and showed how the generalized integral fluctuation theorem and the generalized second law should be modified. This study provides fundamental insights into manipulating energy and information in nonequilibrium systems under fluctuating and correlated environments.

■ ASSOCIATED CONTENT

Supporting Information

The Supporting Information is available free of charge at https://pubs.acs.org/doi/10.1021/acs.jpclett.2c01736.

Derivation of variance of the probability distribution function of the active information engine in steady state, modified generalized integral fluctuation theorem, and peak power at finite cycle time; four figures showing the plot of effective temperature, violation of generalized second law, validation of modified generalized second law, and peak power at finite cycle time due to correlated active noise (PDF)

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The authors declare no competing financial interest.

■ ACKNOWLEDGMENTS

We thank Cheol-Min Ghim for critically reading the manuscript and providing fruitful comments. This research was supported by the Institute for Basic Science (Grant No. IBS-R020-D1).

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