



Research article

The outcome of Newtonian heating on Couette flow of viscoelastic dusty fluid along with the heat transfer in a rotating frame: second law analysis

Dolat Khan^{a,b}, Poom Kumam^{a,b,c,*}, Ata ur Rahman^d, Gohar Ali^d, Kanokwan Sitthithakerngkiet^e, Wiboonsak Waththayu^a, Ahmed M. Galal^{f,g}^a Fixed Point Research Laboratory, Fixed Point Theory and Applications Research Group, Center of Excellence in Theoretical and Computational Science (TaCS-CoE), Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), 126 Pracha Uthit Rd., Bang Mod, Thung Khru, Bangkok 10140, Thailand^b Center of Excellence in Theoretical and Computational Science (TaCS-CoE), Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), 126 Pracha Uthit Rd., Bang Mod, Thung Khru, Bangkok 10140, Thailand^c Department of Medical Research, China Medical University Hospital, China Medical University, Taichung 40402, Taiwan^d Department of Mathematics, City University of Science & Information Technology, Peshawar, KPK, Pakistan^e Intelligent and Nonlinear Dynamic Innovations Research Center, Department of Mathematics, Faculty of Applied Science, King Mongkut's University of Technology North Bangkok (KMUTNB), 1518, Wongsawang, Bangsue, Bangkok 10800, Thailand^f Department of Mechanical Engineering, College of Engineering in Wadi Alddawasir, Prince Sattam Bin Abdulaziz University, Saudi Arabia^g Production Engineering and Mechanical Design Department, Faculty of Engineering, Mansoura University, P.O 35516, Mansoura, Egypt

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ABSTRACT

The outcome of Newtonian heating on the viscoelastic fluid plays a vital role in daily life applications such as conjugate heat transfer around fins, heat exchanger, solar radiation, petroleum industry, etc. Also, rotation of viscoelastic fluid has various importance in product-making industries and engineering. Viscoelastic dusty fluids and Newtonian heating are applicable in nuclear reactors, gas cooling systems, control temperature of the system and centrifugal separators, etc. Therefore, based on this motivation, the present study presents the Newtonian heating effect on the dusty viscoelastic fluid. Additionally, a free convective heat transfer is taken for Couette flow in a rotating frame along with a uniform applied magnetic field. The dust particles possess complex velocities due to rotation and therefore it is the combination of the primary and secondary velocities. For the specified flow, the entropy generation and Bejan number are also computed. Poincare-Light Hill technique has been used for the solution of the system of partial differential equations. The velocity profile for dust particles and fluid are discussed in this article. The influence of different parameters on the Nusselt number, temperature profile, velocity of fluid and dust particle is discussed thoroughly.

1. Introduction

The mechanical demeanor of various natural fluids is well sufficient to illustrate in terms of Newtonian fluid theory. There are various rheological sophisticated fluids like drilling mud, ketchup, shampoo, paints, blood, and the solutions which are related to polymers are partially illustrated by the theory of Newtonian fluids. Newtonian fluids motion has been a significant subject in biomedical, chemical, and engineering of environmental science [1]. In various feasible situations like a contraction, chemical reactions, evaporation, and heat transfer are consistently followed by the mass transfer procedure in Newtonian fluids. The subject of mixed heat transfer and mass transfer is very accessible in exceptional considerable of various statistics of mechanical transfer

procedures because of the evidence. Additionally, in Newtonian fluids the investigation of flows due to free convection with the repercussion of heat transfer and mass transfer over a perpendicular plate have been considered broadly in the literature because of its industrial and engineering utilization in polymer manufacturing and food processing, granular and fiber covering, and geothermal systems [2, 3, 4, 5, 6, 7].

On the other hand, various researchers are very keen to discuss the multiphase flows of different fluids because of their considerable and broad applications in the enhancement of transfer of heat in gas cooling systems, in blood flows, in exhausting of rockets, and also in the atmosphere where the flow of inert particles occur. The reality of multiphase flows is detected in everyday life regularly, like flows of nature, industrial and mechanical flows. Consequently, there is a privilege of options to

* Corresponding author.

E-mail address: poom.kum@kmutt.ac.th (P. Kumam).<https://doi.org/10.1016/j.heliyon.2022.e10538>

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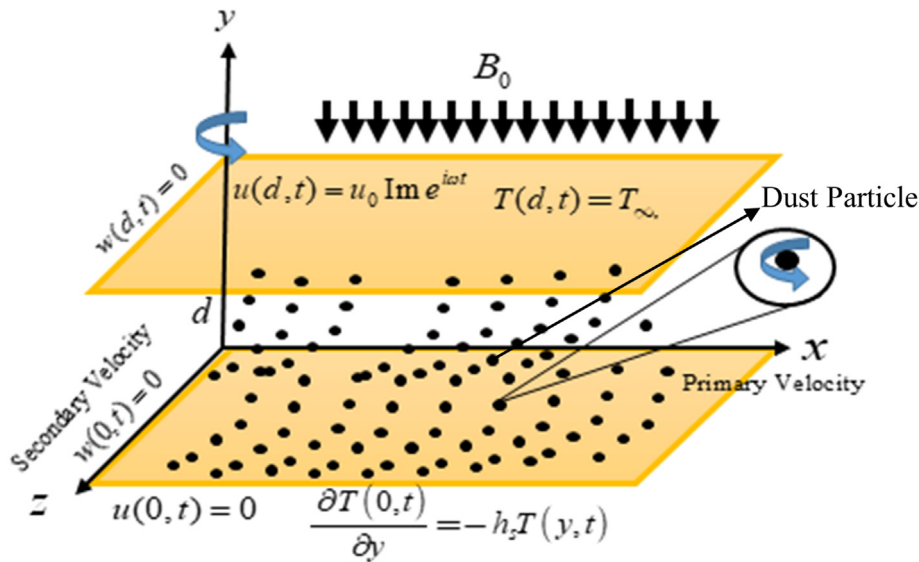


Figure 1. Schematic illustration of the flow.

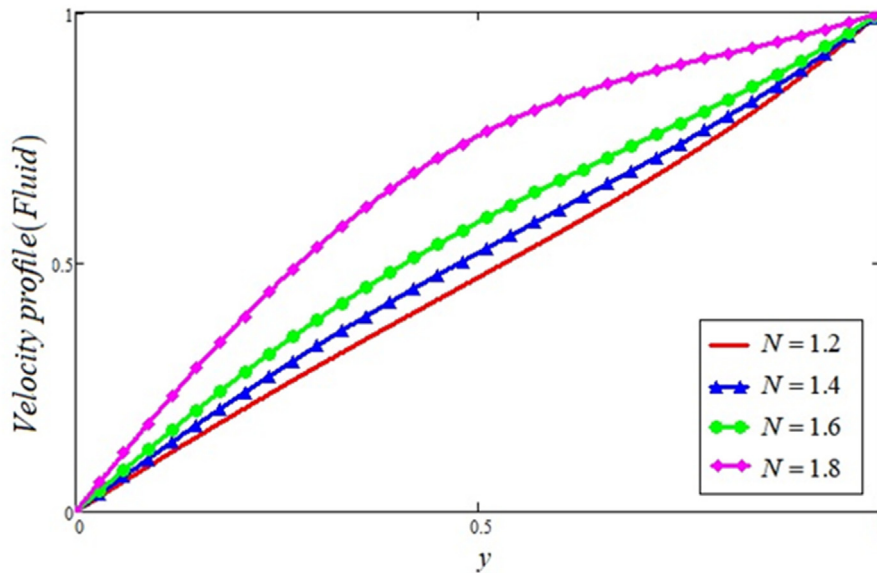


Figure 2. Evolution of velocity (Fluid) against N .

inspect and find out important results for multi-phase flows in terms of any type of fluid even suspended with distinct kinds of particles. A valid amount of exploration can be found in accessible literature which is associated with semi-infinite plans for distinct types of fluid flows covered by various conditions, for example, Palani and Srikanth [8] discussed the flow of a Newtonian fluid under the effect of a magnetic field applied transversely and execute a magnetized mass transfer on a vertical semi-infinite plate. Kumar [9] presented some important results for skin fraction, pressure gradient, and velocities in different directions analytically. He treated the flow of a couple of stress fluids passing through a channel in the inclined form in which low Reynold's number and long wavelength presumption restrained the nonlinear hydromagnetic fluid flow. Similarly, in another paper, the flow of a steady viscous fluid was investigated in a porous medium by Kumar [10]. The basic theory of multiphase flow was introduced by Soo [11] for the first time in 1967, and he studies fluid dynamics in a multiphase system. Moreover, different researchers investigate dusty flows in other consequences in the literature. In this regard, Vimala [12] studies the motion of dusty fluid in

a channel. Similarly, Saffman [13] is presented an article in which they illustrate the balance of gas in the laminar flows. Michael and Miller [14] deliver the solutions for two problems in which they discussed the motion of dusty fluid formed due to the motion of the plate which is infinite. They used the basic formulation derived by Saffman [13] and consider that the gas accommodates dust particles' uniform dissemination. Furthermore, the study of the flows of dusty fluid in cylindrical coordinates and over the flat plate are presented by Healy [15]. Being scientifically tempting and claiming, non-Newtonian fluids have been considered broadly by researchers in different types of non-rotating frames. Abhimanyu et al. [16] investigated microflows of a viscoelastic fluid experiencing transients in rotational electro-hydrodynamics under electrical double layer phenomena. Kaushik et al. [17] studied the double-layer phenomenon for non-Newtonian rotational fluid. Furthermore, the fluid phenomena in a rotating microchannel studied by Kaushik et al. [18] for electroosmotic flow, Balasubramanian et al. [19] for viscoelastic fluid and Kumar Mondal and Wongwises [20] for MHD micropump of nanofluids.

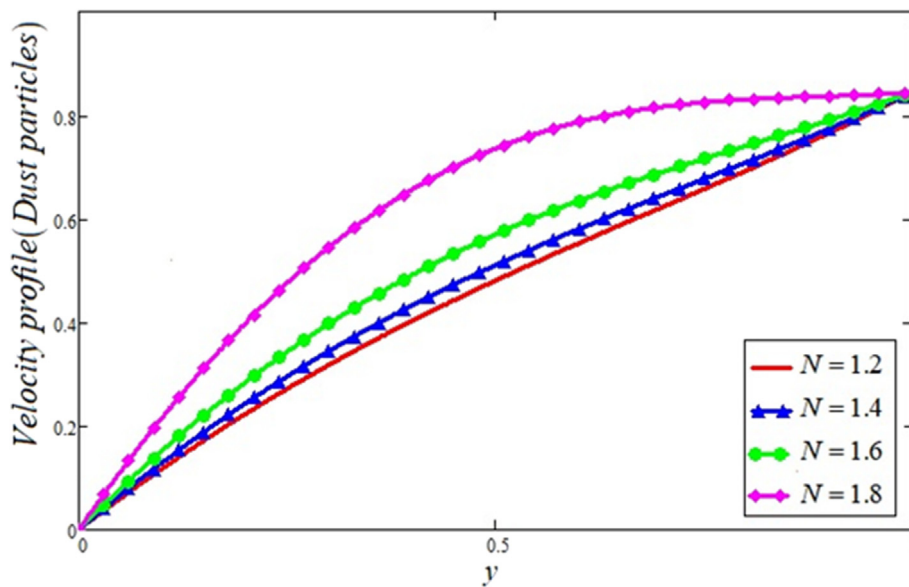


Figure 3. Evolution of velocity (Dust particles) against N .

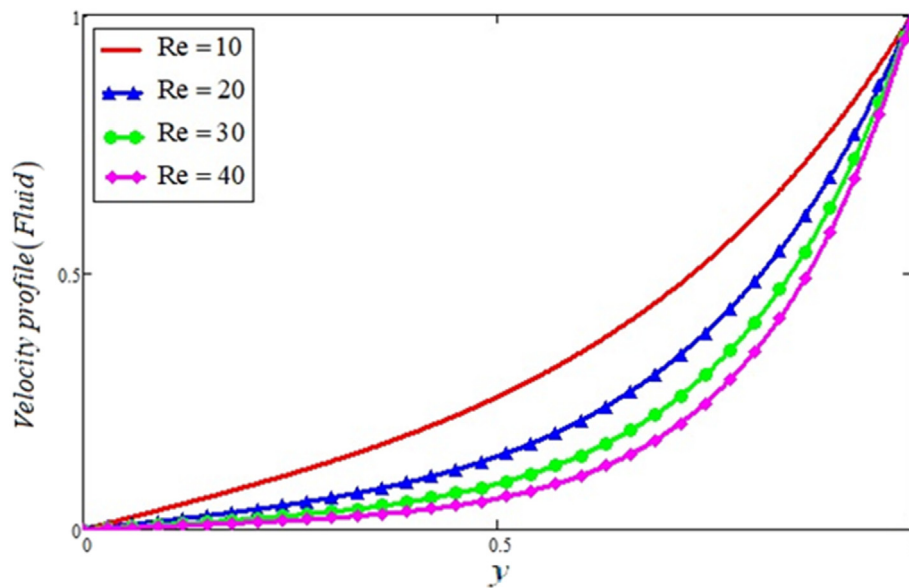


Figure 4. Evolution of velocity (Fluid) against Re .

Similarly, some helpful attempts have also been modeled on the flows of non-Newtonian fluids in rotating frames [21, 22, 23]. Dinesh *et al.* [24] report, recently the influence of Forchheimer, MHD and Radiation Absorption for Unsteady Dusty Viscoelastic Fluid Couette Flow in an Irregular Channel. Computational results are carried out and effect of various parameters are discussed for all profiles. The convective flow with mass and heat transfer with the consequence of a chemical reaction and magnetic field has captivated the researchers with extensive importance because the main reason behind such processes exists in various wings of science and technologies. In a Couette flow model, mass transfer procedures accommodate a fluid that flows according to the laminar regime, which has a variety of applications in chemical engineering processes. Some of the essential applications consist of coating processes, liquid-solid eradication, and a biochemical system like mass transfer actions in oxygenators, dialyzers, and similarly in other membrane processes. The MHD free convection Couette flow was investigated by Reddy *et al.* [25]. While Sinha [26] investigate three dimensional

Couette flow in the presence of chemical reaction and thermo-diffusion. Later, Ali *et al.* [27] investigate a generalized Couette flow with heat and mass transfer.

The physical significance of Newtonian heating is discussed broadly in the literature. According to Newton's law of cooling, the rate of heat loss of a body varies directly to the distinction of temperature between the surrounding and the body. Newtonian heating performs a very decisive role in heat exchanger designing, associate heat transfer about fins, radiation of solar, heating and cooling processes of buildings, and in petroleum industries. The matter of the convection of heat in the cylinders invites a lot of researchers globally due to the enormous number of applications in wires coating and polymer fiber spinning. Merkin [28] discussed four distinct types of heat transfer to the liquid from the surface. Keep in mind Mabood *et al.* [29] study the behavior of the flow of a second-grade fluid with Newtonian heating in a vertical cylinder. He also described these results numerically in the inclined magnetic field with mixed convection. Murthy *et al.* [30] illustrate some important results

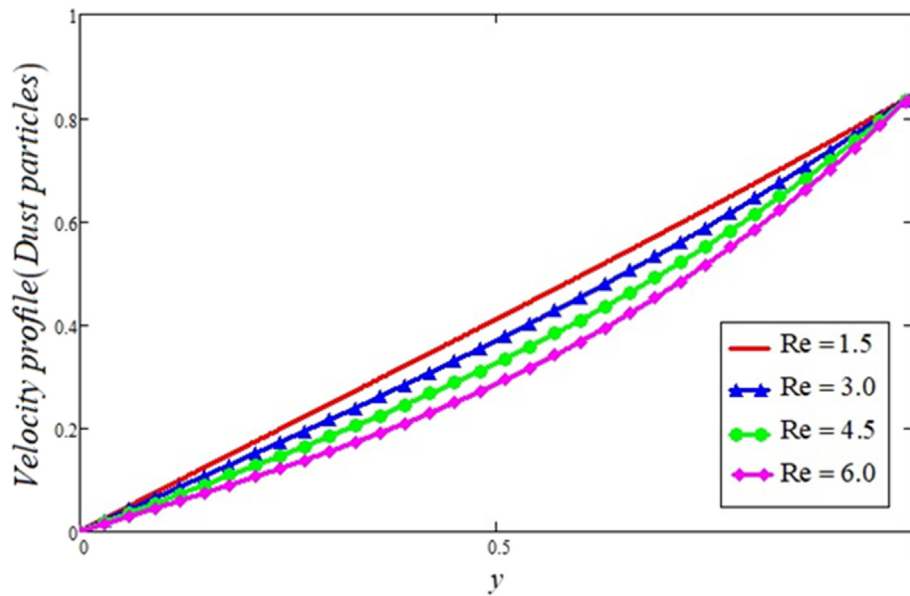


Figure 5. Evolution of velocity (Dust particles) against Re.

related to the flow of Casson fluid with Newtonian heating and slip condition on a stretchable linear cylinder. From these results, it is observed that higher values of Newtonian heating boost up the capacity of heat transfer and temperature of the fluid. Qaiser *et al.* [31] reported the Newtonian heating effect for Walters-B nanofluid through numerical assessment. Later Kamran and Wiwatanapataphee [32] report the influence of Newtonian heating for micropolar fluid along with chemical reaction. While Ahmad and Nadeem [33] highlighted the applications of CNT-based micropolar hybrid nanofluid flow with the effect of Newtonian heating.

As for as dusty viscoelastic fluid is concerned, it is one of the multi-phase flows. The particular importance in various engineering disciplines are heat transfer and flow attitude of viscoelastic fluid among parallel plates. In the aspects of these uses, the study of the perspective of boundary layers has been channeled to viscoelastic fluid. Beard and Walters [34] has shown the boundary layer analysis for a viscoelastic fluid in an idealised state. The transfer of heat of a viscoelastic fluid in the convection flow of Walter's problem was studied by Rajagopal and Na [35]. Many researchers investigate various issues about Couette flow

with heat mass transfers. Some of these problems are discussed in the channel and some over the plate, as mentioned in our above references found in the literature. But to the best of the author's knowledge, the outcome of Newtonian heating on the Couette flow of dusty viscoelastic fluid along with the heat transfer in a rotating frame has not been reported yet. Therefore, this study aims to compose a mathematical model to investigate the different behavior of the influences of heat absorption, viscous dissipation and entropy generation of Couette flow of dusty viscoelastic fluid in a rotating frame.

2. Mathematical modeling

In this article, the incompressible, unidirectional, and one-dimensional unsteady viscoelastic fluid along with ingrained dust particles in spherical shape are considered in a rotating frame. It is thought that the direction of the fluid motion is along x -axis over an infinite plate which is spread out in the direction of x and z . therefore, at $y \geq 0$, the fluid is covered the plane xz . The considered fluid is conducting electrically and a uniform magnetic field B_0 is enforced in the direction of

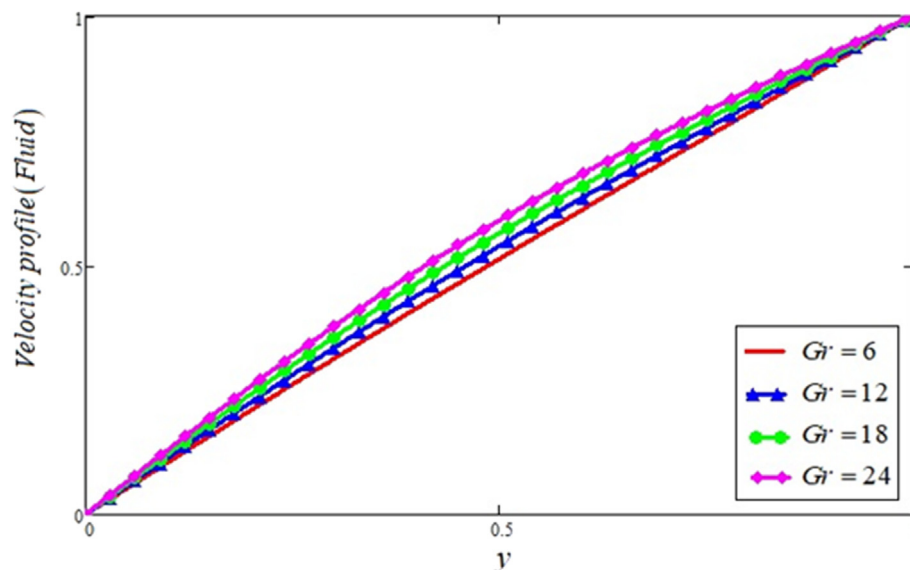


Figure 6. Evolution of velocity (Fluid) against Gr.

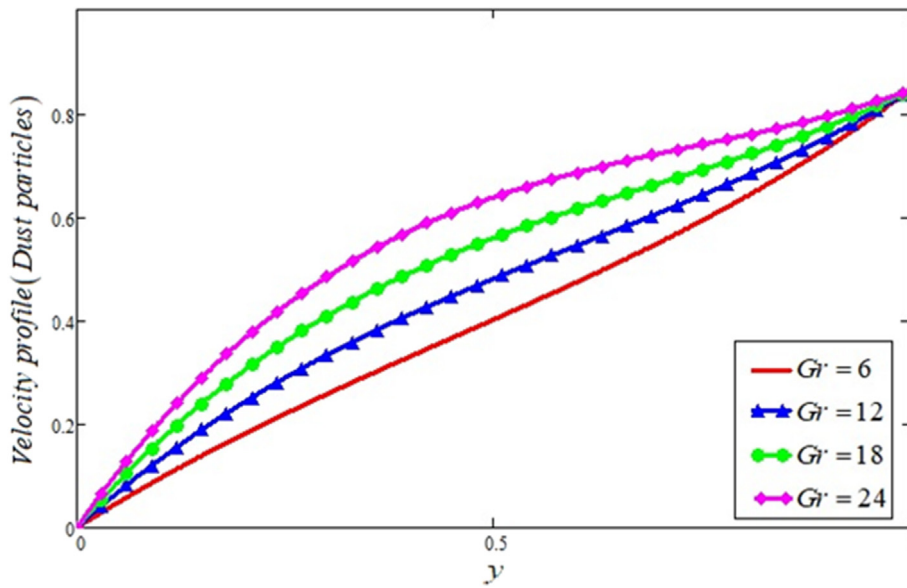


Figure 7. Evolution of velocity (Dust particles) against Gr.

flow transversely. The fallout of thermal radiation with heat transfer is also considered in this problem. The system is treated in solid body rotation with a uniform angular velocity Ω . The lower plate is heated as Newtonian heating ($\frac{\partial T(y,t)}{\partial y} = -h_s T(y,t)$), while the upper plate having embedded temperature. the lower plate is at rest, while the upper plate starts oscillation at $t = 0^+$, with $u_0 \text{Im}e^{i\omega t}$. which is the cause of motion in the fluid, see Figure 1. The secondary velocity is represented by $w(y, t)$.

The momentum equation for the viscoelastic fluid for the rotating flow is modeled as Eq. (1):

$$\left. \begin{aligned} \nabla \cdot \vec{F} &= 0 \\ \rho \left[\frac{d\vec{F}}{dt} + 2(\vec{\Omega} \times \vec{F}) + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}') \right] &= \text{div} \vec{T} + \rho \vec{b}_f + s \\ A_1 &= L^T + L \\ A_2 &= \frac{dA_1}{dt} + LA_1 + A_1L^T \\ \mathbf{T} &= -IP + A_1\mu + A_2\alpha_1 + A_1^2\alpha_2 \end{aligned} \right\} \quad (1)$$

Reference to the pre-published work the basic governing equations for non-Newtonian viscoelastic dusty fluid in a rotating frame in Eqs. (2), (3), (4), (5), (6), and (7) are [36, 37, 38]:

$$\frac{\partial u(y,t)}{\partial t} = -2\Omega w(y,t) + \left(v + \frac{\alpha_1}{\rho} \frac{\partial}{\partial t} \right) \frac{\partial^2 u(y,t)}{\partial y^2} + \frac{K_0 N_0}{\rho} (w_1(y,t) - u(y,t)) - \frac{\sigma B_0^2 u(y,t)}{\rho} + g_x \beta_T (T - T_\infty), \quad (2)$$

$$\frac{\partial w(y,t)}{\partial t} = 2\Omega u(y,t) + \left(v + \frac{\alpha_1}{\rho} \frac{\partial}{\partial t} \right) \frac{\partial^2 w(y,t)}{\partial y^2} + \frac{K_0 N_0}{\rho} (w_2(y,t) - w(y,t)) - \frac{\sigma B_0^2 w(y,t)}{\rho}, \quad (3)$$

$$m \frac{\partial w_1(y,t)}{\partial t} + 2\Omega w_2(y,t) = K_0 (u(y,t) - w_1(y,t)), \quad (4)$$

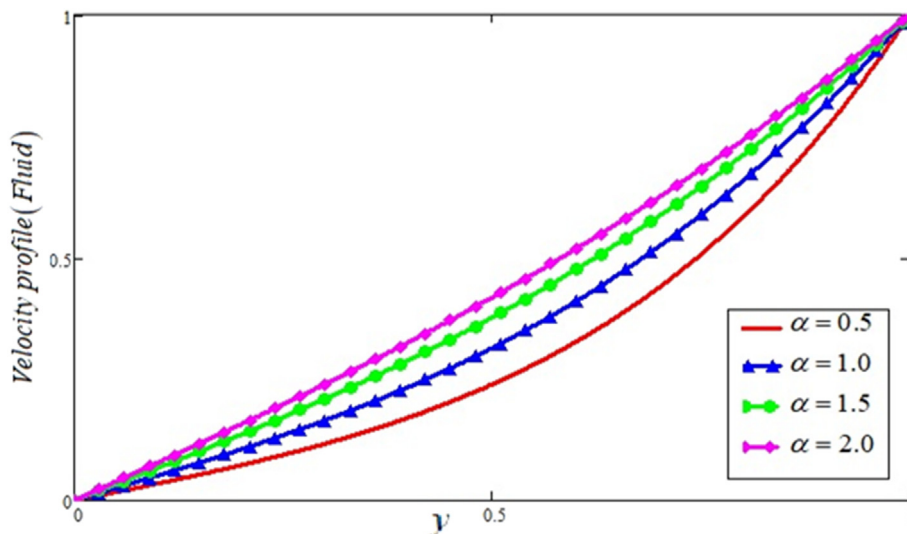


Figure 8. Evolution of velocity (Fluid) against α .

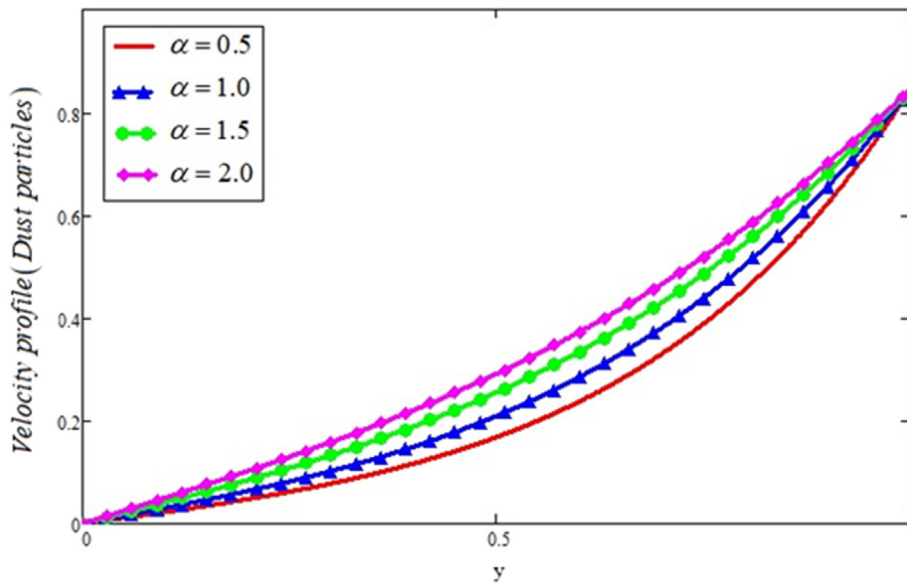


Figure 9. Evolution of velocity (Dust particles) against α .

$$m \frac{\partial w_2(y, t)}{\partial t} = 2\Omega w_1(y, t) + K_0(w(y, t) - w_2(y, t)), \tag{5}$$

$$\frac{\partial T(y, t)}{\partial t} = \frac{k}{\rho c_p} \frac{\partial^2 T(y, t)}{\partial y^2} - \frac{\partial q_r(y, t)}{\partial y}, \tag{6}$$

where, $-\frac{\partial q_r}{\partial y} = 4\alpha_0^2(T - T_\infty)$

The physical boundary and initial conditions are p:

$$\left. \begin{aligned} w(y, 0) = u(y, 0) = 0; \quad t \leq 0, \\ w(0, t) = u(0, t) = 0; \quad y = 0, \\ w(d, t) = 0, \quad u(d, t) = u_0 \text{Im}e^{i\omega t}; \quad y = d, \end{aligned} \right\}, \quad t > 0 \tag{7}$$

Where w and u represent secondary and primary velocities of fluid and similarly w_1 and w_2 shows the primary and secondary velocities of dust particles respectively. The symbols $\nu, T, \rho, \alpha_1, K_0, \sigma, N_0, B_0, g, k, \beta_T, C_p$ and α_0 are kinematic viscosity, fluid temperature, fluid density, material parameter, coefficient of stocks resistance, electrical conductivity, number of density of the dust particles, applied magnetic field, gravitational

acceleration, thermal conductivity, coefficient of thermal expansion, the specific heat capacity of the fluid and coefficient of mean radiation respectively. The initial (primary) and final (secondary) velocities of dust particles and fluid correspond to Eqs. (2), (3), (4), and (5). Similarly, complex velocities have been achieved for both velocities from Eqs. (2), (3), (4), and (5) as Eqs. (8), (9), (10), and (11).

$$\begin{aligned} \frac{\partial \widehat{F}(y, t)}{\partial t} = 2i\Omega \widehat{F}(y, t) + \left(\nu + \frac{\alpha_1}{\rho} \frac{\partial}{\partial t} \right) \frac{\partial^2 \widehat{F}(y, t)}{\partial y^2} + \frac{K_0 N_0}{\rho} (W(y, t) - \widehat{F}(y, t)) \\ - \frac{\sigma B_0^2 \widehat{F}(y, t)}{\rho} + g\beta_T(T - T_\infty), \end{aligned} \tag{8}$$

$$m \frac{\partial W(y, t)}{\partial t} = 2i\Omega W(y, t) + K_0 (\widehat{F}(y, t) - W(y, t)), \tag{9}$$

$$\frac{\partial T(y, t)}{\partial t} = \frac{k}{\rho c_p} \frac{\partial^2 T(y, t)}{\partial y^2} - \frac{\partial q_r(y, t)}{\partial y}, \tag{10}$$

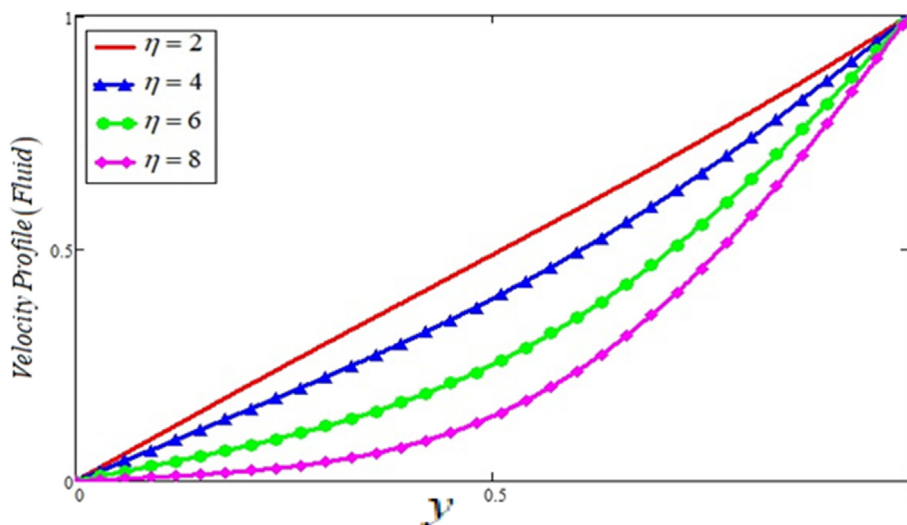


Figure 10. Evolution of velocity (Fluid) against η .

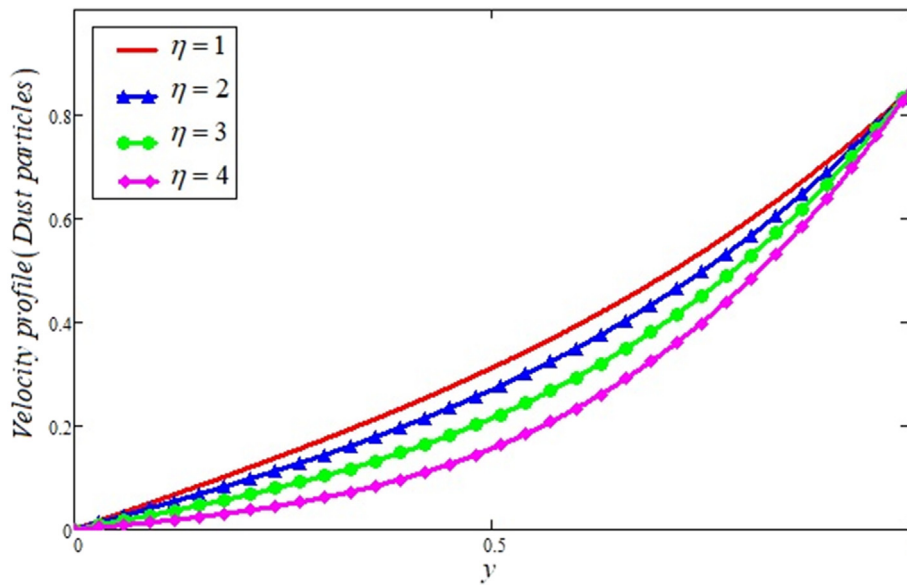


Figure 11. Evolution of velocity (Dust particles) against η .

Where $g_y = 0$, $g = g_x + g_y$, $\widehat{F}(y, t) = iw(y, t) + u(y, t)$ and $W(y, t) = iw_2(y, t) + w_1(y, t)$ are the complex velocities of dust particles and fluid with transformed boundary and initial conditions:

$$\left. \begin{aligned} \widehat{F}(y, 0) = 0, \quad T(y, 0) = T_\infty, \quad t \leq 0, \\ \widehat{F}(y, t) = 0, \quad \frac{\partial T(y, t)}{\partial y} = -h_s T(y, t) \quad y = 0, \quad t > 0, \\ \widehat{F}(d, t) = \widehat{F}_0 \text{Imexp}(i\omega t), \quad T(d, t) = T_\infty, \quad y = d, \quad t > 0, \end{aligned} \right\} \quad (11)$$

Where.

h_s is a heat transfer coefficient and unit is $\frac{1}{m}$.

By considering the solution of Eq. (9), we can assume in Eq. (12)

$$W(y, t) = e^{i\omega t} w_0(y), \quad (12)$$

Therefore, the velocity of dust particles which is in complex form can be expressed by means of the complex velocity of the fluid as follows in Eq. (13):

$$W(y, t) = \frac{K_0 \widehat{F}(y, t)}{mi\omega - 2i\Omega + K_0}, \quad (13)$$

By using the value of W in Eq. (8), we have Eq. (14):

$$\left. \begin{aligned} \frac{\partial \widehat{F}(y, t)}{\partial t} - 2i\Omega \widehat{F}(y, t) = \left(v + \frac{\alpha_1}{\rho} \frac{\partial}{\partial t} \right) \frac{\partial^2 \widehat{F}(y, t)}{\partial y^2} + \\ \frac{K_0 N_0}{\rho} \left(\frac{K_0}{mi\omega - 2i\Omega + K_0} - 1 \right) \widehat{F}(y, t) - \frac{\sigma B_0^2 \widehat{F}(y, t)}{\rho} + g\beta_T(T - T_\infty), \end{aligned} \right\} \quad (14)$$

Eq. (15) denotes the dimensionless variables.

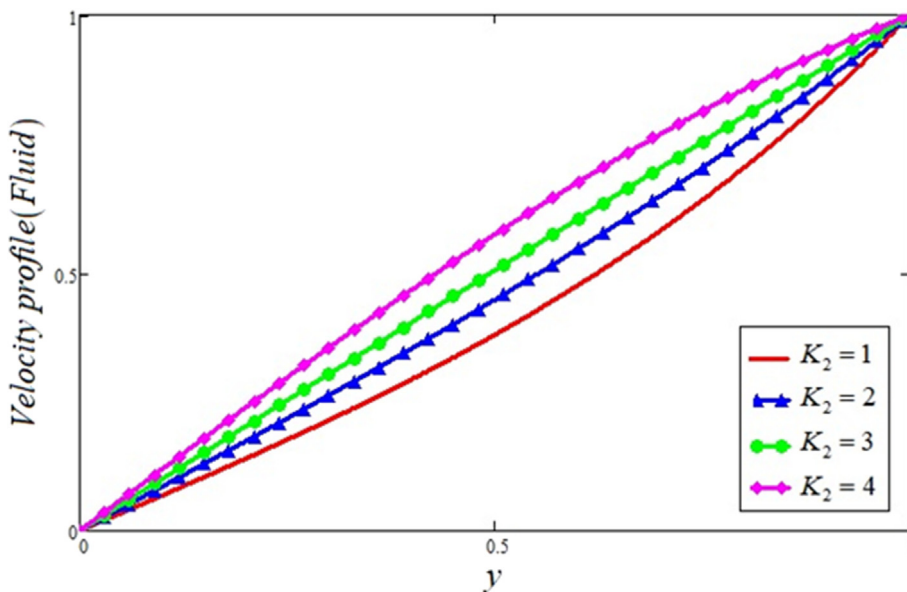


Figure 12. Evolution of velocity (Fluid) against K_2 .

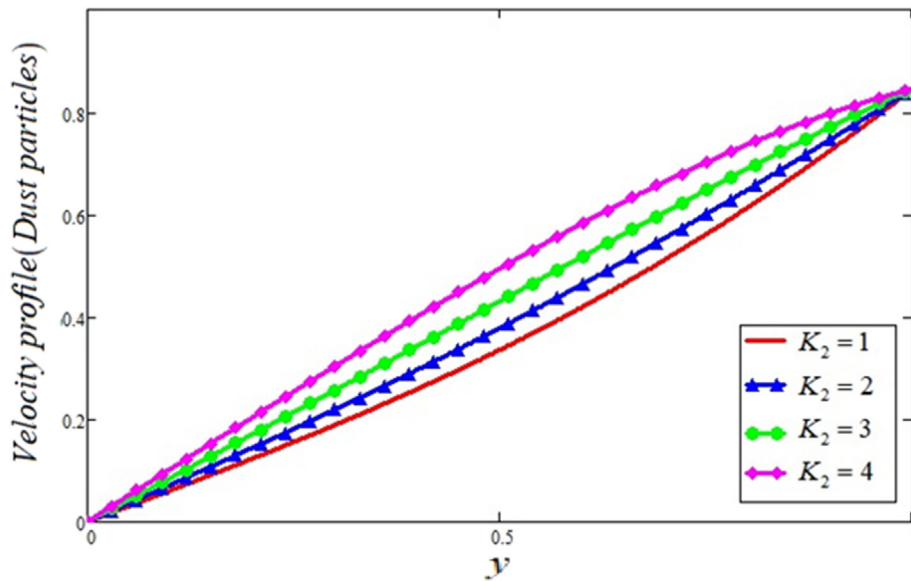


Figure 13. Evolution of velocity (Dust particles) against K_2 .

$$\left. \begin{aligned} \widehat{F}^* &= \frac{\widehat{F}}{\widehat{F}_0}, \quad y^* = yh_s, \quad t^* = \widehat{F}_0 t h_s, \quad \theta = \frac{T - T_\infty}{T_\infty}, \quad \tau^* = \frac{\tau}{\mu v h_s^2} \end{aligned} \right\} \quad (15)$$

By using the above mentioned dimensionless variables in Eqs. (10), (11), and (14) we have Eqs. (16), (17), (18), and (19):

$$\text{Re} \frac{\partial \widehat{F}(y,t)}{\partial t} - 2i\eta \widehat{F}(y,t) = \frac{\partial^2 \widehat{F}(y,t)}{\partial y^2} + \alpha \frac{\partial^3 \widehat{F}(y,t)}{\partial t \partial y^2} + (K_2 - K_1) \widehat{F}(y,t) - M \widehat{F}(y,t) + Gr \theta(y,t), \quad (16)$$

$$Pe \frac{\partial \theta(y,t)}{\partial t} = \frac{\partial^2 \theta(y,t)}{\partial y^2} + N^2 \theta(y,t); \quad (17)$$

$$\left. \begin{aligned} \widehat{F}(0,t) &= 0, \quad \left. \frac{\partial \theta(y,t)}{\partial y} \right|_{y=0} = -(1 + \theta(y,t)) \\ \widehat{F}(1,t) &= \text{Imexp}(i\omega t), \quad \theta(1,t) = 0 \end{aligned} \right\} \quad (18)$$

where

$$\left. \begin{aligned} \text{Re} &= \frac{\widehat{F}_0}{v h_s}, \quad \alpha = \frac{\alpha_1 \widehat{F}_0 h_s}{\rho v}, \quad K_1 = \frac{K_0 N_0}{\rho v h_s^2}, \quad K_2 = \frac{K_0^2 N_0}{\rho v h_s^2 (mi\omega - 2i\Omega + K_0)}, \\ M &= \frac{\sigma B_0^2}{\rho v h_s^2}, \quad Gr = \frac{g \beta_T T_\infty}{v \widehat{F}_0 h_s^2}, \quad Pe = \frac{\rho c_p \widehat{F}_0}{k h_s}, \quad \eta = \frac{\Omega}{v h_s^2}, \quad N^2 = \frac{4\alpha_0^2}{k h_s^2} \end{aligned} \right\} \quad (19)$$

Where $\alpha, \text{Re}, M, Pe, K_1, K_2, N^2, \eta$ and Gr represent the second-grade parameter, dimensionless Reynolds number, magnetic parameter, Peclet number, Dusty parameters (K_1, K_2), radiation parameter, rotational parameter, and Grashof number, respectively. Drop * sign for simplicity.

For the energy equation, consider the following periodic solutions:

$$\theta(y,t) = \theta_0(y) + \varepsilon \theta_1(y) e^{i\omega t} + O(\varepsilon^2). \quad (20)$$

By ignoring the higher order of ε and using the above solution mentioned in Eq. (20), we get.

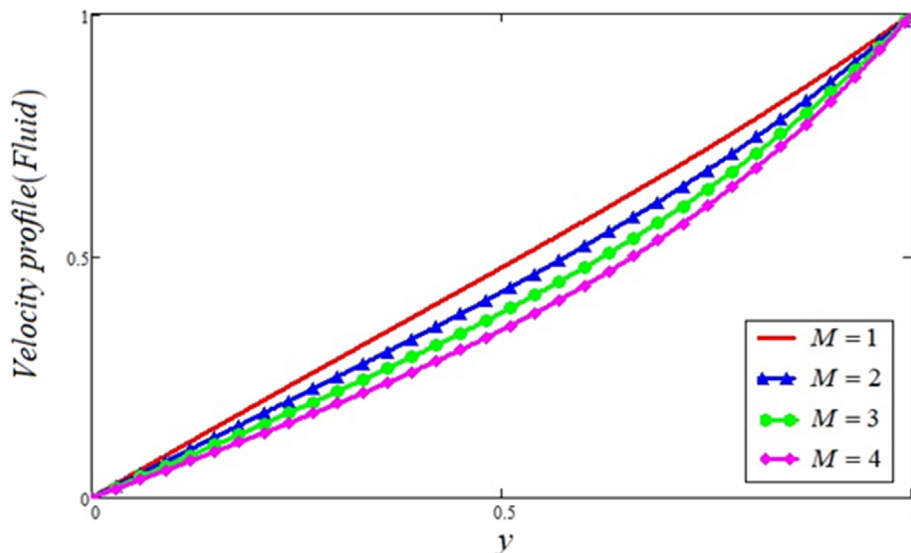


Figure 14. Evolution of velocity (Fluid) against M .

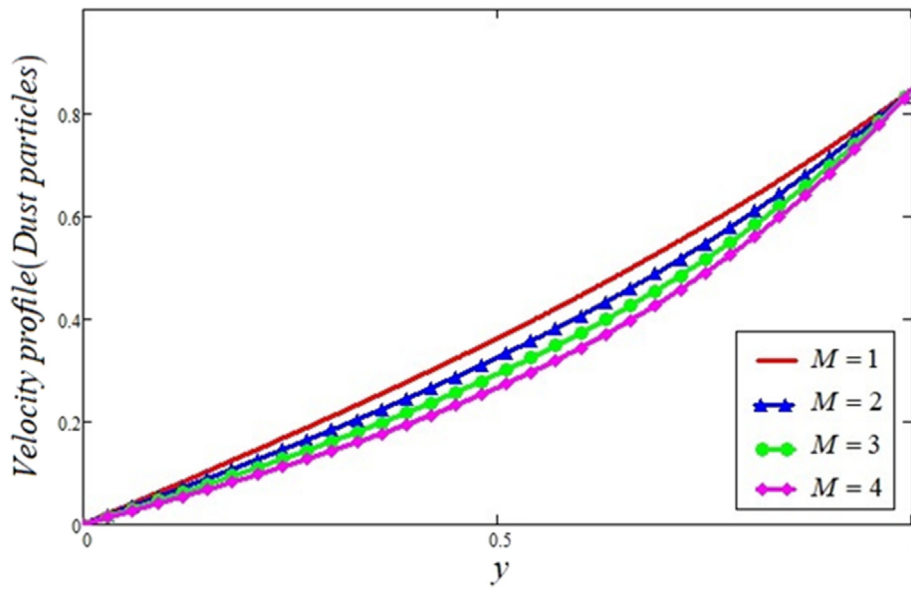


Figure 15. Evolution of velocity (Dust particles) against M .

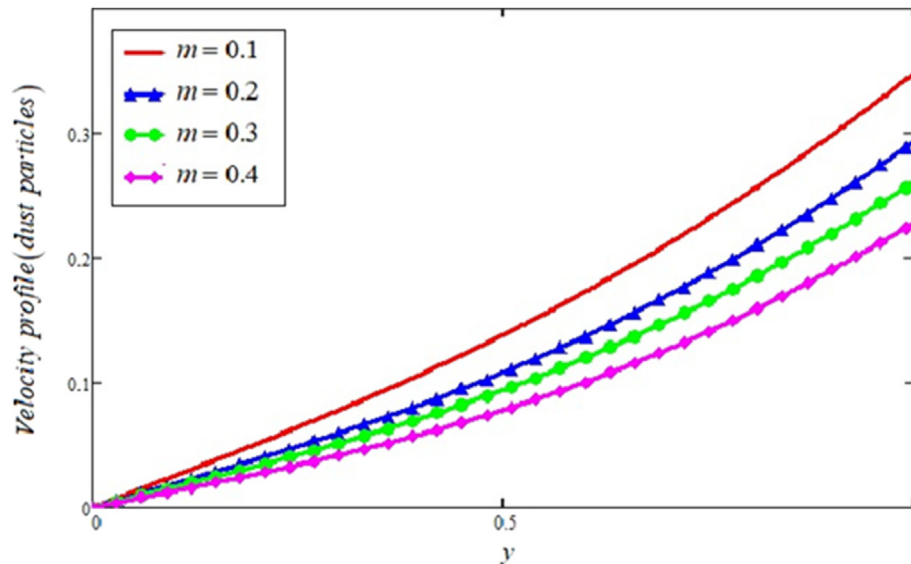


Figure 16. Evolution of velocity (Dust particles) against m .

$$\theta_0(y) = \frac{\sin(N - Ny)}{\sin(N) + N \cos(N)}; \quad \theta_1(y) = 0. \tag{21}$$

Putting the values of $\theta_0(y)$ and $\theta_1(y)$ from Eq. (21) in Eq. (20), we get Eq. (22):

$$\theta(y, t) = \frac{\sin(N - Ny)}{\sin(N) + N \cos(N)}. \tag{22}$$

By considering the periodic solution and pairing the energy equation in momentum equation we obtained the below form:

$$\widehat{F}(y, t) = \widehat{F}_1(y) + \varepsilon \widehat{F}_2(y) e^{i\omega t} + O(\varepsilon^2). \tag{23}$$

$$\left. \begin{aligned} \widehat{F}_1(0) = 0, \widehat{F}_2(0) = 0 \\ \widehat{F}_1(1) = 0, \widehat{F}_2(1) = \frac{1}{\varepsilon} \end{aligned} \right\} \tag{24}$$

By using Eqs. (23) and (24) results in Eq. (16) and split the non-harmonic and harmonic parts, we get Eqs. (25) and (26):

$$\left. \begin{aligned} \widehat{F}_1(y) &= \left\{ \frac{A \sin(N - Ny)}{N \cos(N) + \sin(N)} + H \frac{\cosh(\sqrt{m_2}) \sinh(\sqrt{m_2} y) - H \cosh(\sqrt{m_2} y)}{\sinh(\sqrt{m_2})} \right\}, \\ \widehat{F}_2(y) &= \frac{\sinh(\sqrt{m_1} y)}{\varepsilon \sinh(\sqrt{m_1})} \end{aligned} \right\} \tag{25}$$

$$\widehat{F}(y, t) = \left(\frac{A \sin(N - Ny)}{\sin(N) + N \cos(N)} + H \frac{\cosh(\sqrt{m_2}) \sinh(\sqrt{m_2} y)}{\sinh(\sqrt{m_2})} - H \cosh(\sqrt{m_2} y) \right) + \frac{\sinh(\sqrt{m_1} y)}{\sinh(\sqrt{m_1})} e^{i\omega t} \tag{26}$$

where

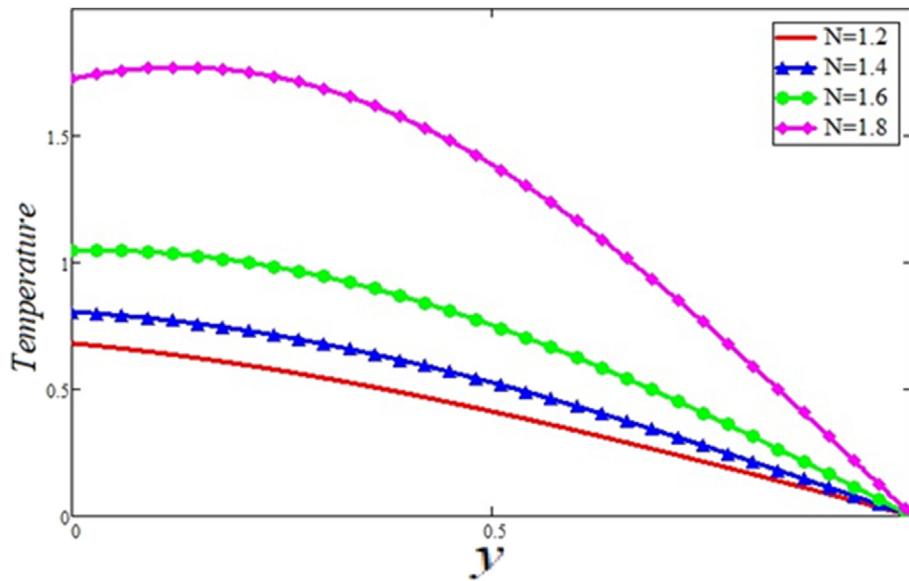


Figure 17. Evolution of temperature against N .

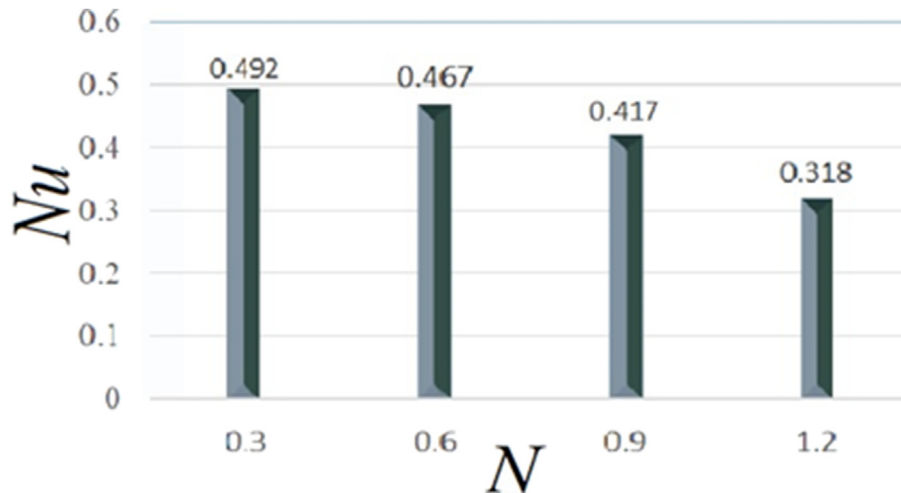


Figure 18. Nusselt number analysis against radiation parameter.

$$A = \frac{Gr}{N^2 + m_2}, \quad H = \frac{A \sin(N)}{N \cos(N) + \sin(N)}, \quad m_0 = K_2 - K_1 - M,$$

$$m_2 = \frac{Rei\omega - 2i\eta - m_0}{1 + \alpha i\omega}, \quad m_2 = K_1 - K_2 + M - 2i\eta,$$

3. Nusselt number

The heat transfer rate is termed as the Nusselt number. Nusselt number in dimensionless form is given by

$$Nu = -\left. \frac{\partial \theta}{\partial y} \right|_{y=0} = \frac{N \cos(N)}{\sin(N) + N \cos(N)}, \tag{27}$$

3.1. Limiting case

By considering the following equation (28)

$$\left. \begin{aligned} \widehat{F}(0, t) = 0, \quad \theta(y, t) = 0 \\ \widehat{F}(1, t) = \text{Imexp}(i\omega t), \quad \theta(1, t) = 1 \end{aligned} \right\} \tag{28}$$

We get the following temperature and velocity Eqs. (29) and (30).

$$\theta(y, t) = \frac{\sin(Ny)}{\sin(N)}. \tag{29}$$

$$\widehat{F}(y, t) = A \left\{ \frac{\sin(Ny)}{\sin(N)} \frac{\sinh(\sqrt{m_2}y)}{\sinh(\sqrt{m_2})} + \frac{\sinh(\sqrt{m_1}y)}{\varepsilon \sinh(\sqrt{m_1})} e^{i\omega t} \right\} \tag{30}$$

Which is similar to the results obtained by [34].

4. Skin friction

Drag force is the force that reduces the motion of the fluid. Skin friction is one of the drag force which occurs between the surface and fluid. In this particular problem at $y = 0$ the skin friction is formed by the fraction between the fluids across the surface of the lower plate. In the case of viscoelastic non-Newtonian fluid, the equation for the skin friction is:

$$\tau = \left(\mu + \alpha_1 \frac{\partial}{\partial t} \right) \frac{\partial \widehat{F}}{\partial y}. \tag{31}$$

Table 1. Physical parameters effects on skin friction.

Gr	K_2	M	N	α	η	$C.f$
6	1	1	0.7	0.5	2	1.065
12						1.153
18						1.240
24						1.327
	2					0.851
	3					1.002
	4					1.193
		2				0.835
		3				0.723
		4				0.630
			1.2			1.024
			1.4			1.130
			1.6			1.341
			1.8			1.931
				0.5		0.003
				1.0		0.053
				1.5		0.164
				2.0		0.172
					2	0.945
					4	0.627
					6	0.242
					8	0.003

Using Eq. (15) in Eq. (31), we obtained the below equation for skin fraction in dimensionless form:

$$\tau = \text{Re} \frac{\partial \widehat{F}}{\partial y} + \alpha \frac{\partial^2 \widehat{F}}{\partial t \partial y}. \tag{32}$$

Putting Eq. (26) in Eq. (32), we obtained the following expression for skin friction as in Eq. (33):

$$\tau = \text{Re}H \left(-N + \sqrt{m_2} \frac{\cosh(\sqrt{m_2})}{\sinh \sqrt{m_2}} \right) + \frac{\sqrt{m_1} e^{i\omega t}}{\sinh(\sqrt{m_1})} \tag{33}$$

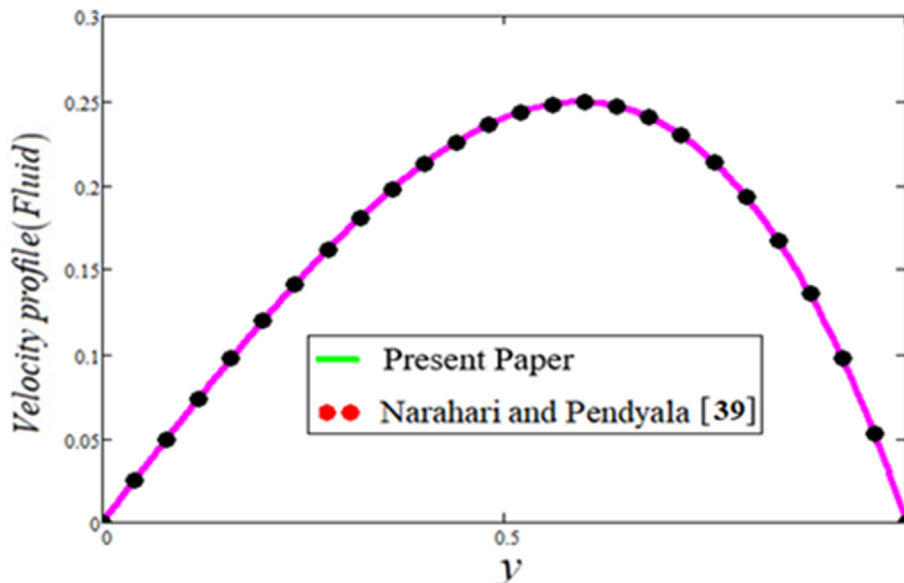


Figure 19. Comparison of the present solution with Narahari and Pendyala [39].

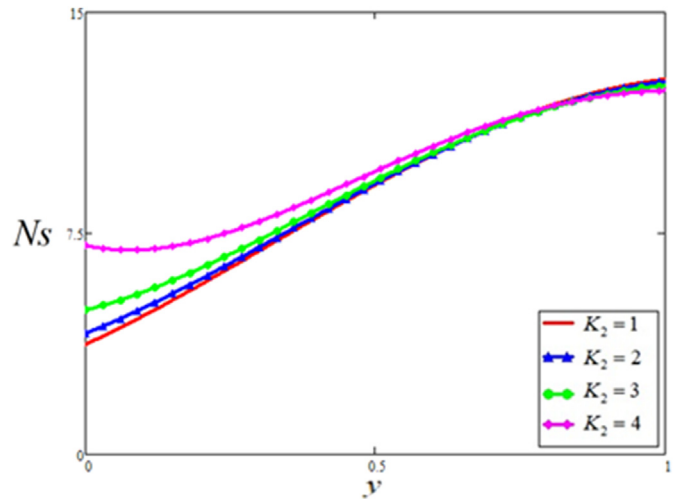


Figure 20. Effect of K_2 on entropy generation.

5. Irreversibility analysis or entropy

Limiting losses or losing useable power in thermodynamical systems is a significant and tough topic for engineers and scientists. In this process, entropy creation is important; in our case, it is given in Eq. (34):

$$Ns = \left(\frac{\partial \theta(y, t)}{\partial y} \right)^2 + \frac{MBr}{\Omega} (u(y, t))^2 + \frac{Br}{\Omega} \left(\frac{\partial u(y, t)}{\partial y} \right)^2 \tag{34}$$

Where Br and Ω is the Brinkman number and dimensionless temperature difference denoted by,

$$Br = \frac{\mu}{\kappa(\theta_w - \theta_\infty)}, \quad \Omega = \frac{\theta_w - \theta_\infty}{\theta_\infty}$$

Moreover, the Bejan number B_e is known as in Eq. (35).

$$B_e = \frac{\left(\frac{\partial \theta(y, t)}{\partial y} \right)^2}{\left(\frac{\partial \theta(y, t)}{\partial y} \right)^2 + \frac{MBr}{\Omega} (u(y, t))^2 + \frac{Br}{\Omega} \left(\frac{\partial u(y, t)}{\partial y} \right)^2} \tag{35}$$

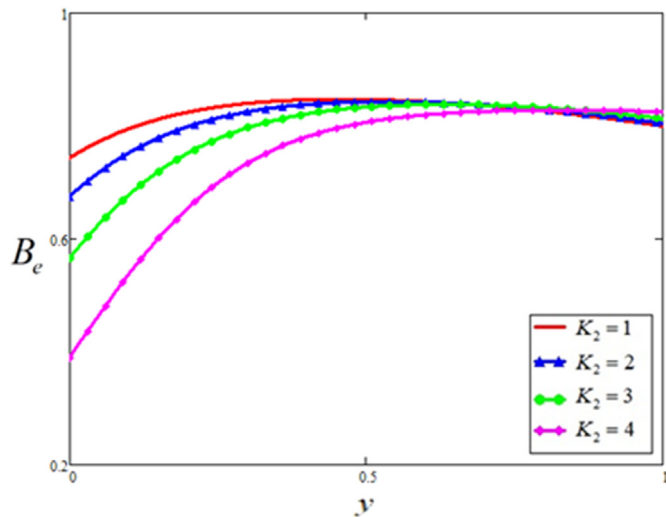


Figure 21. Effect of K_2 on Bejan number.

The Bejan number is well-known for giving a heat transmission concept that is influenced by a fluid fraction and magnetic field management.

6. Discussion and graphical results

To examine the different behaviors of both the velocities, we need to inspect and understand the above-plotted graphs. These graphs give us the other physical conduct of different parameters like parameter of radiation N , Grashof number Gr , Second grade parameter α , rotation parameter η , Magnetic parameter M , Dusty parameters K_2 and Reynolds number Re on both velocities. While Mass of the dust particle m on dust particle velocity. All the figures are plotted with $N = 0.7$, $Gr = 2.5$, $\alpha = 0.1$, $\eta = 0.5$, $M = 5$, $K_2 = K_1 = 0.5$, $Re = 3$, $t = 1$, $\omega = \frac{\pi}{2}$ and $Pe = 1$ fixed values. Figures 2 and 3 corresponds to the behavior of radiations parameter N on the velocity profiles of fluid and dust particles respectively. According to these graphs by increasing the radiation parameter the dust particles and fluid velocity are also increases. It is obvious that by increasing the radiation the temperature of the fluid increases which brings increase in kinetic energy and this why the dust particles and fluid velocity increases. The influence of Reynolds number Re on the velocity of dust particles and fluid and dust particles are shown in Figures 4 and 5 respectively. In these graphs, it is cleared that the Reynolds number is the decreasing function of the fluid velocity and as well as of the dust particles velocity. Reynolds number is used to control the boundary layer, therefore, the Reynolds number retard both the velocities of dust particles and fluid. Moreover, the velocity of fluid retards for larger values of Reynolds numbers while the velocity of dust particles retards for smaller values. Figures 6 and 7 are plotted to investigate the behavior of Grashof Number Gr against the fluid and dust particles velocities correspondingly. These graphs show the direct variation between Grashof number and the velocity of fluid and dust particles. According to the physics of the Grashof number, increasing the Grashof number, the bouncy forces increases. Due to an rise in bouncy forces, the viscosity of fluid decreases, and therefore, the velocity of fluid and dust particles are increasing. Figures 8 and 9 are plotted to study the effect of the parameter of second-grade α for both fluid and dust particles respectively. It is cleared from these plotted graphs that increase occurs in both the dust particles and fluid velocities by increasing in second-grade parameter α . The behavior of rotational parameters on velocities of fluid and dust particles are plotted in Figures 10 and 11 respectively. These graphs show that fluid and dust particles velocities decrease because of the increase in rotational parameters η . The reason behind the retardation in the velocity of fluid and dust particle due to the increase in rotation parameter occurs because it is cleared from the

relation $\eta = \Omega d^2 / \nu$. Furthermore, by increasing rotational parameter, the Coriolis forces are growing, which are actually the inertial forces. It is the fact that the viscosity will be decreasing due to an increase in the rotational parameter, as a result, inertial forces become more muscular and therefore, the decline can be observed in dust particles and fluid velocities. The relation between dusty parameters and the velocities of dust particles and fluid are portrayed in Figures 12 and 13 respectively. These graphs show the increment in the dust particles and fluid velocities by increasing the number of dust particles. According to Stokes drag formula, decrease occurs in the viscous forces of the dusty viscoelastic fluid due to increase in dust particles. Therefore, the plotted graphs show increase in both velocities. The relation between magnetic factor and fluid and dust particles velocities can be observed in Figures 14 and 15, respectively. It is revealed from these graphs that the magnetic parameter is the decreasing function of the velocities of dust particles and fluid.

Physically, greater magnetic parameter values enhance the drag forces called the Lorentz forces, which retards the flow. It is true that the fractional force is motivated to increase by increasing the magnetic parameter values, which contributes to confront the fluid flow and thus reduces its velocity. Figure 16 interrogates the behavioral change in the dust particles velocity profile by changing the dust particles mass, which controls the dust particles velocity. The relation of radiation parameters temperature and N are also discussed in this article. Figure 17 corresponds to show the behavior of radiation parameter N with temperature. This figure shows that there is a direct variation between radiation and temperature. It is the fact that by increasing the radiation parameter N , the kinetic energy is increasing and therefore increase occurs in the temperature of the fluid. Due to the same physics, the radiation parameter N retards the Nusselt number and this behavior is reported and highlighted in Figure 18. Table 1 shows that the skin friction of the fluid can be controlled by increasing η and decreasing the value of Gr , K_2 , N and α . Figure 19 is plotted for the verification of the present solution, by putting $\alpha = \eta = M = K_2 = K_1 = Re = Pe = 0$, and $\omega = 0$ the present solution is coincided with the solution obtained by Narahari and Pendyala [39]. Which is verified our solution. The impact of dusty parameters K_2 against entropy generation and Bejan number is highlighted in Figure 20 and Figure 21 respectively. It is clear that the increasing of dust particle enhance the entropy generation while retard the Bejan number.

7. Conclusion

In this article, the numerical and theoretical behaviors of various physical constraints on the unsteady Newtonian heating Couette flow of dusty viscoelastic fluid along with heat transfer is investigated in a rotational frame. It is considered that the flow is unidirectional, incompressible, one-dimensional and conducting electrically. The dust particles are also conducting electrically and equally scattered in the second-grade fluid. The summary of this article is given in the following key points.

- The Newtonian heating phenomena affect the heating on the plate, which is clear from Figure 17.
- The rate of heat transfer in fluid should be controlled by increasing the radiation parameter.
- The fluid and dust particles velocities gain by increasing N , Gr , α and K_2 while by increasing Re , η and M both velocities are decline.
- The mass of particles retard the velocity of dust particles.
- The entropy generation can be enhance by increasing dust particle parameter.

Declarations

Author contribution statement

Dolat Khan: Conceived and designed the experiments; Performed the experiments; Contributed reagents, materials, analysis tools or data.

Gohar Ali: Conceived and designed the experiments; Wrote the paper.
Wiboonsak Watthayu: Conceived and designed the experiments;
Analyzed and interpreted the data.

Poom Kumam: Performed the experiments; Contributed reagents,
materials, analysis tools or data.

Ahmed M. Galal: Performed the experiments; Analyzed and interpreted
the data.

Kanokwan Sitthithakerngkiet, Ata ur Rahman: Analyzed and interpreted
the data; Contributed reagents, materials, analysis tools or data;
Wrote the paper.

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Data availability statement

Data included in article/supp. material/referenced in article.

Declaration of interest's statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

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