Heliyon 10 (2024) e35848

Contents lists available at ScienceDirect

Heliyon

journal homepage: www.cell.com/heliyon

Research article

Penalized estimation in parametric frailty model

Marwan H. Ahelali [a], Osama Abdulaziz Alamri [b], Anu Sirohi [c,*]

[a] Department of Statistic, University of Tabuk, Tabuk-71491, Kingdom of Saudi Arabia
[b] Department of Statistics, University of Tabuk, Tabuk-71491, Kingdom of Saudi Arabia
[c] Department of Statistics, AIAS, Amity University, Noida, India

ARTICLE INFO

Keywords:
Frailty model
Collinearity
Ridge estimator
Principal component estimator
Infant mortality

ABSTRACT

Frailty model examines the effect of observable and non-observable factors on time to event data. Presence of collinearity produces unstable estimates of parameters. Therefore, this research focus on the penalized estimation of frailty model and proposed the new estimator which is the extension of ridge and principal component estimators. Simulation is run to reveal the performance of proposed estimator. Moreover, the technique is applied on NFHS (National Family Health Survey) data to examine the infant mortality in India.

1. Introduction

In survival analysis frailty model is a random effect model which analyzes the measured and unmeasured heterogeneity simultaneously [21]. It is the extension of proportional hazard regression model which was introduced by Cox [4]. Maximum likelihood estimation is used for the estimation of frailty model but, estimates become unstable, if explanatory variables are highly correlated to each other. In several real life situations, explanatory variables are not independent to each other therefore, alternative inference procedures have been proposed.

Hoerl and Kennard [5], Hoerl et al. [6], introduced the ridge estimator to overcome the problem of collinearity in linear regression model then Schafer et al. [18] extent this approach in logistic regression. Xue et al. [23] implemented the same procedure in Cox regression model furthermore, Albalawi et al. [3] extended it into the frailty model. Accuracy of continuous biomarker is checked through different estimators in different sampling situations, Mahdizadeh and Zamanzade [8,9], Zamanzade et al. [10], Mahdizadeh and Zamanzade [12,11]. Some other estimators are also developed in survival analysis, Ahmad et al. [2] and Michimae and Emura [13]. Ozkale and Arican [16] developed an estimator in logistic regression which is the combined form of ridge and principal component estimator, Sirohi and Rai [19], extent this estimator in proportional hazard regression model. But, this approach is not developed in frailty models.

This research generalizes the Ozkale and Arican's approach in frailty model. Proposed estimator is compared with maximum likelihood (ML), ridge and principal component estimators in respect of different situations. Further, the approach is utilized to examine the infant mortality in India.

* Corresponding author.
E-mail addresses: malhilaly@ut.edu.sa (M.H. Ahelali), oalmughamisi@ut.edu.sa (O.A. Alamri), bsc.ashori@gmail.com (A. Sirohi).

https://doi.org/10.1016/j.heliyon.2024.e35848
Received 10 November 2023; Received in revised form 1 August 2024; Accepted 5 August 2024

Available online 8 August 2024
2405-8440/© 2024 Published by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).

2. Estimation in frailty model

Frailty model is a popular mixed model in survival analysis as it considers measurable and non-measurable factors simultaneously. Mathematical formulation of this model is represented as,

$$\lambda(\tau_i \mid \mathbf{x}_i, \nu_i) = \nu_i \lambda_0(\tau_i)\exp(\gamma' \mathbf{x}_i) \tag{1}$$

In equation (1), $i^{th}(i = 1, 2, ..., n)$ individual's hazard is represented as $\lambda(\tau_i \mid \mathbf{x}_i, \nu_i)$ at time τ_i. Explanatory variables (observable) are represented as the vector $\mathbf{x}_i' = (\mathbf{x}_{i1}, \mathbf{x}_{i2}, ..., \mathbf{x}_{iq})$ and unobserved variable is represented through ν_i which is named as frailty variable. Base-line hazard is represented through the function $\lambda_0(\tau_i)$ and regression coefficients are represented through the vector $\gamma' = (\gamma_1, \gamma_2, ..., \gamma_q)$.

Research is considering the parametric form of frailty model therefore it will have an assumption of the distribution regarding survival time which is taken as Weibull distribution with one parameter η, hence $\lambda_0(\tau_i) = \eta\tau_i^{\eta-1}, \eta > 0$. Likelihood functional form of model-(1) is,

$$L(\gamma) = \Pi_{i=1}^{n}(\nu_i \lambda_0(\tau_i)e^{\gamma' \mathbf{x}_i})^{\Delta_i} \exp(-\nu_i \Psi_0(\tau_i)e^{\gamma' \mathbf{x}_i}) \tag{2}$$

In equation (2), $\Psi_0(\tau_i)$ represents the base-line cumulative hazard and Δ_i is the notation of censoring indicator having the value one (event has been occurred) and zero (observation is right censored). Oakes [14] and Sastry [17] incorporated an assumption that the frailty variable is measured through the variance ρ^2 of gamma distribution $(f(\nu_i) = (1/\rho)e^{-\nu_i/\rho})$, this paper also has the same assumption. Following this the frailty term is integrated out and resulted likelihood function is given below,

$$L(\gamma) = \prod_{i=1}^{n} \left(\frac{\lambda_0(\tau_i)e^{\gamma' \mathbf{x}_i}}{1 + \rho^2 \Psi_0(\tau_i)e^{\gamma' \mathbf{x}_i}} \right)^{\Delta_i} (1 + \rho^2 \Psi_0(\tau_i)e^{\gamma' \mathbf{x}_i})^{-1/\rho^2} \tag{3}$$

On applying log, equation (3) is transformed in log-likelihood function i.e. $\ell(\gamma)$ function,

$$\begin{aligned}\ell(\gamma) = \sum_{i=1}^{n} [\Delta_i[\log(\lambda_0(\tau_i)) + \gamma' \mathbf{x}_i - \log(1 + \rho^2 \Psi_0(\tau_i)e^{\gamma' \mathbf{x}_i})] \\ -(1/\rho^2)\log(1 + \rho^2 \Psi_0(\tau_i)e^{\gamma' \mathbf{x}_i})]\end{aligned} \tag{4}$$

Newtom-Raphson method is utilized to maximize the equation (4),

$$\hat{\gamma}^{(s)} = \hat{\gamma}^{(s-1)} + P(\hat{\gamma}^{(s-1)})^{-1} Q(\hat{\gamma}^{(s-1)}) \tag{5}$$

where s representing the iteration, $Q(\hat{\gamma}^{(s-1)})$ stands for first-order derivative and $P(\hat{\gamma}^{(s-1)})$ stands for the negative of second-order derivative of equation (5) in respect of γ.

$$\begin{aligned}Q(\hat{\gamma}^{(s-1)}) &= \sum_{i=1}^{n} [\Delta_i \mathbf{x}_i - \{(1 + \Delta_i \rho^2)\Psi_0(\tau_i)\mathbf{x}_i e^{\gamma'^{(s-1)} \mathbf{x}_i}\}/\{1 + \rho^2 \Psi_0(\tau_i)e^{\gamma'^{(s-1)} \mathbf{x}_i}\}] \\ P(\hat{\gamma}^{(s-1)}) &= X'VX\end{aligned} \tag{6}$$

In equation (6), V is the notation of diagonal matrix (n x n) and $(i,i)^{th}$ element of it is defined as, $v_{ii} = \tau_i^{\eta} e^{\hat{\gamma}'^{(s-1)} \mathbf{x}_i}(1 + \Delta_i \rho^2)/(1 + \rho^2 \tau_i^{\eta} e^{\hat{\gamma}'^{(s-1)} \mathbf{x}_i})$.

If collinearity presents in the data set, variance matrix raises and leads the unstable estimates. To overcome collinearity this research generalizes the approach of Ozkale and Arican [16] in parametric frailty model. This approach firstly performs principal component estimation then utilizes ridge estimation on the reduced model. Smith and Marx [20] and Aguilera et al. [1] spectrally decomposed the information matrix $\zeta = X'\hat{V}X$ and developed a reduced model. On the basis of this approach, ζ is represented as $\zeta = X'\hat{V}X = U\Omega U'$ here $U = [U_1 ... U_q]$ is an orthogonal matrix of order q x q and $U'\zeta U = A'\hat{V}A = \Omega = \text{diag}(\omega_j)$ is the diagonal matrix (q x q) of eigenvalues of $\zeta(\omega_1 = \omega_{max} \geq \omega_2 \geq ... \geq \omega_q = \omega_{min})$. Orthogonal linear spans with maximum variance of the columns of matrix X is considered as principal component and represented as $A_j = XU_j$, where U_j stand for the eigenvectors of ζ which is associated with their eigenvalues $\omega_j, (j = 1, 2, ..., q)$. These eigenvalues are the variances of principal components. To overcome the problem of collinearity, principal component frailty model is developed on a reduced set of principal components.

Aguilera et al. [1] and Sirohi and Rai [19] expressed logistic regression and Cox proportional hazard regression in matrix form by using logit transformation respectively. In continuation this paper writes frailty model in terms of matrix by using logit transformation.

$$L = X\gamma = XUU'\gamma = A\beta$$

Partitioned form of the matrix A and vector β is given as $A = [A_r A_q - r]$ and $\beta = [\beta_r' \beta_q' - r]'$ here $A_{q-r}(r \leq q)$ includes the principal component which are going to be deleted. Therefor U and Ω is partitioned as, $U = [U_r \;\; U_{q-r}]$ and

$$\begin{bmatrix} \Omega_r & 0 \\ 0 & \Omega_{q-r} \end{bmatrix}$$

respectively, where $A_r'\hat{V}A_r = U_r'X'\hat{V}XU_r = \Omega_r$ and $A_{q-r}'\hat{V}A_{q-r} = U_{q-r}'X'\hat{V}XU_{q-r} = \Omega_{q-r}$. On incorporating decomposition, logit transformation can be represented as, $L = A\beta = A_r\beta_r + A_{q-r}\beta_{q-r}$. Collinearity is addressed by principal component regression, then the model $L = X\gamma = XU_rU_r'\gamma + XU_{q-r}U_{q-r}'\gamma = A_r\beta_r + A_{q-r}\beta_{q-r}$ is shrinked into the model $L = A_r\beta_r$ where $\beta_r = U_r'\gamma$ and $A_r = XU_r$. Shrinked frailty model is written as,

$$\lambda(\tau_i \mid a_{(r)i}, v_i) = v_i\lambda_0(\tau_i)\exp(\beta_r' a_{(r)i})$$

where, $a_{(r)i}' = [a_{i1}, a_{i2}, ..., a_{ir}]$ stands for the ith observations of A_r and $\beta_r' = [\beta_1, \beta_2, ..., \beta_r]$ is the vectors of parameters. Log likelihood function of shrinked frailty model is defined as,

$$\begin{aligned}\ell(\beta_r) = \sum_{i=1}^{n}[\Delta_i[\log(\lambda_0(\tau_i)) + \beta_r' a_{(r)i} - \log(1+\rho^2\Psi_0(\tau_i)e^{\beta_r' a_{(r)i}})]\\ -(1/\rho^2)\log(1+\rho^2\Psi_0(\tau_i)e^{\beta_r' a_{(r)i}})]\end{aligned} \tag{7}$$

Newton-Raphson method is utilized to maximize equation (7),

$$\hat{\beta}_r^{(s)} = \hat{\beta}_r^{(s-1)} + P(\hat{\beta}_r^{(s-1)})^{-1}Q(\hat{\beta}_r^{(s-1)})$$

where $Q(\hat{\beta}^{(s-1)})$ again stands for the first order derivative of equation (10), $P(\hat{\beta}^{(s-1)})^{-1}$ stands for the negative of second order derivative. Estimated parameters are converted into the original form as, $\hat{\gamma}_r = U_r\hat{\beta}_r = U_rU_r'\hat{\gamma}$ and named as principal component frailty (PCF) estimator.

On following the approach of Ozkale and Arican [16], penalty term is introduced into the log-likelihood function of shrinked frailty model.

$$\ell^k(\beta_r) = \ell(\beta_r) - k\frac{1}{2}\|\beta\|_r^{\,2}$$

On maximizing penalized likelihood function

$$\hat{\beta}_r^{k(s)} = \hat{\beta}_r^{k(s-1)} + P^k(\hat{\beta}_r^{k(s-1)})^{-1}Q^k(\hat{\beta}_r^{k(s-1)})$$

where,

$$Q^k(\hat{\beta}_r^{k(s-1)}) = Q(\hat{\beta}_r^{k(s-1)}) - k\hat{\beta}_r^{k(s-1)}$$

$$P^k(\hat{\beta}_r^{k(s-1)}) = P(\hat{\beta}_r^{k(s-1)}) + kI_r$$

I_r is representing the identity matrix (r x r). On using the Cessie and Houwelingen [24] approach,

$$\hat{\beta}_r^{k(s)} = (A_r'\hat{V}A_r + kI_r)^{-1}(A_r'\hat{V}A_r)\hat{\beta}_r^{(s)}$$

which is expressed in the original form as,

$$\hat{\gamma}_r^{k(s)} = U_r(U_r'X'\hat{V}XU_r + kI_r)^{-1}U_r'X'\hat{V}XU_rU_r'\hat{\gamma}^{(s)}$$

and named as $r-k$ frailty estimator. If we take $r \to q$, then $r-k$ converts into ridge frailty estimator and if $k \to 0$, then $r-k$ converts into principal component (PC) frailty estimator. If $r \to q$ and $k \to 0$ then $r-k$ frailty estimator tends to Maximum likelihood (ML) frailty estimator.

3. Comparison of estimators

Asymptotic bias of $r-k$ frailty estimator is,

$$E(\hat{\gamma}_r^k - \gamma) = [U_r(U_r'X'\hat{V}XU_r + kI_r)^{-1}U_r'X'\hat{V}XU_rU_r' - I_q]\gamma$$

On taking $I_q = U_rU_r' + U_{q-r}U_{q-r}'$

$$E(\hat{\gamma}_r^k - \gamma) = [-kU_r(U_r'X'\hat{V}XU_r + kI_r)^{-1}U_r' - U_{q-r}U_{q-r}']\gamma \tag{8}$$

Asymptotic covariance is defined as,

$$\begin{aligned}Cov(\hat{\gamma}_r^k) = U_r(U_r'X'\hat{V}XU_r + kI_r)^{-1}U_r'X'\hat{V}XU_rU_r'Cov(\hat{\gamma})\\ U_rU_r'X'\hat{V}XU_rU_r'(U_r'X'\hat{V}XU_r + kI_r)^{-1}\end{aligned} \tag{9}$$

On the basis of equation (8) and (9), MSE is defined as,

$$\begin{aligned}\mathrm{MSE}(\hat{\gamma}_r^k) = U_r(U_r'X'\hat{V}XU_r + kI_r)^{-1}U_r'X'\hat{V}XU_rU_r'Cov(\hat{\gamma})\\ U_rU_r'X'\hat{V}XU_rU_r'(U_r'X'\hat{V}XU_r + kI_r)^{-1}+\\ [-kU_r(U_r'X'\hat{V}XU_r + kI_r)^{-1}U_r' - U_{q-r}U_{q-r}']\gamma\gamma'\\ [-kU_r(U_r'X'\hat{V}XU_r + kI_r)^{-1}U_r' - U_{q-r}U_{q-r}']\end{aligned}$$

we know that, $(X'\hat{V}X)^{-1}$ can be rewritten as, $U(\Omega)^{-1}U' = U_r(\Omega_r)^{-1}U_r' + U_{q-r}(\Omega_{q-r})^{-1}U_{q-r}'$

$$\begin{aligned}\mathrm{MSE}(\hat{\gamma}_r^k) = U_r(\Omega_r + kI_r)^{-1}\Omega_r(\Omega_r + kI_r)^{-1}U_r' + [-kU_r(\Omega_r + kI_r)^{-1}U_r'\\ -U_{q-r}U_{q-r}']\gamma\gamma'[-kU_r(\Omega_r + kI_r)^{-1}U_r' - U_{q-r}U_{q-r}']\end{aligned}$$

If $r = q$, then MSE of ridge frailty estimator is defined as,

$$\begin{aligned}\mathrm{MSE}(\hat{\gamma}^k) = (X'\hat{V}X + kI_q)^{-1}X'\hat{V}X(X'\hat{V}X + kI_q)^{-1}+\\ k^2(X'\hat{V}X + kI_q)^{-1}\gamma\gamma'(X'\hat{V}X + kI_q)^{-1}\\ \mathrm{MSE}(\hat{\gamma}^k) = U(\Omega + kI_q)^{-1}\Omega(\Omega + kI_q)^{-1}U' + k^2U(\Omega + kI_q)^{-1}U'\\ \gamma\gamma'U(\Omega + kI_q)^{-1}U'\end{aligned}$$

If $k = 0$, then MSE of PC frailty estimator is defined as,

$$\mathrm{MSE}(\hat{\gamma}_r) = U_r\Omega_r^{-1}U_r' + U_{q-r}U_{q-r}'\gamma\gamma'U_{q-r}U_{q-r}'$$

If $k = 0$ and $r = q$, then MSE of ML frailty estimator is defined as,

$$\mathrm{MSE}(\hat{\gamma}) = (X'\hat{V}X)^{-1} = U\Omega^{-1}U'$$

Three choices of k i.e. $1/\hat{\gamma}'\hat{\gamma}$, $q/\hat{\gamma}'\hat{\gamma}$, $(q+1)/\hat{\gamma}'\hat{\gamma}$ are taken on the basis of the approach of Xue et al. [23], Sirohi and Rai [19] and Albalawi et al. [3].

4. Simulation study

Estimators are compared through simulation study. Survival function is defined as,

$$S(\tau_i) = \exp(-\Psi_0(\tau_i)e^{\gamma'\mathbf{x}_i})$$

Distributions function (G) assumed to follow uniform distribution i.e. $U(0,1)$ then $(1-G) \sim U(0,1)$. Therefore,

$$G = \exp(-\Psi_0(\tau_i)e^{\gamma'\mathbf{x}_i}) \sim U(0,1)$$

$$\tau = \Psi_0{}^{-1}(-\log(G)/e^{\gamma'\mathbf{x}_i})$$

On the basis of Weibull distribution assumption with one parameter $\eta = 0.6$, $\Psi_0{}^{-1}(.) = t^{1/\eta}$ and survival time is defined as,

$$\tau = (-\log(G)/e^{\gamma'\mathbf{x}_i})^{1/\eta} \tag{10}$$

Equation (10) is utilized to generate the survival time. Size of samples are taken as, 50, 150, 250, 350, 450, 5000. Censored time is generated through uniform distribution with in the range zero to six and gave approximate 39% censoring rate. Number of explanatory variables are taken as 3, 5, 7, 9, 11 among these, two variables are generated on the basis of the values of correlation coefficients $(p) = 0.60, 0.80, 0.85, 0.90, 0.95, 0.99$. Rest of the variables are generated through standard normal distribution. Initial values of the parameters are taken as, 0.150, 0.263, 0.270 for $(q = 3)$, 0.150, 0.263, 0.270, 0.331, 0.121 for $(q = 5)$, 0.150, 0.263, 0.270, 0.331, 0.121, 0.167, 0.216 for $(q = 7)$, 0.150, 0.263, 0.270, 0.331, 0.121, 0.167, 0.216, 0.450, 0.163 for $q = 9$ and 0.150, 0.263, 0.270, 0.331, 0.121, 0.167, 0.216, 0.450, 0.163, 0.034, 0.045 for $q = 11$. Replication number of the experiment is taken as 1000 for each category. Convergence criteria of the estimated parameter are taken as, 10^{-4} and iteration is terminated at $\left\|\gamma^{(s)} - \gamma^{(s-1)} \leq 10^{-5}\right\|$. Averaged MSE are computed to compare the $r-k$, PC, ridge and ML frailty estimators. Findings of simulation are given in Table 1 to 6.

Result of simulation shows that the $r-k$ frailty estimators have smaller values of MSE as compared to ML, PC and ridge frailty estimator for all categories of p, n, q and k except $n = 5000$ and these MSE are minimum for $k = (q+1)/\hat{\gamma}'\hat{\gamma}$. But ridge frailty estimator for $k = (q+1)/\hat{\gamma}'\hat{\gamma}$, $q/\hat{\gamma}'\hat{\gamma}$, big sample sizes $(n = 5000)$, high correlation $(p = 0.99)$ and $(q \geq 5)$ has smallest MSE as compared to ML, PC and $r-k$ frailty estimators.

5. Application

To show the importance of proposed estimator in real life, this research utilized data from the third phase of NFHS (National Family Health survey) India. But this data is confidential and not allowed to print. Data includes 11,581 infants out of which 10,448

Table 1
Averaged Mean Square error ($n = 50$).

p		$\hat{\gamma}$	$\hat{\gamma}_r$	$\hat{\gamma}^k$	$\hat{\gamma}_r^k$	$\hat{\gamma}^k$	$\hat{\gamma}_r^k$	$\hat{\gamma}^k$	$\hat{\gamma}_r^k$
				$k = 1/\hat{\gamma}'\hat{\gamma}$		$k = q/\hat{\gamma}'\hat{\gamma}$		$k = (q+1)/\hat{\gamma}'\hat{\gamma}$	
0.6	q = 3	0.14633	0.07671	0.02600	0.01817	0.00906	0.00647	0.00650	0.00463
0.8		0.22965	0.07704	0.02985	0.02284	0.01119	0.00903	0.00831	0.00665
0.85		0.28459	0.07627	0.03275	0.02384	0.01270	0.01025	0.00964	0.00781
0.9		0.40190	0.07376	0.03705	0.02706	0.01493	0.01287	0.01155	0.01008
0.95		0.74523	0.07901	0.04296	0.03207	0.01791	0.01632	0.01411	0.01303
0.99		3.50290	0.07731	0.11773	0.05038	0.04229	0.03537	0.03369	0.03118
0.6	q = 5	0.23208	0.17267	0.06643	0.05289	0.01524	0.01187	0.01230	0.00955
0.8		0.32511	0.21391	0.07460	0.06139	0.01829	0.01524	0.01494	0.01243
0.85		0.38361	0.23469	0.06910	0.05673	0.01760	0.01480	0.01456	0.01222
0.9		0.50007	0.26194	0.07257	0.06133	0.01824	0.01535	0.01499	0.01256
0.95		0.86207	0.30457	0.08149	0.07072	0.02304	0.02021	0.01933	0.01687
0.99		3.83314	0.64316	0.16453	0.13585	0.05427	0.05606	0.04820	0.04968
0.6	q = 7	0.32498	0.26469	0.12499	0.10629	0.02543	0.02159	0.02190	0.01856
0.8		0.32511	0.21391	0.07460	0.06139	0.01829	0.01524	0.01494	0.01243
0.85		0.48510	0.35109	0.13811	0.11950	0.02827	0.02419	0.02440	0.02082
0.9		0.62312	0.41296	0.14015	0.12315	0.02987	0.02610	0.02595	0.02261
0.95		1.01744	0.52072	0.17053	0.15094	0.04442	0.03910	0.03940	0.03452
0.99		4.14629	0.91224	0.32952	0.24949	0.09358	0.08720	0.08557	0.07994
0.6	q = 9	0.44262	0.37200	0.19725	0.17430	0.02586	0.02314	0.02244	0.02007
0.8		0.53915	0.43754	0.19416	0.17381	0.02522	0.02272	0.02190	0.01971
0.85		0.64351	0.48788	0.19252	0.17210	0.02493	0.02240	0.02166	0.01945
0.9		0.74140	0.53799	0.18991	0.17298	0.02510	0.02240	0.02181	0.01945
0.95		1.16508	0.69922	0.18161	0.16925	0.02502	0.02244	0.02178	0.01951
0.99		1.25678	1.40652	0.16999	0.16204	0.02448	0.02189	0.02129	0.01903
0.6	q = 11	0.56273	0.48590	0.29244	0.26055	0.04794	0.04294	0.04343	0.03888
0.8		0.67444	0.57390	0.29711	0.26812	0.04638	0.04258	0.04207	0.03861
0.85		2.97974	0.59370	0.28707	0.25988	0.04941	0.04405	0.04540	0.04031
0.9		0.90243	0.70097	0.32626	0.29458	0.05716	0.05270	0.05217	0.04811
0.95		1.32056	0.91030	0.30733	0.28484	0.05338	0.04916	0.04871	0.04485
0.99		4.88150	1.53459	0.22865	0.22249	0.02692	0.02462	0.02394	0.02188

p: Correlation coefficient, q: No. of covariates, γ: regression coefficients.

Table 2
Averaged Mean Square error ($n = 150$).

p		$\hat{\gamma}$	$\hat{\gamma}_r$	$\hat{\gamma}^k$	$\hat{\gamma}_r^k$	$\hat{\gamma}^k$	$\hat{\gamma}_r^k$	$\hat{\gamma}^k$	$\hat{\gamma}_r^k$
				$k = 1/\hat{\gamma}'\hat{\gamma}$		$k = q/\hat{\gamma}'\hat{\gamma}$		$k = (q+1)/\hat{\gamma}'\hat{\gamma}$	
0.6	q = 3	0.04658	0.02347	0.01540	0.01098	0.00561	0.00533	0.00539	0.00411
0.8		0.07363	0.02312	0.01478	0.01111	0.00551	0.00594	0.00537	0.00476
0.85		0.09266	0.02353	0.01851	0.01325	0.00727	0.00788	0.00714	0.00651
0.9		0.12885	0.02374	0.01747	0.01298	0.00674	0.00752	0.00638	0.00616
0.95		0.24191	0.02375	0.02042	0.01468	0.00786	0.00945	0.00765	0.00802
0.99		1.13137	0.02320	0.02879	0.01602	0.01089	0.01120	0.01027	0.00984
0.6	q = 5	0.06969	0.05266	0.02367	0.01901	0.00540	0.00508	0.00518	0.00416
0.8		0.09770	0.06668	0.02668	0.02166	0.00616	0.00595	0.00603	0.00491
0.85		0.11495	0.06927	0.02447	0.02062	0.00529	0.00517	0.00507	0.00422
0.9		0.15275	0.08189	0.02524	0.02168	0.00600	0.00585	0.00573	0.00483
0.95		0.26590	0.09880	0.03113	0.02714	0.00878	0.00883	0.00852	0.00752
0.99		1.16635	0.15840	0.05196	0.04614	0.01713	0.01830	0.01640	0.01614
0.6	q = 7	0.09340	0.07793	0.04007	0.03447	0.00646	0.00690	0.00672	0.00589
0.8		0.12208	0.09536	0.04053	0.03522	0.00692	0.00716	0.00704	0.00614
0.85		0.14211	0.10466	0.04623	0.04034	0.00891	0.00920	0.00926	0.00800
0.9		0.18032	0.11751	0.04661	0.04162	0.00900	0.00924	0.00904	0.00801
0.95		0.29266	0.15590	0.04488	0.04182	0.00927	0.00944	0.00938	0.00823
0.99		1.21029	0.33298	0.06146	0.05988	0.01950	0.01946	0.01871	0.01763
0.6	q = 9	0.11925	0.10372	0.09174	0.08064	0.02898	0.02694	0.02755	0.02456
0.8		0.14839	0.12322	0.09868	0.08642	0.02822	0.02655	0.02695	0.02420
0.85		0.16791	0.13546	0.10071	0.08837	0.02814	0.02643	0.02678	0.02411
0.9		0.20558	0.15491	0.10210	0.08999	0.02766	0.02621	0.02653	0.02393
0.95		0.32227	0.19464	0.09844	0.09024	0.02671	0.02576	0.02582	0.02353
0.99		1.25403	0.44267	0.08287	0.08668	0.02650	0.02515	0.02513	0.02299
0.6	q = 11	0.14611	0.13075	0.11523	0.10384	0.03276	0.03141	0.03187	0.02903
0.8		0.17556	0.15281	0.12315	0.11072	0.03175	0.03063	0.03089	0.02828
0.85		0.19471	0.16495	0.12558	0.11277	0.03149	0.03063	0.03084	0.02831
0.9		0.23580	0.18629	0.12816	0.11578	0.03136	0.03044	0.03056	0.02811
0.95		0.35143	0.23044	0.12456	0.11570	0.03059	0.02981	0.02984	0.02756
0.99		1.29604	0.52297	0.10660	0.10362	0.03023	0.02942	0.02956	0.02722

p: Correlation coefficient, q: No. of covariates, γ: regression coefficients.

Table 3
Averaged Mean Square error ($n = 250$).

p		$\hat{\gamma}$	$\hat{\gamma}_r$	$\hat{\gamma}^k$	$\hat{\gamma}_r^k$	$\hat{\gamma}^k$	$\hat{\gamma}_r^k$	$\hat{\gamma}^k$	$\hat{\gamma}_r^k$
				$k = 1/\hat{\gamma}'\hat{\gamma}$		$k = q/\hat{\gamma}'\hat{\gamma}$		$k = (q+1)/\hat{\gamma}'\hat{\gamma}$	
0.6	q = 3	0.02767	0.01343	0.01251	0.00828	0.00518	0.00517	0.00563	0.00433
0.8		0.04367	0.01360	0.01279	0.00886	0.00489	0.00576	0.00529	0.00484
0.85		0.05466	0.01359	0.01224	0.00876	0.00469	0.00574	0.00497	0.00485
0.9		0.07670	0.01386	0.01262	0.00934	0.00498	0.00650	0.00527	0.00561
0.95		0.14325	0.01363	0.01283	0.00955	0.00505	0.00690	0.00538	0.00605
0.99		0.67649	0.01369	0.02535	0.01087	0.00838	0.00870	0.00794	0.00799
0.6	q = 5	0.04102	0.03124	0.01564	0.01263	0.00332	0.00353	0.00360	0.00290
0.8		0.05716	0.03784	0.01669	0.01397	0.00376	0.00414	0.00410	0.00345
0.85		0.06845	0.04087	0.01677	0.01405	0.00388	0.00415	0.00417	0.00346
0.9		0.09039	0.04457	0.01702	0.01507	0.00406	0.00452	0.00437	0.00378
0.95		0.15758	0.05588	0.01921	0.01817	0.00542	0.00625	0.00575	0.00532
0.99		0.68968	0.09352	0.02970	0.02498	0.00931	0.01056	0.00921	0.00937
0.6	q = 7	0.05514	0.04618	0.02813	0.02417	0.00531	0.00605	0.00607	0.00527
0.8		0.07165	0.05554	0.02752	0.02387	0.00505	0.00571	0.00568	0.00497
0.85		0.07165	0.05554	0.02752	0.02387	0.00505	0.00571	0.00568	0.00497
0.9		0.10553	0.07059	0.03064	0.02736	0.00622	0.00694	0.00699	0.00609
0.95		0.17167	0.09437	0.03142	0.02896	0.00731	0.00780	0.00779	0.00691
0.99		0.71156	0.17449	0.04322	0.04153	0.01410	0.01478	0.01398	0.01350
0.6	q = 9	0.06908	0.06045	0.05903	0.05197	0.02627	0.02410	0.02518	0.02248
0.8		0.08619	0.07143	0.06633	0.05717	0.02561	0.02377	0.02464	0.02213
0.85		0.09713	0.07878	0.06907	0.05951	0.02521	0.02349	0.02419	0.02186
0.9		0.11942	0.08766	0.07261	0.06160	0.02461	0.02330	0.02370	0.02167
0.95		0.18848	0.11539	0.07549	0.06576	0.02399	0.02310	0.02331	0.02151
0.99		0.73688	0.20692	0.06098	0.06788	0.02310	0.02277	0.02259	0.02121
0.6	q = 11	0.08378	0.07525	0.07285	0.06569	0.02999	0.02833	0.02927	0.02668
0.8		0.10084	0.08769	0.08063	0.07189	0.02956	0.02807	0.02885	0.02641
0.85		0.10084	0.08769	0.08063	0.07189	0.02956	0.02807	0.02885	0.02641
0.9		0.13570	0.10851	0.08841	0.07825	0.02850	0.02768	0.02808	0.02604
0.95		0.20394	0.13637	0.09231	0.08169	0.02796	0.02725	0.02763	0.02566
0.99		0.75606	0.25660	0.07763	0.07236	0.02715	0.02682	0.02677	0.02527

p: Correlation coefficient, q: No. of covariates, γ: regression coefficients.

Table 4
Averaged Mean Square error ($n = 350$).

p		$\hat{\gamma}$	$\hat{\gamma}_r$	$\hat{\gamma}^k$	$\hat{\gamma}_r^k$	$\hat{\gamma}^k$	$\hat{\gamma}_r^k$	$\hat{\gamma}^k$	$\hat{\gamma}_r^k$
				$k = 1/\hat{\gamma}'\hat{\gamma}$		$k = q/\hat{\gamma}'\hat{\gamma}$		$k = (q+1)/\hat{\gamma}'\hat{\gamma}$	
0.6	q = 3	0.01969	0.00973	0.01075	0.00690	0.00494	0.00498	0.00563	0.00438
0.8		0.03121	0.00969	0.01046	0.00687	0.00424	0.00506	0.00473	0.00447
0.85		0.03892	0.00968	0.01042	0.00699	0.00409	0.00528	0.00463	0.00468
0.9		0.05474	0.00984	0.01112	0.00741	0.00433	0.00587	0.00485	0.00529
0.95		0.10205	0.00968	0.01060	0.00741	0.00423	0.00599	0.00469	0.00544
0.99		0.48025	0.00960	0.01736	0.00793	0.00594	0.00675	0.00601	0.00632
0.6	q = 5	0.02933	0.02239	0.01367	0.01102	0.00339	0.00371	0.00387	0.00313
0.8		0.04056	0.02661	0.01301	0.01094	0.00302	0.00347	0.00348	0.00291
0.85		0.04842	0.02916	0.01401	0.01198	0.00355	0.00407	0.00407	0.00345
0.9		0.06392	0.03471	0.01365	0.01202	0.00346	0.00395	0.00395	0.00335
0.95		0.11198	0.03955	0.01549	0.01453	0.00447	0.00519	0.00495	0.00446
0.99		0.48918	0.07351	0.02276	0.01930	0.00744	0.00873	0.00748	0.00780
0.6	q = 7	0.03926	0.03283	0.02180	0.01868	0.00468	0.00549	0.00561	0.00485
0.8		0.05066	0.03915	0.02174	0.01903	0.00418	0.00498	0.00497	0.00437
0.85		0.05868	0.04333	0.02246	0.01964	0.00460	0.00543	0.00552	0.00480
0.9		0.07441	0.04960	0.02258	0.02014	0.00476	0.00565	0.00562	0.00501
0.95		0.12183	0.06420	0.02252	0.02148	0.00523	0.00607	0.00600	0.00539
0.99		0.50435	0.13682	0.02670	0.02821	0.00876	0.00932	0.00901	0.00847
0.6	q = 9	0.04914	0.04305	0.04389	0.03862	0.02334	0.02120	0.02245	0.02003
0.8		0.06045	0.05049	0.04985	0.04287	0.02279	0.02098	0.02193	0.01976
0.85		0.06823	0.05557	0.05279	0.04513	0.02243	0.02082	0.02165	0.01960
0.9		0.08443	0.06186	0.05724	0.04747	0.02197	0.02084	0.02130	0.01961
0.95		0.13225	0.08329	0.06197	0.05214	0.02118	0.02025	0.02040	0.01904
0.99		0.51562	0.14699	0.05256	0.05356	0.01996	0.01998	0.01948	0.01880
0.6	q = 11	0.05899	0.05299	0.05343	0.04814	0.02690	0.02505	0.02621	0.02385
0.8		0.07060	0.06136	0.05994	0.05316	0.02626	0.02480	0.02565	0.02357
0.85		0.07853	0.06716	0.06328	0.05591	0.02600	0.02470	0.02550	0.02347
0.9		0.09489	0.07790	0.06833	0.05999	0.02549	0.02421	0.02487	0.02297
0.95		0.14257	0.09859	0.07413	0.06425	0.02476	0.02388	0.02427	0.02269
0.99		0.53127	0.20607	0.06491	0.06370	0.02400	0.02354	0.02352	0.02238

p: Correlation coefficient, q: No. of covariates, γ: regression coefficients.

Table 5
Averaged Mean Square error ($n = 450$).

p		$\hat{\gamma}$	$\hat{\gamma}_r$	$\hat{\gamma}^k$	$\hat{\gamma}_r^k$	$\hat{\gamma}^k$	$\hat{\gamma}_r^k$	$\hat{\gamma}^k$	$\hat{\gamma}_r^k$
				$k = 1/\hat{\gamma}'\hat{\gamma}$		$k = q/\hat{\gamma}'\hat{\gamma}$		$k = (q+1)/\hat{\gamma}'\hat{\gamma}$	
0.6	q = 3	0.01517	0.00752	0.00888	0.00564	0.00435	0.00440	0.00503	0.00399
0.8		0.02423	0.00747	0.00961	0.00590	0.00418	0.00505	0.00472	0.00471
0.85		0.03026	0.00757	0.00960	0.00599	0.00395	0.00513	0.00449	0.00477
0.9		0.04258	0.00764	0.00991	0.00607	0.00394	0.00522	0.00437	0.00487
0.95		0.07937	0.00761	0.00906	0.00616	0.00365	0.00544	0.00409	0.00512
0.99		0.37265	0.00749	0.01468	0.00646	0.00510	0.00585	0.00516	0.00562
0.6	q = 5	0.02250	0.01739	0.01127	0.00908	0.00292	0.00332	0.00349	0.00283
0.8		0.03138	0.02081	0.01094	0.00919	0.00266	0.00315	0.00315	0.00267
0.85		0.03773	0.02346	0.01241	0.01058	0.00329	0.00394	0.00396	0.00339
0.9		0.04968	0.02556	0.01184	0.01059	0.00312	0.00382	0.00368	0.00327
0.95		0.08649	0.03264	0.01330	0.01217	0.00393	0.00477	0.00441	0.00413
0.99		0.38060	0.06167	0.01673	0.01599	0.00579	0.00715	0.00617	0.00639
0.6	q = 7	0.03012	0.02517	0.01691	0.01451	0.00353	0.00427	0.00432	0.00377
0.8		0.03909	0.03080	0.01790	0.01551	0.00377	0.00459	0.00464	0.00407
0.85		0.04519	0.03366	0.01825	0.01598	0.00387	0.00463	0.00471	0.00411
0.9		0.05754	0.03893	0.01846	0.01637	0.00406	0.00484	0.00492	0.00431
0.95		0.09440	0.05021	0.01968	0.01844	0.00496	0.00584	0.00585	0.00526
0.99		0.39060	0.09945	0.02743	0.02679	0.00841	0.00949	0.00894	0.00872
0.6	q = 9	0.03767	0.03303	0.03453	0.03037	0.02063	0.01866	0.01998	0.01780
0.8		0.04669	0.03906	0.04003	0.03429	0.02047	0.01871	0.01971	0.01776
0.85		0.05272	0.04303	0.04289	0.03642	0.02016	0.01852	0.01940	0.01754
0.9		0.06505	0.04990	0.04732	0.03944	0.01965	0.01831	0.01900	0.01732
0.95		0.10210	0.05866	0.05368	0.04184	0.01882	0.01835	0.01821	0.01736
0.99		0.39855	0.12554	0.04793	0.04636	0.01769	0.01776	0.01715	0.01681
0.6	q = 11	0.04535	0.04087	0.04201	0.03793	0.02392	0.02213	0.02331	0.02123
0.8		0.05451	0.04741	0.04790	0.04234	0.02375	0.02227	0.02319	0.02131
0.85		0.06050	0.05192	0.05090	0.04485	0.02347	0.02199	0.02286	0.02103
0.9		0.07283	0.05805	0.05571	0.04759	0.02299	0.02195	0.02251	0.02098
0.95		0.11005	0.07355	0.06326	0.05230	0.02223	0.02157	0.02178	0.02061
0.99		0.40885	0.14253	0.05949	0.05771	0.02119	0.02107	0.02082	0.02014

p: Correlation coefficient, q: No. of covariates, γ: regression coefficients.

Table 6
Averaged Mean Square error ($n = 5000$).

p		$\hat{\gamma}$	$\hat{\gamma}_r$	$\hat{\gamma}^k$	$\hat{\gamma}_r^k$	$\hat{\gamma}^k$	$\hat{\gamma}_r^k$	$\hat{\gamma}^k$	$\hat{\gamma}_r^k$
				$k = 1/\hat{\gamma}'\hat{\gamma}$		$k = q/\hat{\gamma}'\hat{\gamma}$		$k = (q+1)/\hat{\gamma}'\hat{\gamma}$	
0.6	q = 3	0.001365	0.000664	0.001284	0.000648	0.001631	0.000672	0.001213	0.000704
0.8		0.002167	0.000666	0.001896	0.000653	0.001914	0.000700	0.001550	0.000749
0.85		0.002713	0.000667	0.002260	0.000655	0.002039	0.000708	0.001696	0.000761
0.9		0.003806	0.000670	0.002863	0.000660	0.002186	0.000723	0.001813	0.000783
0.95		0.007097	0.000667	0.003989	0.000658	0.002217	0.000729	0.001744	0.000794
0.99		0.033488	0.000668	0.005104	0.000660	0.001648	0.000728	0.001165	0.000792
0.6	q = 5	0.002004	0.001505	0.001798	0.001384	0.001405	0.001270	0.001471	0.001270
0.8		0.002797	0.001828	0.002255	0.001633	0.001466	0.001536	0.001497	0.001538
0.85		0.003339	0.002078	0.002506	0.001788	0.001482	0.001613	0.001503	0.001607
0.9		0.004424	0.002370	0.002831	0.001973	0.001421	0.001772	0.001442	0.001750
0.95		0.007693	0.002688	0.003268	0.002222	0.001327	0.002107	0.001334	0.002066
0.99		0.033815	0.004412	0.003763	0.002919	0.001269	0.002523	0.001224	0.002466
0.6	q = 7	0.001366	0.000663	0.001285	0.000648	0.001636	0.000674	0.001207	0.000708
0.8		0.003466	0.002719	0.002961	0.002418	0.001816	0.001901	0.001997	0.001876
0.85		0.004002	0.002985	0.003212	0.002580	0.001785	0.001976	0.001960	0.001942
0.9		0.005088	0.003449	0.003664	0.002878	0.001795	0.002084	0.001954	0.002035
0.95		0.008358	0.004516	0.004320	0.003372	0.001704	0.002277	0.001847	0.002208
0.99		0.034542	0.006974	0.004916	0.004268	0.001677	0.003032	0.001731	0.002854
0.6	q = 9	0.003311	0.002911	0.003285	0.002889	0.003244	0.002731	0.003077	0.002714
0.8		0.004101	0.003411	0.004040	0.003370	0.003787	0.003090	0.003586	0.003060
0.85		0.004640	0.003764	0.004544	0.003703	0.004092	0.003310	0.003866	0.003270
0.9		0.005735	0.004354	0.005537	0.004249	0.004571	0.003633	0.004311	0.003575
0.95		0.008994	0.005485	0.008278	0.005243	0.005312	0.004131	0.005004	0.004046
0.99		0.035112	0.011353	0.022172	0.009318	0.004923	0.005363	0.004694	0.005174
0.6	q = 11	0.003966	0.003578	0.003939	0.003554	0.003883	0.003344	0.003677	0.003325
0.8		0.004758	0.004146	0.004699	0.004102	0.004406	0.003738	0.004172	0.003707
0.85		0.005307	0.004526	0.005216	0.004463	0.004700	0.003964	0.004447	0.003924
0.9		0.006385	0.005111	0.006203	0.005007	0.005128	0.004267	0.004848	0.004213
0.95		0.009655	0.006703	0.009002	0.006420	0.005776	0.004878	0.005458	0.004788
0.99		0.035771	0.013124	0.023745	0.010864	0.005249	0.006013	0.005067	0.005847

p: Correlation coefficient, q: No. of covariates, γ: regression coefficients.

Table 7
Mean square Error of estimators in NFHS data.

Method	MSE(γ)
MLE	4.214
$\hat{\gamma}^k$	3.124
$\hat{\gamma}^k$ $(k = 1/\hat{\gamma}'\hat{\gamma})$	3.025
$\hat{\gamma}_r^k$ $(k = 1/\hat{\gamma}'\hat{\gamma})$	2.548
$\hat{\gamma}^k$ $(k = q/\hat{\gamma}'\hat{\gamma})$	3.012
$\hat{\gamma}_r^k$ $(k = q/\hat{\gamma}'\hat{\gamma})$	2.037
$\hat{\gamma}^k$ $(k = (q+1)/\hat{\gamma}'\hat{\gamma})$	2.127
$\hat{\gamma}_r^k$ $(k = (q+1)/\hat{\gamma}'\hat{\gamma})$	1.982

Table 8
Estimated parameters in NFHS Data.

Category	$\exp(\hat{\gamma})$	$\exp(\hat{\gamma}_r)$	$\exp(\hat{\gamma}^k)$	$\exp(\hat{\gamma}_r{}^k)$
FS				
One to Three	2.342*	2.331*	2.519*	2.723*
Four to Six#	-	-	-	-
Greater than Six	0.612*	0.659*	0.834*	0.921*
IFA				
No#	-	-	-	-
Yes	1.153*	1.112*	1.261*	1.312*
ME				
Not Educated	10.631*	10.695*	10.732*	10.921*
Primary	10.230*	10.123*	10.342*	10.639*
Secondary	7.461*	7.731*	7.923*	8.036*
Higher#	-	-	-	-
MA (in Years)				
< 18	1.356*	1.389*	1.493*	1.562*
18 - 24#	-	-	-	-
25 - 29	1.412*	1.459*	1.515*	1.623*
30 - 34	1.631*	1.672*	1.722*	1.917*
Greater than 35	1.245*	1.213*	1.316*	1.441*
TT				
No	1.121	1.113	1.261	1.273
One	1.012*	1.013*	1.142*	1.241*
Greater than One#	-	-	-	-
ANC				
No	1.216	1.312	1.161	1.215
One to Two	1.362*	1.316*	1.318*	1.531*
Greater than Two#	-	-	-	-
Marital status				
unmarried	0.413	0.512	0.612	0.712
0 - 4 Years#	-	-	-	-
5 - 9 Years	1.312*	1.415*	1.561*	1.712*
>=10 Years	1.726*	1.816*	1.912*	2.112*
Breastfeeding				
No#	-	-	-	-
Yes	7.881E-05*	7.761E-05*	7.912E-05*	7.325E-05*

Statistical significance: * at 5% level of significance.
Reference Category: #.

are right censored. Here, dependent variable is taken as the survival time of infant in months and eight explanatory variables are considered namely, Family Size (FS), Iron Folic Acid (IFA), Mother Education (ME), Mother Age (MA), Tetanus Toxoid (TT), Antenatal Care (ANC), Marital Status (MS), Breastfeeding (BF). Out of these TT and ANC explain each other upto 60% i.e. $p = 0.6$.

Sometime correlation coefficient misleads collinearity therefore, this research utilizes the Ozkale [15]'s approach to compute condition number. The calculated condition number is 36.45 and it is greater than 10. This is the indication of collinearity [7,22].

Frailty model with different estimation techniques is applied on the data set. Table 7, compares all the estimators in terms of MSE and revealed that the $r-k$ frailty estimator has smaller MSE as compared to the other estimators and the value is minimum for $k = (q+1)/\hat{\gamma}'\hat{\gamma}$. Therefore, Table 8, shows the estimated values of the parameters corresponding to different covariates at $k = (q+1)/\hat{\gamma}'\hat{\gamma}$. If hazard ratio ($exp(\hat{\gamma})$) is less than one, there is a decreased risk of death otherwise there is an increased risk of death.

All the variables have significant impact on infant survival and interpretation is given as: If size of the family is small (one to three members in the family), the risk of infant death is 2.3 times higher as compared to the infant who is having the family of $4-6$ members. If infant is breastfeeding, risk of infant death will be 99% lower as compared to the infant who is not breastfeeding. Uneducated mother has 9.9 times higher risk of infant death as compared to highly educated mother. In the same manner we can see the impact of other variables. Estimated value of ρ^2 is 2.85 which is non zero that means unobserved variables also have impact on infant death.

6. Discussion and conclusion

Frailty model is a mixture model in survival analysis which considers time to event data with observable and un-observable variables simultaneously. In case of collinearity estimated parameters of frailty becomes unstable. Therefore this research proposed the biased $(r-k)$ frailty estimator and compared it with the ML, PC and ridge frailty estimator through simulation study. Findings suggest that the proposed estimator performs better as compared to other estimators for all categories of p, q and k at the sample sizes 50, 150, 250, 350 and 450. Precision of the estimator is more accurate at $k=(q+1)/\hat{\gamma}'\hat{\gamma}$.

Further, the developed approach is applied to examine the infant mortality in NFHS data, India where, collinearity was present in the data set. Here again it is found that the $r-k$ frailty estimator performed better than the other estimators and results are more benefited at $k=(q+1)/\hat{\gamma}'\hat{\gamma}$. Frailty model suggests that breastfeeding has a very positive significant impact on infant mortality. To make less infant mortality, people should believe on joint family system, avoid teenage pregnancy and promote women education. Meanwhile mother should take proper antenatal checkup and IFA tablets.

This research considers the parametric frailty model, in future semi-parametric model can also be considered. Researchers can also work on the estimation of ridge parameter.

CRediT authorship contribution statement

Marwan H. Ahelali: Formal analysis, Conceptualization. **Osama Abdulaziz Alamri:** Conceptualization, Formal analysis. **Anu Sirohi:** Methodology, Software.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] A.M. Aguilera, M. Escabias, M.J. Valderrama, Using principal components for estimating logistic regression with high-dimensional multicollinear data, Comput. Stat. Data Anal. 50 (8) (2006) 1905–1924.

[2] S. Ahmad, M. Aslam, S. Ahmad, Extending the Liu estimator for the Cox proportional hazards regression model with multicollinearity, Commun. Stat., Simul. Comput. (2023) 1–14.

[3] O. Albalawi, A. Sirohi, P.K. Rai, A.R.A. Alanzi, Parametric frailty analysis in presence of collinearity: an application to assessment of infant mortality, Mathematics 10 (13) (2022) 1–10.

[4] D.R. Cox, Regression models and life tables, J. R. Stat. Soc. B 34 (1972) 187–202.

[5] A.E. Hoerl, R.W. Kennard, Ridge regression: biased estimation for nonorthogonal problems, Technometrics 12 (1) (1970) 55–67.

[6] A.E. Hoerl, R.W. Kannard, K.F. Baldwin, Ridge regression: some simulations, Commun. Stat., Theory Methods 4 (2) (1975) 105–123.

[7] M.J. Mackinnon, A.L. Puterman, Collinearity in generalized linear models, Commun. Stat., Theory Methods 18 (9) (1989) 3463–3472.

[8] M. Mahdizadeh, E. Zamanzade, On estimating the area under the ROC curve in ranked set sampling, Stat. Methods Med. Res. 31 (8) (2022) 1500–1514.

[9] M. Mahdizadeh, E. Zamanzade, Smooth estimation of the area under the ROC curve in multistage ranked set sampling, Stat. Pap. 62 (4) (2021) 1753–1776.

[10] E. Zamanzade, M. Mahdizadeh, H.M. Samawi, Efficient estimation of cumulative distribution function using moving extreme ranked set sampling with application to reliability, AStA Adv. Stat. Anal. 104 (3) (2020) 485–502.

[11] M. Mahdizadeh, E. Zamanzade, Dynamic reliability estimation in a rank-based design, Probab. Math. Stat. 39 (2019) 1–18.

[12] M. Mahdizadeh, E. Zamanzade, Reliability estimation in multistage ranked set sampling, REVSTAT Stat. J. 15 (4) (2017) 565–581.

[13] H. Michimae, T. Emura, Bayesian ridge regression for survival data based on a vine copula-based prior, AStA Adv. Stat. Anal. 107 (4) (2023) 755–784.

[14] D. Oakes, A model for association in bivariate survival data, J. R. Stat. Soc., Ser. B, Stat. Methodol. 44 (3) (1982) 414–422.

[15] M.R. Ozkale, The red indicator and corrected VIFs in generalized linear models, Commun. Stat., Simul. Comput. 50 (12) (2021) 4144–4170.

[16] M.R. Ozkale, E. Arican, A new biased estimator in logistic regression model, Statistics 50 (2) (2016) 233–253.

[17] N. Sastry, A nested frailty model for survival data, with an application to the study of child survival in Northeast Brazil, J. Am. Stat. Assoc. 92 (438) (1997) 426–435.

[18] R.L. Schafer, L.D. Roi, R.A. Wolfe, A ridge logistic estimator, Commun. Stat., Theory Methods 13 (1) (1984) 99–113.

[19] A. Sirohi, P.K. Rai, Some $r-k$ class proportional hazard regression models in the presence of collinearity: an evidence from Indian infant mortality, Commun. Stat., Simul. Comput. 52 (10) (2023) 4880–4890.

[20] E.P. Smith, B.D. Marx, III-conditioned information matrices, generalized linear models and estimation of the effects of acid rain, Environmetrics 1 (1) (1990) 57–71.

[21] J. Vaupel, K. Manton, E. Stallard, The impact of heterogeneity in individual frailty on the dynamics of mortality, Demography 16 (3) (1979) 439–454.

[22] L.A. Weissfeld, S.M. Sereika, A multicollinearity diagnostic for generalized linear models, Commun. Stat., Theory Methods 20 (4) (1991) 1183–1198.

[23] X. Xue, M.Y. Kim, E.S. Roy, Cox regression analysis in presence of collinearity: an application to assessment of health risks associated with occupational radiation exposure, Lifetime Data Anal. 13 (2007) 333–350.

[24] S.L. Cessie, J.C.V. Houwelingen, Ridge estimators in logistic regression, Applied Statistics 41 (1992) 191–201.