



Inventory systems with uncertain supplier capacity: an application to covid-19 testing

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Abstract

The COVID-19 pandemic has forced governments to impose crippling restrictions on the day-to-day activities of citizens. To contain the virus and lift these restrictions safely, policymakers need to know quickly where the virus is spreading. This has been possible only through widespread testing. Not long after starting largescale testing in the early stages of the pandemic and more recently with a surge of new variants, countries hit a roadblock—the shortage of swabs used in the testing kits due to disruptions in the supply chain caused by COVID-19. This disruption translates to a variable production capacity of the swab suppliers. As a result, when countries order swabs from a swab supplier, their order might not be fully satisfied. Hence, adopting a proper swab inventory management model can help countries better manage COVID-19 testing and avoid widespread shortages of testing supplies. By considering two different swab demand patterns (i.e., stationary and stochastic) and two different production capacity scenarios for the swab supplier (i.e., ample and variable production capacity), we develop four analytical models, in which we consider all combinations of the above demand and capacity scenarios, to derive the optimal swab-procurement policy for a country. Given the rapid change of COVID-19 infection cases and the limited planning period, countries should aim for reactive scheduling. Through a comprehensive numerical study, we also provide guidelines on how countries should optimally react to these changes in the supply and demand of swabs. The research implications for managing inventory with stochastic supplier capacity and uncertain demand in a finite time horizon extend well beyond the application to COVID-19 testing.

Keywords Disaster management · Inventory management · COVID-19 testing · Supplier capacity uncertainty · Stationary and stochastic demand

1 Introduction

It has been over a year since the COVID-19 pandemic took control of our lives. While essential workers remained on the job, the rest tried to come to terms with life on pause. As of October 20, 2021, based on the WHO situation report, more than 242 million people have been infected globally, and more than 4.91 million people died due to the COVID-19 pandemic (WHO 2021) Furthermore, the United Nations Development Program warns that the COVID-19 pandemic

is far more than a health crisis; in fact, it is changing societies and economies at their core (Amir et al. 2020).

As the COVID-19 pandemic surges worldwide, countries are mobilizing in a united effort to confront, delay, and prevent its spread. Various trials for candidate vaccines and potential therapies have been undertaken. At the beginning of 2021, the FDA issued emergency use authorization for the Pfizer/BioNTech, Moderna, and Johnson & Johnson Vaccines. Similar approval was granted by the European Medicines Agency (EMA) for the AstraZeneca/Oxford vaccine. Although some vaccines have been approved, mass vaccination across the globe is still very slow. It will take months to vaccinate the entire population on different continents (Bushwick 2021). Further, recently reported deaths due to blood clotting complications has caused governments to pause the use of J&J and AstraZeneca vaccines in the U.S. and Europe, making the vaccination pace even slower with fewer supplies (Mahase 2021; Wise 2021). Hence,

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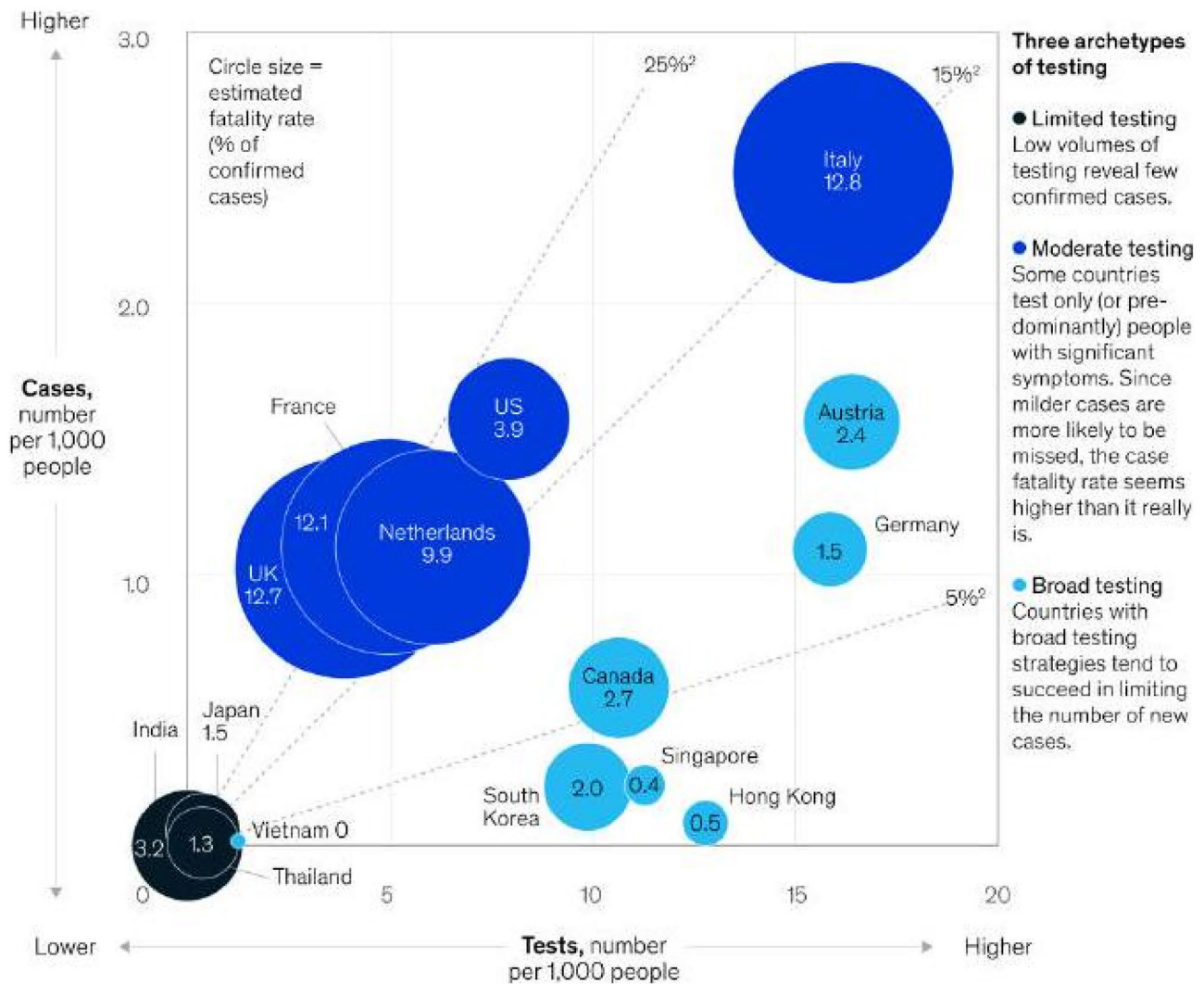


Fig. 1 Illustration of COVID-19 Cases Per Capita Associated with Testing Capacity for Different Countries, adopted from Craven et al. (2020)

until we reach herd immunity created by vaccines, which are currently not widely available for all, testing becomes an essential tool in controlling the spread of COVID-19 (Melwert and Loeb 2021). Note that during the first twelve months of the pandemic, even before vaccines were available, testing was the essential tool to monitor and contain the virus. Therefore, to promptly identify and control the spread of the virus, widespread access to viral testing has played a crucial role as countries and cities plan to loosen distancing measures, and open schools (Eddy 2021). Harvard researchers have also claimed that unless the U.S. could maintain PCR and rapid testing¹ at a significantly high level,

it would take tremendous public health risks to open up the economy (Wise 2020; Melwert and Loeb 2021; Nuzzo and Pond 2021). To reflect the importance of testing in controlling spreads of COVID-19, Fig. 1 shows that countries with the broadest testing tend to have the fewest cases per capita (Craven et al. 2020).

As many health experts suggest (Jones 2020), testing is crucial for containing COVID-19, mainly with the rise of contagious new variants (Salam 2021); yet many countries still grapple with the lack of the necessary materials. Currently, the most common test is a nasal swab test, which involves (1) inserting a swab (i.e., a long stick with a very soft brush on the end) up someone's nose into their sinus, to collect a sample of secretions; and (2) analyzing the sample in the lab for the presence of the virus. The swab test checks for active infections, as opposed to an antibody test, which requires drawing blood to test whether the patient has been

¹ For information about different types of COVID-19 tests, please visit <https://www.fda.gov/consumers/consumerupdates/coronavirus-disease-2019-testing-basics>.

exposed to and/or has recovered from the virus (Chen et al. 2020). Many countries' nasal swab testing rate, including the United States, is not at the level they desire. Many health experts believe that a widespread swab shortage has caused or greatly contributed to this gap (Jones 2020; Nellis 2021). The shortage of swabs started in the early pandemic, and with the recent rise of new COVID-19 variants and the need for more home test kits, it persists (Clark 2021; Zuckerman 2021). For instance, in a recent report published by the College of American Pathologists (CAP) in March 2021, 45% of laboratories nationwide have difficulties obtaining the testing supplies they need, including swabs due to shortages (Stull 2021). Thus, in this study, we tackle the issue of swab shortage in a country². We later show how our modeling framework also extends to similar dilemmas during a pandemic alongside classic inventory problems as additional relevant motivations.

The swabs used to conduct the tests have long been produced mainly by Puritan Medical Products in Maine and Copan Diagnostics in Italy, where COVID-19 hit early and hard, resulting in disruptions in swab supply chains. Consequently, these two suppliers' production capacity (hereafter, production capacity and capacity are used interchangeably) was affected (Pfeiffer et al. 2020). This translates into an unknown capacity for the supplier over time. That is, the more significant the disruption, the more the capacity uncertainty, and vice versa. Therefore, with swab supplier capacity uncertainty, countries must adopt a proper swab inventory management policy to facilitate and well-coordinate the testing procedure. Our paper focuses on an inventory management problem, where the retailer (e.g., a country or a state) orders swabs from a supplier with stochastic capacity. Note that this temporary swab inventory management should continue until most people worldwide are vaccinated because that is when the supply chain disruption is resolved. Due to massive financial support provided by non-governmental organizations and governments worldwide, health authorities expect that vaccination will happen soon in the rest of the world, i.e., following 12 months (Gebrekidan and Apuzzo 2021; Mukherjee 2020). Therefore, our paper considers the time horizon to be finite. This is indeed a key feature for any pandemic problem—the disruption only occurs for a short period of time. Furthermore, because swabs can be ordered at any point in time, we focus on continuous review models.

Additionally, the demand for swab-testing could have different patterns based on the country's situation. For example, in collaboration with public health authorities in Germany, some² researchers have determined the target COVID-19

testing per day should be 200,000 tests (Cohen and Kupferschmidt 2020). In this case, the country orders swabs based on the forecasts; hence, swab demand is deterministic (e.g., 200,000 tests per day). On the other hand, some countries plan to perform a certain number of tests per day. However, they may not be able to control and monitor the spread of COVID-19; thus, frequent outbreaks over time may occur. This leads to changes in swab demand. For instance, the United States plans to perform one million tests per day. However, due to the outbreaks in different regions, the required number of tests varies daily.³ This example, alongside how other countries plan their COVID-19 testing, shows that while the mean of the demand remains the same over time, the actual demand varies. In this case, swab demand is stationarily stochastic. Therefore, in our analytical analysis, we focus on two demand patterns: (1) stationary demand and (2) stochastic demand. On the other hand, in our numerical analysis, we relax the stationarity of the demand and provide guidelines how to optimally react to the changes in the demand distribution parameters. Furthermore, note that any swab shortage results in a COVID-19 testing shortage. If shortages occur, demand for COVID-19 testing does not vanish; rather, it shows itself as back-orders. For example, in the recent surge of COVID-19 in Florida, Arizona, and Texas, people have had to wait in a virtual queue for testing for a couple of days before they could access a test (Weiner 2020). Therefore, in our paper, we consider a case where testing shortages are back-ordered. Although the above discussion focuses on a country, our analysis can be extended to scenarios wherein tests are handled on a state level. For example, Oregon, Missouri, and Arkansas⁴ plan to perform a certain number of tests per day.

To sum up, we study a swab inventory management problem in a finite time horizon setting. Our model considers a country/state/province as the retailer, which orders swabs from an overseas supplier facing supply chain disruption. Depending on the supplier's capacity constraint and the stochasticity of the demand, we study the following four models:

Model 1: In this model, demand is stationary, and the supplier has ample capacity. This model serves us as the first benchmark.

Model 2: In this model, demand is stationary, and the supplier has stochastic capacity. That is, the supplier cannot necessarily fulfill the swab demand.

² Although our analysis is at country-level, similar analysis can be applied to a state, insurance company, or hospital.

³ <https://www.cdc.gov/coronavirus/2019-ncov/cases-updates/testing-in-us.html>.

⁴ <https://rb.gy/fvv4ny> (Oregon), <https://rb.gy/pvm9os> (Missouri), and <https://rb.gy/dgvmov> (Arkansas).

Model 3: In this model, demand is stochastic (i.e., constant mean over time with non-zero variance), and the supplier has ample capacity. This model serves us as the second benchmark.

Model 4: In this model, demand is stochastic (i.e., constant mean over time with non-zero variance), and the supplier has stochastic capacity. That is, the supplier cannot necessarily fulfill the swab demand.

We derive the total cost expression for the above four models. As one may observe, Model 1 is a simplification of Model 3 (i.e., Model 1 can be derived from Model 3 by setting the variance of the demand equal to zero in Model 3). However, Model 1 is the building block of Model 3. For Model 1, we derive the optimal policy. For Model 3, we partially derive the optimal policy and devise an algorithm to find the optimal policy. The starting point of this algorithm is the result of Model 1. Further, Model 2 can be derived from Model 4 by setting the variance of the demand equal to zero in model 4. However, again, Model 2 is the building block of Model 4. For Model 2, we find sufficient conditions to obtain the optimal policy. For Model 4, using a restrictive assumption, which is the result of Model 2, we derive sufficient conditions to partially find the optimal policy. We, then, devise an algorithm to characterize the optimal policy fully. The starting point of this algorithm is the result of Model 2.

Next, through an extensive numerical study, we compare the optimal policies to derive insights for public health administrations regarding how to optimally react to changes in swab supply chain disruptions and changes in daily demand for COVID-19 testing. This study makes three major contributions to the literature:

1. To our knowledge, the problem considered here has not been studied. This problem is essential in practice and has significant implications because it deals with people's lives and livelihoods. As many health experts have noted, better COVID-19 testing results in better strategies to slow the spread of the virus. At the same time, it helps countries and businesses begin to approach their pre-pandemic functions and capacities sooner rather than later.
2. Given the rapid change of COVID-19 infection cases and the limited planning period, governments should aim for data-driven and reactive scheduling. Our results provide guidelines for the decision-makers (e.g., governments) on optimally responding to the changes in the number of daily COVID-19 cases and the disruptions the virus causes in swab supply chains. For example, our results show that under stochastic demand (e.g., when multiple outbreaks occur in the country), as the variation of the supplier's capacity increases, the optimal number of

swabs to be ordered should increase, and consequently, the optimal number of back-orders decreases. Additionally, when the mean of supplier's capacity is large, as the variation of the supplier's capacity decreases, the optimal order quantity should decrease. In other words, when swab supply chains face significantly large disruptions, countries should have a higher swab reorder point.

3. Our managerial insights provide decision-makers with strategies to better control the spread of COVID-19 by reducing the testing back-orders and by optimally ordering swabs under different demand circumstances. When the swab supply chain is disrupted, as the supplier's average swab capacity increases, the country should carry fewer swabs over the time horizon. This implies that as swab-supplier capacity constraint becomes less of an issue for the country, it should decrease its reorder point. This results in a lower holding cost and a higher expected number of back-orders. This observation sheds light on the importance of forecasting. That is, if the country is sure how many swabs it needs during the time horizon (i.e., removing the variability of demand), it can reduce the total cost.

Although this study is motivated by the need for COVID-19 testing, the model and results apply to other situations in which a supply chain is disrupted. The closest example is the shortage of Personal Protective Equipment (PPE) in hospitals and nursing homes that is still persisting (Akhtar 2021; Paulin and Paulin 2021). Another current example is the beef industry in the US. During the pandemic, the meat and poultry processing industries appeared to be walloped. Major meat processing plants (e.g., JBS USA Holdings Incorporation and Tyson Foods Incorporation) have been among major virus hotspots as workers have fallen ill with COVID-19. As a result, their production capacity has been unstable, and retailers have faced widespread meat and poultry shortages. More recently, a lack of crucial semiconductors due to supply disruption during the pandemic has affected most automakers (Reuters 2021). Additionally, the major apparel and footwear brands have requested western governments to support the vaccination of their overseas vital garment manufacturers, which has witnessed a wave of factory shutdowns due to surging coronavirus cases (Hoang 2021). This results in unstable production capacity and brands face product shortages.

One may extend our model to classic inventory problems, wherein a disruption within a finite time interval occurs due to a natural disaster, climate change, or unexpected incidents. Note that our models can be implemented in all these scenarios to find the optimal inventory policies yet under various finite time intervals. However, one may find that the disruption due to pandemics may take longer (e.g., more than two years for COVID-19) than disruption due to climate

change and natural disaster. For instance, an unexpected fire and explosion in Nike's supplier factory disrupted its supply chain for a couple of months (finite horizon planning), wherein the supply capacity was uncertain (Lim and Prakash 2017). The 2011 Tohoku earthquake caused devastating disruptions to the industrial supply chains in Japan (Carvalho et al. 2021). Specifically, it temporarily (almost six months) affected the supplier's capacity. It caused supply disruption of the automotive microcontroller parts manufactured by Renesas Electronics and delivered to Toyota via its first-tier vendors like Denso (Matsuo 2015; Son et al. 2021). Climate change can also cause some capacity uncertainties for the suppliers due to creating severe weather or floods (Leonard 2021; Parker 2021). The extreme winter weather and ensuing electrical power crisis in Texas worsen supply chain woes. For example, severe weather in Austin, the home to Samsung's semiconductor factory—which manufactures and supplies more than 100k logic and flash memory chips per month—caused supply disruptions and uncertainty for numerous tech companies such as Dell and Apple during last winter (Shih 2021).

The remainder of our paper is organized as follows. In §2, we review the relevant literature. We introduce the model set-up in §3. We present the optimal policy facing stationary demand in §4, where we discuss two scenarios, one without any supplier's capacity constraint (benchmark 1) in §4.1 and the other with supplier's stochastic capacity constraint in §4.2. We analyze the policy facing stochastic demand in §5, where we investigate two scenarios, one without any supplier's capacity constraint (benchmark 2) in §5.1, and the other with supplier's stochastic capacity constraint in §5.2. In §6, we further analyze numerically the above-mentioned models to derive more insights. We provide concluding remarks in §7.

2 Literature Review

Our research is related to inventory management literature, which has been extensively studied. The majority of inventory models assume that the product to be ordered is always available (i.e., ample supply availability). Under this assumption, when an order is placed, it is received either immediately or after a lead time. Given this assumption, seminal papers of Harris (1990), Taft 1918, and Wilson (1934) among others (Whitin 1954; Wagner and Whitin 1958) provide various models of lot-sizing and inventory management. Nevertheless, we examine the case wherein the supply is not guaranteed to be available at the desired amount.

There are two streams of literature that consider supply (capacity) uncertainties, which cause the output to be random in inventory systems and procurement environments.

The first stream focuses on random yield, which is the outcome of faulty production processes, which causes part of the processed items to be unusable or defective (Yano and Lee 1995). Silver (1976) and Shih (1980) extend the basic Economic Order Quantity (EOQ) model (in a single period and infinite time horizon), in which the proportion of defective units in the accepted lot is a random variable. Their results include numerical comparisons between these newly developed models with the same systems without defective items. Noori and Keller (1986) investigate the effect of the same phenomenon on an optimal lot size in the order quantity/reorder point inventory system (i.e., (Q, r)). They characterize explicit and approximate solutions for the back-orders case by assuming that the standard deviation of the lot received is linearly related to the quantity requested. Gerchak (1992) extends the (Q, r) model with binomial and stochastically proportional yields to address both back-ordering and lost sales circumstances. Hong et al. (2017) study inventory decisions in an assemble-to-order setting wherein items are subject to defect risk (i.e., supply uncertainty). They characterize the conditions under which dual sourcing is better than single sourcing when a random yield is present. For a thorough review of inventory models with random yields, interested scholars are referred to the excellent paper of Yano and Lee (1995) and a recent review by Tinani and Kandpal (2017), which surveys multiple extensions. While this stream of literature focuses on random yield, in our paper, we focus on 100% yield. That is, there is no faulty production.

In a domain more related to our study, another stream of research has focused on variable capacity, which is often prompted by material shortages, unplanned repairs, or accidental machine breakdowns, which restrict the quantity that could be processed in each cycle (Ciarallo et al. 1994). Wang and Gerchak (1996) consider an infinite horizon to analyze the effect of variable capacity on optimal lot-sizing in the basic EOQ model and continuous review (Q, r) inventory system. They provide the optimal conditions for generally distributed variable capacity. They also develop practical procedures to find optimal solutions when the variable capacity distribution is exponential. Further, Hariga and Haouari (1999) analyze a particular case of Wang and Gerchak (1996) model, wherein the replenishment lead time is zero. For a general capacity distribution, they find that the expected cost per unit of time is a unimodal function and pseudo-convex in the ordering quantity. Moreover, in his technical note, Wang (2010) characterizes the sufficient conditions needed to guarantee that the solution proposed by Wang and Gerchak (1996) is unique and optimal. In the same line of research, while the supplier has a random capacity, Wu (2001) analyzes an infinite horizon (Q, r) model, wherein the order quantity and the reorder point are decision variables. He characterizes the optimal

ordering policy's properties when the lead-time has a normal distribution or is distributed free. Erdem et al. (2006) consider the basic EOQ policy (with deterministic demand) to model multiple suppliers with random capacities, which leads to uncertain yield in orders. They characterize properties of the optimal order quantity when the random capacity of suppliers follows a uniform and exponential distribution. Moon et al. (2012) combine the two scenarios of supply uncertainties (i.e., variable supplier capacity and random yield) to analyze the (Q, r) model when multiple products are considered. They use a distribution-free approach (DFA) to characterize the optimal solution when the storage and investment constraints are involved. They find that variable supplier capacity always increases the optimal lot size compared to unlimited capacity. In a recent study, Gholami and Mirzazadeh (2018) consider a (Q, r) policy and design a mathematical model wherein the demand follows the log-normal distribution, random capacity follows a gamma-type distribution (right-skewed), the ordering cost is a deterministic variable, and it is possible to reduce it by an extra investment. They propose a solution algorithm to find the optimal order quantity. Using a numerical example and under some conditions, they find that savings (i.e., decrease in the total expected cost) due to using their proposed methodology might be more than 20% of using the standard continuous-review inventory model, wherein the demand follows the normal distribution.

Unlike previous research and motivated by COVID-19 testing and swab production, we focus on a finite time horizon problem. Further, we investigate the effect of stochastic capacity constraint on the optimal order quantity in a continuous review setting. One key assumption in the existing literature is that orders in different cycles are equal. However, in our paper, we relaxed this assumption. Moreover, we compare the optimal results of each system with the benchmark wherein there is no stochastic capacity. Our results help decision-makers manage the inventory systems (i.e., minimize the inventory cost) under capacity limitations for a fixed planning horizon.

Although we have differentiated our study from those in the literature on the EOQ and (Q, r) models, there is vast literature on periodic review inventory models focusing on supply uncertainty. Federgruen and Zipkin (1986a, b) study a single-item, periodic-review inventory model with uncertain demands for average-cost and discounted-cost setups while the system has a limited production capacity in each period. They show that a base-stock policy is optimal in these setups. Later, Tayur (1993) provides an algorithm for the computation of the optimal policy or the cost for Federgruen and Zipkin (1986a, b)'s models. Ciarallo et al. (1994) consider both finite (single-period and multiple-period) and infinite horizons inventory systems for a single product with random capacity. They find that the random capacity has no

effect on the optimal policy in the single-period model but results in a unimodal, nonconvex cost function. In multiple-period and infinite-horizon models, they find that the order-up-to policies, which depend on the distribution of capacity, are optimal despite a nonconvex cost. Considering a single item inventory system with uncertain capacity, Güllü (1998) presents a procedure for computing the optimal base stock level under the expected average cost per period criterion. His main contribution is to use the $G/G/1$ queues and their associated random walks to model the class of base stock inventory policies that operate under demand/capacity uncertainty. Further, DeCroix and Arreola-Risa (1998) characterize the optimal policy for the multi-product in an infinite-horizon. Iida (2002) studies a non-stationary periodic review model with stochastic capacity for finite and infinite horizons. He finds the lower bounds and upper bounds of the optimal order up-to levels converge as the planning horizons analyzed become more prolonged. For the sake of brevity, scholars who are interested in periodic review models and related extensions are referred to the recent review by Tinani and Kandpal (2017).

Our research also contributes to and offers insights on how to manage the inventory systems and supply chains resiliently in the case of disruption due to pandemics. In the same vein, a growing body of literature addresses this issue from various perspectives as well. Hosseini and Ivanov (2021) find supply chain disruption triggers and risk events due to the COVID-19 pandemic and quantify the effects of disruption. Scholars also evaluate the supply chain resiliency considering innovative strategies during the pandemic (Moosavi and Hosseini 2021; Khan et al. 2022). Hald and Coslugeanu (2021) and Sharma et al. (2021) discuss opportunities and offer insights on adopting technology in supply chains during pandemics to mitigate the risks and avoid disruption. To further learn about the related works in this area, we refer a reader to see Hosseini et al. (2019), Hosseini and Ivanov (2020), Ardolino et al. (2022), where a thorough literature review are presented.

Closer to the focus of our paper, scholars have studied challenges around COVID-19 and how to use operations and supply chain techniques to tackle them (Kaplan 2020; Nagurney 2021; Nikolopoulos et al. 2021; Kumar et al. 2022; Yu et al. 2021). On topics related to inventory management, Eftekhar and Webster (2020) study inventory models for a single item under two policies (i.e., procures relief items before or after a disaster). They characterize the optimal order quantity and approximate methods by minimizing the total cost subject to budget constraints. Eftekhar et al. (2021) investigate the role of prepositioning inventories and local purchasing for specific products needed to cover demands due to a rapid-onset disaster. They determine the optimal preposition stock by minimizing the total cost of inventory. They also examine how the interplay between

supply, demand, and budget uncertainties affects the optimal inventory levels. Note that the above recent papers mainly focus on cost minimization to find optimal policies to manage the inventories based on the local vs. non-local purchasing or purchasing time (i.e., before or after the incidents) for a disaster. However, our model mainly focuses on stochastic capacity constraint on the optimal order quantity in a continuous review setting while the demand could be different based on the COVID-19 infection pattern. We next present the model setup.

3 Model Setting

We consider a country (i.e., a retailer) that is facing the new wave of COVID-19 and aims to find the optimal policy for ordering its testing kit components (i.e., swabs). Recall from the introduction that: 1) although our model is set up for a country, it is applicable to any state, province, and city that aims to conduct COVID-19 testing; 2) there are two main swab suppliers in the world, whose supply chains are facing disruption due to the spread of COVID-19 globally. This results in uncertain swab-supply capacity. Therefore, in our model, we consider a country that orders swabs from an overseas supplier, which has a stochastic production capacity. Let x be the supplier's random swab capacity at the time of ordering with PDF of $f(x)$ and CDF of $F(x)$.

Moreover, we consider a finite time horizon, T , because an effective vaccine is introduced and vaccination is underway. For example, in the United States, the first case of COVID-19 was observed in February, 2020, and a vaccine was developed by the end of 2020, and vaccination of the whole country is in progress. Next, let h be the per swab-unit holding cost rate, c be the purchasing cost for one swab, and A be the fixed ordering cost (e.g., the shipping cost of a large container from supplier's warehouse to country's facility). In this paper, we focus on finding the optimal policy for two inventory models. In §4, we derive the optimal policy when demand is stationary. This model represents the scenario wherein the country has an accurate constant demand forecast or plans to reach a certain number of tests per day. For example, the United States plans to reach one million tests per day. In this case, let λ be the demand rate for swabs. Due to the importance of COVID-19 testing to stop the spread of the virus, in §4, we assume shortages are not allowed. Subsequently, in §5, we derive the optimal policy when demand is stochastic, which suggests having shortages is inevitable. This model is motivated by the outbreaks in different regions of a country, where demand for COVID-19 testing is unpredictable. For instance, New York faced an outbreak in April, 2020. We define y as the per-time-unit stochastic demand for the swabs with PDF of $d(y)$ and CDF of $D(y)$. To evaluate the performance of optimal policies under each demand

scenario, we analyze a benchmark that does not restrict supplier capacity. We present the benchmarks in §4.1 and §5.1 for stationary and stochastic demand, respectively.

To find the optimal policy, we divide the time horizon into N cycles, and we assume that t_i is the length of cycle i . Within cycle i (where $i = 1, 2, 3, \dots, N$), the country orders Q_i swabs whenever the on-hand swab inventory reaches the reorder point, R_i . Recall that the existing literature assumes that orders in different cycles are equal. However, in our paper, we relaxed this assumption. Next, let τ be the constant lead time for the supplier to ship the swabs to the country. Therefore, if the supplier has no capacity constraint, on-hand inventory increases by Q_i after τ units of time. However, if the supplier is facing the capacity constraint, x , then the on-hand inventory increases by $\min\{x, Q_i\}$ after τ units of time.

To sum up, the decision variables, resulting in the optimal policy, are: (1) number of cycles N ; (2) length of each cycle t_i ; (3) reorder point R_i ; and (4) order quantity Q_i . The objective is to minimize the country's total cost of managing its swab inventory. Next, we analyze optimal policies facing stationary demand in §4 and optimal policies facing stochastic demand in §5.

4 Optimal Policy Facing Stationary Demand

In this section, we consider an inventory management problem with stationary demand in a finite time horizon setting, where the time horizon is divided into N cycles. Without loss of generality, we assume $\tau = 0 \Rightarrow R_i = 0; \forall i \in \{1, 2, \dots, N\}$; otherwise, because λ is constant, $R_i = \lambda\tau, \forall i \in \{1, 2, \dots, N\}$. Recall from the introduction that it is countries' highest priority to avoid COVID-19 test shortages. Therefore, in this section, we do not allow for any shortages. In §4.1, we introduce and analyze a benchmark, wherein there is no capacity constraint for the supplier. Subsequently, in §4.2, we analyze the model where the supplier has stochastic production capacity.

4.1 Supplier with Ample Capacity (Benchmark 1)

In this section, (1) the supplier has no capacity constraint. Therefore, all the swab-orders placed by the country are fulfilled. (2) The demand is stationary. Given the assumptions that $\tau = 0 \Rightarrow R_i = 0$ and shortages are not allowed, the optimal policy is as follows. At the beginning of cycle $i \in \{1, 2, \dots, N-1\}$, order Q_i units of swabs. Whenever the on-hand inventory reaches zero, cycle $i+1$ begins, and an order of Q_{i+1} should be made. One can observe that $t_i = Q_i/\lambda$, where both t_i and Q_i are decision variables.

Given the optimal policy, the costs involved with swab procurement for the country during cycle $i \in \{1, 2, \dots, N\}$ are as follows. (1) Fixed ordering cost equals A . (2)

Purchasing cost equals cQ_i . (3) Holding cost equals $\frac{ht_i Q_i}{2}$. Furthermore, for the entire time horizon $T = \sum_{i=1}^N t_i = \frac{\sum_{i=1}^N 2Q_i}{\lambda}$. Therefore, the country's total swab procurement cost over the entire time horizon is:

$$\sum_{i=1}^N \left[A + cQ_i + \frac{ht_i Q_i}{2} \right]$$

$$\Leftrightarrow_{Q_i = t_i \lambda} NA + cT\lambda + \frac{h\lambda}{2} \sum_{i=1}^N t_i^2$$

The first term in the above total cost function is the total fixed ordering cost. The second term is total purchasing cost. Note that λT is the total demand over the entire time horizon. The last term is the holding cost for N cycles. Finally, the country's problem is:

$$\min_{N, t_i} NA + cT\lambda + \frac{h\lambda}{2} \sum_{i=1}^N t_i^2 \quad (1)$$

$$\text{s.t. } \sum_{i=1}^N t_i = T$$

Note that Schwarz (1972) examines a similar model wherein a production scenario (i.e., production cost) is considered to minimize the total inventory cost. However, we focus on a purchasing scenario (i.e., purchasing cost). Next, the following lemma helps us simplify the country's problem.

Lemma 1 Consider a finite time horizon EOQ model with no capacity constraint for the supplier. To minimize the retailer's total cost, it is optimal to divide the planning horizon into N cycles with the same cycle length. In other words, $t_1 = t_2 = \dots = t_N = \frac{T}{N}$.

Proof: All proofs are relegated to the (Supplementary Material, Appendix).

Using Lemma 1, the country's problem in Eq. (1) can be simplified to:

$$\min_N NA + cT\lambda + \frac{h\lambda}{2} \sum_{i=1}^N \left(\frac{T}{N} \right)^2 = NA + cT\lambda + \frac{h\lambda T^2}{2N} \quad (2)$$

Proposition 1 presents the optimal number of cycles, the optimal order quantity, and the associated optimal total cost for the country (i.e., the retailer).

Proposition 1 Consider a finite time horizon EOQ model with no capacity constraint for the supplier. In this case, the optimal number of cycles over the time horizon, the optimal order quantity, and the associated optimal total cost are:

$$N^* = T \sqrt{\frac{h\lambda}{2A}}$$

$$Q_i^* = \lambda t_i^* = \lambda \frac{T}{N} = \sqrt{\frac{2A\lambda}{h}}$$

$$TC^* = T \sqrt{2Ah\lambda} + cT\lambda$$

Similar to the EOQ model, in this model, the optimal number of cycles and the optimal order quantity increase with the demand. Further, the optimal total cost increases as holding costs and fixed ordering cost increase. In the next section, we analyze a model with stationary demand wherein the supplier has a stochastic capacity.

4.2 Supplier with Stochastic Capacity

In this section, Eq. (1) demand is stationary, and Eq. (2) because the swab supply chain faces disruption, the swab supplier has a stochastic capacity. Recall that x is the supplier's random swab capacity at the time of order, with $f(x)$ and $F(x)$ as its PDF and CDF, respectively. The optimal policy is as follows. At the beginning of cycle $i \in \{1, 2, \dots, N-1\}$, order Q_i units of swabs. However, in the presence of swab-supplier capacity constraint, the actual number of swabs that the country receives is the minimum of Q_i and the supplier's available capacity. Next, whenever the on-hand inventory reaches zero, cycle $i+1$ begins, and an order of Q_{i+1} should be made.

Given the optimal policy, when a lot with the size Q_i is ordered during the i^{th} cycle, the fulfilled number of swabs, z_i , is:

$$z_i = m\{Q_i, x\}$$

Next, note that in this model the demand is stationary, which implies total demand during cycle i is λt_i . Therefore, to balance swab supply and demand over the cycle, $z_i = \lambda t_i$ and, for the entire time horizon, $E\left(\sum_{i=1}^N z_i\right) = \lambda T$. That is,

$$E \sum_{i=1}^N \left[\int_0^{Q_i} xf(x)dx + Q_i(1 - F(Q_i)) \right] = \lambda T$$

Furthermore, the costs involved with swab procurement for the country during cycle $i \in \{1, 2, \dots, N\}$ are as follows. Equation (1) Fixed ordering cost equals A . Equation (2) Purchasing cost equals cz_i . Equation (3) Holding cost equals $ht_i z_i/2$. Therefore, the country's expected total swab procurement cost over the entire time horizon is:

$$\sum_{i=1}^N E \left[A + cz_i + \frac{h}{2} t_i z_i \right] \stackrel{z_i = \lambda t_i \& E(\sum_{i=1}^N z_i) = \lambda T}{\Leftrightarrow} NA + c\lambda T + \frac{h\lambda}{2} \left[\sum_{i=1}^N E(t_i^2) \right]$$

The first term in the country's expected total swab cost is the total fixed ordering cost. The second term is the total purchasing cost of swabs, and the last term is the expected total holding cost of swab inventory. In the next lemma, we show that, similar to the benchmark in §4.1, it is optimal to order the same number of swabs in different cycles.

Lemma 2 Consider a finite time horizon EOQ model where the supplier has stochastic capacity. To minimize the retailer's total inventory cost, it is necessary: $Q_1 = Q_2 = \dots = Q_N = Q$.

Lemma 2 implies that, at optimality, order quantities in different cycles are equal. It helps us simplify the analysis of the country's problem by assuming that all order quantities in all cycles are equal. We use Lemma 2 to write the country's swab problem as:

$$\min_{Q,N} K(Q, N) = NA + c\lambda T + \frac{h}{2\lambda} E \left[\sum_{i=1}^N z_i^2 \right] \tag{3}$$

$$\text{s.t. } N \left[\int_0^Q xf(x)dx + Q(1 - F(Q)) \right] = \lambda T$$

The constraint in Eq. (3) balances expected supply and demand over the entire time horizon, and it can be rewritten as

$$N = \frac{\lambda T}{\int_0^Q xf(x)dx + Q(1 - F(Q))} \tag{4}$$

Plugging Eq. (4) back into the objective function in Equation (3) simplifies the country's problem to:

$$\min_Q K(Q) = T \left[\frac{2A\lambda + h \left(\int_0^Q x^2 f(x)dx + Q^2(1 - F(Q)) \right)}{2 \left(\int_0^Q xf(x)dx + Q(1 - F(Q)) \right)} \right] \tag{5}$$

To find the optimal order quantity, we define $G(Q)$ as one part of the first derivative of $K(Q)$:

$$G(Q) \triangleq \left[Q^2(1 - F(Q)) + 2Q \int_0^Q xf(x)dx - \int_0^Q x^2 f(x)dx - \frac{2A\lambda}{h} \right]$$

Proposition 2 presents the characteristics of the country's expected total cost, along with the optimal order quantity.

Proposition 2 Consider a finite time horizon EOQ model where the supplier has stochastic capacity. In this case, the retailer's total expected cost, $K(Q)$, is convex and unimodal with respect to Q , and the optimal order quantity, Q^* , is the unique solution to the following equation:

$$G(Q) = 0 \iff Q^2(1 - F(Q)) + 2Q \int_0^Q xf(x)dx = \int_0^Q x^2 f(x)dx + \frac{2A\lambda}{h} \tag{6}$$

To solve Eq. (6), note that $G(0) < 0$, $G(\infty) > 0$, and $\frac{\partial G(Q)}{\partial Q} > 0$. Therefore, to find the optimal order quantity, one can set $Q = 0$, and incrementally increase Q . The first Q for which $G(Q) > 0$ is Q^* . Once Q^* is found, plugging it back

Table 1 Optimal Order Quantities, Number of Cycles, and Total Costs for Finite EOQ with/without Stochastic Supplier Capacity Constraint

	Without Capacity Constraint	With Capacity Constraint
Optimal Order Size (Q^*)	60,415,000	72,267,588
Optimal # of Cycles (N^*)	6	8
Optimal Total Cost (TC^*)	\$604,150	\$722,680

into Eq. (4) and $K(Q)$ in Eq. (5) returns the optimal number of cycles and optimal total expected cost, respectively. Next, to study whether this section's results generate benchmark 1's results in §4.1, we consider a case where the variable capacity $x \rightarrow \infty$ with probability of 1. Corollary 1 presents the result of this analysis.

Corollary 1 When the variable capacity $x \rightarrow \infty$ with a probability of 1, the optimal order quantity for the finite time horizon EOQ model with stochastic capacity—i.e., the solution to Eq. (6)—is:

$$Q^* = \sqrt{\frac{2A\lambda}{h}}$$

One can observe that the optimal order quantity derived in Corollary 1 is similar to that of the benchmark 1. To conclude the result in Corollary 1, note that $G(Q) = \left[Q^2 - \frac{2A\lambda}{h} \right]$ when $x \rightarrow \infty$ with probability of 1. Then, solving $G(Q) = 0$ results in the above corollary. To further investigate the effect of the supplier's stochastic capacity on optimal policy, we provide a numerical example in the context of COVID-19 testing in the United States.

Example 1: The United States' government has announced a goal of 1,000,000 COVID-19 daily tests,⁵ which requires a million swabs on a daily basis (i.e., $\lambda = 1,000,000$ swabs per day). Each swab is priced around 10 cents. Similar to traditional inventory management problems, we assume that the annual inventory holding cost is 10% of the purchasing cost. Therefore, daily holding cost is $h = \$\frac{10\% \times 0.1}{365}$. Next, although there are no clear data on fixed ordering cost and capacity constraint of the swab supplier, we assume that

⁵ <https://www.cdc.gov/coronavirus/2019-ncov/cases-updates/testing-in-us.html>

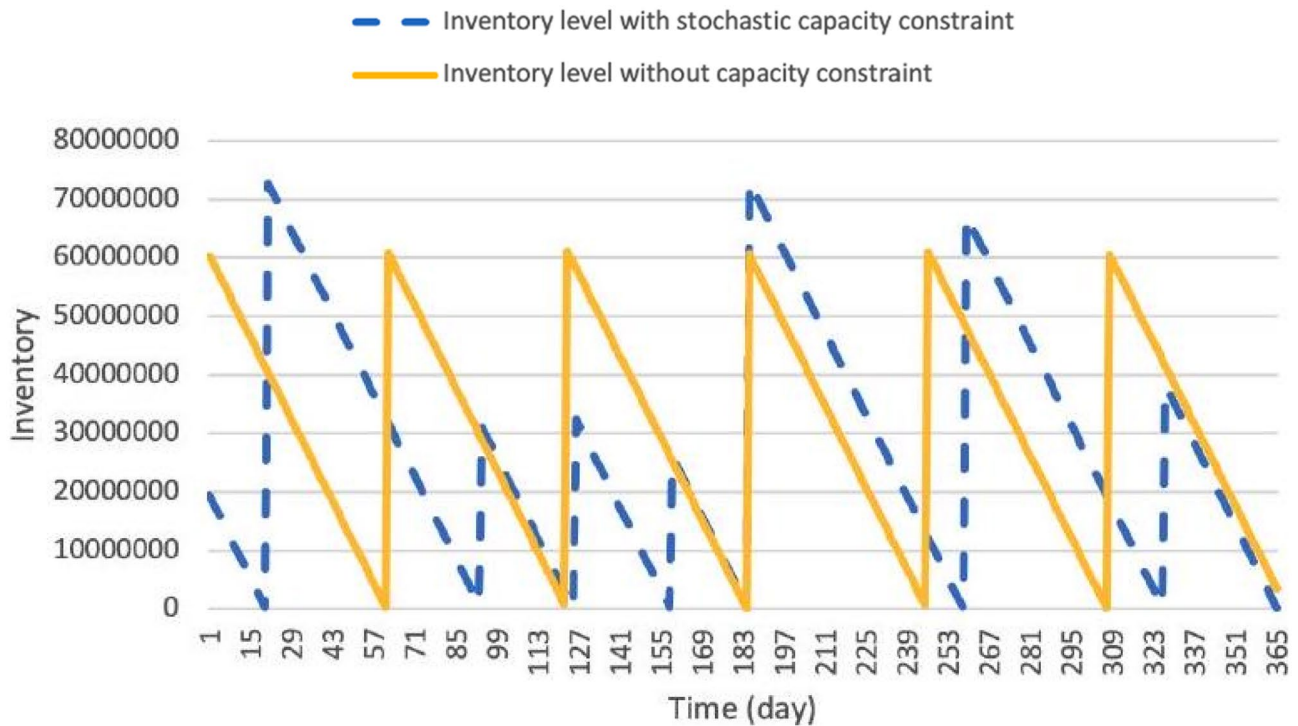


Fig. 2 Illustration of the Cycle Inventory Levels for the Benchmark model (Model with No Capacity Constraint) Discussed in §4.1 and the Model with Stochastic Capacity Constraint Discussed in §4.2, Using Monte Carlo Simulation

1. the fixed cost of shipping swabs from Europe to the US, the cost of shipping a large truck from Europe to Texas, is \$5000⁶; therefore, we set the fixed cost of ordering, $A = \$50000$, and
2. capacity constraint has a uniform distribution. Here we assume $x \sim \text{uniform}[0, 800, 000, 000]$

Using the above-mentioned setup, the results from benchmark 1 in §4.1 and the model with stochastic capacity constraint in §4.2 are summarized in Table 1. This example illustrates the following results. First, when there is no disruption (i.e., no stochastic capacity constraint), it is optimal to order 60,415,000 swabs every two months. When there is disruption, it is optimal to order 72,267,588. However, not all of this order is fulfilled, due to the supplier's capacity constraint. As one may observe, the optimal order quantity increases when there is disruption in the supply chain. Next, the total cost (excluding the purchasing cost), when there is a capacity constraint, is \$722,680, which is 19.6% higher than when there is no capacity constraint. This difference represents the effect of disruption in the swab supply chain.

Next, using the Monte Carlo simulation similar to Hadley and Whitin (1963), we simulate the capacity constraints, and we show the inventory levels of Benchmark 1 in §4.1 and the model in §4.2 on Fig. 2.

The solid line represents Benchmark 1, where the inventory level is similar to the well-known EOQ model. The dashed line represents the case where the supplier faces disruption and, therefore, has a capacity constraint. In this case, although the US always orders 72,267,588 swabs, it receives this amount only two times (out of 6 orders). The other four times, they receive less due to the supplier's capacity constraint.

Still, this question remains unanswered: How do the nature of the supplier's capacity distribution and changes in its variance and mean affect the optimal decision variables? We answer this question in §6 through a numerical study.

5 Optimal Policy Facing Stochastic Demand

In this section, we consider a continuous inventory management problem with stochastic demand in a finite time horizon, where the time horizon is divided into N cycles.

⁶ For information about different types of COVID-19 tests, please visit <https://www.uber.com/us/en/freight/shipper/>.

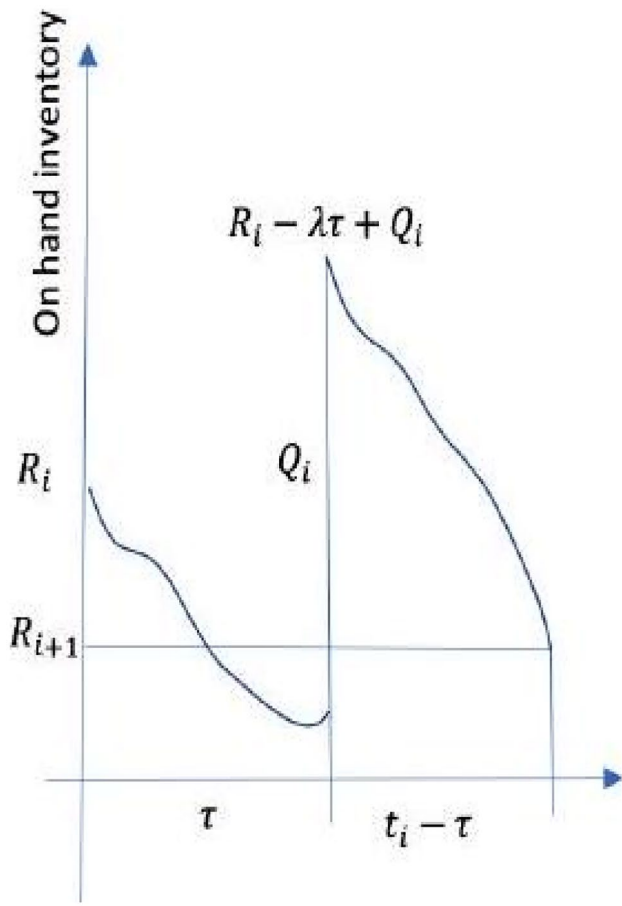


Fig. 3 Illustration of On-hand Inventory in Time for Continuous Review Model Without Capacity Constraint

We define y as the per-timeunit stochastic demand for the swabs with PDF of $d(y)$ and CDF of $D(y)$. Recall that countries or states (e.g., the United States, Oregon, Missouri, and Arkansas) plan to perform a certain number of tests per day. Thus, we assume the mean of stochastic demand is constant over the time horizon. Next, note that because demand is stochastic, having shortages is inevitable. We assume that all the shortages are back-ordered, because the need for COVID-19 testing does not disappear. In §5.1, we introduce and analyze a benchmark, wherein there is no capacity constraint for the supplier. Subsequently, in §5.2, we analyze the same problem while the supplier has stochastic capacity.

5.1 Supplier with Ample Capacity (Benchmark 2)

In this section, (1) the supplier has no capacity constraint. Therefore, all the swab-orders placed by the country are fulfilled. (2) The demand is stochastic. Thus, the optimal policy and timeline are as follows. Cycle $i \in \{1, 2, \dots, N - 1\}$ begins at $\sum_{j=1}^i t_j$ (where $t_1 = 0$) when the on-hand inventory of swabs reaches R_i (i.e., the reorder point). At this point, an order of Q_i should be made. This order will be received at $\sum_{j=1}^i t_j + \tau$, because the on-hand inventory of swabs increases by Q_i . Note that if demand during lead time is greater than R_i , then the country faces a test shortage. Next, cycle $i + 1$ begins when the on-hand inventory of swabs reaches R_{i+1} . An example of the optimal policy during cycle i is shown in Fig. 3.

We define λ as the mean of demand per time unit, which implies the expected demand during cycle i is λt_i . Therefore, to satisfy the expected demand and supply during cycle i , $t_i \lambda = Q_i$. Similarly, for the entire time horizon, we must have:

$$\sum_{i=1}^N Q_i = \lambda T \tag{7}$$

Given the optimal policy, the costs involved with swab procurement for the country during cycle $i \in \{1, 2, \dots, N\}$ are as follows. (1) Fixed ordering cost equals A . (2) Purchasing cost equals cQ_i . (3) To determine the holding cost, we must first find the average inventory of swabs carried during the cycle. To that end, the area under the graph in Fig. 3 is:

$$\begin{aligned} &= \frac{1}{2} [2R_i - \lambda\tau] \tau + \frac{1}{2} [R_i + R_{i+1} - \lambda\tau + Q_i] [t_i - \tau] \\ &= \frac{Q_i}{2\lambda} (R_i + R_{i+1}) + \frac{Q_i^2}{2\lambda} - \tau Q_i + \frac{1}{2} \tau (R_i - R_{i+1}) \end{aligned} \tag{8}$$

(4) Because some of the demand might be back-ordered, back-order cost, also, might be incurred. To that end, we define π as the per-unit-of-demand back-ordered cost. Recall that during cycle i , if swab-demand during lead time is higher than R_i , then the country faces a swab shortage. We assume shortages are not lost, but rather, are back-ordered, because the demand for COVID-19 testing does not disappear. We define z as the demand during lead time; therefore, the number of back-orders during cycle i is $[z - R_i]^+$, and the back-order cost is $\pi [z - R_i]^+$.

Therefore, the country's expected total swab procurement cost is:

$$\begin{aligned} \Gamma &= E \left[\sum_{i=1}^N A + \sum_{i=1}^N cQ_i + \frac{h}{2} \sum_{i=1}^N \left[\frac{Q_i}{\lambda} (R_i + R_{i+1}) + \frac{Q_i^2}{\lambda} - 2\tau Q_i + \tau (R_i - R_{i+1}) \right] + \pi \sum_{i=1}^N [z - R_i]^+ \right] \\ &= NA + (c - h\tau)\lambda T + \frac{h}{2} \sum_{i=1}^N \left[\frac{Q_i}{\lambda} (R_i + R_{i+1}) + \frac{Q_i^2}{\lambda} + \tau (R_i - R_{i+1}) \right] + \pi \sum_{i=1}^N \int_{R_i}^{\infty} (z - R_i) d(z) dz \end{aligned}$$

The country's objective is to minimize Γ with respect to decision variables (i.e., Q_i, R_i and N). However, we do not optimize Γ with respect to R_{N+1} , because it deals with a cycle beyond T . Without loss of generality, we assume that $R_{N+1} = R_1$. The reason is that any swabs left over at the end of the horizon can be used for other medical purposes. Using this point, we simplify the total expected cost, Γ , to:

$$\Gamma = NA + (c - h\tau)\lambda T + \frac{h}{2} \sum_{i=1}^N \left[\frac{Q_i}{\lambda} (R_i + R_{i+1}) + \frac{Q_i^2}{\lambda} \right] + \pi \sum_{i=1}^N \int_{R_i}^{\infty} (z - R_i) d(z) dz \quad (9)$$

We present Lemma 3 to further simplify the expected total cost in Eq. (9).

Lemma 3 Consider a finite time horizon continuous review inventory model with stochastic demand. To minimize the expected total cost shown in Eq. (9), it is necessary that:

$$R_1 = R_2 = \dots = R_N \triangleq R, \text{ and} \\ Q_1 = Q_2 = \dots = Q_N \triangleq Q$$

Lemma 3 implies that, at optimality, order quantities as well as reorder points in different cycles are equal. We use Lemma 3 to simplify the supply and demand balance—which is shown in Eq. (7) as follows:

$$NQ = \lambda T \quad (10)$$

Moreover, by using Lemma 3, the expected total cost, Γ , simplifies to:

$$NA + (c - h\tau)\lambda T + \frac{h}{2} \sum_{i=1}^N \left[\frac{Q}{\lambda} 2R + \frac{Q^2}{\lambda} \right] + \pi N \int_R^{\infty} (z - R) d(z) dz \\ = NA + (c - h\tau)\lambda T + \frac{hNQ^2}{\lambda} + \frac{hNQ^2}{2\lambda} + \pi N \int_R^{\infty} (z - R) d(z) dz \quad (11)$$

Plugging Eq. (10) into Eq. (11) results in the country's cost minimization problem as follows.

$$\min_{Q,R} K(Q, R) \triangleq T \left[\frac{\lambda A}{Q} + (c - h\tau)\lambda + hR + \frac{hQ}{2} + \frac{\pi\lambda}{Q} \int_R^{\infty} (z - R) d(z) dz \right] \\ = T \left[\frac{\lambda A}{Q} + c\lambda + h \left(R + \frac{Q}{2} - \tau\lambda \right) + \frac{\pi\lambda}{Q} b(R) \right]$$

We present the optimal order quantity and reorder point for this model in Proposition 3.

Proposition 3 Consider a finite time horizon continuous review inventory model with stochastic demand. The optimal order quantity and reorder points, respectively, can be calculated from:

$$(Q^*(R^*), R^*(Q^*)) = \left(\sqrt{\frac{2(A\lambda + \pi\lambda b(R^*))}{h}}, D^{-1} \left(1 - \left(\frac{hQ^*}{\pi\lambda} \right) \right) \right) \quad (12)$$

In Eq. (12), because the optimal Q is a function of R^* , and the optimal R is a function of Q^* , we use Hadley and Whitin (1963)'s iterative algorithm to find Q^* and R^* .

Algorithm 1 There are three steps in Hadley and Whitin (1963)'s proposed algorithm to find the optimal order quantity and reorder point:

- Step 1: Find Q^* in Benchmark 1. Name it Q_1 . Set $i = 0$.
- Step 2: Set $i = i + 1$. Further, find
- Step 3: Find $Q_{i+1} = Q^*(R_i)$.
- Step 4: Repeat Step 2 and Step 3 until convergence.

Hadley and Whitin (1963) show that the above algorithm converges to a minimum expected total cost solution. Based on this algorithm: $Q_1 = \sqrt{\frac{A\lambda}{h}}$. Next, $R_1 = D^{-1} \left(1 - \left(\frac{hQ_1}{\pi\lambda} \right) \right)$. Next, $Q_2 = \sqrt{\frac{2(A\lambda + \pi\lambda b(R_1))}{h}}$. We continue this trend till $K(Q_i, R_i)$ converges to a minimum total expected cost. We use the same algorithm to obtain the optimal swab procurement policy for the country. Although the algorithm is straightforward to implement, it does not yield a closed-form solution for Q^* and R^* . In the next section, we bring the notion of the supplier's stochastic capacity to find the country's optimal ordering policy under uncertain supply.

5.2 Supplier with Stochastic Capacity

In this section, (1) demand, y , is stochastic with PDF of $d(y)$ and CDF of $D(y)$; and (2) because the swab supply chain faces disruption, the swab supplier has a stochastic capacity. Recall that x is the supplier's random swab capacity at the time of order, with $f(x)$ and $F(x)$ as its PDF and CDF, respectively. Thus, the optimal policy timeline is as follows. Cycle $i \in \{1, 2, \dots, N\}$ begins at $\sum_{j=1}^N t_j$ (where $t_1 = 0$) when the on-hand inventory of swabs reaches R_i . At this point, an order of Q_i should be made. However, due to the supplier's stochastic capacity, the minimum of Q_i and the supplier's capacity will be fulfilled at $\sum_{j=1}^N t_j + \tau$. Note that if demand during lead time is greater than R_i , then the country faces a testing shortage. Next, cycle $i + 1$ begins when the on-hand inventory of swabs reaches R_{i+1} , when an order of Q_{i+1} should be made. Given the optimal policy, and the stochasticity of the supplier's capacity, we define z_i as the number of swabs fulfilled by the supplier. Therefore,

$$z_i = m\{Q_i, x\}$$

Given the definition of z_i , we make the following observation.

Observation 1 One can observe that the proposed optimal policy for this section is similar to that of §5.1, with the difference being that in this section, the country receives z_i number of swabs during cycle i , whereas in §5.1, the country receives Q_i .

Before we carry on to calculating the expected total cost, we make the following assumption.

Assumption 1: Following the results in §5.1, we assume that $R_1 = R_2 = \dots = R_N = R_{\text{and}} Q_1 = Q_2 = \dots = Q_N = Q$

Given Assumption 1, we define $z = \min\{Q, x\}$. Next, we define λ as the mean swab-demand per unit of time, which implies the expected demand over the entire time horizon is λT . Therefore, to balance supply and the expected demand over the entire time horizon, $(\sum_1^N z) = \lambda T \Rightarrow NE(z) = \lambda T$. Solving this for N gives us:

$$N = \frac{\lambda T}{\int_0^Q x f(x) dx + Q(1 - F(Q))} \tag{13}$$

Given Observation 1, the modified carried inventory—compared to Eq. (8) over one cycle is $\frac{z}{\lambda} \left[\frac{z}{2} + R - \lambda \tau \right]$, and the modified cost expression during cycle i is $E\left(A + cz + h \frac{z}{\lambda} \left(\frac{z}{2} + R - \lambda \tau \right) + \pi [z - R]^+\right)$. Therefore, the country's expected total cost is:

$$K(Q, R, N) = \sum_1^N E\left(A + cz + h \frac{z}{\lambda} \left(\frac{z}{2} + R - \lambda \tau \right) + \pi [z - R]^+\right)$$

$$c = NA + c\lambda T + \frac{Nh}{\lambda} \left(\int_0^Q x \left(\frac{x}{2} + R - \lambda \tau \right) f(x) dx + Q \left(\frac{Q}{2} + R - \lambda \tau \right) (1 - F(Q)) \right) + N\pi \int_R^\infty (z - R) d(z) dz$$

Using Eq. (13), we simplify the expected total cost as follows:

$$K(Q, R) = T \left[c\lambda + \frac{A\lambda + h \left(\int_0^Q x \left(\frac{x}{2} + R - \lambda \tau \right) f(x) dx + Q \left(\frac{Q}{2} + R - \lambda \tau \right) (1 - F(Q)) \right) + \lambda \pi \int_R^\infty (z - R) d(z) dz}{\int_0^Q x f(x) dx + Q(1 - F(Q))} \right]$$

Lemma 4 presents the characteristics of the expected total cost, $K(Q, R)$, with respect to Q .

Lemma 4 Consider a finite time horizon continuous review inventory model with stochastic demand and stochastic capacity for the supplier. In this case, the expected total

cost, $K(Q, R)$, is convex and unimodal with respect to Q . We define $M(Q, R)$ as a part of the first derivative of $K(Q, R)$

$$M(Q, R) = Q^2(1 - F(Q)) - \frac{2\lambda}{h} [A + \pi b(R)] + 2Q \int_0^Q x f(x) dx - \int_0^Q x^2 f(x) dx$$

Lemma presents the characteristics of the expected total cost, $K(Q, R)$, with respect to R .

Lemma 5 Consider a finite time horizon continuous review inventory model with stochastic demand and stochastic capacity for the supplier. In this case, the expected total cost, $K(Q, R)$, is convex and unimodal with respect to R .

Although we show the convexity of $K(Q, R)$ with respect to Q and R separately, the joint convexity of $K(Q, R)$ with respect to (Q, R) is unclear, due to the complexity of the problem. However, Proposition 4 sheds light on finding the optimal (Q, R) .

Proposition 4 Consider a finite time horizon continuous review inventory model with stochastic demand and stochastic capacity for the supplier. In this case, the optimal order quantity with respect to R (i.e., $Q^*(R)$) is the unique solution to:

$$M(Q, R) = 0 \rightarrow Q^2(1 - F(Q)) + 2Q \int_0^Q x f(x) dx = \int_0^Q x^2 f(x) dx + \frac{2\lambda}{h} [A + \pi b(R)] \tag{14}$$

and the optimal reorder point with respect to Q (i.e., $R^*(Q)$) is:

$$R^*(Q) = G^{-1} \left(1 - \frac{h}{\lambda \pi} \left(\int_0^Q x f(x) dx + Q(1 - F(Q)) \right) \right)$$

Although Proposition 4 does not result in the direct calculation of (Q^*, R^*) , one can modify the algorithm dis-

cussed in §5.1 to minimize the expected total cost.

Algorithm 2 The modified algorithm is as follows:

- Step 1: Find Q^* in Benchmark 1. Name it Q_1 . set $i = 0$.
- Step 2: Set $i = i + 1$. Further, find $R_i = R^*(Q_i)$.
- Step 3: Find Q_{i+1}^* by solving $M(Q_i, R_i) = 0$.

Table 2 Optimal Order Quantities, Number of Cycles, and Total Costs for Finite Continuous Review Model with/without Stochastic Supplier Capacity Constraint

	Without capacity constraint	With capacity constraint
Optimal reorder point (R^*)	1,664,700	1,951,800
Optimal order size (Q^*)	60,581,000	72,289,194
Back-order, $b(R^*)$	110,240	469.14
Optimal total cost (TC^*)	\$604,150	\$658,400

Step 4: Repeat Steps 2 and 3 until convergence.

Our numerical study of 459 instances shows that the above algorithm converges to a minimum expected total cost. However, the analytical proof is not tractable. Next, note that Steps 1 and 2 in the above algorithm are similar to that of the algorithm in §5. Step 3, however, requires solving $M(Q_i, R_i) = 0$. To that end, note that $M(0, R_i) < 0, M(\infty, R_i) > 0$, and $\frac{\partial M(Q, R_i)}{\partial Q} > 0$. Therefore, to find the optimal order quantity in Step 3, one can initially set $Q_i = 0$, and incrementally increase it. The first Q_i for which $M(Q_i, R_i) > 0$, is Q_{i+1}^* .

Recall that $G(Q) = [Q^2(1 - F(Q)) + 2Q \int_0^Q xf(x)dx - \int_0^Q x^2 f(x)dx - \frac{2\lambda}{h}]$. In Corollary 2, We compare $G(Q)$ and $M(Q, R)$ to derive some insights.

Corollary 2 Note that $M(Q, R) = G(Q) - \frac{2\lambda\pi b(R)}{h}$, which suggests that the optimal order quantity when demand is stochastic is greater than that of when demand is stationary.

Corollary 2 implies that more variability in demand results in higher order quantities. We study this observation in detail in §6. Next, In Corollary 3, we examine Proposition 4 to study whether the results of our proposed model in this section generate the results of the benchmark model in §5.1 COROLLARY 3. When the variable capacity $x \rightarrow \infty$ with the probability of 1, the optimal solution to Eq. (14) is:

$$(Q^*(R^*), R^*(Q^*)) = \left(\sqrt{\frac{2(A\lambda + \pi\lambda b(R^*))}{h}}, F^{-1}\left(1 - \left(\frac{hQ^*}{\pi\lambda}\right)\right) \right)$$

One can observe that the optimal order quantity derived in Corollary 3 is similar to the optimal order quantity of the benchmark model in § 5.1. To conclude, the result in Corollary 3, when $x \rightarrow \infty$ with the probability of 1, $M(Q, R) = Q^2 - \frac{2\lambda}{h}[A - \pi b(R)]$, and $\frac{\partial M(Q, R)}{\partial R} = \frac{hQ - \lambda\pi(1 - F(R))}{\int_0^Q xf(x)dx + Q(1 - F(Q))}$. Setting these two equations equal to zero results in Corollary 3. To further investigate the effect of supplier's stochastic capacity on optimal policy for the country's swab inventory management, we provide a numerical example with the context of COVID-19 testing in the United States.

Example 2: For this example, we follow the setting in Example 1. Also, we assume demand has Uniform distribution between 100,000 and 1,900,000 swabs and $\pi = 5$ cents. The optimal order quantity, reorder point, number of cycles,

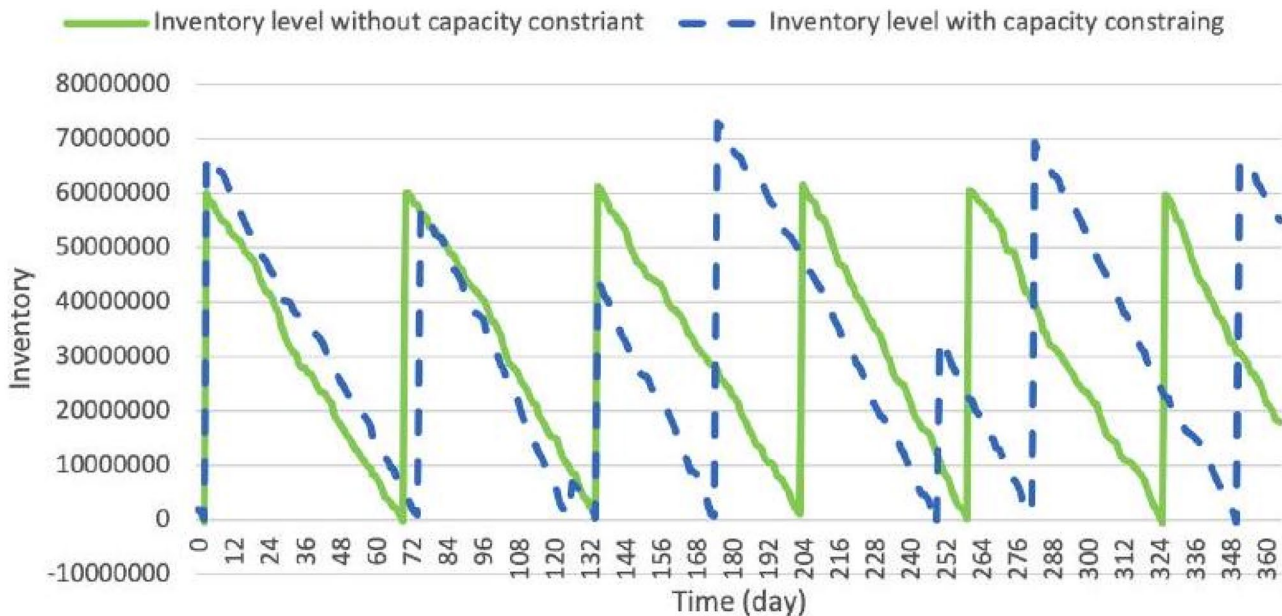


Fig. 4 Illustration of the Inventory Levels of Benchmark 2 in §5.1 and the Model in §5.2 Using Monte Carlo Simulation

and expected total cost for the benchmark model (discussed in §5.1) and continuous review model with stochastic capacity (discussed in §5.2) are reported in Table 2. One may observe that, similar to the example in §4.2, the optimal order quantity, reorder point, and expected total cost are larger when there is a capacity constraint. We investigate these relationships in more detail in §6.

Next, using the Monte Carlo simulation, we simulate the capacity constraints and daily demand. We show the inventory levels of the benchmark model (discussed in §5.1) and continuous review model with stochastic capacity constraint (discussed in §5.2) on Fig. 4. The solid line represents the benchmark, where the inventory level is similar to the well-known continuous review model. The dashed line represents the case where the supplier has a stochastic capacity. In this case, although the country always orders 72, 289, 194 swabs, they receive that amount only once (out of 6 orders) The other five times, they receive less due to supplier's capacity constraint.

In the next section, we present our numerical analysis to compare our optimal policies further and provide managerial insights.

6 Numerical Implications for Reactive Planning

One can observe that the analytical models and proposed policies in previous sections are developed based on proactive planning. However, given the rapid change of COVID-19 infection cases and the limited planning period, governments should aim for data-driven and reactive scheduling. Therefore, one can argue that the only-optimal-inventory plan examined in previous sections may not be able to cope with rapid variations in COVID-19 data. That is why in this section, via an extensive numerical study, we provide guidelines for the governments (e.g., decision makers) as to how optimally respond to changes in the number of daily COVID-19 cases and the disruptions the virus causes in swab supply chains. That is, during one cycle, governments can observe how problem parameters change and optimally react to it in the next cycle. Note that the managerial implications of this section can be extended to other mentioned examples in §1, such as the shortage of Personal Protective Equipment (PPE) in hospitals and nursing homes.

Note that during a pandemic, the number of infected people, increases over time, and then, starts decreasing (Duijzer et al. 2018). This could potentially translate into an increase in the variability of the supplier's capacity constraint, followed by a decrease. The same behavior can decrease the supplier's capacity, followed by an increase. Hence, in §6.1, we first study how the changes in the mean and standard

deviation of the supplier capacity constraint affect the optimal policy. This provides guidelines for the governments regarding how to tackle smaller or larger disruptions in the swab supply chain. Next, in §6.2, we compare the results in §4 and §5, the difference between which is the stochasticity of the demand. This provides guidelines for the governments on addressing the issue of changes in the daily number of COVID-19 cases and subsequently demand for swabs.

In our numerical experiment, because vaccines were developed almost one year after the pandemic started (and the expectation that vaccination of most countries may take up to one year), we set the planning horizon, T , to 1 year (i.e., 365 days). Although it is ideal to set the other problem parameters to that of the COVID-19 situation, the algorithms to find the optimal swab-order quantities and reorder points are complex. Setting the problem parameters to large numbers (e.g., $\lambda = 1,000,000$ swabs per day) results in a very lengthy process, more than 500 h for the number of instances we study. Therefore, we examine a set of not-very-large problem parameters similar to Moon et al. (2012), which are as follows:

As for the stochastic capacity constraint, we set the mean and standard deviation to $\mu \in \{250, 300, 350, 400\}$ and $\sigma \in \{10, 30, \dots, 170\}$, respectively. Here we assume a uniform distribution for the stochastic capacity constraint.

As for the yearly demand, we set the mean and standard deviation, respectively, to: $\lambda \in \{1000, 1500, 2000, 2500\}$ and $\Sigma \in \{250, 300, 400, 500\}$.

We set the other problem parameters to $(h, A, \pi, \tau) = (\$5, \$200, \$50, 2 \text{ days})$.

Therefore, we examine 576 instances. Before we move on to the guidelines, there are three discussions in order. First, note that we only study the effects of demand and capacity constraint on the optimal policies. We could, potentially, study the effects of other problem parameters (i.e., h, A, π , and τ). These effects, however, are well studied in the traditional inventory management literature. For example, as h increases, the optimal number of cycles increases and the total cost increases. Another example is as A increases, the optimal number of cycles decreases and the total cost increases. For such results, we refer the reader to Hadley and Whitin (1963) and Tinani and Kandpal (2017). Second, we only focus on uniform distribution for demand and capacity constraints for this section. However, we can confirm that our results are qualitatively the same for other distributions. Specifically, we examined exponential and normal distribution. Third, note that purchasing cost (i.e., $c\lambda T$) is the same in the total costs in §4.1, §4.2, §5.1, and §5.2. Therefore, we exclude it from all the total costs reported in this section.

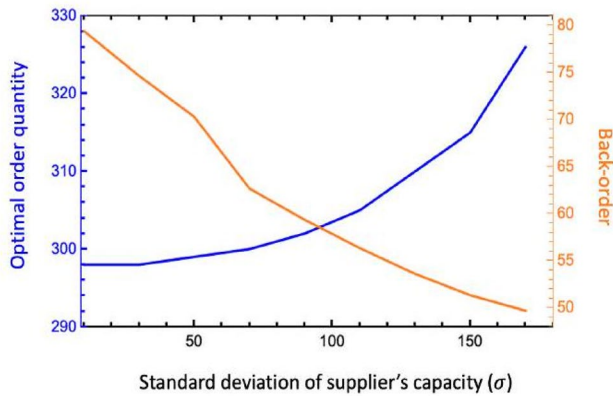


Fig. 5 Illustration of the optimal order quantity versus optimal back-order for different standard deviations of supplier's capacity

6.1 Guidelines on Changes in Supplier's Capacity

Note that provided algorithms in Sects. 2 and 3 (i.e., Algorithm 1 and Algorithm 2) are slow to obtain the optimal policy. Given the fast-changing nature of the COVID-19 infection rate, problem parameters might change from one cycle to another. Hence, governments must know the guidelines on how to respond to changes in problem parameters swiftly. That is why in this section, via studying how changes in σ and λ affect the optimal policy, we provide some guidelines for governments.

Guideline 1 When the supplier's capacity is uncertain, governments should respond to the changes in the variability of the supplier's capacity (i.e., σ) as following:

Governments should increase the reorder point (i.e., R^*) as the variability in supplier's capacity (i.e., σ) increases in presence of stochastic demand. This will result in the optimal expected number of back-orders (i.e., $b(R^*)$) to decrease (i.e., shorter lines to get tested). Moreover, the optimal number of cycles (i.e., N^*) will increase regardless of the demand's stochasticity.

Governments should decrease their order size (i.e., Q^* decreases) when the mean of the supplier's capacity, μ , is large, as its variability (i.e., σ) decreases. As a result, optimal expected total cost, $K(Q^*, N^*)$, will decrease, regardless of the stochasticity of the demand.

Governments should increase their order size (i.e., Q^*) as the supplier's capacity's variability (i.e., σ) increases and when the mean of supplier's capacity, μ , is small. As a result, optimal expected total cost (i.e., $K(Q^*, N^*)$) will increase, regardless of the stochasticity of the demand.

To explain the above guidelines, first, note that higher (lower) σ implies a larger (smaller) variability (i.e., disruption)

Table 3 Effect of changes in the standard deviation of supplier's capacity, σ , on the optimal policy, where $\mu = 300$ and $y \in [0, 2000]$

σ	Q^*	N^*	R^*	$b(R^*)$	$K(Q^*, R^*)$
10	298	3	8.05	79.4	8814.32
30	298	3	8.06	74.64	8815.401
50	299	3	8.06	70.28	8817.684
70	300	4	8.07	62.67	8821.059
90	302	4	8.07	59.36	8825.409
110	305	4	8.07	56.35	8830.612
130	310	4	8.08	53.62	8836.539
150	315	4	8.08	51.35	8843.06
170	326	5	8.09	49.71	8849.432

in the swab supply chain. Therefore, Guideline 1 part 1 suggests that countries should have a higher swab reorder point whenever swab supply chains face significantly large disruptions. As there is more disruption in supply chains due to the uncertainty of the capacity, there is a higher chance the swab supplier cannot fulfill the order placed by the country. It, however, can only fulfill the number of swabs being produced. Therefore, (a) the country has to order more frequently to satisfy the total demand over the time horizon; and (b) the country has to have a higher swab reorder point to avoid stockouts, because of which the expected number of swab back-orders decreases. This adjustment, however, comes at a cost. First, because the country is ordering more frequently, fixed ordering cost increases. Next, because the country is carrying more swab inventory, holding cost increases. Therefore, total cost faces an increase. Figure 5 is the graphical representation for this part of Guideline 1.

Guideline 1 part 2 is in line with Corollary 1 and Corollary 3. It implies, if the mean of swabsupplier capacity is large enough, as the disruption in the supply chain decreases, the optimal swab-order quantity decreases, and it converges to that of the case with *NO* swab supplier capacity constraint. Guideline 1 part 3 refers to the case where the mean of the swab-supplier capacity is small. In this case, as disruption increases, it means that the maximum supplier capacity increases.

Therefore, the country should increase its optimal swab-order quantity to increase the chance of receiving a larger number of swabs. Before we move to Guideline 2, we provide evidence for Guideline 1 in Table 3. Because there is already evidence for Guideline 1 part 2 in Corollary 1 and Corollary 3, we only provide evidence for Guideline 1 part 1 and part 3.

Guideline 2 Governments should respond to the changes in the mean of supplier's capacity (i.e., μ) as following:

Governments should decline their reorder point (i.e., R^* decreases) when the mean of supplier's capacity (i.e., μ)

Table 4 Effect of changes in the mean of supplier's capacity, μ , on the optimal policy, where $\sigma = 10$ and $y \in [0, 2000]$

μ	Q^*	N^*	R^*	$b(R^*)$	$K(Q^*, N^*)$
250	232	5	10.85431	98.8050	11297.51
300	313	4	10.63035	164.0405	11239.31
350	332	3	10.59432	201.9831	11239.14
400	382	2	10.53953	267.2596	11260.59

increases given a stochastic demand. As a result, the optimal expected number of back-orders (i.e., $b(R^*)$) increases (i.e., patients have to stay in longer lines to get tested).

As μ raises, the number of times that governments order decreases (i.e., N^* declines).

To explain the above guidelines, note that higher (lower) μ is associated with higher (lower) swab-supplier average capacity. Guideline 2 part 2 implies that less frequent orders are optimal when swab-supplier average capacity is high. This takes place because, as μ increases, more swabs are available for the country to order, which results in (a) larger fulfillment; (b) larger cycle length (i.e., the smaller number of optimal cycles, N^*); and (c) lower fixed ordering cost. Guideline 2 part 1 is in line with Corollary 3, which implies that as μ increases, the optimal reorder point (i.e., R^*) decreases and converges to the R^* associated with the case where there is no capacity constraint. This means that as swab-supplier capacity constraint becomes less of an issue for the country, the country should decrease its reorder point and carry less swab inventory over each cycle. This results in: (a) lower holding cost and (b) higher expected number of back-orders. Our numerical study suggests that the decrease in fixed ordering costs and holding costs only sometimes dominates the increase in back-order cost, hence a decrease in total cost. In Table 4, we provide the evidence for this guideline.

Table 5 Comparing the optimal policies with and without supplier's capacity constraint under stationary demand, where $\mu = 300$ and $y \in [0, 2000]$

σ	N^*		Q^*		$K(Q^*, N^*)$	
	Without capacity constraint	With capacity constraint	Without capacity constraint	With capacity constraint	Without capacity constraint	With capacity constraint
10	3	3	282	283	1414.214	1414.22
30	3	3	282	284	1414.214	1415.439
50	3	4	282	284	1414.214	1420.198
70	3	4	282	286	1414.214	1429.047
90	3	4	282	289	1414.214	1442.364
110	3	4	282	292	1414.214	1460.559
130	3	4	282	296	1414.214	1484.047
150	3	4	282	303	1414.214	1513.259
170	3	5	282	310	1414.214	1548.628

6.2 Guidelines on Supply Chain Disruptions

As mentioned in the introduction, the swab supply chain was able to satisfy the global demand before the COVID-19 pandemic. However, after the pandemic due to swab supply chain disruptions, global demand surged while global supply plunged. This resulted in a worldwide shortage of swabs, which resulted in the COVID-19 testing shortage in many countries. When such disruptions take place, governments should proactively react to the changes in supplier's capacity. That is why in this section, we provide guidelines for governments on how to respond to supply chain disruptions optimally.

Guideline 3 Governments should respond to supply chain disruptions as follows when the swab demand is stationary.

Governments should increase order quantity when the supplier faces capacity constraints.

Governments should increase the optimal order quantity and optimal reorder point when there is a capacity constraint and demand is stochastic unless the mean and standard deviation of the capacity constraint is relatively smaller than that of the demand. As a result, the expected back-order is generally smaller when there is a capacity constraint.

Regardless of the demand's stochasticity, governments' total inventory cost is higher when there is a capacity constraint.

Guideline 3 part 1 implies, if the demand is stationary, when the swab supplier faces disruption (i.e., when there is a capacity constraint), the country should increase its swab order. To explain it, note that due to capacity constraints, not all swab orders placed are fulfilled. That is, in one cycle, the country might receive a small number of swabs, and in another, it might receive Q^* . Therefore, increasing Q helps the country avoid too

Table 6 Effect of demand (stationary vs. stochastic) on optimal policies where $\mu = 300, \gamma \in [0, 2000]$

σ	N^*		Q^*		$K(N^*, Q^*)$	
	Constant demand	Stochastic demand	Constant demand	Stochastic demand	Constant demand	Stochastic demand
10	3	3	283	313	11239.31	1414.214
30	3	4	284	314	11241.17	1414.214
50	4	4	284	313	11244.58	1414.214
70	4	4	286	314	11248.86	1414.214
90	4	4	289	316	11254.35	1414.214
110	4	4	292	319	11260.32	1414.214
130	4	4	296	323	11266.92	1414.214
150	4	4	303	328	11274.58	1414.214
170	4	5	310	335	11281.52	1414.214

many orders (i.e., short cycle length, hence, high fixed ordering cost). Guideline 3 part 2 implies that if demand is stochastic, then the country should set a high reorder point, which helps tackle the stochasticity of the capacity constraint. If the country receives a number of swabs smaller than Q , in one cycle, the country can use the safety stock to avoid stockouts. Moreover, with the same logic for Guideline 3 part 1, the country should increase its order quantity. All these adjustments come at the cost of the increased total cost, which is observed in Sect. 3. In Table 5, we provide evidence for Guideline 3 part 1.

Finally, Guideline 4 presents the effect of demand stochasticity on optimal order quantity and optimal total cost.

GUIDELINE 4 Governments should increase their optimal order quantity (i.e., Q^*) as uncertainty in demand increases regardless of supply chain disruption. This will result in the optimal number of cycles (i.e., N^*) to decrease.

The above guideline implies that, whether or not the swab supplier is facing capacity constraint, the country should order more swabs if COVID-19 testing demand is stochastic. This results in longer cycle lengths (i.e., a decrease in N) and increased swab inventory carried during each cycle, resulting in higher total cost. This guideline sheds light on the importance of forecasting. That is, if the country is sure how many swabs it needs during the time horizon (i.e., removing the variability of demand), it can reduce the total cost. This guideline is in line with traditional inventory management literature that suggests that more demand variability results in higher optimal order quantities and higher total cost. For this guideline, we provide evidence in Table 6. We offer a summary and conclusion in the next section.

7 Conclusions

Recently, an outbreak of COVID-19 in China's Wuhan resulted in a global pandemic, which crippled many countries' economies. To control the spread of the virus, some

countries (e.g., the United States, varying state-by-state; Italy and Germany, nationwide) have adopted shelter-in-place strategies. To contain COVID-19 and re-open the economy, health experts believe that widespread testing is crucial, mainly with the rise of contagious new variants. However, this is not possible because a critical component of COVID-19 testing, nasal swabs, faces a global shortage. The reason is that swabs are supplied by two primary producers, whose supply chains are facing disruption because of COVID-19. This persistent disruption translates to swab supplier variable production capacity. As a result, when countries order swabs from a swab supplier, their order might not be fully satisfied. Hence, adopting a proper swab inventory management model can help countries to manage COVID-19 testing better. This research is motivated by the above inventory management problem. We developed mathematical models where: (1) demand for COVID-19 testing is assumed to be stationary or stochastic; (2) the supplier's production capacity is assumed to be ample or variable; and (3) because the vaccination will be completed soon worldwide, i.e., following 12 months, the time horizon is finite. To the best of our knowledge, the existing literature has not studied such a problem. Specifically, the model featuring stochastic demand and a supplier with variable capacity is known to be challenging.

First, we study the case with stationary demand. Because of the demand pattern, avoiding swab shortages is possible. We divide the planning horizon into multiple cycles, and we derive the optimal policy for the following two scenarios. (1) We examine a scenario where the supplier has ample capacity, because of which all the swab orders placed by the country are fulfilled. In this scenario, we prove that it is optimal to order the same number of swabs in different cycles, and we derive the optimal order quantity. (2) We examine a scenario where the supplier has a stochastic capacity, because of which it can only fulfill the minimum of the number of swabs ordered and its production capacity. Similarly, in this scenario, we prove that it is optimal to

order the same number of swabs in different cycles, and we derive sufficient conditions to find that number. Notably, due to the stochasticity of the supplier's capacity, cycles have different lengths. Finally, we report the results of these two models for a COVID-19-specific example.

Second, we study the case with stochastic demand. Because of the demand pattern, having swab shortages is inevitable. We divide the planning horizon into multiple cycles, and we derive the optimal policy for the following two scenarios. (1) We examine a scenario wherein the supplier has ample capacity, because of which all the swab orders placed by the country are fulfilled. We prove that, in this scenario, at optimality, the optimal order quantities and the optimal reorder points in different cycles are the same. Although we cannot find the optimal order quantity and the optimal reorder point in closed-form, we derive the optimal order quantity as a function of order quantity, and we also derive the optimal reorder point as a function of order quantity. We then propose an iterative algorithm that finds the optimal order quantity and optimal reorder point. (2) We examine a scenario wherein the supplier has a stochastic capacity, because of which it can only fulfill the minimum number of swabs ordered and its production capacity. We assume that, in this scenario, at optimality, the optimal order quantities and the optimal reorder points in different cycles are the same. Similarly, we derive the optimal order quantity as a function of order quantity, and we also derive the optimal reorder point as a function of order quantity. We then propose an iterative algorithm that finds the optimal order quantity and the optimal reorder point. Finally, we report the results of these two models for a COVID-19-specific example.

Lastly, through a comprehensive numerical study, we provide guidelines to decision-makers (i.e., governments) on optimally reacting to changes in the demand and supply of the swabs. The most notable managerial implications are as follows: (1) As the swab supply chain becomes more disrupted: (a) governments should increase their reorder point, and as a result, the optimal expected number of back-orders decreases (i.e., shorter lines to get tested); and (b) if the supplier's average capacity is small, governments should increase their order size, and as a result, the optimal expected total cost increases. (2) When demand's variation increases, the optimal order quantity and the optimal total cost increase. (3) When supply chain disruptions occur, governments' optimal order quantity and optimal total cost increase.

Admittedly, our research has two limitations, based on which we propose future research possibilities. 1) Algorithms to find the optimal policy for the stochastic demand pattern are complex and time-consuming for large-scale problems. For a real-world example, it could take days to find the optimal solutions on a regular computer (this time could reduce

to only a few hours on an advanced computer). Any algorithm that can reduce the time to find the optimal solution could be beneficial for governments, hospitals and insurance companies. Therefore, for future research, one can develop more efficient heuristics to find the optimal solution promptly. To that end, one might want to focus on the properties of the total cost function expression (i.e., its concavity or convexity) and base their algorithm on these properties. 2) We assumed that the mean of the demand and capacity constraint are constant over time, while in reality this assumption might not hold for some products. For future research, one may relax these assumptions. We suggest using Brownian Motion to model such a problem. However, this problem will be complex, and the optimal solutions might not be tractable.

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