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OPEN Destination choice game: A spatial interaction theory on human mobility

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With remarkable significance in migration prediction, global disease mitigation, urban planning and many others, an arresting challenge is to predict human mobility fluxes between any two locations. A number of methods have been proposed against the above challenge, including the gravity model, the intervening opportunity model, the radiation model, the population-weighted opportunity model, and so on. Despite their theoretical elegance, all models ignored an intuitive and important ingredient in individual decision about where to go, that is, the possible congestion on the way and the possible crowding in the destination. Here we propose a microscopic mechanism underlying mobility decisions, named destination choice game (DCG), which takes into account the crowding effects resulted from spatial interactions among individuals. In comparison with the state-of-the-art models, the present one shows more accurate prediction on mobility fluxes across wide scales from intracity trips to intercity travels, and further to internal migrations. The well-known gravity model is proved to be the equilibrium solution of a degenerated DCG neglecting the crowding effects in the destinations.

Predicting human mobility fluxes between locations is a fundamental problem in transportation science and spatial economics^{1,2}. For more than a hundred years researchers have demonstrated the existence of gravity law in railway passenger movements^{3,4}, highway car flow^{4,5}, cargo shipping volume⁶, commuters' trips⁷, population migration⁸, and so on. Therefore, the corresponding gravity model and its variants become the mostly widely used predictor for mobility fluxes and have found applications in many fields⁹, such as urban planning¹⁰, transportation science^{1,11}, infectious disease epidemiology^{12,13} and migration prediction¹⁴. However, the gravity model is just an analogy to the Newton's law, without any insights about the underlying mechanism leading to the observed mobility patterns. To capture the underlying mechanism of human mobility, some models accounting for individuals' decisions on destination choices were proposed, including the intervening opportunities (IO) model¹⁵, the radiation model¹⁶ and the population-weighted opportunity (PWO) model^{17,18}. Some recently developed novel variants and extensions of the radiation and the gravity model 19-28 can more accurately predict commuting, immigration or long distance travel patterns at different spatial scales. However, all these models assume that individuals are independent of each other when selecting destinations, without any interactions.

In reality, individuals consider not only the destination attractiveness and the travelling cost, but also the crowding caused by the people who choose the same destination ²⁹⁻³¹, as well as the congestion brought by the people on the same way to the destination^{31,32}. The crowding in the destination even happens in migration, because the more people move to a certain place, the competition among job seekers and the living expense become higher. For example, in China, the city with larger population are usually of higher house price. However, so far, to our knowledge, there is no mechanistic model about human mobility taking into account the crowding effects caused by spatial interactions among individuals.

In this paper, we propose a so-called destination choice game (DCG) to model individuals' decision-makings about where to go. In the utility function about destination choice, in addition to the travelling cost and the fixed destination attractiveness, we consider the costs resulted from the crowding effects in the destination and the congestion in the way. Extensive empirical studies from intracity trips to intercity travels, and further to internal migrations have demonstrated the advantages of DCG in accurately predicting human mobility fluxes between any two locations, in comparison with other well-known models including the gravity model, IO model, radiation model and PWO model. We have further proved that the famous gravity model is equivalent to a degenerated

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DCG neglecting the crowding effects in the destination. Therefore, the higher accuracy of the prediction of DCG indicates the existence of the crowding effects on our decision-makings, which also provides a supportive evidence for the underlying hypothesis of the El Farol Bar problem²⁹ and the minority game³⁰.

Results

Model. We introduce the details of the DCG model in the context of travel issues. The number of individuals T_{ij} travelling from the starting location i to the destination j is resulted from the cumulation of destination choices of all individuals at location i. We model such decision-making process by a multiplayer game with spatial interactions, where each individual chooses one destination from all candidates to maximize his utility. Specifically speaking, the utility U_{ij} of an arbitrary individual at location i to choose location j as destination consists of the following four parts. (i) The fixed payoff of the destination $h(A_j)$, where h is intuitively assumed to be a monotonically increasing function of j's attractiveness A_j that is usually dependent on j's population, GDP, environment, and so on³³. (ii) The fixed travelling $\cos C_{ij}$. (iii) The congestion effect $g(T_{ij})$ on the way, where T_{ij} is the target quantity and g is a monotonically non-decreasing function. (iv) The crowding effect $f(D_j)$ at the destination, where f is a monotonically non-decreasing function and $D_j = \sum_i T_{ij}$ is the total number of individuals choosing j as their destination. In a word, the utility function U_{ij} reads

$$U_{ij} = h(A_i) - f(D_i) - C_{ij} - g(T_{ij}),$$
(1)

where destination attractiveness A_j and travelling $\cos t C_{ij}$ are input data, T_{ij} is the model estimated flux from location i to j and destination attraction $D_i = \sum_i T_{ii}$.

tion i to j and destination attraction $D_j = \sum_i T_{ij}$. In the above destination choice game (DCG), if every individual knows complete information, the equilibrium solution guarantees that all O_i individuals at the same starting location i have exactly the same utility no matter which destinations to be chosen. Strictly speaking, the variable T_{ij} has to be continuous to guarantee the existence of an equilibrium solution, which is a reasonable approximation when there are many individuals in each journey $i \rightarrow j$. Figure 1a illustrates a simple game scene. Considering a simple utility function $U_{ij} = A_j - \frac{1}{3}D_j - C_{ij} - T_{ij}$ that takes into account both the congestion effect on the way and the crowding effect in the destination, we can obtain the equilibrium solution based on the equilibrium condition $(U_{i3} = U_{i4})$ and the conservation law $(T_{i3} + T_{i4} = O_i$ and $T_{ij} + T_{2j} = D_j$). The solution is shown in Fig. 1b.

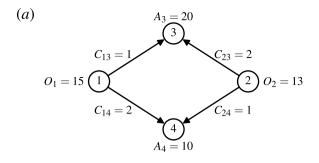
Generally speaking, we cannot obtain the analytical expression of the equilibrium solution, instead, we apply the method of successive averages³⁴ (MSA, see **Methods**) to iteratively approach the solution. Since the Weber-Fechner law³⁵ (see **Methods**) in behavioral economics is a good explanation of how humans perceive the change in a given stimulus, we select the logarithmic form determined by the Weber-Fechner law to express the destination payoff function $h(A_j)$ as $\alpha \ln A_j$, the destination crowding function $f(D_j)$ as $\gamma \ln D_j$ and the route congestion function $g(T_{ij})$ as $\ln T_{ij}$. On the other hand, since travelling cost often follows an approximate logarithmic relationship with distance in multimodal transportation system³⁶, we use $\beta \ln d_{ij}$ instead of C_{ij} , where d_{ij} is the geometric distance between i and j. We then get a practical utility function

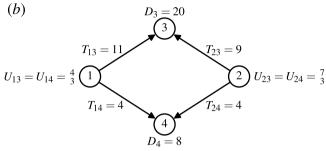
$$U_{ij} = \alpha \ln A_j - \beta \ln d_{ij} - \gamma \ln D_j - \ln T_{ij}, \qquad (2)$$

where α , β and γ are nonnegative parameters that can be fitted by real data (see **Methods**), subject to the largest Sørensen similarity index³⁷ (SSI, see **Methods**). A_j is the location j's attractiveness, which is approximated by the actual number of attracted individuals in the real data.

Prediction. We use three real data sets, including intracity trips in Abidjan, intercity travels in China and internal migrations in US, to test the predictive ability of the DCG model. The data set of intracity trips in Abidjan is extracted from the anonymous Call Detail Records (CDR) of phone calls and SMS exchanges between Orange Company's customers in Côte d'Ivoire³⁸. To protect customers' privacy, the customer identifications have been anonymized. The positions of corresponding base stations are used to approximate the positions of starting points and destinations. The data set of intercity travels in China¹⁸ is extracted from anonymous users' check-in records at Sina Weibo, a large-scale social network in China with functions similar to Twitter. Since here we focus on movements between cities, all the check-ins within a prefecture-level city are regarded as the same with a proxy position being the centre of the city. The data set of internal migrations in US is downloaded from https://www.irs.gov/statistics/soi-tax-stats-migration-data. This data set is based on year-to-year address changes reported on individual income tax returns and presents migration patterns at the state resolution for the entire US, namely for each pair of states *i* and *j* in US, we record the number of residents migrated from *i* to *j*. The fundamental statistics are presented in Table 1. In all the above three data sets and other data sets presented in the Supplementary Information, Table S1, every location can be chosen as a destination.

We use three different metrics to quantify the proximity of the DCG model to the real data. Firstly, we investigate the travel distance distribution, which is the most representative feature to capture human mobility behaviours 36,39,40 . As shown in Fig. 2a–c, the distributions of travel distances predicted by the DCG model are in good agreement with the real distributions. We next explore the probability P(D) that a randomly selected location has eventually attracted D travels (in the model, for any location j, D_j is the total number of individuals choosing j as their destination). P(D) is a key quantity measuring the accuracy of origin-constrained mobility models, because origin-constrained models cannot ensure the agreement between predicted travels and real travels to a location Figure 2d–f demonstrate that the predicted and real P(D) are almost statistically indistinguishable. Thirdly, we directly look at the mobility fluxes between all pairs of locations $^{16-18}$. As shown in Fig. 2g–i, the average fluxes predicted by the DCG model are in reasonable agreement with real observations.

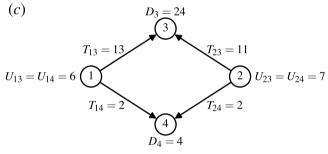




$$U_{13} = A_3 - \frac{1}{3}D_3 - C_{13} - T_{13} = A_4 - \frac{1}{3}D_4 - C_{14} - T_{14} = U_{14}$$

$$U_{23} = A_3 - \frac{1}{3}D_3 - C_{23} - T_{23} = A_4 - \frac{1}{3}D_4 - C_{24} - T_{24} = U_{24}$$

$$T_{13} + T_{14} = O_1, T_{23} + T_{24} = O_2, T_{13} + T_{23} = D_3, T_{14} + T_{24} = D_4$$



$$U_{13} = A_3 - C_{13} - T_{13} = A_4 - C_{14} - T_{14} = U_{14}, T_{13} + T_{14} = O_1$$

$$U_{23} = A_3 - C_{23} - T_{23} = A_4 - C_{24} - T_{24} = U_{24}, T_{23} + T_{24} = O_2$$

Figure 1. Illustration of a simple example of DCG. (a) The game scene. The nodes 1 and 2 represent two starting locations while the nodes 3 and 4 are two destinations. O_i is the number of individuals located in i, A_j is the attractiveness of j, and C_{ij} is the fixed travelling cost from i to j. (b) An example game taking into account both the congestion effect on the way and the crowding effect in the destination, with a utility function $U_{ij} = A_j - \frac{1}{3}D_j - C_{ij} - T_{ij}$. (c) An example game that does not consider the crowding effect in the destination, with a utility function $U_{ij} = A_j - C_{ij} - T_{ij}$. For both (a and b), the equilibrium solutions are shown in the plots while the equations towards the solutions are listed below the plots.

Data set	#individuals	#movements	#locations	positional proxy
intracity trips in Abidjan	154849	519710	381	base station
intercity travels in China	1571056	4976255	340	prefecture-level city
internal migrations in US	N/A	2498464	51	state capital

Table 1. Fundamental statistics of the data sets. The second to fifth columns present the number of individuals, the number of recorded movements, the number of locations and how to estimate the geographical positions of these locations. For migration data, we do not know the precise number of individuals, but it should be close to the number of total records since people usually do not migrate frequently.

We next compare the predicting accuracy on mobility fluxes of DCG with well-known models including the gravity models, the intervening opportunities model, the radiation model and the population-weighted opportunities (PWO) model (see **Methods**). In terms of SSI, as shown in Fig. 3, DCG performs best. Specifically speaking,

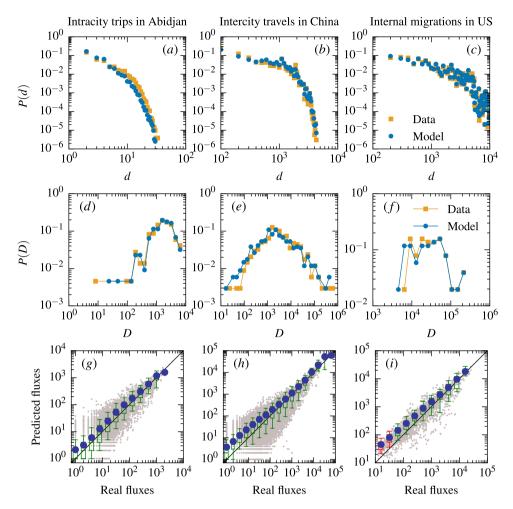


Figure 2. Comparing the predictions of DCG model and the empirical data. (\mathbf{a} - \mathbf{c}) Predicted and real distributions of travel distances P(d). (\mathbf{d} - \mathbf{f}) Predicted and real distributions of locations's attracted travels P(D). (\mathbf{g} - \mathbf{i}) Predicted and observed fluxes. The gray points are scatter plot for each pair of locations. The blue points represent the average number of predicted travels in different bins. The standard boxplots represent the distribution of predicted travels in different bins. A box is marked in green if the line y = x lies between 10% and 91% in that bin and in red otherwise. The data presented in (\mathbf{d} - \mathbf{i}) are binned using the logarithmic binning method.

it is remarkably better than parameter-free models like the radiation model and the PWO model and slightly better than the gravity model with two parameters. Supplementary Information, Additional validation of the DCG model shows extensive empirical comparisons between predicted and real statistics as well as accuracies of different methods for more data sets involving travels inside and between cities in Japan, UK, Belgium, US and Norway. Again, in terms of SSI, DCG outperforms other benchmarks in all cases. Not only that, DCG also better predicts the travel distance distribution P(d) and destination attraction distribution P(D) in most cases (see Figs S5 and S6 and Tables S2 and S3).

Derivation of the gravity model. To further understand the advantage of the DCG model in comparison with the well-adopted gravity models, we give a close look at the key mechanism differentiated from all previous models, that is, the extra cost caused by the crowding effect, as inspired by the famous *minority game*³⁰. Accordingly, we test a simplified model without the term $f(D_j)$ in Eq. (1). Figure 1c illustrates an example with a simple utility function $U_{ij} = A_j - C_{ij} - T_{ij}$ that only takes into account the congestion effect on the way. Similar to the case shown in Fig. 1b, the equilibrium solution can be obtained by the equilibrium condition and the conservation law. For a more general and complicated utility function (by removing the term related to the crowding effect in Eq. (2))

$$U_{ij} = \alpha \ln A_j - \beta \ln d_{ij} - \ln T_{ij}, \tag{3}$$

based on the potential game theory⁴¹, one can prove that the equilibrium solution is equivalent to the solution of the following optimization problem

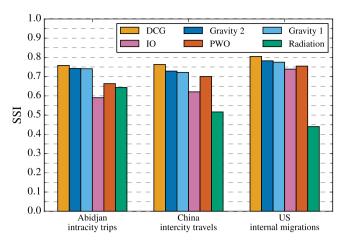


Figure 3. Comparing predicting accuracy of the DCG model and well-known benchmarks in terms of SSI.

$$\max Z(x) = \sum_{j} \int_{0}^{T_{ij}} (\alpha \ln A_{j} - \beta \ln d_{ij} - \ln x) dx,$$
s. t.
$$\sum_{j} T_{ij} = O_{i}, \ T_{ij} \geq 0.$$
(4)

Since the objective function is strictly convex, the solution is existent and unique. Applying the Lagrange multiplier method, we can obtain the solution of Eq. (4), which is exactly the same to the gravity model with two free parameters (i.e., Gravity 2, Eq. (11)), and if we set $\alpha=1$ in Eq. (3), the solution degenerates to the gravity model with one free parameter (i.e., Gravity 1, Eq. (10)). The detailed derivation is shown in Supplementary Information, Derivation of the gravity model using potential game theory. The significance of such interesting finding is threefold. Firstly, it provides a theoretical bridge that connecting the DCG model and the gravity model, which are seemingly two unrelated theories. Indeed, it provides an alternative way to derive the gravity model. Secondly, comparing with the gravity models, the higher accuracy of the prediction from the DCG model suggests the existence of the crowding effect in our decision-making about where to go, which also provides a positive evidence for the validity of the critical hypothesis underlying the minority game. Thirdly, the improvement of accuracy from Gravity 2 to the DCG model can be treated as a measure for the crowding effect, which is, to our knowledge, the first quantitative measure for the crowding effect in human mobility.

Discussion

In summary, the theoretical advantages of DCG are twofold. First of all, it does not require any prerequisite from God's perspective, like the constraint on total costs in the maximum entropy approach 42,43 and the deterministic utility theory⁴⁴, or any oversubtle assumption, like the independent identical Gumbel distribution to generate the hypothetically unobserved utilities associated with travels in the random utility theory⁴⁵. Instead, the two assumptions underlying DCG, namely (i) each individual chooses a destination to maximize his utility and (ii) congestion and crowding will decrease utility, are very reasonable. Therefore, in comparison with the above-mentioned theories, DCG shows a more realistic explanation towards the gravity model by neglecting the crowding effect in destinations (see some other derivations to the gravity model in Supplementary Information, Other derivations of the gravity model). Secondly, the present game theoretical framework is more universal and extendable. As the travelling costs and crowding effects are naturally included in the utility function, DCG is easy to be extended to deal with more complicated spatial interactions that depend on individuals' choices about not only destinations, but also departure time, travel modes, travel routes, and so on $^{46-48}$. Not only that, the utility function of DCG can also be extended in predicting specific mobility behaviours. For example, when predicting the mobility fluxes in a multi-modal transportation system, the logarithmic (or linear logarithmic) function of distance is usually used to calculate the fixed travel cost between locations, while when predicting in a single-modal transportation system, the linear cost-distance function is usually used³⁶. For the destination payoff, destination crowding cost and route congestion cost in the utility function, although the DCG model has obtained better prediction accuracy by using the logarithmic functions inspired by the Weber-Fechner law, the realistic payoff and cost functions may be much more complicated. Therefore if we can mine real cost functions by some machine learning algorithms from real data, the prediction accuracy could be further improved.

In addition to theoretical advantages, DCG could better aid government officials in transportation intervention. For example, if the government would like to raise congestion charges in some areas (e.g., in Beijing, the parking fees in central urban areas are surprisingly high), the parameter-free models like the radiation model and the PWO model cannot predict the quantitative impacts on travelling patterns since the population distribution is not changed, instead, the game theoretical framework could respond to the policy changes by rewriting its utility function. Another example is to forecast and regulate tourism demand⁴⁹. In China, in the vacations of the National Day and the Spring Festival, many people stream in a few most popular tourist spots, leading

to unimaginable crowding and great environmental pressure. Recently, Chinese government forecasts tourism demand before those golden holidays based on the booking information about air tickets, train tickets and entrance tickets, and then the visitors are effectively redistributed to more diverse tourist spots with remarkable decreases of visitors to the most noticed a few spots. Such phenomenon can be explained by the crowding effects in the destination choices, but none of other known models. In a word, DCG is more relevant to real practices and thus of potential to be enriched towards an assistance for decision making.

Methods

Method of successive averages. The method of successive averages (MSA) is an iterative algorithms to solve various mathematical problems³⁴. For a general fixed point problem $\mathbf{x} = \mathbf{F}(\mathbf{x})$, the **n**th iteration in the MSA uses the current solution $\mathbf{x}^{(n)}$ to find a new solution $\mathbf{y}^{(n)} = \mathbf{F}(\mathbf{x}^{(n)})$. The next current solution is an average of these two solutions $\mathbf{x}^{(n+1)} = (1-\lambda^{(n)})\mathbf{x}^{(n)} + \lambda^{(n)}\mathbf{y}^{(n)}$, where $0 < \lambda^{(n)} < 1$ is a parameter. For the DCG model, the MSA contains the following steps:

Step 1: Initialization. Set the iteration index $\mathbf{n} = 1$. Calculate an initial solution for the number of individuals travelling from i to j

$$T_{ij}^{(\mathbf{n})} = O_i \frac{A_j^{\alpha} d_{ij}^{-\beta}}{\sum_j A_j^{\alpha} d_{ij}^{-\beta}},\tag{5}$$

where O_i is an independent variable representing the number of travellers starting from location i, A_j is the attractiveness of location j and d_{ii} is the distance from i to j (O_i , A_i and d_{ii} are all initial input variables).

Step 2: Calculate a new solution for the number of individuals travelling from *i* to *j*

$$F_{ij}^{(\mathbf{n})} = O_i \frac{A_j^{\alpha} d_{ij}^{-\beta} [D_j^{(\mathbf{n})}]^{-\gamma}}{\sum_i A_j^{\alpha} d_{ij}^{-\beta} [D_j^{(\mathbf{n})}]^{-\gamma}},$$
(6)

where $D_j^{(n)} = \sum_i T_{ij}^{(n)}$ is the total number of individuals choosing j as their destination.

Step 3: Calculate the average solution

$$T_{ij}^{(\mathbf{n}+1)} = (1 - \lambda^{(\mathbf{n})}) T_{ij}^{(\mathbf{n})} + \lambda^{(\mathbf{n})} F_{ij}^{(\mathbf{n})}.$$
(7)

If $|T_{ij}^{(\mathbf{n}+1)}-T_{ij}^{(\mathbf{n})}|<\varepsilon$ (ε is a very small threshold, set as 0.01 in the work), the algorithm stops with current solution being the approximated solution; Otherwise, let $\mathbf{n}=\mathbf{n}+1$ and return to **Step 2**.

For simplicity, we use a fixed parameter $\lambda^{(n)} = \lambda = 0.5$.

Weber-Fechner law. Weber-Fechner Law (WFL) is a well-known law in behavioural psychology³⁵, which represents the relationship between human perception and the magnitude of a physical stimulus. WFL assumes the differential change in perception dp to be directly proportional to the relative change dW/W of a physical stimulus with size W, namely $dp = \kappa dW/W$, where κ is a constant. From this relation, one can derive a logarithmic function $p = \kappa \ln(W/W_0)$, where p equals the magnitude of perception, and the constant W_0 can be interpreted as stimulus threshold. This equation means the magnitude of perception is proportional to the logarithm of the magnitude of physical stimulus. The WFL is widely used to determine the explicit quantitative utility function in behavioural economics³⁵, and thus we adopt it in Eq. (2).

Sørensen similarity index. Sørensen similarity index is a similarity measure between two samples³⁷. Here we apply a modified version¹⁷ of the index to measure whether real fluxes are correctly reproduced (on average) by theoretical models, defined as

$$SSI = \frac{1}{N(N-1)} \sum_{i}^{N} \sum_{j \neq i}^{N} \frac{2 \min(T_{ij}, T'_{ij})}{T_{ij} + T'_{ij}},$$
(8)

where T_{ij} is the predicted fluxes from location i to j and T'_{ij} is the empirical fluxes. Obviously, if each T_{ij} is equal to T'_{ij} the index is 1, while if all T_{ij} are far from the real values, the index is close to 0.

Parameter estimation. We use grid search method 50 to estimate the three parameters α , β and γ of the DCG model. We first set the candidate value for each parameter from 0 to 10 at an interval of 0.01, and then exhaust all the candidate parameter sets to calculate the SSI (see Eq. (8)) of the DCG model, and finally select the parameter set that maximizes SSI. The parameter estimation results are shown in Supplementary Information, Table S1.

Benchmark models. We select two classical models, the gravity model and the intervening opportunities model, and two parameter-free models, the radiation model and the population-weighted opportunities model, as the benchmark models for comparison with the DCG model.

(i) The gravity model is the earliest proposed and the most widely used spatial interaction model². The basic assumption is that the flow T_{ij} between two locations i and j is proportional to the population m_i and m_j of the two locations and inversely proportional to the power function of the distance d_{ij} between the two

locations, as

$$T_{ij} = \alpha \frac{m_i m_j}{d_{ij}^{\beta}},\tag{9}$$

where α and β are parameters. To guarantee the predicted flow matrix T satisfies $O_i = \sum_j T_{ij}$, we use two origin-constrained gravity models¹. The first one is called Gravity 1 as it has only one parameter, namely

$$T_{ij} = O_i \frac{A_j d_{ij}^{-\beta}}{\sum_i A_j d_{ij}^{-\beta}},\tag{10}$$

while the second one is named Gravity 2 for it has two parameters, as

$$T_{ij} = O_i \frac{A_j^{\alpha} d_{ij}^{-\beta}}{\sum_j A_j^{\alpha} d_{ij}^{-\beta}}.$$
 (11)

(ii) The intervening opportunities (IO) model¹⁵ argues that the destination choice is not directly related to distance but to the relative accessibility of opportunities to satisfy the traveller. The model's basic assumption is that for an arbitrary traveller departed from the origin i, there is a constant very small probability α/β that this traveller is satisfied with a single opportunity. Assume the number of opportunities at the jth location (ordered by its distance from i) is proportional to its population m_i , i. e. the number of opportunities is βm_i , and thus the probability that this traveller is attracted by the jth location is approximated αm_i . Let $q_i^{(j)} = q_i^{(j-1)}(1-\alpha m_j)$ be the probability that this traveller has not been satisfied by the first to the jth locations (i itself can be treated as the 0th location), we can get the relationship $q_i^{(j)} = e^{-\alpha S_{ij}}/(1-e^{-\alpha M})$ between the probability $q_i^{(j)}$ and the total population S_{ij} in the circle of radius d_{ij} centred at location i, where M is the total population of all locations. Furthermore, we can get the expected fluxes from i to j is

$$T_{ij} = O_i(q_i^{(j-1)} - q_i^{(j)}) = O_i \frac{e^{-\alpha(S_{ij} - m_j)} - e^{-\alpha S_{ij}}}{1 - e^{-\alpha M}}.$$
(12)

(iii) The radiation model ¹⁶ assumes that an individual at location i will select the nearest location j as destination, whose benefits (randomly selected from an arbitrary continuous probability distribution p(z)) are higher than the best offer available at the origin i. The fluxes T_{ij} predicted by the radiation model is

$$T_{ij} = O_i \frac{m_i m_j}{(S_{ij} - m_j) S_{ij}}.$$
(13)

(iv) The population-weighted opportunities (PWO) model¹⁷ assumes that the probability of travel from i to j is proportional to the attractiveness of destination j, inversely proportional to the population S_{ji} in the circle centred at the destination with radius d_{ij} , minus a finite-size correction 1/M. It results to the analytical solution as

$$T_{ij} = O_i \frac{m_j \left(\frac{1}{S_{ji}} - \frac{1}{M}\right)}{\sum_j m_j \left(\frac{1}{S_{ji}} - \frac{1}{M}\right)}.$$
(14)

Data Availability

Data available on request from the authors.

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Author Contributions

X.-Y.Y. and T.Z. designed the research; X.-Y.Y. and T.Z. performed the research; X.-Y.Y. analysed the empirical data; and T.Z. and X.-Y.Y. wrote the paper.

Additional Information

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