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Effects of Submaximal Performances on Critical Speed and Power: Uses of an Arbitrary-Unit Method with Different Protocols

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Abstract: The effects of submaximal performances on critical speed (S_{Crit}) and critical power (P_{Crit}) were studied in 3 protocols: a constant-speed protocol (protocol 1), a constant-time protocol (protocol 2) and a constant-distance protocol (protocol 3). The effects of submaximal performances on S_{Crit} and P_{Crit} were studied with the results of two theoretical maximal exercises multiplied by coefficients lower or equal to 1 (from 0.8 to 1 for protocol 1; from 0.95 to 1 for protocols 2 and 3): coefficient C_1 for the shortest exercises and C_2 for the longest exercises. Arbitrary units were used for exhaustion times (t_{lim}), speeds (or power-output in cycling) and distances (or work in cycling). The submaximal-performance effects on S_{Crit} and P_{Crit} were computed from two ranges of t_{lim} (1–4 and 1–7). These effects have been compared for a low-endurance athlete (exponent = 0.8 in the power-law model of Kennelly) and a high-endurance athlete (exponent = 0.95). Unexpectedly, the effects of submaximal performances on S_{Crit} and P_{Crit} are lower in protocol 1. For the 3 protocols, the effects of submaximal performances on S_{Crit}, and P_{Crit}, are low in many cases and are lower when the range of t_{lim} is longer. The results of the present theoretical study confirm the possibility of the computation of S_{Crit} and P_{Crit} from several submaximal exercises performed in the same session.

Keywords: critical speed; critical power; performance reliability; modelling

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1. Introduction

1.1. Empirical Model of Running and Cycling Performances

Several empirical and descriptive models of performance have been proposed: the power-law model by Kennelly [1], asymptotic hyperbolic models by Hill and Scherrer [2,3], and, more recently, the logarithmic model of Péronnet and Thibault [4] and the 3-parameter asymptotic models by Hopkins [5] and Morton [6]. These empirical models are often used to estimate (i) the improvement in performance (ii) the future performances and running speeds over given distances (iii) the endurance capability, i.e., "the ability to sustain a high fractional utilization of maximal oxygen uptake for a prolonged period of time" [4].

In 1954, Scherrer et al. proposed a linear relationship [3] between the exhaustion time (t_{lim}) of a local exercise (flexions or extensions of the elbow or the knee) performed at different constant power outputs (P) and the total amount of work performed at exhaustion (W_{lim}) for t_{lim} ranging between 3 and 30 min:

$$W_{lim} = a + bt_{lim} = Pt_{lim}$$

Consequently, the relationship between P and t_{lim} is hyperbolic:

$$a = Pt_{lim} - bt_{lim} = (P - b)\ t_{lim}$$

$$t_{lim} = a/(P - b) = a/(P - P_{Crit})$$

where b is a critical power (P_{Crit}).

In 1966, Ettema applied the critical-power concept to world records in running, swimming, cycling, and skating exercises [7] and proposed a linear relationship between the distances (D_{lim}) and world records (t_{lim}) from 1500 to 10000 m:

$$D_{lim} = a + b_{tlim}$$

It was assumed that the energy cost of running was almost independent of speed (1 kcal.kg^{-1}.km^{-1}) under 22 km.h^{-1} [8]. Consequently, D_{lim} and parameter "a" were equivalent to amounts of energy. Therefore, parameter "a" has been interpreted as equivalent to an energy store. Thereafter, parameter "a" was considered as an estimation of maximal Anaerobic Distance Capacity (ADC expressed in metres) for running exercises [7,9]. Slope b was considered as a critical velocity (S_{Crit}).

$$D_{lim} = ADC + S_{Crit}\ t_{lim}$$

In 1981, the linear W_{lim}-t_{lim} relationship was adapted to exercises on a stationary cycle ergometer and it was demonstrated that slope b of the W_{lim}-t_{lim} relationship was correlated with the ventilatory threshold [10]. Therefore, slope b was proposed as an index of general endurance (P_{Crit}). Thereafter, Whipp et al. [11] proposed another linear model with S_{Crit} (or P_{Crit}) and a:

$$S = S_{Crit\ 1/t} + a\ (1/t_{lim})\ \text{Running}$$

$$P = P_{Crit\ 1/t} + a\ (1/t_{lim})\ \text{Cycling}$$

where S is running speed.

Actually, the hyperbolic model is often used as it is the simplest model that corresponds to a linear relationship between exhaustion time (t_{lim}) and distance (D_{lim}) in running and swimming or total work (W_{lim}) in cycling. For many exercise physiologists, S_{Crit} and P_{Crit} are considered as fatigue thresholds [12]. Moreover, the values of S_{Crit} and P_{Crit} become endurance indices when they are normalized to Maximal Aerobic Speed (S_{Crit} /MAS) or Maximal Aerobic Power (P_{Crit} /MAP). Parameter "a" is also called ADC (Anaerobic Distance Capacity) or ARC (Anaerobic Running Capacity) in running [7,9] and AWC (Anaerobic Work Capacity) in cycling [13,14].

However, the relationship between t_{lim} and D_{lim} is not perfectly linear as suggested by the power-law model by Kennelly:

$$D_{lim} = k\ t_{lim}{}^{g}$$

$$S = D_{lim}/t_{lim} = (k\ t_{lim}{}^{g})/t_{lim} = k\ t_{lim}{}^{g-1}$$

where exponent g can be considered as an endurance index and parameter k is equal to the maximal speed corresponding to the unit of t_{lim} [15].

1.2. Variability of the Performances of Exhausting Exercises.

Three protocols are used for the estimation of the performances of exhausting exercises:

(1) to run as long as possible at a constant speed or to cycle as long as possible at a constant power output. This constant-speed protocol is often called Time-to-Exhaustion;

(2) to run as much distance (or to produce as much work on a cycle-ergometer) as they can within a given time period (constant-time protocol);

(3) to run a set distance (or to produce a set work on a cycle-ergometer) as fast as possible (constant-distance protocol).

The first protocol (Time-to-Exhaustion) is used for the estimation of S_{Crit} on a treadmill or the estimation of P_{Crit} on a cycle ergometer. The second protocol was used for prediction of one-hour running performance [16]. The third protocol was used in the studies on the modelling of running performances that were based on the world records [17–20] or performances in the Olympic games [21] or individual performance of elite endurance runners [15]. The reliability of performances in protocol 1 (constant speed) is low, whereas the reliability of the other protocols is higher [22–27]. For example in swimming, the Coefficient of Variation of constant-speed protocol (CV = 6.46 ± 6.24%) was significantly less reliable ($p < 0.001$) than those of constant-time protocol (CV = 0.63 ± 0.54%) and constant-distance protocol (CV = 0.56 ± 0.60%) [27].

In a recent study [28] critical speeds measured on a treadmill with a constant-speed protocol were compared with a Single-Visit Field Test of Critical Speed. The constant-speed runs to exhaustion on treadmill were performed with 3 running speeds during 3 separate sessions. Two single-visit field tests on separate days consisted to the measurement of maximal performances over 3600, 2400, and 1200 m (constant-distance protocol) with 30-min or 60-min recovery. Unexpectedly, there was no difference in S_{Crit} measured with the treadmill and 30-min- and 60-min recovery field tests although the reliability of protocol 1 is lower than that of protocol 3. Thereafter, a Single-Visit Field Test of Critical Speed was tested in trained and untrained runners [29]: the reliability of S_{Crit} was better in the trained runners.

1.3. Purpose of the Present Study

In few studies, the values of S_{Crit} and P_{Crit} are computed from the best maximal performances of several exhausting exercises of the same subjects [15] or world records [17–20] or performances in the Olympic games [21]. In most studies, it is not obvious that the data used in the computation of S_{Crit} and P_{Crit} are maximal. For example, the performance variability is important in protocol 1 that is mainly used in laboratories. Moreover, several exhausting exercises are often performed in the same session with protocols 1, 2 and 3, which could increase the performance variability because of fatigue. As suggested in a review [30], the purpose of the present study was to confirm the interest of S_{Crit} and P_{Crit} computed from exercises whose performances are submaximal.

In the Single-Visit Field Test of Critical Speed, there were trained and untrained runners whose reliability of S_{Crit} was different [29]. Therefore, the effects of submaximal performance on S_{Crit} and P_{Crit} in the present study have been compared between a low-endurance athlete and a high-endurance athlete, i.e., athletes with low and high endurance indices (for example, exponent g or S_{Crit}/MAS or P_{Crit}/MAP). The values of exponent g were about 0.95 in the best elite endurance runners [15,31] as Gebrselassie whose ratio S_{Crit}/MAS was equal to 0.945 (MAS corresponded to the maximal running speed during 7 min). In the low-endurance runners whose ratios S_{Crit}/MAS were equal to 0.764 ± 0.078, exponent g was about 0.80 [31].

As S_{Crit} and P_{Crit} can be computed from two exhausting exercises, the effects of submaximal performances on these indices have been estimated by multiplying both theoretical maximal data by two coefficients: C_1 for the shortest performances and C_2 for the longest performances.

The effects of submaximal performances on S_{Crit} and P_{Crit} were estimated with arbitrary units for t_{lim}, D_{lim} and S (or P) in the present study. Indeed, there are many different cases:

- the range of t_{lim} can be different for each athlete in protocols 1 and 3;
- the range of speeds and distances (or power-output and work in cycling) can be different for each athlete in protocol 2;
- the same endurance indices can correspond to different maximal aerobic speed (MAS) or maximal aerobic power (MAP in cycling).

The effects of submaximal performances on endurance indices have been tested for values of t_{lim} equal to 1 and 4 (arbitrary units), which corresponds to the usual range of t_{lim} (3–12m in [28,29]) in many study. In some studies, the range of t_{lim} is 2–15 min [12,32,33], Therefore, the effects of submaximal performances have also been tested for values of t_{lim} equal to 1–7 (arbitrary unit).

2. Methods

2.1. Arbitrary Units

The values of t_{lim1}, D_{lim1} and S_1 in arbitrary units in protocols 1, 2 and 3 are equal to 1:

$$t_{lim1} = 1 \quad \text{and} \quad S_1 = 1 \quad \text{and} \quad D_{lim1} = 1$$

2.1.1. Arbitrary Units in Protocols 1 and 2

The values of t_{lim2} and t_{lim3} in arbitrary units in protocols 1 and 2 are equal to 4 and 7, respectively.

$$t_{lim2} = 4 \quad \text{and} \quad t_{lim3} = 7$$

The distances and running speeds corresponding to t_{lim2} and t_{lim3} was computed from the power-law model b Kennelly with arbitrary units of t_{lim}:

$$D_{lim1} = k\ t_{lim1}{}^{g} = k\ (1^{g}) = 1 \text{ then } k = 1 \quad \text{and} \quad D_{lim} = t_{lim}{}^{g} \text{ and} \quad S = t_{lim}{}^{g-1}$$

For the high-endurance athlete, exponent g of the power-law model is equal to 0.95. Therefore, the values of D_{lim2}, D_{lim3}, S_2 and S_3 in arbitrary units are equal to:

$$D_{lim2} = t_{lim2}{}^{g} = 4^{0.95} = 3.7321 \quad \text{and} \quad D_{lim3} = t_{lim3}{}^{g} = 7^{0.95} = 6.3510$$

$$S_2 = t_{lim2}{}^{g-1} = 4^{-0.05} = 0.9330 \quad \text{and} \quad S_3 = t_{lim3}{}^{g-1} = 7^{-0.05} = 0.9073$$

For the low-endurance athlete, exponent g of the power-law model is equal to 0.80. Therefore, the values of D_{lim2}, D_{lim3}, S_2 and S_3 in arbitrary units are equal to:

$$D_{lim2} = 4^{0.80} = 3.0314 \quad \text{and} \quad D_{lim3} = 7^{0.80} = 4.7433$$

$$S_2 = 4^{-0.2} = 0.7579 \quad \text{and} \quad S_3 = 7^{-0.2} = 0.6776$$

2.1.2. Arbitrary Units in Protocol 3

The values of constant-distances (D_{lim}) in the present study were equal to the averages of the distances of low and high-endurance athletes, in protocols 1 and 2, i.e., 1 (D_{lim1}), 3.3819 (D_{lim2}) and 5.5471 (D_{lim3}). The values of t_{lim2} and t_{lim3} corresponding to these distances were:

$$D_{lim} = t_{lim}{}^{g}$$

$$(t_{lim}{}^{g})^{1/g} = t_{lim} = D_{lim}{}^{1/g}$$

$$t_{lim1} = D_{lim1}{}^{1/g} = 1^{1/g} = 1 \quad \text{and} \quad t_{lim2} = D_{lim2}{}^{1/g} = 3.3819^{1/g}$$
$$\text{and } t_{lim3} = D_{lim3}{}^{1/g} = 5.5471^{1/g}$$

For the low-endurance athlete:

$$t_{lim2} = 3.3819^{1/0.8} = 4.5862 \quad \text{and} \quad t_{lim3} = 5.5471^{1/0.80} = 8.5130$$

For the high-endurance athlete:

$$t_{lim2} = 3.3819^{1/0.95} = 3.6059 \quad \text{and} \quad t_{lim3} = 5.5471^{1/0.95} = 6.\ 0705$$

2.2. Coefficients C_1 and C_2

The ranges of coefficients C_1 and C_2 used in protocol 2 and 3 were from 0.95 to 1. Indeed, in protocols 2 and 3, the submaximal performances are the results of submaximal speeds (or submaximal power outputs in cycling). If the ratio C_1/C_2 is lower than 0.9330, the speed corresponding to t_{lim2} would be higher than the speed at t_{lim1} in the high-endurance athlete. In protocol 1 (constant-speed protocol), the speed (or power) does not depend on C_1 or C_2 and the variability of performances are higher [27]. Therefore, the ranges of C_1 and C_2 were larger (from 0.8 to 1).

2.3. Computation of the Effects of Submaximal Performances on S_{Crit} and P_{Crit}

2.3.1. Computation in the Constant-Speed (or Constant Power Output) Protocol (Protocol 1)

The submaximal performances are the result of the submaximal values of t_{lim}. The submaximal values of t_{lim} ($t_{lim1\ sub}$ and $t_{lim2\ sub}$) are:

$$t_{lim1\ sub} = C_1\ t_{lim1} \qquad \text{and} \qquad t_{lim2\ sub} = C_2\ t_{lim2}$$

Therefore, the submaximal values of D_{lim} ($D_{lim1\ sub}$ and $D_{lim2\ sub}$) are equal to:

$$D_{lim1\ sub} = S_1\ t_{lim1\ sub} = S_1C_1\ t_{lim1} = C_1\ D_{lim1}$$

$$D_{lim2\ sub} = S_2\ t_{lim2\ sub} = S_2C_2\ t_{lim2} = C_2\ t_{lim2}$$

For running:

$$S_{Crit} = (D_{lim2} - D_{lim1})/(t_{lim2} - t_{lim1})$$

$$S_{Crit\ sub} = (C_2\ D_{lim2} - C_1\ D_{lim1})/(C_2\ t_{lim2} - C_1\ t_{lim1})$$

$$S_{Crit\ sub}/S_{Crit} = [(C_2\ D_{lim2} - C_1\ D_{lim1})/(C_2\ t_{lim2} - C_1\ t_{lim1})]/[(D_{lim2} - D_{lim1})/(t_{lim2} - t_{lim1})] \quad (1)$$

For cycling:

$$P_{Crit\ sub}/P_{Crit} = [(C_2\ W_{lim2} - C_1\ W_{lim1})/(C_2\ t_{lim2} - C_1\ t_{lim1})]/[(W_{lim2} - W_{lim1})/(t_{lim2} - t_{lim1})]$$

2.3.2. Computation in the Constant-Time Protocol (Protocol 2)

The submaximal performances are the result of submaximal speeds.

$$S_{1sub} = C_1S_1 \qquad \text{and} \qquad S_{2sub} = C_2S_2$$

$$D_{lim1\ sub} = C_1S_1\ t_{lim1} = C_1D_{lim1} \qquad \text{and} \qquad D_{lim2\ sub} = C_2S_2\ t_{lim2} = C_2D_{lim2}$$

For cycling, the submaximal performances correspond to lower powers.

$$P_{1sub} = C_1P_1 \qquad \text{and} \qquad P_{2sub} = P_2S_2$$

$$W_{lim1\ sub} = C_1P_1\ t_{lim1} = C_1\ W_{lim1} \qquad \text{and} \qquad W_{lim2\ sub} = C_2P_2\ t_{lim2} = C_2\ W_{lim2}$$

For running:

$$S_{Crit} = (D_{lim2} - D_{lim1})/(t_{lim2} - t_{lim1})$$

$$S_{Crit\ sub} = (D_{lim2\ sub} - D_{lim1sub})/(t_{lim2} - t_{lim1}) = (C_2\ D_{lim2} - C_1\ D_{lim1})/(t_{lim2} - t_{lim1})$$

$$S_{Crit\ sub}/S_{Crit} = [(C_2\ D_{lim2} - C_1\ D_{lim1})/(t_{lim2} - t_{lim1})]/[(D_{lim2} - D_{lim1})/(t_{lim2} - t_{lim1})]$$

$$S_{Crit\ sub}/S_{Crit} = (C_2\ D_{lim2} - C_1\ D_{lim1})\ /(D_{lim2} - D_{lim1}) \tag{2}$$

For cycling:

$$P_{Crit\ sub}/P_{Crit} = (C_2\ W_{lim2} - C_1\ W_{lim1})/(W_{lim2} - W_{lim1})$$

2.3.3. Computation in the Constant-Distance Protocol

The submaximal performances are the result of submaximal speeds.

$$S_{1sub} = C_1S_1 \qquad \text{and} \qquad S_{2sub} = C_2S_2$$

For cycling, the submaximal performances correspond to lower powers.

$$P_{1sub} = C_1P_1 \qquad \text{and} \qquad P_{2sub} = C_2P_2$$

$$\text{Hence:} \qquad t_{lim1sub} = D_{lim1}/S_{1sub} = D_{lim1}/C_1S_1 = t_{lim1}/C_1$$

$$\text{and} \qquad t_{lim2sub} = D_{lim2}/S_{2sub} = D_{lim2}/C_2S_2 = t_{lim2}/C_2$$

For running:

$$S_{Crit} = (D_{lim2} - D_{lim1})/(t_{lim2} - t_{lim1})$$

$$S_{Crit\ sub} = (D_{lim2} - D_{lim1})/(t_{lim2sub} - t_{lim1sub}) = (D_{lim2} - D_{lim1})/(t_{lim2}/C_2 - t_{lim1}/C_1)$$

$$S_{Crit\ sub}\ /S_{Crit} = [(D_{lim2} - D_{lim1})/(t_{lim2}/C_2 - t_{lim1}/C_1)]/[(D_{lim2} - D_{lim1})/(t_{lim2} - t_{lim1})]$$

$$S_{Crit\ sub}\ /S_{Crit} = (t_{lim2} - t_{lim1})/(t_{lim2}/C_2 - t_{lim1}/C_1) \tag{3}$$

For cycling:

$$P_{Crit\ sub}\ /P_{Crit} = (t_{lim2} - t_{lim1})/(t_{lim2}/C_2 - t_{lim1}/C_1)$$

3. Results

The interest of the use of arbitrary units is demonstrated in Tables 1 and 2 for protocol 1. Athletes A, B, C, D, E and F who have the same ratio t_{lim1}/t_{lim2} (4) and the same ratio S_2/S_1 (0.7579) have the same effects ($S_{Critsub}/S_{Crit}$) corresponding to the same coefficients C_1 and C_2 (Table 1). Moreover, the use of the same arbitrary units can also been applied to cycling exercises (Table 2): the effects ($P_{Critsub}/P_{Crit}$) of submaximal performances are the same when ratio t_{lim1}/t_{lim2} and ratio P_2/P_1 are similar as in running exercises.

Table 1. Athletes A, B and C have the same values of t_{lim1} and t_{lim2} but different running speeds (S_1 and S_2). Athletes D, E and F have the same values of running speed as athletes A, B and C, respectively. The values of t_{lim1} and t_{lim2} are higher in athletes D, E and F but ratio t_{lim2}/t_{lim1} is the same and equal to 4.

						$C_1 = 0.9$ and $C_2 = 1$		$C_1 = 1$ and $C_2 = 0.9$	
	S_1	S_2	$t_{lim\ 1}$	$t_{lim\ 2}$	S_{Crit}	$S_{Critsub}$	$S_{Critsub}/S_{Crit}$	$S_{Critsub}$	$S_{Critsub}/S_{Crit}$
	$m.s^{-1}$	$m.s^{-1}$	s	S	$m.s^{-1}$	$m.s^{-1}$		$m.s^{-1}$	
A	4	3.0316	180	720	2.709	2.750	1.015	2.659	0.982
B	5	3.7895	180	720	3.386	3.438	1.015	3.324	0.982
C	6	4.5474	180	720	4.063	4.126	1.015	3.989	0.982
D	4	3.0316	240	960	2.709	2.750	1.015	2.659	0.982
E	5	3.7895	240	960	3.386	3.438	1.015	3.324	0.982
F	6	4.5474	240	960	4.063	4.126	1.015	3.989	0.982

Table 2. Athletes A, B and C have the same values of t_{lim1} and t_{lim2} but different power-outputs (P_1 and P_2). Athletes D, E and F have the same values of power-outputs as athletes A, B and C, respectively. The values of t_{lim1} and t_{lim2} are higher in athletes D, E and F but ratio t_{lim2}/t_{lim1} is the same and equal to 4.

						C_1 = 0.9 and C_2 = 1		C_1 = 1 and C_2 = 0.9	
	P_1	P_2	$t_{lim\,1}$	$t_{lim\,2}$	P_{Crit}	$P_{Critsub}$	$P_{Critsub}/P_{Crit}$	$P_{Critsub}$	$P_{Critsub}/P_{Crit}$
	W	W	s	s	W	W		W	
A	240	182	180	720	163	165	1.015	160	0.982
B	320	243	180	720	217	220	1.015	213	0.982
C	400	303	180	720	271	275	1.015	266	0.982
D	240	182	240	960	163	165	1.015	160	0.982
E	320	243	240	960	217	220	1.015	213	0.982
F	400	303	240	960	271	275	1.015	266	0.982

The effects of submaximal performances on endurance indices are presented in Figure 1 (constant-speed protocol), Figure 2 (constant-time protocol) and Figure 3 (constant-distance protocol).

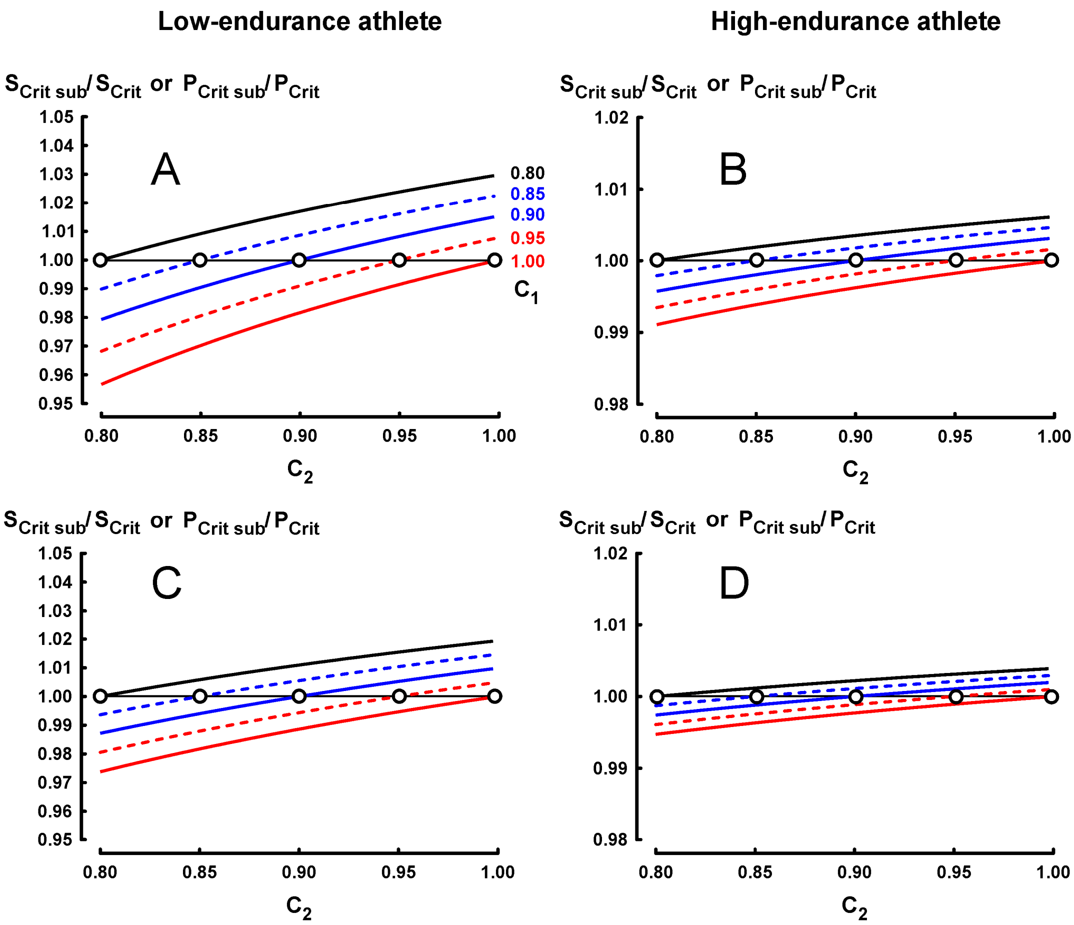

Figure 1. effects of submaximal performances in protocol 1 (constant speed or constant power) on ratios $S_{Critsub}/S_{Crit}$ or $P_{Critsub}/P_{Crit}$ for the different values of C_1 and C_2. Figures **A** and **B** correspond to the range t_{lim1}-t_{lim2} (1–4) whereas Figures **C** and **D** correspond to the range t_{lim1}-t_{lim3} (1–7). The specifications of the lines are presented in Figure 1A. Empty circles correspond to C_1 equal to C_2.

3.1. Results for Protocol 1 (Constant Speed or Power Output Protocol)

For protocol 1, five curves of ratio $S_{Crit\ sub}/S_{Crit}$ corresponding to five values of C_1 (0.80, 0.85, 0.90, 0.95 and 1.00) were computed from equation 1 with an increment of C_2 equal to 0.001.

The effects of submaximal performances on ratio $S_{Crit\ sub}/S_{Crit}$ (or ratio $P_{Crit\ sub}/P_{Crit}$) are lower in the high-endurance athlete (Figure 1B,D) than in the low-endurance athlete (Figure 1A,C).

In Figure 1B,D, the effects of submaximal performances on ratio $S_{Crit\ sub}/S_{Crit}$ are lower when the range of t_{lim} is longer (1–7 instead of 1–4).

In Figure 1A, the lowest and the highest ratios $S_{Crit\ sub}/S_{Crit}$ are equal to 0.9567 and 1.0295, respectively.

When C_1 is equal to C_2 (empty circles), i.e., when the levels of submaximal performances are the same for both exhausting exercises, there is no effect of submaximal performances on ratio $S_{Crit\ sub}/S_{Crit}$ (or ratio $P_{Crit\ sub}/P_{Crit}$) according to Equation (1):

$$S_{Crit\ sub}/S_{Crit} = [(C_2\ D_{lim2} - C_2\ D_{lim1})/(C_2\ t_{lim2} - C_2\ t_{lim1})]/[(D_{lim2} - D_{lim1})/(t_{lim2} - t_{lim1})] = 1$$

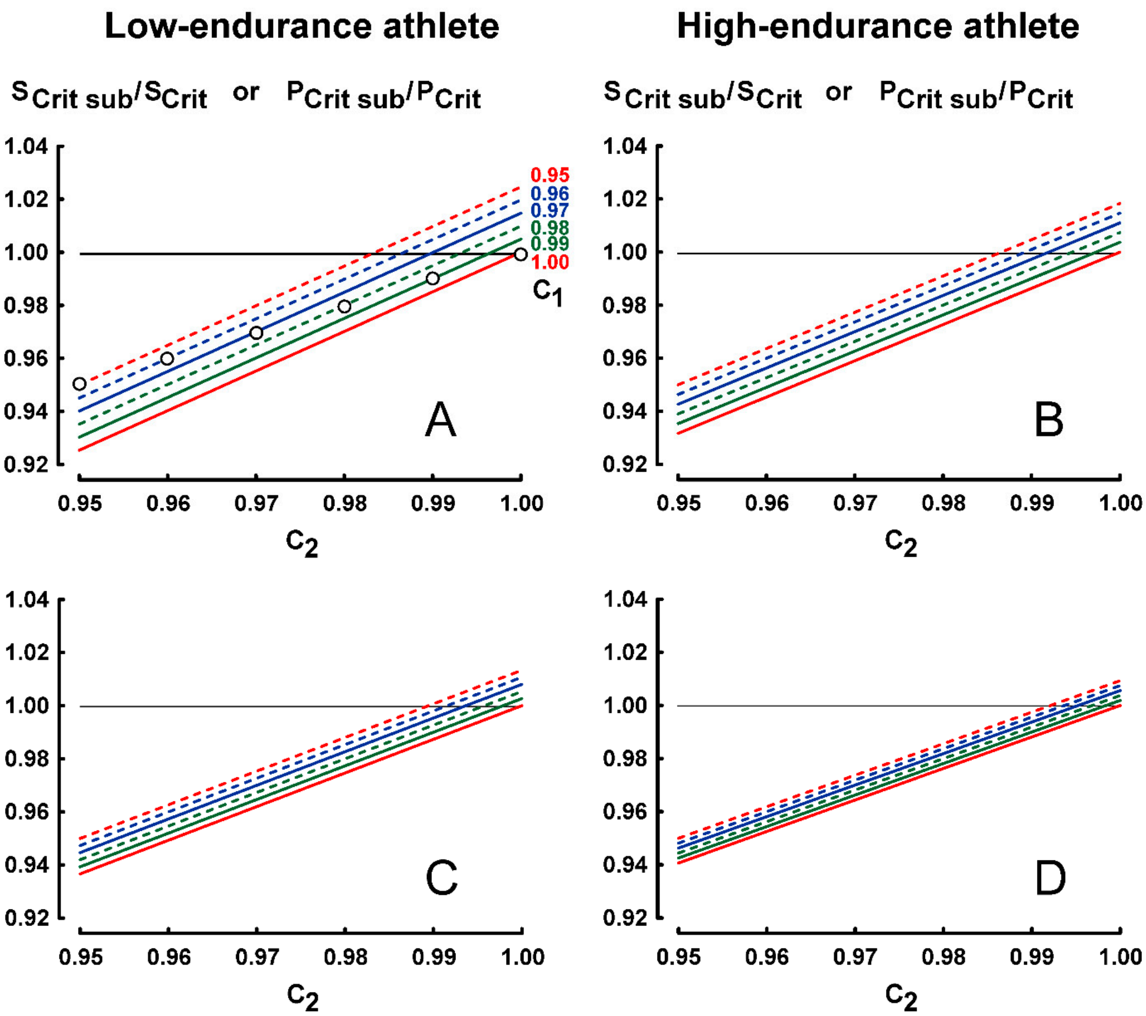

Figure 2. effects of submaximal performances in protocol 2 (constant time-protocol) on ratios $S_{Crit\ sub}/S_{Crit}$ for the different values of C_1 and C_2. Figures **A** and **B** correspond to the range t_{lim1}-t_{lim2} (1–4) whereas Figures **C** and **D** correspond to the range t_{lim1}-t_{lim3} (1–7). The specification of the lines is presented in Figure **A**. Empty circles in Figure **A** correspond to C_1 equal to C_2.

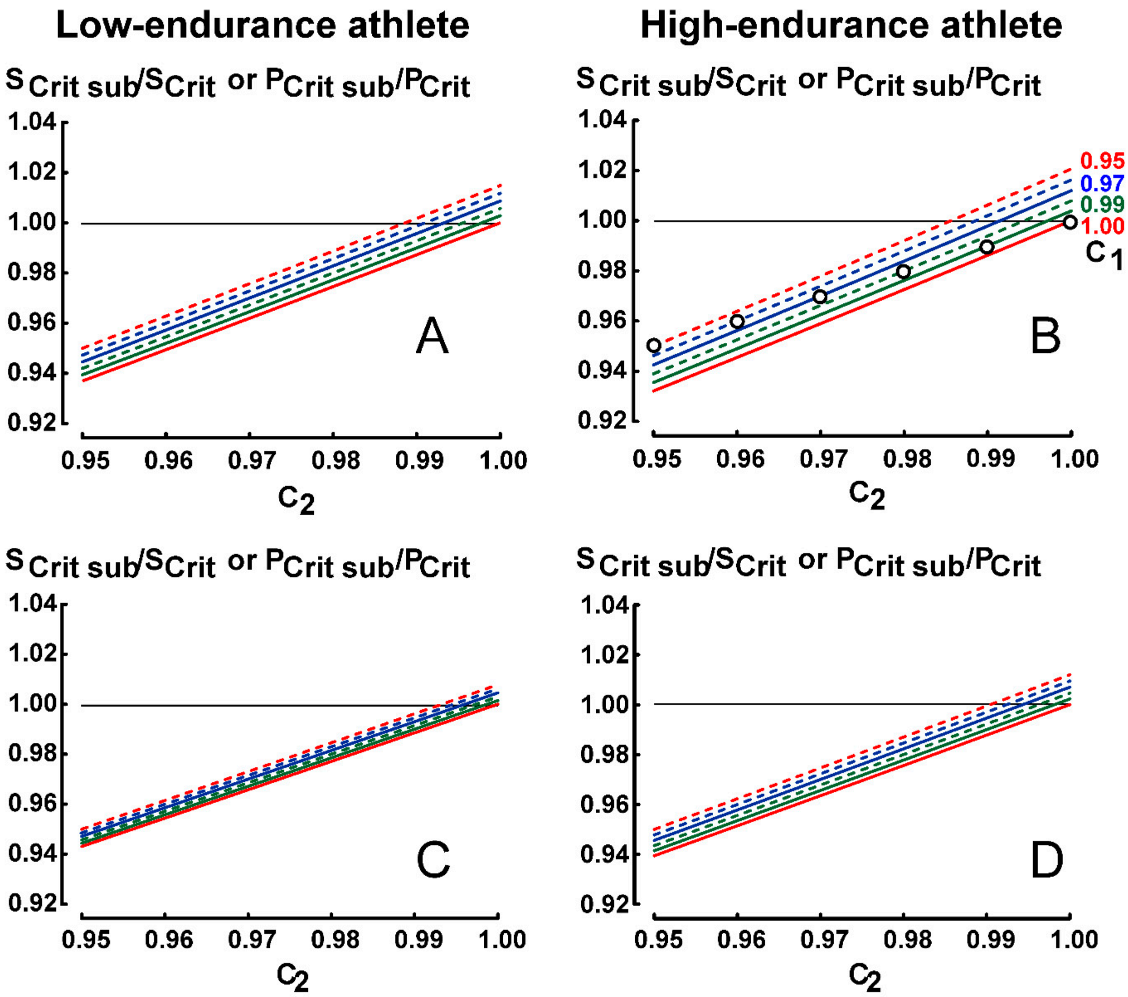

Figure 3. Effects of submaximal performances in protocol 3 (constant distance-protocol) for the different values of C_1 (specification of the lines in Figure **B** are the same as in Figure 2) and C_2. Figures **A** and **B** correspond to the range t_{lim1}-t_{lim2} whereas Figures **C** and **D** correspond to the range t_{lim1}-t_{lim3}. Empty circles in Figure **B** correspond to C_1 equal to C_2.

3.2. Results for Protocol 2 (Constant-Time Protocol)

The values of coefficients C_1 and C_2 were limited from 0.95 to 1. Six curves of ratio $S_{Crit\ sub}/S_{Crit}$ corresponding to six values of C_1 (0.95, 0.96, 0.97, 0.98, 0.99 and 1.00) were computed from Equation (2) with an increment of C_2 equal to 0.001.

As for protocol 1, the effects of submaximal performances on ratio $S_{Crit\ sub}/S_{Crit}$ (or ratio $P_{Crit\ sub}/P_{Crit}$) are lower in the high-endurance athlete (Figure 2B,D) than in the low-endurance athlete (Figure 2A,C).

In Figure 2C,D, the effects of submaximal performances on ratio $S_{Crit\ sub}/S_{Crit}$ are lower when the range of t_{lim} is longer (1–7 instead of 1–4).

In Figure 2A, the lowest and the highest ratios $S_{Crit\ sub}/S_{Crit}$ are equal to 0.9254 and 1.0246, respectively.

When C_1 is equal to C_2 (empty circles in Figure 2A), i.e., when the levels of submaximal performances are the same for both exhausting exercises, the ratios $S_{Crit\ sub}/S_{Crit}$ (or $P_{Crit\ sub}/P_{Crit}$) are equal to C_2 (or C_1) according to Equation (2):

$$S_{Crit\ sub}/S_{Crit} = (C_2\ D_{lim2} - C_2\ D_{lim1})/(D_{lim2} - D_{lim1}) = C_2\ (\text{or}\ C_1)$$

3.3. Results for Protocol 3 (Constant-Distance Protocol)

Six curves of ratio $S_{Crit\ sub}/S_{Crit}$ corresponding to six values of C_1 (0.95, 0.96, 0.97, 0.98, 0.99 and 1.00) were computed from equation 3 with an increment of C_2 equal to 0.001.

In contrast with protocols 1 and 2, the effects of submaximal performance on ratio $S_{Crit\ sub}/S_{Crit}$ (or $P_{Crit\ sub}/P_{Crit}$) are more important in the high-endurance athlete. However, in the high-endurance athlete, the ranges of t_{lim1}–t_{lim2} (1-3.6059) and t_{lim1}–t_{lim3} (1–6.0705) is shorter than the ranges of t_{lim1}–t_{lim2} (1–4.5862) and t_{lim1}-t_{lim3} (1–8.5130) in the low-endurance athlete.

In Figure 3B, the lowest and the highest ratios $S_{Crit\ sub}/S_{Crit}$ are equal to 0.9321 and 1.0206, respectively.

When C_1 is equal to C_2 (empty circles in Figure 3B), i.e., when the levels of submaximal performances are the same for both exhausting exercises, the ratios $S_{Crit\ sub}/S_{Crit}$ (or $P_{Crit\ sub}/P_{Crit}$) are equal to C_2 (or C_1) according to Equation (3):

$$S_{Crit\ sub}/S_{Crit} = (t_{lim2} - t_{lim1})/(t_{lim2}/C_2 - t_{lim1}/C_2) = C_2\ (t_{lim2} - t_{lim1})/(t_{lim2} - t_{lim1}) = C_2\ (\text{or } C_1)$$

4. Discussion

The effects of submaximal performances on $S_{Crit\ 1/t}$ and $P_{Crit\ 1/t}$ in the model proposed by Whipp et al. [11] are not presented in the present study. Indeed, $S_{Crit\ 1/t}$ (or $P_{Crit\ 1/t}$) is equal to S_{Crit} (or P_{Crit}) when both indices are computed only from two exhausting exercises with constant-distance [15] or constant-power [34] protocols in running and cycling. Similarly, in the present study, the effects of submaximal performances were the same for $S_{Crit\ 1/t}$ and S_{Crit} (or $P_{Crit\ 1/t}$ and P_{Crit}) when they were computed from two submaximal exercises whatever the protocol. Consequently, the Figures about the effects of submaximal performances on $S_{Crit\ 1/t}$ or $P_{Crit\ 1/t}$ are not added in the present study.

Previous experimental studies [22–27] showed that performance reliability with constant-speed protocol is significantly lower than those with the other protocols (constant-time or constant-distance protocols). However, for S_{Crit} or P_{Crit} in the present theoretical study, the effects of 20%-submaximal performances in protocol 1 are lower than the effects of 5%-submaximal performances in protocols 2 and 3. For example, the lowest ratio $S_{Crit\ sub}/S_{Crit}$ in protocol 1 is equal to 0.9567 (Figure 1A) whereas the lowest ratio in protocol 2 is equal to 0.9254 (Figure 2A).

Ratio $S_{Cri\ tsub}/S_{Crit}$ (or ratio $P_{Crit\ sub}/P_{Crit}$) is equal to 1 when the maximal and submaximal D_{lim}-t_{lim} relationships are parallel i.e., when the distances of both submaximal performances to the maximal D_{lim}-t_{lim} line are similar:

- C_1 is equal to C_2 for protocol 1(empty circles in Figure 1);
- D_{lim1} $(1 - C_1)$ is equal to D_{lim2} $(1 - C_2)$ for protocol 2;
- t_{lim1} $(1 - 1/C_1)$ is equal to t_{lim2} $(1 - 1/C_2)$ for protocol 3.

In protocol 1, the submaximal performances correspond to similar decreases in D_{lim} and t_{lim} whereas, in protocol 2 and 3, they only correspond to a decrease in D_{lim} or t_{lim} (Figure 4). These simultaneous decreases in D_{lim} and t_{lim} limit the distance between the submaximal performance and the maximal D_{lim}-t_{lim} line, which explain the lower effects of submaximal performances on ratio $S_{Crit\ sub}/S_{Crit}$.

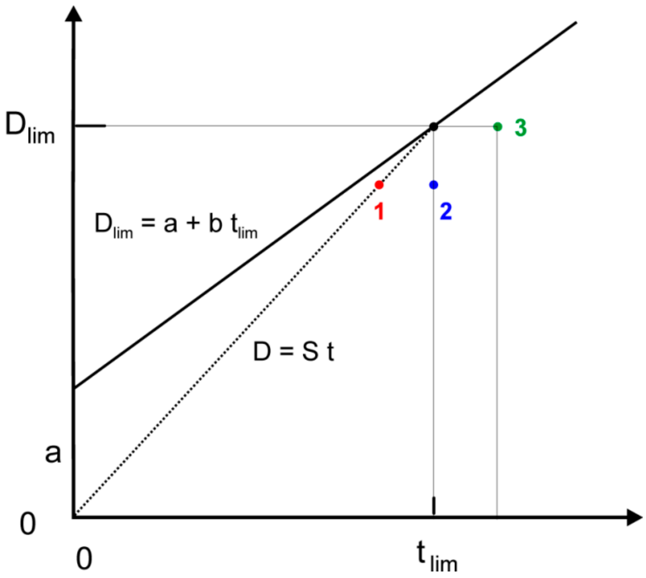

Figure 4. Comparison of the effects of submaximal performances in the 3 protocols. Black dot: maximal performance. Red dot: submaximal performance in protocol 1. Blue dot: submaximal performance in protocol 2. Green dot: submaximal performance in protocol 3. The dotted line corresponds to the relationship between distance (D) and time (t) at a given running speed (S).

In protocols 2 and 3, when C_1 is equal to C_2 (empty circles in Figure 2A, Figure 3B) ratios $S_{Crit\ sub}$ /S_{Crit} are equal to C_1 and are not very low ($S_{Crit\ sub}/S_{Crit} \geq 0.95$). When C_2 is lower than C_1, ratios $S_{Crit\ sub}$ /S_{Crit} are lower and sometimes not negligible in protocols 2 and 3 (for example, in Figure 2A, $S_{Crit\ sub}/S_{Crit} = 0.9254$ for $C_2 = 0.95$ and $C_1 = 1$). However, although the range of C_1-C_2 is much larger (0.8–1.0), ratios $S_{Crit\ sub}/S_{Crit}$ are not very low (≥ 0.9567) in protocol 1, even when C_2 is lower than C_1 (Figure 1A).

The effects of submaximal performances on ratio $S_{Crit\ sub}/S_{Crit}$ (or ratio $P_{Crit\ sub}/P_{Crit}$) may be low ($S_{Crit\ sub}$ /$S_{Crit} \geq 0.95$) for both low-endurance and high-endurance athletes in the three protocols. These possible low effects of submaximal performances on ratio $S_{Crit\ sub}/S_{Crit}$ could explain that it is possible to compute S_{Crit} from the values of t_{lim} of 3 trials performed with protocol 3 in a same session with only 30 min of recovery between the trials as in a Single-Visit Field Test [28,29]. This low sensitivity of S_{Crit} or P_{Crit} to submaximal performances was previously suggested in a study on the comparison of critical speeds of continuous and intermittent running exercise on a track [35] and also in a review [30].

The effects of submaximal performances on ratio $S_{Crit\ sub}/S_{Crit}$ are lower in the high-endurance athlete for constant-speed and constant-time protocols (Figures 1 and 2). That said, the effects of submaximal performances on ratio $S_{Crit\ sub}/S_{Crit}$ in constant-distance protocol (protocol 3) are higher in the high-endurance athlete (Figure 3). However, the effects of submaximal performances on ratio $S_{Crit\ sub}/S_{Crit}$ (or ratio $P_{Crit\ sub}/P_{Crit}$) are lower when the range of t_{lim} is longer (for example, 1–7 instead of 1–4) as illustrated in Figures 1–3. Therefore, in protocol 3, the shorter ranges of t_{lim1}-t_{lim2} and t_{lim1}-t_{lim3} in the high-endurance athlete explain these computed higher effects of submaximal performances on ratio $S_{Crit\ sub}/S_{Crit}$. In contrast, the coefficient of variation of the Single-Visit Field Test that corresponds to this constant-distance protocol was lower in trained runners whose S_{Crit} were faster than untrained runners [29]. It was likely that the reliability of S_{Crit} in the trained runners was higher because of control of the maximal running speed corresponding to a given distance and better recovery.

In the Single-Visit Field Test [28], the running performances were submaximal because the values of D′ (equivalent of ADC) were significantly lower in the 30-min (106.4-m) and 60-min-recovery (102.4-m) than in the 3-session treadmill test (249.7 m). However, S_{Crit} in the Single-Visit Field Test was not different of S_{Crit} in the 3-session treadmill test. These results were consistent with those of a previous experimental study on the effects of a 6-min exhausting exercise on S_{Crit} in cycling [36].

In the present theoretical study, the submaximal performances have also effects on parameter "a" (ADC). For example, in protocol 1, parameter "a" decreases when C_1 and C_2 are equal and lower than 1 (empty circles in Figure 1) but increases when C_1 is equal to 1 and C_2 is lower than 1. These effects of submaximal performances on parameter "a" are not computed in the present study because this parameter is not an endurance index and its meaning is questionable [11].

The present method can also be used for the submaximal-performance effects on the exponent g of the power-law model by Kennelly [1] and the endurance index (EI) of the logarithmic model by Péronnet and Thibault [4] as they are 2-parameter models:

$$D_{lim} = k\ t_{lim}{}^{g} \quad \text{and} \quad S = k\ t_{lim}{}^{g-1} \qquad \text{power-law model}$$

$$100\ S/MAS = 100 - EI \log(t_{lim}/t_{MAS}) \qquad \text{logarithmic model}$$

5. Conclusions

The results of the present theoretical study confirm the interest of S_{Crit} and P_{Crit} computed from exercises whose performances are submaximal and performed in the same session. Indeed, for the 3 protocols, the theoretical effects of submaximal performances on ratio $S_{Crit\ sub}/S_{Crit}$ (or ratio $P_{Crit\ sub}/P_{Crit}$) are low in many cases. The effects of submaximal performances are lower when the ratio t_{lim2}/t_{lim1} is larger. In protocol 3, it is likely that, in practice, the reliability of S_{Crit} is better in trained runners due to the control of the maximal running speed corresponding to a given distance.

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