



## A mumps model with seasonality in China



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### ABSTRACT

**Background:** Mumps, an infectious viral disease, classically manifested by inflammation of salivary glands and is best known as a common childhood viral disease with no specific treatment. Although it can be protected by vaccine, there are more than 100,000 reported mumps cases according to the Chinese Center for Disease Control and Prevention. However, the factors and mechanisms behind the persistence and prevalence of mumps have not been well understood.

**Methods:** A mumps model with seasonal fluctuation is formulated and investigated. We evaluate the basic reproduction number  $\mathcal{R}_0$  and analyze the dynamical behavior of the model. We also use the model to simulate the monthly data of mumps cases and carry out some sensitivity analysis of  $\mathcal{R}_0$  in terms of various model parameters.

**Results:** It is shown that there exists only disease-free solution which is globally asymptotically stable if  $\mathcal{R}_0 < 1$ , and there exists a positive periodic solution if  $\mathcal{R}_0 > 1$ .  $\mathcal{R}_0$  is a threshold parameter, and its magnitude determines the extinction or persistence of the disease.

**Conclusion:** Our analysis shows that vaccination rate and invalid vaccination rate play important roles in the spread of mumps. Hence, Our study suggests to increase the vaccine coverage and make two doses of MMR (Measles, mumps and rubella vaccine) vaccine freely available in China.

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## 1. Background

Mumps, also known as epidemic parotitis, is a viral disease caused by the mumps virus, classically manifested by inflammation of salivary glands and fever (Ennis & Jackson, 1968; Latner & Hickman, 2015). This disease is best known as a common childhood viral disease (Richardson, Elliman, Maguire, Simpson, & Nicoll, 2001). Initial signs and symptoms often include fever, muscle pain, headache, and feeling tired, and there is no specific treatment (Mühlemann, 2004). It is usually followed by painful swelling of one or both parotid glands (Wharton, Chaudhry, & French, 2006). Symptoms in adults are often more severe than in children. Aseptic, meningitis, encephalitis, orchitis and oophoritis are common complications of mumps, which can arise in adult men and women (Latner & Hickman, 2015). About a third of people have mild or no

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symptoms (Unal et al., 1998). Complications may include infections of the covering of the brain (15 percent), pancreatitis (4 percent), permanent deafness, and painful testicular swelling which uncommonly results in infertility (Hviid, Rubin, & Mühlemann, 2008).

In general, many infectious diseases fluctuate over time and show seasonal patterns in the incident rate, such as measles, whooping cough, polio, influenza, chickenpox, mumps, etc (Bjørnstad, Finkenstädt, & Grenfell, 2002; Dowell, 2001; London & Yorke, 1973). We consider the periodic transmission attribute to the following three facts (Ma & Ma, 2006): (i) In winter most children stay at home because the cold weather and while in summer and fall, people especially children have more frequent outdoor activities which increase chances of contact. (ii) Meanwhile, in summer the warm climate contribute to the growth of virus and the spread of disease. (iii) From April to June children usually go to school and study together, and during July and August most schools are closed for summer vacation thus many children play together without supervision of their parents. All these cause the disease spread easily and form a seasonal pattern. Mumps is highly contagious and spreads rapidly among people living in close quarters. The virus is transmitted by respiratory droplets or direct contact with an infected person (Gupta, Best, & Macmahon, 2005). Only humans get and spread the disease. A person infected with mumps is contagious from approximately seven days before the onset of symptoms until about eight days after symptoms start. The incubation period can be 12–26 days, but is typically 16–18 days. 20–40 percent of persons infected with the mumps virus do not show symptoms, so it is possible to be infected and spread the virus without knowing it (Kutty et al., 2010). After an infection a person is typically immune for life. Reinfection is possible but tends to be mild (Senanayake, 2008). Larger outbreaks of disease would typically occur every two to five years. Children between the ages of five and nine were most commonly affected. Among immunized population often those in their early 20s are affected. Around the equator it often occurs all year round while in the more northerly and southerly regions of the world it is more common in the winter and spring (Wharton et al., 2006). About one per ten thousand people who are infected die.

The most common preventative measure against mumps is a vaccination with 2 doses of the mumps vaccine, invented by American microbiologist Maurice Hilleman at Merck (Buynak, Weibel, Whitman, Stokes, & Hilleman, 1969). Before the introduction of a vaccine, mumps was a common childhood disease worldwide. Widespread vaccination has resulted in a more than 90 percent decline in rates of disease. Most of the developed world includes it in their immunization programs, often in combination with measles and rubella vaccine (MMR). Hospitalization may be required if meningitis or pancreatitis develops (Gupta et al., 2005; Senanayake, 2008). The vaccine may be given separately or as part of the MMR immunization vaccine that also protects against measles and rubella. In the US, MMR is now being supplanted by MMRV, which adds protection against chickenpox (varicella, HHV3). The WHO (World Health Organization) recommends the use of mumps vaccines in all countries with well-functioning childhood vaccination programmes. In the United Kingdom it is routinely given to children at age 13 months with a booster at 3–5 years (preschool). This confers lifelong immunity. The American Academy of Pediatrics recommends the routine administration of MMR vaccine at ages 12–15 months and at 4–6 years (Center for Disease Control and Prevention, 2012). In some locations, the vaccine is given again between four and six years of age, or between 11 and 12 years of age if not previously given. The efficacy of the vaccine depends on the strain of the vaccine, but is usually around 80 percent (Schlegel, Osterwalder, Galeazzi, & Vernazza, 1999). Because of the outbreaks within college and university settings, many governments have established vaccination programs to prevent large-scale outbreaks. In Canada, provincial governments and the Public Health Agency of Canada have all participated in awareness campaigns to encourage students ranging from grade one to college and university to get vaccinated.

Not only did it greatly reduce the incidence of mumps but also decreased significantly in patients with encephalitis and meningitis. Mumps vaccination is almost universally used in developed countries nowadays (Guy et al., 2004; Latner & Hickman, 2015). In Beijing, starting from 2000, under the disease immunization program: with Children should get 2 doses of MMR vaccine, a year and a half after the first immunization, the children need to be vaccinated at the age of 6 (Beijing vaccine). But for most of the cities and provinces in China, according to the National Immunization Program by Chinese Center for Disease Control and Prevention (National Immunization Program Chinese Center for Disease Control and Prevention), children just get vaccinated one dose. We also note that there have been outbreaks in vaccinated populations. An outbreak of mumps occurred unexpectedly in May 2005 in Nova Scotia, Canada, followed later by an outbreak in Quebec, Canada (Watson-Creed et al., 2006), and in September 2005, by an outbreak in Iowa (Centers for Disease Control and Prevention, 2006). In 2006 the United States experienced the largest nationwide mumps epidemic in 20 years (Barskey, Glasser, Lebaron, & Charles, 2009).

The organization of this paper is as follows. In the next section, an epidemic model for mumps with seasonal fluctuation is proposed to understand the infectious dynamics. Then we study the global asymptotic stability of the disease-free equilibrium and the existence of positive periodic solutions. Simulations of the model and sensitivity analysis of the basic reproduction number are performed in Section 3. We end this article with model-based suggestion of intervention improvement to control the mumps.

## 2. Methods

### 2.1. Model formulation

We divide the population into seven compartments according to their states: susceptible, vaccinated, exposed, mild infectious, severe infectious, hospitalized and recovered, which are denoted by  $S(t)$ ,  $V(t)$ ,  $E(t)$ ,  $I(t)$ ,  $L(t)$ ,  $H(t)$  and  $R(t)$ , respectively, and we denote the total number of the population by  $N(t)$ , that is  $N(t) = S(t) + V(t) + E(t) + I(t) + L(t) + H(t) + R(t)$ . Assuming that the birth numbers of humans per month are constant. One dose of vaccination is often applied to susceptible individuals. Susceptible individuals who are infected firstly enter into the latent period, during which they do not show symptoms and can not infect others. After about 12–26 days, these people become the infectious. The infectious people are classified into mild infectious  $I(t)$  and severe infectious  $L(t)$ , who have different symptoms. Mild infectious will mostly recover, but there are still some patients who will become severely ill patient and in need of treatment. And we assume that the recovered have lifelong immunity. The transmission dynamics associated with these subpopulations are illustrated in Fig. 1.

In this paper, the periodic incidence  $\beta_1(t)S(I + L)/N$  and  $\beta_2(t)V(I + L)/N$  are applied, an infectious individual can contact a finite number of persons in one time unit in a large population. The transmission rate between  $S(t)$ ,  $I(t)$  and  $L(t)$  is  $\beta_1(t)$ , and the transmission rate between  $V(t)$ ,  $I(t)$  and  $L(t)$  is  $\beta_2(t)$ . Many epidemiological models (Ma & Ma, 2006; Moneim & Grennhalgh, 2005; White et al., 2007; Zhang, Jin, Sun, & Ruan, 2012) were simulated by using sinusoidal function of period one year  $\beta(t) = \beta_0 + \beta(\sin(\omega t + \varphi))$  for the seasonal varying transmission rate. In this model, we use the periodic functions  $\beta_1(t) = a_1 + (1 + b_1 \sin(\pi t/T + c_1))$  and  $\beta_2(t) = a_2 + (1 + b_2 \sin(\pi t/T + c_2))$  with period  $2T$  as the transmission rates. Here  $a_1$ ,  $b_1$ ,  $c_1$ ,  $a_2$ ,  $b_2$  and  $c_2$  are positive constants, where  $a_1$  and  $a_2$  are the baseline contact rates and  $b_1$  and  $b_2$  are the magnitudes of forcing, which can be determined by the least-square fitting in Section 3.

Following the schematic diagram we use a system of ordinary differential equations to model the transmission of mumps:

$$\left\{ \begin{aligned} \frac{dS(t)}{dt} &= \Lambda - \frac{\beta_1(t)S(t)(I(t) + L(t))}{N(t)} + \lambda V(t) - (\epsilon + \mu)S(t), \\ \frac{dV(t)}{dt} &= \epsilon S(t) - \lambda V(t) - \frac{k\beta_2(t)V(t)(I(t) + L(t))}{N(t)} - \mu V(t), \\ \frac{dE(t)}{dt} &= \frac{\beta_1(t)S(t)(I(t) + L(t))}{N(t)} + \frac{k\beta_2(t)V(t)(I(t) + L(t))}{N(t)} - (\alpha + \mu)E(t), \\ \frac{dI(t)}{dt} &= \alpha\rho E(t) - (\delta_1 + \eta)I(t) - \mu I(t), \\ \frac{dL(t)}{dt} &= \alpha(1 - \rho)E(t) + \eta I(t) - (\sigma + \delta_2 + \mu)L(t), \\ \frac{dH(t)}{dt} &= \delta_2 L(t) - (\gamma + \mu)H(t), \\ \frac{dR(t)}{dt} &= \delta_1 I(t) + \sigma L(t) + \gamma H(t) - \mu R(t). \end{aligned} \right. \tag{2.1}$$

where all parameters are positive, the interpretations and values of parameters are described in Table 1 with the range of the parameters suggested from relevant literatures, where  $\beta_1(t) = a_1 + (1 + b_1 \sin(\pi t/T + c_1))$  and  $\beta_2(t) = a_2 + (1 + b_2 \sin(\pi t/T + c_2))$ .

### 2.2. Global stability of the disease-free equilibrium

In this section, we investigate the global stability of disease-free equilibrium and the existence of the positive periodic solution of model (2.1). It is easy to see that model (2.1) always has one disease-free equilibrium. Notice that from the equations in model (2.1), we have

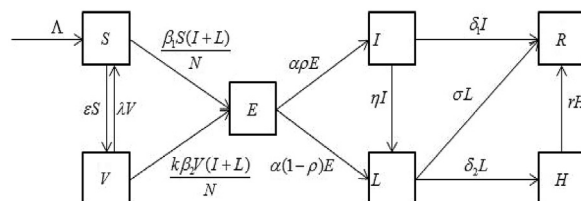


Fig. 1. Flow chart of compartments of mumps model.

**Table 1**  
Descriptions and values of parameters in model (2.1).

Para.	Value	Unit	Interpretation	Source
$\Lambda \in [60,600]$	53.8055	month <sup>-1</sup> million	Human birth population	Fitting
$\lambda \in [0.0001,1]$	0.0935	month <sup>-1</sup>	Loss of vaccination rate	Fitting
$e \in [0.0001,2]$	0.5082	month <sup>-1</sup>	Vaccination rate	Fitting
$\kappa \in [0.0001,0.1]$	0.0529	none	Invalid vaccination rate	Fitting
$\alpha \in [0.0001,2]$	1.9856	month <sup>-1</sup>	Rate moving from $E$	Fitting
$\gamma \in [0.0001,3]$	0.3710	month <sup>-1</sup>	Recovery rate	Fitting
$\rho \in [0.0001,1]$	0.3020	none	Proportion of the infectious $I$ and $L$	Fitting
$\sigma \in [0.0001,30]$	2.3910	month <sup>-1</sup>	Rate moving from $L$	Fitting
$\delta_1 \in [0.0001,30]$	1.0200	month <sup>-1</sup>	Rate moving from $I$	Fitting
$\delta_2 \in [0.0001,30]$	0.0003	month <sup>-1</sup>	Hospitalization rate	Fitting
$\eta \in [0.0001,30]$	1.8595	month <sup>-1</sup>	Deterioration rate	Fitting
$T \in [0.0001,6]$	3.0428	month	The outbreak period	Fitting
$a_1 \in [0.0001,30]$	10.9688	none	The baseline contact rate	Fitting
$a_2 \in [0.0001,30]$	274.6218	none	The baseline contact rate	Fitting
$b_1 \in [-1,1]$	0.9182	none	The magnitude of forcing	Fitting
$b_2 \in [-1,1]$	0.8319	none	The magnitude of forcing	Fitting
$c_1 \in [0.0001,50]$	5.1630	none	Level fluctuations constant	Fitting
$c_2 \in [0.0001,50]$	19.1097	none	Level fluctuations constant	Fitting

$$\frac{dN}{dt} = \Lambda - \mu N. \quad (2.2)$$

Therefore  $X = \left\{ (S, V, E, I, L, H, R) \mid S, V, E, I, L, H, R \geq 0, 0 < S + V + E + I + L + H + R \leq \frac{\Lambda}{\mu} \right\}$  is the feasible region for model (2.1). It can be easily shown that the region  $X$  is positively invariant with respect to system (2.1). It is easy to see that system (1) has one disease-free equilibrium  $P_0 = (\widehat{S}, \widehat{V}, 0, 0, 0, 0, 0)$ , where  $\widehat{S} = \frac{(\mu+\lambda)\Lambda}{\mu(\mu+\lambda+\epsilon)}$ ,  $\widehat{V} = \frac{\epsilon\Lambda}{\mu(\mu+\lambda+\epsilon)}$ .

We can evaluate the basic reproduction number  $\mathcal{R}_0$  for system (2.1) following the definition of Bacaer and Guernaoui (Bacaer & Guernaoui, 2006) and the general calculation procedure in Wang and Zhao (Wang & Zhao, 2008), and the basic reproduction number  $\mathcal{R}_0$  stands for the number of infected during the initial patient's infectious (not sick) period.

**Theorem 2.1.** *The disease-free equilibrium  $P_0$  is globally asymptotically stable when  $\mathcal{R}_0 < 1$ .*

*Proof.* If  $\mathcal{R}_0 < 1$ ,  $P_0$  is locally asymptotically stable by Theorem 2.2 in Wang and Zhao (Wang & Zhao, 2008). To show the solution is globally stable, we need to show that  $P_0$  is globally attractive. Clearly,  $S(t) \leq N(t)$ ,  $V(t) \leq N(t)$ , for all  $t \leq 0$ . Then from system (2.1), we have

$$\begin{cases} \frac{dE}{dt} \leq (\beta(t)_1 + k\beta_2(t))(I + L) - (\alpha + \mu)E, \\ \frac{dI(t)}{dt} = \alpha\rho E - (\delta_1 + \eta) - \mu I, \\ \frac{dL(t)}{dt} = \alpha(1 - \rho)E + \eta I - (\sigma + \delta_2 + \mu)L, \\ \frac{dH(t)}{dt} = \delta_2 L - (\gamma + \mu)H. \end{cases} \quad (2.3)$$

Consider the following comparison system

$$\frac{dh}{dt} = (F(t) - V(t))h(t), \quad h(t) = (E(t), I(t), L(t), H(t))^T. \quad (2.4)$$

Applying Theorem 2.2 in Wang and Zhao (Wang & Zhao, 2008), we know that  $\mathcal{R}_0 < 1$  if and only if  $\rho(\Phi_{F-V}(\omega)) < 1$ . By Lemma 2.1 in Zhang and Zhao (Zhang & Zhao, 2007), it follows that there exists a positive  $\omega$ -periodic function  $\widehat{h}(t)$  such that  $h(t) = e^{pt}\widehat{h}(t)$  is a solution of system (2.4), where  $p = \frac{1}{\omega} \ln \rho(\Phi_{F-V}(\omega))$ . We know when  $\mathcal{R}_0 < 1$ ,  $\rho(\Phi_{F-V}(\omega)) < 1$ . Therefore, we have  $h(t) \rightarrow 0$  as  $t \rightarrow \infty$ . Which implies that the zero solution of system (2.3) is globally asymptotically stable. Applying the comparison principle, we know that for system (2.1),  $E(t) \rightarrow 0$ ,  $I(t) \rightarrow 0$ ,  $L(t) \rightarrow 0$ , and  $H(t) \rightarrow 0$  as  $t \rightarrow \infty$ . By the theory of asymptotic autonomous systems, it is also known that  $S(t) \rightarrow \widehat{S}$  as  $t \rightarrow \infty$ . So  $P_0$  is globally attractive when  $\mathcal{R}_0 < 1$ . It follows that  $P_0$  globally asymptotically stable when  $\mathcal{R}_0 < 1$ .

### 2.3. Existence of positive periodic solutions

Define  $X_0 := \{(S, V, E, I, L, H, R) \in X : E > 0, I > 0\}$  and  $\partial X_0 = X \setminus X_0$ . Denote by  $u(t, x_0)$  the unique solution of system (2.1) with the initial value  $x_0 = (S(0), V(0), E(0), I(0), L(0), H(0), R(0))$ . Let  $X \rightarrow X$  be the Poincaré map associated with system (2.1), i.e.,

$$P(x_0) = u(\omega, x_0), \forall x_0 \in X,$$

where  $\omega$  is the period. Applying the fundamental existence-uniqueness theorem (Perko, 2000), we know that  $u(t, x_0)$  is the unique solution of system (2.1) with  $u(0, x_0) = x_0$ . We notice that  $X$  is positively invariant and  $P$  is point dissipative.

**Lemma 2.2.** When  $\mathcal{R}_0 > 1$ , then there exists a  $\delta > 0$  such that when

$$\|(S(0), V(0), E(0), I(0), L(0), H(0), R(0)) - P_0\| \leq \delta,$$

for any  $(S(0), V(0), E(0), I(0), L(0), H(0), R(0)) \in X_0$ , we have

$$\limsup_{m \rightarrow \infty} d[P^m(S(0), V(0), E(0), I(0), L(0), H(0), R(0)), P_0] \geq \delta,$$

where  $P_0 = (\widehat{S}, \widehat{V}, 0, 0, 0, 0, 0)$ .

*Proof.* If  $\mathcal{R}_0 > 1$ , we obtain  $\rho(\Phi_{F-V}(\omega)) > 1$  by Theorem 2.2 in Wang and Zhao (Wang & Zhao, 2008). Choose  $\omega > 0$  small enough such that  $\rho(\Phi_{F-V-M_\varepsilon}(\omega)) > 1$ , where

$$M_\varepsilon = \begin{pmatrix} 0 & \varepsilon & \varepsilon & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{2.5}$$

Now we proceed by contradiction to prove that

$$\limsup_{m \rightarrow \infty} d[P^m(S(0), V(0), E(0), I(0), L(0), H(0), R(0)), P_0] \geq \delta,$$

If not, then

$$\limsup_{m \rightarrow \infty} d[P^m(S(0), V(0), E(0), I(0), L(0), H(0), R(0)), P_0] < \delta,$$

for some  $(S(0), V(0), E(0), I(0), L(0), H(0), R(0)) \in X_0$ . Without loss of generality, we assume that  $d(P^m(S(0), V(0), E(0), I(0), L(0), H(0), R(0)), P_0) < \delta$  for all  $m \geq 0$ . By the continuity of the solutions with respect to the initial values, we obtain

$$\|u(t, P^m(S(0), V(0), E(0), I(0), L(0), H(0), R(0))) - u(t, P_0)\| \leq \varepsilon, \forall m \geq 0, \forall t_1 \in [0, \omega].$$

For any  $t \geq 0$  let  $t = m\omega + t_1$  where  $t_1 \in [0, \omega]$  and  $m = \lfloor \frac{t}{\omega} \rfloor$ , which is the greatest integer less than or equal to  $\frac{t}{\omega}$ . Then we have

$$\begin{aligned} & \|u(t, P^m(S(0), V(0), E(0), I(0), L(0), H(0), R(0))) - u(t, P_0)\| \\ &= \|u(t_1, P^m(S(0), V(0), E(0), I(0), L(0), H(0), R(0))) - u(t_1, P_0)\| \leq \varepsilon \end{aligned} \tag{2.6}$$

for any  $t \geq 0$ , which implies that  $\widehat{S} - \varepsilon \leq S(t) \leq \widehat{S} + \varepsilon, \widehat{V} - \varepsilon \leq V(t) \leq \widehat{V} + \varepsilon, t \geq 0$ . Then for  $\|(S(0), V(0), E(0), I(0), L(0), H(0), R(0)) - P_0\| \leq \delta$ , we have

$$\begin{cases} \frac{dE}{dt} \geq \frac{\beta_1(t)(\widehat{S} - \varepsilon)(I + L)}{N} + \frac{k\beta_2(t)(\widehat{V} - \varepsilon)(I + L)}{N} - (\alpha + \mu)E, \\ \frac{dI}{dt} = \alpha\rho E - (\delta_1 + \eta)I - \mu I, \\ \frac{dL}{dt} = \alpha(1 - \rho)E + \eta I - (\sigma + \delta_2 + \mu)L, \\ \frac{dH}{dt} = \delta_2 L - (\gamma + \mu)H. \end{cases} \tag{2.7}$$

Next we consider the linear system

$$\begin{cases} \frac{dE}{dt} = \frac{\beta_1(t)(\widehat{S} - \varepsilon)(I + L)}{N} + \frac{k\beta_2(t)(\widehat{V} - \varepsilon)(I + L)}{N} - (\alpha + \mu)E, \\ \frac{dI}{dt} = \alpha\rho E - (\delta_1 + \eta)I - \mu I, \\ \frac{dL}{dt} = \alpha(1 - \rho)E + \eta I - (\sigma + \delta_2 + \mu)L, \\ \frac{dH}{dt} = \delta_2 L - (\gamma + \mu)H. \end{cases} \quad (2.8)$$

Once again by Lemma 2.1 in Zhang and Zhao (Zhang & Zhao, 2007), we conclude that there exists a positive  $\omega$ -periodic function  $\widehat{g}(t)$  such that  $g(t) = e^{pt}\widehat{g}(t)$  is a solution of system (2.8), where  $p = \frac{1}{\omega} \ln \rho(\Phi_{F-V-M_e}(\omega))$ . Because  $\rho(\Phi_{F-V-M_e}(\omega)) > 1$ , when  $g(0) > 0$ ,  $g(t) \rightarrow \infty$  as  $t \rightarrow \infty$ . Applying the comparison principle (Smith and Waltman (Smith & Waltman, 1995)), we know that when  $E(0) > 0$ ,  $I(0) > 0$ ,  $L(0) > 0$  and  $H(0) > 0$ ,  $E(t) \rightarrow \infty$ ,  $I(t) \rightarrow \infty$ ,  $L(t) \rightarrow \infty$  and  $H(t) \rightarrow \infty$  as  $t \rightarrow \infty$ .

**Theorem 2.3.** System (2.1) has at least one positive periodic solution.

*Proof.* We first prove that  $\{P^m\}_{m \geq 0}$  is uniformly persistent with respect to  $(X_0, \partial X_0)$ . It is easy to show that  $X_0$  is positively invariant. Clearly,  $\partial X_0$  is relatively closed in  $X$ . Set

$$M_\partial = \{(S(0), V(0), E(0), I(0), L(0), H(0), R(0)) \in \partial X_0 : P^m(S(0), V(0), E(0), I(0), L(0), H(0), R(0)) \in \partial X_0, \forall m \geq 0\}.$$

It is easy to show that

$$M_\partial = \{(S, V, 0, 0, 0, 0, 0) \in X : S \geq 0, V \geq 0\}. \quad (2.9)$$

Note that

$$\{(S, V, 0, 0, 0, 0, 0) \in X : S \geq 0, V \geq 0\} \subseteq M_\partial. \quad (2.10)$$

We only need to prove that

$$M_\partial \subseteq \{(S, V, 0, 0, 0, 0, 0) \in X : S \geq 0, V \geq 0\}. \quad (2.11)$$

That is, for any  $(S(0), V(0), E(0), I(0), L(0), H(0), R(0)) \in \partial X_0$ , we have  $E(m\omega) = I(m\omega) = 0, \forall m \geq 0$ . If there exists an  $m_1 \geq 0$  such that  $(E(m_1\omega), I(m_1\omega))^T > 0$ . By replacing the initial time 0 with  $m_1\omega$  and  $S(t) > 0, V(t) > 0$ . Analogously, we have  $(E(t), I(t))^T > 0, \forall t > m_1\omega$ . Thus, we have

$$(S(t), V(t), E(t), I(t), L(t), H(t), R(t)) \in X_0, \forall t > m_1\omega, \quad (2.12)$$

which contradicts that  $(S(0), V(0), E(0), I(0), L(0), H(0), R(0)) \in \partial X_0$  that requires

$$P^m(S(0), V(0), E(0), I(0), L(0), H(0), R(0)) \in \partial X_0, \forall m \geq 0. \quad (2.13)$$

So, the equality (2.9) holds, which implies that  $P_0$  is the only fixed point of  $P$  and acyclic in  $\partial X_0$ . Moreover, Lemma 2.4 implies that  $P_0 = (\widehat{S}, \widehat{V}, 0, 0, 0, 0, 0)$  is an isolated invariant set in  $X$  and  $W^S(P_0) \cap X_0 = \emptyset$ . By the acclivity theorem on uniform persistence for maps (Theorem 1.3.1 and Remark 1.3.1 in Zhao (Zhao, 2003)), it follows that  $P$  is uniformly persistent with respect to  $(X_0, \partial X_0)$ . Now Theorem 1.3.6 in Zhao (Zhao, 2003) implies that  $P$  has a fixed point

$$(S^*(0), V^*(0), E^*(0), I^*(0), L^*(0), H^*(0), R^*(0)) \in X_0. \quad (2.14)$$

From the first equation of system (2.1) we have

$$\begin{aligned}
 S^*(t) &= e^{-\int_0^t (\varepsilon + \mu + a(s_1)) ds_1} \left[ S^*(0) + \int_0^t (\Lambda + \lambda V(s_2)) e^{\int_0^{s_2} (\varepsilon + \mu + a(s_1)) ds_1} ds_2 \right] \\
 &= e^{-\int_0^t (\varepsilon + \mu + a(s_1)) ds_1} \int_0^t (\Lambda + \lambda V(s_2)) e^{\int_0^{s_2} (\varepsilon + \mu + a(s_1)) ds_1} ds_2 \\
 &> 0, \forall t > [0, \omega].
 \end{aligned}
 \tag{2.15}$$

The periodicity of  $S^*(t)$  implies  $S^*(t) > 0$  for all  $t > 0$ : Following the processes of the proof, we have  $V^*(0) > 0, E^*(0) > 0, I^*(0) > 0, L^*(0) > 0, H^*(0) > 0, R^*(0) > 0$ , for all  $t \geq 0$ . Therefore,

$$(S^*(t), V^*(t), E^*(t), I^*(t), L^*(t), H^*(t), R^*(t))$$

is a positive  $\omega$ - periodic solution of system (2.1).

### 3. Simulations and sensitivity analysis

In this section, we first use model (2.1) to simulate the reported mumps data of China from January 2013 to December 2015, predict the trend of the disease and seek some control and prevention measures. The data given in Table 2, are obtained mainly from epidemiologic bulletins published by the China's CDC (Chinese Center for Disease). Assume that the natural death follows a uniform distribution, then natural death rate is calculated as  $\mu = 1/(74.83 \times 12) = 0.0011$ , since life expectancy is 74.83 years old in China (National Bureau of Statistics of China). Then, we have to estimate the other 18 parameters and 7 initial values through calculating the minimum sum of chi-square

$$J(\theta) = \sum_{i=1}^{36} \frac{(H(t_i) - \hat{H}(t_i))^2}{\hat{H}(t_i)},$$

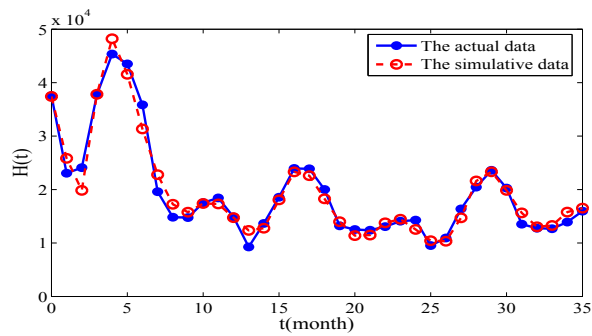
with the MATLAB (the Mathworks, Inc.) tool lsqnonlin that is a part of optimization toolbox. This method is similar to Zhang's (Zhang, Zhao, & Neumann, 2010).

We use system (2.1) to conduct the data fitting to the number of hospitalized, as shown in Fig. 2 and the numerical results are found to be a good match with the data of mumps in China from 2013 to 2015. The optimal values of parameters are listed in Table 1, and the corresponding initial values are shown in the figure caption of Fig. 2. The values of parameters listed in Table 1 are consistent in the reality. For example, invalid vaccination rate ( $k$ ) is 0.0529, the incubation period ( $1/\alpha$ ) are 17.7521 days (i.e. 0.5036 month), the period of mumps ( $2T$ ) are 6.0856 months, there are 30.20% infectious ( $\rho$ ) are mild and there are 69.80% infectious ( $1 - \rho$ ) are severe. By using the parameters value we calculate  $\mathcal{R}_0 \approx 6.5428$  under the current circumstances in China. From Fig. 3, we can see that when  $\mathcal{R}_0 < 1$ , the number of hospitalized mumps cases  $H(t)$  tends to 0. On the contrary, when  $\mathcal{R}_0 > 1$ ,  $H(t)$  tends to a stable periodic solution. This shows that the mumps in China will persist for a long time according to the current situation. When studying the transmission dynamics of periodic epidemic models, some researchers use the average basic reproduction number  $\overline{\mathcal{R}_0}$ , namely the basic reproduction number of the time-averaged autonomous system of the periodic epidemic model over a time (Greenhalgh & Moneim, 2003; Ma & Ma, 2006; Moneim, 2007; Wesley & Allen, 2009; Williams & Dye, 1997). We also calculate the average basic reproduction number

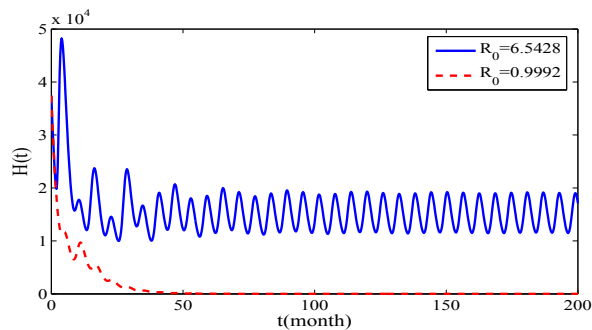
**Table 2**  
Monthly new reported mumps cases from China's CDC.

Month	2013	2014	2015
January	37,565	14,872	14,289
February	23,051	9259	9517
March	24,099	13,658	10,942
April	37,748	18,570	16,371
May	45,354	23,947	20,462
June	43,512	23,874	23,606
July	35,854	20,009	20,267
August	19,596	13,215	13,517
September	14,825	12,518	12,767
October	14,766	12,397	12,675
November	17,547	13,065	13,917
December	18,432	14,085	15,965





**Fig. 2.** The comparison between the reported mumps cases in China from January 2013 to December 2015 and the simulation of our model. The solid curve represents the monthly data reported by China's CDC while the dashed curve is simulated by using our model. The values of parameters are given in Table 1. The initial values used in the simulations were  $S(0) = 3.7021 \times 10^6$ ,  $V(0) = 4.6947 \times 10^4$ ,  $E(0) = 5.5003 \times 10^4$ ,  $I(0) = 1.2245 \times 10^5$ ,  $L(0) = 7.2075 \times 10^4$ ,  $H(0) = 3.7410 \times 10^4$  and  $R(0) = 4.0217 \times 10^6$ .



**Fig. 3.** The tendency of the mumps cases  $H(t)$  in a long time with different values of  $\mathcal{R}_0$ . Choosing the initial values from the figure caption of Fig. 2. When  $\beta_1(t) = a_1 + (1+b_1\sin(c_1+\pi t/T))$  and  $\beta_2(t) = a_2 + (1+b_2\sin(8.0084+\pi t/T))$ , and the values of other parameters in Table 1 do not change,  $\mathcal{R}_0 = 0.9992$  (see lower curve). And the values of parameters from Table 1, and  $\mathcal{R}_0 = 6.5428$  (see upper curve).

$$\overline{\mathcal{R}_0} = \frac{\alpha(\overline{\beta}_1(\lambda + \mu) + \overline{\beta}_2\kappa\varepsilon)}{(\varepsilon + \lambda + \mu)(\alpha + \mu)(\delta_1 + \eta + \mu)} \cdot \left( \frac{\rho\eta + (\delta_1 + \eta + \mu)(1 - \rho)}{\delta_2 + \sigma + \mu} + \rho \right),$$

where

$$\overline{\beta}_1 = \frac{1}{2T} \int_0^{2T} \beta_1(t) dt, \overline{\beta}_2 = \frac{1}{2T} \int_0^{2T} \beta_2(t) dt.$$

Using the parameter values in Table 1, we have  $\overline{\mathcal{R}_0} = 6.6737$ , which is larger than  $\mathcal{R}_0$ . Though sometimes the average basic reproduction number  $\overline{\mathcal{R}_0}$  may overestimate or underestimate infections risks. But  $\overline{\mathcal{R}_0}$  can also reflect the risk to some extent. From 1960 to 1980, the basic reproduction number of mumps in Netherlands, England and Wales was 11–14. In 1943, in Baltimore of USA, the basic reproduction number of mumps was 7–8, and the basic reproduction number of mumps is little smaller in China (Anderson & May 1991).

Next we discover the influence of initial values  $S(0)$ ,  $V(0)$ ,  $E(0)$ ,  $I(0)$ ,  $L(0)$ ,  $H(0)$  and  $R(0)$  on the number of hospitalized mumps cases  $H(t)$ . From Fig. 4(a), we can see that the initial value  $S(0)$  has a short-term strong influence on  $H(t)$  and other initial conditions have little or almost no effect. It implies that decreasing or increasing the number of initial values is insignificant factor for the prevalence and persistence of mumps in China.

By decreasing the seasonal varying transmission rates  $\beta_1(t)$  and  $\beta_2(t)$  to sufficient scales, mumps can be effectively controlled (see Fig. 3). Our simulations also indicate that by changing other parameters, except  $\beta_1(t)$  and  $\beta_2(t)$ ,  $\mathcal{R}_0$  can't be less than 1. And if  $\varepsilon$  and  $\kappa$  change together can educe  $\mathcal{R}_0$  to be less than 1 (see Fig. 5). Finally, we perform some sensitivity analyses to determine the influence of parameters  $\varepsilon$  and  $\kappa$  on  $\mathcal{R}_0$ . This analysis indicates that mumps in China can be eradicated by increasing vaccination rate and decreasing invalid vaccination rate simultaneously. This is contrast to the observation that the mumps has never been controlled by the single dose of mumps vaccine (MMR) from National Immunization Programmes in China in recent years.



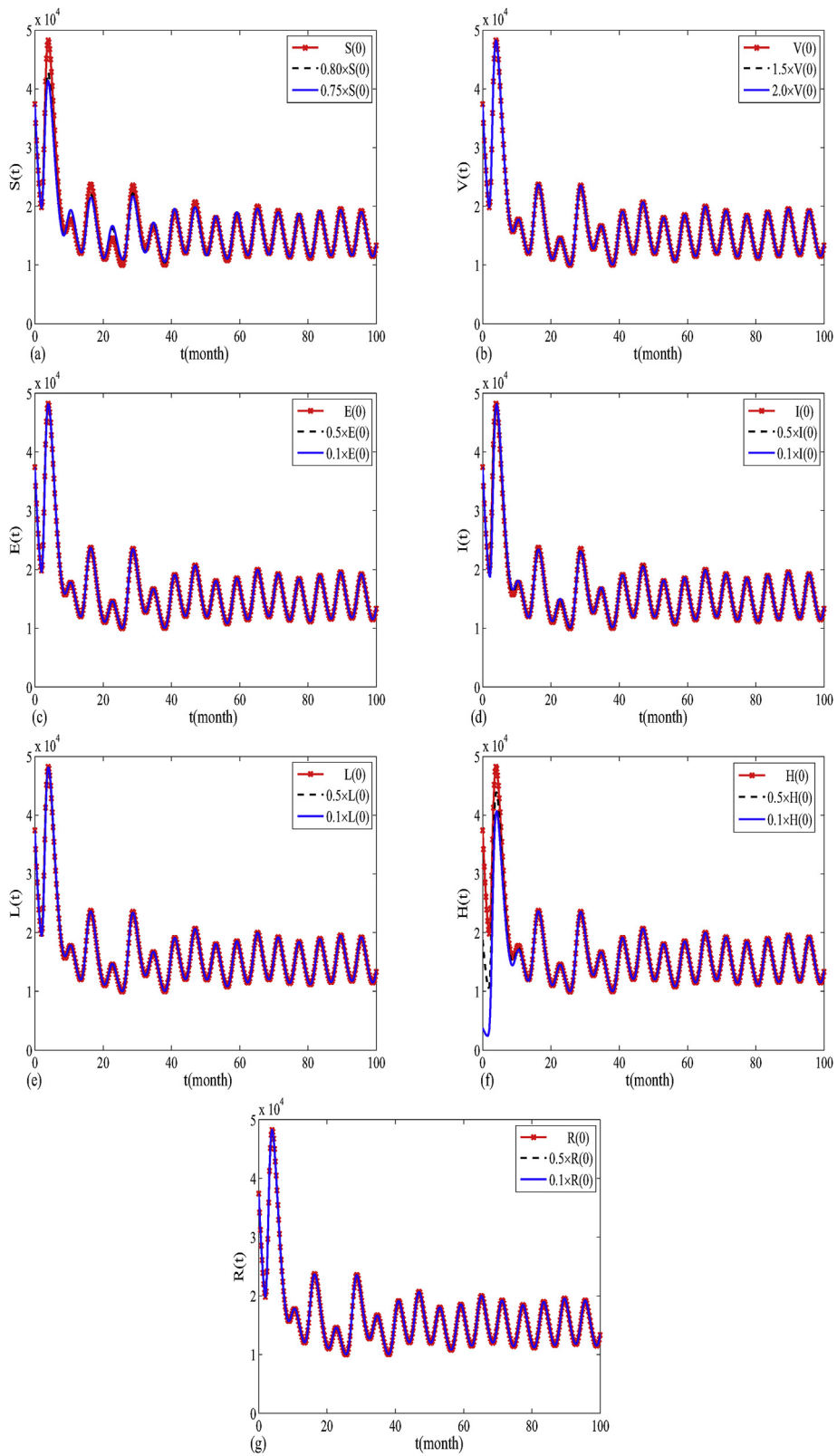
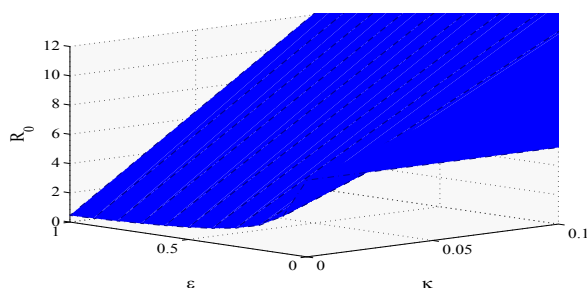


Fig. 4. Simulations of mumps cases  $H(t)$  with different values of  $S(0)$ ,  $V(0)$ ,  $E(0)$ ,  $I(0)$ ,  $L(0)$ ,  $H(0)$  and  $R(0)$  in China.



**Fig. 5.** The graph of  $\mathcal{R}_0$  in terms of  $\kappa$  and  $\epsilon$ . Other parameter values in Table 1 do not change.

The WHO, the American Academy of Pediatrics, the Advisory Committee on Immunization Practices of the Centers for Disease Control and Prevention, the American Academy of Family Physicians, the British Medical Association and the Royal Pharmaceutical Society of Great Britain currently recommend routine vaccination of children against mumps. The British Medical Association and Royal Pharmaceutical Society of Great Britain had previously recommended against general mumps vaccination, changing that recommendation in 1987. The mumps vaccine was introduced into the United States in December 1967: since its introduction there has been a steady decrease in the incidence of mumps and mumps virus infection. There were 151,209 cases of mumps reported in 1968. From 2001 to 2008, the case average was only 265 per year, excluding an outbreak of less than 6000 cases in 2006 attributed largely to university contagion in young adults (McNabb et al., 2008; Mumps despite shots, 2008). Children get 2 doses of MMR vaccine, the first is at 12–15 months, then the second is at 4–6 years. To sum up, the effect of two doses of vaccine is very significant. When children get 2 doses, the invalid vaccination rate ( $k$ ) will be smaller and vaccination rate ( $\epsilon$ ) will be bigger than get only one dose. Of course, if we will describe two doses of vaccine accurately, need to use pulse model, and this will be our future work.

But for most of the cities and provinces in China, according to the National Immunization Program by Chinese Center for Disease Control and Prevention strictly (National Immunization Program Chinese Center for Disease Control and Prevention), children get only one dose. After a period of time, the person's immunity vaccination will lose. Therefore we encourage the Chinese government should carry out two doses of MMR vaccine free for people.

#### 4. Discussion

The transmission of mumps has been a growing concern in China. In this paper, using the mumps data from China from 2013 to 2015, we constructed and parameterized a dynamic model for mumps transmission in China, and constructed a SVEILHR model with periodic transmission rates to investigate the spread of mumps. We evaluate the corresponding basic reproduction number  $\mathcal{R}_0 = 6.5428$  and average basic reproduction number  $\overline{\mathcal{R}_0} = 6.6737$ , analyze the dynamical behavior of the model. It is shown that there exists only disease-free solution which is globally asymptotically stable if  $\mathcal{R}_0 < 1$ , and there exists a positive periodic solution if  $\mathcal{R}_0 > 1$ .  $\mathcal{R}_0$  is a threshold parameters, its magnitude determines the extinction or the persistence of the disease. We also use the model to simulate the monthly data of mumps cases and carry out some sensitivity analysis of  $\mathcal{R}_0$  in terms of various model parameters.

We conclude that mumps will persist in China under the current conditions, which is presented in Fig. 3. By carrying out some sensitivity analysis of the average basic reproduction number in terms of some parameters, we found in addition to reduce the transmission rates  $\beta_1(t)$ ,  $\beta_2(t)$  and that vaccination rate and invalid vaccination rate play important roles in the spread of mumps while the other parameters have a little effect to control the disease.

Therefore, health-care education such as washing hands before meals and after using the toilet, and making air fresh indoors and so on, should be carried out in kindergartens, schools, hospitals and other places to popularize health knowledge and advocate good personal hygiene habits. Kindergartens should clean and disinfect toys and appliances every day. In addition, hospitals should strengthen infection control practices to avoid nosocomial cross infection. That is to say, reduce the transmission rates  $\beta_1(t)$ ,  $\beta_2(t)$ , the outbreak will be mitigated, or even eliminated. Although the efficacy of mumps vaccine is good but it is still very prevalent in China, Chinese children generally have mumps vaccine only one dose.

#### Conflict of interests

The authors declare that there is no conflict of interests regarding the publication of this article.

#### Authors' contributions

These authors contributed equally to this work. QQ, FC and LY conceptualized and designed the study, drafted the initial manuscript, and approved the final manuscript as submitted. FC, WJ analyzed the data and simulated parameters. QQ, ZL and JW carried out the initial analyses, reviewed and revised the manuscript. LY gave some suggestions.

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