SCIENTIFIC REPORTS

OPEN

SUBJECT AREAS:
QUANTUM INFORMATION
QUANTUM MECHANICS

Received
4 March 2014

Accepted
23 April 2014

Published
12 May 2014

Correspondence and requests for materials should be addressed to J.Q.Y. (jqyou@csrc.ac.cn)

Probing the non-locality of Majorana fermions via quantum correlations

Jun Li[1,2], Ting Yu[3], Hai-Qing Lin[1] & J. Q. You[1]

[1]Beijing Computational Science Research Center, Beijing 100084, China, [2]College of Physics and Electronic Engineering, Dezhou University, Dezhou 253023, China, [3]Center for Controlled Quantum Systems and Department of Physics and Engineering Physics, Stevens Institute of Technology, Hoboken, New Jersey 07030, USA.

Majorana fermions (MFs) are exotic particles that are their own anti-particles. Recently, the search for the MFs occurring as quasi-particle excitations in solid-state systems has attracted widespread interest, because of their fundamental importance in fundamental physics and potential applications in topological quantum computation based on solid-state devices. Here we study the quantum correlations between two spatially separate quantum dots induced by a pair of MFs emerging at the two ends of a semiconductor nanowire, in order to develop a new method for probing the MFs. We find that without the tunnel coupling between these paired MFs, quantum entanglement cannot be induced from an unentangled (i.e., product) state, but quantum discord is observed due to the intrinsic nonlocal correlations of the paired MFs. This finding reveals that quantum discord can indeed demonstrate the intrinsic non-locality of the MFs formed in the nanowire. Also, quantum discord can be employed to discriminate the MFs from the regular fermions. Furthermore, we propose an experimental setup to measure the onset of quantum discord due to the nonlocal correlations. Our approach provides a new, and experimentally accessible, method to study the Majorana bound states by probing their intrinsic non-locality signature.

Quantum entanglement is a quantum correlation that has no classical analog. Dynamical entanglement has exhibited some exotic properties such as the early-stage decoherence[1] and revival[2]. Recently, it was found that there exists another type of nonclassical correlation coined as quantum discord[3,4]. A remarkable feature of the quantum discord is that it can characterize some fundamental processes of physics, including the quantum phase transitions[5] and the discrimination of quantum and classical Maxwell's demons[6,7]. In addition, it can be used as a new resource for quantum information processing protocols[8–10], and be harnessed as a measurable quantity to implement quantum-enhanced tasks in the absence of quantum entanglement[11–13].

Majorana fermions (MFs) as quasi-particle excitations were anticipated to occur in different solid-state systems, such as the 5/2 fractional quantum Hall system[14], the *p*-wave superconductor[15], and the hybrid systems of a topological insulator[16]. Recently, semiconductor nanowires with strong spin-orbit coupling are attracting increasing interest. In addition to the application in spin-orbit qubit[17,18], these semiconductor nanowires provide a platform for demonstrating MFs[19–22]. Theoretically, it is shown that MFs can occur at the ends of a semiconductor nanowire with strong spin-orbit coupling when the nanowire is placed in proximity to an *s*-wave superconductor[19,20]. It is expected that the nanowire-superconductor system based on conventional materials is more accessible to experimentally detecting the MFs[21,22]. In solid-state systems, MFs are coherent superpositions of electron and hole excitations. Two spatially separated MFs can form one fermionic level, which can be either occupied or empty, and thus defines a nonlocal qubit. The paired MFs may exhibit intrinsic nonlocal properties, so the search for the quantum effects induced by such nonlocal paired MFs can provide an important signature for the existence of MFs. Several schemes have been proposed to test the non-locality of MFs by studying the correlation behaviors of electron tunneling through individual MFs[23–25]. However, for the completely separated MFs, the tunneling events through each MF are shown to be fully independent[23–25]. Therefore, the intrinsic non-locality of the paired MFs cannot be demonstrated in such a context.

A hybrid quantum system combining two or more physical systems can often utilize the strengths of different systems to better explore new phenomena and potentially bring about novel quantum technologies[26]. For the two MFs emerging at the ends of a semiconductor nanowire, because their energies are zero, it is difficult to directly demonstrate their intrinsic non-locality, particularly when the tunnel coupling between them becomes zero. Thus, to probe the emerged two MFs in the nanowire, in this work, we consider a hybrid quantum system consisting of two quantum dots (QDs) interacting with these two MFs. To characterize the quantum correlations between the two QDs mediated by the paired MFs, we use both quantum entanglement and quantum discord. We

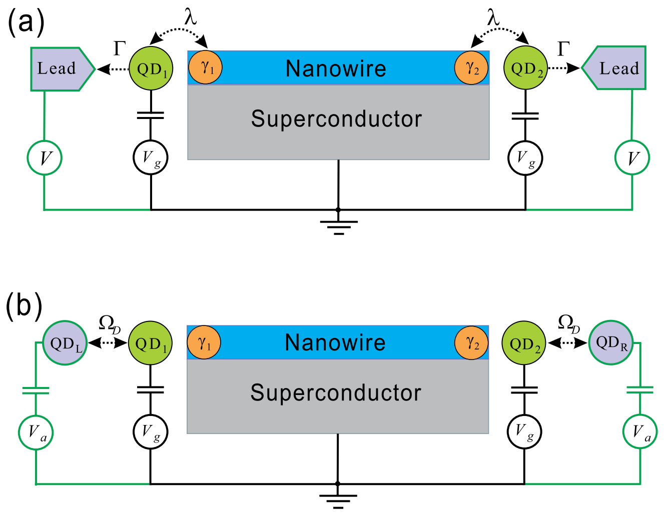

Figure 1 | Schematic setup for probing quantum correlations between two QDs mediated by a pair of MFs. A semiconductor nanowire with strong spin-orbit coupling is placed in proximity to an *s*-wave superconductor, with a pair of MFs emerging at the two ends of the nanowire. (a) Two electrode leads are introduced to investigate the decoherence effects on the quantum correlations. (b) Two auxiliary QDs (denoted by QD_L and QD_R) are introduced to measure the quantum correlations. The arrows indicate the tunneling directions of electrons. The energy levels of the QDs and the chemical potentials of the electrode leads are tuned by the voltages applied on them.

show that when the paired MFs are tunnel-coupled via the wavefunction overlapping, the maximally entangled states can be induced from an unentangled (i.e., product) state at specific dynamical time points. However, when the paired MFs are completely separated (i.e., no tunneling between them), quantum entanglement cannot be induced from this product state, but remarkably, quantum discord is shown to still persist owing to the intrinsic nonlocal correlation of MFs. This indicates that the quantum discord can indeed reflect the non-locality of the paired MFs. Furthermore, we propose an experimentally accessible approach to measuring the onset of quantum discord. Our protocol provides a new method to probe the MFs emerging at the two ends of the nanowire.

Results

Quantum dynamics. We study a hybrid structure shown in Fig. 1, where a semiconductor nanowire with strong spin-orbit coupling is placed in proximity to an *s*-wave superconductor. This nanowire is predicted to be driven into a topological superconducting phase[19,20]. A pair of zero-energy MFs, γ_1 and γ_2, are anticipated to appear at the two ends of the nanowire. The Hamiltonian of the paired MFs is described by $H_M = i\epsilon_m\gamma_1\gamma_2/2$, where $\epsilon_m \sim e^{-L/\xi}$ describes the tunnel coupling between the two MFs due to the overlap of wavefunctions. This inter-MF coupling damps exponentially with L (the length of the nanowire) and ξ (the superconducting coherent length). Because the paired MFs have zero energies, it is difficult to directly demonstrate their quantum correlations, particularly when the tunnel coupling ϵ_m becomes zero. Thus, we introduce two QDs (i.e., QD_1 and QD_2) on both sides of the nanowire, each of which is tunnel-coupled to a MF neighboring to it. In addition, we consider the strong Coulomb blockade regime for each QD, so that only one electron is allowed therein when the tunneling process takes place. The Hamiltonian of the two spatially separated quantum dots is written as $H_{DD} = \sum_{j=1,2} \varepsilon_j d_j^\dagger d_j$, where $d_j^\dagger \left(d_j\right)$ is the electron creation (annihilation) operator of the dot j, with the corresponding energy ε_j. The tunneling Hamiltonian between the QDs and the MFs is described by[23–25] $H_{TM} = \sum_{j=1,2} \lambda_j \left(d_j^\dagger - d_j\right)\gamma_j$, with λ_j (j = 1, 2) denoting the tunnel coupling of the jth dot to its neighboring MF.

The total Hamiltonian of the system is a sum of the above three parts: $H_{\rm sys} = H_M + H_{DD} + H_{TM}$. For practical calculations, it is more convenient to switch from the Majorana representation to the regular fermion one via the transformations: $\gamma_1 = i(f - f^\dagger)$, and $\gamma_2 = f + f^\dagger$, where f is the regular fermion operator, satisfying the anticommutation relation $\{f, f^\dagger\} = 1$. After an additional local gauge transformation $d_1 \rightarrow id_1$, the total Hamiltonian of the system can be written in the new representation as[23–25]

$$\begin{aligned} H_{\rm sys} = & \epsilon_m\left(f^\dagger f - 1/2\right) + \sum_{j=1,2}\left[\varepsilon_j d_j^\dagger d_j + \lambda_j\left(d_j^\dagger f + f^\dagger d_j\right)\right] \\ & -\lambda_1\left(d_1^\dagger f^\dagger + f d_1\right) + \lambda_2\left(d_2^\dagger f^\dagger + f d_2\right). \end{aligned} \tag{1}$$

For simplicity, we adopt a symmetric setup with $\lambda_1 = \lambda_2 = \lambda$. Moreover, the two QDs are adjusted in resonance with the two MFs (i.e., $\varepsilon_1 = \varepsilon_2 = 0$) via the gate voltages. Also, we use λ ($1/\lambda$) as the energy (time) unit throughout the paper.

The QD-MF system considered in this paper includes the two QDs (i.e., QD_1 and QD_2) and the MFs. For the new fermion representation, it is convenient to use the state basis $|n_1, n_2, n_M\rangle$, where $n_l = 0$ or 1, with $l = 1, 2, M$, denoting the electron occupation number in the left QD (i.e., QD_1), the right QD (i.e., QD_2) and the paired MFs, respectively. From Hamiltonian (1), one may notice that the electron numbers in both QDs and the paired MFs are not conserved, because a pair of electrons can be extracted out from the superconductor or absorbed by it. As a consequence, the parity of the isolated QD-MF system becomes conserved, and the Hilbert space can be split into two subspaces: $|1, 1, 1\rangle$, $|1, 0, 0\rangle$, $|0, 1, 0\rangle$, and $|0, 0, 1\rangle$ with odd parity; $|1, 1, 0\rangle$, $|1, 0, 1\rangle$, $|0, 1, 1\rangle$, and $|0, 0, 0\rangle$ with even parity. Thus, starting from a product state $|0, 0, 1\rangle$, only odd-parity states are involved in the state evolution:

$$\begin{aligned} |\Psi(t)\rangle = & C_1(t)|1,1,1\rangle + C_2(t)|1,0,0\rangle \\ & + C_3(t)|0,1,0\rangle + C_4(t)|0,0,1\rangle. \end{aligned} \tag{2}$$

With this initial condition, the time-dependent coefficients can be obtained, via directly solving Schrödinger equation, as follows:

$$\begin{aligned} C_1(t) &= \frac{-\Delta e^{-i\Omega t} + \Delta\cos(\Delta t) - i\Omega\sin(\Delta t)}{2\Delta}, \\ C_2(t) &= \frac{i\lambda\sin(\Delta t)}{\Delta}, \quad C_3(t) = -\frac{i\lambda\sin(\Delta t)}{\Delta}, \\ C_4(t) &= \frac{\Delta e^{-i\Omega t} + \Delta\cos(\Delta t) - i\Omega\sin(\Delta t)}{2\Delta}, \end{aligned} \tag{3}$$

where $\Delta = \sqrt{\epsilon_m^2 + 16\lambda^2}\Big/2$, and $\Omega = \epsilon_m/2$. From equation (2), we can easily obtain the density operator $\rho(t) = |\Psi(t)\rangle\langle\Psi(t)|$. After tracing over the degrees of freedom of the MFs, the reduced density operator $\rho_d(t)$ for the two QDs is obtained. This reduced density operator contains the nonlocal information of the paired MFs and serves as the starting point for calculating the quantum correlations (see Methods).

In order to investigate the decoherence effect on quantum correlations, we also introduce two electrode leads, each weakly coupled to its neighboring QD. The two electrode leads can be treated as electron reservoirs described by the Hamiltonian $H_{\rm leads} = \sum_{j=1,2}\sum_k \varepsilon_{kj} c_{kj}^\dagger c_{kj}$, where $c_{kj}^\dagger\left(c_{kj}\right)$ is the creation (annihilation) operator for the electron with wave vector k in the jth electrode lead and ε_{kj} is the corresponding energy. The tunneling Hamiltonian between the electrode leads and the QDs is

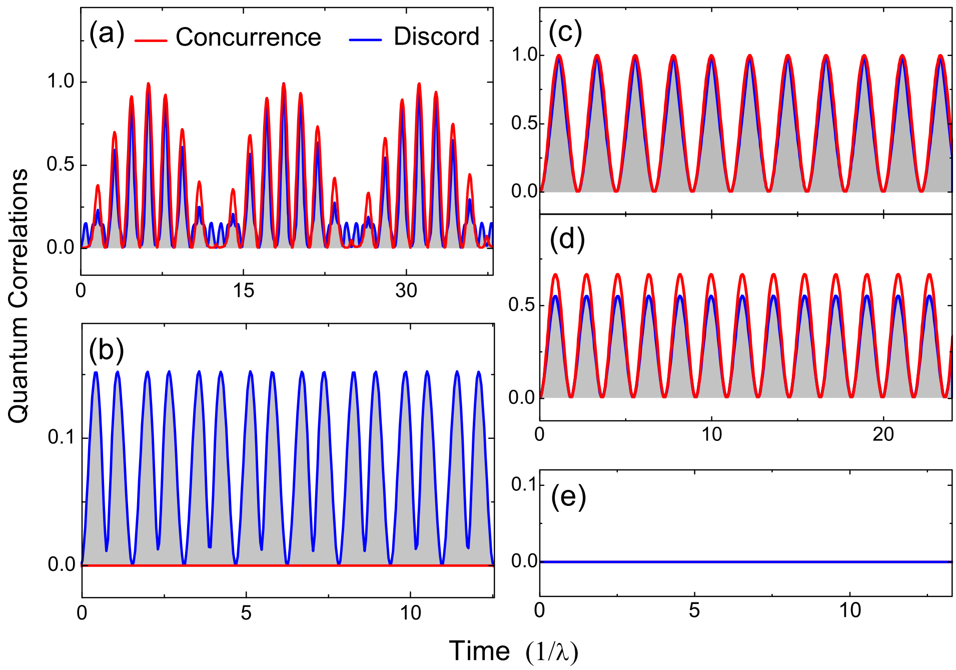

Figure 2 | Quantum dynamics of the quantum correlations between two QDs mediated by different kinds of fermions in the absence of quantum decoherence. We assume a symmetric setup with $\lambda_1 = \lambda_2 = \lambda$, and $\varepsilon_1 = \varepsilon_2 = 0$. (a) Two QDs mediated by a pair of MFs with an inter-MF coupling $\epsilon_m = 0.5\lambda$. (b) Two QDs mediated by a pair of MFs without an inter-MF coupling (i.e., $\epsilon_m = 0$). (c) Two QDs mediated by a regular fermion with energy $\epsilon_c = 0$. (d) Two QDs mediated by a regular fermion with $\epsilon_c = \lambda$, (e) Two QDs mediated by a pair of non-interacting regular fermions with energies $\epsilon_1 = \epsilon_2 = 0$.

$H_{TL} = \sum_{j=1,2}\sum_k \left[t_j d_j^\dagger c_{kj} + \text{h.c.}\right]$. Here we employ the master equation approach to solving this problem. For the whole setup, the leads are regarded as an environment, and the central QD-MF device is described by a reduced density operator $\rho(t)$. We assume that the chemical potentials of the two leads, both denoted by μ, are well below the energy levels of the dots, i.e., the energy difference $\varepsilon_1 - \mu$ and $\varepsilon_2 - \mu$ are much larger than the level broadening of the QDs. In this case, electrons can only jump outwards from the central device to the electrodes. In addition, we consider the regime with a weak coupling between the QDs and the leads. Thus, we can use the standard Born-Markov master equation[27]. In the present case, the master equation can be written as

$$\dot{\rho}(t) = -i[H_{\text{sys}}, \rho] + \Gamma(\mathcal{D}[d_1]\rho(t) + \mathcal{D}[d_2]\rho(t)), \qquad (4)$$

where $\mathcal{D}[A]\rho \equiv A\rho A^\dagger - \frac{1}{2}\{A^\dagger A, \rho\}$, and the tunneling rate Γ is associated with both the tunnel-coupling strengths and the densities of states of the two electrode leads. With the density operator $\rho(t)$ solved from equation (4), we can then obtain the reduced density operator $\rho_d(t)$ of the two QDs by tracing over the degrees of freedom of the MFs.

Quantum discord versus quantum entanglement. To measure quantum correlations, here we use both quantum entanglement and quantum discord (see Methods). We show the evolutions of quantum entanglement and quantum discord for the two QDs (i.e., QD_1 and QD_2) in Fig. 2. As a typical example, we choose $\epsilon_m = 0.5\lambda$ in Fig. 2a. In this case, the two MFs are tunnel-coupled, in addition to the intrinsic nonlocal correlation of the paired MFs. Equation (3) shows that there are two frequencies determining the oscillating behaviors of the quantum state $|\Psi(t)\rangle$. The fast frequency $\Delta/2\pi$ determines the period of the oscillations. From equations (2) and (3), it can be seen that the state of the system becomes $C_1(t)|1, 1, 1\rangle + C_4(t)|0, 0, 1\rangle$ at $t = n\pi/\Delta$ ($n = 1,2,3\cdots$), i.e., the state of the two QDs is just an entangled state $C_1(t)|1, 1\rangle + C_4(t)|0, 0\rangle$. As shown in Fig. 2a, the quantum entanglement peaks at $t = n\pi/\Delta$. The slow frequency $\Omega/2\pi$ modulates the height of oscillating peaks and determines the period of the modulating cycles. It can be seen that the quantum discord exhibits similar behaviors of oscillations except for the very short initial and final periods of each cycle, where the quantum entanglement becomes close to zero. Also, as shown in Fig. 2a, the quantum entanglement (discord) can reach its maximal value $C_d = 1$ ($D_d = 1$) at some specific time points. When $\epsilon_m = 0$, which corresponds to the long-length limit of the nanowire, the two MFs are completely separated in space. In this case, there is no direct tunnel coupling between the two MFs, but their intrinsic nonlocal correlation still exists. As shown in Fig. 2b, the temporal behaviors of quantum entanglement and quantum discord are very different at $\epsilon_m = 0$. While the quantum entanglement remains zero with time, the quantum discord oscillates with a smaller amplitude. Previous studies have found it hard to demonstrate the intrinsic nonlocal correlation between the two MFs in the electrical measurement signals, including the electrical current and the current noise[23–25]. Thus, it will be important to select a physical quantity that can significantly reflect the non-locality of the MFs. Interestingly, our results reveal that the quantum discord can clearly serve as an indicator showing the intrinsic nonlocal correlation of the MFs in contrast to the insensitive quantum entanglement considered in our system.

For the purpose of discriminating MFs from regular fermions, we also show the quantum correlations of the two QDs (i.e., QD_1 and QD_2) induced by regular fermions that couple to them. We first consider the case with the two QDs coupled by a common regular fermion. Different from equation (1), the Hamiltonian of this QD-fermion system is given by $H_{\text{sys}} = \epsilon_c c^\dagger c + \sum_{j=1,2}\left[\varepsilon_j d_j^\dagger d_j + \lambda_j\left(d_j^\dagger c + c^\dagger d_j\right)\right]$, where $c^\dagger$ (c) is the electron creation (annihilation) operator of the regular fermion with energy ϵ_c, and λ_j ($j = 1, 2$) denotes the tunnel coupling between the jth dot and the regular fermion. We assume that the two QDs is initially empty, but the fermionic level ϵ_c is occupied by a single regular fermion such as the fermionic quasi-particle excitation in the nanowire. When the fermionic level ϵ_c is resonant to the energy levels ε_i of the two QDs, both quantum discord and quantum entanglement of the two QDs induced by the fermion have identical dynamical evolutions (see Fig. 2c); when ϵ_c is off-resonant to ε_i, these quantum discord and quantum entanglement have different amplitudes, but still exhibit very similar temporal behaviors. Moreover, we consider the case involving a pair of non-interacting regular fermions, where each fermion couples to a QD. The Hamiltonian of this QD-fermion system is $H_{\rm sys} = \sum_{j=1,2}\left[\epsilon_j d_j^\dagger d_j + \varepsilon_j c_j^\dagger c_j + \lambda_j\left(d_j^\dagger c_j + c_j^\dagger d_j\right)\right]$, where $c_j^\dagger\,(c_j)$ is the electron creation (annihilation) operator of the regular fermion with energy ϵ_j, and λ_j (j = 1, 2) denotes the tunnel coupling between the jth quantum dot and the regular fermion that couples to it. Also, we assume that the two QDs are initially empty, but each fermionic level ϵ_i is occupied by a single regular fermion. Because there is no intrinsic non-locality between these two non-interacting regular fermions, both quantum discord and quantum entanglement of the two QDs remains zero with time (see Fig. 2e). As compared with Figs. 2a and 2b, these very different temporal behaviors in Figs. 2c–2d can be used to discriminate the MFs from the regular fermions.

We also study the temporal behaviors of both quantum entanglement and quantum discord for the QD-MF system in the presence of two electrode leads. Here we choose a tunneling rate Γ = 0.05λ to ensure a weak coupling between the central QD-MF system and the two electrode leads. These two leads play the role of a fermionic environment which induces decoherence on the evolutions of quantum correlations. With tunnel coupling between the paired MFs, e.g., $\epsilon_m = 0.5\lambda$ in Figs. 3a, both quantum entanglement and quantum discord can be generated from the product state, but the oscillating quantum entanglement decays faster with time than the quantum discord. This implies that the quantum discord, as a new resource of quantum information processing, is more robust than the quantum entanglement in this model system. Moreover, without tunnel coupling between the paired MFs, i.e., $\epsilon_m = 0$, the quantum entanglement still remains zero with time, but the oscillating quantum discord decays due to the decoherence induced by the electric leads (see Fig. 3b).

A measurement scheme. Note that equation (2) can be rewritten as

$$|\Psi(t)\rangle = b_1(t)e^{i\phi_1(t)}|1,1,1\rangle + b_2(t)|1,0,0\rangle + b_2(t)|0,1,0\rangle + b_3(t)e^{i\phi_3(t)}|0,0,1\rangle, \quad (5)$$

where the real parameters $b_1(t)$, $b_2(t)$, $b_3(t)$, $\phi_1(t)$ and $\phi_3(t)$ are determined by $C_i(t)$, i = 1, 2, 3, and 4, in equation (3). After tracing over the degrees of freedom of the MFs, the reduced density operator of the two QDs (i.e., QD_1 and QD_2) can be obtained via equation (5) as

$$\rho_d(t) = \begin{pmatrix} b_1^2(t) & 0 & 0 & b_1(t)b_3(t)e^{i\Delta\phi(t)} \\ 0 & b_2^2(t) & b_2^2(t) & 0 \\ 0 & b_2^2(t) & b_2^2(t) & 0 \\ b_1(t)b_3(t)e^{-i\Delta\phi(t)} & 0 & 0 & b_3^2(t) \end{pmatrix}, \quad (6)$$

with $\Delta\phi(t) = \phi_1(t) - \phi_3(t)$. Here $b_1^2(t)$, $b_2^2(t)$ and $b_3^2(t)$, i.e., the diagonal matrix elements in equation (6), correspond to the probabilities of both dots occupied, either dot occupied, and both dots empty, respectively. These probabilities can be directly obtained by joint measurements on the electron occupation of QD_1 and QD_2 (see, e.g., Ref. 30). In order to obtain the values of the phase difference $\Delta\phi(t)$, we introduce, as Fig. 1b shows, two auxiliary QDs (denoted by QD_L and QD_R) adjacent to QD_1 and QD_2, respectively. First, instead of $|0\rangle$, these two auxiliary QDs are initially prepared in the superposition state: $\left|\psi_{L(R)}\right\rangle = \left(|0\rangle_{L(R)} + |1\rangle_{L(R)}\right)/\sqrt{2}$. Second, the QD_L (QD_R) is adjusted in resonance with the QD_1 (QD_2) (i.e., $\varepsilon_L = \varepsilon_1 = 0$, and $\varepsilon_R = \varepsilon_2 = 0$) at the moment t, via tuning the gate voltages. Simultaneously, the tunnel coupling between QD_1 (QD_2) and its adjoining MF is switched off by tuning the voltages of the electrical gates that control the tunnel barrier between them. The Hamiltonian of the total system becomes $H_{TD} = T_1\left(d_L^\dagger d_1 + d_1^\dagger d_L\right) + T_2\left(d_R^\dagger d_2 + d_2^\dagger d_R\right)$, where T_1 (T_2) is the tunnel coupling between QD_L (QD_R) and QD_1 (QD_2) and assume $T_1 = T_2 = T$. Therefore, the system further evolves under the Hamiltonian H_{TD}, from the initial product state $|\psi_L\rangle \otimes |\psi_R\rangle \otimes |\Psi(t)\rangle$. Third, along with the evolution of the total system, we choose a specific moment, $t + \Delta t$, to jointly measure the occupation probabilities of both QD_1 and QD_2, i.e., P_{11}, P_{10}, P_{01}, and P_{00} which denote the probabilities of both dots occupied, single QD_1 occupied, single QD_2 occupied, and both dots empty, respectively. When $\Delta t = \pi/4T$, these probabilities can be obtained as

$$\begin{aligned} P_{11}(t) &= \frac{1}{16}\left[9b_1^2(t) + 8b_2^2(t) + b_3^2(t) + 2b_1(t)b_3(t)\cos\Delta\phi(t)\right], \\ P_{00}(t) &= \frac{1}{16}\left[b_1^2(t) + 8b_2^2(t) + 9b_3^2(t) + 2b_1(t)b_3(t)\cos\Delta\phi(t)\right], \\ P_{10}(t) &= \frac{1}{16}\left[3b_1^2(t) + 8b_2^2(t) + 3b_3^2(t) - 2b_1(t)b_3(t)\cos\Delta\phi(t)\right], \end{aligned} \quad (7)$$

and $P_{01}(t) = P_{10}(t)$. This measurable occupation probabilities include the phase difference $\Delta\phi(t)$, in addition to $b_1(t)$, $b_2(t)$, and $b_3(t)$ that have previously been obtained via joint measurements as well. Thus, it is experimentally feasible to obtain the reduced density operator $\rho_d(t)$ of the two QDs mediated by the paired MFs. Once the reduced density operator $\rho_d(t)$ is determined experimentally, one can then derive the quantum entanglement and the quantum discord of the two QDs using this reduced density operator (see Methods).

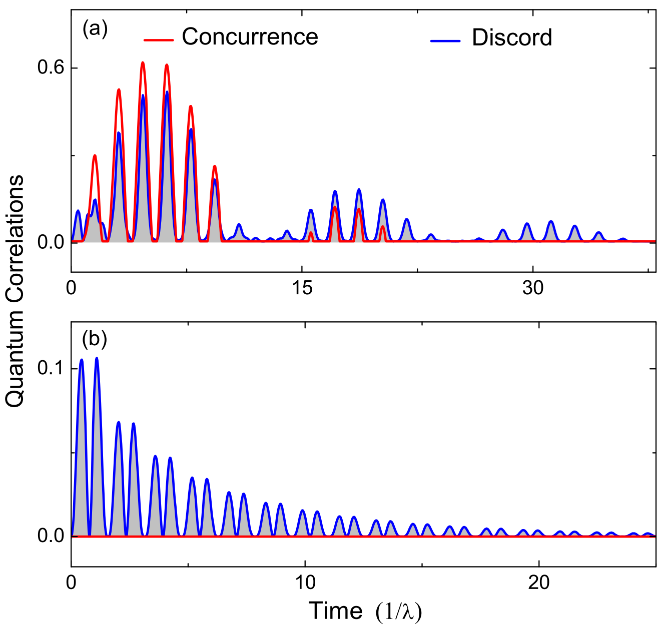

Figure 3 | Quantum dynamics of the quantum correlations between two QDs mediated by a pair of MFs in the presence of quantum decoherence. We assume a symmetric setup with $\lambda_1 = \lambda_2 = \lambda$, and $\varepsilon_1 = \varepsilon_2 = 0$. The tunneling rate between the central QD-MF device and the two electrode leads is chosen as Γ = 0.05λ. (a) $\epsilon_m = 0.5\lambda$, and (b) $\epsilon_m = 0$.

Discussion

We have investigated the dynamics of quantum correlations between two QDs mediated by a pair of MFs. We find that in the presence of tunnel coupling between the two MFs, maximally entangled states of the two QDs can be generated from an unentangled state under proper dynamical conditions. Notably, in the absence of tunnel coupling between MFs, while the quantum entanglement remains zero, quantum discord can be induced from this unentangled state owing to the intrinsic nonlocal correlations of the MFs. While the conventional methods fail to reveal the non-locality information of the MFs, our demonstrated new feature indicates that the quantum discord can serves as a novel quantum signature reflecting the intrinsic non-locality of the MFs. Physically, it shows that quantum discord captures some interesting correlation features of the system that is insensitive to entanglement. Also, we show that these features do not exist when replacing MFs with regular fermions. Therefore, they can be used to discriminate the MFs from the regular fermions. Furthermore, we have proposed a scheme for experimentally determining the quantum discord of the two QDs mediated by the paired MFs. In addition, we further investigate the decoherence effects on the quantum correlations, and the results show that the quantum discord is more robust than the quantum entanglement. Because the quantum discord is a measurable physical quantity and the intrinsic nonlocal nature is an extraordinary character of the MFs, our approach provides a new, and experimentally feasible, method to probe the MFs by demonstrating their intrinsic non-locality via the quantum discord.

Methods

With the reduced density operator ρ_d of the two QDs (i.e., QD_1 and QD_2), the concurrence[28], as a measure of the quantum entanglement, can be obtained as

$$C_d = \max\left\{0, \sqrt{\Lambda_1} - \sqrt{\Lambda_2} - \sqrt{\Lambda_3} - \sqrt{\Lambda_4}\right\}, \tag{8}$$

where $\Lambda_1, \Lambda_2, \Lambda_3$ and Λ_4 are the square roots of the eigenvalues in decreasing order for the matrix

$$G = \rho_d (\sigma_y \otimes \sigma_y) \rho_d^* (\sigma_y \otimes \sigma_y), \tag{9}$$

where σ_y is a Pauli matrix, and ρ_d^* is the complex conjugate of ρ_d.

Quantum discord is a measure of quantum correlations obtained by subtracting the classical correlations from the total correlations[3,4]. For a bipartite quantum system including subsystems A and B, the total correlations are defined, in terms of quantum mutual information, as $I(\rho^A : \rho^B) = S(\rho^A) + S(\rho^B) - S(\rho_{AB})$, where ρ_{AB} denotes the density operator of the bipartite system, with ρ^A (ρ^B) being the reduced density operator of the subsystem A (B), and $S(\rho_{AB}) = -\mathrm{Tr}(\rho_{AB}\log_2\rho_{AB})$ is the entropy of the bipartite quantum system. The total classical correlations based on the POVM measurement of the subsystem B are defined as[3,29] $J(\rho_{AB}|\{B_k\}) = S(\rho^A) - \min_{\{B_k\}} \sum_k p_k S(\rho_k)$, where $\{B_k\}$ is a complete set of projectors of local measurements performed on the subsystem B; $p_k = \mathrm{Tr}[(I \otimes B_k)\rho_{AB}(I \otimes B_k)]$ denotes the probability of obtaining the measurement outcome k, with I being the identity operator for the subsystem A; $\rho^k = (I \otimes B_k)\rho_{AB}(I \otimes B_k)/p_k$ is the state of the bipartite system, conditioned on the measurement outcome labeled by k. The quantum discord is defined as[3,4]

$$D_B(\rho_{AB}) = \min_{\{B_k\}} \left[I(\rho^A : \rho^B) - J(\rho_{AB}|\{B_k\})\right]. \tag{10}$$

For our system, the A and B parts are the two QDs (i.e., QD_1 and QD_2), and $\rho_{AB} = \rho_d$. To compute classical correlations, we adopt the specific case of von Neumann local measurements, which are provided by the orthogonal projectors $B_k = R\,\Pi_k\,R^\dagger$, where $\{\Pi_k\} = \{|k\rangle\langle k|\}$ defines the computational basis R, with $k = 0, 1$ corresponding to the empty and occupied states, respectively. As a rotation operator, V can be parameterized as

$$R = \begin{pmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2} e^{-i\phi} \\ -\sin\frac{\theta}{2} e^{i\phi} & -\cos\frac{\theta}{2} \end{pmatrix} \tag{11}$$

where $\theta \in [0, \pi]$, and $\phi \in [0, 2\pi]$. Hence, the quantum discord given in equation (10) can be directly obtained by minimizing over all angles θ and ϕ.

1. Zurek, W. H. Decoherence, einselection, and the quantum origins of the classical. *Rev. Mod. Phys.* **75**, 715–775 (2003).
2. Yu, Ting & Eberly, J. H. Sudden death of entanglement. *Science* **323**, 598–601 (2009).
3. Henderson, L. & Vedral, V. Classical, quantum and total correlations. *J. Phys. A* **34**, 6899–6905 (2001).
4. Ollivier, H. & Zurek, W. H. Quantum discord: a measure of the quantumness of correlations. *Phys. Rev. Lett.* **88**, 017901 (2001).
5. Werlang, T., Trippe, C., Ribeiro, G. A. P. & Rigolin, G. Quantum correlations in spin chains at finite temperatures and quantum phase transitions. *Phys. Rev. Lett.* **105**, 095702 (2010).
6. Zurek, W. H. Quantum discord and Maxwell's demons. *Phys. Rev. A* **67**, 012320 (2003).
7. Maruyama, K., Nori, F. & Vedral, V. Colloquium: The physics of Maxwell's demon and information. *Rev. Mod. Phys.* **81**, 1–23 (2009).
8. Datta, A., Shaji, A. & Caves, C. M. Quantum discord and the power of one qubit. *Phys. Rev. Lett.* **100**, 050502 (2008).
9. Luo, S. & Sun, W. Decomposition of bipartite states with applications to quantum no-broadcasting theorems. *Phys. Rev. A.* **82**, 012338 (2010).
10. Madhok, V. & Datta, A. Interpreting quantum discord through quantum state merging. *Phys. Rev. A* **83**, 032323 (2011).
11. Lanyon, B. P., Barbieri, M., Almeida, M. P. & White, A. G. Experimental quantum computing without entanglement. *Phys. Rev. Lett.* **101**, 200501 (2008).
12. Cavalcanti, D. *et al.* Observing the operational significance of discord consumption. *Nat. Phys.* **8**, 666–670 (2012).
13. Dakic, B. *et al.* Quantum discord as resource for remote state preparation. *Nat. Phys.* **8**, 671–675 (2012).
14. Moore, G. & Read, N. Paired states of fermions in two dimensions with breaking of parity and time reversal symmetries and the fractional quantum Hall effect. *Phys. Rev. B* **61**, 10267 (2000).
15. Kitaev, A. Y. Unpaired Majorana fermions in quantum wires. *Phys.-Usp.* **44**, 131–136 (2001).
16. Fu, L. & Kane, C. Superconducting proximity effect and Majorana fermions at the surface of a topological insulator. *Phys. Rev. Lett.* **100**, 096407 (2008).
17. Nadj-Perge, S., Frolov, S. M., Bakkers, E. P. A. M. & Kouwenhoven, L. P. Spin-orbit qubit in a semiconductor nanowire. *Nature* **468**, 1084–1087 (2010).
18. Li, R., You, J. Q., Sun, C. P. & Nori, F. Controlling a nanowire spin-orbit qubit via electric-dipole spin resonance. *Phys. Rev. Lett.* **111**, 086805 (2013).
19. Sau, J. D., Lutchyn, R. M., Tewari, S. & Das Sarma, S. Generic new platform for topological quantum computation using semiconductor heterostructures. *Phys. Rev. Lett.* **104**, 040502 (2010).
20. Oreg, Y., Refael, G. & von Oppen, F. Helical liquids and Majorana bound states in quantum wires. *Phys. Rev. Lett.* **105**, 177002 (2010).
21. Mourik, V., Zuo, K., Frolov, S. M., Plissard, S. R., Bakkers, E. P. A. M. & Kouwenhoven, L. P. Signatures of Majorana fermions in hybrid superconductor-semiconductor nanowire devices. *Science* **336**, 1003–1007 (2012).
22. Das, A., Ronen, Y., Most, Y., Oreg, Y., Heiblum, M. & Shtrikman, H. Zero-bias peaks and splitting in an Al-InAs nanowire topological superconductor as a signature of Majorana fermions. *Nat. Phys.* **8**, 887–895 (2012).
23. Tewari, S., Zhang, C., Das Sarma, S., Nayak, C. & Lee, D. H. Testable signatures of quantum nonlocality in a two-dimensional chiral p-wave superconductor. *Phys. Rev. Lett.* **100**, 027001 (2008).
24. Bolech, C. J. & Demler, E. Observing Majorana bound states in p-wave superconductors using noise measurements in tunneling experiments. *Phys. Rev. Lett.* **98**, 237002 (2007).
25. Wang, P. Y., Cao, Y. S., Gong, M., Xiong, G. & Li, X. Q. Cross-correlations mediated by Majorana bound states. *Europhys. Lett.* **103**, 57016 (2013).
26. Xiang, Z. L., Ashhab, S., You, J. Q. & Nori, F. Hybrid quantum circuits: Superconducting circuits interacting with other quantum systems. *Rev. Mod. Phys.* **85**, 623–653 (2013).
27. Sun, H. B. & Milburn, G. J. Quantum open-systems approach to current noise in resonant tunneling junctions. *Phys. Rev. B* **59**, 10748 (1999).
28. Wootters, W. K. Entanglement of formation of an arbitrary state of two qubits. *Phys. Rev. Lett.* **80**, 2245 (1998).
29. Henderson, L. & Vedral, V. Information, relative entropy of entanglement, and irreversibility. *Phys. Rev. Lett.* **84**, 2263 (2000).
30. Gustavsson, S., Leturcq, R., Simovic, B., Schleser, R., Ihn, T., Studerus, P., Ensslin, K., Driscoll, D. C. & Gossard, A. C. Counting statistics of single electron transport in a quantum dot. *Phys. Rev. Lett.* **96**, 076605 (2006).

Acknowledgments

J.L. and J.Q.Y. are partially supported by the National Natural Science Foundation of China Grant Nos. 91121015 and 11304031, the National Basic Research Program of China Grant No. 2014CB921401, and the NSAF Grant No. U1330201. T.Y. is partially supported by the NSF PHY-0925174 and AFOSR No. FA9550-12-1-0001. H.Q.L. is partially supported by the National Natural Science Foundation of China Grant No. 91230203, the National Basic Research Program of China Grant No. 2011CB922200 and the NSAF Grant No. U1230202.

Author contributions

J.L. carried out all calculations under the guidance of J.Q.Y. and both T.Y. and H.Q.L. participated in the discussions. All authors contributed to the interpretation of the work and the writing of the manuscript.

Additional information

Competing financial interests: The authors declare no competing financial interests.

How to cite this article: Li, J., Yu, T., Lin, H.-Q. & You, J.Q. Probing the non-locality of Majorana fermions via quantum correlations. *Sci. Rep.* **4**, 4930; DOI:10.1038/srep04930 (2014).