Heliyon 9 (2023) e16729

Contents lists available at ScienceDirect

Heliyon

journal homepage: www.cell.com/heliyon

Super-rational aspiration promotes cooperation in the asymmetric game with peer exit punishment and reward

Si-Yi Wang [a], Xin Yao [a], Yi-Mei Yang [b], Daniel Chen [c], Rui-Wu Wang [d], Feng-Jie Xie [a,*]

[a] School of Modern Posts, Xi'an University of Posts & Telecommunications, Xi'an, Shaanxi, 710061, China
[b] Department of Applied Mathematics, Northwestern Polytechnical University, Xi'an, 710129, China
[c] The High School Affiliated to Renmin University of China, Beijing, 100097, China
[d] School of Ecology and Environment, Northwestern Polytechnical University, Xi'an, 710129, China

ARTICLE INFO

Keywords:
Exit rights
Asymmetry
Super-rational aspiration
Peer punishment and reward

ABSTRACT

Super-rational aspiration induced strategy updating with exit rights has been considered in some previous studies, in which the players adjust strategies in line with their payoffs and aspirations, and they have access to exit the game. However, exit payoffs for exiting players are automatically allocated, which is clearly contrary to reality. In this study, evolutionary cooperation dynamics with super-rational aspiration and asymmetry in the Prisoner's Dilemma game is investigated, where exit payoffs are implemented by local peers. The results show that for different population structures, the asymmetry of the system is always contributive to the participation of the players. Furthermore, we show that under different exit payoffs, super-rationality and asymmetry are conductive to the evolution of cooperation.

1. Introduction

The existence and stability maintenance mechanisms of cooperative behavior have attracted the attention of scholars in different fields [1–3]. Cooperation requires players to make contributions to the collective. It may cause players to give up their interests in part or in whole, which means a conflict between personal interests and collective welfare. Such conflicts may lead to the disintegration of the cooperation system, which is called "social dilemma" [4–7]. The focus of the question is the conditions of cooperation, that is, under what conditions, players would like to offer public goods (i.e. cooperate) instead of taking the "hitchhiking" behavior (i.e. defect) [8–11]. Five rules including kin selection, direct reciprocity, indirect reciprocity, network reciprocity and group reciprocity have been proposed to explain why cooperation is possible, but these mechanisms cannot cover all situations, that is, the mechanisms for the emergence and maintenance of cooperative behavior are not yet perfect [12,13].

In most previous studies, based on matrix game or public goods game, the interaction between players is set as mandatory [14–17] although this seems not to be in line with reality. Recently, some studies have considered that players have access to exit the game and they get payoffs based on the abundance of public goods [18]. Specifically, the exit players will be punished (or rewarded) if the public goods are not enough (or are abundant), which we call the punishments, or rewards, as the exit payoffs. Although exit payoffs based on public goods seem to be allocated automatically, in fact, it should be implemented by the players. It is particularly important to point out that when all players exit the game, no one will punish or reward them, that is, they will get nothing. Obviously, the exit option will undoubtedly affect the choice of strategy of the players. Therefore, it should be significant to consider the impact of peer-based exit

* Corresponding author.
E-mail address: fengjie_xie@163.com (F.-J. Xie).

https://doi.org/10.1016/j.heliyon.2023.e16729
Received 31 December 2022; Received in revised form 19 May 2023; Accepted 25 May 2023
Available online 1 June 2023

punishment and reward on the evolution of cooperation.

We also noted that most theoretical models and experimental studies on the evolution of cooperation assume that the interaction between players is symmetric, but in real systems it is often asymmetric [19–23]. In addition, most previous game studies have relied on the assumption of complete rationality, which requires all players to have perfect information and cognition, and to choose the strategy to maximize benefits [24–26]. However, this is often impossible. Simon [27] proposed the concept of "bounded rationality" as an alternative, in which players choose satisfactory strategies in the strategic environment based on objective variables, but there is no consensus on the choice of the objective variables [28,29]. Hofstadter [30] proposed the concept of super-rationality. It means that players make decisions based solely on payoffs, and this rule is accepted by all players as a universal law [18,22,31]. Some studies that combined the super-rationality and aspiration level proposed the strategy updating rule induced by super-rational aspiration [18,31]. In this rule, players compare their current factual payoffs with their aspiration levels. If the factual payoffs reach or surpass the aspiration level, they retain the strategies, or they imitate neighbors' strategies. Some studies also considered the impact of super-rational aspiration with exit rights on the evolution of cooperation [18]. However, the effect of super-rational aspiration with peer exit punishment and reward on the evolution of cooperation is still unclear.

The classic evolutionary game theory usually assumes that the population is well-mixed [32–36]. However, with the expansion of population size, we are inevitably faced with the problem of localization, that is, the emergence of spatial structure [37–40]. In this process, a global group is divided into several local groups, and players can only interact with neighbors of the local group. When the players of the local group exit the game, the partners of the local group implement exit punishment (or exit reward) to improve the cooperation rate in the group to solve the "social dilemma".

In this study, based on the asymmetric PD game in different spatial structures, not only the stability of the replicator dynamics in the well-mixed population is investigated, but also the influence of spatial structure on the evolution of cooperation is investigated using Monte Carlo simulations and robustness tests. Further, we used the mean-field theory to approximate the spatial structure and compared the results of different structures. Our main goal is to explore the evolutionary cooperation dynamics of the super-rational aspiration and the peer-based exit payoffs in the asymmetric game, so as to provide new outlooks for solving the "social dilemma".

2. Model and analysis

The social dilemma is generally described by the public goods game (PGG) where N players put resources into a common pool. The resource in the common pool is multiplied and then distributed equally among all players [41,42]. Moreover, for the PD game with pairwise interactions [19,32,33,37,43], a cooperator will pay a cost c and receive a benefit b (nothing) when he meets a cooperator (defector), and a defector will pay nothing and receive benefit b (nothing) when he meets a cooperator (defector). Thus, the payoff of a cooperator in a mutual cooperation should be $b-c$; when a defector interacts with a cooperator, the defector will receive a benefit b (i.e. "temptation to defect") and the cooperator will pay a cost c; and the payoff of a defector in a mutual defection is 0 (see Table 1 for payoff matrix of PD game).

For the asymmetric PD game with exit rights, there are four strategy types: strong/weak cooperator (*SC/ WC*), defector (D), and loner (L) [18]. Cooperators have asymmetries (k) in resource allocation and the higher the asymmetry, the higher the payoffs for strong cooperators and the lower the payoffs for weak cooperators. Since asymmetric systems are assumed to have higher productivity, the payoffs for defectors are higher in asymmetric systems [18,31]. Since the payoff of strong cooperator is higher than that of weak cooperator, the asymmetry degree, denoted by k, is defined in the interval $0 \le k < \frac{1}{2}$, where the case with $k=0$ corresponds to the symmetric PD game. Moreover, in some previous studies [14,15,18], the exit payoff (σ) is automatically distributed based on the abundance of public resources, while this is not in line with the reality that the exit payoffs are implemented by the local-interacting neighbors (i.e. peers). Here we made an improvement that the exit payoffs are implemented by peers. Specifically, if a player exits the game, his partner will implement exit punishment or reward, and they will get the same exit payoff; and if both players exit the game, then they will get nothing because neither of them has the right to implement the exit payoff. The payoff matrix corresponding to the asymmetric PD game with exit rights is given in Table 2.

In Table 2, the interaction between the defector and the loner is considered as a classic coordination game [44]. For the case with exit punishment ($\sigma < 0$), the equilibrium strategy is (D, D) and (L, L), which means that the same strategies are used through the coordination between players. On the other hand, for the case with exit reward ($\sigma > 0$), the equilibrium strategy is (D, L) and (L, D), which means that different strategies are taken through coordination. In $(\cdot, \cdot)$, the first element is the equilibrium strategy taken by player 1 in Table 2, and the second element is the equilibrium strategy taken by player 2. In the coordination game, given a player's strategy, the other player has no incentive to deviate from the equilibrium strategy. Even if the player's strategy is not given, the players always tend to "coordinate" their strategy because the result of coordination is always superior.

Aspiration dynamics was first proposed by Macy and Flache [45]. The basic idea is from the "win-stay-lose-shift" strategy proposed by Nowak and Sigmund, and it is often used to study the evolution of behavior in nature and human society [46–50]. The aspiration

Table 1
Payoff in symmetric prisoner's dilemma game.

	player 2		
player 1		C	D
	C	$b-c$	$-c$
	D	b	0

Table 2
Payoff in asymmetric prisoner's dilemma game with peer exit punishment and reward.

	player 2				
player 1		*SC*	*WC*	*D*	*L*
	SC	$b-c$	$\frac{b-c}{1-2k}$	$\frac{-c}{1+2k}$	σ
	WC	$\frac{b-c}{1+2k}$	$b-c$	$\frac{-c}{1+2k}$	σ
	D	$\frac{b}{1-2k}$	$\frac{b}{1-2k}$	0	σ
	L	σ	σ	σ	0

dynamic model is described as follows: a player has an intrinsic determination called aspiration level to measure whether he is satisfied with the factual payment. The player is more likely to change strategy if the factual payoff does not meet the aspiration level [51–55]. In the model, the parameter A_i represents the aspiration level of player *i*. The parameter A is called super-rationality degree with $A \geq 0$. $P_{i,\max}$ denotes the maximum payoff that the player i may receive [18,31]. Let

$$A_i = (1-A)P_{i,\max} \tag{1}$$

for all possible $i = 1, 2, \ldots$, where the super-rationality degree, A, is uniform in the population. The aspiration level (A_i) is uniform for the same strategic players. In each game round, the player i updates his strategy by comparing his factual payoff P_i with the aspiration level A_i. If $P_i \geq A_i$, then the player will keep his original strategy. Otherwise, if $P_i < A_i$, then the player will imitate the strategy of one of his neighbors. Moreover, for the situation with $A = 0$, all members of the population are completely rational, that is, they are eager to get the maximum payoff (i.e. players have rational aspiration). For $A > 0$, the population is super-rational, and the aspiration level decreases with the increase in parameter A (i.e. players have super-rational aspiration). Specially, $A = 1$ means that the players are dissatisfied with any loss; and $A > 1$ means that the players can withstand some losses.

In this study, we first consider the replicator dynamics in an infinite well-mixed population, and then, the super-rational aspiration-induced strategy updating rule in spatially finite population. Our main goal is to explore the maintenance mechanism of the cooperation system. We noted that the parameter "temptation to defection" b only affects the trajectory and velocity toward the equilibrium point, but does not affect the properties of the equilibrium point. Therefore, we always take $b = 2$ and $c = 1$ in the following analysis.

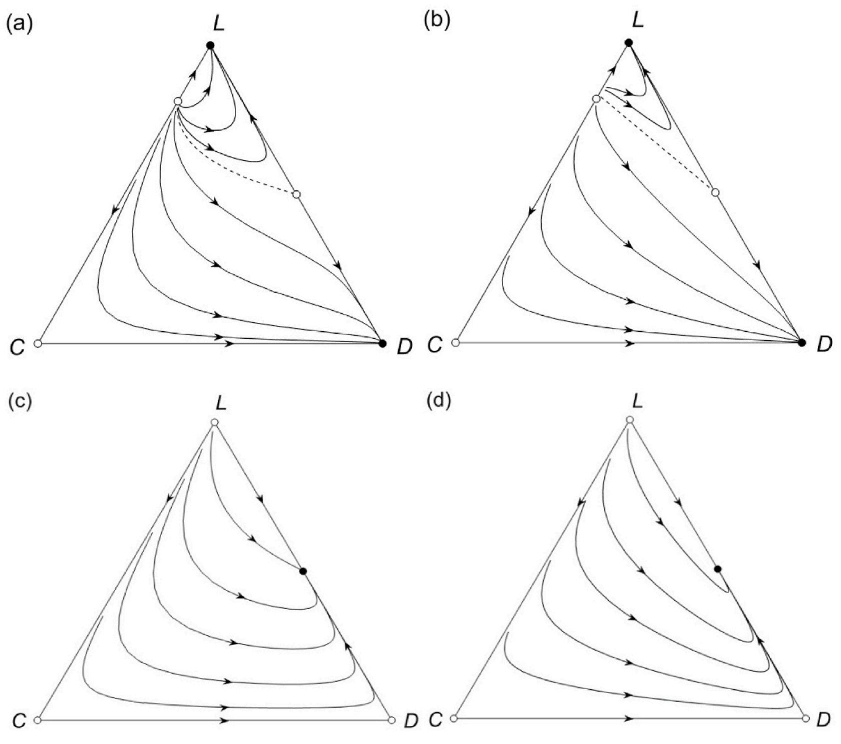

Fig. 1. Phase diagrams of replicator dynamics with peer exit punishment and reward. (a) and (b) $\sigma = -0.3$; (c) and (d) $\sigma = 0.3$; (a) and (c) $k = 0$; (b) and (d) $k = 0.3$. It shows that exit punishment and exit reward have different evolutionary dynamics in essence, while asymmetry affects trajectories but does not affect the properties of equilibrium points.

2.1. Replicator dynamics in a well-mixed population

The core concept of Evolutionary Game Theory is the evolutionarily stable strategy (*ESS*), which gives the precise condition that a strategy is evolutionarily stable [19]. The replicator dynamics proposed by Taylor and Jonker [56] depicts the evolutionary process of the stable strategy. Specifically, the rate of change in frequency of a given strategy depends on the difference between the expected payoff of the strategy and the average payoff of the population [32,33]. This implies that in a large population with size N ($N \to \infty$), the player i compares his factual payoff with that of a randomly chosen player j. Then, the player i updates his strategy by taking j's strategy with a probability proportional to their payoff difference. The stability analysis of replicator dynamics is given in the Supplementary Materials, and the phase diagram is shown in Fig. 1. We can see that *SC* and *WC* will eventually disappear but *WC* converges faster than *SC* (i.e. the proportion of *WC* evolves to zero in a shorter time). To facilitate analysis without ignoring our concerns, we unified *SC* and *WC* as cooperator (*C*).

In Fig. 1(a) and (b), corner *C* is a saddle point (unstable), and corners *D* and *L* are nodes (stable). The equilibrium points on *C-L* and *D-L* sides are saddle points (unstable). In Fig. 1(c) and (d), corners *C* and *D* are saddle points (unstable), while corner *L* is a node (unstable), and the equilibrium point on *D-L* side is a node (stable).

For the case with exit punishment ($-\frac{1}{2} < \sigma < 0$, Fig. 1(a) and (b)), the different initial conditions may lead to that the system state tends to different equilibrium points. On the other hand, for the case with exit reward ($0 < \sigma < \frac{1}{2}$, Fig. 1(c) and (d)), the system state eventually reaches the equilibrium point $E_1\left(0, 0, \frac{1}{2}, \frac{1}{2}\right)$, which indicates that half of the players defect, while the other half exit the game.

To explore the effects of parameters k and σ on the equilibrium points, we chose a fixed initial point near the boundary of the internal attraction region in Fig. 1(a) and (b). The evolutionary outcomes of the system are shown on the k-σ plane (see Fig. 2) [8,57]. We tried other initial points and the results were robust. The results show that for $\sigma > 0$ (exit reward) and possible k values, the system state tends to an equilibrium point $E_1\left(0, 0, \frac{1}{2}, \frac{1}{2}\right)$, which means the coexistence of defectors and loners. For $\sigma < 0$ (exit punishment), the system state tends to $E_4(0, 0, 0, 1)$ if the asymmetry degree is low ($0 \le k \le 0.03$) and to $E_5(0, 0, 1, 0)$ if the asymmetry degree is high ($0.29 \le k \le 0.49$). This means that the population is occupied by the loners or by the defectors. Furthermore, for moderate asymmetry ($0.04 \le k \le 0.28$), the system state in the symmetric system with high exit punishment evolves to $E_4(0, 0, 0, 1)$, and the system state in the asymmetric system with low exit punishment evolves to $E_5(0, 0, 1, 0)$. All of these results (see Fig. 2) strongly suggest that for exit punishment, the low asymmetry (i.e., small k) is more beneficial to the loners, while the high asymmetry (i.e., large k) is more beneficial to the defectors (see Fig. 2). These results are also consistent with some previous conclusions [18,58,59].

The above analysis of replicator dynamics clearly shows that the cooperation has no chance of long-term existence in the population under any conditions. Therefore, we will use super-rational aspiration induced strategy updating in structured populations to explore the conditions of promoting cooperation.

2.2. Evolution of cooperation in the structured population

The network structure has an impact on the outcome of evolution, which is not our focus. For simplicity, here we only consider the asymmetric game with exit punishment and with exit reward on a regular network (the 100×100 spatial structure with periodic boundaries), in which each node is inhabited by a player who uses one of the strategies *SC*, *WC*, *D* or *L*, and interacts with local adjacent players. We take the von Neumann neighborhood with $n = 4$. In this way, the factual payoff of the central player i is the whole payoffs

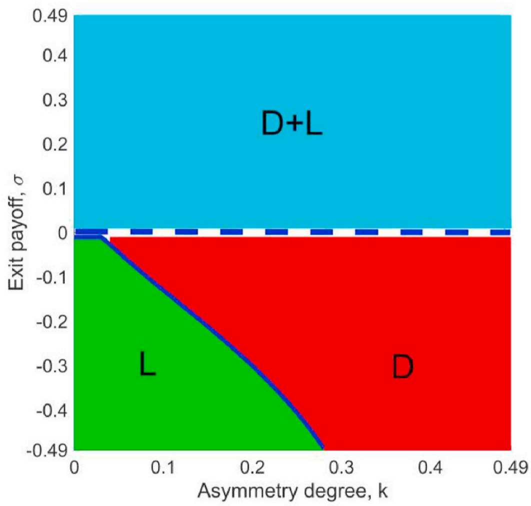

Fig. 2. Different equilibrium states at the same initial point in the parameter region k-σ. The fixed initial proportion is $(0.08, 0.08, 0.1, 0.74)$. It shows that exit punishment and exit reward have different evolutionary outcomes, and asymmetry only affects the evolutionary outcomes with exit punishment.

obtained by interacting with 4 neighbors. This implies that the probability of a player interacting with another one depends not only on the relative frequency in the population, but also on the geometry of the spatial network [38].

A player's aspiration level (A_i) should be proportional to his neighborhood size [18,31]. To include the aspiration level into the payoff values, the super-rationality (A) is set to be $0 \le A \le 1 - \frac{P_{i,\min}}{P_{i,\max}}$, where $P_{i,\min}$ ($P_{i,\max}$) represents the minimum (maximum) payoff that a player may receive. For two extreme cases, $A = 0$ means that players are fully rational and aspire to gain the maximum payoff, while $A = 1 - \frac{P_{i,\min}}{P_{i,\max}}$ means that players are fully super-rational and are satisfied with any potential payoff. When all neighbors of SC and WC display D (or WC), they will get the minimum (or maximum) payoff. If all neighbors of D display D (or C), he will get the minimum (or maximum) payoff. The minimum (or maximum) payoff of L is determined by the abundance of public resources, where he will get $1/2$ when the public resources are sufficient, and $-1/2$ when the public resources are insufficient. Since defectors have the temptation of defection, the super-rationality degree is set to be $A = 0$ (i.e., completely rational).

Since each player has the same quantity of neighbors, the aspiration level A_i depends solely on the strategy type $s_i \in \{SC, WC, D, L\}$. Take B_X to be the aspiration level of every strategy type, where $X \in \{SC, WC, D, L\}$ and $A_i = B_{s_i}$. Specifically, the aspiration level for SC is $B_{SC} = \frac{4(b-c)(1-A)}{1-2k}$, where $0 \le A \le 1 + \frac{c}{b-c}\frac{1-2k}{1+2k}$. The aspiration level for WC is $B_{WC} = 4(b-c)(1-A)$, where $0 \le A \le 1 + \frac{c}{b-c}\frac{1}{1+2k}$. The aspiration level for D is $B_D = \frac{4b}{1-2k}$. The aspiration level for L is $B_L = 2(1-A)$, where $0 \le A \le 2$. Particularly, if B_X is less than or equal to $P_{i,\min}$, then players are satisfied and keep their original strategies. In the simulations, we take $\sigma = -0.3$ (exit punishment) and $\sigma = 0.3$ (exit reward). To allow the aspiration level of every strategy type B_X to cover all possible payoffs, we also take the super-rational degree to be in the interval $0 \le A \le 2$.

We use the synchronous update rule, that is, player i changes his strategy into a randomly chosen neighbor j's strategy with probability α_{ij}, which is the Fermi update rule

$$\alpha_{ij} = \frac{1}{1 + \exp\left[\left(P_i - P_j\right)/K\right]} \tag{2}$$

[60,61], where K depicts the noise effect and $1/K$ is defined as the selection intensity. The weak selection intensity means that players update strategies more randomly and the strong selection intensity means that players are more likely to imitate the strategies of players with higher payoffs. For each set of parameter values, players update strategies for 1000 Monte Carlo steps to make the strategies reach a steady proportion.

We explored player proportions in the parameter region k-A and exit payoffs, where players of different strategies are randomly distributed on the initial lattice. The results show that all players participate but defect at low super-rationality ($0 \le A \le 0.7$). Higher super-rationality and asymmetry contribute to cooperation at the mid level of super-rationality ($0.8 \le A \le 1.5$). Within this parameter range, players all participate in the game with $0.8 \le A \le 1.2$, but in the range of $1.3 \le A \le 1.5$, higher super-rationality lead to a

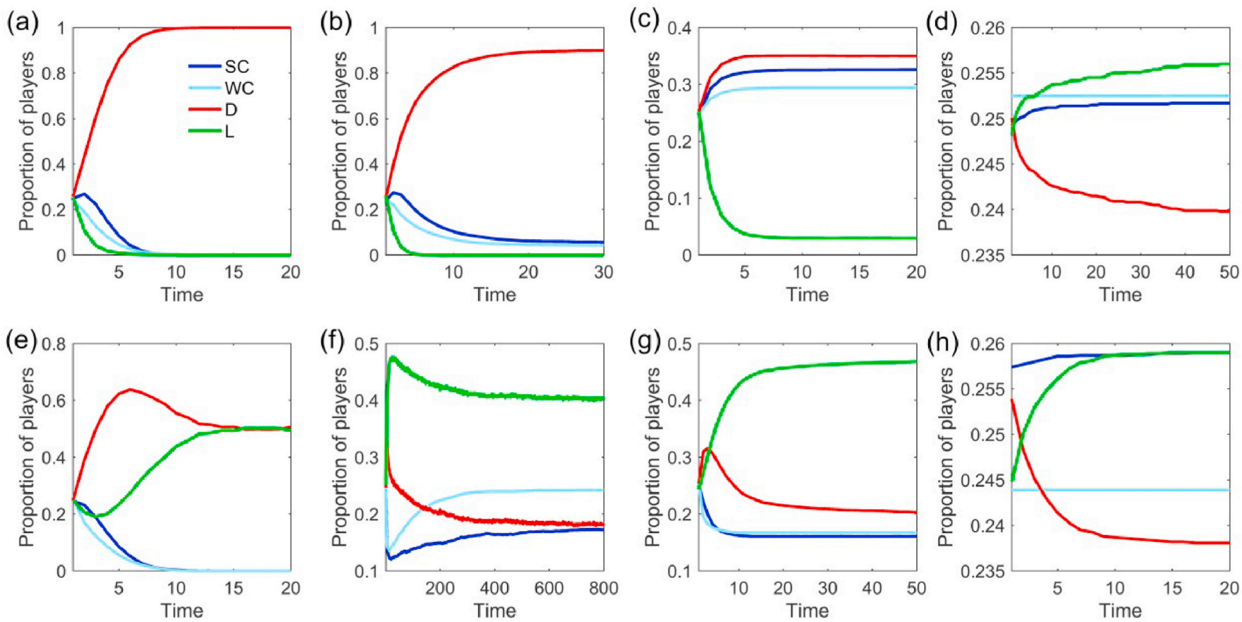

Fig. 3. Time trajectories for possible patterns at the steady-state with exit punishment and reward and different parameter A values, where $k = 0.3$. Blue, light blue, red and green represent SC, WC, D and L. The initial proportion is $(0.25, 0.25, 0.25, 0.25)$. (a)–(d) $\sigma = -0.3$, $A = 0.5, 0.8, 1.3, 1.8$; (e)–(h) $\sigma = 0.3$, $A = 0.5, 0.8, 0.9, 1.8$. It shows that different degrees of super-rationality produce different patterns, and high degrees of super-rationality are contributive to cooperation. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

higher proportion of loners. The initial proportion is remains with $1.6 \le A \le 2$.

With exit reward at $0 \le A \le 0.5$, cooperators disappear, while defectors and loners occupy half of the spatial grid, which is consistent with the conclusions in the well-mixed population. With an increase in super-rationality degree ($0.6 \le A \le 0.7$), the proportion remains at (0,0,0.47,0.53), which indicates that a few players exit the game with the increasing of A. At $A = 0.8$, cooperators arise. The proportion of participating players is higher, but they all defect. The proportion of participating players is lower, but it is more advantageous to promote cooperation. At $A = 0.9$, the proportion of cooperators decreases, and the proportion of loners increases. The proportion of defectors in the asymmetric system ($0.39 \le k \le 0.49$) decreases but in the symmetric system ($0 \le k \le 0.38$) increases. The proportion of cooperators is higher in the asymmetric system, which indicates that asymmetry contributes to cooperation. At $1 \le A \le 2$ (i.e. high super-rationality), the proportion of participating players is higher, which is more beneficial to cooperation.

Since the asymmetry degree has no essential influence on the equilibrium point of the system, we fixed $k = 0.3$ to explore the possible patterns on the spatial lattice when the system state tends to a stable equilibrium for different values of A. The time trajectories for different patterns are shown in Fig. 3.

For the exit punishment with $\sigma = -0.3$, the possible patterns with different levels of super-rationality are shown in Fig. 3(a)–(d). For $A = 0.5$ (see Fig. 3(a)), the defectors occupy the whole lattice, and the system state reaches the steady-state at time step $T = 15$. For $A = 0.8$, (see Fig. 3(b)), SC and WC converge into small gatherings and live together with D. The system state reaches the steady-state of $(0.054, 0.041, 0.905, 0)$ at $T = 26$. For $A = 1.3$ (see Fig. 3(c)), we can see that the total proportion of SC, WC and D increases, while the proportion of D decreases, and the system state reaches a steady state $(0.326, 0.294, 0.35, 0.03)$ at $T = 12$. For $A = 1.8$ (see Fig. 3(d)), the proportion of WC keeps unchanged, the proportions of SC and L increase slightly over time, and the proportion of D decreases over time. At the time step $T = 40$, the system state reaches the steady-state of $(0.252, 0.253, 0.24, 0.255)$.

For the exit reward with $\sigma = 0.3$, the possible patterns with different levels of super-rationality are shown in Fig. 3(e)–(h). For $A = 0.5$ (see Fig. 3(e)), we can see that the total proportion of SC and WC decrease over time until they disappear, and the system reaches $(0, 0, 0.498, 0.502)$ at $T = 16$ as the steady-state. The defectors and loners on the lattice form into “strips” and each of them occupies half of the population. This result is consistent with the conclusion in the well-mixed population. For $A = 0.8$ (see Fig. 3(f)), we can see that the total proportion of SC and WC decreased first and then increased, while the total proportion of D and L increased first and then decreased. The system state reaches a steady-state $(0.17, 0.242, 0.184, 0.404)$ at $T = 600$, where different strategies form into “strips” and coexist. For $A = 0.9$ (see Fig. 3(g)), we can see that the total proportion of SC and WC decreased, the proportion of D first increased and then decreased, and the proportion of L increased with time. The system state reaches a steady-state $(0.161, 0.167, 0.225, 0.447)$ at $T = 14$. For $A = 1.8$ (see Fig. 3(h)), SC increases slightly over time, WC remains unchanged, D decreases, and L increases. The system state reaches $(0.259, 0.244, 0.238, 0.259)$ at $T = 15$ as the steady-state.

Note that a defector as a central player will be satisfied if his four neighbors are all cooperators (SC or WC); otherwise, he will be dissatisfied, but will not imitate the neighbor’s strategy. This implies that for a defector, if his randomly chosen neighbor is also a defector, then he will not need to imitate this neighbor’s strategy; however, if the randomly chosen neighbor is not a defector, he will also not imitate this neighbor’s strategy because the payoff of D is at least not lower than that of other strategies in any case. Therefore, the evolution of SC and WC on the spatial lattice is mainly determined by the initial spatial distribution and the randomness of the strategy updating rule.

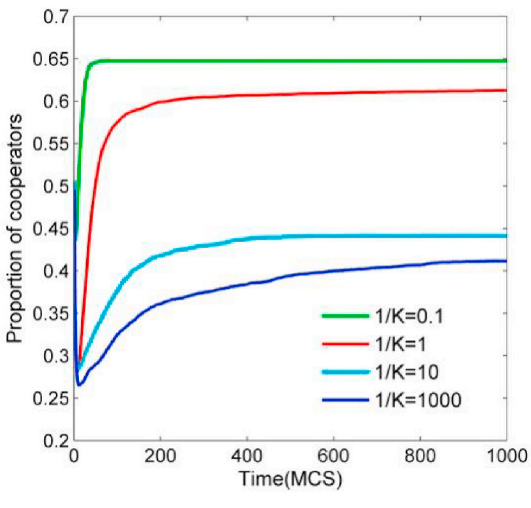

Fig. 4. Time trajectories for the proportion of cooperators with different selection intensities, where $\sigma = -0.3$, $A = 1.3$, $k = 0.3$. Green, red, cyan and blue lines represent selection intensities of $1/K = 0.1, 1, 10, 1000$, respectively. Players are randomly distributed on the lattice initially. It shows that low selection intensity is contributive to cooperation. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

2.3. Robustness test with different selection intensities

In the above subsection, the selection intensity of the Fermi update rule is taken as $1/K = 10$. Here, to verify the effects of different selection intensities on the evolution of cooperation, we take $1/K = 2$ and $1/K = 100$. This implies that under weak selection, the probability of a player imitating his neighbor's strategy should be close to 1/2; however, as the selection intensity increases, the probability of a player imitating the strategy of the neighbor with high (or low) payoff will also increase (or decrease). The robustness with the exit punishment and with the exit reward on the *k*-*A* parameter plane is verified. The results show that for both the exit punishment and exit reward, the basic pattern of the system state on the parameter plane is similar, but the proportions of different strategies at steady-state is different, that is, the selection intensities have only a quantitative effect on the system state, but not a qualitative effect. Furthermore, in order to explore whether the selection intensity affects the evolution rate of the system, the time trajectories of system state for different selection intensities are shown in Fig. 4, where $k = 0.3$, $A = 1.3$, and the selection intensities are taken as $1/K = 0.1, 1, 10$ and 1000, respectively. We can see that corresponding to these different selection intensities, the total proportions of *SC* and *WC* are 0.6472, 0.6126, 0.4411 and 0.4115, respectively at the steady-state. This implies that the low selection intensity should be better for the evolution of cooperation because the defectors are "tempted to defect" and they are never satisfied with their payoffs. However, for situations with high selection intensity, the defectors should be less likely to imitate the cooperators.

2.4. Mean-field approximation of the spatially structured population

Here, we use the mean-field theory to approximate the model in a spatially structured population. Specifically, we assume that the probability that a player imitates the strategy of one of his neighbors is independent of his other neighbors' payoffs. Based on this assumption, the local density and other probability measures of each player's neighbors are considered to be the same as the global average [62–64].

The transition probability of an *X* strategy type player to a *Y* strategy type player is

$$W_{X\rightarrow Y} = \sum_{\substack{M=0 \\ (P_{X|M} < B_X)}}^{m-1} \binom{n}{n_{SC}, n_{WC}, n_D, n_L} x_1^{n_{SC}} x_2^{n_{WC}} x_3^{n_D} x_4^{n_L} \frac{n_{Y|M}}{n} \frac{1}{1+\exp\left[\left(P_{X|M} - \overline{P}_{Y|n_X \geq 1}\right)/K\right]} \tag{3}$$

[18,31], where m is the number of possible neighbor patterns ($m = 35$ in our model); M denotes the M-th neighbor pattern of the central player; $P_{X|M}$ is the payoff of strategy Y in the M-th neighbor pattern, n_X is the number of the neighbors using strategy X with $n_{SC} + n_{WC} + n_D + n_L = n$; x_X is the proportion of strategy X ($0 \leq x_X \leq 1, X \in \{SC, WC, D, L\}$); $\frac{n_{Y|M}}{n}$ is the probability that a neighbor using strategy Y is chosen in the M-th neighbor pattern; and $\overline{P}_{Y|n_X \geq 1}$ is the average payoff of all possible neighbor patterns that satisfy $n_X \geq 1$ for player displaying Y and $\overline{P}_{Y|n_X \geq 1} = \frac{1}{N_{n_X \geq 1}} \sum_{\substack{M=0 \\ n_X \geq 1}}^{m-1} P_{Y|M}$.

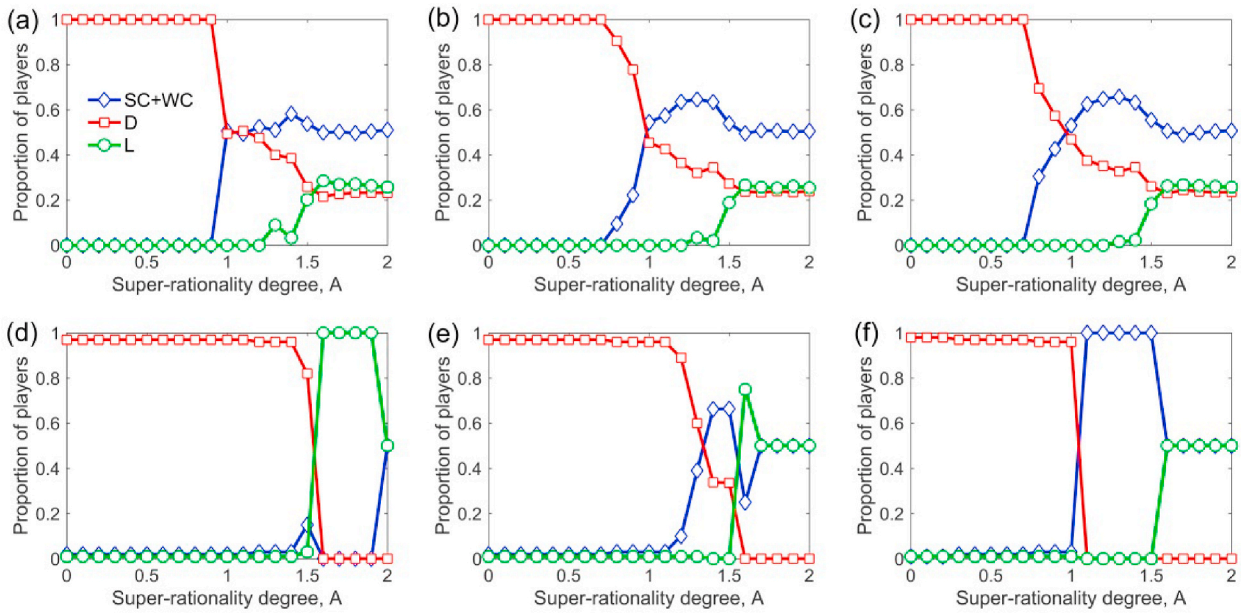

Fig. 5. Player proportions at the steady-state with exit punishment and different *A*-*k* values, where $\sigma = -0.3$. (a)–(c) spatial simulation; (d)–(f) mean-field; (a) and (d) $k = 0$; (b) and (e) $k = 0.25$; (c) and (f) $k = 0.49$. It shows that the conclusion is similar in different population structures; Super-rationality and asymmetry have promotion effect to cooperation.

Based on the mean-field approximation, the time evolution of the proportions of SC, WC, D and L, denoted by x_{SC}, x_{WC}, x_D, and x_L, respectively, can be given by

$$\begin{cases} \dfrac{dx_{SC}}{dt} = x_{WC}W_{WC\to SC} + x_D W_{D\to SC} + x_L W_{L\to SC} - x_{SC}(W_{SC\to WC} + W_{SC\to D} + W_{SC\to L}) \triangleq f_1(x) \\ \dfrac{dx_{WC}}{dt} = x_{SC}W_{SC\to WC} + x_D W_{D\to WC} + x_L W_{L\to WC} - x_{WC}(W_{WC\to SC} + W_{WC\to D} + W_{WC\to L}) \triangleq f_2(x) \\ \dfrac{dx_D}{dt} = x_{SC}W_{SC\to D} + x_{WC} W_{WC\to D} + x_L W_{L\to D} - x_D(W_{D\to SC} + W_{D\to WC} + W_{D\to L}) \triangleq f_3(x) \\ \dfrac{dx_L}{dt} = x_{SC}W_{SC\to L} + x_{WC} W_{WC\to L} + x_D W_{D\to L} - x_L(W_{L\to SC} + W_{L\to WC} + W_{L\to D}) \triangleq f_4(x) \end{cases} \tag{4}$$

Note that a stable equilibrium of this system, denoted by $(x_{SC}, x_{WC}, x_D, x_L)$, must be the solution of equation $f_1(x) = f_2(x) = f_3(x) = f_4(x) = 0$. Thus, we can use the stable equilibrium of Eq. (4) to compare it with the simulation results of the spatial lattice. In the calculation, when some types of strategies coexist, the player proportion fluctuates slightly when they reach the steady-state. We chose the solution that satisfies the restriction condition of $\min(f_1^2(x) + f_2^2(x) + f_3^2(x) + f_4^2(x))$ as the solution $(x_{SC}, x_{WC}, x_D, x_L)$ of the system [65]. When there were multiple steady-state solutions, we use the average values of possible solution values in the analysis. For the mean-field approximation and the simulations based on the spatial lattice, under the different parameters of exit punishment and exit reward, the steady-state proportions of strategies are presented in Figs. 5 and 6. For the simulations on the spatial lattice, the players are randomly and uniformly distributed initially.

For $\sigma = -0.3$ and different A values and k values, the simulation results show that for small A and possible k ($0 \le A \le 0.9$ in Fig. 5(a), $0 \le A \le 0.7$ in Fig. 5(b) and in Fig. 5(c)), the defectors almost occupy the whole population (see Fig. 5(a)–(c)); and for large A and possible k, the proportion of the cooperators is larger than that of the defectors and loners, and the coexistence of different strategies appears (see also Fig. 5(a)–(c)). This implies that in the structured population with exit punishment, the large super-rationality degree is not only conductive to the long-term presence of cooperators in the population, but also conductive to the coexistence of different strategies. Similarly, the analysis results based on the mean-field approximation show that for small A and possible k ($0 \le A \le 1.4$ in Fig. 5(d), $0 \le A \le 1.1$ in Fig. 5(e) and $0 \le A \le 1$ in Fig. 5(f)), the defectors almost occupy the whole population (see Fig. 5(d)–(f)); and, however, for large A and possible k, the coexistence of the cooperators and loners appears and the defectors disappear (see also Fig. 5(d)–(f)). Therefore, for evolutionary dynamics with exit punishment based on the mean-field approximation, the large degree of super-rationality is more contributive to the evolution of cooperation. This result is consistent with the simulation result on the spatial lattice.

Similarly, for $\sigma = 0.3$ and different A values and k values, the simulation results show that for small A and possible k ($0 \le A \le 0.7$), the coexistence of the defectors and loners appears and the cooperators disappear (see Fig. 6(a)-(c)); and for large A and possible k ($0.8 \le A \le 1.4$), the coexistence of the cooperators, defectors and loners appears (see also Fig. 6(a)–(c)). This implies that, similar to the situation with exit punishment, in the structured population with exit reward, the large super-rationality degree is also conductive

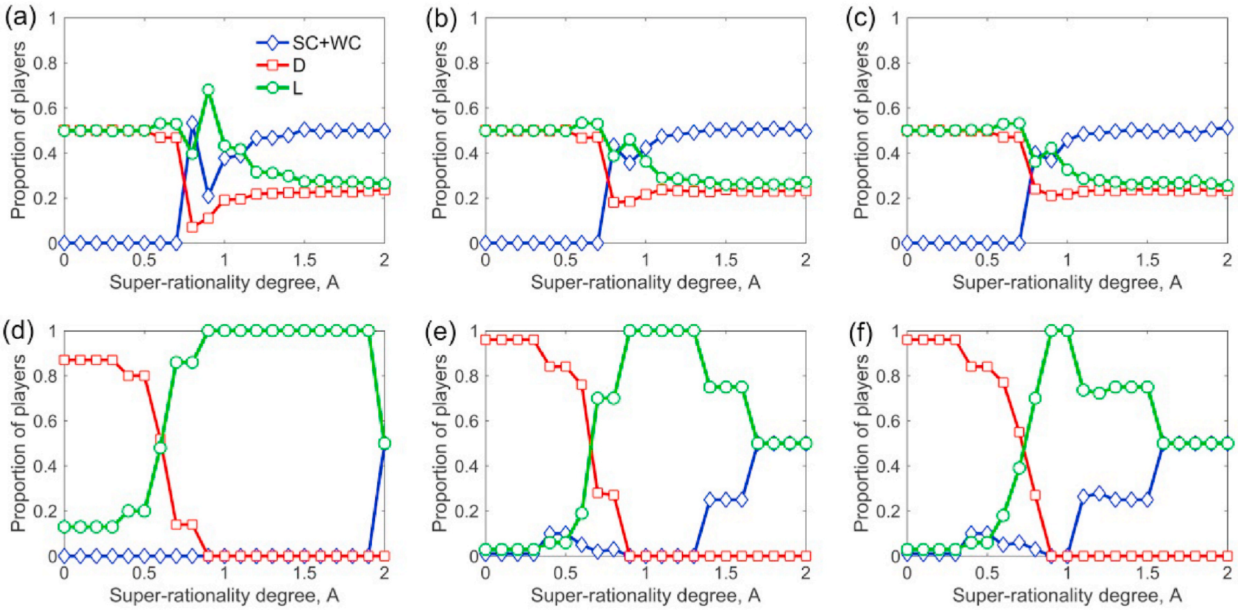

Fig. 6. Player proportions at the steady-state with exit reward and different A-k parameters, where $\sigma = 0.3$. (a)–(c) spatial simulation; (d)–(f) mean-field; (a) and (d) $k = 0$; (b) and (e) $k = 0.25$; (c) and (f) $k = 0.49$. It shows that the conclusion is similar in different population structures; Super-rationality and asymmetry have promotion effect to cooperation.

to the coexistence of different strategies. In addition, asymmetry helps increase player participation, but only at moderate levels of super-rationality. The analysis results based on the mean-field approximation show that for small A and possible k ($0 \leq A \leq 0.3$), the cooperators disappear (see Fig. 6(d)–(f)); and, however, for large A and possible k, the coexistence of the cooperators and loners appears and the defectors disappear (see also Fig. 6(d)–(f)). It is easy to see that for evolutionary dynamics with exit reward based on the mean-field approximation, the large degree of super-rationality contributes to the long-term presence of cooperation. This result is consistent with the simulation result for the spatial lattice.

The above results strongly suggest that for both the exit punishment and exit reward and the possible k values, a large degree of super-rationality is always conducive to the evolution of cooperation.

Comparing two groups of results in Figs. 5 and 6, we obtained the same conclusions in the structured and well-mixed population, which further validates the reliability of the conclusion. However, the promoting effect is stronger in the structured population, which indicates that the geometric limitation of the spatial structure makes the cooperators form clusters to resist the invasion of other strategic players, thus having the promotion effect on cooperation. Similar observations were made in the last two decades, which is called network reciprocity.

3. Conclusion and analysis

In this paper, the effects of super-rational aspiration induced updating rule and peer exit punishment and reward on the evolution of cooperation are investigated, in which the asymmetric PD game in well-mixed and structured populations is considered. For the well-mixed population, the replicator dynamics with exit punishment has two stable equilibrium states called “all D” and “all L″, respectively, and the evolutionary outcome of the system depends on its initial state. However, the replicator dynamics with exit reward has only one stable equilibrium state, in which the defectors and loners account for half of the population. These results differ from previous studies because of the different rules of the game [41]. The asymmetry degree (k) of the system affects the trajectory and velocity at which the system state tends to the stable equilibrium point. The low asymmetry may cause the players to exit the game, while the high asymmetry may promote the players to participate in the game.

In order to further explore the cooperation evolution, super-rational aspiration induced strategy updating on spatial networks is introduced. For low exit punishment, the defectors will occupy the entire lattice with low super-rationality, while high super-rationality and asymmetry are both contributive to cooperation. For the same super-rationality degree, the increase in exit punishment is contributive to cooperation, and for the same exit punishment, the mid super-rationality is more contributive to cooperation. For the exit reward, the defectors and loners occupy half of the population, respectively, if the degree of super-rationality is low, which is consistent with the results in the well-mixed population. However, for the arbitrary exit reward, the increase of super-rationality should be contributive to cooperation.

The robustness test shows that the selection intensity may affect the proportions of different strategies at the equilibrium, but it cannot affect the property of the equilibrium. Furthermore, the low selection intensity should be contributive to cooperation. On the other hand, the conclusion of the mean-field approximation is similar to that of the structured population, which verifies the reliability of the conclusion that super-rationality and asymmetry promote cooperation, and asymmetry improves the participation rate of the population. In addition, super-rational aspiration has positive significance for maintaining the diversity of strategies in the system.

In summary, the asymmetry is beneficial to player participation for different population structures. Super-rationality and asymmetry have promotion effect on the evolution of cooperation in structured populations. The effects of exit reward and exit punishment on the evolutionary cooperation dynamics are different. Therefore, the exit punishment or reward based on the abundance of public resources and the super-rationality degree of the system can maximize the level of cooperation.

Historically, the human population was small in primitive hunter-gatherer societies, where players had the same interacting probability and it is similar to the well-mixed population. After the agricultural revolution, the scale of human social groups became larger due to increased productivity and settlement, which inevitably led to the emergence of localized villages and communities [66]. Subsequently, punishments and rewards are introduced to increase productivity, which is based on the abundance of common resources. However, it cannot be distributed automatically and needs to be implemented by players, which requires costs and limits the number of implementing players and is also naturally limited by localization. For example, players in a village or community can usually implement punishment or reward to their neighbors because of geopolitical isolation, which is the practical significance of introducing spatial structure in the model. Therefore, the more players commit to a public project, the harder it is to adequately punish or reward those “free-riders”. Based on the above arguments, we combined the localized spatial structure with peer exit punishment and reward in the model is more realistic. On this basis, we discussed the impact of super-rationality and asymmetry on the evolution of cooperation with peer exit punishment and reward and the extent to which they can help solve the global social dilemma [67,68].

In this paper, we considered the uniform super-rational aspiration. It is worth further studying heterogeneous aspirations and other types of network structures on the evolution of cooperation. In addition, we considered adequate or inadequate resources in the model, but ignored the impact of environmental random fluctuations on the abundance of public resources, which is worth further exploring. In addition, a node in the structured population is assumed to represent a player and one unit of public resources, but in reality, players and public resources are often separated, such as natural resources (minerals, fisheries), roads, parks, and other public facilities. Elinor Ostrom gave an empirical conclusion: if the organization is relatively small and players live near the common pool for long-term residential, the cooperative organization will be formed [69]. Therefore, we can further consider the impact of player spatial distribution and resource heterogeneity on the evolution of cooperation, which has practical guiding significance to the sharing economy. In addition, it is of enlightening significance to the economic growth of remote areas that lack capital and industrial clusters.

Author contribution statement

Si-Yi Wang: Conceived and designed the experiments; Wrote the paper.
Xin Yao, Yi-Mei Yang: Performed the experiments.
Daniel Chen: Analyzed and interpreted the data.
Rui-Wu Wang, Feng-Jie Xie: Contributed reagents, materials, analysis tools or data.

Data availability statement

No data was used for the research described in the article.

Additional information

No additional information is available for this paper.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

The work is supported by the Natural Science Basic Research Plan in Shaanxi Province of China (2023-JC-QN-0261), the NSFC-Yunnan Joint Fund (U2102221), and the Shaanxi Provincial Education Department (No. 22JK0563).

Appendix B. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.heliyon.2023.e16729.

References

[1] A.M. Colman, The puzzle of cooperation, Nature 440 (7085) (2006) 744–745.
[2] N.J. Raihani, A. Thornton, R. Bshary, Punishment and cooperation in nature, Trends Ecol. Evol. 27 (5) (2012) 288–295.
[3] I.P.A. Lee, O.T. Eldakar, J.P. Gogarten, C. P Andam, Bacterial cooperation through horizontal gene transfer, Trends Ecol. Evol. (2021) 223–232.
[4] R.M. Dawes, Social dilemmas, Annu. Rev. Psychol. 31 (1) (1980) 169–193.
[5] Z. Wang, M. Jusup, R.W. Wang, et al., Onymity promotes cooperation in social dilemma experiments, Sci. Adv. 3 (3) (2017), e1601444.
[6] J. Tanimoto, H. Sagara, Relationship between dilemma occurrence and the existence of a weakly dominant strategy in a two-player symmetric game, Biosystems 90 (1) (2007) 105–114.
[7] Z. Wang, S. Kokubo, M. Jusup, et al., Universal scaling for the dilemma strength in evolutionary games, Phys. Life Rev. 14 (2015) 1–30.
[8] D. Helbing, A. Szolnoki, M. Perc, et al., Evolutionary establishment of moral and double moral standards through spatial interactions, PLoS Comput. Biol. 6 (4) (2010), e1000758.
[9] A. Szolnoki, M. Perc, Evolutionary advantages of adaptive rewarding, New, J. Phys. 14 (9) (2012), 093016.
[10] A. Szolnoki, M. Perc, Correlation of positive and negative reciprocity fails to confer an evolutionary advantage: phase transitions to elementary strategies, Phys. Rev. X 3 (4) (2013), 041021.
[11] M. Perc, J.J. Jordan, D.G. Rand, et al., Statistical physics of human cooperation, Phys. Rep. 687 (2017) 1–51.
[12] M.A. Nowak, Five rules for the evolution of cooperation, Science 314 (5805) (2006) 1560–1563.
[13] H. Ito, J. Tanimoto, Scaling the phase-planes of social dilemma strengths shows game-class changes in the five rules governing the evolution of cooperation, R. Soc. Open Sci. 5 (10) (2018), 181085.
[14] G. Szabó, C. Hauert, Phase transitions and volunteering in spatial public goods games, Phys. Rev. Lett. 89 (11) (2002), 118101.
[15] H.C. Jeong, S.Y. Oh, B. Allen, M.A. Nowak, Optional games on cycles and complete graphs, J. Theor. Biol. 356 (2014) 98–112.
[16] D. Jia, C. Shen, H. Guo, et al., The impact of loners' participation willingness on cooperation in voluntary prisoner's dilemma, Chaos, Solitons Fract. 108 (2018) 218–223.
[17] H. Guo, Z. Song, S. Geček, et al., A novel route to cyclic dominance in voluntary social dilemmas, J. R. Soc. Interface 17 (164) (2020), 20190789.
[18] S.Y. Wang, Y.P. Liu, M.L. Li, et al., Super-rational aspiration induced strategy updating helps resolve the tragedy of the commons in a cooperation system with exit rights, Biosystems 208 (2021), 104496.
[19] J.M. Smith, Evolution and the Theory of Games, Cambridge university press, 1982.
[20] R. Bshary, A.S. Grutter, Asymmetric cheating opportunities and partner control in a cleaner fish mutualism, Anim. Behav. 63 (3) (2002) 547–555.
[21] R.W. Wang, J.Z. He, Y.Q. Wang, et al., Asymmetric interaction will facilitate the evolution of cooperation, Sci. China E 53 (8) (2010) 1041–1046.
[22] J.Z. He, R.W. Wang, X.J. Jensen Christopher, et al., Asymmetric interaction paired with a super-rational strategy might resolve the tragedy of the commons without requiring recognition or negotiation, Sci. Rep. 5 (1) (2015) 7715.
[23] H. Guo, X. Li, K. Hu, et al., The dynamics of cooperation in asymmetric sub-populations, New J. Phys. 22 (8) (2020), 083015.
[24] A. Smith, The Wealth of Nations: an Inquiry into the Nature and Causes of the Wealth of Nations, Harriman House Limited, 2010.
[25] M.A. Javarone, F. Battiston, The role of noise in the spatial public goods game, 2016, J. Stat. Mech-Theory. E 7 (2016), 073404.
[26] M.A. Amaral, M.A. Javarone, Strategy equilibrium in dilemma games with off-diagonal payoff perturbations, Phys Rev. E 101 (6) (2020), 062309.
[27] H.A. Simon, Models of Man; Social and Rational, Wiley Press, 1957.
[28] R. Radner, Bounded rationality, indeterminacy, and the theory of the firm, Econ. J. 106 (438) (1996) 1360–1373.
[29] D. Kahneman, A perspective on judgment and choice: mapping bounded rationality, Am. Psychol. 58 (9) (2003) 697.

[30] D.R. Hofstadter, The calculus of cooperation is tested through a lottery, Sci. Am. 248 (6) (1983) 14.
[31] S.Y. Wang, Y.P. Liu, F. Zhang, et al., Super-rational aspiration induced strategy updating promotes cooperation in the asymmetric prisoner's dilemma game, Appl. Math. Comput. 403 (2021), 126180.
[32] J.W. Weibull, Evolutionary Game Theory, MIT press, 1997.
[33] J. Hofbauer, K. Sigmund, Evolutionary Games and Population Dynamics, Cambridge university press, 1998.
[34] C.P. Roca, J.A. Cuesta, A. Sánchez, Evolutionary game theory: temporal and spatial effects beyond replicator dynamics, Phys. Life Rev. 6 (4) (2009) 208–249.
[35] J.Z. He, R.W. Wang, Y.T. Li, Evolutionary stability in the asymmetric volunteer's dilemma, PLoS One 9 (8) (2014), e103931.
[36] J. Tanimoto, Sociophysics Approach to Epidemics, Springer, Singapore, 2021.
[37] M.A. Nowak, R.M. May, Evolutionary games and spatial chaos, Nature 359 (6398) (1992) 826–829.
[38] C. Hauert, M. Doebeli, Spatial structure often inhibits the evolution of cooperation in the snowdrift game, Nature 428 (6983) (2004) 643–646.
[39] C.P. Roca, J.A. Cuesta, A. Sánchez, Effect of spatial structure on the evolution of cooperation, Phys. Rev. E. 80 (4) (2009), 046106.
[40] Q. Su, A. Li, L. Wang, et al., Spatial reciprocity in the evolution of cooperation, 1900, P. Roy. Soc. Biol. Sci. 286 (2019), 20190041.
[41] C. Hauert, S.D. Monte, J. Hofbauer, et al., Volunteering as red queen mechanism for cooperation in public goods games, Science 296 (5570) (2002) 1129–1132.
[42] J.M. Du, Redistribution promotes cooperation in spatial public goods games under aspiration dynamics, Appl. Math. Comput. 363 (2019), 124629.
[43] X.S. Liu, M. F He, Y. Kang, et al., Aspiration promotes cooperation in the prisoner's dilemma game with the imitation rule, Phys. Rev. E 94 (1) (2016), 012124.
[44] Y.T. Bian, L. Xu, J.S. Li, Evolving dynamics of trading behavior based on coordination game in complex networks, Physica A 449 (2016) 281–290.
[45] M.W. Macy, A. Flache, Learning dynamics in social dilemmas, Proc. Natl. Acad. Sci. USA 99 (2002) 7229–7236.
[46] M. Nowak, K. Sigmund, A strategy of win-stay, lose-shift that outperforms tit-for-tat in the Prisoner's Dilemma game, Nature 364 (6432) (1993) 56–58.
[47] M.A. Amaral, L. Wardil, M. Perc, et al., Stochastic win-stay-lose-shift strategy with dynamic aspirations in evolutionary social dilemmas, Phys. Rev. E. 94 (3) (2016), 032317.
[48] L.A. Imhof, D. Fudenberg, M.A. Nowak, Tit-for-tat or win-stay, lose-shift? J. Theor. Biol. 247 (3) (2007) 574–580.
[49] X. Chen, L. Wang, Promotion of cooperation induced by appropriate payoff aspirations in a small-world networked game, Phys. Rev. E. 77 (1) (2008), 017103.
[50] X. Chen, F. Fu, L. Wang, Promoting cooperation by local contribution under stochastic win-stay-lose-shift mechanism, Physica. A. 387 (22) (2008) 5609–5615.
[51] H.X. Yang, Z.X. Wu, B.H. Wang, Role of aspiration-induced migration in cooperation, Phys. Rev. E. 81 (6) (2010), 065101.
[52] Y. Liu, X. Chen, L. Wang, et al., Aspiration-based learning promotes cooperation in spatial prisoner's dilemma games, EPL 94 (6) (2011), 60002.
[53] Y. Peng, X.W. Wang, Q. Lu, et al., Effects of aspiration-induced adaptation and migration on the evolution of cooperation, Int. J. Mod. Phys. C 25 (7) (2014), 1450025.
[54] Z.H. Rong, Q. Zhao, Z.X. Wu, et al., Proper aspiration level promotes generous behavior in the spatial prisoner's dilemma game, Eur. Phys. J. B 89 (7) (2016) 166.
[55] T. Płatkowski, P. Bujnowski, Cooperation in aspiration-based n-person prisoner's dilemmas, Phys. Rev. E. 79 (3) (2009), 036103.
[56] P.D. Taylor, L.B. Jonker, Evolutionary stable strategies and game dynamics, Math. Biosci. 40 (1–2) (1978) 145–156.
[57] A. Szolnoki, G. Szabó, M. Perc, Phase diagrams for the spatial public goods game with pool punishment, Phys. Rev. E. 83 (3) (2011), 036101.
[58] W.B. MacLeod, The Role of exit costs in the theory of cooperative teams: a theoretical perspective, J. Comp. Econ. 17 (2) (1993) 521–529.
[59] C. Shen, M. Jusup, L. Shi, et al., Exit rights open complex pathways to cooperation, J. R. Soc. Interface. 18 (174) (2021), 20200777.
[60] A. Szolnoki, M. Perc, G. Szabó, Topology-independent impact of noise on cooperation in spatial public goods games, Phys. Rev. E. 80 (5) (2009), 056109.
[61] K. Nagashima, J. Tanimoto, A stochastic pairwise fermi rule modified by utilizing the average in payoff differences of neighbors leads to increased network reciprocity in spatial prisoner's dilemma games, Appl. Math. Comput. 361 (2019) 661–669.
[62] M. Nakamaru, H. Matsuda, Y. Iwasa, The evolution of cooperation in a lattice-structured population, J. Theor. Biol. 184 (1) (1997) 65–81.
[63] L.P. Kadanoff, More is the same; phase Transitions and mean field theories, J. Stat. Phys. 137 (5–6) (2009) 777–797.
[64] J. Zhang, Y.P. Fang, W.B. Du, et al., Promotion of cooperation in aspiration-based spatial prisoner's dilemma game, Physica. A. 390 (12) (2011) 2258–2266.
[65] H.T. Jongen, K. Meer, E. Triesch, Optimization Theory, Springer Science & Business Media, 2007.
[66] M.A. Javarone, D. Marinazzo, Evolutionary dynamics of group formation, PLoS One 12 (11) (2017), e0187960.
[67] J.M. Diamond, Guns Germs and Steel: a Short History of Everybody for the Last 13,000 Years, Random House, 1998.
[68] H. Ozono, Y. Kamijo, K. Shimizu, The role of peer reward and punishment for public goods problems in a localized society, Sci. Rep. 10 (1) (2020) 1–8.
[69] E. Ostrom, Governing the Commons: the Evolution of Institutions for Collective Action, Cambridge university press, 1990.