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Research article

Decision analysis with IDOCRIW-QUALIFLEX approach in the 2TLq-ROF environment: An application of accident prediction models in Pakistan

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ABSTRACT

In Pakistan, the assessment of road safety measures within road safety management systems is commonly seen as the most deficient part. Accident prediction models are essential for road authorities, road designers, and road safety specialists. These models facilitate the examination of safety concerns, the identification of safety improvements, and the projection of the potential impact of these modifications in terms of collision reduction. In the context described above, the goal of this paper is to utilize the 2-tuple linguistic q-rung orthopair fuzzy set (2TLq-ROFS), a new and useful decision tool with a strong ability to address uncertain or imprecise information in practical decision-making processes. In addition, for dealing with the multi-attribute group decision-making problems in road safety management, this paper proposes a new 2TLq-ROF integrated determination of objective criteria weights (IDOCRIW)-the qualitative flexible multiple criteria (QUALIFLEX) decision analysis method with a weighted power average (WPA) operator based on the 2TLq-ROF numbers. The IDOCRIW method is used to calculate the weight of attributes and the QUALIFLEX method is used to rank the options. To show the viability and superiority of the proposed approach, we also perform a case study on the evaluation of accident prediction models in road safety management. Finally, the results of the experiments and comparisons with existing methods are used to explain the benefits and superiority of the suggested approach. The findings of this study show that the proposed approach is more practical and compatible with other existing approaches.

1. Introduction

The assessment of road safety measures seems to be the least robust element within Pakistan's road safety management (RSM) systems. The integration of road safety measures into the cultural fabric and routine operations of road safety programs, including the allocation of dedicated financial resources is observed in only a limited number of countries. In instances where this scenario

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Table 1
Some existing research work about RSM.

Reference	Approach	Application
Deveci et al. [1]	Fuzzy Einstein-based logarithmic methodology of additive weights and TOPSIS method	Evaluation of metaverse traffic safety implementations
Benallou et al. [2]	Fuzzy Bayesian approach	Evaluation of the accidents risk caused by truck drivers
Xie et al. [3]	Fuzzy logic	Comprehensive evaluation of freeway driving risks
Gaber et al. [4]	Fuzzy logic algorithm	Analysis and modeling of rural roads traffic safety data
Garnaik et al. [5]	Fuzzy inference system	Impact of highway design on traffic safety
Mohammadi et al. [6]	Fuzzy-analytical hierarchy process	Pedestrian road traffic accidents in metropolitan areas
Jafarzadeh Ghoushchi et al. [7]	Integrated SWARA and MARCOS approaches under spherical fuzzy environment	Road safety assessment and risks prioritization
Gecchele et al. [8]	Fuzzy Delphi analytic hierarchy process	A flexible approach to select road traffic counting locations
Ahammad et al. [9]	Crash prediction model technique	A novel approach to avoid road traffic accidents and develop safety rules for traffic
Stevic et al. [10]	Integrated multi-criteria decision-making model	Evaluation of two-lane road sections in terms of traffic risk
Koçar et al. [11]	Fuzzy logic	A risk assessment model for traffic crashes problem
Mitrovic Simic et al. [12]	A novel CRITIC-fuzzy full consistency method-data envelopment analysis-fuzzy MARCOS model	Safety evaluation of road sections based on geometric parameters of road
Zaranezhad et al. [13]	Artificial neural network, fuzzy system, genetic algorithm, and ant colony optimization algorithm	Development of prediction models for repair and maintenance-related accidents at oil refineries
Al-Omari et al. [14]	Fuzzy logic and geographic information system	Prediction of traffic accidents hot spots
Cubranic-Dobrodolac et al. [15]	Using the interval type-2 fuzzy inference systems	Compare the impact of speed and space perception on the occurrence of road traffic accidents

arises, the examination typically focuses on matters about infrastructure and the implementation of legal measures, while comprehensive assessments of road safety programs are notably rare. Prediction tools, referred to as accident prediction models (APMs) are essential for road authorities, designers, and road safety practitioners. These tools are used to analyze safety issues, identify potential improvements, and estimate the anticipated safety outcomes of these enhancements. Their purpose is to enhance the management and safety of road infrastructure. APMs have been shown to be essential instruments in the field of RSM. These complex analytical frameworks use historical data, traffic patterns, weather conditions, and various other pertinent elements to predict the chance of accidents occurring in certain areas or under specified situations. Road safety authorities can proactively allocate resources, perform targeted interventions, and design comprehensive programs to reduce the probability of accidents by identifying high-risk locations and possible danger zones. These models not only improve overall road safety but also lead to more efficient resource allocation and a reduction in the economic and human toll of accidents. As technology advances, these prediction models will become ever more accurate and successful, playing a critical role in ensuring safer and more secure streets for all. The literature review about RSM is given in Table 1.

To choose the best choice, it is standard policy in our daily lives to compare and assess related or interchangeable items from a variety of angles [16–19]. In contemporary management science, multi-attribute decision-making (MADM) is a widely prominent topic of research. Making decisions is a daily necessity for many aspects of our lives. Due to the subjective nature of qualitative attributes or the expensive cost associated with obtaining precise numerical data, people are typically forced to express their thoughts in language. In many areas, including enterprise strategy planning, quality assessment, the choice of investment strategy, and linguistic decision-making which analyzes linguistic data as the values of linguistic variables has made significant progress. In actual decision-making situations, decision makers (DMs) can express their preferences for the evaluated options using linguistic terms like good, fair, or bad, and then choose the best strategy by applying the appropriate decision-making approaches [20–22]. In the context of evaluating the feasibility of a company's investment plan, experts may employ a linguistic term set (LTS) denoted as $S=\{s_0$: extremely poor, s_1: poor, s_2: slightly poor, s_3: fair, s_4: slightly good, s_5: good, s_6: extremely good$\}$. These seven terms serve as a means for experts to express their subjective assessments. If a qualified specialist assesses the company's profitability as good, it may be represented as $\{s_5\}$. The presence of cognitive disparities among specialists, along with the intricacy of the decision-making context can result in varying assessment criteria for the same problem within the realm of actual decision-making scenarios. In the exact mathematical set, the no and yes of a thing's illustration are presented by the numbers 0 and 1, but the representation of the real world commonly displays uncertainty. One of the foremost concerns that necessitates resolution pertains to the representation of assessment values within the context of decision-making, since the information frequently exhibits qualities of vagueness and unpredictability.

Thus, Zadeh [23] introduced the concept of fuzzy set (FS) to cope with uncertainties in actual decision-making scenarios. Subsequently, the introduction of FS extensions and FS-based multi-attribute group decision-making (MAGDM) approaches took place. When representing uncertain information, the non-membership degree (NMD) is not considered in the context of FS, where only the membership degree (MD) is utilized. Consequently, Atanassov [24] formulated the concept of intuitionistic FS (IFS), whereby each element is characterized by two components: the MD, denoted as μ, and the NMD, denoted as ν. To overcome the drawback of the IFS, Yager [25] created the Pythagorean FS (PyFS) with the constraints $\mu+\nu\geq 1$ but $\mu^2+\nu^2\leq 1$. Similar to IFS, MDs and NMDs are also used to create PyFS. PyFS has laxer constraints than IFS, which means that both MD and NMD cannot have squares that are greater than one. Individuals cannot effectively navigate situations wherein the sum of the squared values of MD and NMD exceeds 1. Yager [26] proposed the concept of q-rung orthopair FS (q-ROFS) as an extension of the conventional FS theory. They imposed the condition that $\mu^q + \nu^q \leq 1$ and $q \geq 1$ in order to address the aforementioned constraint of PyFS. Subsequently, the emergence of q-ROFS-based MAGDM approaches has attracted significant attention as a new area of research, leading to the introduction of many innovative decision-making strategies. Thus, q-ROFS is better equipped to handle ambiguous information suitably and flexibly. Many scholars have studied the q-ROFS, and their work has generated several innovations [27–29]. The aggregation operators (AOs) are important tools for dealing with MADM or MAGDM problems, and these AOs also have benefits. Three different implementation strategies for driverless cars in the virtual world were examined by Deveci et al. [30]. Twelve unique attributes were used to evaluate those alternatives using the suggested MADM method. The attributes were divided into four groups: technological, sociological, legal and ethical, and transportation. In the COVID-19 period, Demir Uslu et al. [31] highlighted the critical challenges for a sustainable healthcare policy. Deveci et al. [32] proposed a hybrid decision-making model called the combined compromise solution, which incorporated weighted q-ROF Hamacher average and weighted q-ROF Hamacher geometric mean AOs. Fetanat and Tayebi [33] developed a novel decision support system known as q-ROFS-based multi-attributive ideal-real comparative analysis.

Herrera and Martínez [34] established significant contributions to the field of information extraction with their proposal of the 2-tuple linguistic (2TL) representation approach. The core element of 2TL representation model is comprised of a LT and an associated numerical value. It can successfully avoid both data loss and misrepresentations, which used to happen in earlier language modelizations. Experts have chosen this model for application in many decision-making scenarios that have been seen in real-world contexts. The efficacy of a 2TL information processing method in mitigating information loss and distortion has been demonstrated by Herrera and Martínez. To examine system safety and dependability, Yazdi [35] designed an extension of the 2TL model. Yu et al. [36] created a new integrated MADM framework to more effectively assess and rank offshore wind farm sites to support the healthy development of offshore wind generation. Naz et al. [37] presented the 2TL complex q-ROF concept by combining the complex q-ROFS with the 2TL terms, which include the core definition, operational guidelines, score, and accuracy functions. Several enlarged AOs, such as IF AOs and generalized AOs have been developed to enable decision-making based on standard AOs, like operators weighted average (WA), order WA (OWA), order weighted geometric (OWG), etc. In order to address the issue of decision information, Liu and Wang [38] proposed the q-ROF WA operator and the q-ROF WG operator. The authors have provided a comprehensive demonstration of the attributes associated with these AOs. However, a significant portion of expanded argumentation frameworks fails to adequately address the task of establishing meaningful connections between the input arguments. Yager [39] introduced the power average (PA) operator and the power OWA operator. These operators were designed to mitigate the adverse impact of negative data provided by DMs and incorporated attribute values for mutual support. Furthermore, Xu and Yager [40] introduced the concepts of the power geometric operator and power OWG operator. Numerous scholars have further elaborated on these AOs in diverse situations. In the realm of interval-valued FS, Xu [41] introduced the concept of IF power AOs. Wan and Dong [42] put forth the notion of power geometric AOs within the context of trapezoidal interval-valued fuzzy environments. Liu and Wang [43] proposed a generalized framework for power AOs that can accommodate linguistic interval-valued fuzzy numbers. Zhang [44] presented a generalized IF power geometric operator. Consequently, the utilization of the PA operator can effectively mitigate the adverse consequences arising from irrational data components. This is achieved by assigning distinct weights according to the support degree, accommodating scenarios where specialist values may deviate significantly from the optimal range within the decision-making context.

In 2016, Zavadskas and Podvezko [45] proposed the IDOCRIW approach. Zavadskas and Podvezko have integrated the most advantageous attributes of the Entropy method and the criteria impact loss (CILOS) approach in order to develop a novel method referred to as IDOCRIW. The utilization of the Entropy and CILOS methodologies in assessing the data structure is a common practice. This method utilizes the Entropy and CILOS methods in conjunction with other approaches to compute the relative impact loss and weights of attributes. By combining relative impact loss and weights of attributes, this combination aims to calculate the weights of attributes. By combining two approaches, IDOCRIW outcomes are implied to be reliable and accurate [46]. Eghbali-Zarch et al. [47] employed the IDOCRIW approach to update and use triangular fuzzy numbers to determine the weights of the sustainable development attributes. For the best selection of transmitting generation's optimal waste-to-energy technologies, Alao et al. [48] developed a novel hybrid multi-criteria methodology based on TOPSIS and IDOCRIW. The QUALIFLEX [49] is a highly well-known decision-making approach for simultaneously handling fundamental and numerical information in the decision-making process. In order to address challenges related to group decision-making, Wan et al. [50] developed a novel methodology called the interval-valued q-ROFS. This approach incorporated Dombi operators and introduced the interval-valued q-ROF QUALIFLEX decision analysis method. To resolve the problem of green suppliers where there were significantly fewer alternatives than there were attributes, Liu et al. [51] applied the QUALIFLEX approach. Sahin [52] expanded the classical QUALIFLEX approach to the neutrosophic environment and created a neutrosophic QUALIFLEX method that made use of a newly developed distance-based comparison method. To evaluate the quality of operations workers in engineering projects, He et al. [53] combined the Pythagorean 2TLFS and QUALIFLEX method. Pythagorean 2TL fuzzy numbers were used to convey the DM's judgment of each scheme along with the QUALIFLEX approach to decision-making. To manage decisions with numerous attributes, Chen [54] created a new QUALIFLEX-based model to assess the degree of concordance of the complete preference order.

Despite the scientifically robust basis of the APM, which facilitates the evaluation and selection of road safety measures and enables efficient decision-making under budgetary constraints, it is imperative to enhance the adoption of APMs among national road administrations, designers, and road safety engineers in Pakistan. The objective can be accomplished through the utilization of research in APM that exhibits a significant need for implementation but is currently lacking in availability. FS theory is capable of addressing issues of uncertain, interpretive, and vague assessments. q-ROFS is a type of FS that is used to handle uncertainty and imprecision in decision-making. It is a generalization of the PyFS which allows us to represent and analyze complex information. q-ROFS introduces a new parameter, q, which controls the degree of fuzziness in the set. This parameter can be adjusted to suit different decision-making scenarios, allowing us to handle situations where different degrees of uncertainty are present. q-ROFS is a powerful tool for decision-making that enables us to handle complex and uncertain situations more effectively. 2TLq-ROFS is a type of FS that combines 2TL variables and q-ROFS. It allows DMs to represent and analyze complex linguistic information in decision-making scenarios, taking into account both the uncertainty and the imprecision of the data. In 2TLq-ROFS, the values of the linguistic variables are represented in tuples of two elements, which are interpreted as the numerical value and symbolic translation of the variables. The weighting methods are great tools that allow DMs to make unemotional and calculated decisions. They are also a great way to communicate and justify any decisions that come from different attributes characterized by DMs. Utilizing a weighting method can help add a whole new angle to the strategic planning process. IDOCRIW is a well-known weighting method that benefits from the Entropy and CILOS methods to determine the weights of attributes in combination with the two methods. Hence, the IDOCRIW method is a well-suited and simple weighting method other than MEREC and LOPCOW methods for calculating weights of attributes. Ranking methods in FS theory offer a unique way to handle imprecise information. Fuzzy ranking allows for the sorting and prioritization of data when exact values are not available, enabling a more flexible approach to decision-making. These methods help in dealing with vagueness and uncertainty, providing a means to establish order among elements based on degrees of membership to various sets, and contributing to a more comprehensive analysis of complex and ambiguous data. The QUALIFLEX approach is founded on an analytical strategy for ranking alternatives, where the approach conducts pairwise assessments of options about each attribute over all conceivable permutations of the options. When the decision data is either accurate or fuzzy, all current versions of the IDOCRIW and QUALIFLEX methods are efficient and adequate. However, they cannot be employed in decision situations involving the 2TLq-ROF information. The following reasons motivate us to construct the 2TLq-ROF-IDOCRIW-QUALIFLEX approach:

- The 2TLq-ROF-IDOCRIW-QUALIFLEX approach extends the structure of previous models by providing a valid and compatible framework for describing the 2TLq-ROF information.
- The approach is first based on 2TLq-ROF assessments, which are then converted into fuzzy data, which greatly improves the accuracy of making choices and offers decision information.
- The flexibility of the 2TL representation model and q-ROFS in previous research models allows for a broad range of applications for the proposed methodology and provides a better understanding of how to use qualitative and quantitative q-ROF information in decision-making problems.
- The choice of the optimal alternative by comparing its superiority, equality, and inferiority relations to other alternatives, as well as the visual depiction, make the approach more admirable and appropriate for generating exciting ranking results.

The adaptability of the proposed work addresses the limitations of the previous approaches and helps to clarify the concerns. The proposed 2TLq-ROF-IDOCRIW-QUALIFLEX approach is used in a case study to evaluate APMs for RSM in terms of superiority, equity, and inadequacy using some competing attributes. The 2TLq-ROF-IDOCRIW-QUALIFLEX method is then used to generate a comprehensive and sorted list of options. We choose the APMs for RSM utilizing the existing models to compare the authenticity of the innovative approach. The comparison results show that the provided approach can be successfully applied to MAGDM problems and offer a decreasing ranking of the options. This study contributes to the growth and evolution of the decision-making scenario through an innovative approach. The following are the contributions to the paper:

- We introduce the concept of WPA operator under the 2TLq-ROF environment.
- We develop an innovative approach using IDOCRIW and QUALIFLEX methodologies in a 2TLq-ROF environment.
- We utilize a MAGDM methodology to solve a case study to evaluate APMs for RSM.
- Comparison analysis is done with different existing approaches and concluding remarks are given.

The remainder of this paper is organized as follows:

Section 2 discusses some fundamental concepts related to the 2TL terms, q-ROFS, 2TLq-ROFS, 2TLq-ROF normalized Hamming distance, and PA operator. The 2TLq-ROF-IDOCRIW-QUALIFLEX decision analysis method is detailed in Section 3 to resolve the MAGDM problem. A case study related to APMs for RSM in Pakistan is presented in Section 4 and solved utilizing the steps described in Section 3. The same Section 4 includes the comparative study with discussion and advantages to demonstrate the feasibility and superiority of the proposed method. The conclusions of this research are provided in Section 5 along with limitations and future directions.

2. Preliminaries

In this section, numerous key concepts related to the proposed study, including the 2TL terms, q-ROFS, 2TLq-ROFS, 2TLq-ROF normalized Hamming distance as well as PA operator are described to facilitate comprehension of subsequent sections.

Definition 1. [34] Let $S = \{s_t | t = 0, 1, \ldots, k\}$ be a set defined as a discrete LTS. Let η be a quantitative value ranging from 0 to k, representing the outcome of a symbolic aggregating process. The corresponding 2-tuple representing the same content as η is provided below:

$$\Delta : [0, k] \rightarrow \bar{S},$$

$$\Delta(\eta) = (s_{\text{round}(\eta)}, \eta - \mathfrak{t})$$

where $\bar{S} = S \times [-0.5, 0.5)$, round(.) is the standard round function which assigns to fuzzy value η a numeric integer $\mathfrak{t} \in \{0, 1, \ldots, k\}$ closest to η, and $\upsilon = \eta - \mathfrak{t}$ is termed as the symbolic translation.

Remark 1. [34] The aforementioned Δ mapping is a one-to-one function, ensuring its invertibility. As a result, there exists an inverse function $\Delta^{-1} : \bar{S} \to [0, k]$, which yields the same fuzzy result. This means that for any $\eta \in [0, k] \subset R$, there exists a corresponding 2-tuple where $\Delta^{-1}(s_{\mathfrak{t}}, \upsilon) = \eta = \mathfrak{t} + \upsilon$.

Remark 2. The 2TL term is obtained from a linguistic term $s_{\mathfrak{t}}$ by incorporating a symbolic translation of 0. This can be expressed as follows: If $s_{\mathfrak{t}}$ belongs to the set S, then the pair $(s_{\mathfrak{t}}, 0)$ also belongs to the set $S \times [-0.5, 0.5)$.

In addition, Herrera and Martínez [34] conducted a comparative analysis of 2TL information with the conventional lexicographic ordering method.

Definition 2. Consider two 2TL variables denoted as $(s_{\mathfrak{t}}, \upsilon_{\mathfrak{t}})$ and $(s_{\varphi}, \upsilon_{\varphi})$.

- if $\mathfrak{t} < \varphi \Rightarrow (s_{\mathfrak{t}}, \upsilon_{\mathfrak{t}}) < (s_{\varphi}, \upsilon_{\varphi})$.
- if $\mathfrak{t} = \varphi$ then,
  (1) if $\upsilon_{\mathfrak{t}} = \upsilon_{\varphi} \Rightarrow (s_{\mathfrak{t}}, \upsilon_{\mathfrak{t}}) = (s_{\varphi}, \upsilon_{\varphi})$;
  (2) if $\upsilon_{\mathfrak{t}} < \upsilon_{\varphi} \Rightarrow (s_{\mathfrak{t}}, \upsilon_{\mathfrak{t}}) < (s_{\varphi}, \upsilon_{\varphi})$;
  (3) if $\upsilon_{\mathfrak{t}} > \upsilon_{\varphi} \Rightarrow (s_{\mathfrak{t}}, \upsilon_{\mathfrak{t}}) > (s_{\varphi}, \upsilon_{\varphi})$.

Definition 3. [34] Consider a finite cardinal LTS $S = \{s_{\mathfrak{t}} | \mathfrak{t} = 0, 1, \ldots, k\}$ with $k + 1$ elements. The negative operator for a 2-tuple is defined as follows:

$$Neg(s_{\mathfrak{t}}, \upsilon) = \Delta(k - (\Delta^{-1}(s_{\mathfrak{t}}, \upsilon))).$$

The 2TL variable and translating function were presented in an expanded version by Chen and Tai [55].

Definition 4. [55] Let $S = \{s_{\mathfrak{t}} | \mathfrak{t} = 0, 1, \ldots, k\}$ be a finite cardinal LTS with $k + 1$ elements, and consider η as the fuzzy result, where $\eta \in [0, 1]$. The conversion of η into a 2TL variable is accomplished through the following steps:

$$\Delta : [0, 1] \to \bar{S},$$

$$\Delta(\eta) = (s_{\mathfrak{t}}, \upsilon) \quad \text{with} \quad \begin{cases} s_{\mathfrak{t}}, & \mathfrak{t} = \text{round}(\eta k), \\ \upsilon = \eta - \mathfrak{t}/k, & \upsilon \in [-0.5/k, 0.5/k) \end{cases}$$

where $\bar{S} = S \times [-0.5, 0.5)$, round(.) is the basic round function, $s_{\mathfrak{t}}$ being the most nearby indexing labeling to η, and υ denotes the symbolic translation.

In an alternative perspective, the function $\Delta^{-1} : \bar{S} \to [0, 1]$ can be defined as the operation of transforming a 2TL variable into its matching fuzzy outcome η with η belonging to the interval $[0, 1]$.

$$\Delta^{-1}(s_{\mathfrak{t}}, \upsilon) = \eta = \mathfrak{t}/k + \upsilon.$$

Definition 5. [26] Consider a standard set $\mathcal{P}$. A q-ROFS $\mathfrak{X}$ on $\mathcal{P}$ can be defined in Equation (1):

$$\mathfrak{X} = \{\langle \mathfrak{f}, \Upsilon_{\mathfrak{X}}(\mathfrak{f}), \Upsilon'_{\mathfrak{X}}(\mathfrak{f}) \rangle | \mathfrak{f} \in \mathcal{P}\} \tag{1}$$

where $\mathfrak{f} \in \mathcal{P}$, and $\Upsilon_{\mathfrak{X}}(\mathfrak{f})$ and $\Upsilon'_{\mathfrak{X}}(\mathfrak{f})$ denote the MD and NMD of the element $\mathfrak{f}$ to the set $\mathfrak{X}$, respectively. It is required that $0 \le \Upsilon_{\mathfrak{X}}(\mathfrak{f}), \Upsilon'_{\mathfrak{X}}(\mathfrak{f}) \le 1$, and $\Upsilon_{\mathfrak{X}}(\mathfrak{f})^q + \Upsilon'_{\mathfrak{X}}(\mathfrak{f})^q \le 1$ for $q \ge 1$. The degree of indeterminacy, denoted as $\pi_{\mathfrak{X}}(\mathfrak{f})$, is given by $(1 - \Upsilon_{\mathfrak{X}}(\mathfrak{f})^q - \Upsilon'_{\mathfrak{X}}(\mathfrak{f})^q)^{\frac{1}{q}}$.

Liu and Wang [38] referred to the ordered pair $(\Upsilon_{\mathfrak{X}}(\mathfrak{f}), \Upsilon'_{\mathfrak{X}}(\mathfrak{f}))$ as a q-ROFN, which can be represented as $\chi = (\Upsilon, \Upsilon')$.

Definition 6. [56] Let $S = \{s_{\mathfrak{t}} | \mathfrak{t} = 0, 1, \ldots, k\}$ be a LTS with odd cardinality. If $(s_{\mathfrak{p}}(\mathfrak{f}), \Psi(\mathfrak{f})), (s_{\mathfrak{l}}(\mathfrak{f}), \Phi(\mathfrak{f}))$ is defined for $s_{\mathfrak{p}}(\mathfrak{f}), s_{\mathfrak{l}}(\mathfrak{f}) \in S$, $\Psi(\mathfrak{f}), \Phi(\mathfrak{f}) \in [-0.5, 0.5)$, where $(s_{\mathfrak{p}}(\mathfrak{f}), \Psi(\mathfrak{f}))$ and $(s_{\mathfrak{l}}(\mathfrak{f}), \Phi(\mathfrak{f}))$ represent the MD and NMD using 2TL numbers, respectively. The 2TLq-ROFS can be defined in Equation (2):

$$\aleph = \{\langle \mathfrak{f}, ((s_{\mathfrak{p}}(\mathfrak{f}), \Psi(\mathfrak{f})), (s_{\mathfrak{l}}(\mathfrak{f}), \Phi(\mathfrak{f}))) \rangle | \mathfrak{f} \in \mathcal{P}\} \tag{2}$$

where $0 \le \Delta^{-1}(s_{\mathfrak{p}}(\mathfrak{f}), \Psi(\mathfrak{f})) \le k, 0 \le \Delta^{-1}(s_{\mathfrak{l}}(\mathfrak{f}), \Phi(\mathfrak{f})) \le k$, and $0 \le (\Delta^{-1}(s_{\mathfrak{p}}(\mathfrak{f}), \Psi(\mathfrak{f})))^q + (\Delta^{-1}(s_{\mathfrak{l}}(\mathfrak{f}), \Phi(\mathfrak{f})))^q \le k^q$.

In order to conduct a comparison between two 2TLq-ROF numbers (2TLq-ROFNs), it is necessary to establish their respective score function and accuracy function.

Definition 7. [56] Let $\Lambda=((s_{\mathfrak{p}},\Psi),(s_{\mathfrak{l}},\Phi))$ be represented the 2TLq-ROFN. The score function Sc of a 2TLq-ROFN can be represented in Equation (3):

$$Sc(\Lambda)=\Delta\left(\frac{k}{2}\left(1+\left(\frac{\Delta^{-1}(s_{\mathfrak{p}},\Psi)}{k}\right)^{q}-\left(\frac{\Delta^{-1}(s_{\mathfrak{l}},\Phi)}{k}\right)^{q}\right)\right),\ \Delta^{-1}(Sc(\Lambda))\in[0,k], \tag{3}$$

and its accuracy function $\beth$ can be defined in Equation (4):

$$\beth(\Lambda)=\Delta\left(k\left(\left(\frac{\Delta^{-1}(s_{\mathfrak{p}},\Psi)}{k}\right)^{q}+\left(\frac{\Delta^{-1}(s_{\mathfrak{l}},\Phi)}{k}\right)^{q}\right)\right),\ \Delta^{-1}(\beth(\Lambda))\in[0,k]. \tag{4}$$

Definition 8. [56] Consider two 2TLq-ROFNs: $\Lambda_1=((s_{\mathfrak{p}_1},\Psi_1),(s_{\mathfrak{l}_1},\Phi_1))$ and $\Lambda_2=((s_{\mathfrak{p}_2},\Psi_2),(s_{\mathfrak{l}_2},\Phi_2))$. These two 2TL$q$-ROFNs can be compared based on the following rules:

(1) If $Sc(\Lambda_1)>Sc(\Lambda_2)$, then $\Lambda_1\succ\Lambda_2$;
(2) If $Sc(\Lambda_1)<Sc(\Lambda_2)$, then $\Lambda_1\prec\Lambda_2$;
(3) If $Sc(\Lambda_1)=Sc(\Lambda_2)$, then
  - If $\beth(\Lambda_1)>\beth(\Lambda_2)$, then $\Lambda_1\succ\Lambda_2$;
  - If $\beth(\Lambda_1)<\beth(\Lambda_2)$, then $\Lambda_1\prec\Lambda_2$;
  - If $\beth(\Lambda_1)=\beth(\Lambda_2)$, then $\Lambda_1\sim\Lambda_2$.

The operational laws of the 2TLq-ROFNs have various mathematical operations including addition, multiplication, scalar multiplication, and power rules. These laws can be defined as follows:

Definition 9. [56] Let $\Lambda=((s_{\mathfrak{p}},\Psi),(s_{\mathfrak{l}},\Phi))$, $\Lambda_1=((s_{\mathfrak{p}_1},\Psi_1),(s_{\mathfrak{l}_1},\Phi_1))$, and $\Lambda_2=((s_{\mathfrak{p}_2},\Psi_2),(s_{\mathfrak{l}_2},\Phi_2))$ be three 2TLq-ROFNs, where $q\geq1$, then

1. $\Lambda_1\oplus\Lambda_2=\left(\Delta\left(k\sqrt[q]{1-\left(1-\left(\frac{\Delta^{-1}(s_{\mathfrak{p}_1},\Psi_1)}{k}\right)^{q}\right)\left(1-\left(\frac{\Delta^{-1}(s_{\mathfrak{p}_2},\Psi_2)}{k}\right)^{q}\right)}\right),\Delta\left(k\left(\frac{\Delta^{-1}(s_{\mathfrak{l}_1},\Phi_1)}{k}\right)\left(\frac{\Delta^{-1}(s_{\mathfrak{l}_2},\Phi_2)}{k}\right)\right)\right)$;
2. $\Lambda_1\otimes\Lambda_2=\left(\Delta\left(k\left(\frac{\Delta^{-1}(s_{\mathfrak{p}_1},\Psi_1)}{k}\right)\left(\frac{\Delta^{-1}(s_{\mathfrak{p}_2},\Psi_2)}{k}\right)\right),\Delta\left(k\sqrt[q]{1-\left(1-\left(\frac{\Delta^{-1}(s_{\mathfrak{l}_1},\Phi_1)}{k}\right)^{q}\right)\left(1-\left(\frac{\Delta^{-1}(s_{\mathfrak{l}_2},\Phi_2)}{k}\right)^{q}\right)}\right)\right)$;
3. $\lambda\Lambda=\left(\Delta\left(k\sqrt[q]{1-\left(1-\left(\frac{\Delta^{-1}(s_{\mathfrak{p}},\Psi)}{k}\right)^{q}\right)^{\lambda}}\right),\Delta\left(k\left(\frac{\Delta^{-1}(s_{\mathfrak{l}},\Phi)}{k}\right)^{\lambda}\right)\right),\ \lambda>0$;
4. $\Lambda^{\lambda}=\left(\Delta\left(k\left(\frac{\Delta^{-1}(s_{\mathfrak{p}},\Psi)}{k}\right)^{\lambda}\right),\Delta\left(k\sqrt[q]{1-\left(1-\left(\frac{\Delta^{-1}(s_{\mathfrak{l}},\Phi)}{k}\right)^{q}\right)^{\lambda}}\right)\right),\ \lambda>0.$

Definition 10. [56] Let $\Lambda_1=((s_{\mathfrak{p}_1},\Psi_1),(s_{\mathfrak{l}_1},\Phi_1))$ and $\Lambda_2=((s_{\mathfrak{p}_2},\Psi_2),(s_{\mathfrak{l}_2},\Phi_2))$ be two 2TLq-ROFNs. The 2TLq-ROF normalized Hamming distance can be described in Equation (5):

$$d(\Lambda_1,\Lambda_2)=\Delta\left(\frac{k}{2}\left(\left|\left(\frac{\Delta^{-1}(s_{\mathfrak{p}_1},\Psi_1)}{k}\right)^{q}-\left(\frac{\Delta^{-1}(s_{\mathfrak{p}_2},\Psi_2)}{k}\right)^{q}\right|+\left|\left(\frac{\Delta^{-1}(s_{\mathfrak{l}_1},\Phi_1)}{k}\right)^{q}-\left(\frac{\Delta^{-1}(s_{\mathfrak{l}_2},\Phi_2)}{k}\right)^{q}\right|\right)\right). \tag{5}$$

Definition 11. Consider a set of non-negative real numbers $\mathfrak{a}_\sigma$, where $\sigma=1,2,\ldots,\breve{n}$. The PA [39] operator can be defined as:

$$PA(\mathfrak{a}_1,\mathfrak{a}_2,\ldots,\mathfrak{a}_{\breve{n}})=\frac{\sum_{\sigma=1}^{\breve{n}}(1+T(\mathfrak{a}_\sigma))\mathfrak{a}_\sigma}{\sum_{\varrho=1}^{\breve{n}}(1+T(\mathfrak{a}_\varrho))}$$

where $T(\mathfrak{a}_\sigma)=\sum_{\sigma,\varrho=1,\sigma\neq\varrho}^{\breve{n}}\mathrm{Sup}(\mathfrak{a}_\sigma,\mathfrak{a}_\varrho)$, where $\mathrm{Sup}(\mathfrak{a}_\sigma,\mathfrak{a}_\varrho)=1-d_{2\mathrm{TL}q\text{-ROFNHD}}(\mathfrak{a}_\sigma,\mathfrak{a}_\varrho)$ and $\mathrm{Sup}(\mathfrak{a},\mathfrak{b})$ is the support for $\mathfrak{a}$ from $\mathfrak{b}$ that satisfies the three characteristics:

(1) Sup $(\mathfrak{a},\mathfrak{b})\in[0,1]$.
(2) Sup $(\mathfrak{a},\mathfrak{b})=$ Sup $(\mathfrak{b},\mathfrak{a})$.

(3) $\text{Sup}\,(\mathfrak{a},\mathfrak{b}) \geq \text{Sup}\,(\mathfrak{x},\mathfrak{y})$, if $|\mathfrak{a}-\mathfrak{b}| < |\mathfrak{x}-\mathfrak{y}|$.

The support function (Sup) serves as a similarity factor. The closer two values are to each other, the more they support each other, indicating higher similarity. Furthermore, we can represent $\frac{1+T(\mathfrak{a}_\sigma)}{\sum_{\sigma=1}^{\breve{n}}(1+T(\mathfrak{a}_\sigma))}$ as ζ_σ.

3. Decision analysis with IDOCRIW-QUALIFLEX approach in MAGDM environment

This section presents a framework that combines the QUALIFLEX and IDOCRIW approaches within the 2TLq-ROF environment. Both QUALIFLEX and IDOCRIW methods are crucial in the context of MAGDM. Using the 2TLq-ROFWPA operator, the research study introduces a new MAGDM approach to resolve group decision-making problems in the 2TLq-ROF environment. It is advantageous to consider AOs when tackling MAGDM problems, as MAGDM methods that employ AOs typically outperform conventional methods.

Below is a comprehensive explanation of the 2TLq-ROF-QUALIFLEX method in which the weight vector is determined by the IDOCRIW method.

Step 1. Construct the 2TLq-ROF decision matrix as below:

Let there be a set of '$\breve{m}$' alternatives denoted by $\mathbb{A}=\{\mathbb{A}_1,\mathbb{A}_2,\ldots,\mathbb{A}_{\breve{m}}\}$ and a set of '$\breve{n}$' attributes denoted by $\mathbb{N}=\{\mathbb{N}_1,\mathbb{N}_2,\ldots,\mathbb{N}_{\breve{n}}\}$. The attributes have corresponding weight values in the vector $\omega=(\omega_1,\omega_2,\ldots,\omega_{\breve{n}})^T$, where $\omega\in[0,1]$ and $\sum_{\sigma=1}^{\breve{n}}\omega_\sigma=1$. A group of DMs $\mathcal{D}=\{\mathcal{D}_1,\mathcal{D}_2,\ldots,\mathcal{D}_{\mathfrak{e}}\}$ is formed, and they express their views on each alternative $\mathbb{A}_\varrho$ concerning the attributes $\mathbb{N}_\sigma$ in terms of 2TLq-ROFNs. The weight vector of DMs is represented as $\omega'=(\omega'_1,\omega'_2,\ldots,\omega'_{\mathfrak{e}})^T$, where $\omega'\in[0,1]$ and $\sum_{\kappa=1}^{\mathfrak{e}}\omega'_\kappa=1$. Each decision maker $\mathcal{D}_\kappa$ provides his assessment information in the form of a 2TLq-ROF decision matrix denoted by $\hbar^\kappa=[\Lambda^\kappa_{\varrho\sigma}]_{\breve{m}\times\breve{n}}=((s^\kappa_{\mathfrak{p}_{\varrho\sigma}},\Psi^\kappa_{\varrho\sigma}),(s^\kappa_{\mathfrak{l}_{\varrho\sigma}},\Phi^\kappa_{\varrho\sigma}))$ where $(\varrho=1,2,\ldots,\breve{m},\sigma=1,2,\ldots,\breve{n},\kappa=1,2,\ldots,\mathfrak{e})$.

$$\hbar^\kappa=[\Lambda^\kappa_{\varrho\sigma}]_{\breve{m}\times\breve{n}}$$
$$=\begin{bmatrix} ((s_{\mathfrak{p}_{11}},\Psi_{11})^\kappa,(s_{\mathfrak{l}_{11}},\Phi_{11})^\kappa) & ((s_{\mathfrak{p}_{12}},\Psi_{12})^\kappa,(s_{\mathfrak{l}_{12}},\Phi_{12})^\kappa) & \ldots & ((s_{\mathfrak{p}_{1\breve{m}}},\Psi_{1\breve{m}})^\kappa,(s_{\mathfrak{l}_{1\breve{m}}},\Phi_{1\breve{m}})^\kappa) \\ ((s_{\mathfrak{p}_{21}},\Psi_{21})^\kappa,(s_{\mathfrak{l}_{21}},\Phi_{21})^\kappa) & ((s_{\mathfrak{p}_{22}},\Psi_{22})^\kappa,(s_{\mathfrak{l}_{22}},\Phi_{22})^\kappa) & \ldots & ((s_{\mathfrak{p}_{2\breve{m}}},\Psi_{2\breve{m}})^\kappa,(s_{\mathfrak{l}_{2\breve{m}}},\Phi_{2\breve{m}})^\kappa) \\ \vdots & \vdots & \ddots & \vdots \\ ((s_{\mathfrak{p}_{\breve{n}1}},\Psi_{\breve{n}1})^\kappa,(s_{\mathfrak{l}_{\breve{n}1}},\Phi_{\breve{n}1})^\kappa) & ((s_{\mathfrak{p}_{\breve{n}2}},\Psi_{\breve{n}2})^\kappa,(s_{\mathfrak{l}_{\breve{n}2}},\Phi_{\breve{n}2})^\kappa) & \ldots & ((s_{\mathfrak{p}_{\breve{n}\breve{m}}},\Psi_{\breve{n}\breve{m}})^\kappa,(s_{\mathfrak{l}_{\breve{n}\breve{m}}},\Phi_{\breve{n}\breve{m}})^\kappa) \end{bmatrix}_{\breve{m}\times\breve{n}}$$

Step 2. Calculate the support degree $\text{Sup}(\Lambda^\kappa_{\varrho\sigma},\Lambda^{\mathfrak{d}}_{\varrho\sigma})$ as defined in Equation (6):

$$\text{Sup}(\Lambda^\kappa_{\varrho\sigma},\Lambda^{\mathfrak{d}}_{\varrho\sigma})=1-d(\Lambda^\kappa_{\varrho\sigma},\Lambda^{\mathfrak{d}}_{\varrho\sigma})(\kappa,\mathfrak{d}=1,2,\ldots,\mathfrak{e};\kappa\neq\mathfrak{d}) \tag{6}$$

where the expression $d(\Lambda^\kappa_{\varrho\sigma},\Lambda^{\mathfrak{d}}_{\varrho\sigma})$ denotes the normalized Hamming distance between $\Lambda^\kappa_{\varrho\sigma}$ and $\Lambda^{\mathfrak{d}}_{\varrho\sigma}$, which is computed using Equation (5).

Step 3. Calculate the synthesis support matrices $[\mathfrak{T}(\Lambda^\kappa_{\varrho\sigma})]_{\breve{m}\times\breve{n}}$ as defined in Equation (7):

$$\mathfrak{T}(\Lambda^\kappa_{\varrho\sigma})=\sum_{\kappa,\mathfrak{d}=1;\mathfrak{d}\neq\kappa}^{\mathfrak{e}}\text{Sup}(\Lambda^\kappa_{\varrho\sigma},\Lambda^{\mathfrak{d}}_{\varrho\sigma}). \tag{7}$$

Step 4. Calculate the comprehensive power weight matrices $[\zeta^\kappa_{\varrho\sigma}]_{\breve{m}\times\breve{n}}$ as defined in Equation (8):

$$\zeta^\kappa_{\varrho\sigma}=\frac{\omega'\left(1+\mathfrak{T}(\Lambda^\kappa_{\varrho\sigma})\right)}{\sum_{\kappa=1}^{\mathfrak{e}}\omega'\left(1+\mathfrak{T}(\Lambda^\kappa_{\varrho\sigma})\right)}. \tag{8}$$

Step 5. To form the aggregated 2TLq-ROF decision matrix, the individual decisions made by the DMs must be merged into a collective decision using the 2TLq-ROFWPA operator as defined in Equation (9). This operator combines the individual assessments provided by the DMs to generate a comprehensive decision represented as $\hbar=[\Lambda_{\varrho\sigma}]_{\breve{m}\times\breve{n}}$, where

$$\text{2TL}q\text{-ROFWPA}(\Lambda_1,\Lambda_2,\ldots,\Lambda_{\breve{n}})$$
$$=\left(\Delta\left(k\left(1-\prod_{\sigma=1}^{\breve{n}}\left(1-\left(\frac{\Delta^{-1}(s_{\mathfrak{p}_\sigma},\Psi_\sigma)}{k}\right)^q\right)^{\zeta_\sigma}\right)^{\frac{1}{q}}\right),\Delta\left(k\prod_{\sigma=1}^{\breve{n}}\left(\frac{\Delta^{-1}(s_{\mathfrak{l}_\sigma},\Phi_\sigma)}{k}\right)^{\zeta_\sigma}\right)\right). \tag{9}$$

Step 6. Normalize the aggregated matrix $\hbar=[\Lambda_{\varrho\sigma}]_{\breve{m}\times\breve{n}}$ using a computation based on each attribute:

For benefit attributes, Equation (10) can be used:

$$N_{\varrho\sigma} = \Lambda_{\varrho\sigma} = ((s_{\mathfrak{p}_{\varrho\sigma}}, \Psi_{\varrho\sigma}), (s_{\mathfrak{l}_{\varrho\sigma}}, \Phi_{\varrho\sigma})), \ \varrho = 1, 2, \dots, \breve{\mathfrak{m}}, \ \sigma = 1, 2, \dots, \breve{\mathfrak{n}}. \tag{10}$$

For cost attributes, Equation (11) can be used:

$$N_{\varrho\sigma} = \Lambda^{c}_{\varrho\sigma} = ((s_{\mathfrak{l}_{\varrho\sigma}}, \Phi_{\varrho\sigma}), (s_{\mathfrak{p}_{\varrho\sigma}}, \Psi_{\varrho\sigma})), \ \varrho = 1, 2, \dots, \breve{\mathfrak{m}}, \ \sigma = 1, 2, \dots, \breve{\mathfrak{n}}. \tag{11}$$

Next, we will use the Entropy weighting method.

Step 7. To determine Entropy weights, we first create a decision matrix $\mathfrak{R}$ that contains $\breve{\mathfrak{n}}$ attributes $\mathbb{N}_\sigma(\sigma = 1, 2, \dots, \breve{\mathfrak{n}})$ and $\breve{\mathfrak{m}}$ alternatives $\mathbb{A}_\varrho(\varrho = 1, 2, \dots, \breve{\mathfrak{m}})$ and then combined it by adding their respective MDs and NMDs. The elements of this decision matrix represent the scores or performances of each alternative concerning the evaluation attribute.

Step 8. To eliminate ambiguity in attribute units, we normalize the attribute values in the decision matrix, bringing every attribute to the same base. To achieve this, we use Equation (12) to make the negative attributes positive in the decision matrix. Subsequently, the normalized attribute values are computed using Equation (13).

$$\chi'_{\varrho\sigma} = \frac{\min\limits_{\varrho} \mathfrak{R}_{\varrho\sigma}}{\mathfrak{R}_{\varrho\sigma}}, \tag{12}$$

$$\chi_{\varrho\sigma} = \frac{\mathfrak{R}_{\varrho\sigma}}{\sum\limits_{\varrho,\sigma=1}^{\breve{\mathfrak{m}}} \mathfrak{R}_{\varrho\sigma}} \tag{13}$$

where $\chi_{\varrho\sigma}$ represents the normalized (unit-less) values of the σ-th attribute for the ϱ-th alternative.

Step 9. We calculate the degree of Entropy $\mathcal{E}_\sigma(\sigma = 1, 2, \dots, \breve{\mathfrak{n}}; 0 \le \mathcal{E}_\sigma \le 1)$ for each attribute using Equation (14).

$$\mathcal{E}_\sigma = -\frac{1}{ln\breve{\mathfrak{n}}} \sum_{\varrho,\sigma=1}^{\breve{\mathfrak{n}}} \chi_{\varrho\sigma} ln(\chi_{\varrho\sigma}). \tag{14}$$

Step 10. We quantify the degree of differences ($\mathbb{D}_\sigma$) for each attribute as follows (see Equation (15)):

$$\mathbb{D}_\sigma = 1 - \mathcal{E}_\sigma. \tag{15}$$

Step 11. Finally, the Entropy weight $\mathcal{W}_\sigma$ of the attribute can be determined as a normalized value of $\mathbb{D}_\sigma$ using the following Equation (16):

$$\mathcal{W}_\sigma = \frac{\mathbb{D}_\sigma}{\sum\limits_{\sigma=1}^{\mathfrak{n}} \mathbb{D}_\sigma}, \tag{16}$$

it should be noted that $\sum\limits_{\sigma=1}^{\mathfrak{n}} \mathcal{W}_\sigma = 1, \sigma = 1, 2, \dots, \breve{\mathfrak{n}}$.

Next, we will use the CILOS weighting method.

Step 12. We construct a square matrix $\mathcal{B}$ by selecting the values of $\mathbb{X}_{\mathbb{K}_{\varrho\sigma}}$ from matrix $\mathcal{X}$ that correspond to the maximum values of the ϱ-th attribute with $\mathbb{K}_\varrho$ rows, as defined in Equation (17). In the resulting square matrix $\mathcal{B}$, the highest values of all attributes are located along the leading diagonal. We then formulate the matrix of the relative loss of attribute significance $\mathfrak{p}$ using Equation (18).

$$\mathcal{B} = ||\mathbb{C}_{\varrho\sigma}||, \mathbb{C}_{\varrho\varrho} = \mathbb{X}_\varrho, \mathbb{C}_{\varrho\sigma} = \mathbb{X}_{\mathbb{K}_{\varrho\sigma}}, \tag{17}$$

$$\mathfrak{p} = ||\mathbb{P}_{\varrho\sigma}||. \tag{18}$$

Step 13. The determination of the relative impact loss matrix is as follows (see Equation (19)):

$$\mathbb{P}_{\varrho\sigma} = \frac{\chi_\sigma - \mathbb{C}_{\varrho\sigma}}{\chi_\sigma} = \frac{\mathbb{C}_{\varrho\varrho} - \mathbb{C}_{\varrho\sigma}}{\mathbb{C}_{\varrho\varrho}}, \ (\mathfrak{p}_{\varrho\sigma} = 0; \varrho, \sigma = 1, 2, \dots, \breve{\mathfrak{n}}), \tag{19}$$

in this context, $\mathbb{P}_{\varrho\sigma}$ represents the relative loss of the σ-th attribute when the ϱ-th attribute is chosen as the best.

Step 14. The weight system matrix $\mathcal{F}$ is constructed as shown by Matrix (20). Subsequently, the weights $\mathbb{Q}_\sigma = (\mathbb{Q}_1, \mathbb{Q}_2, \dots, \mathbb{Q}_\mathfrak{n})^T$ for each attribute are determined by solving a system of linear homogeneous equations as presented in Equation (21).

$$\mathcal{F}=\begin{bmatrix} -\sum_{\varrho=1}^{\breve{n}}\mathbb{P}_{\varrho 1} & \mathbb{P}_{12} & \cdots & \mathbb{P}_{1\breve{n}} \\ \mathbb{P}_{21} & -\sum_{\varrho=1}^{\breve{n}}\mathbb{P}_{\varrho 2} & \cdots & \mathbb{P}_{2\breve{n}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{P}_{\breve{n}1} & \mathbb{P}_{n2} & \cdots & -\sum_{\varrho=1}^{\breve{n}}\mathbb{P}_{\varrho\breve{n}} \end{bmatrix}_{\breve{n}\times\breve{n}} \tag{20}$$

$$\mathcal{F}\times\mathbb{Q}_\sigma=0. \tag{21}$$

Step 15. The weights of attributes $\mathbb{Q}_\sigma$ are computed from the formulated system of homogeneous linear equations using the approach proposed by Ali et al. [57] as defined in Equation (22).

$$\mathbb{Q}_\sigma=\mathcal{F}^{-1}\mathcal{A}, \tag{22}$$

$\mathcal{A}$ is a vector close to zero (see Equation (23)). To determine the value of $\mathcal{A}$, we assume that the first element of $\mathcal{A}$ is close to zero, while the rest of the elements are zeros, resulting in the vector $\mathcal{A}$ taking the form:

$$\mathcal{A}=\begin{bmatrix}0.29 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}^T \tag{23}$$

therefore, the weight vector $\mathbb{Q}_\sigma$ is normalized to ensure that $\sum_{\sigma=1}^{\breve{n}}\mathbb{Q}_\sigma=1$.

Step 16. The IDOCRIW values are acquired by combining the Entropy and CILOS methods through the use of Equation (24).

$$\omega_\sigma=\frac{\mathbb{Q}_\sigma\mathcal{W}_\sigma}{\sum_{\sigma=1}^{n}\mathbb{Q}_\sigma\mathcal{W}_\sigma}, \tag{24}$$

the values of ω_σ are such that $\sum_{\sigma=1}^{\breve{n}}\omega_\sigma=1$.

Next, we will use the QUALIFLEX method to rank the alternatives.

Step 17. Compute the initial permutation of alternatives by generating all possible permutations using the existing alternatives. For instance, if there are 8 alternatives, then $\mathbb{A}=8$, and therefore, $\mathbb{A}!=8!=40,320$.

Step 18. Establish the initial ranking of alternatives by evaluating the 2TLq-ROF decision matrix provided by the DMs based on its strengths. An alternative that outperforms the others in an attribute is assigned the number 1 and the remaining alternatives are ranked accordingly.

Step 19. Determine the values of dominance and being dominated. If the permutation corresponds to the ranking order, the value is 1; otherwise, the value is −1. In cases where two alternatives are identical in one attribute, the value 0 is assigned.

Step 20. Calculate the permutation values of attributes by aggregating the values computed in the previous step for all permutations and attributes separately.

Step 21. Determine the permutation values for the given set of options. The permutation value is calculated by multiplying the permutation value of each characteristic by its corresponding weight and then aggregating these values together to obtain the overall permutation value.

Step 22. Determine the ultimate ranking of each alternative.

The visual depiction of the research framework is given in Fig. 1.

4. Case study

The adaptability and effectiveness of the suggested approach are illustrated with a case study in this section. We take on the difficult task of determining which APMs are most useful for Pakistani RSM to validate our work. APMs are tools that can aid in managing traffic safety by predicting the anticipated frequency and severity of collisions at a specific road intersection. The effectiveness of road safety measures can be assessed using APMs and investments in road safety can be prioritized. APMs are based on a statistical analysis of crash issues data and aspects of the road network including its geometry, volume of traffic, speed limit, and pavement quality, among others. Road accidents in Pakistan result in more than 30,000 fatalities and 500,000 injuries annually making them significant economic and public health issues. APMs are not, however, routinely used in Pakistan to manage traffic safety. The lack of data on crashes and road infrastructure as well as their poor quality are some of the causes for not using the APMs in Pakistan. Another factor is the lack of knowledge and expertise regarding the advantages and applications of APMs among road authorities, road designers, and practitioners of road safety. For different types of roads in Pakistan, including urban roads, rural roads, and motorways, some studies have tried to develop and apply APMs. Various approaches and data sources, including regression models, machine learning algorithms, cluster analysis, etc., have been used in the existing studies. These studies' scope and validity are constrained, and neither their findings nor their implementation has received much attention. As a result, there is a need for more thorough and reliable APMs that can capture the unique characteristics and difficulties of road safety in Pakistan.

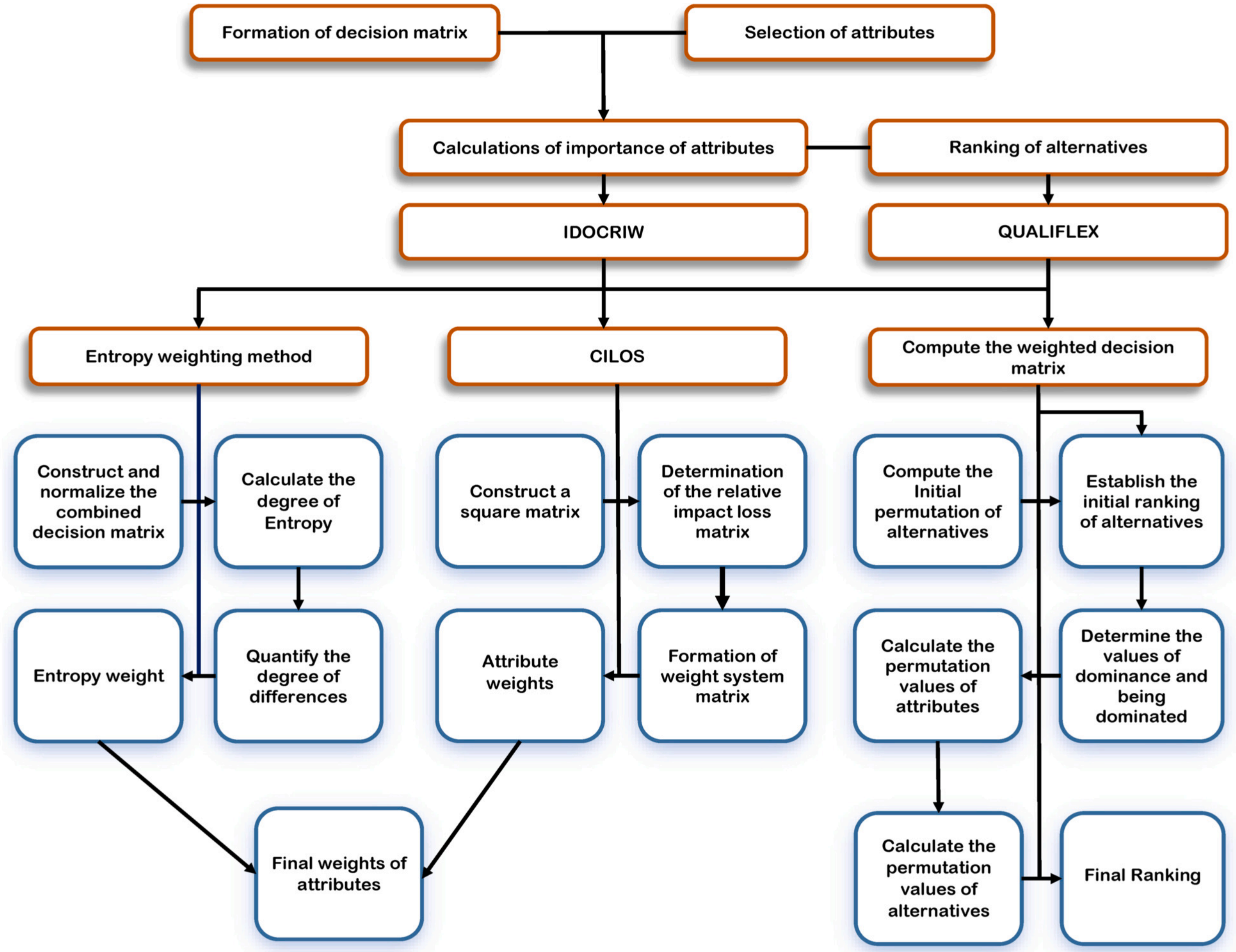


Fig. 1. Visual depiction of the research framework.

4.1. Ethical considerations related to RSM decisions

Ethical considerations for RSM are crucial to ensuring the safety of all individuals on the road. One ethical consideration is the responsibility of drivers to follow traffic laws and regulations. This includes obeying speed limits, stopping at red lights and stop signs, and yielding to pedestrians. Drivers who fail to follow these laws not only put themselves at risk but also endanger others on the road. Another ethical consideration is the responsibility of the government and transportation agencies to provide safe and well-maintained roads. This includes ensuring that roads are properly designed and constructed, regularly inspected and maintained, and equipped with appropriate signage and traffic controls. It is also important for these agencies to collect and analyze data on road safety to identify areas where improvements can be made. Ethical considerations for RSM are essential to promoting safe driving practices and ensure the safety of all individuals on the road. Ethical considerations related to RSM decisions have various aspects that involve moral principles, fairness, and responsibility in making choices concerning road safety. Some ethical considerations related to RSM decisions are:

- Human life and well-being: The primary ethical concern is prioritizing human life and well-being. Decisions should aim to minimize harm, injuries, and fatalities on roads, considering the impact on individuals, families, and communities.
- Equity and fairness: Ensuring fairness in road safety decisions is crucial. This includes addressing disparities in safety measures across different demographics or regions, aiming for equitable distribution of resources, and avoiding decisions that disproportionately affect certain groups.
- Transparency and accountability: Ethical RSM requires transparency in decision-making processes, including public involvement and open communication about policies, regulations, and their enforcement. Holding accountable those responsible for implementing and enforcing road safety measures is vital.
- Balancing priorities: Ethical considerations involve balancing various needs and priorities. For instance, the need for safety might sometimes conflict with other interests like economic considerations or personal freedoms. Finding a fair balance without compromising safety is key.

Table 2
Linguistic variables and 2TLq-ROFNs.

Linguistic variables	2TLq-ROFNs
Extremely high (EH) value	$((s_8,0),(s_0,0))$
Very high (VH) value	$((s_7,0),(s_1,0))$
High (H) value	$((s_6,0),(s_2,0))$
Medium high (MH) value	$((s_5,0),(s_3,0))$
Fair (F) value	$((s_4,0),(s_4,0))$
Medium low (ML) value	$((s_3,0),(s_5,0))$
Low (L) value	$((s_2,0),(s_6,0))$
Very low (VL) value	$((s_1,0),(s_7,0))$
Extremely low (EL) value	$((s_0,0),(s_8,0))$

– Risk assessment and mitigation: Ethical decision-making involves accurately assessing risks and taking measures to mitigate them. This includes implementing appropriate safety standards, maintaining infrastructure, and employing effective enforcement methods while considering the costs and benefits of these measures.
– Legal and regulatory compliance: Compliance with laws, regulations, and industry standards is an integral ethical consideration. Striving to exceed minimum legal requirements to enhance safety whenever possible demonstrates a commitment to ethical RSM.
– Continuous improvement and adaptation: Ethical RSM necessitates a commitment to continuous improvement and adaptation based on evolving technologies, data, and best practices. Being open to innovation and advancements for safer road systems is crucial.
– Public engagement and education: Ethical considerations emphasize the importance of engaging the public and educating individuals about road safety. Empowering communities with knowledge and fostering a culture of responsible behavior on the road is essential.

By incorporating these ethical considerations, RSM decisions can be more comprehensive, fair, and aimed at preserving lives and well-being while addressing the complexities and challenges of a modern transportation system.

4.2. Criteria for case study

This part explains the criteria for choosing the data for selecting APMs for RSM in Pakistan and how our suggested framework can be implemented in other countries to prevent road accidents. The selection of APMs for RSM in Pakistan can be categorized as a conventional MAGDM problem. The present study introduces the 2TLq-ROF-IDOCRIW-QUALIFLEX approach to assess the effectiveness of APMs in the context of RSM in Pakistan. Here are eight possible APMs: curve-fitting models ($\mathbb{A}_1$), artificial intelligence techniques ($\mathbb{A}_2$), text mining analysis ($\mathbb{A}_3$), modern data sources ($\mathbb{A}_4$), road infrastructure and design factors ($\mathbb{A}_5$), vehicle-related factors ($\mathbb{A}_6$), environmental-related factors ($\mathbb{A}_7$), and man-related factors ($\mathbb{A}_8$). APMs can be selected based on the following attributes: visualization and reporting ($\mathbb{N}_1$), feature importance analysis ($\mathbb{N}_2$), machine learning algorithms ($\mathbb{N}_3$), and real-time updates ($\mathbb{N}_4$). Moreover, the weights of four attributes calculated with the IDOCRIW method are as follows: $\omega = (0.2413, 0.2879, 0.1148, 0.3560)^T$. However, DMs believed that 2TL information would be a better choice for them. To select the optimal APMs in RSM, five DMs $\mathcal{D} = \{\mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3, \mathcal{D}_4, \mathcal{D}_5\}$ are invited to provide their assessments using the 2TL terms as described in Table 2 and their abbreviations according to different perspectives of DMs are listed in Table 3. The weight vector for these five DMs is $\omega' = (0.2192, 0.2134, 0.1930, 0.1906, 0.1838)^T$. Hence, DMs can assess the APMs for RSM in Pakistan by providing 2TLq-ROFNs according to their preferences. This case study presents a strategic way to evaluate different APMs utilizing the different viewpoints of DMs. By using the linguistic variables five DMs provide their 2TLq-ROFNs according to four attributes in which symbolic translation is always zero for the sake of simplicity to aggregate data. The assessment values provided by the five DMs for each attribute of each alternative are represented in the decision matrix $\hbar^\kappa = [\Lambda^\kappa_{\varrho\sigma}]_{8\times 4} (\kappa = 1, 2, 3, 4, 5)$ as shown in Tables 4–8.

4.3. Evaluation of case study

This subsection outlines the evaluation approach employed for the selection of APMs in RSM. The 2TLq-ROF-IDOCRIW-QUALIFLEX method which utilizes the 2TLq-ROFWPA operator is used for this purpose as detailed in Section 3.

Step 1. We construct the 2TLq-ROF evaluation matrix $\hbar^\kappa = [\Lambda^\kappa_{\varrho\sigma}]_{8\times 4} = ((s_{\mathfrak{p}_{\varrho\sigma}}, \Psi_{\varrho\sigma})^\kappa, (s_{\mathfrak{l}_{\varrho\sigma}}, \Phi_{\varrho\sigma})^\kappa)_{8\times 4} (\varrho = 1, 2, 3, \ldots, 8, \sigma = 1, 2, 3, 4,$ and $\kappa = 1, 2, 3, 4, 5)$. This matrix describes the assessments of five DMs as computed in Tables 4-8.

Step 2. According to Equation (6), we have the support $\mathrm{Sup}(\Lambda^\kappa_{\varrho\sigma}, \Lambda^{\mathfrak{d}}_{\varrho\sigma}) (\varrho = 1, 2, \ldots, 8;\ \sigma = 1, 2, 3, 4;\ \kappa = 1, 2, 3, 4, 5;\ \mathfrak{d} = 1, 2, 3, 4, 5$ and $\kappa \neq \mathfrak{d})$. For ease of use, we represent $\mathrm{Sup}(\Lambda^\kappa_{\varrho\sigma}, \Lambda^{\mathfrak{d}}_{\varrho\sigma})$ by $\mathrm{Sup}^{\kappa\mathfrak{d}} (\kappa = 1, 2, 3, 4, 5;\ \mathfrak{d} = 1, 2, 3, 4, 5$ and $\kappa \neq \mathfrak{d})$. So, we can get (suppose $q = 4,\ k = 8$):

Table 3
Linguistic assessing matrix by five DMs.

DMs	Alternatives	Attributes				DMs	Alternatives	Attributes			
		$\mathbb{N}_1$	$\mathbb{N}_2$	$\mathbb{N}_3$	$\mathbb{N}_4$			$\mathbb{N}_1$	$\mathbb{N}_2$	$\mathbb{N}_3$	$\mathbb{N}_4$
	$\mathbb{A}_1$	F	L	VH	VL		$\mathbb{A}_1$	VL	F	H	ML
	$\mathbb{A}_2$	H	ML	L	MH		$\mathbb{A}_2$	L	VH	VL	MH
	$\mathbb{A}_3$	VH	VL	F	H		$\mathbb{A}_3$	F	H	ML	VL
$\mathcal{D}_1$	$\mathbb{A}_4$	VL	MH	L	VH	$\mathcal{D}_2$	$\mathbb{A}_4$	F	ML	L	VH
	$\mathbb{A}_5$	L	H	ML	F		$\mathbb{A}_5$	VH	VL	ML	H
	$\mathbb{A}_6$	L	VH	VL	MH		$\mathbb{A}_6$	H	ML	L	MH
	$\mathbb{A}_7$	ML	L	MH	F		$\mathbb{A}_7$	MH	L	VH	F
	$\mathbb{A}_8$	MH	F	H	ML		$\mathbb{A}_8$	ML	H	F	L
	$\mathbb{A}_1$	F	ML	L	MH		$\mathbb{A}_1$	H	L	ML	F
	$\mathbb{A}_2$	L	VH	F	H		$\mathbb{A}_2$	L	H	VL	ML
	$\mathbb{A}_3$	VH	VL	H	F		$\mathbb{A}_3$	ML	VH	F	H
$\mathcal{D}_3$	$\mathbb{A}_4$	VH	F	ML	VL	$\mathcal{D}_4$	$\mathbb{A}_4$	F	MH	VL	L
	$\mathbb{A}_5$	H	MH	VL	ML		$\mathbb{A}_5$	VL	ML	F	L
	$\mathbb{A}_6$	MH	L	VL	H		$\mathbb{A}_6$	MH	H	L	VL
	$\mathbb{A}_7$	L	ML	H	VH		$\mathbb{A}_7$	VH	VL	MH	H
	$\mathbb{A}_8$	VL	H	VH	ML		$\mathbb{A}_8$	H	F	VH	MH
	$\mathbb{A}_1$	VL	H	F	L						
	$\mathbb{A}_2$	H	L	MH	VL						
	$\mathbb{A}_3$	ML	VL	MH	L						
$\mathcal{D}_5$	$\mathbb{A}_4$	F	ML	H	VH						
	$\mathbb{A}_5$	ML	H	VH	L						
	$\mathbb{A}_6$	ML	L	VL	MH						
	$\mathbb{A}_7$	L	F	H	VL						
	$\mathbb{A}_8$	H	VH	MH	ML						

Table 4
The assessing matrix with 2TLq-ROFNs by $\mathcal{D}_1$.

Alternatives	Attributes			
	$\mathbb{N}_1$	$\mathbb{N}_2$	$\mathbb{N}_3$	$\mathbb{N}_4$
$\mathbb{A}_1$	$((s_4,0),(s_4,0))$	$((s_2,0),(s_6,0))$	$((s_7,0),(s_1,0))$	$((s_1,0),(s_7,0))$
$\mathbb{A}_2$	$((s_6,0),(s_2,0))$	$((s_3,0),(s_5,0))$	$((s_2,0),(s_6,0))$	$((s_5,0),(s_3,0))$
$\mathbb{A}_3$	$((s_7,0),(s_1,0))$	$((s_1,0),(s_7,0))$	$((s_4,0),(s_4,0))$	$((s_6,0),(s_2,0))$
$\mathbb{A}_4$	$((s_1,0),(s_7,0))$	$((s_5,0),(s_3,0))$	$((s_2,0),(s_6,0))$	$((s_7,0),(s_1,0))$
$\mathbb{A}_5$	$((s_2,0),(s_6,0))$	$((s_6,0),(s_2,0))$	$((s_3,0),(s_5,0))$	$((s_4,0),(s_4,0))$
$\mathbb{A}_6$	$((s_2,0),(s_6,0))$	$((s_7,0),(s_1,0))$	$((s_1,0),(s_7,0))$	$((s_5,0),(s_3,0))$
$\mathbb{A}_7$	$((s_3,0),(s_5,0))$	$((s_2,0),(s_6,0))$	$((s_5,0),(s_3,0))$	$((s_4,0),(s_4,0))$
$\mathbb{A}_8$	$((s_5,0),(s_3,0))$	$((s_4,0),(s_4,0))$	$((s_6,0),(s_2,0))$	$((s_3,0),(s_5,0))$

Table 5
The assessing matrix with 2TLq-ROFNs by $\mathcal{D}_2$.

Alternatives	Attributes			
	$\mathbb{N}_1$	$\mathbb{N}_2$	$\mathbb{N}_3$	$\mathbb{N}_4$
$\mathbb{A}_1$	$((s_1,0),(s_7,0))$	$((s_4,0),(s_4,0))$	$((s_6,0),(s_2,0))$	$((s_3,0),(s_5,0))$
$\mathbb{A}_2$	$((s_2,0),(s_6,0))$	$((s_7,0),(s_1,0))$	$((s_1,0),(s_7,0))$	$((s_5,0),(s_3,0))$
$\mathbb{A}_3$	$((s_4,0),(s_4,0))$	$((s_6,0),(s_2,0))$	$((s_3,0),(s_5,0))$	$((s_1,0),(s_7,0))$
$\mathbb{A}_4$	$((s_4,0),(s_4,0))$	$((s_5,0),(s_3,0))$	$((s_2,0),(s_6,0))$	$((s_7,0),(s_1,0))$
$\mathbb{A}_5$	$((s_7,0),(s_1,0))$	$((s_1,0),(s_7,0))$	$((s_3,0),(s_5,0))$	$((s_6,0),(s_2,0))$
$\mathbb{A}_6$	$((s_6,0),(s_2,0))$	$((s_3,0),(s_5,0))$	$((s_2,0),(s_6,0))$	$((s_5,0),(s_3,0))$
$\mathbb{A}_7$	$((s_5,0),(s_3,0))$	$((s_2,0),(s_6,0))$	$((s_7,0),(s_1,0))$	$((s_4,0),(s_4,0))$
$\mathbb{A}_8$	$((s_3,0),(s_5,0))$	$((s_6,0),(s_2,0))$	$((s_4,0),(s_4,0))$	$((s_2,0),(s_6,0))$

$$
\mathrm{Sup}^{12}=\mathrm{Sup}^{21}=
\begin{array}{c}
\\
\mathbb{A}_1\\ \mathbb{A}_2\\ \mathbb{A}_3\\ \mathbb{A}_4\\ \mathbb{A}_5\\ \mathbb{A}_6\\ \mathbb{A}_7\\ \mathbb{A}_8
\end{array}
\begin{array}{c}
\begin{array}{cccc}
\quad\mathbb{N}_1\quad & \quad\mathbb{N}_2\quad & \quad\mathbb{N}_3\quad & \quad\mathbb{N}_4\quad
\end{array}\\
\begin{bmatrix}
(s_6,-0.3438) & (s_7,-0.2500) & (s_7,-0.0938) & (s_6,0.1875)\\
(s_6,-0.5000) & (s_5,0.1250) & (s_7,-0.0938) & (s_8,0.0000)\\
(s_6,-0.3438) & (s_4,0.4063) & (s_7,0.4688) & (s_4,0.4063)\\
(s_6,-0.3438) & (s_8,0.0000) & (s_8,0.0000) & (s_8,0.0000)\\
(s_4,0.4063) & (s_4,0.4063) & (s_8,0.0000) & (s_7,-0.2500)\\
(s_6,-0.5000) & (s_5,0.1250) & (s_7,-0.0938) & (s_8,0.0000)\\
(s_7,-0.0625) & (s_8,0.0000) & (s_6,0.1875) & (s_8,0.0000)\\
(s_7,-0.0625) & (s_7,-0.2500) & (s_7,-0.2500) & (s_7,0.2813)
\end{bmatrix}
\end{array}
$$

Table 6
The assessing matrix with 2TLq-ROFNs by $\mathcal{D}_3$.

Alternatives	Attributes			
	$\mathbb{N}_1$	$\mathbb{N}_2$	$\mathbb{N}_3$	$\mathbb{N}_4$
$\mathbb{A}_1$	$((s_4,0),(s_4,0))$	$((s_3,0),(s_5,0))$	$((s_2,0),(s_6,0))$	$((s_5,0),(s_3,0))$
$\mathbb{A}_2$	$((s_2,0),(s_6,0))$	$((s_7,0),(s_1,0))$	$((s_4,0),(s_4,0))$	$((s_6,0),(s_2,0))$
$\mathbb{A}_3$	$((s_7,0),(s_1,0))$	$((s_1,0),(s_7,0))$	$((s_6,0),(s_2,0))$	$((s_4,0),(s_4,0))$
$\mathbb{A}_4$	$((s_7,0),(s_1,0))$	$((s_4,0),(s_4,0))$	$((s_3,0),(s_5,0))$	$((s_1,0),(s_7,0))$
$\mathbb{A}_5$	$((s_6,0),(s_2,0))$	$((s_5,0),(s_3,0))$	$((s_1,0),(s_7,0))$	$((s_3,0),(s_5,0))$
$\mathbb{A}_6$	$((s_5,0),(s_3,0))$	$((s_2,0),(s_6,0))$	$((s_1,0),(s_7,0))$	$((s_6,0),(s_2,0))$
$\mathbb{A}_7$	$((s_2,0),(s_6,0))$	$((s_5,0),(s_3,0))$	$((s_6,0),(s_2,0))$	$((s_7,0),(s_1,0))$
$\mathbb{A}_8$	$((s_1,0),(s_7,0))$	$((s_6,0),(s_2,0))$	$((s_7,0),(s_1,0))$	$((s_3,0),(s_5,0))$

Table 7
The assessing matrix with 2TLq-ROFNs by $\mathcal{D}_4$.

Alternatives	Attributes			
	$\mathbb{N}_1$	$\mathbb{N}_2$	$\mathbb{N}_3$	$\mathbb{N}_4$
$\mathbb{A}_1$	$((s_6,0),(s_2,0))$	$((s_2,0),(s_6,0))$	$((s_3,0),(s_5,0))$	$((s_4,0),(s_4,0))$
$\mathbb{A}_2$	$((s_2,0),(s_6,0))$	$((s_6,0),(s_2,0))$	$((s_1,0),(s_7,0))$	$((s_3,0),(s_5,0))$
$\mathbb{A}_3$	$((s_3,0),(s_5,0))$	$((s_7,0),(s_1,0))$	$((s_4,0),(s_4,0))$	$((s_6,0),(s_2,0))$
$\mathbb{A}_4$	$((s_4,0),(s_4,0))$	$((s_5,0),(s_3,0))$	$((s_1,0),(s_7,0))$	$((s_2,0),(s_6,0))$
$\mathbb{A}_5$	$((s_1,0),(s_7,0))$	$((s_3,0),(s_5,0))$	$((s_4,0),(s_4,0))$	$((s_6,0),(s_2,0))$
$\mathbb{A}_6$	$((s_5,0),(s_3,0))$	$((s_6,0),(s_2,0))$	$((s_2,0),(s_6,0))$	$((s_1,0),(s_7,0))$
$\mathbb{A}_7$	$((s_7,0),(s_1,0))$	$((s_1,0),(s_7,0))$	$((s_5,0),(s_3,0))$	$((s_6,0),(s_2,0))$
$\mathbb{A}_8$	$((s_6,0),(s_2,0))$	$((s_4,0),(s_4,0))$	$((s_7,0),(s_1,0))$	$((s_5,0),(s_3,0))$

Table 8
The assessing matrix with 2TLq-ROFNs by $\mathcal{D}_5$.

Alternatives	Attributes			
	$\mathbb{N}_1$	$\mathbb{N}_2$	$\mathbb{N}_3$	$\mathbb{N}_4$
$\mathbb{A}_1$	$((s_1,0),(s_7,0))$	$((s_6,0),(s_2,0))$	$((s_4,0),(s_4,0))$	$((s_2,0),(s_6,0))$
$\mathbb{A}_2$	$((s_6,0),(s_2,0))$	$((s_2,0),(s_6,0))$	$((s_5,0),(s_3,0))$	$((s_1,0),(s_7,0))$
$\mathbb{A}_3$	$((s_3,0),(s_5,0))$	$((s_1,0),(s_7,0))$	$((s_5,0),(s_3,0))$	$((s_2,0),(s_6,0))$
$\mathbb{A}_4$	$((s_4,0),(s_4,0))$	$((s_3,0),(s_5,0))$	$((s_6,0),(s_2,0))$	$((s_7,0),(s_1,0))$
$\mathbb{A}_5$	$((s_5,0),(s_3,0))$	$((s_6,0),(s_2,0))$	$((s_7,0),(s_1,0))$	$((s_2,0),(s_6,0))$
$\mathbb{A}_6$	$((s_3,0),(s_5,0))$	$((s_2,0),(s_6,0))$	$((s_1,0),(s_7,0))$	$((s_5,0),(s_3,0))$
$\mathbb{A}_7$	$((s_2,0),(s_6,0))$	$((s_4,0),(s_4,0))$	$((s_6,0),(s_2,0))$	$((s_1,0),(s_7,0))$
$\mathbb{A}_8$	$((s_6,0),(s_2,0))$	$((s_7,0),(s_1,0))$	$((s_5,0),(s_3,0))$	$((s_3,0),(s_5,0))$

$$\mathrm{Sup}^{13}=\mathrm{Sup}^{31}=\begin{array}{c} \\ \mathbb{A}_1\\ \mathbb{A}_2\\ \mathbb{A}_3\\ \mathbb{A}_4\\ \mathbb{A}_5\\ \mathbb{A}_6\\ \mathbb{A}_7\\ \mathbb{A}_8\end{array}\begin{array}{c}\begin{array}{cccc}\mathbb{N}_1 & \mathbb{N}_2 & \mathbb{N}_3 & \mathbb{N}_4\end{array}\\ \begin{bmatrix}(s_8,0.0000) & (s_7,0.2813) & (s_4,0.4063) & (s_5,0.1250)\\ (s_6,-0.5000) & (s_5,0.1250) & (s_7,-0.2500) & (s_7,0.2813)\\ (s_8,0.0000) & (s_8,0.0000) & (s_7,-0.2500) & (s_7,-0.2500)\\ (s_3,0.3125) & (s_7,0.4688) & (s_7,0.2813) & (s_3,0.3125)\\ (s_6,-0.5000) & (s_7,0.2813) & (s_6,0.1875) & (s_7,0.4688)\\ (s_6,0.2188) & (s_4,0.4063) & (s_8,0.0000) & (s_7,0.2813)\\ (s_7,0.2813) & (s_6,0.2188) & (s_7,0.2813) & (s_6,-0.3438)\\ (s_5,0.1250) & (s_7,-0.2500) & (s_7,-0.0938) & (s_8,0.0000)\end{bmatrix}\end{array}$$

$$\mathrm{Sup}^{14}=\mathrm{Sup}^{41}=\begin{array}{c} \\ \mathbb{A}_1\\ \mathbb{A}_2\\ \mathbb{A}_3\\ \mathbb{A}_4\\ \mathbb{A}_5\\ \mathbb{A}_6\\ \mathbb{A}_7\\ \mathbb{A}_8\end{array}\begin{array}{c}\begin{array}{cccc}\mathbb{N}_1 & \mathbb{N}_2 & \mathbb{N}_3 & \mathbb{N}_4\end{array}\\ \begin{bmatrix}(s_7,-0.2500) & (s_8,0.0000) & (s_5,0.1250) & (s_6,-0.3438)\\ (s_6,-0.5000) & (s_6,0.2188) & (s_7,-0.0938) & (s_7,-0.0625)\\ (s_5,0.1250) & (s_3,0.3125) & (s_8,0.0000) & (s_8,0.0000)\\ (s_6,-0.3438) & (s_8,0.0000) & (s_7,-0.0938) & (s_4,0.4063)\\ (s_7,-0.0938) & (s_6,0.2188) & (s_7,0.4688) & (s_7,-0.2500)\\ (s_6,0.2188) & (s_7,-0.0938) & (s_7,-0.0938) & (s_5,0.1250)\\ (s_5,0.1250) & (s_7,-0.0938) & (s_8,0.0000) & (s_7,-0.2500)\\ (s_7,0.2813) & (s_8,0.0000) & (s_7,-0.0938) & (s_7,-0.0625)\end{bmatrix}\end{array}$$

$$
\mathrm{Sup}^{15}=\mathrm{Sup}^{51}=\begin{array}{c} \\ \mathbb{A}_1\\ \mathbb{A}_2\\ \mathbb{A}_3\\ \mathbb{A}_4\\ \mathbb{A}_5\\ \mathbb{A}_6\\ \mathbb{A}_7\\ \mathbb{A}_8 \end{array}
\begin{array}{c} \begin{array}{cccc} \mathbb{N}_1 & \mathbb{N}_2 & \mathbb{N}_3 & \mathbb{N}_4 \end{array}\\
\begin{bmatrix}
(s_6,-0.3438) & (s_6,-0.5000) & (s_6,-0.3438) & (s_7,-0.0938)\\
(s_8,0.0000) & (s_7,0.2813) & (s_6,0.2188) & (s_5,0.1250)\\
(s_5,0.1250) & (s_8,0.0000) & (s_7,0.4688) & (s_6,-0.5000)\\
(s_6,-0.3438) & (s_7,-0.0625) & (s_6,-0.5000) & (s_8,0.0000)\\
(s_6,0.2188) & (s_8,0.0000) & (s_5,0.1250) & (s_7,-0.2500)\\
(s_7,0.2813) & (s_4,0.4063) & (s_8,0.0000) & (s_8,0.0000)\\
(s_7,0.2813) & (s_7,-0.2500) & (s_7,0.2813) & (s_6,-0.3438)\\
(s_7,0.2813) & (s_6,-0.3438) & (s_7,0.2813) & (s_8,0.0000)
\end{bmatrix}\end{array}
$$

$$
\mathrm{Sup}^{23}=\mathrm{Sup}^{32}=\begin{array}{c} \\ \mathbb{A}_1\\ \mathbb{A}_2\\ \mathbb{A}_3\\ \mathbb{A}_4\\ \mathbb{A}_5\\ \mathbb{A}_6\\ \mathbb{A}_7\\ \mathbb{A}_8 \end{array}
\begin{array}{c} \begin{array}{cccc} \mathbb{N}_1 & \mathbb{N}_2 & \mathbb{N}_3 & \mathbb{N}_4 \end{array}\\
\begin{bmatrix}
(s_6,-0.3438) & (s_7,0.4688) & (s_6,-0.5000) & (s_7,-0.0625)\\
(s_8,0.0000) & (s_8,0.0000) & (s_6,-0.3438) & (s_7,0.2813)\\
(s_6,-0.3438) & (s_4,0.4063) & (s_6,0.2188) & (s_6,-0.3438)\\
(s_6,-0.3438) & (s_7,0.4688) & (s_7,0.2813) & (s_3,0.3125)\\
(s_7,-0.0938) & (s_5,0.1250) & (s_6,0.1875) & (s_6,0.2188)\\
(s_7,0.2813) & (s_7,0.2813) & (s_7,-0.0938) & (s_7,0.2813)\\
(s_6,0.2188) & (s_6,0.2188) & (s_7,-0.0938) & (s_6,-0.3438)\\
(s_6,0.1875) & (s_8,0.0000) & (s_6,-0.3438) & (s_7,0.2813)
\end{bmatrix}\end{array}
$$

$$
\mathrm{Sup}^{24}=\mathrm{Sup}^{42}=\begin{array}{c} \\ \mathbb{A}_1\\ \mathbb{A}_2\\ \mathbb{A}_3\\ \mathbb{A}_4\\ \mathbb{A}_5\\ \mathbb{A}_6\\ \mathbb{A}_7\\ \mathbb{A}_8 \end{array}
\begin{array}{c} \begin{array}{cccc} \mathbb{N}_1 & \mathbb{N}_2 & \mathbb{N}_3 & \mathbb{N}_4 \end{array}\\
\begin{bmatrix}
(s_4,0.4063) & (s_7,-0.2500) & (s_6,0.2188) & (s_7,0.4688)\\
(s_8,0.0000) & (s_7,-0.0938) & (s_8,0.0000) & (s_7,-0.0625)\\
(s_7,0.4688) & (s_7,-0.0938) & (s_7,0.4688) & (s_4,0.4063)\\
(s_8,0.0000) & (s_8,0.0000) & (s_7,-0.0938) & (s_4,0.4063)\\
(s_3,0.3125) & (s_6,0.1875) & (s_7,0.4688) & (s_8,0.0000)\\
(s_7,0.2813) & (s_6,0.2188) & (s_8,0.0000) & (s_5,0.1250)\\
(s_6,0.1875) & (s_7,-0.0938) & (s_6,0.1875) & (s_7,-0.2500)\\
(s_6,0.2188) & (s_7,-0.2500) & (s_6,-0.3438) & (s_6,0.2188)
\end{bmatrix}\end{array}
$$

$$
\mathrm{Sup}^{25}=\mathrm{Sup}^{52}=\begin{array}{c} \\ \mathbb{A}_1\\ \mathbb{A}_2\\ \mathbb{A}_3\\ \mathbb{A}_4\\ \mathbb{A}_5\\ \mathbb{A}_6\\ \mathbb{A}_7\\ \mathbb{A}_8 \end{array}
\begin{array}{c} \begin{array}{cccc} \mathbb{N}_1 & \mathbb{N}_2 & \mathbb{N}_3 & \mathbb{N}_4 \end{array}\\
\begin{bmatrix}
(s_8,0.0000) & (s_7,-0.2500) & (s_7,-0.2500) & (s_7,0.2813)\\
(s_6,-0.5000) & (s_4,0.4063) & (s_5,0.1250) & (s_5,0.1250)\\
(s_7,0.4688) & (s_4,0.4063) & (s_7,-0.0625) & (s_7,-0.0938)\\
(s_8,0.0000) & (s_7,-0.0625) & (s_6,-0.5000) & (s_8,0.0000)\\
(s_6,0.1875) & (s_4,0.4063) & (s_5,0.1250) & (s_6,-0.5000)\\
(s_6,0.2188) & (s_7,0.2813) & (s_7,-0.0938) & (s_8,0.0000)\\
(s_6,0.2188) & (s_7,-0.2500) & (s_7,-0.0938) & (s_6,-0.3438)\\
(s_6,0.2188) & (s_7,-0.0938) & (s_7,0.4688) & (s_7,0.2813)
\end{bmatrix}\end{array}
$$

$$
\mathrm{Sup}^{34}=\mathrm{Sup}^{43}=\begin{array}{c} \\ \mathbb{A}_1\\ \mathbb{A}_2\\ \mathbb{A}_3\\ \mathbb{A}_4\\ \mathbb{A}_5\\ \mathbb{A}_6\\ \mathbb{A}_7\\ \mathbb{A}_8 \end{array}
\begin{array}{c} \begin{array}{cccc} \mathbb{N}_1 & \mathbb{N}_2 & \mathbb{N}_3 & \mathbb{N}_4 \end{array}\\
\begin{bmatrix}
(s_7,-0.2500) & (s_7,0.2813) & (s_7,0.2813) & (s_7,0.4688)\\
(s_8,0.0000) & (s_7,-0.0938) & (s_6,-0.3438) & (s_6,0.2188)\\
(s_5,0.1250) & (s_3,0.3125) & (s_7,-0.2500) & (s_7,-0.2500)\\
(s_6,-0.3438) & (s_7,0.4688) & (s_6,0.1875) & (s_7,-0.0938)\\
(s_4,0.4063) & (s_7,-0.0625) & (s_6,-0.3438) & (s_6,0.2188)\\
(s_8,0.0000) & (s_6,-0.5000) & (s_7,-0.0938) & (s_4,0.4063)\\
(s_4,0.4063) & (s_5,0.1250) & (s_7,0.2813) & (s_7,-0.0938)\\
(s_4,0.4063) & (s_7,-0.2500) & (s_8,0.0000) & (s_7,-0.0625)
\end{bmatrix}\end{array}
$$

$$
\mathrm{Sup}^{35}=\mathrm{Sup}^{53}=\begin{array}{c} \\ \mathbb{A}_1\\ \mathbb{A}_2\\ \mathbb{A}_3\\ \mathbb{A}_4\\ \mathbb{A}_5\\ \mathbb{A}_6\\ \mathbb{A}_7\\ \mathbb{A}_8 \end{array}
\begin{array}{c} \begin{array}{cccc} \mathbb{N}_1 & \mathbb{N}_2 & \mathbb{N}_3 & \mathbb{N}_4 \end{array}\\
\begin{bmatrix}
(s_6,-0.3438) & (s_6,0.2188) & (s_7,-0.2500) & (s_6,0.2188)\\
(s_6,-0.5000) & (s_4,0.4063) & (s_7,0.4688) & (s_4,0.4063)\\
(s_5,0.1250) & (s_8,0.0000) & (s_7,0.2813) & (s_7,-0.2500)\\
(s_6,-0.3438) & (s_7,0.4688) & (s_6,0.2188) & (s_3,0.3125)\\
(s_7,0.2813) & (s_7,0.2813) & (s_3,0.3125) & (s_7,0.2813)\\
(s_7,-0.0625) & (s_8,0.0000) & (s_8,0.0000) & (s_7,0.2813)\\
(s_8,0.0000) & (s_7,0.4688) & (s_8,0.0000) & (s_3,0.3125)\\
(s_4,0.4063) & (s_7,-0.0938) & (s_6,0.1875) & (s_8,0.0000)
\end{bmatrix}\end{array}
$$

$$\mathrm{Sup}^{45} = \mathrm{Sup}^{54} = \begin{array}{c} \\ \mathbb{A}_1 \\ \mathbb{A}_2 \\ \mathbb{A}_3 \\ \mathbb{A}_4 \\ \mathbb{A}_5 \\ \mathbb{A}_6 \\ \mathbb{A}_7 \\ \mathbb{A}_8 \end{array} \begin{array}{c} \begin{array}{cccc} \mathbb{N}_1 & \mathbb{N}_2 & \mathbb{N}_3 & \mathbb{N}_4 \end{array} \\ \begin{bmatrix} (s_4, 0.4063) & (s_6, -0.5000) & (s_7, 0.4688) & (s_7, -0.2500) \\ (s_6, -0.5000) & (s_6, -0.5000) & (s_5, 0.1250) & (s_6, 0.1875) \\ (s_8, 0.0000) & (s_3, 0.3125) & (s_7, 0.4688) & (s_6, -0.5000) \\ (s_8, 0.0000) & (s_7, -0.0625) & (s_4, 0.4063) & (s_4, 0.4063) \\ (s_5, 0.1250) & (s_6, 0.2188) & (s_6, -0.3438) & (s_6, -0.5000) \\ (s_7, -0.0625) & (s_6, -0.5000) & (s_7, -0.0938) & (s_5, 0.1250) \\ (s_4, 0.4063) & (s_6, -0.3438) & (s_7, 0.2813) & (s_4, 0.4063) \\ (s_8, 0.0000) & (s_6, -0.3438) & (s_6, 0.1875) & (s_7, -0.0625) \end{bmatrix} \end{array}$$

Step 3. Using Equation (7), we obtain the synthesis support matrices $\mathfrak{T}(\Lambda^{\kappa}_{\rho\sigma})$ of the 2TLq-ROFN. To simplify, we represent the values $\mathfrak{T}(\Lambda^{\kappa}_{\rho\sigma})(\rho = 1, 2, \ldots, 8;\ \sigma = 1, 2, 3, 4, 5;\ \kappa = 1, 2, 3, 4, 5)$ as a matrix $\mathfrak{T}^{\kappa}(\kappa = 1, 2, 3, 4, 5)$, shown below:

$$\mathfrak{T}^1 = \begin{array}{c} \\ \mathbb{A}_1 \\ \mathbb{A}_2 \\ \mathbb{A}_3 \\ \mathbb{A}_4 \\ \mathbb{A}_5 \\ \mathbb{A}_6 \\ \mathbb{A}_7 \\ \mathbb{A}_8 \end{array} \begin{array}{c} \begin{array}{cccc} \mathbb{N}_1 & \mathbb{N}_2 & \mathbb{N}_3 & \mathbb{N}_4 \end{array} \\ \begin{bmatrix} (s_5, 0.0772) & (s_5, 0.3814) & (s_4, 0.3406) & (s_5, -0.3430) \\ (s_5, -0.2461) & (s_5, -0.3936) & (s_5, 0.2359) & (s_5, 0.3767) \\ (s_5, -0.3302) & (s_5, -0.4139) & (s_6, -0.2059) & (s_5, -0.2213) \\ (s_4, -0.0359) & (s_6, -0.0514) & (s_5, 0.4397) & (s_5, -0.3433) \\ (s_4, 0.4611) & (s_5, 0.0013) & (s_5, 0.2669) & (s_5, 0.4091) \\ (s_5, -0.1025) & (s_4, 0.0703) & (s_6, -0.1955) & (s_6, -0.4403) \\ (s_5, 0.2009) & (s_5, 0.4644) & (s_6, -0.4112) & (s_5, 0.1250) \\ (s_5, 0.1957) & (s_5, 0.3076) & (s_5, 0.4280) & (s_6, -0.1095) \end{bmatrix} \end{array}$$

$$\mathfrak{T}^2 = \begin{array}{c} \\ \mathbb{A}_1 \\ \mathbb{A}_2 \\ \mathbb{A}_3 \\ \mathbb{A}_4 \\ \mathbb{A}_5 \\ \mathbb{A}_6 \\ \mathbb{A}_7 \\ \mathbb{A}_8 \end{array} \begin{array}{c} \begin{array}{cccc} \mathbb{N}_1 & \mathbb{N}_2 & \mathbb{N}_3 & \mathbb{N}_4 \end{array} \\ \begin{bmatrix} (s_5, -0.3583) & (s_5, 0.4483) & (s_5, 0.0013) & (s_5, 0.4571) \\ (s_5, 0.2853) & (s_5, -0.2064) & (s_5, 0.0723) & (s_5, 0.4231) \\ (s_5, 0.1278) & (s_4, -0.0575) & (s_6, -0.4640) & (s_4, 0.1667) \\ (s_5, 0.3267) & (s_6, -0.0050) & (s_5, 0.4861) & (s_5, -0.2969) \\ (s_4, 0.0674) & (s_4, -0.0558) & (s_5, 0.3133) & (s_5, 0.2155) \\ (s_5, 0.1417) & (s_5, 0.0523) & (s_6, -0.3591) & (s_6, -0.3939) \\ (s_5, 0.0433) & (s_6, -0.4892) & (s_5, 0.1379) & (s_5, 0.1714) \\ (s_5, 0.0432) & (s_6, -0.4205) & (s_5, 0.0221) & (s_6, -0.4751) \end{bmatrix} \end{array}$$

$$\mathfrak{T}^3 = \begin{array}{c} \\ \mathbb{A}_1 \\ \mathbb{A}_2 \\ \mathbb{A}_3 \\ \mathbb{A}_4 \\ \mathbb{A}_5 \\ \mathbb{A}_6 \\ \mathbb{A}_7 \\ \mathbb{A}_8 \end{array} \begin{array}{c} \begin{array}{cccc} \mathbb{N}_1 & \mathbb{N}_2 & \mathbb{N}_3 & \mathbb{N}_4 \end{array} \\ \begin{bmatrix} (s_5, 0.2868) & (s_6, -0.2793) & (s_5, -0.2320) & (s_5, 0.1704) \\ (s_5, 0.4485) & (s_5, -0.0432) & (s_5, 0.1375) & (s_5, 0.1450) \\ (s_5, -0.1206) & (s_5, -0.2043) & (s_5, 0.4315) & (s_5, 0.2138) \\ (s_4, 0.0508) & (s_6, 0.0273) & (s_5, 0.4722) & (s_3, 0.3582) \\ (s_5, -0.1425) & (s_5, 0.3503) & (s_4, 0.3636) & (s_5, 0.4878) \\ (s_6, -0.2831) & (s_5, 0.0384) & (s_6, 0.0141) & (s_5, 0.3280) \\ (s_5, 0.2334) & (s_5, 0.0398) & (s_6, -0.0720) & (s_4, 0.3721) \\ (s_4, 0.0935) & (s_6, -0.2573) & (s_5, 0.3830) & (s_6, 0.1001) \end{bmatrix} \end{array}$$

$$\mathfrak{T}^4 = \begin{array}{c} \\ \mathbb{A}_1 \\ \mathbb{A}_2 \\ \mathbb{A}_3 \\ \mathbb{A}_4 \\ \mathbb{A}_5 \\ \mathbb{A}_6 \\ \mathbb{A}_7 \\ \mathbb{A}_8 \end{array} \begin{array}{c} \begin{array}{cccc} \mathbb{N}_1 & \mathbb{N}_2 & \mathbb{N}_3 & \mathbb{N}_4 \end{array} \\ \begin{bmatrix} (s_5, -0.4675) & (s_6, -0.3898) & (s_5, 0.2285) & (s_6, -0.4842) \\ (s_5, 0.4677) & (s_5, 0.1807) & (s_5, 0.2547) & (s_5, 0.3386) \\ (s_5, 0.1768) & (s_3, 0.4480) & (s_6, 0.0229) & (s_5, 0.0075) \\ (s_6, -0.4909) & (s_6, 0.1774) & (s_5, -0.0083) & (s_4, 0.0489) \\ (s_4, 0.0131) & (s_5, 0.1655) & (s_5, 0.3623) & (s_5, 0.3979) \\ (s_6, -0.2639) & (s_5, -0.0867) & (s_6, -0.1767) & (s_4, 0.0095) \\ (s_4, 0.1041) & (s_5, 0.0164) & (s_6, -0.1824) & (s_5, 0.0628) \\ (s_5, 0.2439) & (s_6, -0.4636) & (s_5, 0.4022) & (s_5, 0.4618) \end{bmatrix} \end{array}$$

$$\mathfrak{T}^5 = \begin{array}{c} \\ \mathbb{A}_1 \\ \mathbb{A}_2 \\ \mathbb{A}_3 \\ \mathbb{A}_4 \\ \mathbb{A}_5 \\ \mathbb{A}_6 \\ \mathbb{A}_7 \\ \mathbb{A}_8 \end{array} \begin{array}{c} \begin{array}{cccc} \mathbb{N}_1 & \mathbb{N}_2 & \mathbb{N}_3 & \mathbb{N}_4 \end{array} \\ \begin{bmatrix} (s_5,-0.1215) & (s_5,-0.1054) & (s_5,0.4066) & (s_6,-0.4456) \\ (s_5,0.0371) & (s_4,0.4351) & (s_5,-0.1249) & (s_4,0.2468) \\ (s_5,0.2312) & (s_5,-0.1307) & (s_6,-0.0536) & (s_5,0.0304) \\ (s_6,-0.4365) & (s_6,-0.2351) & (s_4,0.4193) & (s_5,-0.0601) \\ (s_5,0.0657) & (s_5,0.2845) & (s_4,-0.0655) & (s_5,0.1069) \\ (s_6,-0.4156) & (s_5,0.1120) & (s_6,0.0877) & (s_6,-0.1571) \\ (s_5,0.3070) & (s_5,0.4396) & (s_6,0.0016) & (s_4,-0.0740) \\ (s_5,0.2983) & (s_5,0.1246) & (s_6,-0.4366) & (s_6,0.1737) \end{bmatrix} \end{array}$$

Step 4. Using Equation (8), we obtain the weighted power matrix of decision maker $\mathcal{D}_\kappa(\kappa = 1,2,3,4,5)$ associated with the 2TLq-ROFN. For simplicity, we represent the values $\zeta(\Lambda^\kappa_{\varrho\sigma})(\varrho = 1,2,\ldots,8;\ \sigma = 1,2,3,4;\ \kappa = 1,2,3,4,5)$ as a matrix $\zeta^\kappa(\kappa = 1,2,3,4,5)$, shown below:

$$\zeta^1 = \begin{array}{c} \\ \mathbb{A}_1 \\ \mathbb{A}_2 \\ \mathbb{A}_3 \\ \mathbb{A}_4 \\ \mathbb{A}_5 \\ \mathbb{A}_6 \\ \mathbb{A}_7 \\ \mathbb{A}_8 \end{array} \begin{array}{c} \begin{array}{cccc} \mathbb{N}_1 & \mathbb{N}_2 & \mathbb{N}_3 & \mathbb{N}_4 \end{array} \\ \begin{bmatrix} (s_2,-0.2202) & (s_2,-0.2508) & (s_2,-0.3262) & (s_2,-0.3256) \\ (s_2,-0.3043) & (s_2,-0.2718) & (s_2,-0.2308) & (s_2,-0.2130) \\ (s_2,-0.2920) & (s_2,-0.2092) & (s_2,-0.2396) & (s_2,-0.2523) \\ (s_2,-0.3686) & (s_2,-0.2508) & (s_2,-0.2122) & (s_2,-0.2032) \\ (s_2,-0.2490) & (s_2,-0.2362) & (s_2,-0.1931) & (s_2,-0.2354) \\ (s_2,-0.3114) & (s_2,-0.3488) & (s_2,-0.2541) & (s_2,-0.2097) \\ (s_2,-0.2171) & (s_2,-0.2250) & (s_2,-0.2578) & (s_2,-0.1959) \\ (s_2,-0.2171) & (s_2,-0.2662) & (s_2,-0.2365) & (s_2,-0.2379) \end{bmatrix} \end{array}$$

$$\zeta^2 = \begin{array}{c} \\ \mathbb{A}_1 \\ \mathbb{A}_2 \\ \mathbb{A}_3 \\ \mathbb{A}_4 \\ \mathbb{A}_5 \\ \mathbb{A}_6 \\ \mathbb{A}_7 \\ \mathbb{A}_8 \end{array} \begin{array}{c} \begin{array}{cccc} \mathbb{N}_1 & \mathbb{N}_2 & \mathbb{N}_3 & \mathbb{N}_4 \end{array} \\ \begin{bmatrix} (s_2,-0.3250) & (s_2,-0.2886) & (s_2,-0.2833) & (s_2,-0.2668) \\ (s_2,-0.2804) & (s_2,-0.2926) & (s_2,-0.2989) & (s_2,-0.2543) \\ (s_2,-0.2771) & (s_2,-0.3457) & (s_2,-0.3182) & (s_2,-0.3800) \\ (s_2,-0.2309) & (s_2,-0.2914) & (s_2,-0.2535) & (s_2,-0.2443) \\ (s_2,-0.3492) & (s_2,-0.4225) & (s_2,-0.2348) & (s_2,-0.3069) \\ (s_2,-0.3249) & (s_2,-0.2617) & (s_2,-0.3205) & (s_2,-0.2511) \\ (s_2,-0.2850) & (s_2,-0.2661) & (s_2,-0.3602) & (s_2,-0.2374) \\ (s_2,-0.2843) & (s_2,-0.2776) & (s_2,-0.3351) & (s_2,-0.3296) \end{bmatrix} \end{array}$$

$$\zeta^3 = \begin{array}{c} \\ \mathbb{A}_1 \\ \mathbb{A}_2 \\ \mathbb{A}_3 \\ \mathbb{A}_4 \\ \mathbb{A}_5 \\ \mathbb{A}_6 \\ \mathbb{A}_7 \\ \mathbb{A}_8 \end{array} \begin{array}{c} \begin{array}{cccc} \mathbb{N}_1 & \mathbb{N}_2 & \mathbb{N}_3 & \mathbb{N}_4 \end{array} \\ \begin{bmatrix} (s_2,-0.4078) & (s_2,-0.4209) & (s_2,-0.4753) & (s_2,-0.4659) \\ (s_2,-0.4257) & (s_2,-0.4361) & (s_2,-0.4539) & (s_2,-0.4539) \\ (s_2,-0.4712) & (s_2,-0.3969) & (s_2,-0.4907) & (s_2,-0.4088) \\ (s_1,0.4468) & (s_2,-0.4512) & (s_2,-0.4221) & (s_1,0.4197) \\ (s_2,-0.4092) & (s_2,-0.4054) & (s_1,0.4826) & (s_2,-0.4372) \\ (s_2,-0.4188) & (s_2,-0.4296) & (s_2,-0.4395) & (s_2,-0.4506) \\ (s_2,-0.4263) & (s_2,-0.4865) & (s_2,-0.4277) & (s_1,0.4974) \\ (s_1,0.4387) & (s_2,-0.4235) & (s_2,-0.4525) & (s_2,-0.4251) \end{bmatrix} \end{array}$$

$$\zeta^4 = \begin{array}{c} \\ \mathbb{A}_1 \\ \mathbb{A}_2 \\ \mathbb{A}_3 \\ \mathbb{A}_4 \\ \mathbb{A}_5 \\ \mathbb{A}_6 \\ \mathbb{A}_7 \\ \mathbb{A}_8 \end{array} \begin{array}{c} \begin{array}{cccc} \mathbb{N}_1 & \mathbb{N}_2 & \mathbb{N}_3 & \mathbb{N}_4 \end{array} \\ \begin{bmatrix} (s_1,0.4832) & (s_2,-0.4530) & (s_2,-0.4399) & (s_2,-0.4453) \\ (s_2,-0.4430) & (s_2,-0.4289) & (s_2,-0.4595) & (s_2,-0.4506) \\ (s_2,-0.4554) & (s_1,0.4164) & (s_2,-0.4439) & (s_2,-0.4531) \\ (s_2,-0.3982) & (s_2,-0.4541) & (s_2,-0.4973) & (s_1,0.4874) \\ (s_1,0.4678) & (s_2,-0.4470) & (s_2,-0.4176) & (s_2,-0.4669) \\ (s_2,-0.4362) & (s_2,-0.4640) & (s_2,-0.4799) & (s_1,0.3788) \\ (s_1,0.4215) & (s_1,0.4920) & (s_2,-0.4596) & (s_2,-0.4387) \\ (s_2,-0.4440) & (s_2,-0.4665) & (s_2,-0.4695) & (s_1,0.4849) \end{bmatrix} \end{array}$$

Table 9
Collective 2TLq-ROF evaluation matrix utilizing the 2TLq-ROFWPA operator.

Alternatives	Attributes			
	$\mathbb{N}_1$	$\mathbb{N}_2$	$\mathbb{N}_3$	$\mathbb{N}_4$
$\mathbb{A}_1$	$((s_4, 0.4201)(s_4, 0.3832))$	$((s_4, 0.2930)(s_4, 0.3708))$	$((s_6, -0.3624)(s_3, -0.0896))$	$((s_4, -0.2514)(s_5, -0.1798))$
$\mathbb{A}_2$	$((s_5, -0.0968)(s_4, -0.1065))$	$((s_6, 0.1808)(s_2, 0.2343))$	$((s_4, -0.3702)(s_5, 0.2113))$	$((s_5, -0.1086)(s_4, -0.4588))$
$\mathbb{A}_3$	$((s_6, -0.0062)(s_2, 0.4848))$	$((s_5, 0.4120(s_4, -0.1720))$	$((s_5, -0.2426)(s_3, 0.4860))$	$((s_5, 0.0322)(s_4, -0.3676))$
$\mathbb{A}_4$	$((s_5, 0.2160)(s_3, 0.4893))$	$((s_5, -0.3639)(s_3, 0.4790))$	$((s_4, 0.1013)(s_5, -0.0740))$	$((s_6, 0.4790)(s_2, -0.0292))$
$\mathbb{A}_5$	$((s_6, -0.3313)(s_3, -0.0005))$	$((s_5, 0.1759)(s_3, 0.3164))$	$((s_5, 0.0442)(s_4, -0.1292))$	$((s_5, 0.0456)(s_3, 0.3996))$
$\mathbb{A}_6$	$((s_5, -0.1181)(s_4, -0.4913))$	$((s_6, -0.4616)(s_3, 0.2265))$	$((s_2, -0.3731)(s_7, -0.4185))$	$((s_5, 0.0881)(s_3, 0.2095))$
$\mathbb{A}_7$	$((s_5, 0.2095)(s_4, -0.3883))$	$((s_4, -0.3346)(s_5, 0.0233))$	$((s_6, 0.0469)(s_2, 0.0492))$	$((s_6, -0.4285)(s_3, -0.0326))$
$\mathbb{A}_8$	$((s_5, 0.1402)(s_3, 0.3378))$	$((s_6, -0.1334)(s_2, 0.3443))$	$((s_6, 0.2542)(s_2, -0.0913))$	$((s_4, -0.3633)(s_5, -0.2758))$

Table 10
Matrix for calculating weights of attributes.

	Attributes			
	$\mathbb{N}_1$	$\mathbb{N}_2$	$\mathbb{N}_3$	$\mathbb{N}_4$
$\mathbb{A}_1$	$((s_3, 0)(s_2, 0))$	$((s_1, 0)(s_3, 0))$	$((s_3, 0)(s_4, 0))$	$((s_4, 0)(s_1, 0))$
$\mathbb{A}_2$	$((s_1, 0)(s_4, 0))$	$((s_4, 0)(s_5, 0))$	$((s_2, 0)(s_5, 0))$	$((s_4, 0)(s_3, 0))$
$\mathbb{A}_3$	$((s_2, 0)(s_6, 0))$	$((s_3, 0)(s_4, 0))$	$((s_6, 0)(s_2, 0))$	$((s_6, 0)(s_1, 0))$
$\mathbb{A}_4$	$((s_4, 0)(s_7, 0))$	$((s_4, 0)(s_3, 0))$	$((s_1, 0)(s_2, 0))$	$((s_5, 0)(s_4, 0))$

Table 11
The combined normalized matrix.

	Attributes			
	$\mathbb{N}_1$	$\mathbb{N}_2$	$\mathbb{N}_3$	$\mathbb{N}_4$
$\mathbb{A}_1$	0.6250	0.5000	0.8750	0.6250
$\mathbb{A}_2$	0.6250	1.1250	0.8750	0.8750
$\mathbb{A}_3$	1.0000	0.8750	1.0000	0.8750
$\mathbb{A}_4$	1.3750	0.8750	0.3750	1.1250

$$
\zeta^5 = \begin{array}{c} \\ \mathbb{A}_1 \\ \mathbb{A}_2 \\ \mathbb{A}_3 \\ \mathbb{A}_4 \\ \mathbb{A}_5 \\ \mathbb{A}_6 \\ \mathbb{A}_7 \\ \mathbb{A}_8 \end{array}
\begin{array}{c} \begin{array}{cccc} \mathbb{N}_1 & \mathbb{N}_2 & \mathbb{N}_3 & \mathbb{N}_4 \end{array} \\
\begin{bmatrix}
(s_1, 0.4697) & (s_1, 0.4133) & (s_2, -0.4753) & (s_2, -0.4964) \\
(s_1, 0.4534) & (s_1, 0.4294) & (s_1, 0.4430) & (s_1, 0.3718) \\
(s_1, 0.4956) & (s_2, -0.4646) & (s_1, 0.4924) & (s_1, 0.4943) \\
(s_2, -0.4492) & (s_1, 0.4474) & (s_1, 0.3852) & (s_2, -0.4596) \\
(s_2, -0.4605) & (s_2, -0.4889) & (s_1, 0.3629) & (s_1, 0.4463) \\
(s_1, 0.4913) & (s_2, -0.4960) & (s_1, 0.4939) & (s_2, -0.4674) \\
(s_2, -0.4930) & (s_1, 0.4856) & (s_2, -0.4948) & (s_1, 0.3746) \\
(s_2, -0.4934) & (s_1, 0.4338) & (s_1, 0.4936) & (s_2, -0.4923)
\end{bmatrix} \end{array}
$$

Step 5. Using the 2TLq-ROFWPA operator from Equation (9), we combine all the individual 2TLq-ROF decision matrices into a collective decision matrix as demonstrated in Table 9.

Step 6. Since all the attributes are of benefit type there is no need to normalize the aggregated decision matrix. Therefore, by utilizing Equation (10) the computed results in Table 9 remain unchanged.

Step 7. We construct a decision matrix $\mathfrak{R}$ that contains 4 attributes and 4 alternatives as shown in Table 10.

Step 8. After obtaining the combined values we normalize the matrix (since all the attributes are of benefit type there is no need to normalize the combined decision matrix) as shown in Table 11.

Step 9. Using Equation (14), we calculate the degree of Entropy $\mathcal{E}_\sigma$ for each attribute $\mathbb{N}_\sigma$ as shown in Table 12.

Step 10. Using Equation (15), we determine the degree of differences $\mathbb{D}_\sigma$ for each attribute $\mathbb{N}_\sigma$ as shown in Table 12.

Step 11. Using Equation (16), we calculate the Entropy weights $\mathcal{W}_\sigma$ for each attribute $\mathbb{N}_\sigma$ as presented in Table 12.

Step 12. The square matrix $\mathcal{B}$ is formed using Equation (17) and the results are shown below:

$$
\mathcal{B} = \begin{array}{c} \\ \mathbb{N}_1 \\ \mathbb{N}_2 \\ \mathbb{N}_3 \\ \mathbb{N}_4 \end{array}
\begin{array}{c} \begin{array}{cccc} \mathbb{N}_1 & \mathbb{N}_2 & \mathbb{N}_3 & \mathbb{N}_4 \end{array} \\
\begin{bmatrix}
0.3793 & 0.2593 & 0.1200 & 0.321 \\
0.1724 & 0.3333 & 0.2800 & 0.250 \\
0.2759 & 0.2593 & 0.3200 & 0.250 \\
0.3793 & 0.2593 & 0.1200 & 0.321
\end{bmatrix} \end{array}
$$

Table 12
Weights calculated by the Entropy method.

	Degree of Entropy	Degree of differences	Entropy weights
	$\mathcal{E}_\sigma$	$\mathbb{D}_\sigma$	$\mathcal{W}_\sigma$
$\mathbb{N}_1$	0.1079	0.8921	0.3130
$\mathbb{N}_2$	0.3230	0.6770	0.2375
$\mathbb{N}_3$	0.4339	0.5661	0.1986
$\mathbb{N}_4$	0.2849	0.7151	0.2509

Table 13
The CILOS weights.

	$\mathbb{Q}_1$	$\mathbb{Q}_2$	$\mathbb{Q}_3$	$\mathbb{Q}_4$
$\mathbb{Q}_\sigma$	0.1937	0.3045	0.1452	0.3565

Table 14
The IDOCRIW weights.

	ω_1	ω_2	ω_3	ω_4
ω_σ	0.2413	0.2879	0.1148	0.3560

Table 15
The possible permutations are created using the existing $\check{m}$ alternatives.

Permutations			
$\wp_1$	$\mathbb{A}_1 > \mathbb{A}_2 > \mathbb{A}_3 > \mathbb{A}_4$	$\wp_{13}$	$\mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_2 > \mathbb{A}_4$
$\wp_2$	$\mathbb{A}_1 > \mathbb{A}_2 > \mathbb{A}_4 > \mathbb{A}_3$	$\wp_{14}$	$\mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_4 > \mathbb{A}_2$
$\wp_3$	$\mathbb{A}_1 > \mathbb{A}_3 > \mathbb{A}_2 > \mathbb{A}_4$	$\wp_{15}$	$\mathbb{A}_3 > \mathbb{A}_2 > \mathbb{A}_1 > \mathbb{A}_4$
$\wp_4$	$\mathbb{A}_1 > \mathbb{A}_3 > \mathbb{A}_4 > \mathbb{A}_2$	$\wp_{16}$	$\mathbb{A}_3 > \mathbb{A}_2 > \mathbb{A}_4 > \mathbb{A}_1$
$\wp_5$	$\mathbb{A}_1 > \mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_2$	$\wp_{17}$	$\mathbb{A}_3 > \mathbb{A}_4 > \mathbb{A}_1 > \mathbb{A}_2$
$\wp_6$	$\mathbb{A}_1 > \mathbb{A}_4 > \mathbb{A}_2 > \mathbb{A}_3$	$\wp_{18}$	$\mathbb{A}_3 > \mathbb{A}_4 > \mathbb{A}_2 > \mathbb{A}_1$
$\wp_7$	$\mathbb{A}_2 > \mathbb{A}_1 > \mathbb{A}_3 > \mathbb{A}_4$	$\wp_{19}$	$\mathbb{A}_4 > \mathbb{A}_1 > \mathbb{A}_2 > \mathbb{A}_3$
$\wp_8$	$\mathbb{A}_2 > \mathbb{A}_1 > \mathbb{A}_4 > \mathbb{A}_3$	$\wp_{20}$	$\mathbb{A}_4 > \mathbb{A}_1 > \mathbb{A}_3 > \mathbb{A}_2$
$\wp_9$	$\mathbb{A}_2 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_4$	$\wp_{21}$	$\mathbb{A}_4 > \mathbb{A}_2 > \mathbb{A}_1 > \mathbb{A}_3$
$\wp_{10}$	$\mathbb{A}_2 > \mathbb{A}_3 > \mathbb{A}_4 > \mathbb{A}_1$	$\wp_{22}$	$\mathbb{A}_4 > \mathbb{A}_2 > \mathbb{A}_3 > \mathbb{A}_1$
$\wp_{11}$	$\mathbb{A}_2 > \mathbb{A}_4 > \mathbb{A}_1 > \mathbb{A}_3$	$\wp_{23}$	$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1 > \mathbb{A}_2$
$\wp_{12}$	$\mathbb{A}_2 > \mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_1$	$\wp_{24}$	$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_2 > \mathbb{A}_1$

Step 13. Using Equation (19), we can construct the relative impact loss matrix $\mathbb{P}$ as follows:

$$\mathbb{P} = \begin{array}{c} \\ \mathbb{N}_1 \\ \mathbb{N}_2 \\ \mathbb{N}_3 \\ \mathbb{N}_4 \end{array} \begin{array}{c} \begin{array}{cccc} \mathbb{N}_1 & \mathbb{N}_2 & \mathbb{N}_3 & \mathbb{N}_4 \end{array} \\ \begin{bmatrix} 0 & 0.2222 & 0.6250 & 0 \\ 0.5455 & 0 & 0.1250 & 0.2222 \\ 0.2727 & 0.2222 & 0 & 0.2222 \\ 0 & 0.2222 & 0.6250 & 0 \end{bmatrix} \end{array}$$

Step 14. The attribute weight system matrix $\mathcal{F}$ is computed using the Matrix (20) as follows:

$$\mathcal{F} = \begin{array}{c} \\ \mathbb{N}_1 \\ \mathbb{N}_2 \\ \mathbb{N}_3 \\ \mathbb{N}_4 \end{array} \begin{array}{c} \begin{array}{cccc} \mathbb{N}_1 & \mathbb{N}_2 & \mathbb{N}_3 & \mathbb{N}_4 \end{array} \\ \begin{bmatrix} -0.8182 & 0.2222 & 0.6250 & 0 \\ 0.5455 & -0.6667 & 0.1250 & 0.2222 \\ 0.2727 & 0.2222 & -1.3750 & 0.2222 \\ 0 & 0.2222 & 0.6250 & -0.4444 \end{bmatrix} \end{array}$$

Step 15. The weights of attributes are computed using Equation (22). Consequently, the weights of attributes for the CILOS method are presented in Table 13.

Step 16. Once the weights are calculated using the Entropy and CILOS methods, the final step is to compute the aggregate weights using Equation (24). Therefore, the weights obtained from the IDOCRIW method are presented in Table 14.

Step 17. Compute the initial permutation of alternatives. The available alternatives are used to generate all possible permutations. For example, if there are 4 alternatives, then $\mathbb{A} = 4$ and $\mathbb{A}! = 4! = 24$. As a result, with the assumption of four alternatives, the permutations are listed in Table 15.

Table 16
The initial ranking of alternatives.

Alternatives	Attributes			
	$\mathbb{N}_1$	$\mathbb{N}_2$	$\mathbb{N}_3$	$\mathbb{N}_4$
$\mathbb{A}_1$	4	4	1	4
$\mathbb{A}_2$	3	1	4	3
$\mathbb{A}_3$	1	2	2	2
$\mathbb{A}_4$	2	3	3	1

Table 17
Compute the dominant and dominated values.

The dominant	The dominant values for $\mathbb{N}_1$	The dominant values for $\mathbb{N}_2$	The dominant values for $\mathbb{N}_3$	The dominant values for $\mathbb{N}_4$
$\mathbb{A}_1 > \mathbb{A}_2$	−1	−1	1	−1
$\mathbb{A}_1 > \mathbb{A}_3$	−1	−1	1	−1
$\mathbb{A}_1 > \mathbb{A}_4$	−1	−1	1	−1
$\mathbb{A}_2 > \mathbb{A}_3$	−1	1	−1	−1
$\mathbb{A}_2 > \mathbb{A}_4$	−1	1	−1	−1
$\mathbb{A}_3 > \mathbb{A}_4$	1	1	1	−1

Table 18
The permutation values for all attributes.

Permutations	Attributes				Permutations	Attributes			
	$\mathbb{N}_1$	$\mathbb{N}_2$	$\mathbb{N}_3$	$\mathbb{N}_4$		$\mathbb{N}_1$	$\mathbb{N}_2$	$\mathbb{N}_3$	$\mathbb{N}_4$
$\wp_1$	−4	0	2	−6	$\wp_{13}$	0	0	2	−2
$\wp_2$	−6	−2	0	−4	$\wp_{14}$	2	−2	4	0
$\wp_3$	−2	−2	4	−4	$\wp_{15}$	2	2	0	0
$\wp_4$	0	−4	6	−2	$\wp_{16}$	4	4	−2	2
$\wp_5$	−2	−6	4	0	$\wp_{17}$	4	0	2	2
$\wp_6$	−4	−4	2	−2	$\wp_{18}$	6	2	0	4
$\wp_7$	−2	2	0	−4	$\wp_{19}$	−2	−2	0	0
$\wp_8$	−4	0	−2	−2	$\wp_{20}$	0	−4	2	2
$\wp_9$	0	4	−2	−2	$\wp_{21}$	0	0	−2	2
$\wp_{10}$	2	6	−2	0	$\wp_{22}$	2	2	−4	2
$\wp_{11}$	−2	2	−4	0	$\wp_{23}$	2	−2	0	−4
$\wp_{12}$	0	4	−6	2	$\wp_{24}$	4	0	−2	6

Step 18. Establish the initial ranking of alternatives. The 2TLq-ROF decision matrix supplied by the DMs is ranked at this stage based on its strengths. The number 1 is assigned to an alternative that outperforms the others in an attribute, and the remaining alternatives are ranked accordingly. The initial ranking of alternatives based on the expected value of 2TLq-ROFS is presented in Table 16.

Step 19. Compute the values that indicate dominance and being dominated. If the permutation corresponds to the rankings, the value is 1; otherwise, it is −1. When two alternatives are identical in one attribute, the value 0 is assigned (refer to Table 17).

The dominant and dominated values for other permutations can be calculated in a similar manner.

Step 20. Calculate the permutation values of attributes. The values computed in the previous step are aggregated together and separately calculated for all permutations and attributes. The permutation values for all attributes are presented in Table 18.

Step 21. Compute the permutation values for the current alternatives. The permutation value is obtained by multiplying the permutation value of each attribute by its weight, aggregating them together, and introducing it as the permutation value. The permutation values of alternatives are shown in Table 19.

Step 22. Determine the ultimate ranking of options. According to the alternatives' permutations, the permutations of 24 are chosen:

$$\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_2 > \mathbb{A}_1$$

Hence, the best choice is $\mathbb{A}_4$.

4.4. Comparative analysis in MAGDM innovation

In this subsection, we employ the different existing MAGDM methodologies to address the proposed MAGDM problem and compare the results with our proposed framework to assess its practicality and efficacy. We find that our approach can provide more accurate and robust predictions as well as more transparent and rational decision-making processes for RSM in Pakistan. Therefore,

Table 19
The permutation values of alternatives.

Permutations	Values	Permutations	Values
$\wp_1$	−2.7516	$\wp_{13}$	−0.4424
$\wp_2$	−3.3676	$\wp_{14}$	0.3660
$\wp_3$	−0.7824	$\wp_{15}$	1.0584
$\wp_4$	−1.9432	$\wp_{16}$	2.5592
$\wp_5$	−1.1348	$\wp_{17}$	1.8668
$\wp_6$	−1.7508	$\wp_{18}$	3.3676
$\wp_7$	−2.5592	$\wp_{19}$	−1.0584
$\wp_8$	−1.8668	$\wp_{20}$	−0.2500
$\wp_9$	0.2500	$\wp_{21}$	0.4424
$\wp_{10}$	1.9804	$\wp_{22}$	1.2712
$\wp_{11}$	−0.3660	$\wp_{23}$	−1.4372
$\wp_{12}$	1.1348	$\wp_{24}$	2.7516

we address the aforementioned issue by applying several MAGDM methodologies and showing the outcomes in Tables 20 and 21. The visual depiction of results from Tables 20 and 21 are also provided.

4.4.1. Comparative analysis with 2TLIF-TOPSIS and 2TLq-ROF-TOPSIS methods

We make a comparison of our proposed approach with existing approaches such as the 2TLIF-TOPSIS method introduced by Cheng et al. [58] and the 2TLq-ROF-TOPSIS method introduced by Liu et al. [59] in order to assess its practicality and efficacy. When we compare the proposed approach with 2TLIF-TOPSIS method then the ranking is $\mathbb{A}_6 > \mathbb{A}_8 > \mathbb{A}_3 > \mathbb{A}_5 > \mathbb{A}_7 > \mathbb{A}_2 > \mathbb{A}_1 > \mathbb{A}_4$, and the best alternative according to the 2TLIF-TOPSIS method is $\mathbb{A}_6$. Next, we compare the proposed approach with 2TLq-ROF-TOPSIS method then the ranking is $\mathbb{A}_5 > \mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_8 > \mathbb{A}_6 > \mathbb{A}_1 > \mathbb{A}_2 > \mathbb{A}_4$, and the best alternative according to the 2TLq-ROF-TOPSIS method is $\mathbb{A}_5$.

4.4.2. Comparative analysis with 2TLq-ROF-DEMATEL-TOPSIS method and 2TLq-ROF-PMSM operator

We make a comparison of our proposed approach with existing approaches such as the 2TLq-ROF-DEMATEL-TOPSIS method proposed by Naz et al. [60] and the 2TLq-ROF power Maclaurin symmetric mean (2TLq-ROF-PMSM) operator established by Naz et al. [61] in order to assess its practicality and efficacy. When we compare the proposed approach with 2TLq-ROF-DEMATEL-TOPSIS method then the ranking is $\mathbb{A}_6 > \mathbb{A}_1 > \mathbb{A}_2 > \mathbb{A}_4 > \mathbb{A}_5 > \mathbb{A}_7 > \mathbb{A}_3 > \mathbb{A}_8$, and the best alternative according to the 2TLq-ROF-DEMATEL-TOPSIS method is $\mathbb{A}_6$. Next, we compare the proposed approach with 2TLq-ROF-PMSM operator then the ranking is $\mathbb{A}_8 > \mathbb{A}_4 > \mathbb{A}_5 > \mathbb{A}_2 > \mathbb{A}_7 > \mathbb{A}_3 > \mathbb{A}_6 > \mathbb{A}_1$, and the best alternative according to the 2TLq-ROF-PMSM operator is $\mathbb{A}_8$.

4.4.3. Comparative analysis with 2TLPyF-MABAC and 2TLq-ROF-CODAS methods

We make a comparison of our proposed approach with existing approaches such as the 2TLPyF-MABAC method introduced by Zhang et al. [62] and the 2TLq-ROF-CODAS method proposed by Naz et al. [56] in order to assess its practicality and efficacy. When we compare the proposed approach with 2TLPyF-MABAC method then the ranking is $\mathbb{A}_4 > \mathbb{A}_8 > \mathbb{A}_3 > \mathbb{A}_5 > \mathbb{A}_2 > \mathbb{A}_7 > \mathbb{A}_6 > \mathbb{A}_1$, and the best alternative according to the 2TLPyF-MABAC method is $\mathbb{A}_4$. Next, we compare the proposed approach with 2TLq-ROF-CODAS method then the ranking is $\mathbb{A}_1 > \mathbb{A}_6 > \mathbb{A}_2 > \mathbb{A}_7 > \mathbb{A}_4 > \mathbb{A}_5 > \mathbb{A}_3 > \mathbb{A}_8$, and the best alternative according to the 2TLq-ROF-CODAS method is $\mathbb{A}_1$.

4.4.4. Comparative analysis with 2TLFF-CODAS and 2TLPyF-CODAS methods

We make a comparison of our proposed approach with existing approaches such as the 2TL Fermatean fuzzy CODAS (2TLFF-CODAS) method proposed by Akram et al. [63] and the 2TLPyF-CODAS method proposed by He et al. [64] in order to assess its practicality and efficacy. When we compare the proposed approach with 2TLFF-CODAS method then the ranking is $\mathbb{A}_1 > \mathbb{A}_6 > \mathbb{A}_7 > \mathbb{A}_2 > \mathbb{A}_5 > \mathbb{A}_3 > \mathbb{A}_8 > \mathbb{A}_4$, and the best alternative according to the 2TLFF-CODAS method is $\mathbb{A}_1$. Next, we compare the proposed approach with 2TLPyF-CODAS method then the ranking is $\mathbb{A}_1 > \mathbb{A}_6 > \mathbb{A}_7 > \mathbb{A}_2 > \mathbb{A}_5 > \mathbb{A}_3 > \mathbb{A}_8 > \mathbb{A}_4$, and the best alternative according to the 2TLPyF-CODAS method is $\mathbb{A}_1$.

4.4.5. Comparative analysis with 2TLFF-WASPAS and 2TLPyF-EDAS methods

We make a comparison of our proposed approach with existing approaches such as the 2TLFF-WASPAS method proposed by Akram et al. [65] and the 2TLPyF-EDAS method proposed by Zhang et al. [66] in order to assess its practicality and efficacy. When we compare the proposed approach with 2TLFF-WASPAS method then the ranking is $\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_5 > \mathbb{A}_2 > \mathbb{A}_8 > \mathbb{A}_7 > \mathbb{A}_6 > \mathbb{A}_1$, and the best alternative according to the 2TLFF-WASPAS method is $\mathbb{A}_4$. Next, we compare the proposed approach with 2TLPyF-EDAS method then the ranking is $\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_8 > \mathbb{A}_2 > \mathbb{A}_5 > \mathbb{A}_7 > \mathbb{A}_6 > \mathbb{A}_1$, and the best alternative according to the 2TLPyF-EDAS method is $\mathbb{A}_4$.

4.4.6. Comparative analysis with 2TLFF-MULTIMOORA and 2TLPyF-MULTIMOORA methods

We make a comparison of our proposed approach with existing approaches such as the 2TLFF-MULTIMOORA method proposed by Akram et al. [67] and the 2TLPyF-MULTIMOORA method proposed by Akram et al. [68] in order to assess its practicality and efficacy. When we compare the proposed approach with 2TLFF-MULTIMOORA method then the ranking is $\mathbb{A}_1 > \mathbb{A}_8 > \mathbb{A}_7 > \mathbb{A}_6 > \mathbb{A}_3 > \mathbb{A}_2 > \mathbb{A}_5 > \mathbb{A}_4$, and the best alternative according to the 2TLFF-MULTIMOORA method is $\mathbb{A}_1$. Next, we compare the proposed approach with 2TLPyF-MULTIMOORA method then the ranking is $\mathbb{A}_1 > \mathbb{A}_8 > \mathbb{A}_7 > \mathbb{A}_2 > \mathbb{A}_3 > \mathbb{A}_6 > \mathbb{A}_5 > \mathbb{A}_4$, and the best alternative according to the 2TLPyF-MULTIMOORA method is $\mathbb{A}_1$.

4.4.7. Comparative analysis with some road safety papers

We make a comparison of our proposed approach with existing approaches involving the case studies such as reducing of human risks on the regional road network of Calabria to improve road safety [7], safe E-scooter operation alternative prioritization [69], measuring road safety advance for OAS countries [70], evaluation of the route selection in international freight transportation [71], and a lesson system of legislation and regulation for the United States [72]. When we compare the proposed approach with reference [7] the ranking is $\mathbb{A}_6 > \mathbb{A}_1 > \mathbb{A}_7 > \mathbb{A}_2 > \mathbb{A}_8 > \mathbb{A}_4 > \mathbb{A}_5 > \mathbb{A}_3$, with reference [69] the ranking is $\mathbb{A}_1 > \mathbb{A}_8 > \mathbb{A}_7 > \mathbb{A}_6 > \mathbb{A}_2 > \mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_5$, with reference [70] the ranking is $\mathbb{A}_4 > \mathbb{A}_3 > \mathbb{A}_2 > \mathbb{A}_8 > \mathbb{A}_5 > \mathbb{A}_7 > \mathbb{A}_6 > \mathbb{A}_1$, with reference [71] the ranking is $\mathbb{A}_1 > \mathbb{A}_6 > \mathbb{A}_2 > \mathbb{A}_4 > \mathbb{A}_7 > \mathbb{A}_5 > \mathbb{A}_3 > \mathbb{A}_8$, with reference [72] the ranking is $\mathbb{A}_6 > \mathbb{A}_1 > \mathbb{A}_2 > \mathbb{A}_7 > \mathbb{A}_4 > \mathbb{A}_5 > \mathbb{A}_8 > \mathbb{A}_3$, the best alternatives are $\mathbb{A}_6, \mathbb{A}_1, \mathbb{A}_4, \mathbb{A}_1,$ and $\mathbb{A}_6$, respectively.

4.5. Discussion

The application of various existing MAGDM methodologies is pivotal in addressing the complex problem outlined. By systematically employing these methodologies, a comprehensive assessment of their efficacy in resolving the proposed MAGDM issue is conducted. This rigorous comparative analysis serves as a tool to evaluate the practicality and effectiveness of the newly proposed framework. Our study shows that our method is better at making accurate and strong predictions. Additionally, it highlights the inherent strength of our framework in facilitating transparent and rational decision-making processes, a crucial aspect of RSM in Pakistan. Through the utilization of multiple MAGDM methodologies, the research aims to offer a thorough evaluation of decision-making strategies related to the RSM context in Pakistan. Tables 20 and 21 present a concise yet comprehensive overview of the outcomes derived from the application of these methodologies. Adding visual representations to these tables makes it easier for everyone to understand the results better. It helps DMs see and compare things more clearly. Such visual depictions enhance the accessibility of complex data, facilitating a more comprehensive grasp of differences in outcomes among the methodologies employed. Combining real-world data with pictures makes the suggested strategy more trustworthy, helping people make better decisions about managing resources.

4.6. Advantages

The following advantages of the suggested method over the current ones can be presented: (i) The approach constructed in this research is commonly utilized when dealing with imprecise data, and it adheres to the condition that the sum of the squared MD and NMD in 2TL expressions is not equal to one. When presented with such information, the DMs find it challenging to effectively handle the 2TLIFS and 2TLPyFS. The 2TLq-ROFS approach is capable of effectively addressing this particular scenario. (ii) The utilization of the 2TLq-ROF framework can enhance the precision, reliability, and comprehensiveness of the decision-making process by integrating the experts' degrees of confidence in their familiarity and understanding of the evaluated options. (iii) The IDOCRIW-QUALIFLEX approach is frequently employed for addressing MAGDM challenges. The construction of the IDOCRIW-QUALIFLEX methodology combined with the 2TLq-ROFS is also adequate. (iv) In conclusion, our method is flexible and well-suited to addressing problems in 2TLq-ROF-MAGDM and can handle fuzzy data more competently.

5. Conclusions

APMs are critical components of modern RSM techniques. These models use data and predictive analytics to anticipate possible accident hazards, allowing authorities to take proactive measures to minimize collisions and improve overall road safety. These models identify high-risk regions and periods by assessing past accident data, traffic patterns, weather conditions, and other pertinent variables, allowing resources to be allocated where they are most needed. Road safety officials can then execute targeted interventions such as improved signage, traffic flow changes, or increased law enforcement to alleviate the identified risks. Finally, APMs provide a data-driven foundation for making educated decisions, minimizing accidents, and making roads safer for both vehicles and pedestrians. This work employed the notion of 2TLq-ROFS and examined its basic operations and associated features. Furthermore, a WPA operator was proposed with 2TLq-ROFNs to aggregate the individual decision preferences into a collective one. In order to address the MAGDM problem inside a 2TLq-ROF environment, a novel approach known as the extended IDOCRIW-QUALIFLEX method was proposed. This method utilized the 2TLq-ROFNs and was demonstrated through the application of a real-life scenario. In order to ascertain the superiority and effectiveness of the proposed strategy, a comparative analysis was conducted. The present investigation and comparison analysis demonstrate that the approach employed in this paper exhibits greater adaptability and a broad capacity for effectively communicating ambiguous information.

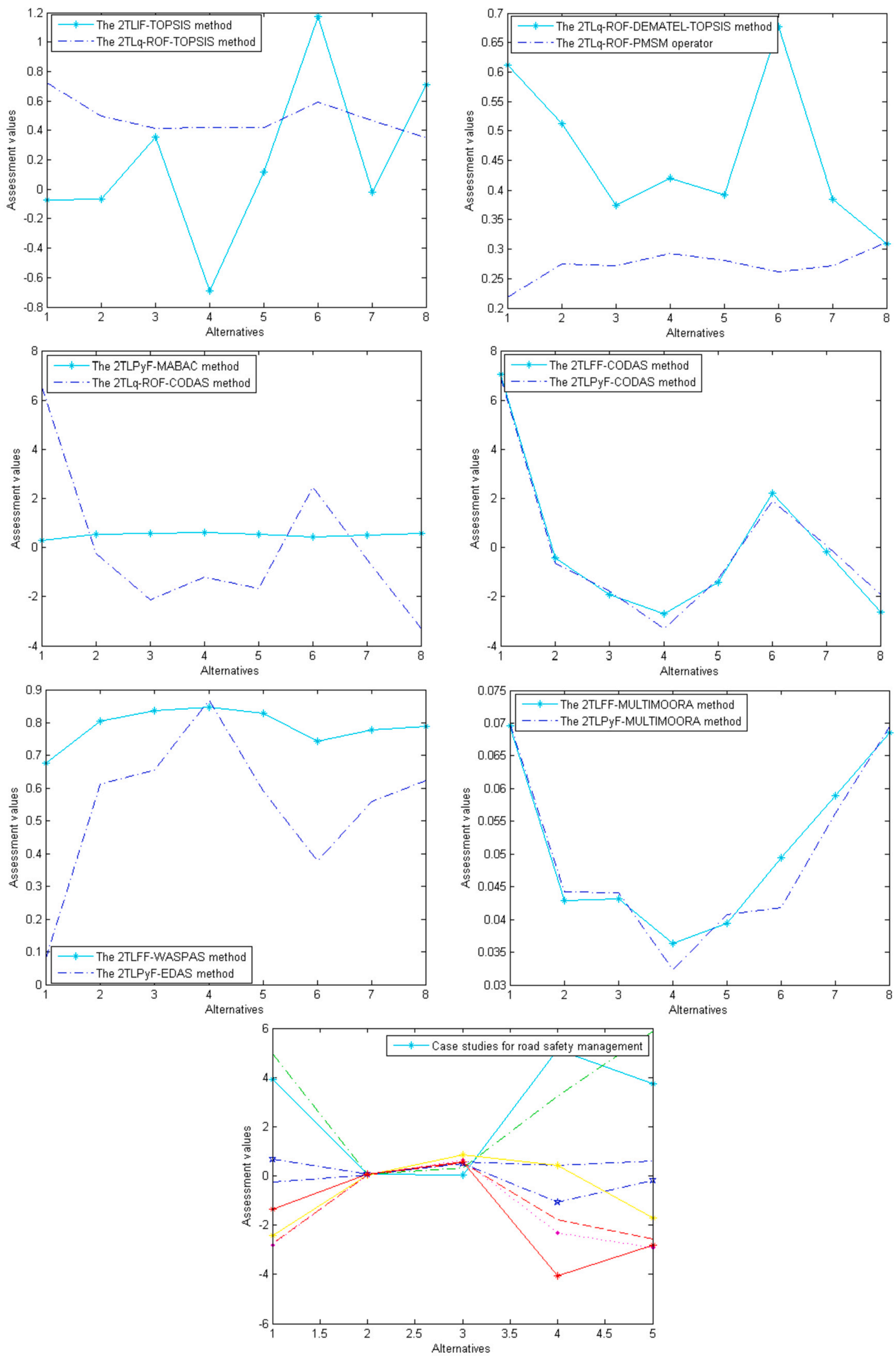
The utilization of the proposed approach is seen as appropriate for consolidating vague and uncertain data within the context of decision-making, as it allows for a more accurate and conclusive representation of the offered information. APMs help identify high-risk regions and facilitate preemptive solutions, which are vital for managing road safety. However, these models mainly rely on past accident data, which may not accurately reflect new trends or modifications in driving behavior. Furthermore, such models

Table 20
Alternatives ranking by various approaches.

Approaches	Assessment values	Ranking
The 2TLIF-TOPSIS method introduced by Cheng et al. [58]	$\mathbb{A}_1 = -0.0734$,	6
	$\mathbb{A}_2 = -0.0641$,	8
	$\mathbb{A}_3 = 0.3561$,	3
	$\mathbb{A}_4 = -0.6877$,	5
	$\mathbb{A}_5 = 0.1197$,	7
	$\mathbb{A}_6 = 1.1733$,	2
	$\mathbb{A}_7 = -0.0219$,	1
	$\mathbb{A}_8 = 0.7123$.	4
The 2TLq-ROF-TOPSIS method suggested by Liu et al. [59]	$\mathbb{A}_1 = 0.7227$,	1
	$\mathbb{A}_2 = 0.4949$,	6
	$\mathbb{A}_3 = 0.4158$,	2
	$\mathbb{A}_4 = 0.4199$,	7
	$\mathbb{A}_5 = 0.4212$,	5
	$\mathbb{A}_6 = 0.5902$,	4
	$\mathbb{A}_7 = 0.4686$,	3
	$\mathbb{A}_8 = 0.3498$.	8
The 2TLq-ROF-DEMATEL-TOPSIS method proposed by Naz et al. [60]	$\mathbb{A}_1 = 0.6122$,	6
	$\mathbb{A}_2 = 0.5123$,	1
	$\mathbb{A}_3 = 0.3747$,	2
	$\mathbb{A}_4 = 0.4202$,	4
	$\mathbb{A}_5 = 0.3916$,	5
	$\mathbb{A}_6 = 0.6772$,	7
	$\mathbb{A}_7 = 0.3854$,	3
	$\mathbb{A}_8 = 0.3084$.	8
The 2TLq-ROF-PMSM operator established by Naz et al. [61]	$\mathbb{A}_1 = 0.2187$,	8
	$\mathbb{A}_2 = 0.2742$,	4
	$\mathbb{A}_3 = 0.2714$,	5
	$\mathbb{A}_4 = 0.2925$,	2
	$\mathbb{A}_5 = 0.2802$,	7
	$\mathbb{A}_6 = 0.2609$,	3
	$\mathbb{A}_7 = 0.2717$,	6
	$\mathbb{A}_8 = 0.3113$.	1
The 2TLPyF-MABAC method introduced by Zhang et al. [62]	$\mathbb{A}_1 = 0.2878$,	4
	$\mathbb{A}_2 = 0.5235$,	8
	$\mathbb{A}_3 = 0.5587$,	3
	$\mathbb{A}_4 = 0.6080$,	5
	$\mathbb{A}_5 = 0.5439$,	2
	$\mathbb{A}_6 = 0.4451$,	7
	$\mathbb{A}_7 = 0.5021$,	6
	$\mathbb{A}_8 = 0.5639$.	1
The 2TLq-ROF-CODAS method proposed by Naz et al. [56]	$\mathbb{A}_1 = 6.5600$,	1
	$\mathbb{A}_2 = -0.2349$,	6
	$\mathbb{A}_3 = -2.1224$,	2
	$\mathbb{A}_4 = -1.2173$,	7
	$\mathbb{A}_5 = -1.6499$,	4
	$\mathbb{A}_6 = 2.4754$,	5
	$\mathbb{A}_7 = -0.4984$,	3
	$\mathbb{A}_8 = -3.3124$.	8
The 2TLFF-CODAS method proposed by Akram et al. [63]	$\mathbb{A}_1 = 7.0538$,	1
	$\mathbb{A}_2 = -0.4056$,	6
	$\mathbb{A}_3 = -1.9023$,	7
	$\mathbb{A}_4 = -2.6906$,	2
	$\mathbb{A}_5 = -1.4350$,	5
	$\mathbb{A}_6 = 2.1992$,	3
	$\mathbb{A}_7 = -0.1730$,	8
	$\mathbb{A}_8 = -2.6464$.	4
The 2TLPyF-CODAS method proposed by He et al. [64]	$\mathbb{A}_1 = 6.9230$,	1
	$\mathbb{A}_2 = -0.6191$,	6
	$\mathbb{A}_3 = -1.7583$,	7
	$\mathbb{A}_4 = -3.3233$,	2
	$\mathbb{A}_5 = -1.2746$,	5
	$\mathbb{A}_6 = 1.8888$,	3
	$\mathbb{A}_7 = 0.0656$,	8
	$\mathbb{A}_8 = -1.9022$.	4
	$\mathbb{A}_1 = 0.6773$,	4
	$\mathbb{A}_2 = 0.8039$,	3
	$\mathbb{A}_3 = 0.8370$,	5
	$\mathbb{A}_4 = 0.8485$,	2

Table 21
Alternatives ranking by various approaches.

Approaches	Assessment values	Ranking
The 2TLFF-WASPAS method proposed by Akram et al. [65]	$\mathbb{A}_5 = 0.8290$,	8
	$\mathbb{A}_6 = 0.7432$,	7
	$\mathbb{A}_7 = 0.7787$,	6
	$\mathbb{A}_8 = 0.7889$.	1
The 2TLPyF-EDAS method proposed by Zhang et al. [66]	$\mathbb{A}_1 = 0.0759$,	4
	$\mathbb{A}_2 = 0.6137$,	3
	$\mathbb{A}_3 = 0.6552$,	8
	$\mathbb{A}_4 = 0.8695$,	2
	$\mathbb{A}_5 = 0.5903$,	5
	$\mathbb{A}_6 = 0.3778$,	7
	$\mathbb{A}_7 = 0.5583$,	6
	$\mathbb{A}_8 = 0.6240$.	1
The 2TLFF-MULTIMOORA method proposed by Akram et al. [67]	$\mathbb{A}_1 = 0.0696$,	1
	$\mathbb{A}_2 = 0.0429$,	8
	$\mathbb{A}_3 = 0.0432$,	7
	$\mathbb{A}_4 = 0.0364$,	6
	$\mathbb{A}_5 = 0.0394$,	3
	$\mathbb{A}_6 = 0.0494$,	2
	$\mathbb{A}_7 = 0.0589$,	5
	$\mathbb{A}_8 = 0.0685$.	4
The 2TLPyF-MULTIMOORA method proposed by Akram et al. [68]	$\mathbb{A}_1 = 0.0700$,	1
	$\mathbb{A}_2 = 0.0442$,	8
	$\mathbb{A}_3 = 0.0441$,	7
	$\mathbb{A}_4 = 0.0323$,	2
	$\mathbb{A}_5 = 0.0407$,	3
	$\mathbb{A}_6 = 0.0418$,	6
	$\mathbb{A}_7 = 0.0562$,	5
	$\mathbb{A}_8 = 0.0694$.	4
Reducing of human risks on the regional road network of Calabria [7]	$\mathbb{A}_1 = 3.9174$,	6
	$\mathbb{A}_2 = -0.2365$,	1
	$\mathbb{A}_3 = -2.8021$,	7
	$\mathbb{A}_4 = -2.4161$,	2
	$\mathbb{A}_5 = -2.7650$,	8
	$\mathbb{A}_6 = 4.9649$,	4
	$\mathbb{A}_7 = 0.6797$,	5
	$\mathbb{A}_8 = -1.3424$.	3
Safe E-scooter operation alternative prioritization [69]	$\mathbb{A}_1 = 0.0643$,	1
	$\mathbb{A}_2 = 0.0438$,	8
	$\mathbb{A}_3 = 0.0389$,	7
	$\mathbb{A}_4 = 0.0408$,	6
	$\mathbb{A}_5 = 0.0352$,	2
	$\mathbb{A}_6 = 0.0561$,	4
	$\mathbb{A}_7 = 0.0576$,	3
	$\mathbb{A}_8 = 0.0628$.	5
Measuring road safety advance for OAS countries [70]	$\mathbb{A}_1 = 0.0526$,	4
	$\mathbb{A}_2 = 0.5737$,	3
	$\mathbb{A}_3 = 0.6266$,	2
	$\mathbb{A}_4 = 0.8465$,	8
	$\mathbb{A}_5 = 0.5457$,	5
	$\mathbb{A}_6 = 0.3037$,	7
	$\mathbb{A}_7 = 0.4853$,	6
	$\mathbb{A}_8 = 0.5537$.	1
Evaluation of the route selection in international freight transportation [71]	$\mathbb{A}_1 = 5.1128$,	1
	$\mathbb{A}_2 = 0.4441$,	6
	$\mathbb{A}_3 = -2.3068$,	2
	$\mathbb{A}_4 = 0.4416$,	4
	$\mathbb{A}_5 = -1.7974$,	7
	$\mathbb{A}_6 = 3.2422$,	5
	$\mathbb{A}_7 = -1.0787$,	3
	$\mathbb{A}_8 = -4.0578$.	8
A lesson system of legislation and regulation for the United States [72]	$\mathbb{A}_1 = 3.7385$,	6
	$\mathbb{A}_2 = 0.6018$,	1
	$\mathbb{A}_3 = -2.9337$,	2
	$\mathbb{A}_4 = -1.7216$,	7
	$\mathbb{A}_5 = -2.5703$,	4
	$\mathbb{A}_6 = 5.8592$,	5
	$\mathbb{A}_7 = -0.1632$,	8
	$\mathbb{A}_8 = -2.8106$.	3


The 2TLIF-TOPSIS method
The 2TLq-ROF-TOPSIS method
The 2TLq-ROF-DEMATEL-TOPSIS method
The 2TLq-ROF-PMSM operator
The 2TLPyF-MABAC method
The 2TLq-ROF-CODAS method
The 2TLFF-CODAS method
The 2TLPyF-CODAS method
The 2TLFF-WASPAS method
The 2TLPyF-EDAS method
The 2TLFF-MULTIMOORA method
The 2TLPyF-MULTIMOORA method
Case studies for road safety management
Assessment values
Alternatives

frequently struggle to take into account the complex interplay between numerous variables that affect road safety, such as weather conditions, road upkeep, and real-time traffic dynamics. The dynamic nature of road conditions makes it much more difficult for these models to make precise and timely forecasts. Preventative measures need to work in APMs to be helpful, but they might not always work well due to human reasons, policy gaps, or money issues. While 2TLq-ROFS is a powerful tool for handling complex linguistic information, it can be challenging to determine the appropriate linguistic variables for a given scenario. This might provide a particular challenge in circumstances where there are several factors or perspectives at play. Additionally, the set may not be suitable for all types of decision-making scenarios as it is designed specifically for handling linguistic information.

Although QUALIFLEX involves the permutations of alternatives for ranking, it is very difficult to set the permutation sequence for a greater number of alternatives. Hence, the computational procedure for the QUALIFLEX method becomes difficult to handle by DMs. IDOCRIW weighting method can be a powerful tool for handling uncertain and imprecise information in decision-making scenarios, but it may not be suitable for all types of decision-making scenarios in order to calculate the weights of attributes. The method is designed specifically for handling uncertain and imprecise information, and may not be effective in situations where the data is clear or the decision-making criteria are well-defined. Furthermore, the IDOCRIW weighting method can be computationally intensive which can slow down the decision-making process. This can be a challenge in scenarios where decisions need to be made quickly. Future studies should use a comprehensive strategy that takes into account both quantitative data and qualitative insights in order to mitigate these constraints and develop RSM tactics. In the future, we can define and discuss various AOs for 2TLq-ROFS, such as OWA, OWG AOs, distance measures, and similarity measures in a defined environment. We can also address additional real-world challenges in a variety of industries by combining 2TLq-ROFNs with other MAGDM problems, such as medical diagnosis, material selection, pattern recognition, information fusion, and green supplier selection.

Ethics statement

This research was conducted in collaboration with the principles and approval of the Ethical Committee of Division of Science and Technology, University of Education, Lahore, Pakistan.

CRediT authorship contribution statement

Sumera Naz: Conceptualization. **Aqsa Shafiq:** Writing – original draft, Conceptualization. **Shariq Aziz Butt:** Writing – original draft. **Shahzra Mazhar:** Writing – review & editing. **Diaz Jorge Martinez:** Visualization. **Emiro De la Hoz Franco:** Validation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The data that support the findings of this study have been enclosed in this paper.

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