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A reformulation of mechanics and electrodynamics

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Abstract

Classical mechanics, as commonly taught in engineering and science, are confined to the conventional Newtonian theory. But classical mechanics has not really changed in substance since Newton formulation, describing simultaneous rotation and translation of objects with somewhat complicate drawbacks, risking interpretation of forces in non-inertial frames. In this work we introduce a new variational principle for out-of-equilibrium, rotating systems, obtaining a set of two first order differential equations that introduces a thermodynamic-mechanistic time into Newton's dynamical equation, and revealing the same formal symplectic structure shared by classical mechanics, fluid mechanics and thermodynamics. The results is a more consistent formulation of dynamics and electrodynamics, explaining natural phenomena as the outcome from a balance between energy and entropy, embedding translational with rotational motion into a single equation, showing centrifugal and Coriolis force as derivatives from the transport of angular momentum, and offering a natural method to handle variational problems, as shown with the brachistochrone problem. In consequence, a new force term appears, the topological torsion current, important for spacecraft dynamics. We describe a set of solved problems showing the potential of a competing technique, with significant interest to electrodynamics as well. We expect this new approach to have impact in a large class of scientific and technological problems.

Keywords: Mechanics, Nonlinear physics, Plasma physics, Statistical physics

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1. Introduction

Classical mechanics, as commonly taught in engineering science are confined to the conventional Newtonian theory. In this work we propose an alternative theory that completes the actual paradigm. The new formulation was elaborated [1] using the tools of conventional vectorial algebra and a variational principle introduced by Landau & Lifshitz [2], but not fully explored by these authors, since they have not included the fundamental equation of thermodynamics $dU = TdS - \mathbf{F} \cdot d\mathbf{r}$ that would link energy (motion) with entropy (dissipation). This new formulation leads to a set of two first-order differential equations that govern all physical processes. At the time of Sir Isaac Newton, the development of thermodynamics was still faraway to express the second law of thermodynamics, and naturally, Newton could not embed his formulation of natural phenomena within the universal balance between energy and entropy [3, 4]. The second law of thermodynamics has contributed greatly to the development of theoretical chemistry and physics, including chemical kinetics and transition states, Boltzmann kinetic theory, Max Planck pioneering quantum theory, and Einstein theory of stimulated and spontaneous emission, to mention a few. However, thermodynamics does not give time evolution equations for thermodynamic states as Hamilton's equations do. This situation has been unsatisfactory and Ilya Prigogine in his Nobel Lecture emphasized that "[...] that 150 years after its formulation the second law of thermodynamics still appears to be more a program than a well-defined theory in the usual sense, as nothing precise (except the sign) is said about the S production".

We propose a new variational procedure initially sketched but not fully explored by Landau/Lifshitz in their Course on Theoretical Physics. It leads to a modified dynamical equation which integrates the second law of Newtonian mechanics with the fundamental equation of thermodynamics. In this new framework, external forces are imposed to a physical system but it reacts back to the external constraint with rotational, vortical motion, dissipative motion. Engineering problems can be solved more straightforwardly, including the paradigmatic brachistochrone problem. In addition, it reveals the existence of a new force term, the topological torsion current, enlightening the puzzle of the flyby anomaly. The extension to electrodynamics shows to be useful to spintronics and a large class of scientific and technological issues.

It is clear by the selected problems solved that this theory introduces an innovative and consistent formulation to comprehend the dynamics of physical systems, at the same time offering a unifying approach. We have shown before that the gravitational field has the nature of an entropic force [5] and obtained the basic equations of electrodynamics with a new plasma balance equation [6], predicting the direct conversion of angular to linear motion by the agency of a topological torsion

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current [1]. Here, we will dwell on its applications in the framework of classical mechanics, but further development of the theory could lead ultimately to an explanation of quantum mechanical phenomena, and a fundamental contribution was advanced by R. Jackiw et al. [7]. The mathematical description of physical systems are defined by the following system of equations:

- Canonical momentum :

$$T\frac{\partial \bar{S}}{\partial \mathbf{p}} = -\frac{\mathbf{p}}{m} + \frac{q}{m}\mathbf{A} + \mathbf{v}_e + [\boldsymbol{\omega} \times \mathbf{r}]. \tag{1}$$

- Fundamental equation of motion :

$$T\frac{\partial \bar{S}}{\partial \mathbf{r}} = \nabla U + m\frac{\partial \mathbf{v}}{\partial t}. \tag{2}$$

Eq. (1) gives the quantity of motion, a measure of the vector velocity (with geometric nature) and the quantity of matter (with energetic content). In the case of a charged particle, it can be shown [8] that the momentum is given by

$$\mathbf{p} = m\mathbf{v}_e + q\mathbf{A} - mT\frac{\partial \bar{S}}{\partial \mathbf{p}}. \tag{3}$$

However, Eq. (3) represents a radically different kind of momentum when compared to the Newtonian concept of force and momentum, because it predicts mutual interaction between interacting systems, self-regulating exchange of energy (see also, Ref. [8]).

Eq. (2) gives the fundamental equation of dynamics and has the form of a general local balance equation having as source term the spatial gradient of entropy, $\nabla_a S > 0$. At thermodynamic equilibrium the total entropy of the body has a maximum value and the equality holds, a feature in general absent in the usual dynamical determinism of Newtonian formulation (although Newton recognizes friction as a source of asymmetry). The mentioned self-regulating mechanism of angular and linear momentum is represented by the last term of Eq. (2) and extends the validity of Newton's fundamental equation of motion to accelerated (non-inertial) frames of reference (see Section 2.4). In Eqs. (1)–(2) it appears one of the most important thermodynamic quantity, the *temperature* T. However, with the exception of Section 2.2, in this work we don't address the extraordinary diversified phenomena of the conversion of heat into work (and vice-versa).

The set of Eqs. (1)–(2) open a major change in the so-far classical paradigm of dynamics, and as well electrodynamics (see Ref. [6]), since new physics may be brought by the set of two first order differential equations, related to the interplay

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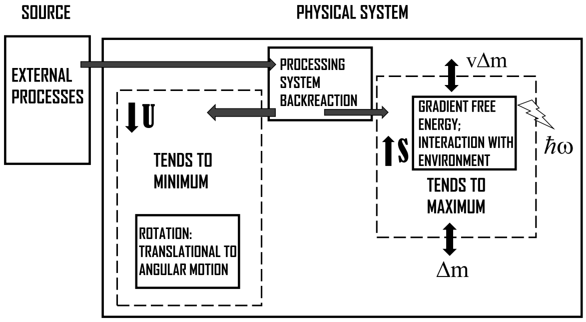

Figure 1. System process chain – a physical system has, at least, two ways to process the external action (it is not enslaved to it), by means of two processes chain: energy tends to a minimum by means of conversion of linear or transversal motion to rotational motion; Entropy tends to a maximum by means of decreasing free energy (e.g., ejection of mass, acoustic or electromagnetic radiation).

between the tendency of energy to attain a minimum, whilst entropy seeks to maximize its value. According to the proposed variational method, Eq. (2) leads to the fundamental equation of (classical) dynamics, now presented under the form:

$$m\frac{d\mathbf{v}}{dt} = \mathbf{F}^{ext} - \nabla\left(\frac{J^2}{2I} - \boldsymbol{\omega}\cdot\mathbf{J} - \Delta F\right). \tag{4}$$

It is a consequence of requiring that the energy *and* entropy attain an optimal balance. This new principle is, therefore, different from Hamilton's principle since this last one is equivalent to $F = ma$.

The external force term can be a deterministic or fluctuating force, but cannot be entropy-dependent. A central role is played by the gradient of rotational energy (and free energy) which drive the system from one state to another. The system is not enslaved by the external force $\mathbf{F}^{ext}$, which we may envisage as an input to the system, but can respond to changes by converting linear or angular motion into angular motion, which we may consider the system output, affecting dynamically the system, as illustrated in Figure 1. Moreover, according to Eq. (4), motion will not necessarily follow in a straight line and in the direction of the external force (see Section 2.9). Angular momentum acts as a damper to dissipate a disturbance, a well-known redressing mechanism in biomechanics and robotics. The free energy term is essential to study the thermomechanical behavior of any continuous media.

In addition, considering the traditional hierarchy of agencies responsible for the motion of matter, on a logic and axiomatic point of view we may establish an operational relationship between linear and angular motion, with possible relevance for the understanding of the trajectory of Near-Earth Objects [9, 10].

Formulating the problem in this novel way, the temporal evolution of a system is wholly contained without the need to add the torque equation, separately, and in particular, giving immediately the theorem work-energy, $E_{mec} = K + U + I\omega^2/2$.

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Variational mass systems can be handled by means of the gradient of free energy. From the selected number of classical problems solved in Section 2, it is clear that Eq. (4) gives a more complete and simple description of dynamical processes in nature. This variational framework, as also shown in a previous work, apparently link the gravitational force to entropic arguments [5].

From discrete particles to the analysis of the kinematics and the mechanical behavior of materials modeled as a continuous mass, first formulated by the French mathematician Augustin-Louis Cauchy in the 19th century, an intensive research has been conducted, and we must cite the work of Clifford Truesdell on modern rational mechanics [11], or all subsequent work particularly focused on the development of coupled atomistic/continuum material modeling from the nanoscale to the macroscale, surface and semi-continuum theories [12, 13, 14, 15, 16, 17, 18, 19], or help to design new technology for space advanced programs [20]. These departures from the standard theories use integral-type formulations with weighted spatial averaging or by implicit or explicit gradient models, enriching the theories of elasticity, elastoplasticity and damage mechanics. Our variational method [5] provides a starting place for the development of new methods that can be incorporated in theories of continuum mechanics.

2. Results and discussion

The investigation as to whether and how far this new formulation represents usable technique is exemplified now with practical application problems.

2.1. The rolling body in an inclined plane

One standard example of classical mechanics is this one: a rigid body of mass M rolling down an inclined plane making an angle θ with the horizontal (see, e.g., p. 97 of Ref. [21]). Eq. (4) can be applied to solve the problem, with $\omega = 0$ (there is no rotation of the frame of reference) and considering that only the gravitational force acts on the rolling body, with inertial moment relative to its own center of mass given by $I_c = \beta M R^2$. Hence, we obtain:

$$M\ddot{x} = Mg\sin\theta - \partial_x w. \tag{5}$$

Here, $w \equiv \frac{(\mathbf{J_c})^2}{2I_c}$. Assuming that the x-axis is directed along the inclined plane, and considering that the angular momentum relative to the rigid body center of mass is given by $J_c = I_c\omega'$, with $\omega' = d\theta/dt$, and noticing that $dx = v_x dt$ (holonomic constraint), it is readily obtained

$$M\ddot{x} = Ma_x = Mg\sin\theta - I_c\omega'\frac{d\omega'}{v_x dt} = Mg\sin\theta - \beta M R^2 \frac{\omega'}{\omega' R}\alpha. \tag{6}$$

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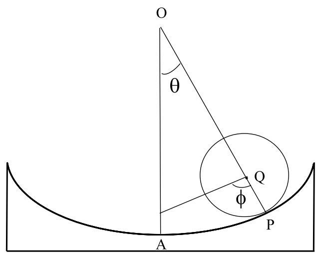

Figure 2. The spherometer.

Since $\alpha = a/R$, then it results the well-known equation

$$a_x = \frac{g \sin\theta}{(1+\beta)}. \tag{7}$$

The usual approach is overridden by the new framework, since the linear and angular momenta are embedded in a unified equation, Eq. (4).

The usual approach in textbooks begins by solving the 2nd Law of dynamics for the center of mass (assuming a point particle) and then, in a separate way, solve the torque equation for an extended particle, with the intermediate holonomic condition for rolling. It looks like a collection of theories, inconsistent under the logical point of view.

2.2. Oscillations of a sphere rolling on concave surface

When a sphere is displaced from its position of equilibrium on the surface of a concave spherical surface, and if be then set free, it will roll backwards and forwards, provided the coefficient of friction between the sphere and the surface is sufficiently great to ensure the sphere is not sliding. This is the problem of a sphere rolling on concave surface and is a current process used to measure the curvature of a mirror or lens in astronomy (spherometer).

Let O be the center of curvature of the mirror, Q the center of the rolling sphere, A the point on the mirror vertically below O, and let P be the point of contact of the sphere with the mirror. The angle POA is denoted θ, R is the radius of the mirror, r is the radius of the sphere (Figure 2).

We have $\boldsymbol{J}_Q = I_Q \boldsymbol{\Omega}$ with $I_Q = \frac{2}{5} M r^2$. The rotational energy is $\mathcal{E} = \frac{1}{2} I_Q \Omega^2 - I_Q \omega \Omega$. The gravitational energy is considered an external force. The acceleration of the CM point Q is given by

$$\mathbf{a}_Q = -(R-r)\dot{\theta}^2 \mathbf{u}_r + (R-r)\ddot{\theta}\mathbf{u}_\theta, \tag{8}$$

and the gradient of the energy in cylindrical coordinates is

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$$\nabla\mathcal{E} = \frac{1}{R-r}\frac{\partial}{\partial\theta}\left[\frac{1}{2}I_Q\omega(\omega-2\omega) - \nabla F\right]\mathbf{u}_\theta. \tag{9}$$

We haven't took into account the radial coordinate so far because it is supposed not to change. It is important to notice that ω and Ω have different sign since they rotate in different direction, supposing that the sphere rolls, not slides, and the constraint equation is

$$-\Omega = \frac{R-r}{r}\omega. \tag{10}$$

Hence,

$$\nabla\mathcal{E} = \frac{Mr^2}{5(R-r)}\left(2\frac{\Omega}{\omega}\dot{\Omega} - 2\frac{\Omega}{\omega}\dot{\omega} - 2\dot{\Omega}\right)\mathbf{u}_\theta \tag{11}$$

since $-\dot{\Omega} = \frac{R-r}{r}\dot{\omega}$. Thus we obtain

$$\nabla\mathcal{E} = \frac{2M}{5}(R-r)\ddot{\theta}\mathbf{u}_\theta. \tag{12}$$

Finally, the equations of motion, when observed in an inertial frame (the mirror) are the following

$$M(R-r)\dot{\theta}^2 + Mg\cos\theta - N = -\frac{\partial}{\partial R}(\Delta F) \tag{13}$$

$$\ddot{\theta} + \frac{5g}{7(R-r)}\sin\theta = 0. \tag{14}$$

Eq. (13) takes into account external forces, such as the weight and the normal reaction N. Eq. (14) gives the well-known period of oscillation, on the approximation of small angles

$$T = 2\pi\sqrt{\frac{7(R-r)}{5g}}. \tag{15}$$

Highly finished hard steel spheres are more suitable for the experiment and more accurate results will be obtained with large spheres than with small. In general, the proposed solution of the present problem is given resorting to Lagrangian or energy considerations, but we give here a new approach. In addition, notice that Eq. (13) points to the possible release (or absorption) of heat during the oscillation at the expense of the gravitational field (or dissipative forces):

$$\frac{\partial\Delta F}{\partial R} \approx \frac{12}{7}Mg \tag{16}$$

The classical solution of this problem using Newtonian Mechanics includes a friction force, and again solving separately the extended body by means of the torque equation. In practice, we do need at the beginning a friction force for the ball start rolling, but once the rolling condition is met the ball experiences no friction. The introduction of the friction force can be misleading and within the new framework it is not taken into account. In textbooks the Lagrangian formulations is frequently used to solve this problem and also certainly has practical advantages.

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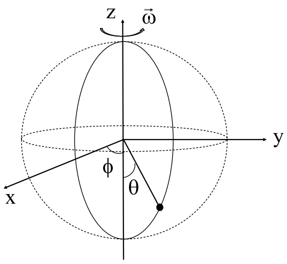

Figure 3. Bead on the hoop.

2.3. Bead on the hoop

Just to illustrate with another example the proposed method, let us consider a bead of mass m, moving without friction on a circular hoop of radius a. The hoop rotates about its vertical axis oz with constant angular velocity $\boldsymbol{\omega}$ (see Figure 3). The position of mass m is given by the pair of angles θ and ϕ in spherical coordinates, and its moment of inertia relative to axis Oz is $I_z = m(r\sin\theta)^2$. Considering the θ-component of the velocity in the rotating frame (and avoiding the normal reaction force $\mathbf{N}$), we have

$$\frac{dv_\theta}{dt} = a\ddot{\theta} = -g\sin\theta - \frac{1}{r}\frac{\partial}{\partial\theta}\left[\frac{1}{2}a^2\sin^2\theta\dot{\theta}^2 - a^2\sin^2\theta\dot{\theta}^2\right], \tag{17}$$

and, finally, we obtain the well-known result, but from a new perspective:

$$\ddot{\theta} = \sin\theta\left(\dot{\theta}^2 - \frac{g}{a}\right). \tag{18}$$

This is the mechanical analogue of an ion inside a Paul Trap with importance for quantum computation [22]. To solve this problem, the usual procedure uses the Lagrangian mechanics which considerably simplifies the physical problem. However, using Newtonian mechanics one would have a complicated set of equations of motion in the rotating frame, such as

$$m\mathbf{g} + \mathbf{N} = m\left[\left(\frac{d^2\mathbf{r}}{dt^2}\right)_{S'} + 2\boldsymbol{\omega}\times\left(\frac{d\mathbf{r}}{dt}\right)_{S'} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r})\right], \tag{19}$$

where $\mathbf{r} = a\mathbf{e}_r$ is the position vector relative to the center of the hoop, S' is the rotating frame of the wire and $\boldsymbol{\omega}$ is the angular velocity vector. After rather cumbersome calculations, the following result is derived:

$$m\mathbf{g} + \mathbf{N} = m[-a\dot{\theta}^2\mathbf{e}_r + a\ddot{\theta}\mathbf{e}_\theta + 2a\dot{\theta}\boldsymbol{\omega}\times\mathbf{e}_\theta + a(\boldsymbol{\omega}\cdot\mathbf{e}_r)\boldsymbol{\omega} - a(\boldsymbol{\omega}\cdot\boldsymbol{\omega})\mathbf{e}_r]. \tag{20}$$

Then, since $\mathbf{N}\cdot\mathbf{e}_\theta = 0$, we obtain

$$m\mathbf{g}\cdot\mathbf{e}_\theta = m[a\ddot{\theta} + a(\boldsymbol{\omega}\cdot\mathbf{e}_r)(\boldsymbol{\omega}\cdot\mathbf{e}_\theta)], \tag{21}$$

and then, after a few more steps, the final result, Eq. (18), is obtained.

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2.4. The origin of the so called "pseudo-forces"

Within this new formulation, the transformation from an inertial coordinate system to a rotating system shows that the so called "fictitious forces", although certainly not related to interacting fields, they are the outcome of the transport of angular momentum, they are real forces in the rotating medium. We should have in mind that there is active transformations (implying its motion, strongly or weakly coupled to other systems) and passive transformations (method of describing transitions to different reference systems, e.g., Galileo or Lorentz transformations) [23]. Any mathematical procedure can be reliable unless it is completely understood the purpose and how it should be applied, otherwise becomes a source of confusion.

In order to justify what we have state above, one therefore has to work out the resultant term $\nabla_r(\boldsymbol{\omega}\cdot\mathbf{J})$ in Eq. (4), developing it in the following manner:

$$\frac{1}{2}\nabla_{\boldsymbol{r}}(\boldsymbol{\omega}\cdot\mathbf{J})=\frac{1}{2}[\boldsymbol{\omega}\times\nabla\times\mathbf{J}]+\frac{1}{2}(\boldsymbol{\omega}\cdot\nabla)\mathbf{J}+\frac{1}{2}[\mathbf{J}\times\nabla\times\boldsymbol{\omega}]+\frac{1}{2}(\mathbf{J}\cdot\nabla)\boldsymbol{\omega}. \tag{22}$$

We may test the above Eq. (22) with the well-known problem of rotating coordinate frame, without translation, considering a rotating system (x', y', z') whose origin coincides with the origin of an inertial system (x, y, z). We suppose in addition, that the z and z' axes always coincide and, therefore, the angular velocity of the rotating system, $\boldsymbol{\omega}$, lies along the z axis. We take $\mathbf{J} = I\boldsymbol{\Omega}$ and $\boldsymbol{\Omega} = \boldsymbol{\omega}$, since we consider the "body" in rotational motion around the z axis. An easy calculation (considering $I_z = mr^2$) show that

$$\nabla\times\mathbf{J}=2m\mathbf{v}_{rot}, \tag{23}$$

where $\mathbf{v}_{rot}$ denotes the particle velocity in the rotating frame. Also, following the analogy

$$\nabla\times\mathbf{v}=\boldsymbol{\Omega}=2\boldsymbol{\omega}, \tag{24}$$

and using a mathematical identity, we obtain

$$\nabla\times\left(\frac{\mathbf{J}}{r^2}\right)=\frac{1}{r^2}\nabla\times\mathbf{J}+\left[\nabla\left(\frac{1}{r^2}\right)\times\mathbf{J}\right], \tag{25}$$

which gives

$$\nabla\times\left(\frac{\mathbf{J}}{r^2}\right)=\frac{2m}{r^2}(\mathbf{v}_{rot})+[\boldsymbol{\omega}\times\mathbf{r}]. \tag{26}$$

Then

$$\frac{1}{2}\left[\mathbf{J}\times\nabla\times\left(\frac{\mathbf{J}}{r^2}\right)\right]=m\boldsymbol{\omega}\times[\mathbf{v}_{rot}+[\boldsymbol{\omega}\times\mathbf{r}]], \tag{27}$$

and, therefore, adding together all the terms, it is finally obtained the well-known formula of Newtonian dynamics:

$$\left(m\frac{d\mathbf{v}}{dt}\right)_{rot}=\mathbf{F}^{ext}-2m[\boldsymbol{\omega}\times\mathbf{v}_{rot}]-m(\boldsymbol{\omega}\times[\boldsymbol{\omega}\times\mathbf{r}]). \tag{28}$$

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We stress that in Eq. (28) the force term is calculated in the non-inertial frame inserting the acceleration force inside the symbol $(...)_{rot}$. The term $(d\boldsymbol{\omega}/dt \times \mathbf{r})$ does not appear because it was assumed $d\omega/dt = 0$. We may notice that the well-known force terms have not a cinematic origin, but are intrinsically dynamical, resulting from the balance between the minimization of energy and maximization of entropy. The gradient term of the rotational energy acts like a pullback process to obtain cinematic from dynamics. Within this formulation, the fundamental equation of dynamics, Eq. (4), is valid in non-inertial frames of reference as well. We may also remark that on the rhs of Eq. (22), the second and fourth terms points to the possible existence of an axial particle motion when the "body" has a characteristic of a viscous fluid, generating phenomena called Taylor structures.

2.5. Newton's bucket experiment

The Newton's bucket experiment, described in his *Opus Magnum* "The Principia" in 1689, is one of the most important experiments in the history of physics having a lasting impact in the thinking of physicists. The experiment can be done with a bucket filled with water, hanging it by a rope, twisting the rope tightly and let it unwind freely. As we may notice if we do ourselves the experiment, at first the bucket starts to spin but the surface of the water (or any other regular fluid) remains practically flat. But as the bucket increases its rotating speed, the water starts gaining momentum due to friction with the walls and at the end, water and bucket rotates at the same speed, and the water's surface acquires a concave shape.

But here we point to another interpretation that we hope to clarify with the application of our fundamental equation of dynamics written in vectorial form:

$$m\frac{d\mathbf{v}}{dt} = \mathbf{F}^{ext} - \nabla\left[\frac{1}{2}I\Omega^2 - I(\boldsymbol{\omega}\cdot\boldsymbol{\Omega}) - \Delta F\right] - \nabla p. \tag{29}$$

The gradient term shows that differential rotation may be a source of thermodynamic free energy in a flow. It is the source of transport of angular momentum outward through the bucket, transferring matter to regions of higher pressure, and this is the signature of the second law of thermodynamics.

To simplify, we assume that there is a "body" inside a "container" that rotates with the container at the same angular velocity, like a solid body – the bucket's experiment. Hence, $\boldsymbol{J} = I\boldsymbol{\Omega}$ with $\Omega = \omega$. To simplify, we doesn't include the buoyant force. At the end, when the rotation attained a uniform angular velocity, each element of fluid has no acceleration, moves at a given constant radial distance from the polar axis. It behaves like a stone tied to the end of the rope in a circular motion at constant speed. We invite the reader to perform the experiment, as Newton described in Scholium to Book I of the Principia. An observer at rest in the rotating coordinate frame (not withstanding the physiological impressions) will have no acceleration

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although from the point of view of an inertial observer at rest, let's say, in the Earth's coordinate frame, there is a circular motion. But since the "body" has no tangential acceleration, nor moves radially, nor is in anyway distorted in its form but keeps simply describing its circular motion, we must state that, dynamically, in the rotating frame, $\mathbf{a} = 0$. Then, we obtain from Eq. (29), and *inside* the rotating coordinate frame

$$0 = -\rho g \mathbf{u}_z - \nabla \left(-\frac{1}{2} I \omega^2 \right) - \frac{\partial p}{\partial r} \mathbf{u}_r - \frac{\partial p}{\partial z} \mathbf{u}_z. \tag{30}$$

The gradient of the angular momentum represents the transport of angular momentum across swirling matter, with a crucial role in the formation of planetary and proto-stellar accretion disks. It gives

$$\nabla \left(\frac{1}{2} I \omega^2 \right) = \frac{1}{2} \omega^2 \nabla (\rho r^2) = \rho \omega^2 r \mathbf{u}_r. \tag{31}$$

We put $I = \rho r^2$ as the moment of inertia of the "body" swirling around the axis. Then, considering $\frac{\partial p}{\partial \theta} = 0$, we have

$$0 = -\rho g \mathbf{u}_z + \rho \omega^2 r \mathbf{u}_r - \frac{\partial p}{\partial r} \mathbf{u}_r - \frac{\partial p}{\partial z} \mathbf{u}_z. \tag{32}$$

This gives for each component: $\frac{\partial p}{\partial r} = \rho \omega^2 r$, $\frac{\partial p}{\partial z} = -\rho g$, $\frac{\partial p}{\partial \theta} = 0$. Integrating these three equations, we obtain the well-known solution $p(r, \theta, z) = \frac{1}{2} \rho \omega^2 r - \rho g z + Const$. However, the interpretation we may give to this experiment is now diverse from the previously given by Newton, Berkeley, Leibniz (see, e.g., Ref. [24] for a clear presentation of traditional theories). It doesn't matter if there is rotation relative to absolute space (Newton) or the background of galaxies (Mach), or if the motion is relational (Leibniz, Torres Assis [24]). What does matter is the transport of angular momentum (imposing a balance between the centrifugal force, pushing the element of fluid to outside) counterbalanced by the fluid pressure. As it is clear from Eq. (31), the centrifugal forces are the outcome of the gradient of rotational energy. Within the scope of the present formulation, in the rotating frame it is not necessary to introduce new, fictitious, forces to account for this curved motion.

As pointed out by Torres Assis [24], even in the instance that all matter around the swirling body is annihilated, nothing else would change in the mathematical prediction.

2.6. Electrodynamical equilibrium and the dynamic effect of the topological torsion current: rotating plasma and flyby anomaly

Extremum conditions imposed on entropy or internal energy, does not only provide criteria for the evolution of the system. They also determine a novel condition for the stability of thermodynamic systems. With the new formulation, we have shown its capability to work out the usual sources and fields of the electromagnetic theory

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in a previous work [6], but we have obtained a general equation of dynamics for electromagnetic-gravitational systems with a new term that we called the topological torsion current (TTC), see Ref. [1]:

$$\rho \frac{d\mathbf{v}}{dt} = \rho \mathbf{E} + [\mathbf{J} \times \mathbf{B}] - \nabla \phi_g - \nabla p + \rho [\mathbf{A} \times \boldsymbol{\omega}]. \tag{33}$$

The symbols have the usual meaning. The last term in Eq. (33) is the topological torsion current and to our best knowledge it is a sofar unforseen force, producing work by means of a direct conversion of angular motion into linear motion. As a consequence, it appears an electric field induced by motion

$$\mathbf{E}^m = -\nabla \phi - \partial_t \mathbf{A} - [\boldsymbol{\omega} \times \mathbf{A}], \tag{34}$$

where ϕ represents an electric potential, $\mathbf{A}$ the vector potential, and $\boldsymbol{\omega}$ is the angular velocity, playing the role of a spin connection, a signature of the torsion field. For a charged particle q, its Liénard–Wiechert potentials are given by $\phi(\mathbf{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{1}{(r - \mathbf{r} \cdot \mathbf{v}/c)}$ where $\mathbf{r}$ is the vector from the retarded position to the field point $\mathbf{r}$ and $\mathbf{v}$ is the velocity of the charge at retarded time, and $\mathbf{A}(\mathbf{r}, t) = \frac{\mathbf{v}}{c^2} \phi(\mathbf{r}, t)$. Notice that we may write $[\boldsymbol{\omega} \times \mathbf{A}] = \widetilde{\boldsymbol{\omega}} \phi$. Inserting into Poisson's equation, which is a second-order elliptical differential equation describing the electrostatic potential caused by a charge distribution, $\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$, it gives back a new form of the Poisson's equation which feature the possibility of resonant phenomena (e.g., in dusty-plasma medium):

$$\nabla^2 \phi + (\nabla \phi \cdot \widetilde{\boldsymbol{\omega}}) + (\nabla \cdot \widetilde{\boldsymbol{\omega}}) \phi = -\frac{\rho}{\epsilon_0}. \tag{35}$$

Eq. (35) is structurally intermediate between the Poisson and Helmholtz equations, having an additional damping term that can be reduced to a one-dimensional differential equation of the forced, damped linear harmonic oscillator. Poisson's equation has extensive applications to engineering problems dealing, e.g., with steady state heat conduction in heat generating media, theory of torsion of prismatic elastic bodies, quantum chemistry, gravitational phenomena, and electrostatics. Evans [25] obtained the same result using a different approach.

Assume to simplify, only radial dependency, such as, $\widetilde{\omega} = \widetilde{\omega}(r)$. Then

$$\partial_{rr}^2 \Phi + \left(\frac{2}{r} + \widetilde{\omega}_r \right) \partial_r \Phi + \left[\frac{1}{r^2} \partial_r (r^2 \widetilde{\omega}_r) \right] \Phi = -\frac{Ze}{\epsilon_0} \tag{36}$$

The analytical solution of Eq. (36) is represented in Figure 4 for the special case $\widetilde{\omega} = 1$.

It is similar to the potential interatomic potential or the effective potential for the one-dimensional radial Schrödinger equation for a system with total angular momentum l, the result of an optimal arrangement between repulsive and attractive forces.

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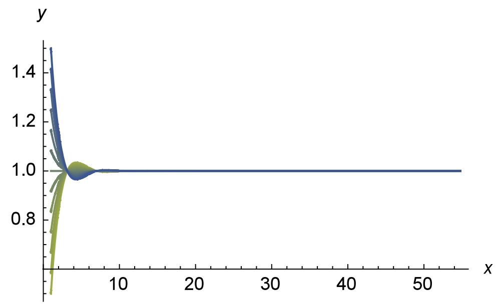

Figure 4. Set of potential curves similar to the interatomic potential or hydrogen atom potential vs. distance, in arbitrary units.

The relevance of Eq. (33) in phenomena as the plasma rotation mechanism in thermonuclear fusion reactors [1] and the possible role of TTC on the flyby anomaly of a spacecraft [9] was elucidated.

2.7. The spin-orbit planetary coupling

The rotational and orbital period of a planet or a satellite is known as spin-orbit coupling. From Eq. (4) another effect will result from the gradient of the energy, representing the transport of angular momentum. This energy is given by (now omitting the free energy term)

$$E = \frac{1}{2} I \Omega^2 - I(\boldsymbol{\omega} \cdot \boldsymbol{\Omega}). \tag{37}$$

Seeking for the radial minimum of transport of angular momentum

$$\frac{dE}{dr} = I \left[(\Omega - \omega) \frac{\partial \Omega}{\partial r} - \Omega \frac{\partial \omega}{\partial r} \right]. \tag{38}$$

It can be shown (see, e.g., Ref. [26]) that $\frac{\partial \omega}{\partial r} \ll \frac{\partial \Omega}{\partial r}$. Therefore, from the equilibrium condition, it follows $\omega = \Omega$. This is the well-known $1 : 1$ resonance verified by most of the outer planet satellites, with the puzzling exception of Mercury's $3 : 2$ spin-orbit resonance [27]. One possible cause of the mysterious Mercury's commensurability is the intense solar radiation actuating by means of the ΔF term [28, 29].

2.8. Period of oscillation of the simple pendulum

The calculation of the natural frequency of the simple pendulum of mass m, fixed length R, for small amplitude oscillations is done by solving the equation:

$$m(R\ddot{\theta}\mathbf{u}_\theta - R\dot{\theta}^2\mathbf{u}_r) = mg\cos\theta\mathbf{u}_r - mg\sin\theta\mathbf{u}_\theta - T\mathbf{u}_r - \frac{\partial}{\partial r}\left[\frac{1}{2}mR^2\dot{\theta}^2\right]\mathbf{u}_r - \frac{1}{r}\frac{\partial}{\partial \theta}\left[\frac{1}{2}mR^2\dot{\theta}^2\right]\mathbf{u}_\theta, \tag{39}$$

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where we have put $I_0 = mR^2$ and $J = I_0\dot{\theta}$ and denoting by T the tension of the rope of length R. It follows immediately the well known equations:

$$\ddot{\theta} + \frac{g}{R}\sin\theta = 0 \tag{40}$$

and

$$T(\theta) = mg\cos\theta + mR\dot{\theta}^2. \tag{41}$$

The exact solution of the Eq. (39) is obtained by the usual method, multiplying by $2\dot{\theta}dt$ and integrating, obtaining

$$\dot{\theta}^2 = 2\omega^2\cos\theta + C \tag{42}$$

where C is the integration constant. At the maximum angular displacement θ_m, $\dot{\theta} = 0$, and then $C = -2\omega^2\cos\theta_m$. Hence

$$\dot{\theta} = \omega\sqrt{2(\cos\theta - \cos\theta_m)} = 2\omega\sqrt{\sin^2(\frac{\theta_m}{2}) - \sin^2(\frac{\theta}{2})}, \tag{43}$$

where we have made use of the trigonometric identity $\cos\theta = 1 - 2\sin^2(\theta/2)$. Separating the variables and integrating between $t = 0$ and t, we obtain

$$\theta(t) = 2\arcsin(k\ JacobiSN(\omega, t, k)) \tag{44}$$

with $k = \sin(\theta_m/2)$. The pendulum period follows:

$$T = \frac{4}{\omega}K(k), \tag{45}$$

with

$$K(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2\sin^2\theta}}, \tag{46}$$

where $JacobiSN(u, k)$ is the Jacobian elliptic sine function, and $K(k)$ is the complete integral of the first kind.

2.9. Additional remarks on the variational problem according to the proposed reformulation

In 1686, Newton solved the problem of determining the shape of a rotationally symmetric body of least resistance, but the beginning of the calculus of variations is set in the year 1696 with the formulation of the brachistochrone problem by Johann Bernoulli in the *Acta Eruditorum*. Without recurring to minimum-time assumptions, or Euler–Lagrange equations, this reformulation has the advantage to yield an answer to the problem of the brachistochrone for uniform (e.g., gravitational) field and be adequate to handle variational problems.

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We now pass on to a description of the brachistochrone. Let us consider a Cartesian reference frame and a particle with mass m accelerated by gravity from one point to another with no friction and solve the problem using Eq. (4). First, notice that the Cartesian frame is not rotating, hence, $\omega = 0$. Noting that Ω is independent from coordinates (x, y), we may assume $x(0) = -R$, $\frac{dx}{dt} = R\Omega t$ and $\frac{dy}{dt}(0) = 0$, and $y(0) = R$. It gives

$$\frac{d^2y}{dt^2} = -g - y\Omega^2 \tag{47}$$

$$\frac{d^2x}{dt^2} = -x\Omega^2 \tag{48}$$

since, $\nabla_y(\frac{1}{2}mr^2\Omega^2) = my\Omega^2$ (with similar result for x component), and plugging the identity $dr = \sqrt{dx^2 + dy^2}$. The solutions of the above system of equations are the well-known equations of a cycloid:

$$x(t) = R(\Omega t - \sin \Omega t) \tag{49}$$

$$y(t) = -\frac{g}{\Omega^2}(1 - \cos \Omega t) + R \cos \Omega t. \tag{50}$$

The cycloid equations were straightforwardly obtained because the fundamental equation of dynamics, Eq. (4), retains consistently the two extremal principles of nature, the tendency to minimize energy and maximize entropy. There is no need to make further hypothesis.

The standard solution of one of the earliest problems in the calculus of variations starts with the calculation of the time to travel from a point P_1 to another point P_2:

$$t_{12} = \int_{P_1}^{P_2} \frac{ds}{v}, \tag{51}$$

obtaining the speed at any point by using the equation of conservation of energy, $v = \sqrt{2gy}$. Plugging the arc length $ds = \sqrt{1 + y'^2}dx$, the integral of t_{12} is converted into

$$t_{12} = \int_{P_1}^{P_2} \sqrt{\frac{1 + y'^2}{2gy}} dx = \int_{P_1}^{P_2} f(y, y')dx. \tag{52}$$

Normally, the Euler–Lagrange differential equation is used, and, after some maths, another differential equation is obtained

$$[1 + (\frac{dy}{dx})^2]y = k^2, \tag{53}$$

where k^2 is a positive constant. The solution is given by the parametric equations, Eqs. (49)–(50). Our foundational project aims at finding a coherent and possibly true representation of what is physically possible without involving extraneous

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idealizations or empirical generalizations. In this sense, our formulation seems more natural than the Lagrangian formulation, since it represents a new synthesis of two different propensities in nature (provided by energy and entropy).

3. Conclusion

It follows, as a result of theory, that dynamics needs irreversibility as an elementary inherent mechanism to consistently describe and comprehend physical systems. The theory represents a fundamental conceptual departure from Newtonian mechanics, showing that a physical system is not only actuated by "forces", acting as a "slave system". It can have mutual interaction with sub-systems and self-regulated mechanisms. The present approach offers consistent solutions to well-known problems from different perspectives, revealing leading principles of Nature's operating system.

The integrations of the gradient of a new term that contains the rotational energy and thermodynamical free energy, besides the usual force term which has a non-entropic character, plays a vital role for an adequate understanding of dynamical and electrodynamical processes in nature. The set of two first order differential equations, related to the interplay between the tendency of energy to attain a minimum, whilst entropy seeks to maximize its value, opens a major change in the description of dynamical and electrodynamical processes in nature. The description of physical phenomena by ergontropic dynamics is remarkably simple and these foundations support applications that extend well beyond the starting concepts, in particular integrating in one equation translational and rotational motion. Considering the embryonic stage of thermodynamics at the 17th century, this synthesis was an impossible task at the time of Sir Isaac Newton. But this formulation embraces Newton's law of dynamics with entropy in a novel way. Of course, at the present stage, this formalism and concepts constitute a research program needing further development. It does not replace Hamiltonian formulation but, depending on the nature of the problem, its approach can be an essential tool for investigation.

Declarations

Author contribution statement

Mario Pinheiro: Conceived and designed the analysis; Analyzed and interpreted the data; Contributed analysis tools or data; Wrote the paper.

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Competing interest statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

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