



## Review article

## Weibull analysis of ceramics and related materials: A review

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## ABSTRACT

It has been realized throughout the years that an ideal combination of high toughness, hardness and strength is required in many engineering applications that need load-bearing capabilities. Ceramics and related materials have significant constraints for structural and particular non-structural applications due to their low toughness and limited strength while having substantially superior hardness than typical metallic materials. For example, hydroxyapatite (HAp) has gained attention for applications in orthopaedic implants, dental materials, drug delivery, etc. Researchers have continued to strive to produce HAp materials with reliable properties within the acceptable Weibull modulus ( $m$ ) for load bearing. The Weibull analysis (WA) is a statistical analysis adopted widely in reliability applications to detect failure periods. Researchers have confirmed it to be an effective technique to get results on the reliability of materials at a moderately low rate with assured reliability of the material or component. This review summarizes the WA and the steps in the Weibull method for its reliability analysis to predict the failure rate of ceramics like HAp and other related materials. Also, the applications of WA for these materials were reviewed. From the review, it was discovered that Weibull distribution is proven to confer to the feeblest-link concept. For brittle materials, it was revealed that the Weibull Modulus ranges from 2 to 40, and environment, production processes, and comparative factors are well-thought-out contributing factors for reliability. In addition, the confidence interval can be up to 95 %. The frequently used technique for reliability valuation is to syndicate the Weibull statistics. Also, a very narrow distribution is desirable to offer the expected likelihood. Furthermore, when paired with trials, Monte Carlo simulations prove to be a very helpful tool for forecasting the dependability of different estimate techniques and their optimization. Finally, if

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the equivalent  $m$  is anticipated to be high, it signifies that the material has a high degree of homogeneity of properties and high reliability. WA can find application in predicting the dependability and lifetime of materials, making it widely utilized in engineering and other disciplines. It is especially useful for analysing data in which the likelihood of failure per unit of time varies over time.

## 1. Introduction

The restoration of bone defects and osteoporosis are two significant challenges in bone repair activities that require the reliability of the implant material [1–3]. Several techniques, including plasma spraying, and hydroxyapatite (HAp) coatings are used to achieve osseointegration and graft external treatment [4–7]. However, HAp and other porous coating materials are the most often used remedies today [8]. Using a coating layer aims to achieve effective osseointegration and critical bone-implant interaction. Researchers have continued to investigate the production of HAp scaffolds for bone tissue applications in recent years because of their essential bioactivity and osteoconductivity [9]. HAp is an inorganic bio-ceramic component of natural bones and teeth with high biocompatibility, osteoinductivity, surface area ( $100 \text{ m}^2 \text{ g}^{-1}$ ), and bioactivity [10–12]. HAp has the following characteristics: The apatite molecular formula is  $\text{Ca}_5(\text{PO}_4)_6(\text{OH})_2$ , similar to human apatite, which has a Calcium - Phosphate ratio of 1.67.

HAp has been confirmed by different researchers to be the most stable biomaterial with these basic qualities (biocompatibility, osteoinductivity, and bioactivity) when employed in bone repair [13]. The bones of cattle or bovine, camels, horses, and fish can produce natural HAp. Egg shells, plants, algae, and limestone can also produce it [14–22]. In the work conducted by [23], some of the physiochemical, mechanical, and biological characteristics of HAp, and some of the methods for producing HAp powder were listed. The composite biomaterials like synthetic or natural polymers plus HAp have mechanical properties most similar to human bone tissue [24]. HAp has low fracture toughness and flexural strength, which prevents it from being used as a graft material on a large scale in orthopaedics and dentistry [25]. As a result, in the field of biomedical engineering for bone implants, their mechanical qualities and reliability are critical. One way to determine the reliability of ceramic materials is through Weibull statistical analysis by knowing the Weibull modulus,  $m$  [26–29]. Numerous methods have been employed to measure the Weibull modulus,  $m$ , and its physical consequences on the statistical distribution of fracture strengths in brittle materials [30]. Despite the importance of the Weibull modulus,  $m$ , as a measure of the mechanical reliability of ceramic materials, only a few research reports are available on the Weibull modulus and fracture strength of HAp. Weibull Analysis (WA) is a statistical approach for analyzing a material's life cycle statistics. The results of measuring a product's life are known as life statistics [31]. Failure analysis is a dynamic system of understanding a system's consistent features and activities, employing a minimal model scope of the area [32–37]. WA is a valuable tool for determining a product's lifetime performance. Weibull techniques can be used to evaluate sample data acquired concerning failures and time to aid in responding to major concerns [38]. The capacity to investigate failure tendencies and provide failure predictions based on known sample data sets is the main value of WA, which is related to its flexibility and ability to apply to small sample sets quickly. It also provides a visual and graphical representation of failure data [39–41].

A technique for assessing life data is WA. Measurements of a product's life yield its life data. Product life data is measured in hours, miles, number of cycles, or other metrics that are used to determine a product's efficient functioning, depending on the product or industry [42]. Weibull analysis's main benefits are as follows: It may provide fairly precise failure assessment and failure predictions with tiny data samples, enabling solutions to be implemented as soon as a flaw is detected [43–46,47]. The Weibull distribution is extensively employed in failure time prediction due to the large range of probability curve shapes that may be produced by varying the two parameters,  $\beta$  (shape parameter), and  $\alpha$  (scaling parameter) [48,49]. Even though the procedure of the normal distribution normally involves at least 20 failures or facts from previous experience, WA works exceedingly fine when there are as limited as 2 or 3 failures, which is serious when the result of a failure comprises safety costs. Other advantages of Weibull distribution include [50–52]:

- (i) It is expansively used to assess mechanical, chemical, and material failures.
- (ii) Afford discreetly precise failure analysis and failure predictions with very small data samples, making results likely at the earliest signs of a problem.
- (iii) Afford modest and valuable graphical plots for distinct failure modes that can be simply read and understood, even when data insufficiencies occur.
- (iv) Signify a wide array of distribution shapes so that the distribution with the finest fit can be selected.
- (v) Afford physics-of-catastrophe signs according to the slope of the Weibull possibility.

Some research has been done to buttress these advantages. To evaluate the mechanical reliability of porous HAp, Pires et al. [53] employed WA to investigate the fracture toughness of dense polycrystalline HAp bioceramic made from cow bones after adding  $\text{ZnO}/\text{TiO}_2$  nanoparticles and  $\text{TiO}_2$  nanotubes. The WA results indicated that adding 5 % of  $\text{TiO}_2$  nanoparticles improved the Weibull parameter ( $m$ ), but there was no statistically significant variation from the pure HAp.  $\text{ZnO}_2$  nanomaterials at a 5% proportion reduced the HAp characteristic strength without altering  $m$ . Fan et al. [54] studied the strength of extremely porous HAp. The partly sintered HAp samples were cracked in biaxial flexure using a ring-on-ring examination fitting. The fracture strength declined monotonically with declining sintering temperature from 4.8 MPa for samples sintered at  $1025^\circ\text{C}$ –0.66 MPa for samples sintered at  $350^\circ\text{C}$ . However, the value of  $m$  rose unexpectedly, extending from 6.6 to 15.5. Ćurković et al. [55] employed WA to investigate the flexural strength of

alumina ceramics. The three-point bend test was used to determine the flexural strength of standard purity alumina ceramics. Flexural strength was determined to be between 266.7 and 357.5 MPa. The numerical randomness of flexural strength calculated by the three-point bend test was investigated using a two-parameter Weibull distribution ( $W_d$ ) function. Flexural strength was measured at 17.4 Wm. They concluded that this restriction can be used to characterize the variability in the tested material's flexural strength as well as its consistency. What matters most for the mechanical characteristics of brittle materials (bioceramics) is the Weibull modulus,  $m$  [56]. In the production of bioceramics, the  $m$  can also be influenced by the following factors in addition to the microstructure: powder treating methods [57], strength evaluation methods [58], rate of loading [59,60], particle size and form [61], and finishes on the surface [62,63]. A two-parameter WA can be employed to evaluate the fracture data of valid cracked samples [64,65]. Generally speaking, only moderately dense bioceramic samples are covered in the literature when discussing the  $m$  of brittle materials [66].

Bioceramics belong to a family of biomaterials employed in biomedical engineering. Because of their versatility in fabrication, high compressive strength, variable porosity, and bioactive qualities in the body, ceramics are frequently employed as implant materials [67]. This type of biomaterial can be synthesized from bovine and catfish bones [68–72]. The boom in bovine and catfish farming in Nigeria has led to a substantial upsurge in the creation of bone biowastes. These biowastes can be used as raw material for producing HAp, a pervasive calcium phosphate substance used in biomedical engineering [73]. The economy, environment and general health could all significantly gain from this conversion of biowastes [74]. The conversion of these biowastes into HAp is a good development, but the analysis to determine or predict its reliability using WA is of great importance and is challenging. This review summarizes the overview of WA and some steps in the Weibull method for its reliability analysis to predict the failure rate of HAp. Also, the applications of HAp are presented. This review thus serves as a reference for further advanced studies on determining the  $m$  of naturally derived HAp from an array of sources like bovine and catfish bones to predict its reliability.

## 2. Overview of Weibull analysis (WA)

Weibull distribution is proven to confer to the feeblest-link concept [75–78]. For brittle materials, the literature revealed that environs, production processes, and comparative factors are well-thought-out contributing factors for reliability [79–82]. The frequently used technique for reliability valuation is to syndicate the Weibull statistics [83]. In this condition, the strength is defined as a definite distribution but not a single numeral. The estimated  $m$  from strength distribution can reveal the strength potential of the material investigated. A very narrow distribution is desirable to offer likelihood. The equivalent  $m$  is anticipated to be high, signifying the material has a high degree of homogeneity of properties and high reliability [84–87]. For materials with a homogeneous flaw density, Weibull analyzes fracture statistics [39].

WA is a widely utilized distribution in the analysis of reliability and durability data. The Weibull distribution may describe diminishing, cumulative, or constant risk functions, allowing it to define any stage of the item's life cycle [88–93]. Waloddi Weibull introduced a statistical distribution in 1939 that frequently defines identified failures. Weibull's distribution was given as a single specialized subject. The distribution was based on Pierce's concept of the "weakest link," and since then, it's been broadly used to evaluate the fracture-related mechanical properties of ceramics and metals. Equation (1) below gives the cumulative probability function of Weibull two parameters,  $W_d$  [94–97]:

$$P = 1 - \exp\left[-\left(\frac{x}{x_0}\right)^m\right] \quad (1)$$

where  $P$  = probability of failure at a given fatigue life,  $x$ , or lower;  $x_0$  and  $m$  are scale and shape parameters, respectively.

The following assumptions support the Weibull model [98]:

1. The sum of defects alleged in one intermission is autonomous for a fixed group of time. The number of detected failures is indiscriminate at this point.
2. At the start of the period in which the package is noticed, there is a fixed number of flaws ( $N$ ) in the package. In life testing, taking the logarithms of the failure times is usual. The log failure times have a normal distribution if the log-normal distribution is the assumed distribution. The assumption that the lifespan follows a Weibull distribution is more widely held. In this instance, the distribution of the log failure times is location-scale rather than typical.
3. The period of defects of failure is scattered as a Weibull distribution with parameters  $x_0$  and  $m$ . In Weibull regression, it is typically assumed that the shape parameter is constant and the scale parameter depends on the predictor variables.
4. A defect is amended instantly deprived of presenting new defects in the package. When choosing an experimental design, one should take the number of parameters that need to be estimated into consideration. All of the model's parameters should be estimable by the design.

We can formally simulate the interaction if assuming a common shape parameter,  $x_0$ , for the Weibull distribution (or constant scale parameter  $m$  in the distribution). For instance, Dey et al. [99] disclosed that the individual data of nano-hardness and Young's modulus were estimated with the aid of Weibull statistics. As the indentation stress was increased from 10 to 1000 mN, the Weibull moduli data for both the nano-hardness and Young's modulus of the MIPS-HAp coating increased. The  $W_d$  fixtures for the nano-hardness value of the coating are presented for the low (10–100 mN) loads and the high (300–1000 mN) loads. Recently, Tiryakioglu and Campbell [100, 101] presented procedures for understanding Weibull probability plots as well as the three-parameter Weibull distribution and Weibull combinations. Their result showed that ceramics, with their low-fracture toughness, are also likely to yield fracture properties that will

trail the two-parameter Weibull distribution. According to Ref. [102], the Weibull distribution ( $W_d$ ) is elastic to diverse distributions.

In conclusion, the use of  $W_d$  has three great advantages [103]:

1. It can precisely model quality and performance characteristics (PC), and its elasticity brands it supreme for use in analyzing a dataset with an unknown distribution.
2. Following that, it denotes a significant quantity of a task.
3. It gives precise failure investigation and hazard forecasts with trivial models.

The parameters have actual denotation and  $m$  shows whether the rate of the measured PC is increasing, steady, or falling at its current rate. A  $\beta < 1.0$  shows that the feature is decreasing, while an  $\alpha > 1.0$  shows that it is an upward rate (Fig. 1).

### 2.1. Graphical representation of Weibull parameters

The shape,  $\beta$ , parameter depicts the distribution's shifts from 0, with a negative shape indicating a shift to the left of 0 and a positive shape indicating a shift to the right. The scale parameter,  $\alpha$ , is the statistics' 63.2 percentile, and it represents the Weibull arc's link to the shape in the same way as the mean characterizes the place on a standard curve. The Weibull curve's shape is determined by the shape parameter. The properties of distinct dissimilar life dispersals can be perfected by modifying the shape [109,110]. The 63.2 percentile of the distribution is the Weibull scale parameter ( $\alpha$ ) [111]. This indicates that, for instance, 63.2 % of the observed values will be less than 2 if a Weibull distribution with  $\alpha = 2$  is used. The image that follows illustrates how the scale parameter changes while maintaining a constant shape parameter ( $\beta = k = 3.5$ ) (Fig. 2).

As illustrated by Liu et al. [112] (Fig. 3a), the scale parameter for 80 % of the 205,873 fitted Weibull Probability Density Function (PDF) was less than 70 s. This indicates that 63.2 % of all observations for those 80 % of probabilities had a dwell time value of less than 70 s. The distribution's plan is modelled by the shape parameter,  $\beta$ . An exponential distribution replaces the Weibull distribution if  $\beta = 1$ . A normal distribution is comparable to the Weibull probability density function (PDF) when  $\beta$  is between 3 and 4. Consider Fig. 3b which represents a Weibull PDF with a steady shape parameter ( $\beta = 20$ ) varied from 0.5 to 2, 3.5, and 8 [112].

#### 2.1.1. Weibull Distribution with shape < 1

When  $\beta$  is between 0 and 1, the graph displays that the likelihood declines exponentially from infinity. About the catastrophe rate, the value that this distribution has is a greater number of early failures, which declines over the period as the faulty samples are removed from the model. Since these failures occur in the early period of an item's life, it is referred to as "infant mortality" [113–117]. In this review hereafter, the scale parameter,  $\alpha$  is replaced with  $x_0$ , the shape parameter,  $\beta$  is replaced with  $m$ , and the threshold parameter,  $\gamma$  is replaced with  $y_0$ . Fig. 4a shows a typical Weibull distribution with the shape parameter less than 1.

#### 2.1.2. Weibull Distribution with shape = 1

If  $m$  is equal to 1, the  $W_d$  declines exponentially from  $1/x_0$ , where  $x_0 =$  the scale parameter. This proves that with time, the failure degree is steady. This shape is suitable for haphazard catastrophes and multiple-cause catastrophes, which can be adapted to perfect the valuable life of items [118]. Fig. 4b shows a typical Weibull distribution with the shape parameter equal to 1.

#### 2.1.3. Weibull Distribution with Shape between 1 & 2

When  $m$ , is between 1 and 2, the Weibull Distribution increases to its highest rapidly, before declining over time. The failure rate

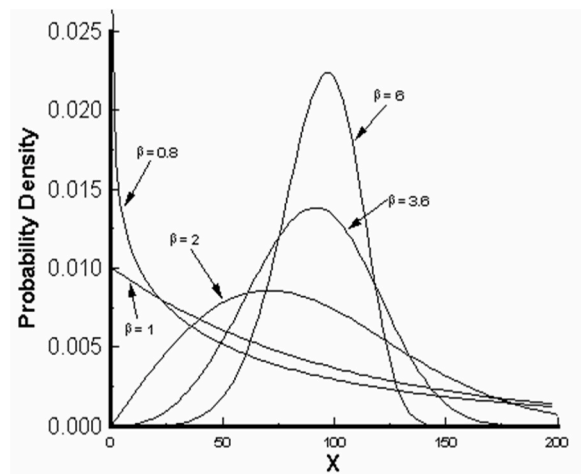


Fig. 1. Typical of Weibull distribution showing the shape parameters [104–108].

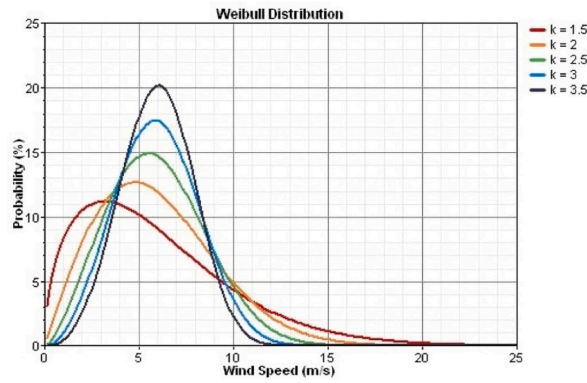


Fig. 2. Typical Weibull distribution plot [111].

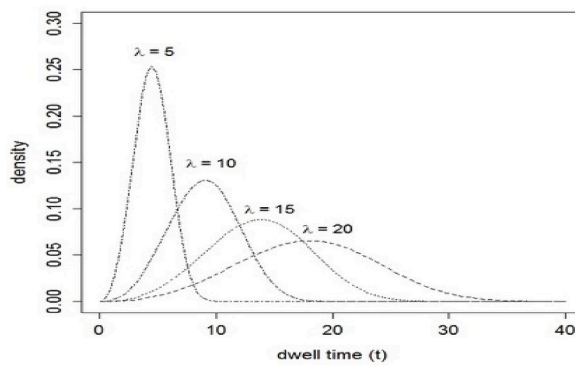


Fig. 3a. Typical 63.2 % Weibull distribution showing shape parameters [112].

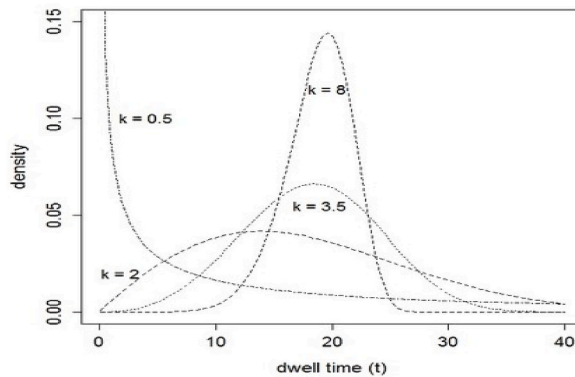


Fig. 3b. Typical 63.2 % Weibull distribution with varied shape parameters [112].

rises completely, with the quickest rise happening first. This shape is revealing premature failures due to wear and tear [116,119]. Fig. 4c shows a typical Weibull distribution with the shape parameter between 1 and 2.

2.1.4. Weibull Distribution with shape = 2

The direct growing failure rate is when  $m = 2$ , in which the risk of wear-out failure climbs progressively during the item's lifetime. The Rayleigh distribution describes the Wd's characteristics [120–122]. Fig. 4d shows a typical Weibull distribution with the shape parameter equal to 2.

2.1.5. Weibull Distribution with Shape between 3 & 4

If  $m$  is between 3 and 4, the Wd is symmetrical and bell-shaped, just like the typical curve. This Wd approach simulates quick wear-

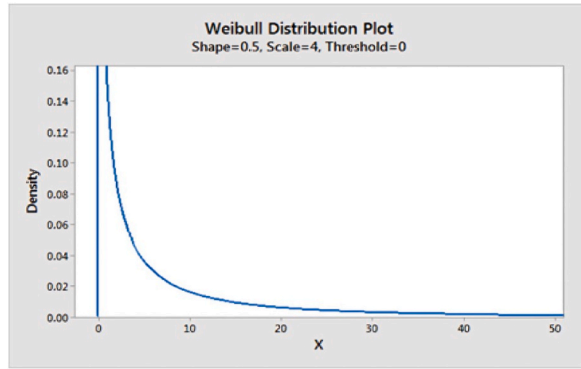


Fig. 4a. Typical Weibull distribution with shape parameter less than 1 [116].

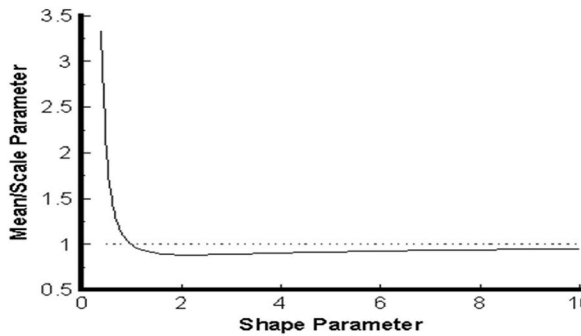


Fig. 4b. Typical Weibull distribution with shape parameter equal to 1 [116,118].

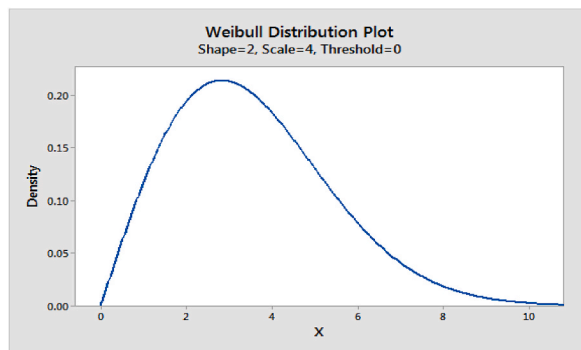


Fig. 4c. Typical Weibull distribution with shape parameters between 1 and 2 [116].

out failures in the final stages of an item’s life when maximum disasters occur [120]. Fig. 4e shows a typical Weibull distribution with the shape parameter between 3 and 4.

2.1.6. Weibull Distribution with shape > 10

When  $m > 10$ , the Wd resembles an end data distribution, and this distribution system can perfect the ultimate passé of a product lifespan [117,123,124]. Fig. 4f shows a typical Weibull distribution with the shape parameter greater than 10.

Considering the bathtub graph shown in Fig. 5, a typical failure pattern and several stages can be noticed. One specific kind of failure rate graph is the bathtub curve. The deterioration prediction and reliability engineering both make use of this graph. The term ‘bathtub’ describes a line with two edges of the curve that resemble a bathtub. Three areas make up the bathtub curve [125,126]:

- (i) Because of these early failures, the first region has a declining failure rate. Where shape parameter  $< 1$ .
- (ii) Because of sporadic failures, the middle section has a steady failure rate. Where  $m = 1$ .
- (iii) The final area shows a rising failure rate as the result of wear-out issues. Where shape parameter  $> 1$ .

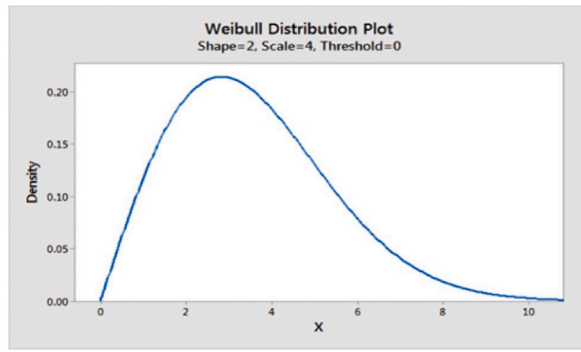


Fig. 4d. Typical Weibull distribution with shape parameter equal to 2 [121].

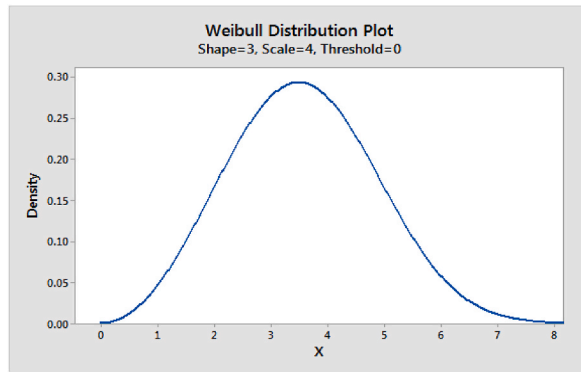


Fig. 4e. Typical Weibull distribution with shape parameters between 3 and 4 [116].

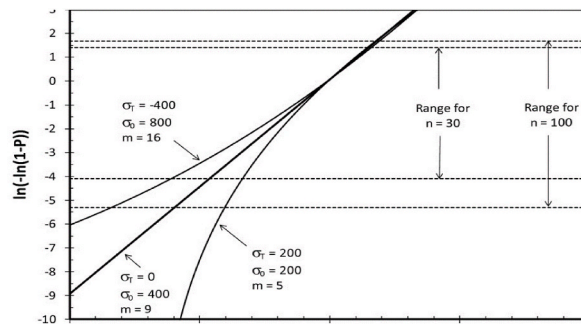


Fig. 4f. Typical Weibull distribution with shape parameter greater than 10 [117,124].

Not every product has a bathtub curve failure rate. A product is considered to follow the bathtub curve when early causes of potential failures, like manufacturing defects or damage received during transit, are identified and addressed. Throughout a product’s midlife, failure rates are consistent. As a product age, wear out increases the failure rate. The life cycles of many consumer electronics goods exhibit the bathtub curve [115,127,128].

2.2. Procedure for the 2-parameter Weibull analysis (2-PWA)

The 2-parameter  $W_d$  serves as the foundation for Load and Resistance Factor Computations (LRF). This method begins by applying a 2-parameter  $W_d$  to an entire data set or a subset of data in place of the distribution’s lower end [120,129]. The  $W_d$  is categorized by two parameters:  $m$  (dimensionless) and  $x_0$  (m/s) [130–132]. Several approaches to assessing Weibull parameters have been proposed by various researchers. Dodson [133] presented various procedures to evaluate the  $m$  (dimensionless) and  $x_0$  (m/s), plus the highest probability estimator and likelihood plotting. The highest probability technique is suggested for ceramics [130]. The cumulative distribution function is denoted by Equation (1) stated above [130]. Fig. 6 is a typical 2-parameter Weibull plot.

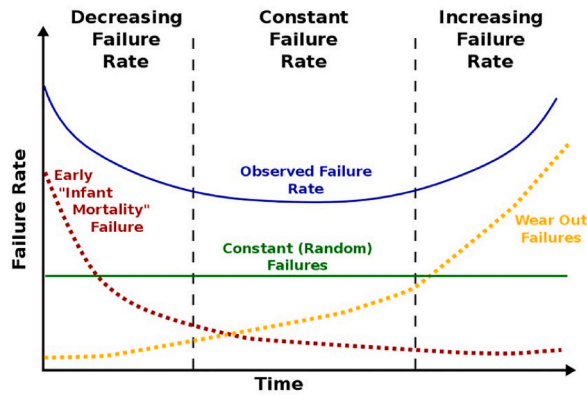


Fig. 5. A typical bathtub curve showing failure stages [125,126].

The Probability Density Function (PDF) is represented by equation (2) [130]:

$$P(x) = \frac{dP}{dx} = \frac{m}{x_0} \left(\frac{x}{x_0}\right)^{m-1} \exp\left[-\left(\frac{x}{x_0}\right)^m\right] \tag{2}$$

For two parameters only, Tiryakioğlu and Hudak [101] in a study presented a stepwise way of conducting Weibull analysis that depends upon the linear regression procedure, which is normally applied in the analysis of fracture value. In what follows, these steps are highlighted.

2.2.1. Allocate probability to Individually statistics point

Several probability estimators (alternatively referred to as simple rank-estimator functions) can be found in the literature [134–137]. For large data, the formula should be used according to Equation 3 (a) [135]:

$$P(x) = \frac{t - u}{n + v} \tag{3a}$$

where  $t$  = ascending order rank,  $n$  = sample size, and  $u$  and  $v$  = empirical constants.

In reality, the plotting locations in equation 3 (b) and (c) below are generally used [138–143]:

$$P(x) = \frac{t - 3/8}{n + 1/4} \text{ for } n \geq 10 \tag{3b}$$

$$P(x) = \frac{t - 1/2}{n} \text{ for } n \geq 11 \tag{3c}$$

Monte Carlo simulation investigations [144–150] disclosed that probability estimators yield partial evaluations of the  $W_m$  (the mean of predictable Weibull moduli) and are dissimilar from the factual  $W_m$  (model sizes ( $n$ ) among 5 and 100). The extent of the bias rests on the values of  $u$  and  $v$  as well as the sample size. With a small number of statistics, other distributions, particularly the normal

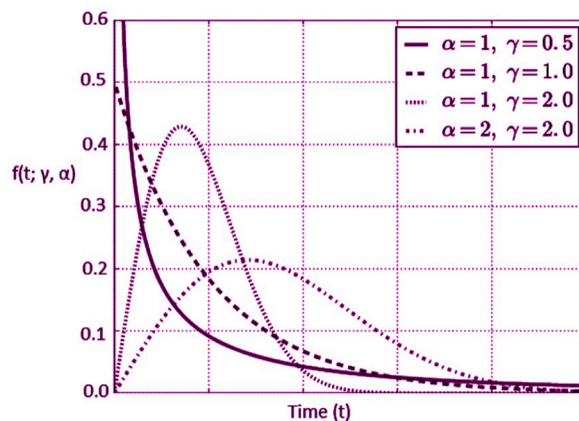


Fig. 6. Typical 2-parameter Weibull distribution plot [101,130].



distribution, resolve and possibly afford suitable turns. Monte Carlo simulation investigation is the commonly used method in probabilistic analysis in engineering. It is used to generate Weibull data points and for the validation of the reliability analysis and easy approximation of the analysis for the accuracy of the results [151–156]. Since it allows for operational or correlation-type interactions of the data being used, Monte Carlo simulation is significantly more flexible [157]. Also, for the estimation of standard errors [158–161]. it makes it easier to understand the multipart mathematical equations with accurate prediction [162–166].

2.2.2. Obtain linear regression fit

That is the plot of  $\ln(x)$  against  $\ln[-\ln(1 - P)]$  [167]. The most frequently applied approach to solving the Weibull is the Weibull probability graph. Changing equation (1) to linear form, we have equation (4) [101]:

$$\ln[-\ln(1 - P(x))] = m \ln(x) - m \ln(x_0) \tag{4}$$

From the above equation,  $m$  is the slope, and  $m \ln(x_0)$  is the intercept of the plot. Henceforth, the best-fit line for this probability plot signifies the linear regression way for assessing the two Weibull parameters (i.e.,  $m =$  shape and  $x_0 =$  scale).

2.2.3. Conduct goodness-of-fit (GOF) test

A GOF test is a numerical trial that regulates whether the analysis figures trail the distribution model. If the figures excel in the GOF test, it proves that it trails the typical form thoroughly enough that forecasts can be decided and founded on that model. If the figures fail the GOF test, it shows that the figures do not trail the model thoroughly enough to make forecasts and that the figures do not look to trail a definite form [168–170]. Weibull outcomes are effective when GOF tests are fulfilled. GOF tests for a  $W_d$  comprise the following [171].

- (i)  $R^2$  linear regression (least squares). If  $R^2 > 0.9$ , it is an acceptable fit for linear regression.
- (ii) Kolmogorov-Smirnov: uses the confidence level and P-value to determine if the figure is a good fit.

If  $P > 1 -$  confidence level, the test passes [172].

The GOF of the Weibull design can also be evaluated using the straightness of the Weibull plot, according to Park et al. [173–176]. The sample correlation coefficient of the matching points can be used to determine the degree of linearity in the Weibull plot as shown in Equation (5) [101,176]:

$$\ln x, \ln[-\ln(1 - P(x))] \tag{5}$$

Let:  $U_x = \ln x$ ;  $V_x = \ln[-\ln(1 - P(x))]$ ;  $\bar{U}_x = \frac{\sum U_x}{n}$  and  $\bar{V}_x = \frac{\sum V_x}{n}$  [176]:

then; The sample correlation coefficient,  $R$ , from the plot, is denoted by Equation (6) [176]:

$$R = \frac{\sum_{i=1}^n (U_x - \bar{U}_x)(V_x - \bar{V}_x)}{\sqrt{\sum_{i=1}^n (U_x - \bar{U}_x)^2 \cdot \sum_{i=1}^n (V_x - \bar{V}_x)^2}} \tag{6}$$

The application of a probability plot, nevertheless, is independent and inadequate, and so, it is assuredly suggested that probability plots are constantly supported by other GOF tests [177]. However, rich rules for the exploitation of  $R^2$  as a GOF test have only recently been established by Tiryakioğlu et al. [178], who conducted Monte Carlo simulations to fix the points of  $R^2$  at  $x_0 = 0.05$  ( $R_{0.05}^2$ ). Tiryakioğlu et al. stated that equation (7) may be applied for trial sizes between 5 and 100 [178]:

$$R_{0.05}^2 = 1.0637 - \frac{0.4174}{n^{0.3}} \tag{7}$$

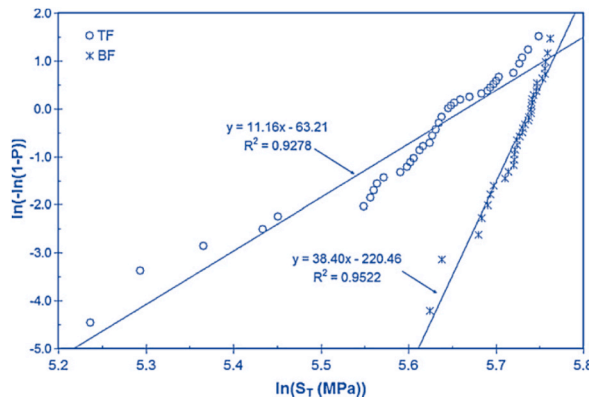


Fig. 7a. Typical Weibull graph for two samples [178].

The proposition that the statistical set trails the tried distribution is excluded once the p-value for the estimated  $R^2 < \alpha$  quantified data for the Category I error, which is characteristically set as 0.05. Otherwise, if the estimated  $R^2 > R^2_{0.05}$ , it can be resolved that the figure originates from a  $W_d$ . Hence, the WA must be rejected and stages 4.4 through 4.66 will not be engaged [178].

**2.2.3.1. Real life examples.** The real-life example here is two datasets reported by Ref. [178] that were used to show the application of the GOF methods suggested by the researchers. The two datasets were extracted from a study by Green and Campbell [179,180], who revealed that the tensile strength (TS) of alloys is influenced to a high degree in the mould-filling phase. If the mould is filled gently, TS is greater and has better reliability. On the other hand, tensile strength has a lesser average and greater unpredictability when the mould filling is intense. The two datasets exemplify these two kinds of mould filling: top-filled which is relatively intense, and bottom-filled, which is gentle. The mould filling sample size is 45 and 36 for top filling and bottom filling, respectively. For the graph point formula (equation (3c)),  $v$  is 0.481 and 0.466 for top filling and bottom filling, respectively, and  $u = 0$  [181]. The Weibull probability plots are shown in Fig. 7a and the predictable parameters as well as goodness-of-fit measures are presented in Table 1.

For top fill,  $R^2 < R^2_{0.05}$ , signifying that the Weibull fit has to be unacceptable. It can be observed that in Fig. 7a, the slope for the last five points appears to be less when compared with other data. This variation in slope demonstrates a reliable instrument to estimate the GOF to the Weibull distribution. The important data for  $R^2$  with  $\alpha = 0.05$  ( $R^2_{0.05}$ ) were estimated for each dataset size (Fig. 7b) using equation (7). The distribution of the mechanical testing data is Weibull if  $R^2 \geq R^2_{0.05}$ . Hence, they concluded that equation (7) be used for the GOF test [178].

**2.2.4. Determine confidence intervals (CI) for the calculated Weibull parameters**

It remains imperative to understand that the calculated Weibull parameters have their statistical distributions. Therefore, it is necessary to compute assurance intervals for the two Weibull parameters, particularly. The calculated  $W_m$  distribution does not trail at all in the prescribed distribution. Hence, the practice of percentage points is essential to estimate the confidence intervals. If the distribution of the projected scale parameter is normal, then the use of percentage point tables is not essential [181].

The standard deviation,  $S_{x_o}$ , of the predictable scale parameter (after standardization) is computed using the probability estimators according to equation (8) [181]:

$$S_{x_o} = \frac{0.359}{\sqrt{n}} \tag{8}$$

Thus, confidence intervals for the factual  $x_o$  can be gotten using equation (9):

$$\frac{\hat{x}_o}{1.000 + Z_{1-\frac{\alpha}{2}} \cdot \frac{0.359}{\sqrt{n}}} \leq x_o \leq \frac{\hat{x}_o}{1.000 + Z_{\frac{\alpha}{2}} \cdot \frac{0.359}{\sqrt{n}}} \tag{9}$$

Let  $\alpha = 0.05$  (95 % confidence),  $Z_{1-\frac{\alpha}{2}}$  and  $Z_{\frac{\alpha}{2}}$  are 1.96 and  $-1.96$ , respectively [101].

Let's examine the bottom fill data with  $n = 36$ ,  $m = 38.4$ , and  $\hat{x}_o = 311.4$  from the real-life example (subsection 2.2.3.1) to demonstrate how to evaluate the lower limit values. A single-side 99 % confidence interval, according to Ref. [181], occurs when  $m > 26.6$ . Calculated from  $38.4/1.446$ , the lower bound of a single-side 99 % confidence interval for  $m = 1.446$ . Equally,  $\hat{x}_o > 273.4$  is the single-sided 99 % confidence interval for a neutral estimate for  $\hat{x}_o$ . The single-sided 97.5 % confidence interval for the population 10th percentile ( $P' = 0.10$ ) is found to be 0.60 when Inserting  $m = 26.6$ ,  $\hat{x}_o = 1.0$  is used.

Determining the confidence interval with results for the 10th percentile in Table 2 (Appendix A) of Ref [178], provides a 97.5 % confidence that the  $P^* > 0.82$ . This means we are 97.5 % confident that 90 % of the population is  $> 0.82$ . In conclusion, different confidence intervals are added to get the final estimate. Thus, if  $\hat{x}_o = 1.0$ , then there is a 99 % confidence that  $m > 26.6$  and we can be 97.5 % confident that the  $P^* > 0.82$ . In other words, when  $\hat{x}_o = 1.0$ , we are around 96 % ( $0.99 \times 0.975$ ) certain that the population's  $P^* > 0.82$ . Furthermore, we can be around 95 % ( $0.96 \times 0.99$ ) convinced that the  $P^* > 224.2$  ( $273.4 \times 0.82$ ) because there is 99 % confidence that  $\hat{x}_o > 273.4$ .

**2.2.5. Estimate lesser assured values**

As soon as the Weibull parameters are known, it is necessary to get conventional estimates in design ideals, by calculating a certain percentile (pct) of the  $W_d$  [182]. To find a certain pct of the Weibull distribution, equation (1) can be reorganized to give equation (10) [101]:

$$x_p = x_o [ - \ln(1 - P) ]^{\frac{1}{m}} \tag{10}$$

**Table 1**  
Typical statistics of two Weibull data [178].

Serial	Samples	n	$R^2_{0.05}$	m
1	Top Fill	45	0.9305	11.16
2	Bottom Fill	36	0.9213	38.40

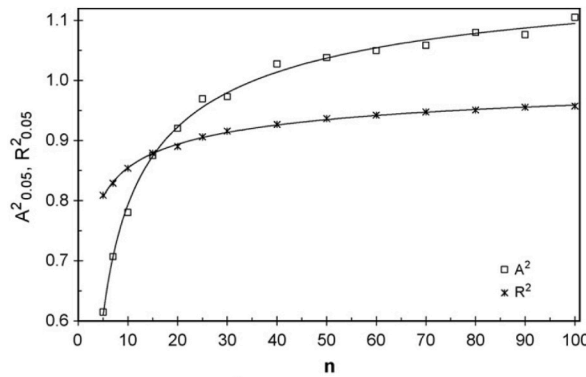


Fig. 7b. The important data of  $R^2 = 0.05$  for model sizes between 5 and 100 [178].

where  $x_p$  is the  $P_{th}$  percentile. Since  $x_0$  &  $m$  are unknown, corresponding approximations,  $\hat{x}_0$  and  $\hat{m}$  have to be applied to estimate the percentile. Because  $\hat{x}_0$  and  $\hat{m}$  have their distributions, the appraised percentiles will take a distribution, which is anticipated to be controlled by the distribution of  $\hat{m}$ , due to the weighty consequence of  $m$  on the percentile. Hudak and Tiryakioğlu [181] recently established a way to evaluate the percentiles of a  $W_d$  at a certain confidence level (CL). The authors presented tables for several sample sizes obtained from Monte Carlo simulations by substituting  $x_0 = 1$  and  $m = 1$ . When  $m$  is not equal to one (1), the pct (columns) for a certain CL (rows) is calculated by Equation (11.1) [181]:

$$P = 1 - \exp\left(-\left\{-\ln(1 - P')\right\}^{\frac{1}{m}}\right) \tag{11.1}$$

where  $P^*$  = percentile of attention. For example, the authors in Ref. [181] studied a situation where  $\hat{m} = 3$ ; and  $n = 40$  to compute the distribution for the 10th percentile. Using Equation (10), Putting  $\hat{m} = 3$ ; and  $P' = 0.10$ , the value of  $P^* = 0.376$ . This means that the distribution for the 37.6th pct when  $\hat{m} = 1$  and  $m = 1$  gives the same result for the distribution as the tenth pct when  $\hat{m} = 1$  and  $\hat{m} = 3$ . Afterwards, interpolation can be applied to evaluate the percentile standards (or values).

The failure probability  $P_f(x)$  can also be found using Equation (11.2) [182–188]:

$$P_f(x) = \frac{a - 0.3}{N + 0.4} \tag{11.2}$$

Where  $N$  = Sum of trials;  $a$  = the failure sequential number, and  $P_f(x)$  is the failure probability.

The Coefficient of Deviation can be calculated as given in equation (11.3):

$$\left/CD\right/ = \frac{S_d}{MTTF} \tag{11.3}$$

According to some researchers,  $S_d$  and  $MTTF$  can be calculated as given in equation (11.4) [189–192]:

$$\text{Standard deviation, } S_d = x_0 \sqrt{\partial \left(1 + \frac{2}{m}\right) - \partial^2 \left(1 + \frac{1}{m}\right)} \tag{11.4}$$

$\partial$  = gamma function and  $\partial^2$  is the function of parameter  $m$  [193].

Also according to Equation (11.5) [193]:

$$\text{Mean Time to Failure (MTTF)} = x_0 \partial \left(1 + \frac{1}{m}\right) \tag{11.5}$$

According to Equation (11.6) [193],

$$\text{Assume } m > 8, CV = \frac{1.2}{m} \tag{11.6}$$

Revol et al. [193] confirmed that the correlation  $CV = 0.78/\sqrt{n}$  offered a perfect connection with experimental results. To identify an appropriate number of samples to produce a restricted variation of the anticipated strength, the influence of the fluctuation of  $m$  on the predicted value of strength was explored.

### 2.2.6. Comparison of two Weibull moduli

As the  $W_m$  has been applied as a degree of reliability of manufacturers, a proper process is essential to evaluate the Weibull moduli

from two groups of manufacturers. The authors have recently presented the outcomes of their Monte Carlo simulations for weighing two Weibull moduli for sample scopes between 10 and 100. It was observed that 92.5 and 97.5 percent of the distributions for  $m_1/m_2$ , where  $n_1 \geq n_2$ . The values can be adopted to examine the proposition that the two Weibull moduli are equal at  $x_o = 0.05$  [101] (Table 3, Appendix A).

### 2.3. Weibull Distribution for three-parameter

The reality of failures covers critical data around the firmness of the system. Particularly for very reliable systems, the application of this data to forecast a likely uncertainty is important. Uncertainty in a product can be viewed as an alteration of the dispersal of period among failures; the distribution may vary or a change may occur. These conditions can be detected using a statistical control plan [194–196]. Nevertheless, generalities of the exponential distribution (like Weibull and Gamma) are beneficial in showing reliability [197–205]. Three-parameter Weibull distributions are beneficial in the sense that their logic implies that verge parameters are considered in modelling [206–208]. This section will review the three-parameter Weibull Distribution.

Let  $\beta = m$ ;  $\alpha = x_o$  and  $\gamma = y_o$ , then; the PDF for the three-parameter Weibull distribution is denoted by Equation (12) [206,208]:

$$P(x; x_o, m, y_o) = 1 - \exp\left\{-\left(\frac{x - y_o}{x_o}\right)^m\right\} \tag{12}$$

where  $x_o > 0$  = scale parameter,  $m > 0$  = shape parameter and  $y_o < x$  = threshold parameter. Then Equation (13.1) gives the corresponding PDF [131,209,210]:

$$p(x; x_o, m, y_o) = \frac{m}{x_o} \left(\frac{x - y_o}{x_o}\right)^{m-1} \exp\left\{-\left(\frac{x - y_o}{x_o}\right)^m\right\} \tag{13.1}$$

For an unknown shape parameter,  $m$ ,  $x_o$ , and  $y_o$ , they have to be found by an iteration system because of uncontrollable nonlinear terms in the likelihood calculations. Ref [211] found clear modified maximum likelihood (MML) estimators of the threshold and scale parameters  $y_o$  and  $x_o$  which are asymptotically equivalent to extreme probability estimators.

Using the first two terms in the Taylor series expansion they got the subsequent estimators given in equations (13.2)–(13.5) [212, 213]:

$$\hat{m} = \frac{T + \sqrt{T^2 + 4nZ}}{2\sqrt{n(n-1)}} \tag{13.2}$$

$$x_o = U + V\hat{m} \tag{13.3}$$

where:

$$W_{(i)}^{-1} \cong \tau_{1i} - \rho_{1i}W_i, \tau_{1i} = 2k_{(i)}^{-1}; \rho_{1i} = k_{(i)}^{-2}; k_i = G(W_{(i)})$$

$$W_{(i)}^{y_o - 1} \cong \tau_{2i} + \rho_{2i}W_i, \tau_{2i} = (2 - y_o)k_{(i)}^{y_o - 1}$$

$$\rho_{2i} = (y_o - 1)k_{(i)}^{y_o - 2}, (1 \leq i \leq n)$$

$$t = (y_o - 1) \sum_{i=1}^n \rho_{1i} + y_o \sum_{i=1}^n \rho_{2i}$$

$$U = \frac{\left\{ (y_o - 1) \sum_{i=1}^n \rho_{1i} + y_o \sum_{i=1}^n \rho_{2i} x_{(i)} \right\}}{t}$$

$$V = \frac{\left\{ y_o \sum_{i=1}^n \tau_{2i} - (y_o - 1) \sum_{i=1}^n \tau_{1i} \right\}}{t}$$

$$T = y_o \sum_{i=1}^n \rho_{2i} (x_{(i)} - U) - (y_o - 1) \sum_{i=1}^n \rho_{1i} (x_i - U)$$

$$Z = (y_o - 1) \sum_{i=1}^n \rho_{1i} (x_{(i)} - U)^2 + y_o \sum_{i=1}^n \rho_{2i} (x_i - U)^2 \tag{13.4}$$

$$\text{Note : } k_i = G(W_{(i)}) \approx k_{(i)} \cong M^{-1}\left(i/(n+1)\right) \tag{13.5}$$

where  $F_{(x)}^{-1}$  is known as the inverse distribution function for the 3-parameter  $W_d$ . Though the Fisher data matrix was applied to get asymptotic alterations and covariance as it occurs for  $y_o > 2$ , it is also possible to find the MML estimators for  $1 < y_o < 2$  [214,215].

The symmetry settings are not fulfilled for the maximum likelihood (ML) estimate of the three-parameter Weibull distribution since the backing of the PDF rests on the unidentified parameter once the threshold parameter  $y_o$  is not known. The ML estimators may not be available at that point, and even if they are, they may not have the standard asymptotic properties [216–221]. Arising from this, several different procedures have been sought after in the literature. Different researchers have specified comprehensive explanations of several procedures for parameter estimation of the 3-parameter  $W_d$  [216–229]. For example, Ahmad [230], Juki'c et al. [231], and Markovic et al. [232] investigated several types of least-squares estimators for the three-parameter Weibull distributions. The least squares estimate (LSE) based on doubly Type-II censored samples was investigated by Nagatsuka [233]. Cousineau [234] provides a succinct overview of parameter estimation for a three-parameter Weibull distribution based on a full sample.

2.3.1. Estimation of the shape parameter,  $m$

Moderately, limited approaches have been projected to evaluate the unidentified shape parameter,  $m$ , for the three-parameter Weibull distribution [235–237]. Sürücü & Sazak [194] used a calibration procedure [195] to determine  $m$  by estimating  $\ln l$  of some values of  $m$  and the equivalent estimates  $\hat{x}_o$  and  $\hat{y}_o$  (which have unambiguous algebraic formulae), as given [238]:

$$V_{(x)} \cong \prod_{i=1}^n \frac{y_o}{\hat{m}} \left( \frac{x_i - x_o}{\hat{m}} \right)^{y_o - 1} \exp \left\{ - \left( \frac{x_i - x_o}{\hat{m}} \right)^{y_o} \right\} \tag{13.6}$$

$$x \geq x_o ; \hat{m}, y_o > 0$$

Tiku & Akkaya [238] estimated (Equation (13.6)) at points  $y_o = i \times D, i = 0, 1, \dots, n$  and  $D = 0.1$ . The statistics of  $y_o$ , which take full advantage of the likelihood role, will be the estimate for  $m$ . Sürücü & Sazak [194] recommend that since  $m$  affects the estimates for  $x_o$  and  $y_o$ , the early model width should be as large as possible (say, greater than or equal to 30) to get an appreciable evaluation of 'm'. Some researchers noted that when  $x_o$  approaches the first-order statistic, the likelihood can tend to infinity which could result in inconsistent MLEs of the other two parameters. They suggested a fix regarding the possibility of solving this issue [239,240]. Using Monte Carlo simulations, different methods were adopted by Cousineau [228] to estimate the Weibull shape parameter.

To determine failure analysis measures, Shafiq et al. [241] employed an intelligent numerical computer solution that was reliant on artificial neural networks (ANN) (software). Using the software, the study investigates a dependability model based on the inverse power law model and the exponential Weibull distribution. Based on their findings, they came to the remarkable conclusion that when combined with the appropriate statistical model, ANNs can practically compute dependability metrics. In another study, a model of reliability designed on inverse power law and a generalized inverse Weibull model was proposed. The work gave a perfect structure for showing the effectiveness and functionality life cycle of equipment. It was observed that the valuation of the recommended distribution differs from the conventional model of inverse Weibull, and that impacts the average time to failure of the component considered [242]. In the study, reliability was modelled according to equation (14.1) [242]:

$$H_r = \frac{1}{kr^\delta} \tag{14.1}$$

where:  $H_r$  = Failure mean time;  $r$  = degree of stress;  $k$  = characteristic that can be influenced by design; and  $\delta$  = stress:

It is feasible to show how different condition levels can affect the lifespan of equipment by using equation (14.2) [242].

$$f(x \setminus m, k, r, \delta) = \frac{1}{kr^\delta} m x^{-(m+1)} x_o^m \exp \left\{ - \frac{1}{kr^\delta} \left( \frac{x_o}{x} \right)^m \right\} \tag{14.2}$$

2.3.2. Models and hyperparameters in Weibull Distribution

Hyperparameters are parameters whose values dictate the model parameters that a learning algorithm ultimately learns and regulates the learning process. The prefix "hyper" implies that these are "top-level" parameters that govern the process of learning and the resulting model parameters [243–246]. Algorithms for machine learning (ML) have been applied extensively in many different fields and applications. An ML model's hyper-parameters need to be adjusted to suit it to various tasks. The performance of ML models is directly affected by the choice of optimal hyperparameter configuration. It frequently calls for in-depth familiarity with hyper-parameter optimization methods and ML algorithms [246]. A model is defined or represented by the model parameters [243].

Conversely, parameters are found inside the model. Because hyperparameters cannot be altered by the model when it is being trained or learned, they are referred to be extrinsic to the model [244]. Hyperparameters regulate the model's training process, whereas parameters enable the model to learn the rules from the data. Data is how parameters determine their values. Hyperparameters, on the other hand, do not get their values from data. Before training the model, they must be manually specified [245–249]. The importance of hyperparameters in ML includes: determining how any ML model turns out, and they aid in achieving this. It has a significant impact on how quickly any ML algorithm converges [250,251]. When optimizing hyperparameters, the set of hyperparameters is frequently chosen based on the generalization performance, or score, of a validation set after being fitted on a training set. Nevertheless, there's a chance that this process will overfit the hyperparameters to the validation set.

Reliability distribution and data-driven models are the two general categories into which failure estimation approaches for materials may be divided [252]. Based on the reliability model and the distribution of the equipment's entire life period, fault data, as well

as other reliability information, the failure estimation technique is established. Then, using the statistical approach, the failure and error data are computationally investigated to calculate the equipment’s reliability measurement [251]. The exponential distribution [253] and the Weibull distribution [49] are the two primary life distributions that are frequently utilized in material reliability estimation. Weibull distribution is commonly employed in the reliability modelling of materials [52]. The advantages of using a reliability model include a straightforward model, quick training times, and good predictive power; nevertheless, the method’s fitting capabilities and prediction accuracy are limited in situations with irregular distributions and insufficient data [51].

The parameter calculation approaches in traditional statistics are frequently utilized to calculate the parameters of Weibull distribution. The usually used techniques include MLE, moment estimation, and the least square technique. Some researchers [254] related the features of the mentioned estimation techniques, and the results reveal that the value evaluated by the MLE technique can well equate to the real necessities. Hence, the tiny material failure data is processed using the maximum likelihood estimate technique.

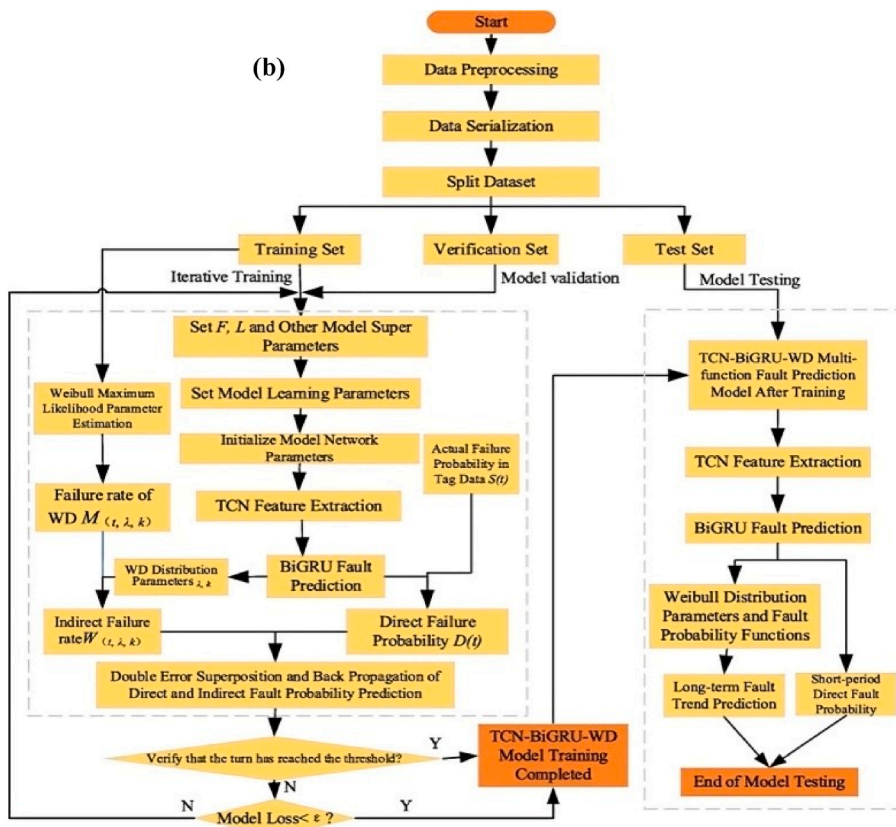
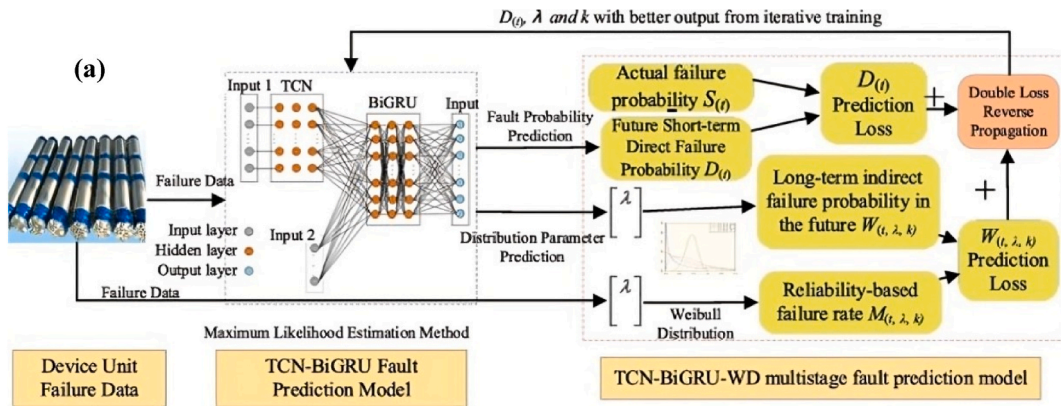


Fig. 8. A simple diagram for modelling in Weibull distribution (a) Plan and (b) ML Execution [51].

First, the Weibull distribution's shape parameter ( $m$ ) and scale parameter ( $x_0$ ) are taken into account. This method has the benefit of being able to predict the failure rate for a long period after the proper failure rate density function is identified; however, it has the disadvantage of requiring sufficient failure data and adhering to a particular distribution rule [51–53,254].

Deep learning is typically utilized in the data-driven modelling space to structure fault estimation models. Different researchers have recently used a variety of more advanced deep learning systems for fault prediction. To get around the LSTM model's poor training pace, Liu et al. [255] developed a Gated Recurrent Unit (GRU), which can achieve a higher convergence rate and produce prediction results that are comparable to the LSTM model. Wan et al. [256] suggested that a CNN architecture that excels at handling time series issues is the Temporal Convolutional Network (TCN). It addresses the "degradation" caused by the rise in network hierarchy and speeds up the feedback and convergence of deep networks through residual connectivity. Guo Ling et al. [257] used TCN to determine the distinctive characteristics of the time series data and then used the non-linear fit ability of the GRU neural network to create the TCN-GRU model to further improve the accuracy of prediction of time series data. Short-term forecasting derived from manufacturing load data shows that its estimation ability is notably more accurate compared to other point prediction models.

**2.3.2.1. Hyperparameter model in Weibull Distribution.** According to Ref. [51] the TCN side uses two hubs in the Weibull distribution fault estimation model to extract structures. Primarily the background involves level number (F) and lowest order length (L) in the hyperparameter (H). The volume of training parameters will rise if F is fixed large enough, which could cause the model to overfit and impair its performance. F may be set too small too in which case the model couldn't fit well and the extracted features might not be significant enough. L is the total of the convolution's lengths at and before the current point. The prior L-1 data and the current position make up the convolution data. An excessively large value for F contributes an excessive amount of information about the positions that came before it, which could result in more pointless calculations and less improvement in the model's efficiency. An excessively small value for F could result in insufficient prefix information being introduced and a decreased capacity to fit the model.

Orthogonal experiments adequately determine the proportions of both super-parameters. For example, to create a two-component, four-level experiment, F and L, the two orthogonal experiment factors, establish an array of parameters for each factor based on experience [258]. They then choose four suitable discrete values from the range. L needs to be an odd number to guarantee a fixed convolution centre anchor location [259]. A simple diagram for modelling in Weibull distribution is given in Fig. 8 (a) and (b). Adjust the model's learning parameters based on the level and design considerations. By varying the convolution layer F and sequence length L values, many fault prediction models are constructed. The accuracy and recall rate on the validation set are utilized as test results to assess the suitability of the parameter combination [51].

## 2.4. Weibull Modulus for different ceramics materials

In materials science, Weibull analysis was initially used almost solely for ceramics and glasses [260–262]. The unpredictability in measured material strength of brittle materials is defined by the dimensionless Weibull modulus, a parameter of the Weibull distribution [263–267]. In this scenario, the Weibull distribution model's form parameter, the Weibull modulus, accounts for the likelihood of a component failing under different loads. One should exercise caution when relying on the precision of the Weibull moduli that are obtained from a small number of tests. The Weibull modulus and its physical effects on the statistical distribution of fracture strengths in brittle materials have been estimated in several ways. For example, the theoretical value of the Weibull modulus labelling the dispersion in fracture toughness is proven to be a constant, i.e., Weibull modulus ( $m$ ) = 4 [268], under the assumption that the fracture probability is proportionate to the volume of a fracture progression region in a brittle material. The application of fractal theory has revealed a relationship between the Weibull modulus, fractal dimension, and  $m < 6$  in highly fractured rocks [30,269–271]. Weibull modulus elaborates on the mechanical behaviours of materials [70,269].

A higher  $m$  value suggested a narrower range of fracture stresses and a higher dependability/reliability because the  $m$  value represented the degree of variation in the strength of the tested samples [272–274]. Table 4 lists the typical values of the  $m$  for a few materials, such as bulk metallic glasses based on magnesium which were compiled.

## 2.5. Summary of steps in Weibull Distribution and its benefits

Brittle materials are widely characterized in the biomedical sector by Weibull statistics [281,282]. Ceramics, particularly high-performance ceramics like alumina, zirconia, or HAp, are routinely examined using the Weibull modulus ( $m$ ) to ensure the consistency and dependability of their structure. Despite the rather robust probabilistic criterion for using the Weibull distribution for

**Table 4**  
Weibull modulus values for certain materials

S/N	Material	Weibull Modulus, $m$	References
1	Local ceramics (Chalk, Brick, Pottery)	>2	[275,276]
2	Advanced ceramics (Al <sub>2</sub> O <sub>3</sub> , HAp, SiC)	≤10	[275,277]
3	Mg glass	>40	[277,278]
4	Glass Ceramics	<7	[279]
5	Zirconia	<7	[280]
6	Graphite	<13	[280]

brittle materials, an appropriately uncomfortable parameter estimate made its practical implementation difficult [281,283]. Much progress has been made recently in terms of making the Weibull distribution accessible. Elastic general-purpose statistical models exist that can be used to calculate the two-parameter Weibull distribution utilizing least squares (LS) or maximum likelihood (ML) approaches, or both [284–286]. A number of them offer tests for parameter variation among factor levels, appropriate probability graphs, and 95 % confidence intervals for the Weibull parameters. Additionally, a free open-source Excel calculator that facilitates a computerized LS calculation of Weibull parameters and the associated 95 % CI is available [287]. Despite this outstanding advancement, data analysis is still a source of unpredictability [288].

In summary, WA has the following advantages [289–291]: (a) It can be used to describe the data of materials without the restrictions of a pre-defined distribution hypothesis because it is robust enough to assume a variety of various distributions, such as the normal, exponential, and beta distributions. (b) The parameters  $m$ ,  $x_0$ , and  $y_0$  predict the distribution's index rate(s). (c) The analysis can deliver precise performance analysis and risk predictions even with very tiny samples.

The steps for WA using a worksheet (MS Excel) can be summarized as given [291].

1. Production of the materials and preparing the samples for mechanical testing
2. Collection of the data
3. Ranking of data in ascending order for the plots
4. Calculating the PDF ( $\ln x, \ln[-\ln(1 - P(x))]$ ) for the distribution and plotting of the graphs
5. Determine the  $m$  and other parameters

## 2.6. Monte Carlo simulation

A popular way for assessing the performance of statistical methods is Monte Carlo simulation. When paired with trials, Monte Carlo simulations prove to be a very helpful tool for forecasting the dependability of different estimate techniques and their optimization [284,292]. It comprises generating random numbers (strength data) for a particular modulus ( $m$ ) and character strength in the framework of the two-parameter Weibull distribution. It is certain that the basic Weibull sampling distribution assumption holds and that the precise parameter values ( $m, x_0$ ) of this distribution are known thanks to the utilization of Monte Carlo techniques. This kind of technique has several benefits [287,292].

1. Real experiments do not need to be conducted to create strength information based on a Weibull distribution using specific parameters  $m$  and  $x_0$
2. Parameter values are freely selectable; for example, they can be set to values often noticed in dental material research.
3. The number of samples can be arbitrarily adjusted and as many results as desired can be obtained.
4. A statistical method can be used in the same manner as measured data to analyze a particular simulated sample.

On the other hand, estimations of parameters generated from the simulated sample ( $\bar{m}, \bar{x}_0$ ) could be juxtaposed to the observed true values ( $m, s$ ) to assess the statistical technique's effectiveness in terms of relative error and bias, unlike measured data [287].

## 2.7. Case study: related works on the application of Weibull analysis on HAP and related materials

In a Weibull modulus analysis, the samples should be cracked with a standard test procedure like the 4-point bend test or a biaxial flexure test [293–295]. Weibull Analysis has been extensively adopted to analyze the failures in an estimated property, such as fracture strength [296,297]. Consequently, the Weibull Modulus,  $m$  is directly connected to the material's reliability. Naturally, WA is carried out on a set of samples that are supposedly alike. Aguirre et al. [26] reinforced HAP with boron nitride platelets to increase the fracture toughness of the HAP samples and achieved 2.3 MPa  $m^{1/2}$  and 79.79 MPa for fracture toughness and flexural strength, respectively. The Weibull distribution showed a low failure probability and a safety factor. The factor of Safety (FOS) = Yield Stress/Working Stress. If the FOS = 1, then it signifies that the design load = to the safety load [298,299]. Abifarin et al. [300] considered a two-parameter Weibull distribution on the mechanical properties of HAP to ascertain the reliability of the produced scaffold.

The fracture toughness, flexural strength, compressive, and hardness data/statistics can be analyzed with the aid of the Weibull probability density function (PDF). Equation (2) and the probability of failure,  $P(x)$ , for a sample under a given stress can be calculated using the Weibull cumulative distribution function (CDF): equation (1) [301–308]. Notwithstanding the usefulness of the  $m$ , as a measure of the mechanical reliability of brittle materials, there is little research on  $m$  for porous HAP [293]. It has long been acknowledged that there is a strong relationship between strength and flaw distribution. The extent of its impact on strength is one implication of statistical strength behaviour in brittle materials. Its continued existence has been predicted hypothetically [308–310] and shown in numerous exploratory trials [148,311]. Using WA, the reliability of a ceramic material like HAP can be predicted.

In another study by Fan et al. [66],  $m$  was estimated for 441 sintered HAP samples cracked in biaxial flexure for  $0.08 \leq P \leq 0.62$ . The  $m$  against  $P$  graph was “U-shaped” with a varied band of  $m$  data for  $P < 0.1$  (Area 1) and  $P > 0.55$  (Area 3), and a thinner band of  $m$  data in the transitional porosity area of  $0.1 < P < 0.55$  (Area 2). The restricted array of  $m$  ( $\sim 4 < m < 11$ ) in Area 2 has vital inferences as Area 2 comprises the  $P$  array for the majority of the uses of porous brittle materials. The grain size versus density path of the HAP samples revealed a distinct value with limited grain development for samples with relative densities of less than about 0.8–0.9. Fig. 9 is a typical Weibull plot as given by Fan et al. Ćurković et al. [55] used Weibull analysis to investigate the flexural strength of alumina ceramics. The three-point bend test was used to determine the flexural strength of standard purity alumina ceramics. Flexural strength



has been determined to be between 266.7 and 357.5 MPa. The numerical unpredictability of flexural strength calculated by the three-point bend test was investigated using a two-parameter Wd function. Flexural strength was measured at 17.4 Wm. This restriction can be used to characterize the variability in the tested material’s flexural strength as well as its consistency.

Nevarez-Rascon et al. [312] investigated the inconsistency in mechanical properties of a nanocomposite using Weibull statistics. Uniaxial compression examinations at room temperature of specimens  $6.35 \pm 0.03$  mm in diameter and  $12.50 \pm 0.63$  mm in length and Vickers hardness readings on polished surfaces were conducted. The indentation fracture toughness ( $K_{IC}$ ) was obtained from the average crack length and WA was made on the statistics. The ATZ2 (18.0 wt %  $Al_2O_3$  + 2.0 wt % (w) + 80.0 wt %  $ZrO_2$  (TZ-3Y)) nanocomposite gave the maximum average compressive load of 1200 MPa, the maximum data of distinctive strength,  $\sigma_o$ , of 1340 MPa with  $m$  of 3.25 and moderately high fracture toughness ( $4.7 \pm 0.7$  MPa  $m^{1/2}$ ), signifying that with the wide variety of mechanical properties gotten in the study, the material is characterized by diverse dental usages. However, zigzagged cracks were observed. In the two composites, the fracture shape was a mixture of transgranular and intergranular fractures [313–316]. The notch of intergranular fracture rises with the growing  $Al_2O_3$  content leading to low fracture toughness.

A 3D-printed HAp scaffold’s dependability was predicted with the aid of Weibull analysis using compressive strength values [317]. With a correlation coefficient ( $R^2$ ) above 0.90, the Weibull plots demonstrated good linearity (Fig. 10 (a) – (d)) [317]. All the scaffolds had a survival probability greater than 90 %. In their work, the highest value of  $m$  obtained was 1.0 [317,318].

Isaacson et al. [319] performed a compressive failure of porous gyroid scaffolds. The Weibull moduli of the scaffolds were discovered to be self-reinforcing, therefore initial failures brought on by minor manufacturing irregularities didn’t seem to be the main reason for the scaffold’s early catastrophe. HAp was reinforced with graphene and silver for dental applications. The Weibull analysis was used to determine the reliability of the binding strength to dentin. Four samples were tested and the obtained Weibull modulus ranged from 2.0 to 3.4. The Weibull plots are presented in Fig. 11 [320]. Using Mussel shells, Galotta et al. [321] produced nanocrystalline HAp. A Weibull Modulus of 9.1 was obtained for the HAp sample. D’Andrea et al. [322] produced HAp via the photopolymerization process and the mechanical reliability of ceramic scaffolds was assessed using the Weibull modulus. For the estimation of the Weibull modulus, strength information from the cantilever and notched beams was combined. The Weibull modulus was determined as the slope of the linear fit of the data. From the plot, the obtained  $m$  at 143 MPa was 2.6. The value of  $m$  can be improved by post-sintering surface machining which removes or strappingly decreases surface defects [323]. The assessment of a novel commercially obtainable ion-releasing HAp and their connections to establish anti-carcinogenic sources was done by Marovic et al. The reliability of the material was evaluated using WA. The result showed that the glass ionomer had the highest water sorption and the least predictable distribution of mechanical characteristics or properties [324].

Zhao et al. [325] performed the Weibull analysis of carbon-fibre-reinforced HAp composites. Huang et al. added HAp nanowires to produce an insulation network. Weibull modulus of 2.65 was obtained for the reliability analysis [326]. Karimi & Paydar [327] studied an anode electrode’s fracture pattern and mechanical performance for solid use. Porosity and compressive strength measurements were between 59 and 75 % and 2.7 and 14.02 MPa, respectively. The Weibull study revealed a direct correlation between the Weibull modulus and cooling rate. Using quasi-static and high strain rates, an experiment was conducted to assess the transverse tensile strength of red deer. There was an 83 MPa improvement in tensile strength. The WA revealed that the osteo progress trend had greater tensile ductility than the osteon progress trend’s transverse [328]. Przystupa [329] evaluated the reliability and durability of the coating surfaces of roofing materials. Higher shape and scale parameters were obtained which meant higher durability and reliability of the materials. Par et al. studied the flexural behaviour of an experimental composite. Flexural strength and modulus were appraised using a three-point bending test and WA was done to estimate material reliability. A higher Weibull modulus of 5.0 was obtained, signifying that the material is reliable for the intended applications [330].

Monteiro et al. [331] investigated the bond strength of various ceramic and dental materials to estimate the reliability of the merged edges. WA was executed to find the probability of failure (POF), Weibull modulus ( $m$ ), and characteristic strength (CSS). At a bond strength of 24.88 MPa lower POF, greater  $m$ , and greater CSS were obtained. The highest data were obtained at 20.07 MPa. Lira et al. [332] studied the WA of fibre supports diluted with diverse cement forms and mechanically aged. Weibull behaviour strength of the materials was considerably high. The effect of laser sintering was observed to advance the bioactivity of a modified HAp. The

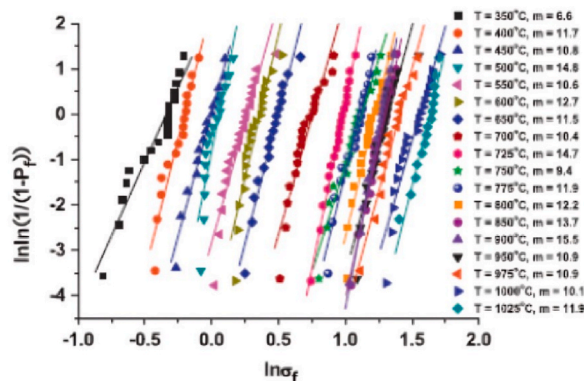


Fig. 9. Typical Weibull plot of HAp at different temperatures [66].

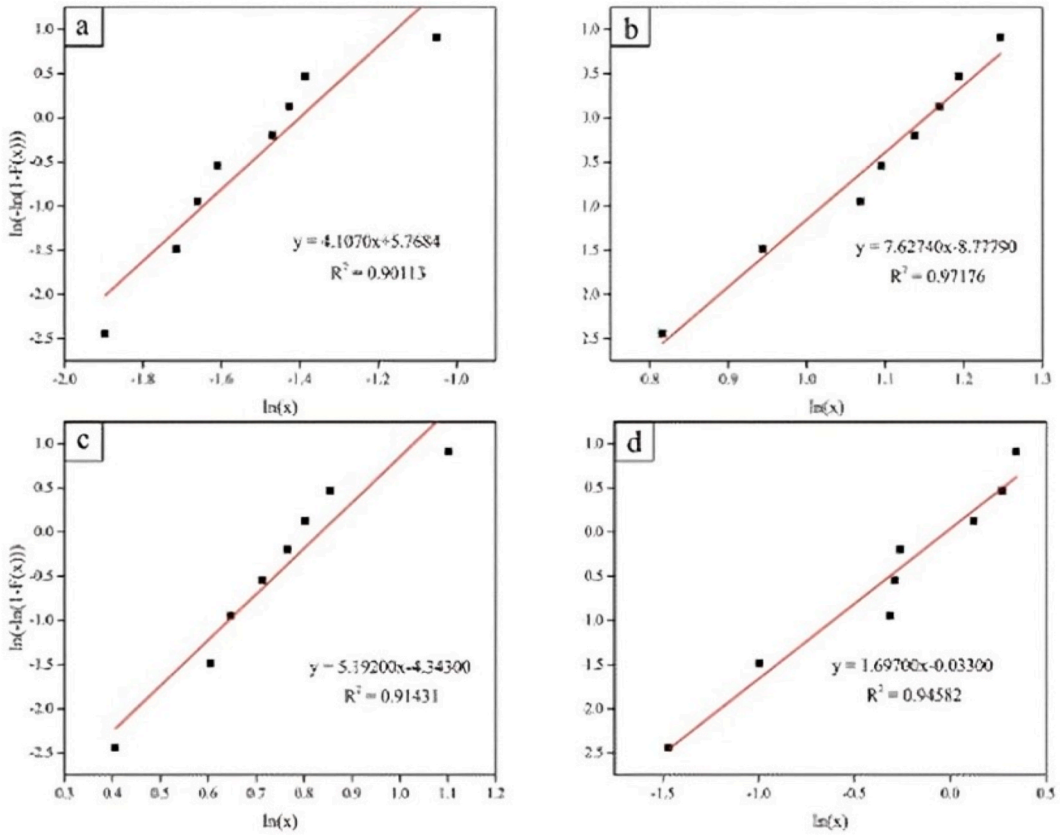


Fig. 10. Typical Weibull plots of 3D printed HAp [317].

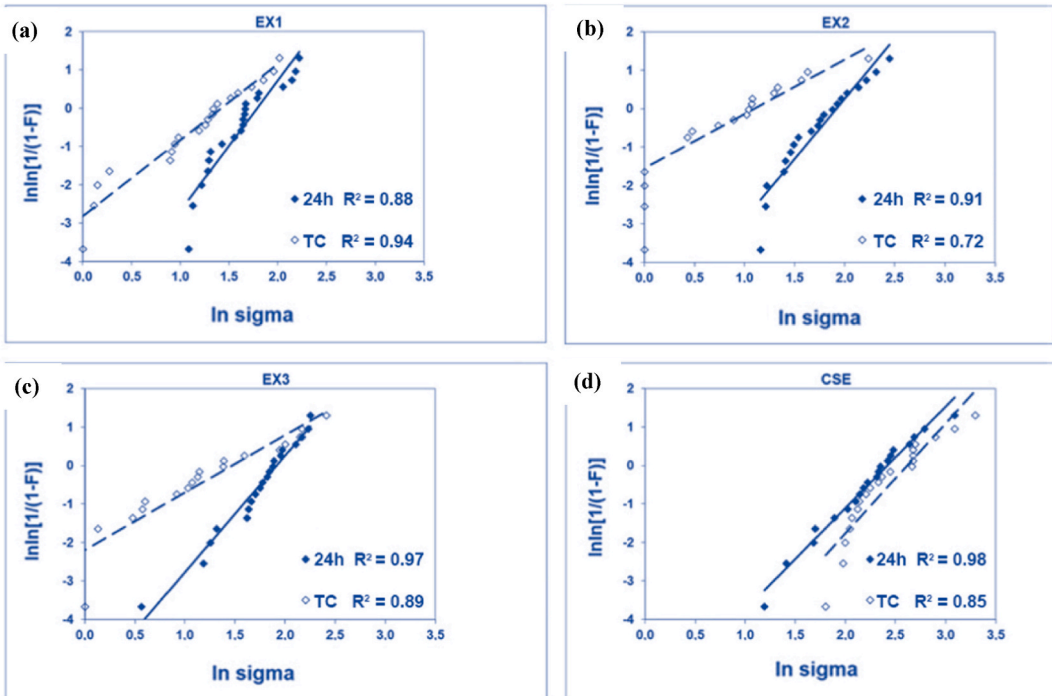


Fig. 11. Typical Weibull plots of reinforced HAp [320].

bioactive coat's reliability was inveterated by severe high-energy ultrasonic cavitation experiments and friction experiments against bovine bone, indicating no transferal of HAp to the bone. Additionally, WA was done to establish the changeability of the documented strength figures for every set. Equation (1) was employed to compute the Weibull modulus,  $m$ . The results showed a higher value of the  $m$  which points to larger homogeneity in the obtained strength values of the set [333]. Ilie [334] compared the manner contemporary resin-based composites react to mechanical stress associated with the tooth structure they are intended to substitute. WA was performed to determine the value of  $m$  and 5.2 was obtained.

Liao et al. [335] employed equation (4) in determining the reliability of a metallic glass wire and a Weibull modulus of 81.0 was obtained. They concluded that the Weibull modulus  $m$  reveals the reliability of the material's samples and a higher  $m$  value signifies a narrow distribution of the fracture strength and thus higher reliability. For the reliability analysis of metallic glass fibres, Liu et al. [336] performed two and three – Weibull analyses. They obtained  $m$  values of 5.71, 3.73, 4.27, & 4.03 and 3.36, 2.07, 2.10, & 2.30 for the two and three Weibull analyses respectively. In another study by Wang et al. [337], the WA was used to obtain the Weibull modulus  $m$  of an amorphous alloy with a special structure and the value of  $m$  represented the dissemination range of the fracture strength of the material.

In a related study, we extracted HAp from bovine and catfish bones to synthesize a novel mix of HAp that can be useful for biomedical applications [69–74,82,338,339]. The samples were prepared and the compressive test was performed using a Universal Testing Machine. After completing the compressive strength tests, equation (4) was used to determine the strength values of the produced HAp samples, Plotting  $\ln(\ln(1/\text{survival probability or the median rank}))$  versus  $\ln(\text{compressive strength, } x_0)$  produced the distinctive Weibull modulus and distribution. The results showed that the novel combination of these powders is a good and promising biomaterial for high-strength biomedical applications. Examining the correlation between the characteristic compressive strength and the Weibull modulus ( $m$ ), a higher  $m$  was obtained for all the samples indicating a more favourable test design [82]. Sample C100 had the highest value of  $m$  (5.29) with a standard deviation of 0.92, while sample B75/C25 had the lowest value of  $m$  (2.67) with a standard deviation of 1.0. The high value of  $m$  ( $m > 1$ ) indicates the materials might not fail in their early stage when relating the obtained  $m$  value with the bathtub [283,340]. Fig. 12 depicts the procedures employed in the production of the HAp samples and the WA of the novel mixture of HAp. The generated scaffolds are somewhat reliable by the acquired results, making them appropriate for use in biological fields. The Weibull plots from the work are presented in Fig. 13.

### 3. Conclusion and Future works

In this paper, we have reviewed the steps in Weibull distribution to critically analyze the strength data of brittle materials, such as ceramics and related materials. The WA and the steps for the analysis were highlighted. With the aid of WA, the  $m$  of the samples can be investigated to predict their failure rates. From the review, it was discovered that Weibull distribution is proven to confer to the feeblest-link concept. For brittle materials, the literature revealed that environs, production processes, and comparative factors are well-thought-out contributing factors for reliability. The Weibull modulus and characteristic strength from the analyses are appropriate in many circumstances, but it should be noted that they are estimations. The literature can provide confidence limits or uncertainty for these estimates. As the number of specimens increases to 10 or more, estimations of the characteristic strength quickly

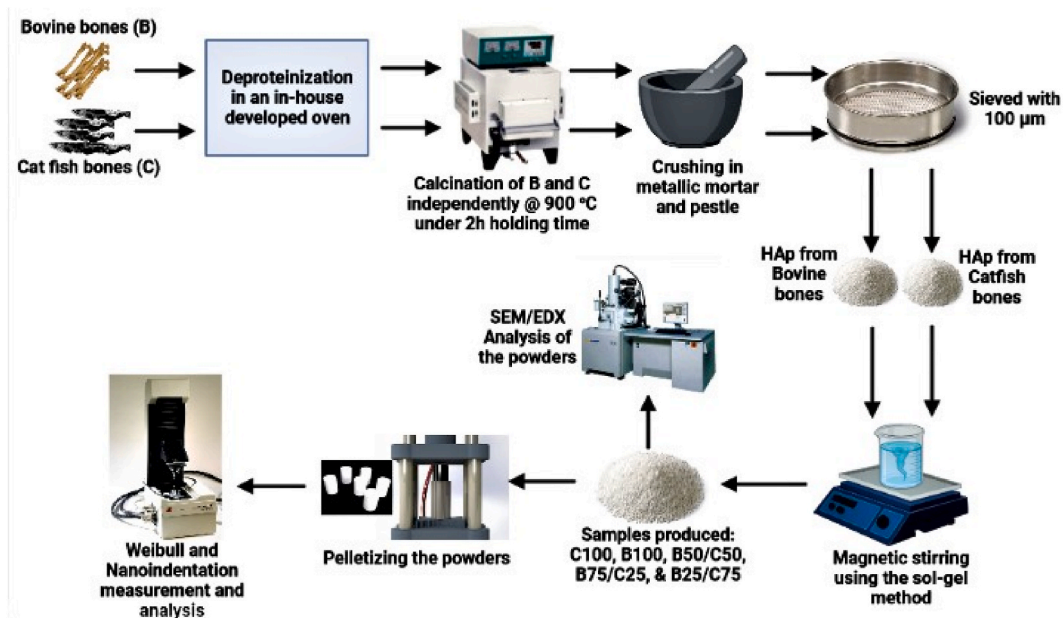


Fig. 12. Schematic details of the synthesis, characterization, and computational procedures [82].

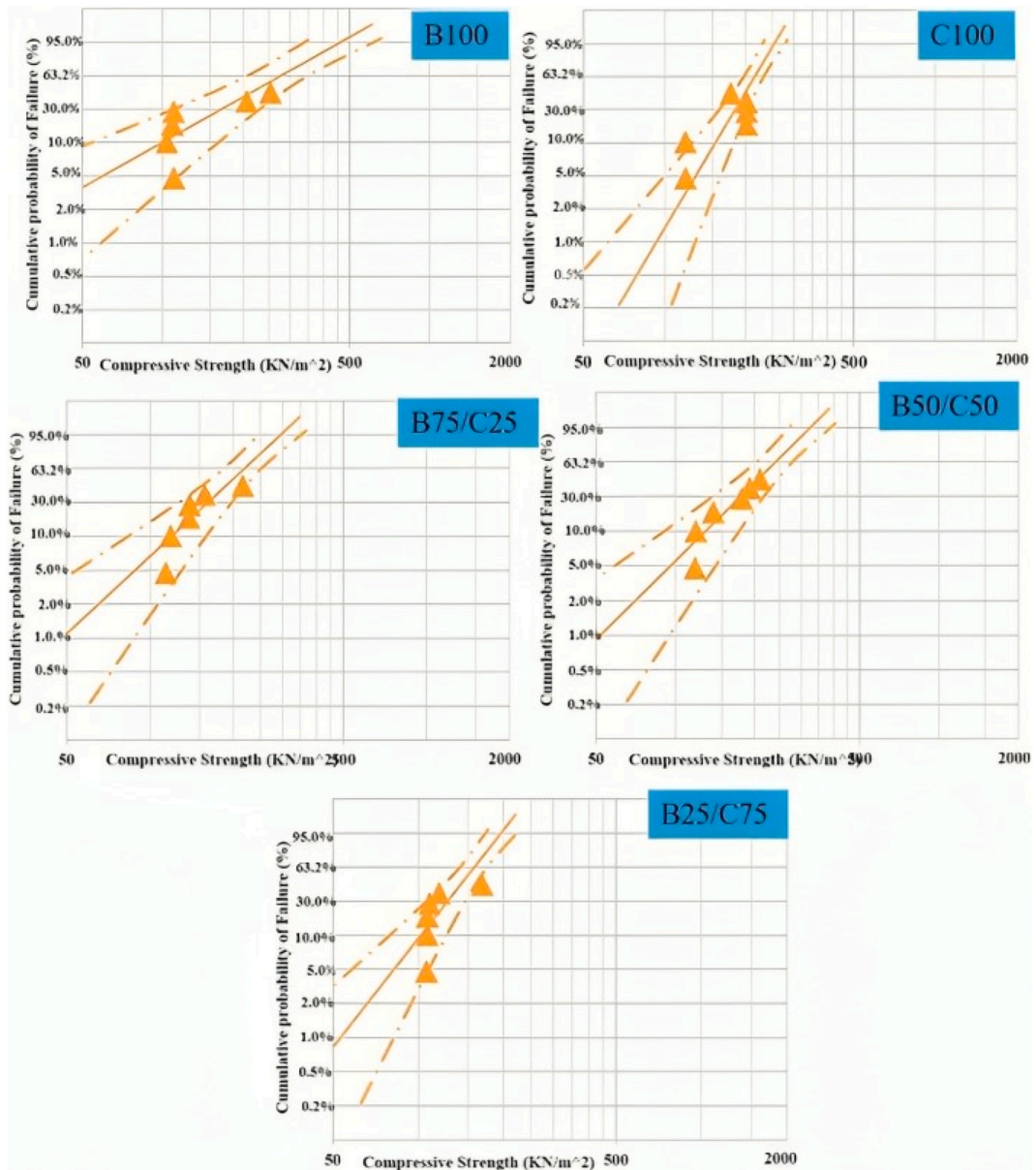


Fig. 13. Weibull plots of hydroxyapatite with high  $m$  [82].

converge on population values. Weibull modulus estimations, on the other hand, can vary when the sample set contains only a small number of test specimens or when the data does not fall on a single line. To acquire good estimations of the Weibull modulus, at least 10 test specimens, preferably thirty, are often required.

**Abbreviations**

ANN	Artificial Neural Network
CI	Confidence Interval
GRU	Gated Recurrent Unit
GOF	Good-of-Fit
HAP	Hydroxyapatite
LRFC	Load and Resistance Factor Computations
MLEs	Maximum Likelihood Estimators
MLs	Maximum Likelihoods
ML	Machine Learning
PC	Performance Characteristics
PDF	Probability Density Function
TCN	Temporal Convolutional Network
WA	Weibull Analysis

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**4. Data availability**

This is a review paper. No raw data was used.

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**Declaration of competing interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

**APPENDIX A. TYPICAL WEIBULL RESULTS**

**Table 2**  
Typical Weibull fits of two values [178].

Data Set	n	Plotting Position		Parameter Estimates		Goodness - of Fit			m	95 % CL		
		u	v	m	x <sub>0</sub> (MPa)	R <sup>2</sup> 0.05	R <sup>2</sup>	Weibull Confirmation		x <sub>0</sub> (MPa)		
TF	45	0.481	0	11.16	288.3	0.931	0.928	Not Agreed				
BF	36	0.466	0	38.40	311.4	0.921	0.952	Agreed	28.16	55.36	278.7	352.8
BFmod	37	0.468	0	50.71	279.7	0.922	0.980	Agreed	37.34	72.73	250.7	316.3

**Table 3**  
A Table showing two Moduli [101].

n2	n1																							
	10	15		20		25		30		35		40		50		60		70		80		90		100
0.412	2.434	2.262	0.475	2.222	0.488	2.151	0.494	2.124	0.505	2.100	0.507	2.089	0.517	2.058	0.524	2.049	0.535	2.042	0.534	2.027	0.536	2.011	0.540	2.030
	0.489	2.069	0.511	1.976	0.531	1.951	0.543	1.906	0.550	1.881	0.559	1.877	0.574	1.854	0.581	1.827	0.591	1.821	0.594	1.823	0.599	1.798	0.595	1.798
			0.526	1.889	0.553	1.851	0.571	1.799	0.575	1.798	0.585	1.767	0.600	1.722	0.609	1.708	0.614	1.703	0.621	1.690	0.629	1.693	0.629	1.693
					0.567	1.771	0.581	1.728	0.598	1.720	0.599	1.677	0.615	1.661	0.624	1.647	0.632	1.630	0.639	1.620	0.644	1.618	0.649	1.600
							0.596	1.677	0.608	1.662	0.621	1.622	0.634	1.612	0.643	1.597	0.653	1.583	0.660	1.573	0.661	1.559	0.668	1.549
									0.614	1.624	0.623	1.605	0.643	1.568	0.658	1.560	0.671	1.545	0.674	1.523	0.683	1.516	0.684	1.505
											0.636	1.569	0.655	1.552	0.668	1.531	0.674	1.515	0.685	1.510	0.693	1.492	0.694	1.489
													0.665	1.504	0.678	1.483	0.689	1.466	0.694	1.458	0.702	1.447	0.708	1.432
															0.690	1.444	0.702	1.439	0.709	1.420	0.713	1.412	0.721	1.402
																	0.710	1.415	0.714	1.399	0.724	1.389	0.731	1.380
																			0.722	1.384	0.731	1.382	0.739	1.360
																					0.737	1.361	0.741	1.355
																							0.752	1.331

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