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Historical Perspective

Complexity and self-organized criticality in liquid foams. A short review

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ABSTRACT

This short review deals with the work done on liquid foams within the framework of the physics of complexity. It aims to stimulate new theoretical and experimental work on foam dynamics as complex dynamical systems. In particular, it examines these systems in relation to Self-Organized Criticality (SOC), for which foams could be used as an accessible experimental model system.

Contents

1. Foreword

Before starting this review, written for an issue honoring Professor Ramón González Rubio, let me first tell you why I have chosen complexity in foams as the topic to write about. Ramón has published more than 200 papers in the area of liquid interfaces, surface rheology, surfactants and polymer-surfactant complexes in bulk and at interfaces, among others. We have co-authored several papers on these subjects [1–15]. Thus, when I was invited to write this article, I thought first to write about one of those subjects, in the line of our joint work. Then, I realized that the rest of the researchers participating in this special issue would probably do the same, and I had a second thought. I spent five years in Ramón's lab as a postdoctoral fellow, from 2006 to almost 2011; during those years, I was encouraged by him and other staff members, to pursue my research interests freely. One of these interests was foam dynamics within the framework of complexity and Self-Organized Criticality (SOC) [16], which I had started studying in a previous postdoctoral position and continued in Ramon's group at the Universidad Complutense de Madrid. The opportunity to choose my own line of research following my own interests was very important for me at that time, and that is why I finally decided to write about complexity in liquid foams. All my gratitude to Ramón for his support during all those years.

2. A brief introduction to liquid foams

This review deals with the dynamics of foams within the framework of complexity and Self-Organized Criticality (SOC), thus let me start by introducing the physics of liquid foams in this section.

Foams are ubiquitous systems in nature and everyday life [17]. The physics of foams has been reviewed from different points of view in

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many books and reviews [17–21]. Here, I will just provide a brief description for those who are not familiar with the field of foam physics.

Liquid foams are two-phase systems formed by gas dispersed in a liquid matrix [18,20]. The gas is dispersed in the form of bubbles, portions of gas enclosed by liquid films, packed together to form a closed cell structure: the foam (see Fig. 1). Both the shape of the bubbles and the whole structure of the foam depend on the relative contents of gas and liquid [22]. This is represented by the liquid volume fraction (or gas volume fraction, φ_g), $\varphi_l = V_l/V_f$, being V_l and V_f the volumes of liquid and foam, respectively ($\varphi_g = 1\text{-}\varphi_l$). For large liquid fractions, the bubbles are spherical (minimum area for a given gas volume), but as φ_l is reduced, bubbles deform adopting shapes of polyhedra, they pack together and compress against each other as liquid drains and films get thinner (see Fig. 1). In the limit of dry foams, $\varphi_l < 0.05$, the liquid films separating bubbles smoothly meet three at a time, at 120 degrees angles (first Plateau equilibrium rule [18]). The intersection of these films forms liquid channels between adjacent bubbles; these channels are called Plateau borders. Plateau borders meet four at a time in a vertex or node, at angles of approximately 109° (second Plateau equilibrium rule [18]). Plateau borders and nodes configure a network through which liquid can flow by gravity and capillarity (see Fig. 1).

The structure just described is not at equilibrium; foams are out of equilibrium systems. To create foams we need to do some work [23] (e.g.to agitate them). This work is used to create the interfaces enclosing the bubbles. The energy needed to create the interfaces is given by the product of the surface tension, γ, and the total area created A. This excess of interfacial energy continuously drives the whole system to minimize its area and towards the complete phase separation, i.e. foam destruction, which is the real (absolute minimum) thermodynamic equilibrium state. That is why additives, such as detergent, are needed for foaming. These additives are surface-active agents, generally surfactants, but proteins, polyelectrolytes, and nano- and microparticles as well as mixtures of them can be used. Surfactants [24] are molecules that spontaneously adsorb at liquid-air interfaces; they reduce the surface tension and thus the energy needed to create the bubble interfaces. Foams, when created, are trapped in one of many metastable states; the selected state depends on the system history, but for a given liquid fraction, the geometry of both the bubbles and the foam structure in such metastable state is determined by the minimization (local minimum) of the interfacial energy [22] (i.e. the interfacial area). However, surface tension alone cannot explain foam stability [20]. Surface active agents also confer certain properties to the interfaces, such as surface viscoelasticity [25] for instance, helping to stabilize the liquid film against rupture. The presence of surfactant molecules not only reduces the surface tension but also helps to kinetically stabilize the system by slowing down the three main dynamical processes that drive liquid foams to their final end, namely drainage, coarsening and coalescence [18]. Drainage refers to liquid flow through the Plateau border network driven by gravity and capillarity. Immediately after freshly forming, the liquid begins to drain out of the foam due to gravity; the top of the foam becomes dry while the bottom, in contact with the solution from which the foam was formed, remains wet (Fig. 1). A vertical profile of liquid content develops along the height of the foam in such a way that, in the metastable state, the force of gravity is balanced by the vertical pressure gradient [26]. Drainage, for which theoretical models exist [21,27], is the most widely understood of the three mentioned dynamical phenomena.

Coarsening or disproportionation refers to the continuous change in bubble sizes as foam ages, due to gas diffusion among them. Large bubbles grow at the expense of smaller bubbles in contact with them, which shrink. The driving force for this process is the difference in pressures between bubbles of different sizes. The Young-Laplace equation. [28] states that the internal bubble pressure varies as the inverse of its radius, thus the gas tends to diffuse from small bubbles to large ones. In order to diffuse, the gas has to traverse the liquid films separating the bubbles; to accomplish this, the gas needs to solubilize in the liquid; thus, for a perfectly insoluble gas, coarsening is not possible. Coarsening is also arrested if the bubbles are all exactly the same shape and size (monodisperse). In this case, all the bubbles have the same internal pressure, and no driving exist for gas diffusion (except for the bubble layer in contact with the atmosphere). The rate of disproportionation not only depends on the gas solubility and the distribution of bubble sizes but also on the kind of surface-active agent used. The gas has to go through the interface covered by surfactant molecules (or polymer, particles, etc.) that could act as barriers. Moreover, in the case of foams stabilized by solid particles, they can completely arrest the coarsening process [29]. Coarsening is also arrested when the surface compression modulus of the surfactant monolayer reaches a value equal to about half the surface tension [30]. There exist theoretical models for coarsening in 2D foams (foams sandwiched between two glass plates

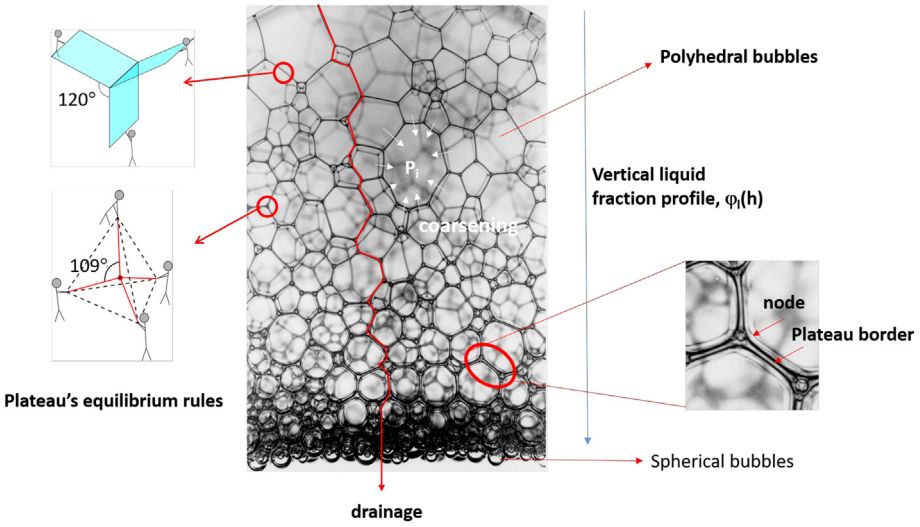

Fig. 1. Foam in the gravity field. Liquid drains out of the foam though the Plateau borders and nodes. A vertical liquid profile develops: at the bottom, where the liquid fraction is high, bubbles are spherical; at the top, they are polyhedral. For dry foams, at the top, Plateau's local equilibrium rules are maintained. Due to pressure differences between neighboring bubbles (Laplace, $\Delta P \sim 1/r$), they coarsen, and gas flows from the small into the large bubbles. Photo courtesy of Professor Douglas Durian.

separated by distances smaller than the bubble size) [18,21], and they are quite well understood. For 3D foams, the situation is different [31]. In general terms, coarsening is less well understood than drainage.

The process of foam coalescence is the least understood of the three [32]. It refers to the rupture of the liquid films separating bubbles in a foam structure. The first model of rupture of a single isolated liquid film was proposed by Sheludko and extended by Vrij [33], and it is based on the assumption that film ruptures because of thermal fluctuations in the film thickness. At a certain critical thickness, van der Waals forces act between both gas/liquid faces of the film producing an instability that cannot be damped and grows leading to film rupture, being the process controlled by surface tension and disjoining pressure [19]. Exerowa et al. [34] and de Gennes [35] proposed that a single film ruptures via thermal fluctuations of the surfactant concentration at the film interfaces. These fluctuations could produce holes (i.e. regions without surfactant molecules) at the interfaces. If the size of these regions is greater than a certain critical value (typically in the order of half the film thickness), the hole grows and the film breaks. This process is controlled by compression surface elasticity. Both models are developed for single isolated films. However, no clear correlation exists between the dynamics of a single isolated film rupture and the dynamics of the coalescence of those films in macroscopic 3D foams [36]. For them, it was reported that coalescence occurs when the bubble radius reaches a critical value (by coarsening) [37]; other authors reported that bubbles coalesce when the pressure difference between the gas and the liquid within Plateau borders reaches a critical value [38]. Foam destabilization mechanisms in which the dynamics of fast rearrangements of films triggered by coarsening are involved were also proposed [39,40]. Additionally, a film rupture within a foam might trigger a cascade of ruptures, a phenomenon that makes it more difficult to correlate single film stability with macroscopic foam stability. Coalescence in foams is by far less well understood than drainage and coarsening.

After all the above description of foam dynamics, we conclude that, in fact, most liquid foams are not even metastable; they evolve continuously by drainage, coarsening and coalescence. Each of these processes, when considered in isolation, acts on different timescales. The rupture of a single film occurs in fraction of seconds; the metastable vertical liquid profile is reached by drainage in minutes; finally, coarsening might last for hours. Despite this separation of timescales, in macroscopic foams, the three processes affect each other. For instance, the rupture of a film releases liquid that is collected by Plateau borders and films, locally increasing the liquid fraction and the film thicknesses, which in turn modifies the coarsening rates and the film stability against coalescence. Quite recently, a new numerical simulation approach, under certain simplifying assumptions, has been presented; it simultaneously takes into account the occurrence of the three processes in the modeling of foam dynamics [41].

I have already mentioned the occurrence of bubble rearrangements or topological changes. They refer to any process that changes the number of neighbors of a bubble (i.e. the number of faces of that bubble). In 2D foams, all rearrangements can be expressed as the combination of two elementary topological changes (Fig. 2). One of them is film switching (Fig. 2a), known as T1 process. The second elementary topological change is the disappearance of a bubble or cell (Fig. 2a, b), called T2 process. In 3D, the topological changes are a bit more complex, but the general idea is the same [21].

These topological changes often occur in cascades. These avalanche-like dynamics that, as I have mentioned earlier, can also be seen in the rupture of films when foams coalescence are the main subject of these review.

3. Complexity, SOC and liquid foams

Some years ago, Weaire and Hutzler [42] published an article entitled 'Foams as complex systems' in which they summarized very briefly the existing papers on 2D foams within the framework of complex systems; since then, the literature on the subject has been scarce. Before discussing the research on foams as complex systems, and having introduced the reader to foam physics in the previous section, let me now introduce *complexity* in physics.

The word complexity derives from the Latin word *plectere*, which means to weave and/or entwine. But what do we mean for complexity in physics? To answer the question, let me propose you to examine some particular examples [43]. First, consider the behavior of a small number of ants, say twenty. If you release them into a wood, they will go around endlessly doing nothing, just walking until they die of exhaustion. However, release one hundred thousand of them and you will find an 'emergent collective behavior' that ensures the survival of the colony by collecting food, eliminating enemies and building shelters for breeding their larvae. Each of these ants is a blind and unintelligent animal that communicates by means of a few simple chemical signals. However, thousands of them behave as if they were an intelligent superorganism. The very simple rules that govern the behavior of a single ant produce an emergent structure and a collective behavior that is much more than the sum of the features of the interacting entities (ants). Let me give you a second example, think of our brain, a group of hundreds of thousands of neurons arranged in a network, which can be essentially in one of only two possible states: inactive or firing

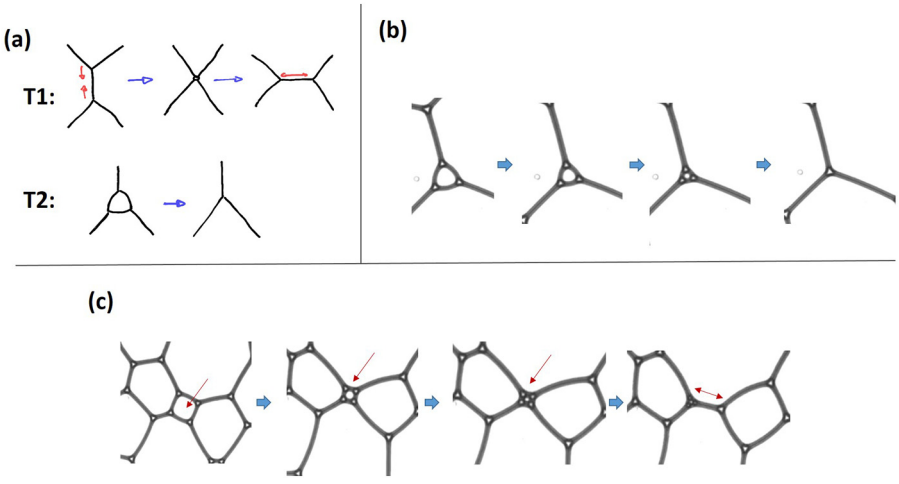

Fig. 2. Topological changes in 2D foams. (a) Schematic representation of T1 and T2 processes. (b) Sequence of a T2 process in a real 2D foam. (c) A combination of T2 and T1 topological changes in a real 2D foam.

a signal to other neurons. A particular neuron sends a signal if it receives enough signals from other neurons. These simple rules produce signal patterns (electric and chemical) in the neuron network that give place to an emergent collective behavior, which in turn results in our intelligence, thoughts, learning capacity, feelings and consciousness. Again, the emergent result of simple interactions among individual entities in the system is, by far, much more than the sum or the average behavior of these individual entities. The same can be said of the immune system, the stock market (in this case, the single entities are humans acting by self-interest) and the spread of a disease (quite relevant in the current COVID-19 crisis) [43]. All the examples I have given so far involve some living entity; but, of course, complexity is not restricted to living systems. Complexity can be found in a pile of rice or sand, in forest fires, in an earthquake, in droplets on a glass window and in vortices in superconductors [44], as well as in solar flares and in other astrophysical systems [45], to name just a few. The emergent complexity of these inanimate systems arises from their statistical behavior (what we mean by this will be clarified later on). All the mentioned examples are many-body systems, interacting via some kind of signals or information that organizes themselves without any tuning from outside the system, resulting in an emergent collective behavior that cannot be described simply by averaging or summing. Thus, let us define a *complex system* in physics, one that exhibits *complexity*, as *a many body system exhibiting nontrivial emergent and self-organized behavior* [43].

All the systems mentioned are dynamical (i.e. systems that change and evolve in time). In these dynamical systems 'the whole is not the sum of its parts' which means that they are nonlinear dynamical systems. These nonlinearities could eventually lead to chaos. Chaotic systems are those that exhibit a sensitive dependence on initial conditions; that is, even small uncertainties in the measurements of initial conditions can result in huge errors in long-term predictions. Chaos, bifurcation, period-doubling cascades, attractors, renormalization, critical phenomena, avalanches, fractals, scale invariants, power laws, self-organization and random networks are all terms and concepts related to complexity and complex systems [43], and almost all of them can be found in relation with liquid foams. Among these concepts, we are particularly concerned here with SOC.

Self-organized criticality attracted a lot of attention the last two decades [44,46]. This concept was introduced by Bak, Tang and Wiesenfeld in 1987 [47], and it proposes that complex behavior can develop spontaneously in certain nonequilibrium systems with many-body interactions. These systems are complex in the sense that there is not a single time or spatial scale characterizing the system behavior, but the statistical properties are well described by simple power laws. The absence of the characteristic length and temporal scales is what is observed in the context of equilibrium thermodynamics at critical temperature in a continuous (critical) phase transition and the reason why the word 'critical' is used in SOC. The self-organized part in the name SOC implies that the system reaches the critical state by itself without any tuning from outside (unlike to what happens in a critical phase transition).

When Bak, Tang and Wiesenfeld (BTW) introduced the concept of SOC [47], they aimed to explain the origin (and mechanism) of self-similar fractal structures, in time and space, found in so many physical systems and phenomena. The following is the main idea [44]: in many-body systems, a signal can travel through if there is a connected path above a certain threshold for that signal to travel. The region above the threshold forms a dynamic random network that changes continuously by the combination of the internal relaxation and the continuous (slow) driving by the external field. The signal stops when it cannot find a region above the threshold to continue travelling, and the system reaches a new metastable state. Then, by the action of the external field, some regions of the system are slowly driven above the threshold once more, and the internal relaxation is restarted. The dynamics is intermittent, with periods of activity and inactivity. The authors suggested that the dynamical network formed by the path followed by the signal has a percolating-like fractal geometry. The fractals could be of any size, as well as the time of the internal relaxation processes that lead the system to a new metastable state (i.e. power law distribution functions and lack of spatial and temporal characteristic scales).

Systems exhibiting SOC behavior and SOC models are defined in terms of some dynamical variable (e.g. the stress on the earthquake fault [48–50] or the slope in a pile of rice [51]). These dynamical variables evolve over time by the presence of a 'field' (e.g. the slow movement of tectonic plates or the slow addition of a rice grain to the pile). The field slowly drives the system to undergo an event or, using the previous vocabulary, the signal (e.g. an earth movement, the displacement of a rice grain on the slope of the pile) when a certain threshold (stress or slope) is locally overcome. These individual events could produce avalanches of events of different sizes leading, for instance, to an earthquake. The statistical size distribution (energy realized in an earthquake or the number of rice grains involved in an avalanche) of these events follows power laws (scale-invariant, long-range spatio-temporal correlations). Thus, the key ingredients for the SOC behavior are: the power laws, the presence of thresholds and metastability (e.g. the friction force between the plates for earthquakes), and finally a slow external driving when compared to the internal relaxation times of the system (e.g. the movement of the tectonic plates that increases the stress lasts decades or centuries, but the internal relaxation -an earthquake- occurs in minutes).

All these ingredients are present in the dynamics of liquid foams. Foams are many-body metastable systems that are continuously driven to equilibrium by drainage and coarsening. This dynamics lasts from minutes to several days, but the local internal relaxation, bubble bursting or topological changes occur in the range of seconds or fractions of a second [52].

In foams, the dynamical variables of the system could be the bubble radius or the pressure difference between the gas and the liquid in Plateau borders. For example, for foam collapse, as it was mentioned in §2, some researchers suggested that coalescence occurs when the bubble radius reaches a critical value (by coarsening) [37]; others reported that bubbles coalesce when the pressure difference between the gas and the liquid within Plateau borders reaches a critical value [38]. In both cases, coarsening acts as 'field' driving parts of the system above a local threshold that produces the first bubble rupture or rearrangement. If the foam is in a SOC state, this first event, rupture or topological change, could produce avalanches of events of any size, in which the macroscopic foam dynamics is now controlled and dominated by the collective dynamics regardless of the microscopic features of the liquid films and bubbles. If this is the case, the size and temporal probability densities of events should follow power laws. For the limit of infinite system sizes, an exponent < 2 for the power law implies that the average of the distribution does not exist, and that for exponents <3, the standard deviation is infinite. In general, for finite size systems, the distribution of the avalanche sizes s (or duration, τ) should be [46],

$$P(s)s^{\tilde{-}\beta}f(L) \tag{1}$$

with a certain lower cutoff, s_0 (e.g. in bubble rupture, an event involves at least one bubble, then $s \geq s_0 = 1$). The function $f(L)$ is a certain function that tends to 1 as the linear system size, L, tends to infinity [46]. For example, some systems exhibit a crossover from power law to exponential behavior as s increases above a certain value s_1, in such a way that $P(s) \sim exp\,[-s/s_1]$ for $s > s_1$, where s_1 scales as $s_1 \sim L^{\omega}$, with $\omega > 0$.

It is within the framework of SOC dynamics that liquid foams could be thought as complex dynamical systems. The search of power laws distributions in the size and temporal distributions of bubble ruptures or topological changes in liquid foams and its relation with SOC is reviewed in the following sections.

4. Foams as complex systems

Foams can be considered as complex systems in several ways, for instance focusing in the fractal structure of foams [53,54] and foams flowing through porous media [55] and in Hele Shaw radial cells [56] (see §4, Fig. 8) or even the fractal-like patterns in chaotic light scattering by foams [57]. Ensembles of soap films were even used as synthetic systems exhibiting some characteristics of the statistics of human mortality [58]. Foam production can also serve as an experimental realization of the 'period doubling route to chaos' (bifurcation) mimicking the behavior of the logistic map [59], as observed in the production of bubbles in microfluidic devices [60] (see Fig. 3 and its caption for a brief explanation).

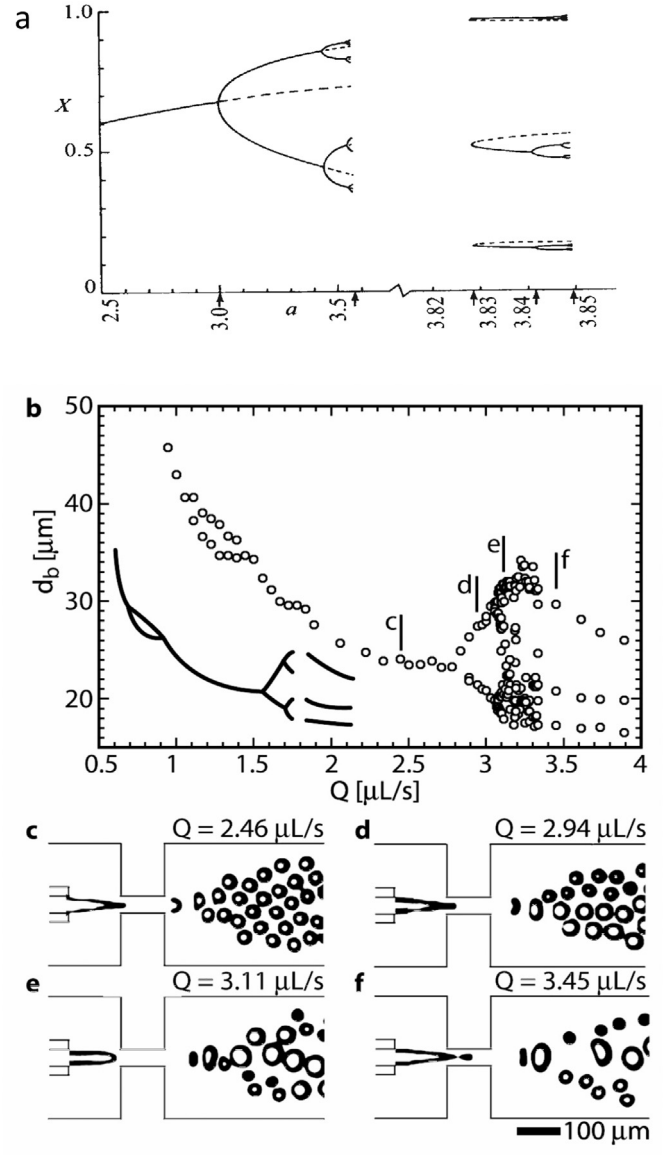

Fig. 3. Period doubling route to chaos. (a) Representation of the fixed points (attractors) for the logistic map. The logistic map is a deterministic equation, $X_{t+1} = a\,X_t(1 - X_t)$ for which each value of X (X_{t+1}) is calculated from the previous value (X_t). In the figure, X (the attractors) is represented as a function *a*. Depending on the value of *a*, the iteration converges to a single fixed point, oscillates between *2, 4, 8,* etc. *different values or fluctuates apparently* at random (bifurcation to chaos). The chaotic behavior occurs at values *a* ~ 3.57. Adapted with permission from reference [59]. (b) Representation of the bubble diameter (d_b) produced in a flow focusing microfluidic device. Bifurcation diagram showing the diameters of the bubbles as a function of the flow rate Q. Period-doubling and period-halving bifurcations are clearly observed, with a region of flow rates (Q ~ 3.2) with chaotic dynamics. *The solid line shows an outline of the bifurcation diagram (not to scale).* (c)-(f) *Micrographs of the generation of bubbles in the device, associated to a flow rate (Q), shown on panel (b). Adapted with permission from reference* [60].

However, hereinafter I will focus mainly on avalanche-like dynamics of bubble ruptures and topological changes within the framework of SOC.

In 1995, Brunet et al. [61] described the dynamics of breaking foams stabilized with SDS surfactant and sandwiched between two Plexiglas plates. They used a light lamp for gently heating these 2D foams to trigger film ruptures whose dynamics was followed by means of a CCD camera. The authors clearly identified a regime where the dynamics is controlled by a collective, cooperative behavior: cascades of film ruptures.

Probably the first article on the subject of foam collapse related to foams and SOC is the one by Müller and di Meglio [62] published in 1999. They studied foams made from SDS solutions at two concentrations below the critical micelle concentration (cmc) by recording the sound emitted by the bursting of bubbles at the top of three-dimensional foam floating on the surface of the surfactant solution and placed in a fish box. From the sound signal, they were able to count the number of events (bubble ruptures) as a function of time. They found that, for aged (dried) foams, the events (cascades of bubble ruptures) were correlated, and the probability distribution functions of avalanche sizes were deviated from the Poisson distribution. They also implemented a highly simplified numerical model from which they obtained results qualitatively similar to those obtained from the experiments and from which they found that the probability distribution of avalanche sizes followed a power law, $P(s) \sim 1/s^{\alpha}$ with $\alpha = 1.75$.

In a similar acoustic experiment, Vandewalle et al. [63] found power laws for both the probability distribution of events sizes, h(E), (energy released, E, calculated from the sound signal by integrating the square of the amplitude) and the time elapsed between bubble ruptures, h (τ). For the released energy, they found $h(E) \sim E^{-\nu}$ with the exponent ν between 1.5 and 3, depending on the surfactant concentration. For the time distribution function shown in Fig. 4, they also found a power law $h(\tau) \sim \tau^{-\alpha}$, with $\alpha = 1 \pm 0.1$ regardless of the chemical system.

Vandewalle and Lentz [64] continued the work done in reference [63], studying 2D and 3D foams by imaging the foams with a CCD camera. The 3D foams were produced in a cylinder vessel and observed from above at the air/foam interface. The rupture of bubbles was detected by subtracting two successive images. This procedure also allowed observing topological changes such as edge and vertex movements. They found that these dynamics are temporally and spatially correlated (avalanches). The 2D foams were studied in a vertical Hele-Shaw cell

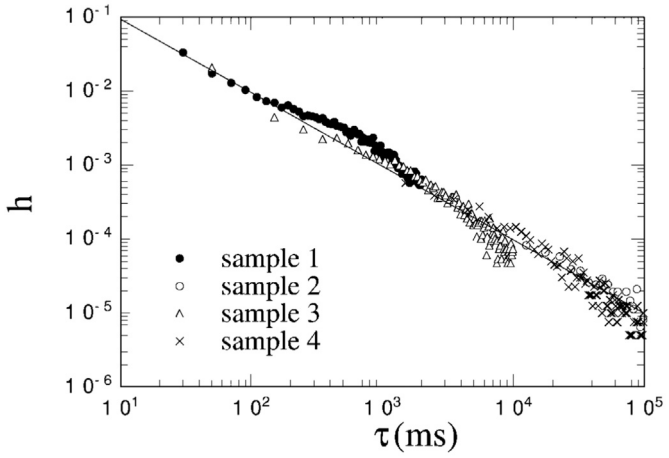

Fig. 4. Scaling law for the time elapsed between successive bubble ruptures. Samples 1, 2, 3 and 4 correspond to different surfactant concentrations. All the curves collapse on the same power law, $h(\tau) \sim \tau^{1\pm0.1}$. Reproduced with permission from Vandewalle et al. [63].

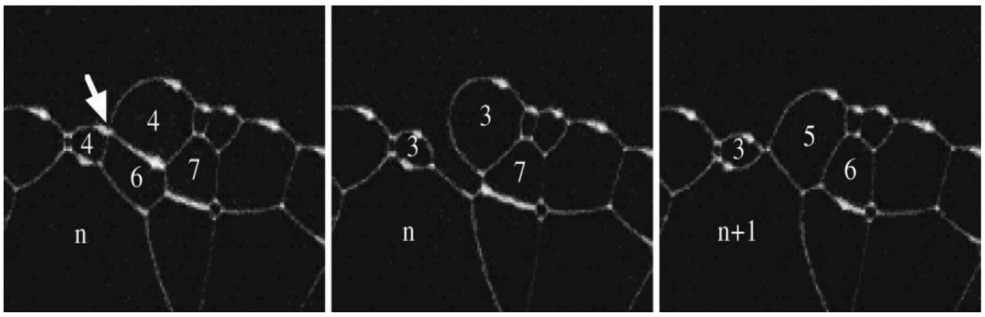

Fig. 5. Three successive images of a 2D foam in a Hele-Shaw cell. The white arrow indicates the film that is going to break. The numbers correspond to the number of sides of each bubble (or cell) and show the topological rearrangements. From [64] with permission.

that allowed observing the dynamics not only at the foam/air interface, as it was the case of 3D foams, but also in the bulk. This allowed them to observe film ruptures and topological changes (side switching, T1, and vertex disappearance, T2; See Fig. 2) triggered by the film rupture. Fig. 5, reproduced from reference [64], shows a film rupture followed by topological rearrangements. For these 2D foams, they also found that, as in 3D foams, film ruptures and topological changes are temporally and spatially correlated. They also observed that film ruptures and topological changes are independent of film size and curvature. This independence from the local (microscopic) characteristics and the existence of cooperativity and avalanches are the hallmark of SOC. However, the authors neither statistically analyzed these events (ruptures and topological changes) nor mentioned that possibility (SOC).

Ritacco et al. [16] used CDD cameras to investigate the cooperativity of bubble ruptures in bubble rafts composed of almost monodisperse bubbles packed in an hexagonal lattice sitting on the surfactant solution. They used SDS at twice the critical micelle concentration and added different amounts of glycerol to change the bulk viscosity. In this configuration, coarsening and drainage are not present allowing the collapse dynamics to be studied on its own. Using a fast CCD camera, they showed the mechanical perturbation produced on the bubble neighbors by a rupturing bubble (see §5, Fig. 10). This led the authors to propose a quite simple cellular automaton model to mimic the foam behavior. The central idea is that if the energy transferred to a neighboring bubble by a rupturing one is greater than a certain energy threshold, the bubble breaks and cascades of ruptures could follow. By changing the bulk viscosity and therefore the energy dissipation, the authors had a certain control over the energy transferred to the neighboring bubbles. The main result of the work is reproduced in Fig. 6, which shows the event size distribution P(s), being *s* the normalized number of bubbles involved in a cascade for the first simulated and experimentally measured avalanche. The process is compatible with SOC for intermediate viscosity values in which the distribution follows power laws, $P(s) \sim s^{-(1\pm0.2)}$. For higher viscosity values, the distribution of avalanche sizes becomes exponential. This scenario is quite similar to the case of cascades on piles of rice [51], for which a change from power law behavior to exponential is observed when the aspect ratio of the rice is changed from elongated grains to more rounded ones (see §54 for a more complete discussion). The aspect ratio plays here the role of bulk viscosity in the bubble raft.

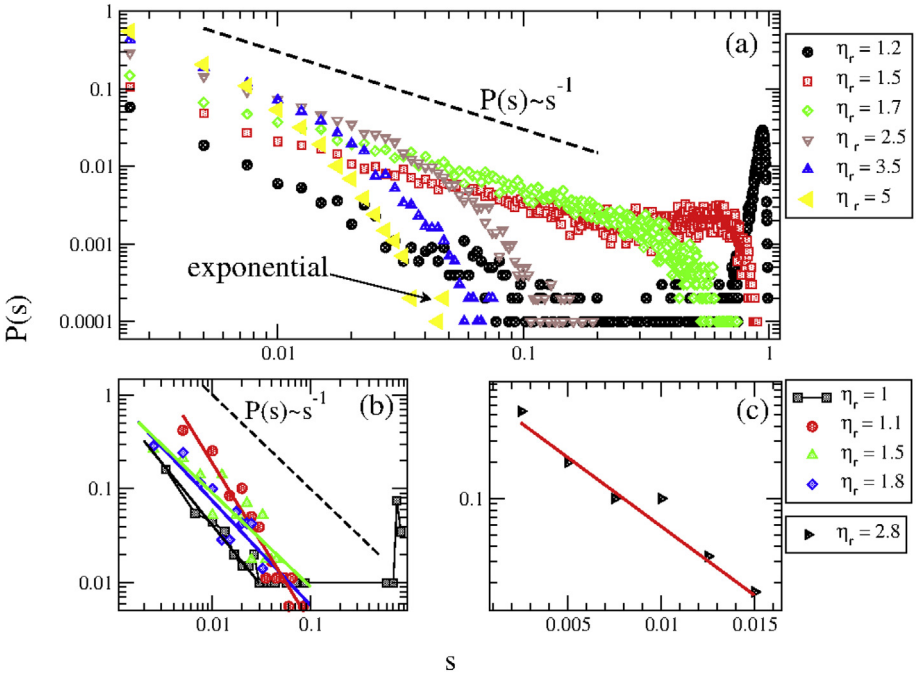

Fig. 6. (a) Avalanche size distribution simulated with a cellular automata model (see [16]) for different values of relative bulk viscosities, η_r. (b) Size distribution for the first avalanche size experimentally measured following power laws. (c) The same as (b), but for $\eta_r = 2.8$, showing that P(s) follows an exponential decay. Both experimental and simulation results show a transition from power laws to exponential decays as the dissipation parameter (viscosity) changes. From [16] with permission.

Dominguez et al. [65] conducted similar experiments on bubble rafts composed of about 500 bubbles. The surfactant used was Gemini 12-2-12 at different concentrations, all above the cmc. As in the work performed by Vandewalle et al.[63], they measured the sound signal emitted by the rupturing bubbles. A preliminary result is shown in Fig. 7. They found power laws for both the time elapsed between bubble ruptures, P(t), and for the size of the avalanches, P(s). For P(t), they found exponents very close to 1 and independent of surfactant concentrations. For P(s), they found that the exponents of the power law increased from 2 to 3 as the surfactant concentration increased. Note that the behavior is similar to the one found by Vandewalle et al. [63], with similar exponents. Although the results are still very preliminary, there seems to be an exponential cutoff that depends on the system size. However, further experiments are needed to confirm these results because of the way it was done. Instead of changing the size of the foam container, the authors changed the average size of the bubbles using the same container, which could change the conditions of the experiments in an uncontrolled way.

Cascades of bubble rearrangements were also observed when shearing foams in rheology experiments. Foams can withstand small shear forces by responding elastically like a solid (linear response). This is because the shear distorts the bubbles, increasing the surface area, which is opposed by surface tension. If stress is increased, the response becomes nonlinear, and if it further increased, localized cascades of bubble rearrangements occur and the foam flows as a simple viscous fluid.

Park and Durian [56] performed experiments on the flow of foams using a commercial shaving cream in a Hele Shaw radial cell (two plates separated a distance b, with a hole in the center through which the foam is injected). They observed and analyzed the different flow patterns, fingering features and fractal dimension of the patterns (Fig. 8) as a function of flow conditions such as plate separation, driving pressure gradient and shear strain rate. They tried to relate pattern morphology to foam rheology. Although the fractal dimension of all patterns is the same (about 7/4) they identified two different morphologies (A and B in Fig. 8; see caption for an explanation) and concluded that the transition between them depends only on the shear strain rate $\dot{\gamma}$. For large $\dot{\gamma}$ ($> 9\ s^{-1}$), the dissipation occurs in liquid films between sliding bubbles, shear stress is viscous in origin, and the flow pattern corresponds to type A. For small $\dot{\gamma}$ ($< 9\ s^{-1}$), the shear stress is elastic due to surface tension, and type B patterns are observed. They suggested a similarity between foam flows, for which nonlinearities arise from large distortions of bubbles and avalanche-like rearrangements, and the flow dynamics in granular systems and earthquakes. Although the authors did not refer to SOC behavior explicitly when comparing the behaviors of foam flows and granular materials, they indirectly suggested it. Hatzler et al. [66] explicitly suggested the possibility of SOC dynamics for bubble rearrangements in simulations of disordered 2D foams subjected to extension. They found that for large volume liquid fractions, the frequency, f(s), of occurrence of cascades involving a number s of topological changes follows the power law $f(s) \sim s^{-1}$.

In line with the work done by Park and Durian [56], Okuzono and Kawasaki [67] conducted computer simulation experiments on the rheology under steady shear of 2D foams both in the absence of

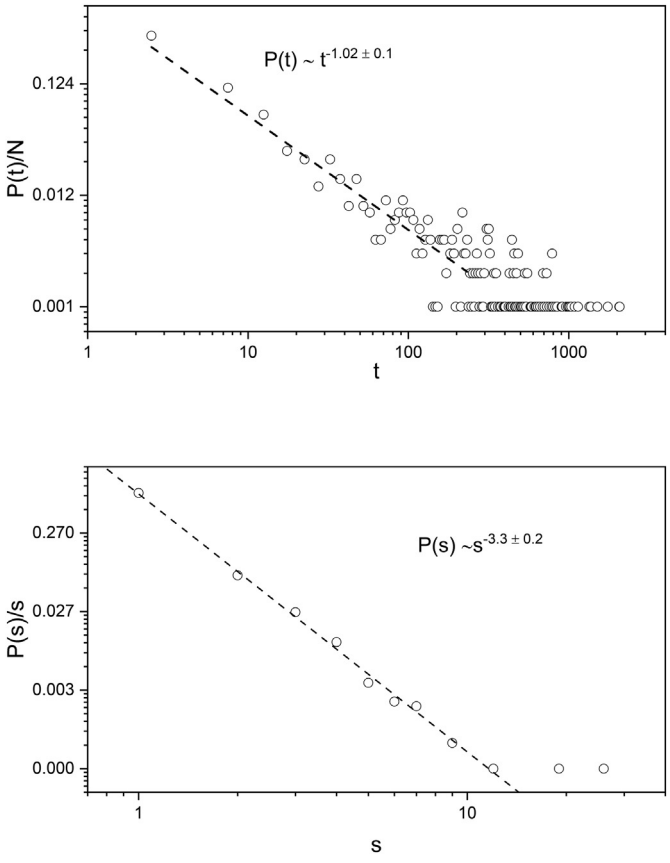

Fig. 7. Power laws in the dynamics of bubble raft collapse. The foam was stabilized with a Gemini 12-2-12 surfactant at a concentration of 10 times the cmc. P(s) is the distribution of avalanche sizes and P(t) is the distribution of times elapsed between bubble ruptures. From [65].

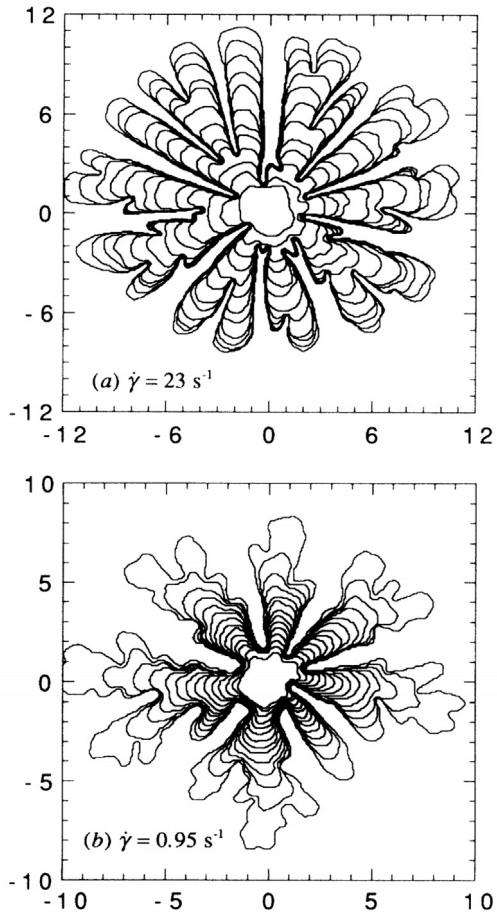

Fig. 8. Patterns of foam flow. The line traces correspond to the observed pattern at different times. (a) Type A pattern corresponds to high shear strain rate, for which fingers advance at constant velocity as it happens in normal viscous fluids. (b) Type B pattern corresponds to low shear strain rate, for which a side branching less aligned in the radial direction as in the fracture of solids, is observed. With permission from [56].

coarsening and in the dry limit. They used the so-called vertex model, which considers the viscous dissipation in the liquid. In their simulations, the authors found an intermittent dynamics with avalanches of topology changes. The probability density, P(s), of the avalanche sizes, s, followed power laws with an exponent 1.5. They compared their results with models exhibiting SOC behavior, in particular with stick-slip models for earthquakes [48,50], and suggested that foams indeed could self-organize in a critical state. However, Gopal and Durian [68], using Diffusing-Wave Spectroscopy (DWS), performed experiments on 3D foams made from shaving cream (wet foam, $\varphi_l = 8\%$) to study the dynamics of topological rearrangements in those foams. They conducted DWS experiments before, during and after an applied shear strain. They found that, although the dynamics of topological changes is a nonlinear stick-slip process similar to the avalanches observed in piles of granular media and in earthquakes, DWS showed that the events are temporal and spatially uncorrelated; thus, a characteristic time and spatial scale can be identified in the foam dynamics contradicting the SOC scenario. Despite this, the authors stressed at the end of the article that the resolution of contradictions between the results of DWS and those of computer simulations [67] should lead to a deeper understanding of the dynamics not only of foams but also of other disordered materials. In this line, Durian presented computer simulation results of the complex macroscopic rheological behavior of foams [69] by implementing the so-called bubble model [70]. He found that the distribution function of released energy is a power law for small events but exhibits an exponential cutoff independent of system size (see Fig. 9a). This result contrasts those obtained by Okuzono and Kawasaki with the vertex model [67] for which, as we have already seen, the behavior is compatible with SOC. Kawasaki and Okuzono [71] extended their work by performing simulations with the vertex model exploring the effects of shear rate and system size. They again found results compatible with SOC (see Fig. 9b).

The difference in the results of both models (the vertex model and the bubble model) in relation to SOC probably comes from the fact that the vertex model is an expansion about the dry limit (polyhedral foams cells) whereas the bubble model is an expansion about the wet limit (spherical foam cells). This topic is discussed in §5.

Bubble geometry depends on the liquid content. In this respect, Hutzler et al. experimentally observed cascades of topological rearrangements in 3D foams as the volume liquid fraction increased [72] in an experiment of forced drainage in highly polydisperse foams, where liquid is added to the top of the foam column. The liquid flow induces movements of the bubbles: the large ones go up and the small ones go down. The small bubbles move irregularly; that is, small arrays of bubbles move down in an avalanche-like way with moments of activity when they descend several centimeters, separated by periods where they stay at the same column height for about 20 s. Dunne et al. [73] studied by computer simulation (using PLAT software) the occurrence of these topological changes but in 2D foams, as a function of the liquid fraction. They found that the histograms of the frequency of occurrence of the avalanches are well fitted with exponentials, contradicting the SOC scenario for which power laws are expected. However, as the authors stated, their simulations are quasi-static and do not involve dynamical variables, which are essential ingredients for SOC.

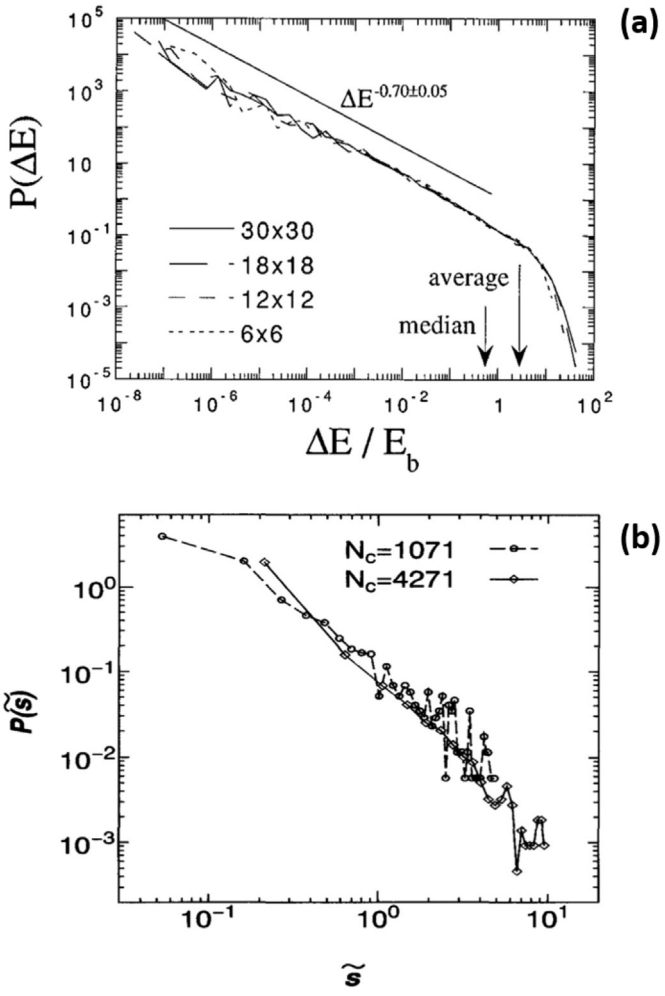

Fig. 9. (a) Probability density, P(ΔE), for bubble rearrangement events releasing an energy ΔE. Only small events exhibit a power-law behavior; exponential cutoffs independent of system size are found for large events. Data from simulations with the bubble numerical model, reproduced with permission from reference [69]. (b) Probability density P(s) of the avalanche size s calculated by simulations using the vertex model for two system sizes (N_c). The results are compatible with SOC. Reproduced with permission from [71].

5. Foam dynamics in the light of SOC in other physical systems

So, do foams evolve into a SOC dynamical state? Before discussing this question let me first comment on SOC in other physical systems. I will specifically discuss two physical systems, piles of granular materials and earthquakes, trying to draw a parallel between them and the dynamics of bubble rearrangements and collapse in foams.

In their original paper, BTW [47] introduced SOC referring to avalanches in sandpiles. They did not perform a real experiment; they implemented instead a cellular automaton simulation mimicking a sandpile. In the simulations, they found power laws both for the size distribution of the avalanches $P(s) \sim s^{-1.35}$ and for the distribution of the avalanche lifetimes $P(t) \sim t^{-0.9}$. However, experiments on real sandpiles [74–76] showed that only the sizes of small avalanches are distributed according to power laws, exhibiting (pseudo) SOC behavior. This behavior is cut off and buried for large avalanches. The latter dominate the whole dynamics of the pile slope; for them, the size and duration of the avalanches are narrowly distributed [76]. It seems that this behavior has its origin in inertial effects: once the grains role down the pile slope, they gain momentum and they get harder and harder to stop. The dynamics develops an oscillatory behavior where the slope, represented by the angle between the horizontal and the free surface of the sandpile, changes between two values: the angle of repose (θ_r), below which no flow of sand can occur, and an upper angle (θ_u) above which large avalanches occur returning the slope to the angle of repose; both θ_r and θ_u are very narrowly distributed, making the behavior of the sandpile predictable. It could be concluded that real physical sandpiles are not in SOC state. However, when the sand grains are replaced by rice grains, the situation is different, at least for certain types of rice grains. Frette et al. [51] performed experiments with piles of three different types of rice grains, one of which was quasi-spherical with a smooth surface; the other two had an elongated shape, but they differed in the roughness of their surfaces. They found that the piles of elongated grains, regardless of the roughness of their surfaces, organize

themselves in a critical state, and the size distribution of the avalanches follows power laws with exponents ~2; they are in SOC state. For the rounded rice grains, the size distribution of the avalanches follows a stretched exponential; thus, the authors concluded that this system is not in critical state. Rounded grains role down the slope and the behavior is dominated by inertia, as in piles of sand grains. For elongated rice grains, their shape anisotropy restricts the way the grains move, increasing the frictional effect (local thresholds), suppressing inertial effects, and leading to a variety of metastable configurations, essential ingredients for SOC behavior. The obvious conclusion is that for piles of granular materials, SOC is not a universal phenomenon; it depends on some microscopic characteristics that ensure the existence of thresholds and a large number of metastable configurations.

I consider that the parallelism of the previous discussion about piles of granular materials and dynamics of bubble rearrangements in foams is quite straightforward. The bubble model, an expansion about the wet limit (spherical bubbles), for which power laws were found for small avalanches but not for the large ones (exponential cutoff independent of the system size), behaves the same as piles of sand grains or spherical rice grain. The behavior is dominated by 'inertia'; the local thresholds are eliminated reducing the number of possible metastable configurations. In contrast, the vertex model, an expansion about the dry limit (polyhedral cells), behaves like the elongated rice grains, exhibiting the features of systems in SOC state. Regarding the experiments in real foams, the only system with a complete and relevant statistical analysis I found in the literature is the one by Gopal and Durian [68] in shaving foams. These are wet foams formed by micrometer sized spherical bubbles, whose dynamical behavior of bubble rearrangements under shear agreed with the results of the simulations with the bubble model, contradicting the SOC scenario. Unfortunately and surely due to experimental difficulties involved, I did not find any studies on the dynamics of topological changes in 3D dry foams. Will they behave as in the vertex model displaying SOC?

Now, I would like to introduce a nonconservative cellular automaton algorithm to model earthquakes dynamics [50]. This model is relevant in our discussion about SOC dynamics in cascades of film ruptures in foams. Earthquakes occur by the abrupt release of the elastic energy accumulated during decades by the relative motion of tectonic plates along the contact interfaces between them (the faults). Friction between the plates hinders the relative movement of the plates and is responsible for the slow, continuous accumulation of elastic energy. The energy storage increases until the build-up stress exceeds the friction force (the threshold); at that moment, the plates suddenly slip with respect to each other, the energy stored for decades is released in seconds and an earthquake is triggered.

The morphology of the faults is fractal-like and all the ingredients of SOC are present in the dynamics described: slow driving, threshold and metastability. The probability of energy released in real earthquakes is indeed distributed in a power law $P(E) \sim E^{-B}$ with $B \sim 2$ (the Gutemberg-Richter law). Olami, Feder and Christensen (OFC) [50] proposed a nonconservative cellular automaton to model earthquake dynamics. Their algorithm is based on the Burridge-Knopoff spring-block model[1] for earthquake faults [77]. The OFC algorithm is as follows: a 2D lattice is defined and a dynamical variable E_i is randomly assigned to each lattice site i, being E_i a force (or an energy). Now, all sites are driven simultaneously at the same rate $E_i = E_i + a$, being a a constant. When in a simulation step the dynamical variable of a site i becomes larger than a certain critical value E*c*, the site is relaxed according to the rule,

$$\begin{cases} E_i \to 0 \\ E_{n,n} \to E_{n.n} + \alpha E_i \end{cases} \quad if\ E_i > E_c \tag{2}$$

where $E_{n,n}$ refers to the neighboring sites of the lattice site i. The excess energy of the overcritical site i is distributed among its neighboring sites. Depending on the value of α, part of the energy is lost (dissipated), the model is nonconservative, except when $\alpha = 1/q_i$ being q_i the coordination number of the site i. The OFC model exhibits SOC behavior, and the distribution of size events followed power laws $P(E) \sim E^{-B}$. The B values were found to be dependant on α and a transition from power laws to exponential decay was found for a certain value of α ($\alpha \le 0.05$). This crossover from power law to exponential is what was observed in bubble rafts as bulk viscosity changes [16] (see Fig. 6). Here viscosity plays the role of the parameter α (dissipation) in the OFC model. Similarly, the change in the exponents of the power laws as the dissipation parameter changes in the OFC model is what was observed in the experiments with Gemini 12-2-12 shown in section §43 (Fig. 7 and associated text), for which the exponent varies from 2 to 3 as the surfactant concentration increases.

In fact, the OFC model could be directly applied to the dynamics of bubble ruptures in bubble rafts. We only need to imagine that each site of the lattice is occupied by a bubble. When the dynamical variable of a particular bubble goes over the threshold by slow driving ($E_i = E_i + a$, $E_i > E_c$), it breaks; the energy (the dynamical variable) of the site goes to zero and part of the released (interfacial) energy is transferred to neighboring bubbles (rules in eq.(2)). This is what we did in our cellular automaton in reference [16]. Fig. 10, reproduced from [16], shows a sequence of images taken with a fast CCD camera that shows the effect of a bubble rupture on its neighbors on the bubble raft. After the bubble rupture, a cascade of ruptures could follow due to the mechanical perturbation produced on the neighbors (see video in [78]). The relation between the phenomenon observed and the OFC model is obvious.

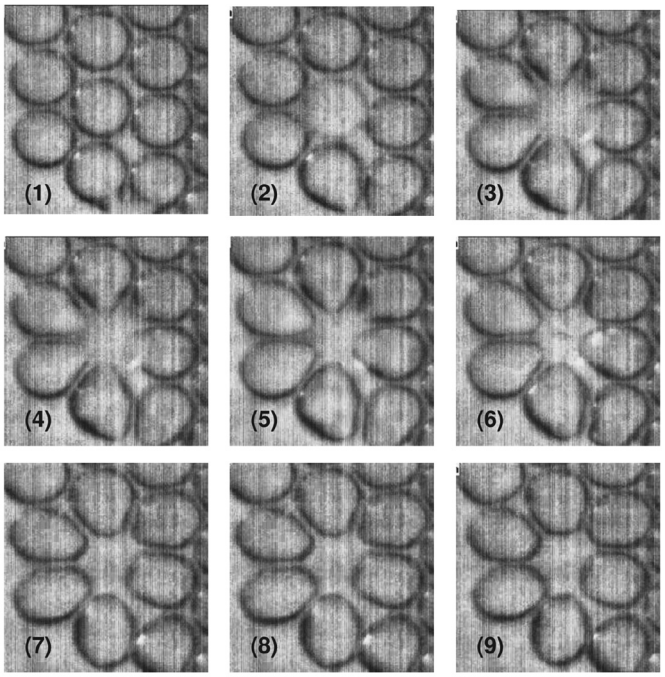

Fig. 10. Sequence of a bubble rupture on a raft of bubbles and its effect on its neighbors. The images were taken with a fast CCD camera, being the time between snapshots of 0.00225 s. The energy released by the rupturing bubble is transferred to the neighbors that in turn can rupture, triggering cascades of ruptures (see video in reference [78]). This is exactly the mechanism proposed in the cellular automaton OFC model (see text). From [16].

[1] The model for the fault consists of a 2D sheet of blocks connected by linear springs to their nearest neighbors. The blocks are also connected to a moving (tectonic) plate by linear springs, and they rest on a surface (the other tectonic plate) with friction forces.

Now, the question with which we started this section can be answered: not all foams organize themselves in a critical state, but some of them can indeed be in SOC dynamical state for both topological rearrangements and macroscopic foam collapse. Foams could set in SOC dynamical state depending on the particular mechanisms of energy dissipation and thresholds of the system.

6. Concluding remarks

Fractal, chaos, unpredictability, SOC, power laws, cooperativity and avalanches are all terms and concepts associated to complex nonlinear dynamical systems that can also be found in relation to foam dynamics. The concept of SOC is of particular interest. Self-organized criticality applied to foams does not necessarily have to capture all the details of the phenomena nor does it need to explain everything about foam dynamics. It could be for example, relevant for small scales, such as small avalanches of bubble rearrangements that are power law distributed exhibiting SOC (or pseudo-SOC), but not for large scales for which "inertia" dominates. At this respect, the result of the bubble model [69] and the DWS experiments on shaving foams [68] are similar to the random OFC model for sandpiles [79] for which and exponential cutoff, independent of system size, exists. The same might be said of the collective dynamics of the collapse of macroscopic foams. For them, the bulk viscosity of the foaming solutions as well as the interfacial properties, such as surface viscoelasticity, could operate to set the foam in a critical state or out of it. If by modifying one of those properties, say the bulk viscosity as in [16], we induce a transition from SOC to non-SOC behavior, or vice versa, without noticing it, any attempts to understand the effect of viscosity on foam stability will be difficult or impossible. This could be the origin of the problems to correlate the stability of single isolated liquid films and macroscopic foam stability in some experimental systems. If a foam is in critical state, its dynamics will be independent of the microscopic features of the films, at least in a certain range. However, if the modification of these microscopic properties induces a transition to a noncritical state, there might be a correlation between the properties and the macroscopic behavior of the foam.

From the point of view of SOC, foams might be a perfect experimental system to test ideas and models. We know how to change the threshold/dissipation by adjusting bulk viscosity or surface elasticity and viscosity by modifying the chemical systems used to stabilize them; in this way, the experimental counterpart of the parameter α (or E_c) in the OFC model is changed. Moreover, by changing and controlling the liquid volume fraction in foams, we can change the equivalent to the aspect ratio of grains in piles of granular materials [51]. The slow driving, which is mainly coarsening in foams, can also be changed by using, for example, more or less soluble gases.

In view of the small number of works that have appeared on the subject, it seems that the community of foam physics researchers has ruled out the possibility that the foams are SOC dynamical systems. Certainly, some foams are not in critical state but, as it happens in piles of granular materials, some others could be. Studying how the transition from SOC to non-SOC behavior occurs will help understand more deeply not only foam dynamics and its relation with microscopic features, but also the emergence of SOC behavior in nature.

Declaration of Competing Interest

None.

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References

[1] Guzmán E, Ritacco H, Ortega F, Svitova T, Radke CJ, Rubio RG. Adsorption kinetics and mechanical properties of ultrathin polyelectrolyte multilayers: liquid-supported versus solid-supported films. J Phys Chem B 2009;113:7128–37. https://doi.org/10.1021/jp811178a.
[2] Guzmán E, Ritacco H, Rubio JEF, Rubio RG, Ortega F. Salt-induced changes in the growth of polyelectrolyte layers of poly(diallyl-dimethylammonium chloride) and poly(4-styrene sulfonate of sodium). Soft Matter 2009;5:2130. https://doi.org/10.1039/b901193e.
[3] Guzmán E, Ritacco H, Ortega F, Rubio RG. Evidence of the influence of adsorption kinetics on the internal reorganization of polyelectrolyte multilayers. Colloids Surf A Physicochem Eng Asp 2011;384:274–81. https://doi.org/10.1016/j.colsurfa.2011.04.005.
[4] Bonales LJ, Rubio JEF, Ritacco H, Vega C, Rubio RG, Ortega F. Freezing transition and interaction potential in monolayers of microparticles at fluid interfaces. Langmuir 2011;27:3391–400. https://doi.org/10.1021/la104917e.
[5] Maestro A, Bonales LJ, Ritacco H, Fischer TM, Rubio RG, Ortega F. Surface rheology: macro- and microrheology of poly(tert-butyl acrylate) monolayers. Soft Matter 2011;7:7761. https://doi.org/10.1039/c1sm05225j.
[6] Guzmán E, Ritacco HA, Ortega F, Rubio RG. Growth of polyelectrolyte layers formed by poly(4-styrenesulfonate sodium salt) and two different Polycations: new insights from study of adsorption kinetics. J Phys Chem C 2012;116:15474–83. https://doi.org/10.1021/jp304522t.
[7] Mendoza AJ, Guzmán E, Martínez-Pedrero F, Ritacco H, Rubio RG, Ortega F, et al. Particle laden fluid interfaces: dynamics and interfacial rheology. Adv Colloid Interface Sci 2014;206:303–19. https://doi.org/10.1016/j.cis.2013.10.010.
[8] Ritacco HA, Kurlat D. G. Rubio R, Ortega F, Langevin D. stationary electric birefringence of flexible polyelectrolyte solutions: experimental evidence of different Counterion polarization mechanisms. Macromolecules 2009;42:5843–50. https://doi.org/10.1021/ma900438u.
[9] Miranda B, Hilles HM, Rubio RG, Ritacco H, Radic D, Gargallo L, et al. Equilibrium and surface rheology of monolayers of insoluble Polycations with side chains. Langmuir 2009;25:12561–8. https://doi.org/10.1021/la901762u.
[10] Ortega F, Ritacco H, Rubio RG. Interfacial microrheology: particle tracking and related techniques. Curr Opin Colloid Interface Sci 2010;15:237–45. https://doi.org/10.1016/j.cocis.2010.03.001.
[11] Ritacco HA, Ortega F, Rubio RG, Ivanova N, Starov VM. Equilibrium and dynamic surface properties of trisiloxane aqueous solutions. Colloids Surf A Physicochem Eng Asp 2010;365:199–203. https://doi.org/10.1016/j.colsurfa.2010.01.053.
[12] Ritacco HA, Fainerman VB, Ortega F, Rubio RG, Ivanova N, Starov VM. Equilibrium and dynamic surface properties of trisiloxane aqueous solutions. Part 2. Theory and comparison with experiment. Colloids Surf A Physicochem Eng Asp 2010;365:204–9. https://doi.org/10.1016/j.colsurfa.2010.01.052.
[13] Ivanova N, Starov V, Rubio R, Ritacco H, Hilal N, Johnson D. Critical wetting concentrations of trisiloxane surfactants. Colloids Surf A Physicochem Eng Asp 2010;354:143–8. https://doi.org/10.1016/j.colsurfa.2009.07.030.
[14] Guzmán E, Ortega F, Prolongo MG, Rubio MA, Rubio RG. On the way to functional coatings : polyelectrolyte multilayers. J Mater 2010;4:1–13.
[15] Maestro A, Bonales LJ, Ritacco H, Rubio RG, Ortega F. Effect of the spreading solvent on the three-phase contact angle of microparticles attached at fluid interfaces. Phys Chem Chem Phys 2010;12:14115. https://doi.org/10.1039/c0cp00570c.
[16] Ritacco H, Kiefer F, Langevin D. Lifetime of bubble rafts: cooperativity and avalanches. Phys Rev Lett 2007;98:244501. https://doi.org/10.1103/PhysRevLett.98.244501.
[17] Hill C, Eastoe J. Foams: from nature to industry. Adv Colloid Interface Sci 2017;247:496–513. https://doi.org/10.1016/j.cis.2017.05.013.
[18] Weaire DL, Hutzler S. The physics of foams. New York: Clarendon Press - Oxford; 1999.
[19] Exerowa DR. Kruglīakov PM. Foam and foam films: Theory, experiment, application. Vol. 5. 1st ed. Amsterdam: Elsevier Science B.V; 1998. https://doi.org/10.1016/S1383-7303(98)80004-5.
[20] Langevin D. Aqueous foams: a field of investigation at the frontier between chemistry and physics. ChemPhysChem 2008;9:510–22. https://doi.org/10.1002/cphc.200700675.
[21] Cantat I, Cohen-Addad S, Elias F, Graner F, Höhler R, Pitois O, et al. Foams: Structure and Dynamics. , 6OUP Oxford; 2013. https://doi.org/10.1093/acprof:oso/9780199662890.001.0001.
[22] Drenckhan W, Hutzler S. Structure and energy of liquid foams. Adv Colloid Interface Sci 2015;224:1–16. https://doi.org/10.1016/j.cis.2015.05.004.
[23] Drenckhan W, Saint-Jalmes A. The science of foaming. Adv Colloid Interface Sci 2015;222:228–59. https://doi.org/10.1016/j.cis.2015.04.001.
[24] Rosen MJ. Surfactants and interfacial phenomena. . 3rd ed.New Jersey: Wiley-Interscience; 2004.
[25] Langevin D. Influence of interfacial rheology on foam and emulsion properties. Adv Colloid Interface Sci 2000;88:209–22. https://doi.org/10.1016/S0001-8686(00)00045-2.
[26] Maestro A, Drenckhan W, Rio E, Höhler R. Liquid dispersions under gravity: volume fraction profile and osmotic pressure. Soft Matter 2013;9:2531–40. https://doi.org/10.1039/c2sm27668b.
[27] Saint-Jalmes A. Physical chemistry in foam drainage and coarsening. Soft Matter 2006;2:836. https://doi.org/10.1039/b606780h.
[28] Adamson AW, Gast AP. Physical chemistry of surfaces. . 6th ed.New York: John Wiley & Sons, Inc; 1997.
[29] Cervantes Martinez A, Rio E, Delon G, Saint-Jalmes A, Langevin D, Binks BP. On the origin of the remarkable stability of aqueous foams stabilised by nanoparticles:

link with microscopic surface properties. Soft Matter 2008;4:1531. https://doi.org/10.1039/b804177f.
[30] Gibbs W. The Collected Works of J. Willard Gibbs. New York. London. Toronto: Yale Univ. Press; 1928.
[31] Briceño-Ahumada Z, Langevin D. On the influence of surfactant on the coarsening of aqueous foams. Adv Colloid Interface Sci 2017;244:124–31. https://doi.org/10.1016/j.cis.2015.11.005.
[32] Langevin D. Bubble coalescence in pure liquids and in surfactant solutions. Curr Opin Colloid Interface Sci 2015;20:92–7. https://doi.org/10.1016/j.cocis.2015.03.005.
[33] Vrij A. Possible mechanism for the spontaneous rupture of thin, free liquid films. Discuss Faraday Soc 1966;42:23. https://doi.org/10.1039/df9664200023.
[34] Exerowa D, Kashchiev D, Platikanov D. Stability and permeability of amphiphile bilayers. Adv Colloid Interface Sci 1992;40:201–56. https://doi.org/10.1016/0001-8686(92)80077-B.
[35] de Gennes P-G. Some remarks on coalescence in emulsions or foams. Chem Eng Sci 2001;56:5449–50. https://doi.org/10.1016/S0009-2509(01)00170-1.
[36] Monin D, Espert A, Colin A. New analysis of foam coalescence: from isolated films to three-dimensional foams. Langmuir 2000;16:3873–83. https://doi.org/10.1021/la981733o.
[37] Georgieva D, Cagna A, Langevin D. Link between surface elasticity and foam stability. Soft Matter 2009;5:2063. https://doi.org/10.1039/b822568k.
[38] Khristov K, Exerowa D, Minkov G. Critical capillary pressure for destruction of single foam films and foam: effect of foam film size. Colloids Surf A Physicochem Eng Asp 2002;210:159–66. https://doi.org/10.1016/S0927-7757(02)00377-1.
[39] Carrier V, Colin A. Coalescence in draining foams. Langmuir 2003;19:4535–8. https://doi.org/10.1021/la026995b.
[40] Biance AL, Delbos A, Pitois O. How topological rearrangements and liquid fraction control liquid foam stability. Phys Rev Lett 2011;106:1–4. https://doi.org/10.1103/PhysRevLett.106.068301.
[41] Saye RI, Sethian JA. Multiscale modeling of membrane rearrangement, drainage, and rupture in evolving foams. Science (80-) 2013;340:720–4. https://doi.org/10.1126/science.1230623.
[42] Weaire D, Hutzler S. Foam as a complex system. J Phys Condens Matter 2009;21:474227. https://doi.org/10.1088/0953-8984/21/47/474227.
[43] Mitchel M. Complexity. A guided tour. 1 st. Oxford New York: Oxford University Press; 2009.
[44] Jensen HJ. Self-organized criticality: Emergent complex behaviour in physical and biological systems. . 1st ed.Cambridge: Cambridge University Press; 1998.
[45] Aschwanden MJ, Crosby NB, Dimitropoulou M, Georgoulis MK, Hergarten S, McAteer J, et al. 25 years of self-organized criticality: solar and astrophysics. Space Sci Rev 2016;198:47–166. https://doi.org/10.1007/s11214-014-0054-6.
[46] Pruessner G. Self-Organized Criticality: Theory, Models and Characterization. . 1st ed. Cambridge - Ne York - Melbourne - Madrid: Cambridge University Press; 2012.
[47] Bak P, Tang C, Wiesenfeld K. Self-organized criticality: an explanation of the 1/f noise. Phys Rev Lett 1987;59:381–4. https://doi.org/10.1103/PhysRevLett.59.381.
[48] Carlson JM, Langer JS. Properties of earthquakes generated by fault dynamics. Phys Rev Lett 1989;62:2632–5. https://doi.org/10.1103/PhysRevLett.62.2632.
[49] Carlson JM, Langer JS. Mechanical model of an earthquake fault. Phys Rev A 1989;40:6470–84. https://doi.org/10.1103/PhysRevA.40.6470.
[50] Olami Z, Feder HJS, Christensen K. Self-organized criticality in a continuous, nonconservative cellular automaton modeling earthquakes. Phys Rev Lett 1992;68:1244–7. https://doi.org/10.1103/PhysRevLett.68.1244.
[51] Frette V, Christensen K, Malthe-Sørenssen A, Feder J, Jøssang T, Meakin P. Avalanche dynamics in a pile of rice. Nature 1996;379:49–52. https://doi.org/10.1038/379049a0.
[52] Durand M, Stone HA. Relaxation time of the topological T1 process in a two-dimensional foam. Phys Rev Lett 2006;97:226101. https://doi.org/10.1103/PhysRevLett.97.226101.
[53] Herdtle T, Aref H. Relaxation of fractal foam. Philos Mag Lett 1991;64:335–40. https://doi.org/10.1080/09500839108214630.
[54] Weairet D, Rivier N. Soap, cells and statistics-random patterns in two dimensions. Contemp Phys 1984;25:59. https://doi.org/10.1080/00107518408210979.
[55] Wang F, Du D, Bi H, Wang H, Chen H, Li H. Quantitative characterization of foam transient structure in porous media and analysis of its flow behavior based on fractal theory. Ind Eng Chem Res 2020;59:5158–66. https://doi.org/10.1021/acs.iecr.9b06941.
[56] Park SS, Durian DJ. Viscous and elastic fingering instabilities in foam. Phys Rev Lett 1994;72:3347–50. https://doi.org/10.1103/PhysRevLett.72.3347.
[57] Tufaile APB, Tufaile A, Liger-Belair G. Hyperbolic kaleidoscopes and Chaos in foams and Hele-Shaw cell. J Phys Conf Ser 2011;285. https://doi.org/10.1088/1742-6596/285/1/012006.
[58] Haffner B, Lalieu J, Richmond P, Hutzler S. Can soap films be used as models for mortality studies? Phys A Stat Mech Appl 2018;508:461–70. https://doi.org/10.1016/j.physa.2018.05.044.
[59] May RM. Simple mathematical models with very complicated dynamics. Nature 1976;261:459–67. https://doi.org/10.1038/261459a0.
[60] Garstecki P, Fuerstman MJ, Whitesides GM. Nonlinear dynamics of a flow-focusing bubble generator: an inverted dripping faucet. Phys Rev Lett 2005;94:234502. https://doi.org/10.1103/PhysRevLett.94.234502.
[61] Burnett GD, Chae JJ, Tam WY, de Almeida RMC, Tabor M. Structure and dynamics of breaking foams. Phys Rev E 1995;51:5788–96. https://doi.org/10.1103/PhysRevE.51.5788.
[62] Müller W, Di Meglio J-M. Avalanches in draining foams. J Phys Condens Matter 1999;11:L209–15. https://doi.org/10.1088/0953-8984/11/21/101.
[63] Vandewalle N, Lentz JF, Dorbolo S, Brisbois F. Avalanches of popping bubbles in collapsing foams. Phys Rev Lett 2001;86:179–82. https://doi.org/10.1103/PhysRevLett.86.179.
[64] Vandewalle N, Lentz JF. Cascades of popping bubbles along air/foam interfaces. Phys Rev E 2001;64:021507. https://doi.org/10.1103/PhysRevE.64.021507.
[65] Dominguez C, Fernández Leyes M, Cuenca VE, Ritacco HA. Scaling Laws in the dynamics of collapse of single bubbles and 2D foams. Unpublished 2020.
[66] Hutzler S, Weaire D, Bolton F. The effects of plateau borders in the two-dimensional soap froth iii. Further results. Philos mag B Phys Condens matter. Stat Mech Electron Opt Magn Prop 1995;71:277–89. https://doi.org/10.1080/13642819508239034.
[67] Okuzono T, Kawasaki K. Intermittent flow behavior of random foams: a computer experiment on foam rheology. Phys Rev E 1995;51:1246–53. https://doi.org/10.1103/PhysRevE.51.1246.
[68] Gopal AD, Durian DJ. Nonlinear bubble dynamics in a slowly driven foam. Phys Rev Lett 1995;75:2610–3. https://doi.org/10.1103/PhysRevLett.75.2610.
[69] Durian DJ. Bubble-scale model of foam mechanics: melting, nonlinear behavior, and avalanches. Phys Rev E - Stat Phys Plasmas Fluids Relat Interdiscip Top 1997;55:1739–51. https://doi.org/10.1103/PhysRevE.55.1739.
[70] Durian DJ. Foam mechanics at the bubble scale. Phys Rev Lett 1995;75:4780–3. https://doi.org/10.1103/PhysRevLett.75.4780.
[71] Kawasaki K, Okuzono T. Self-organized critical behavior of two-dimensional foams. Fractals 1996;4:339–48. https://doi.org/10.1142/S0218348X96000455.
[72] Hutzler S, Weaire D, Shah S. Bubble sorting in a foam under forced drainage. Philos Mag Lett 2000;80:41–8. https://doi.org/10.1080/095008300176443.
[73] Dunne FFF, Bolton F, Weaire D, Hutzler S. Statistics and topological changes in 2D foam from the dry to the wet limit. Philos Mag 2017;97:1768–81. https://doi.org/10.1080/14786435.2017.1312585.
[74] Held GA, Solina DH, Solina H, Keane DT, Haag WJ, Horn PM, et al. Experimental study of critical-mass fluctuations in an evolving sandpile. Phys Rev Lett 1990;65:1120–3. https://doi.org/10.1103/PhysRevLett.65.1120.
[75] Rosendahl J, Vekić M, Kelley J. Persistent self-organization of sandpiles. Phys Rev E 1993;47:1401–4. https://doi.org/10.1103/PhysRevE.47.1401.
[76] Jaeger HM, Liu CH, Nagel SR. Relaxation at the angle of repose. Phys Rev Lett 1989;62:40–3. https://doi.org/10.1103/PhysRevLett.62.40.
[77] Burridge R, Knopoff L. Model and theoretical seismicity. Bull Seismol Soc Am 1967;57:341–71.
[78] Ritacco HA. Avalanche of bubble ruptures. YouTube 2020. https://youtu.be/CsFZYscXUrA.
[79] Christensen K, Olami Z. Sandpile models with and without an underlying spatial structure. Phys Rev E 1993;48:3361–72. https://doi.org/10.1103/PhysRevE.48.3361.