Since January 2020 Elsevier has created a COVID-19 resource centre with free information in English and Mandarin on the novel coronavirus COVID-19. The COVID-19 resource centre is hosted on Elsevier Connect, the company's public news and information website.

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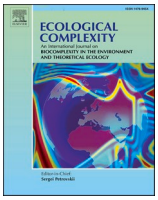

A mathematical model to optimize the available control measures of COVID – 19

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ABSTRACT

In the absence of valid medicine or vaccine for treating the pandemic Coronavirus (COVID-19) infection, other control strategies like; quarantine, social distancing, self- isolation, sanitation and use of personal protective equipment are effective tool used to prevent and curtail the spread of the disease. In this paper, we present a mathematical model to study the dynamics of COVID-19. We then formulate an optimal control problem with the aim to study the most effective control strategies to prevent the proliferation of the disease. The existence of an optimal control function is established and the Pontryagin maximum principle is applied for the characterization of the controller. The equilibrium solutions (DFE & endemic) are found to be locally asymptotically stable and subsequently the basic reproduction number is obtained. Numerical simulations are carried out to support the analytic results and to explicitly show the significance of the control. It is shown that Quarantine/isolating those infected with the disease is the best control measure at the moment.

1. Introduction

The highly contagious corona virus (COVID-19) infection that cause the current global pandemic was first identified in the late December, 2019 in Wuhan city, China (European Center for Disease Prevention and Control (ECDC)). It is significantly less severe than the other two corona viruses; Severe Acute Respiratory Syndrome (SARS-COV) and Middle East Respiratory Syndrome (MERS-COV) that caused an outbreak in 2002 and 2008 respectively (1mg).Although the source of the virus is not yet known, but genetic investigation revealed thatCOVID-19 virus has the same genetic characteristics with SARS-COV which were likely to be originated from bats (European Center for Disease Prevention and Control (ECDC)).

The COVID-19 virus spread from infected human to healthy human through eyes, nose or mouth via a droplet produced of coughing, sneezing or contact with contaminated surfaces, object and equipment of personal use. After the incubation period of 2-14 days (1mg) the infection developed further to cause mild symptoms that include respiratory symptoms, fever, cough, and shortness of breath and breathing difficulties. In more severe cases, infection can cause pneumonia, SARS, kidney failure and even death (World Health Organization (WHO)).

Scientists have not yet developed a vaccine or medicine to cure the COVID-19 infection, but the standard recommendation to prevent the spread of the infection include; quarantine, social distancing, self-isolation, use of personal protective equipment (such as face mask, hand globes, overall gown, e.t.c) regular hand washing using sanitizer, avoiding having contact with person showing the symptoms, reporting any suspected case, and compliance with orientation exercises.

However, many governments engaged in widely public orientation on distancing from public gathering that include social and religious, banning both the local and international trip except for an essential purpose, closing both public and private institutions that may attract large gathering, contact tracing and isolation of infected individuals, providing sanitizers at public domains like markets and car park, fumigating exercise, and to the large extent imposing stay at home curfew.

Since the inception of SIR epidemiological model by Kermack and Merckendric (Kermack and Mckendrick, 1991) in 1927, significant contributions in modeling human diseases and their controls have been offered, as such models of HIV-AIDS (Naik et al., 2020, Naik et al.,

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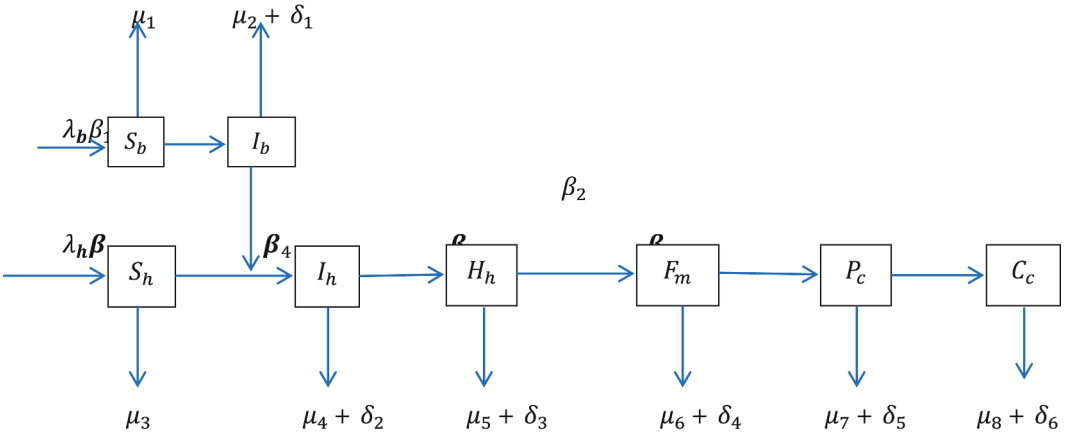

Figure 1. schematic diagram describing the transmission dynamics of COVID-19.

2020), illicit drug use (Berat et al.,), multi-mutation and drug resistance (Owolabi and Shikongo, 2020), e.t.c . In fact, a collection of such related articles can be found in Kumar and Singh (Kumar and Singh, July 10, 2020).

The impact of the ongoing global pandemic of COVID-19 epidemic in terms of morbidity, mortality and socio-economic aspects require urgent and effective control measures. However, to decline the apocalyptic proportion of such predicaments, mathematical model should be applied to determine the transmission dynamics and optimize the possible control measures (Naik et al., 2020, Yavuz and Bonyah, 1 July 2019, Naik et al., 2020, Naik et al., 2020, Yavuz and Ozdemir, 2020).

Optimal control is an effective mathematical tool use to optimize the control problems arising in different field including epidemiology, aeronautic engineering, economics and finance, robotic, e.t.c. (Becerra, 2008).

In an effort to have a quick response about the situations, several contributions have been made as such: Mishra et al (Mishra et al., 2020) who considered asymptomatic and quarantine classes for SARS-COV2.

Sunhwa et al (Sunhwa and Ki, 2020), established deterministic mathematical model (SEIHR) to suit Korean outbreak, in which he estimated the reproduction number and the effect of preventive measures.

S. Zhao et al. (Zhao and Chen, 2020 March 11) developed a Susceptible, Un-quarantined infected, Quarantined infected, Confirmed infected (SUQC) model to characterized the dynamic od COVID-19 and explicitly parameterized the intervention effects of control measures.

In (Yang and Wang, 2020), C. Yang et al., proposed a new model to study the current outbreak COVID-19 in Wuhan, China. M. Tahir et al. (Tahir et al., 2019) developed mathematical model (for MERS) in form of nonlinear system of differential equations, in which he considered a camel to be the source of infection that spread the virus to infective human population, then human to human transmission, then to clinic center then to care center. However, they constructed the Lyapunov candidate function to investigate the local and global stability analysis of the equilibriums solution and subsequently obtained the basic reproduction number or roughly, a key parameter describing transmission the infection.

T. M Chen et al. (Chen et al., 2020) Developed a Bats-Hosts-Reservoir-People (BHRP) transmission network model for the potential transmission from the infection source (probably bats) to the human infection,which focus on calculating R_0.

Q. lin et al. (Lin et al.,) modeled (based on SEIR) the outbreak in Wuhan with individual reaction and governmental action (holiday extension, city lockdown, hospitalization and quarantine) in which they estimated the preliminary magnitude of different effect of individual reaction and governmental action.

M. A. Khan and A. Atangana (Khan and Atangana, February 2020) developed a fractional order model to study the brief interaction among the bats and unknown hosts, and then among people and infection reservoir. Their concern is on the origin of the virus; bats and sea food market.

Manyof the models in literature have a general shortcoming of not taking into consideration the time dependent control strategies. Some of these time dependent control strategies were studied (Yang and Wang, 2020; Tahir et al., 2019; Chen et al., 2020; Lin et al.; World Health Organization (WHO); Baleanu et al., September 2016; Qureshi and Yusuf, 2019; Qureshi and Yusuf, 2019; Qureshi and Yusuf, 2019).This type of control strategy can be used to suggest or design epidemic control programs (Baba et al., 2020; Akman et al., December 2018; Akman et al., December 2018).

To mimic the ongoing global pandemic of COVID-19, we modified the model by M. Tahir et al. in which we incorporated the susceptible human population. We also formulated anoptimal control problem subject to the model with the aim of minimizing the transmission in infected human population I_h, in human to human transmission population H_h, in infected individuals to family members F_m, in patient to clinic center transmission population P_c, and in the patient to care center transmission population C_c.

In the absence of valid medicine or vaccine for treating the pandemic Coronavirus (COVID-19) infection, other control strategies like; quarantine, social distancing, self- isolation, sanitation and use of personal protective equipment are effective tool used to prevent and curtail the spread of the disease. The main contribution of this paper is incorporating these available control measures in the model as function of time and studying the end result if they were to be applied optimally.

The paper is arranged in the following order: chapter one gives an introduction, chapter two deals with the model formulation, chapter three studies the optimal control problem, chapter four discusses local stability analysis and the derivation of the reproduction number and lastly chapter five gives numerical simulation to support the analytic

Table 1
Model variables and their description.

Model Variables	Descriptions
$S_b = S_b(t)$	Susceptible bats population
$I_b = I_b(t)$	Infected bats population
$S_h = S_h(t)$	Susceptible human population
$I_h = I_h(t)$	Infected human population
$H_h = H_h(t)$	Human to human transmission population
$F_m = F_m(t)$	Infected individual to family members
$P_c = P_c(t)$	Patient to clinic center transmission population
$C_c = C_c(t)$	Patient to care center transmission population
$Z = Z(t)$	Total model population

Table 2
Model parameters and their descriptions.

Model parameters	Description
$\mu_i,\ i=1,\ 2,\ldots,\ 8$	Natural death rates in $S_b,\ I_b,\ S_h,\ I_h,\ H_h,\ F_m,\ P_c,$ *and* C_c compartments
δ_1	Disease induced death in I_b
$\delta_i,\ i=2,3,\ldots,5$	Disease induced death in $S_h,\ I_h,\ H_h,\ F_m,\ P_c,$ *and* C_c compartments
λ_b	Birth rates of bats
λ_h	Birth rates of human
$\beta_i,\ i=1,2,\ldots,6$	Transmission rates

result, and then the discussion follows.

2. Model formulation

With bats as the origin of the novel Covid-19 virus, it is assumed that the new born of bats are born into susceptible class S_b, at the rate λ_b, which joined the infectious classI_b at rate β_1.It is also assumed that the new born of humans are born into susceptible class S_h which later became infectious I_h as a result of contact with an infected bats at the rate β_2. Then the virus spreads from an infected human to human H_h, to a family member F_m, then to clinic center P_c and care center C_c at the rates β_3, β_4, β_5 and β_6 respectively. The dynamics of the model is illustrated in figure 1, and the meaning of parameters and variables are given in table 1 and 2 respectively.

The transmission dynamics can be described by the nonlinear system of the first order differential equations;

$$\frac{dS_b(t)}{dt}=\lambda_b-\ \mu_1 S_b-\beta_1 S_b I_b, \tag{1}$$

$$\frac{dI_b(t)}{dt}=\beta_1 S_b I_b-(\mu_2+\delta_1)I_b-\beta_2 S_h I_b, \tag{2}$$

$$\frac{dS_h(t)}{dt}=\lambda_h-\mu_3 S_h-\beta_2 S_h I_b, \tag{3}$$

$$\frac{dI_h(t)}{dt}=\beta_2 S_h I_b-(\mu_4+\delta_2)I_h-\beta_3 I_h H_h, \tag{4}$$

$$\frac{dH_h(t)}{dt}=\beta_3 I_h H_h-(\mu_5+\delta_3)H_h-\beta_4 H_h F_m, \tag{5}$$

$$\frac{dF_m(t)}{dt}=\beta_4 H_h F_m-(\mu_6+\delta_4)F_m-\beta_5 P_c F_m, \tag{6}$$

$$\frac{dP_c(t)}{dt}=\beta_5 P_c F_m-(\mu_7+\delta_5)P_c-\beta_6 P_c C_c, \tag{7}$$

$$\frac{dC_c(t)}{dt}=\beta_6 P_c C_c-(\mu_8+\delta_6)C_c. \tag{8}$$

Where,

$S_b(0)\geq 0,\ I_b(0)\geq 0,\ S_h(0)\geq 0,\ I_h(0)\geq 0,\ H_h(0)\geq 0,\ F_m(0)\geq 0,\ P_c(0)\geq 0,\ C_c(0)\geq 0.$

3. Optimal Control

In this chapter we give detail of the formation of the optimal control problem together with the analysis of the control function.

3.1. Formulation of Optimal Control Problem

The goal of the control strategies is to reduce or minimize the infected human population I_h, the human to human transmission population H_h, the infected individuals to family members F_m, patient to clinic center transmission population P_c, and the patient to care center transmission population C_c.

Let the control functions, $u_1(t)$ be the rate of quarantine of infected individuals which assume to minimize the infected human $I_h, u_2(t)$ be the rate of social distance which assume to minimize human to human transmission population $H_h, u_3(t)$ be the rate of self- isolation which assume to minimize transmission to family member F_m,$u_4(t)$ be the rate of sanitation which assume to minimize patient to clinic center transmission population P_c, and $u_5(t)$ be the rate of use of personal protective equipment which is assumed to minimize patient to care center transmission population C_c.

The dynamics of control system can be described by the following system of nonlinear ODE;

$$\frac{dS_b(t)}{dt}=\lambda_b-\ \mu_1 S_b-\beta_1 S_b I_b, \tag{9}$$

$$\frac{dI_b(t)}{dt}=\beta_1 S_b I_b-(\mu_2+\delta_1)I_b-\beta_2 S_h I_b, \tag{10}$$

$$\frac{dS_h(t)}{dt}=\lambda_h-\mu_3 S_h-\beta_2 S_h I_b, \tag{11}$$

$$\frac{dI_h(t)}{dt}=\beta_2 S_h I_b-(\mu_4+\delta_2+a_1 u_1(t))I_h-\beta_3 I_h H_h, \tag{12}$$

$$\frac{dH_h(t)}{dt}=\beta_3 I_h H_h-(\mu_5+\delta_3+a_2 u_2(t))H_h-\beta_4 H_h F_m, \tag{13}$$

$$\frac{dF_m(t)}{dt}=\beta_4 H_h F_m-(\mu_6+\delta_4+a_3 u_3(t))F_m-\beta_5 P_c F_m, \tag{14}$$

$$\frac{dP_c(t)}{dt}=\beta_5 P_c F_m-(\mu_7+\delta_5+a_4 u_4(t))P_c-\beta_6 P_c C_c, \tag{15}$$

$$\frac{dC_c(t)}{dt}=\beta_6 P_c C_c-(\mu_8+\delta_6+a_5 u_5(t))C_c, \tag{16}$$

$a_i\geq 0,\ i=1,2,\ldots,5$ are constants.

$$I_h(0)=I_{h0}, H_h(0)=H_{h0},\ F_m(0)=F_{m0},\ C_c(0)=C_{c0},\ P_c(0)=P_{c0}. \tag{17}$$

$U_i(t),\ i=1,\ \ldots,\ 5$are defined on the closed interval $[0,t_f]$.The control functions $u_1,\ \ldots,\ u_5$ are bounded and Lebesgue integrable. The coefficients$A_i,\ C_i\ (i=1,\ \ldots,\ 5)$ denote the corresponding weight constants which balance the cost elements on the basis of their size and importance in the parts of the objective functional. Our goal is to reduce the number of infectious population and the cost for implementing strategy. The objective functional is defined as;

$$J(u_1,u_2,u_3,u_4,u_5)=\int_0^{t_f}\left[A_1 I_h+A_2 H_h+A_3 F_m+A_4 P_c+A_5 C_c+C_1 u_1^2(t)+C_2 u_2^2(t)+C_3 u_3^2(t)+C_4 u_4^2(t)+C_5 u_5^2(t)\right]dt. \tag{18}$$

the goal is to find the optimal control $U_i(t),\ i=1,\ \ldots,\ 5$ such that,

$$J(u_1{}^*,u_2{}^*,u_3{}^*,u_4{}^*,u_5{}^*)=\min_{(u_1,\ldots,\ u_5)\in\Gamma}J(u_1,u_2,u_3,u_4,u_5).$$

where the constraint is $\Gamma=\{(u_1,u_2,u_3,u_4,u_5)\in L^1[0,t_f],\ 0\leq u_1,u_2,u_3,u_4,u_5\leq 1\}$. The existence, uniqueness and the characteristics will be discussed in the following subsection.

3.2. Existence of optimal solutions

Theorem 1. There exist optimal controls $(u_1{}^*,u_2{}^*,u_3{}^*,u_4{}^*,u_5{}^*)\in\Gamma$

such that;

$$J(u_1^*, u_2^*, u_3^*, u_4^*, u_5^*) = \min_{(u_1,\ldots,\ u_5)\in\Gamma} J(u_1, u_2, u_3, u_4, u_5).$$

satisfy the control system (9)–(16) with the initial condition (17).

Proof. : This result will be yielded by the general discussion in (Lin et al.) and the following Lemma.

Lemma 1. The control system (9) – (16) with the initial condition (17) satisfy the following conditions

i Γ is non – empty and the interval $[0,\ 1]$ is closed and convex.

ii Let $X = [S_b, I_b, S_h, I_h, H_h, F_m, P_c, C_c]^T$, $U = [u_1, u_2, u_3, u_4, u_5]^T$. Then the right – hand function of the control system (9) – (16) has the form $\psi(X) + \phi(X)U$, where

$$\psi(X) = [],$$

$$\phi(X) = .$$

iii $L(X, U) = A_1 I_h + A_2 H_h + A_3 F_m + A_4 P_c + A_5 C_c + C_1 u_1^2(t) + C_2 u_2^2(t) + C_3 u_3^2(t) + C_4 u_4^2(t) + C_5 u_5^2(t)$

is a convex function of U and satisfies $L(X, U) \geq min\{c_1, c_2, c_3, c_4, c_5\} \| U \|_2^2$. **Proof.** : It is clear that 1 and 2 are trivial and 3 comes from the positivity of the state variables.

3.3. Characterization of optimal control

The control goal is to search the optimal control function $(u_1^*, u_2^*, u_3^*, u_4^*, u_5^*)$ such that,

$$J(u_1^*, u_2^*, u_3^*, u_4^*, u_5^*) = \min_{(u_1,\ldots,\ u_5)\in\Gamma} J(u_1, u_2, u_3, u_4, u_5),$$

reaches its minimum. The necessary condition for the optimal solution is given by Pontryagin's maximum principle, which converts the optimal control (9)–(16) into a problem of minimizing the Hamiltonian function. Let us consider the Hamiltonian function given by;

$$H = A_1 I_h + A_2 H_h + A_3 F_m + A_4 P_c + A_5 C_c + C_1 u_1^2(t) + C_2 u_2^2(t) + C_3 u_3^2(t) + C_4 u_4^2(t) + C_5 u_5^2(t)$$

where $\lambda = (\lambda_1, \ldots, \lambda_{13})^T$ is the adjoint variable and $w = (w_1, \ldots, w_5)^T$ is the relaxing variable. By Pontryagin's maximum principle in (Becerra, 2008) and the existence result for the optimal control, the following theorem is formulated.

Theorem 2. There exists an optimal control $(u_1^*, u_2^*, u_3^*, u_4^*, u_5^*)$ and corresponding solutions $S_b, I_b, S_h, I_h, H_h, F_m, P_c, C_c$ which minimizes $J(u_1, u_2, u_3, u_4, u_5)$ *over* Γ. Furthermore, there exist adjoint functions $\lambda_1(t), \ldots, \lambda_{13}(t)$ such that the adjoint equations:

$$\dot{\lambda}_{13} = -\frac{\partial H}{\partial S_b},\ \ldots,\ \dot{\lambda}_{13} = -\frac{\partial H}{\partial w_5},$$

with the transversality conditions;

$$\lambda_i(t_f) = 0,\ i = 1, \ldots, 13,$$

then the characteristic values $u_1^*, u_2^*, u_3^*, u_4^*, u_5^*$ are given by;

$$u_1^*(t) = \min\left\{b,\ \max\left(0,\ \frac{\lambda_4}{A_6} I_h\right)\right\}, \tag{19}$$

$$u_2^*(t) = \min\left\{b,\ \max\left(0,\ \frac{\lambda_5}{A_7} H_h\right)\right\}, \tag{20}$$

$$u_3^*(t) = \min\left\{b,\ \max\left(0,\ \frac{\lambda_6}{A_8} F_m\right)\right\}, \tag{21}$$

$$u_4^*(t) = \min\left\{b,\ \max\left(0,\ \frac{\lambda_7}{A_9} P_c\right)\right\}, \tag{22}$$

$$u_5^*(t) = \min\left\{b,\ \max\left(0,\ \frac{\lambda_8}{A_{10}} C_c\right)\right\}. \tag{23}$$

Proof. : In view of Pontryagin's maximum principle, we only need to consider the extremum conditions $u_i \frac{\partial y}{\partial u_i} = 0\ i = 1, 2, 3, 4, 5$ which implies that,

$$u_1 \frac{\partial H}{\partial u_1} = (A_6 u_1 - I_h \lambda_4) u_1 = 0, \tag{24}$$

$$u_2 \frac{\partial H}{\partial u_2} = (A_7 u_2 - H_h \lambda_5) u_2 = 0, \tag{25}$$

$$u_3 \frac{\partial H}{\partial u_3} = (A_8 u_3 - F_m \lambda_6) u_3 = 0, \tag{26}$$

$$u_4 \frac{\partial H}{\partial u_4} = (A_9 u_4 - P_c \lambda_7) u_4 = 0, \tag{27}$$

$$u_5 \frac{\partial H}{\partial u_5} = (A_{10} u_5 - C_c \lambda_8) u_5 = 0, \tag{28}$$

namely;

$$u_1^*(t) = \min\left\{b,\ \max\left(0,\ \frac{\lambda_4}{A_6} I_h\right)\right\}, \tag{29}$$

$$u_2^*(t) = \min\left\{b,\ \max\left(0,\ \frac{\lambda_5}{A_7} H_h\right)\right\}, \tag{30}$$

$$u_3^*(t) = \min\left\{b,\ \max\left(0,\ \frac{\lambda_6}{A_8} F_m\right)\right\}, \tag{31}$$

$$u_4^*(t) = \min\left\{b,\ \max\left(0,\ \frac{\lambda_7}{A_9} P_c\right)\right\}, \tag{32}$$

$$u_5^*(t) = \min\left\{b,\ \max\left(0,\ \frac{\lambda_8}{A_{10}} C_c\right)\right\}. \tag{33}$$

Through the range of $(u_1^*, u_2^*, u_3^*, u_4^*, u_5^*)$we can obtain the properties of$(u_1^*, u_2^*, u_3^*, u_4^*, u_5^*)$. Furthermore, the second derivative of the optimal control $u_i (i = 1, \ldots,\ 5)$ is positive, which means that the optimal problem arrives minimum at controls. Then combined with the adjoint equations, the state equations, the initial and transversality conditions, the optimal system can be formulated as;

$$\frac{dS_b(t)}{dt} = \lambda_b - \ \mu_1 S_b - \beta_1 S_b I_b, \tag{34}$$

$$\frac{dI_b(t)}{dt} = \beta_1 S_b I_b - (\mu_2 + \delta_1) I_b - \beta_2 S_h I_b, \tag{35}$$

$$\frac{dS_h(t)}{dt} = \lambda_h - \mu_3 S_h - \beta_2 S_h I_b, \tag{36}$$

$$\frac{dI_h(t)}{dt} = \beta_2 S_h I_b - \left(\mu_4 + \delta_2 + min\left\{b,\ \max\left(0,\ \frac{\lambda_4}{A_6} I_h\right)\right\}\right) I_h - \beta_3 I_h H_h, \tag{37}$$

$$\frac{dH_h(t)}{dt} = \beta_3 I_h H_h - \left(\mu_5 + \delta_3 + min\left\{b,\ \max\left(0,\ \frac{\lambda_5}{A_7} H_h\right)\right\}\right) H_h - \beta_4 H_h F_m, \tag{38}$$

$$\frac{dF_m(t)}{dt} = \beta_4 H_h F_m - \left(\mu_6 + \delta_4 + min\left\{b,\ \max\left(0,\ \frac{\lambda_6}{A_8}F_m\right)\right\}\right)F_m - \beta_5 P_c F_m, \tag{39}$$

$$\frac{dP_c(t)}{dt} = \beta_5 P_c F_m - \left(\mu_7 + \delta_5 + min\left\{b,\ \max\left(0,\ \frac{\lambda_7}{A_9}P_c\right)\right\}\right)P_c - \beta_6 P_c C_c, \tag{40}$$

$$\frac{dC_c(t)}{dt} = \beta_6 P_c F_m - \left(\mu_8 + \delta_6 + min\left\{b,\ \max\left(0,\ \frac{\lambda_8}{A_{10}}C_c\right)\right\}\right)C_c. \tag{41}$$

4. Stability Analysis

In this chapter we study boundedness of the solutions, obtain disease free and endemic equilibria, calculate the basic reproduction ratio and carry out local and global stability analysis of the equilibria.

4.1. Boundedness

Theorem 3. All positive solutions of the system have the ultimate upper bound in $\mathbb{R}^8$

Proof. : Let $N = S_b + I_b + S_h + I_h + H_h + F_m + P_c + C_c$.

Then,

$$\frac{dN}{dt} = \frac{dS_b}{dt} + \frac{dI_b}{dt} + \frac{dS_h}{dt} + \frac{dI_h}{dt} + \frac{dH_h}{dt} + \frac{dF_m}{dt} + \frac{dP_c}{dt} + \frac{dC_c}{dt},$$

$$= \lambda_b - \mu_1 S_b - \beta_1 S_b I_b + \beta_1 S_b I_b - (\mu_2 + \delta_1) I_b - \beta_2 S_h I_b + \lambda_h - \mu_3 S_h - \beta_2 S_h I_b + \beta_2 S_h I_b - (\mu_4 + \delta_2 + a_1 u_1(t)) I_h - \beta_3 I_h H_h + \beta_3 I_h H_h$$
$$-(\mu_5 + \delta_3 + a_2 u_2(t)) H_h - \beta_4 H_h F_m + \beta_4 H_h F_m - (\mu_6 + \delta_4 + a_3 u_3(t)) F_m - \beta_5 P_c F_m + \beta_5 P_c F_m - (\mu_7 + \delta_5 + a_4 u_4(t)) P_c -$$
$$\beta_6 P_c C_c + \beta_6 P_c C_c - (\mu_8 + \delta_6 + a_5 u_5(t)) C_c$$

$$\leq \lambda_b + \lambda_h - \mu_1 S_b - \mu_2 I_b - \mu_3 S_h - \mu_4 I_h - \mu_5 H_h - \mu_6 F_m - \mu_7 P_c - \mu_8 C_c.$$

Let $\mu := \min\{\mu_i\},\ i = 1, \ldots, 8$

Then,

$$\frac{dN}{dt} \leq \lambda_b + \lambda_h - \mu N,$$

which implies,

$$N \leq \frac{\lambda_b + \lambda_h}{\mu} + ce^{-\mu t},$$

and,

$$\lim_{t\to\infty} N(t) \leq \frac{\lambda_b + \lambda_h}{\mu}.$$

So, the biologically feasible region of the model is;

$$'\Omega = \left\{(S_b, I_b, S_h, I_h, H_h, F_m, P_c, C_c) \in R_+^8 : S_b + I_b + S_h + I_h + H_h + F_m + P_c + C_c \leq \frac{\lambda_b + \lambda_h}{\mu}\right\}.$$

4.2. Equilibria

Disease free equilibrium $E_0$is obtained by equating I_b, I_h, H_h, F_m, P_c *and* C_c to zero. Hence we get;

$$E_0 = (S_b^0,\ I_b^0,\ S_h^0,\ I_h^0,\ H_h^0,\ F_m^0,\ P_c^0,\ C_c^0) = \left(\frac{\lambda_b}{\mu_1},\ 0, \frac{\lambda_h}{\mu_3},\ 0,\ 0,\ 0,\ 0,\ 0\right).$$

The endemic equilibrium is obtained by taking all the variables to be different from zero, and solving the system simultaneously;

Solving (9)–(16)

$$P_c^* = \frac{(\mu_8 + \delta_6 + u_5)}{\beta_6}, \tag{42}$$

$$H_h^* = \frac{1}{\beta_4}\left[\mu_6 + \delta_4 + \beta_5\frac{(\mu_8 + \delta_6 + u_5)}{\beta_6}\right], \tag{43}$$

$$S^*{}_b = \frac{\lambda_b}{\mu_1 + \beta_1 I_b^*}, \tag{44}$$

$$S^*{}_h = \frac{\lambda_h}{\mu_3 + \beta_2 I_b^*}, \tag{45}$$

$$\beta_1 S^*{}_b - (\mu_2 + \delta_1) - \beta_1 S^*{}_h = 0. \tag{46}$$

Putting (42) *and* (43) *into* (46) yield

$$\beta_1\lambda_b\mu_3 - \beta_2\lambda_h\mu_1 - (\mu_2 + \delta_1)\mu_1\mu_3 - \beta_1\beta_2(\mu_2 + \delta_1) I_b^{*2} + [\beta_1\beta_2(\lambda_b - \lambda_h) + (\mu_2 + \delta_1)(\mu_1\beta_2 + \mu_3\beta_1)] I_b^* = 0,$$

$$\beta_1\lambda_b\mu_3\left(1 - \frac{1}{R_0}\right) + [\beta_1\beta_2(\lambda_b - \lambda_h) + (\mu_2 + \delta_1)(\mu_1\beta_2 + \mu_3\beta_1)] I_b^* - \beta_1\beta_2(\mu_2 + \delta_1) I_b^{*2} = 0,$$

$$I_b^* = \frac{1}{2}\left[\frac{\lambda_b - \lambda_h}{\mu_2 + \delta_1} + \frac{\mu_1}{\beta_1} + \frac{\mu_1}{\beta_2} \pm \sqrt{\left(\frac{\lambda_b - \lambda_h}{\mu_2 + \delta_1} + \frac{\mu_1}{\beta_1} + \frac{\mu_1}{\beta_2}\right)^2 + \frac{4\lambda_b\mu_3}{\beta_2(\mu_2 + \delta_1)}\left(1 - \frac{1}{R_0}\right)}\right], \tag{47}$$

$$I_h^* = \frac{\beta_2\beta_4\beta_6\lambda_h I_b^*}{[\beta_4\beta_6(\mu_6 + \delta_4 + u_3) + \beta_3\beta_6(\mu_6 + \delta_4 + u_3) + \beta_3\beta_5(\mu_8 + \delta_6 + u_5)]\left(\mu_3 + \beta_2 I_b^*\right)}, \tag{48}$$

$$F_m^* = \frac{1}{\beta_4}\left[\beta_3 I_h^* - (\mu_5 + \delta_3 + u_2)\right],$$

$$F_m^* = \frac{\beta_2\beta_3\beta_6\lambda_h I_b^*}{[\beta_4\beta_6(\mu_4 + \delta_2 + u_1) + \beta_3\beta_6(\mu_6 + \delta_4 + u_3) + \beta_3\beta_5(\mu_8 + \delta_6 + u_5)]\left(\mu_3 + \beta_2 I_b^*\right)} - \frac{1}{\beta_4}(\mu_5 + \delta_3 + u_2), \tag{49}$$

$$C_c^* = \frac{1}{\beta_6}\left[\beta_5 F_m^* - (\mu_7 + \delta_5 + u_4)\right],$$

$$C_c^* = \frac{\beta_2\beta_3\beta_5\lambda_h I_b^*}{[\beta_4\beta_6(\mu_4+\delta_2+u_1)+\beta_3\beta_6(\mu_6+\delta_4+u_3)+\beta_3\beta_5(\mu_8+\delta_6+u_5)](\mu_3+\beta_2 I_b^*)} - \frac{\beta_5}{\beta_4\beta_6}(\mu_5+\delta_3+u_2) - \frac{1}{\beta_6}(\mu_7+\delta_5+u_4). \quad (50)$$

4.3. Local Stability of the Equilibria

From equation (9) through (16) we obtained the Jacobian matrix

$$J = \begin{bmatrix} N_1 & -\beta_1 S_b & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 I_b & N_2 & -\beta_2 I_b & 0 & 0 & 0 & 0 & 0 \\ 0 & -\beta_2 S_h & N_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_2 S_h & \beta_2 I_b & N_4 & -\beta_3 I_h & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_3 H_h & N_5 & -\beta_3 H_h & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta_4 F_m & N_6 & -\beta_4 F_m & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_5 P_c & N_7 & -\beta_6 C_c \\ 0 & 0 & 0 & 0 & 0 & 0 & \beta_6 C_c & N_8 \end{bmatrix}$$

$N_1 = -\mu_1 - \beta_1 I_b,\ N_2 = \beta_1 S_b - \mu_2 - \delta_1 - \beta_2 S_h,\ N_3 = -\mu_3 - \beta_2 I_b,\ N_4 = -\mu_4 - \delta_2 - u_1 - \beta_3 H_h,\ N_5 = \beta_3 I_h - \mu_5 - \delta_3 - u_2 - \beta_4 F_m,\ N_6 = \beta_4 H_h - \mu_6 - \delta_4 - u_3 - \beta_5 P_c,\ N_7 = \beta_5 F_m - \mu_7 - \delta_5 - u_4 - \beta_6 C_c,$ *and* $N_8 = \beta_6 P_c - \mu_8 - \delta_6 - u_5.$

where

Theorem 4. The disease free equilibrium E_0 is locally asymptotically stable

Proof

$$J_{E_0} = \begin{bmatrix} & \beta_1\lambda_b/\mu_1 & & 0 & 0 & 0 & 0 & 0 \\ -\mu_1 & N_2^0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & & 0 & & 0 & 0 & 0 & 0 \\ 0 & -\beta_2\lambda_h/\mu_3 & -\mu_3 & 0 & & 0 & 0 & 0 \\ 0 & & 0 & N_4^0 & & 0 & 0 & 0 \\ 0 & \beta_2\lambda_h/\mu_3 & 0 & & N_5^0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & & N_6^0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & & 0 & 0 \\ 0 & & 0 & & 0 & 0 & N_7^0 & 0 \\ & 0 & & 0 & 0 & 0 & 0 & N_8^0 \end{bmatrix},$$

where,

$N_2^0 = \beta_1\lambda_b/\mu_1 - \mu_2 - \delta_1 - \beta_2\lambda_h/\mu_3,\ N_4^0 = -\mu_4 - \delta_2 - u_1,\ N_5^0 = -\mu_5 - \delta_3 - u_2,\ N_6^0 = -\mu_6 - \delta_4 - u_3,\ N_7^0 = -\mu_7 - \delta_5 - u_4,\ N_8^0 = -\mu_8 - \delta_6 - u_5.$

Setting,

$$det\left|J_{E_0} - KI\right| = 0,$$

we get the eigenvalues

$K_1 = -\mu_1,$

$K_2 = \beta_1\frac{\lambda_b}{\mu_1} - \mu_2 - \delta_1 - \beta_2\frac{\lambda_h}{\mu_3},$

$K_3 = -\mu_3,$

$K_4 = -\mu_4 - \delta_2,$

$K_5 = -\mu_5 - \delta_3,$

$K_6 = -\mu_6 - \delta_4,$

$K_7 = -\mu_7 - \delta_5,$

$K_8 = -\mu_8 - \delta_6.$

4.4. Basic Reproduction Number

For the DFE to be locally asymptotically stable, we need the eigenvalue $K_2 < 0$

$$\beta_1\frac{\lambda_b}{\mu_1} - \mu_2 - \delta_1 - \beta_2\frac{\lambda_h}{\mu_3} < 0.$$

Simplifying, we get

$$\frac{\beta_1\lambda_b\mu_3}{\mu_1(\mu_3(\mu_2+\delta_1)+\beta_2\lambda_h)} < 0.$$

We let basic reproduction ratio (R_0) to be $R_0 < 1$.

$$R_0 = \frac{\beta_1\lambda_b\mu_3}{\mu_1(\mu_3(\mu_2+\delta_1)+\beta_2\lambda_h)}.$$

Theorem 5. The endemic equilibrium (S_b^*, I_b^*, S_h^*, I_h^*, H_h^*, F_m^*, P_c^*, C_c^*) is stable if $R_0 > 1$.

Proof

Since all the equilibrium points in the endemic equilibrium depend on I_b^* then it suffices to investigate the stability or otherwise of I_b^*

Since $I_b^* > 0$, then from (46)

Either

$$\frac{\lambda_b-\lambda_h}{\mu_2+\delta_1}+\frac{\mu_1}{\beta_1}+\frac{\mu_1}{\beta_2} > \sqrt{\left(\frac{\lambda_b-\lambda_h}{\mu_2+\delta_1}+\frac{\mu_1}{\beta_1}+\frac{\mu_1}{\beta_2}\right)^2+\frac{4\lambda_b\mu_3}{\beta_2(\mu_2+\delta_1)}\left(1-\frac{1}{R_0}\right)}$$

$$0 > \frac{4\lambda_b\mu_3}{\beta_2(\mu_2+\delta_1)}\left(1-\frac{1}{R_0}\right)$$

$$0 > \left(1-\frac{1}{R_0}\right)$$

$R_0 < 1.$

This may lead to a chance of I_b^* to be complex.

Or

$$\sqrt{\left(\frac{\lambda_b-\lambda_h}{\mu_2+\delta_1}+\frac{\mu_1}{\beta_1}+\frac{\mu_1}{\beta_2}\right)^2+\frac{4\lambda_b\mu_3}{\beta_2(\mu_2+\delta_1)}\left(1-\frac{1}{R_0}\right)} > -\left(\frac{\lambda_b-\lambda_h}{\mu_2+\delta_1}+\frac{\mu_1}{\beta_1}+\frac{\mu_1}{\beta_2}\right)$$

$$\frac{4\lambda_b\mu_3}{\beta_2(\mu_2+\delta_1)}\left(1-\frac{1}{R_0}\right) > 0$$

$$\left(1-\frac{1}{R_0}\right) > 0$$

$R_0 > 1.$

Thus, only

$$I_b^* = \frac{1}{2}\left[\frac{\lambda_b-\lambda_h}{\mu_2+\delta_1}+\frac{\mu_1}{\beta_1}+\frac{\mu_1}{\beta_2}+\sqrt{\left(\frac{\lambda_b-\lambda_h}{\mu_2+\delta_1}+\frac{\mu_1}{\beta_1}+\frac{\mu_1}{\beta_2}\right)^2+\frac{4\lambda_b\mu_3}{\beta_2(\mu_2+\delta_1)}\left(1-\frac{1}{R_0}\right)}\right]. \quad (51)$$

4.5. Global Stability Analysis

Here we study global stability analysis of the equilibria.

Theorem6. : The disease free equilibrium is globally asymptotically stableif $Q > (\lambda_b + \lambda_h)$.

Proof

Let the Lyapunov candidate function be

$$G(S_b,\ I_b,\ S_h,\ I_h,\ H_h,\ F_m,\ P_c,\ C_c) = \frac{1}{2}[(S_b - S_b^*) + I_b + (S_h - S_h^*) + I_h + H_h + F_m + P_c + C_c]^2.$$

e:inline-figure))"^?^(1)[?tal=1]> Clearly the above function $W(S_b,\ I_b,\ S_h,\ I_h,\ H_h,\ F_m,\ P_c,\ C_c) \geq 0$,

Also, $G(S_b,\ I_b,\ S_h,\ I_h,\ H_h,\ F_m,\ P_c,\ C_c) = 0$ at

$$\left(S_b^0,\ I_b^0,\ S_h^0,\ I_h^0,\ H_h^0,\ F_m^0,\ P_c^0,\ C_c^0\right) = \left(\frac{\lambda_b}{\mu_1},\ 0, \frac{\lambda_h}{\mu_3},\ 0,\ 0,\ 0,\ 0,\ 0\right),$$

$$\frac{dG}{dt} = [(S_b - S_b^*) + I_b + (S_h - S_h^*) + I_h + H_h + F_m + P_c + C_c]\left[\frac{dS_b}{dt}+\frac{dI_b}{dt}+\frac{dS_h}{dt}+\frac{dI_h}{dt}+\frac{dH_h}{dt}+\frac{dF_m}{dt}+\frac{dP_c}{dt}+\frac{dC_c}{dt}\right]$$

$$\frac{dG}{dt} = [(S_b - S_b^*) + I_b + (S_h - S_h^*) + I_h + H_h + F_m + P_c + C_c]$$
$$[\lambda_b + \lambda_h - \{\mu_1 S_b + (\mu_2+\delta_1)I_b + \beta_2 S_h I_b + \mu_3 S_h + (\mu_4+\delta_2+u_1)I_h$$
$$+(\mu_5+\delta_3+u_2)H_h + (\mu_6+\delta_4+u_3)F_m \pm (\mu_7+\delta_5+u_4)P_c$$
$$+(\mu_8+\delta_6+u_5)C_c\}],$$

$$\frac{dG}{dt} = -[(S_b - S_b^*) + I_b + (S_h - S_h^*) + I_h + H_h + F_m + P_c + C_c][\ Q - (\lambda_b + \lambda_h)].$$

Where,

$$Q = \mu_1 S_b + (\mu_2+\delta_1)I_b + \beta_2 S_h I_b + \mu_3 S_h + (\mu_4+\delta_2+u_1)I_h + (\mu_5+\delta_3+u_2)H_h + (\mu_6+\delta_4+u_3)F_m + (\mu_7+\delta_5+u_4)P_c + (\mu_8+\delta_6+u_5)C_c)\}$$

Clearly,

$$\frac{dG}{dt} = 0,\ if\ \left(S_b^0,\ I_b^0,\ S_h^0,\ I_h^0,\ H_h^0,\ F_m^0,\ P_c^0,\ C_c^0\right) = \left(\frac{\lambda_b}{\mu_1},\ 0, \frac{\lambda_h}{\mu_3},\ 0,\ 0,\ 0,\ 0,\ 0\right).$$

$$\frac{dG}{dt}\langle 0,\ if\ Q\rangle(\lambda_b + \lambda_h).$$

Theorem7. : The endemic equilibrium is globally asymptotically stableif $W > \lambda_b + \lambda_h$.

Proof

Let the Lyapunov candidate function be

Table 3
Variable values.

Variable Values
$S_b = 00 - 600$
$I_b = 200 - 500$
$S_h(0) = 10{,}000{,}000$
$I_h = 240 - 440$
$H_h = 100 - 400$
$F_m = 40 - 200$
$P_c = 00 - 300$
$C_c = 00 - 300$

Table 4
Parameter values.

Parameter value
$\beta_1 = 1.2300$
$\beta_2 = 0.1000$
$\beta_3 = 0.0060$
$\beta_4 = 1.0090$
$\beta_5 = 0.0040$
$\beta_6 = 0.0900$
$\lambda_b = 1.5000$
$\lambda_h = 1.25$
$\mu_1 = 1.7000$
$\mu_2 = 0.1340$
$\mu_3 = 0.5$
$\mu_4 = 0.1343$
$\mu_5 = 0.0024$
$\mu_6 = 0.0074$
$\mu_7 = 0.3440$
$\mu_8 = 0.5410$
$\delta_1 = 0.0143$
$\delta_2 = 0.3002$
$\delta_3 = 0.0054$
$\delta_4 = 0.0019$
$\delta_5 = 0.0640$
$\delta_6 = 0.4400$

$$V(S_b,\ I_b,\ S_h,\ I_h,\ H_h,\ F_m,\ P_c,\ C_c) = \frac{1}{2}[(S_b - S_b^*) + (I_b - I_b^*) + (S_h - S_h^*) + (I_h - I_h^*) + (H_h - H_h^*) + (F_m - F_m^*) + (P_c - P_c^*) + (C_c - C_c^*)]^2.$$

Clearly the above function $V(S_b,\ I_b,\ S_h,\ I_h,\ H_h,\ F_m,\ P_c,\ C_c) \geq 0$,

Also,$V(S_b,\ I_b,\ S_h,\ I_h,\ H_h,\ F_m,\ P_c,\ C_c) = 0$ if $S_b = S_b^*,\ I_b = I_b^*,\ S_h = S_h^*,\ I_h = I_h^*,\ H_h = H_h^*,\ F_m = F_m^*,\ P_c = P_c^*,\ C_c = C_c^*$.

The remaining result is,

$$\frac{dV}{dt} = [(S_b - S_b^*) + (I_b - I_b^*) + (S_h - S_h^*) + (I_h - I_h^*) + (H_h - H_h^*) + (F_m - F_m^*) + (P_c - P_c^*) + (C_c - C_c^*)]\left[\frac{dS_b}{dt}+\frac{dI_b}{dt}+\frac{dS_h}{dt}+\frac{dI_h}{dt}+\frac{dH_h}{dt}+\frac{dF_m}{dt}+\frac{dP_c}{dt}+\frac{dC_c}{dt}\right].$$

Substituting equation (9) – (16) in above we get

$$\frac{dV}{dt} = [(S_b - S_b^*) + (I_b - I_b^*) + (S_h - S_h^*) + (I_h - I_h^*) + (H_h - H_h^*) + (F_m - F_m^*) + (P_c - P_c^*) + (C_c - C_c^*)]$$
$$[\lambda_b + \lambda_h - \{\mu_1 S_b + (\mu_2+\delta_1)I_b + \beta_2 S_h I_b + \mu_3 S_h + (\mu_4+\delta_2+u_1)I_h + (\mu_5+\delta_3+u_2)H_h + (\mu_6+\delta_4+u_3)F_m + -(\mu_7+\delta_5+u_4)P_c + (\mu_8+\delta_6+u_5)C_c)\}],$$

$$\frac{dV}{dt} = -[(S_b - S_b^*) + (I_b - I_b^*) + (S_h - S_h^*) + (I_h - I_h^*) + (H_h - H_h^*) + (F_m - F_m^*) + (P_c - P_c^*) + (C_c - C_c^*)][W - (\lambda_b + \lambda_h)].$$

Where,

$$W = \mu_1 S_b + (\mu_2 + \delta_1) I_b + \beta_2 S_h I_b + \mu_3 S_h + (\mu_4 + \delta_2 + u_1) I_h + (\mu_5 + \delta_3 + u_2) H_h + (\mu_6 + \delta_4 + u_3) F_m + (\mu_7 + \delta_5 + u_4) P_c + (\mu_8 + \delta_6 + u_5 C_c)\}$$

Clearly

$$\frac{dV}{dt} = 0, \ if \ S_b = S_b^*, \ I_b = I_b^*, \ S_h = S_h^*, \ I_h = I_h^*, \ H_h = H_h^*, \ F_m = F_m^*, \ P_c = P_c^*, \ C_c = C_c^*$$

$$\frac{dV}{dt} \langle 0, \ if \ W \rangle \lambda_b + \lambda_h.$$

Hence the endemic equilibrium is globally asymptotically stable.

5. Numerical Simulation and Discussions

In this chapter, numerical simulations are carried out. Variable and parameter values in table 3 and 4 were adopted from (Tahir et al., 2019).

The following can be observed from the graphs; Figure 2, when no any control measure is taken, it can clearly be seen that there will be epidemics. Figure 3, shows the dynamics of the optimal control functions. Figure 4, shows how effective these control measures can be when applied optimally. That is when infected individuals enter quarantine, people applied social distancing, family of infected individuals undergo self-isolation, sanitation is adopted for medical center individuals and use of personal protective equipment for care center individuals. Although all these control measures can't be applied at a time in some settings, Figure 5 shows that if all the measures will be taken except quarantining the infected individuals, there will still be epidemics. Lastly, from Figure 6, we can understand how important quarantine is, that if it can be applied optimally with time the epidemics will be

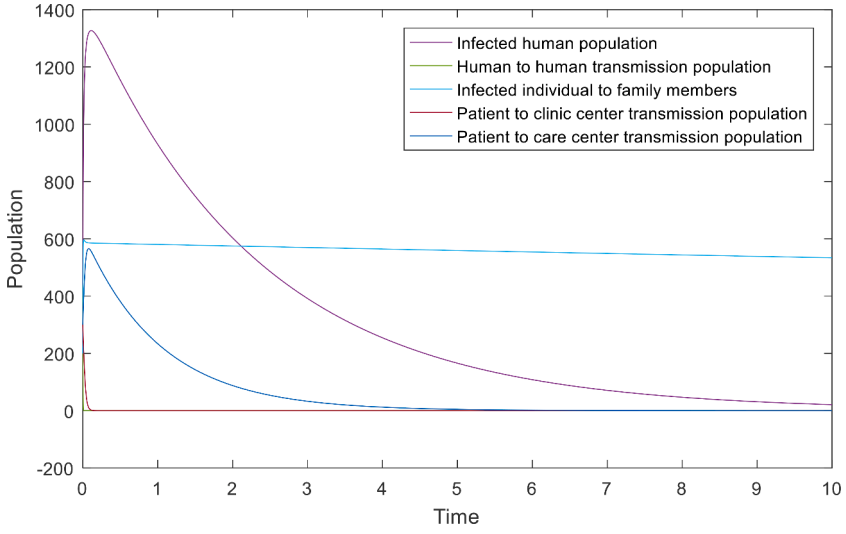

Figure 2. No control is applied to any of the population.

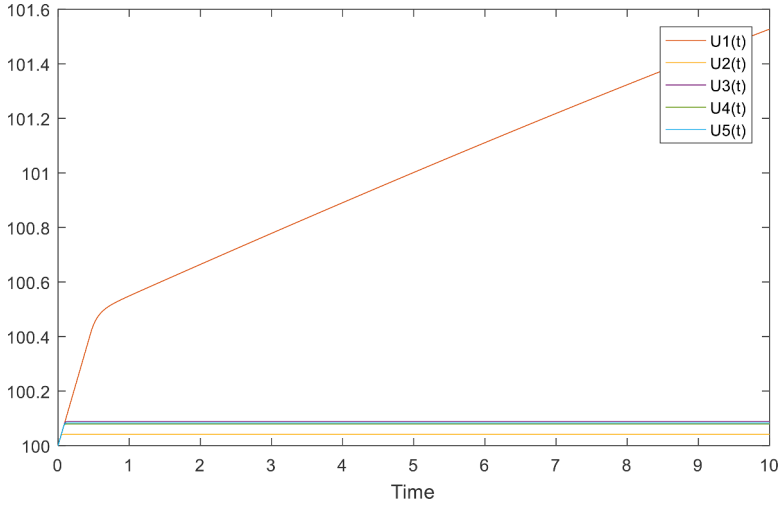

Figure 3. Dynamics of the optimal control functions.

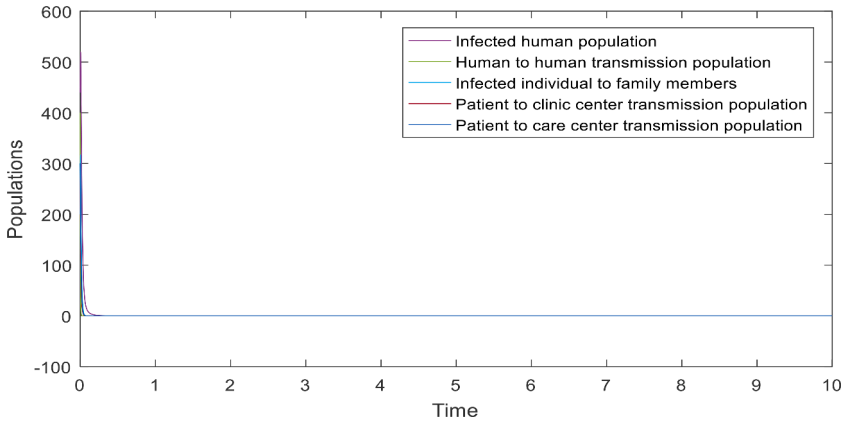

Figure 4. Dynamics of the populations when control functions were applied optimally.

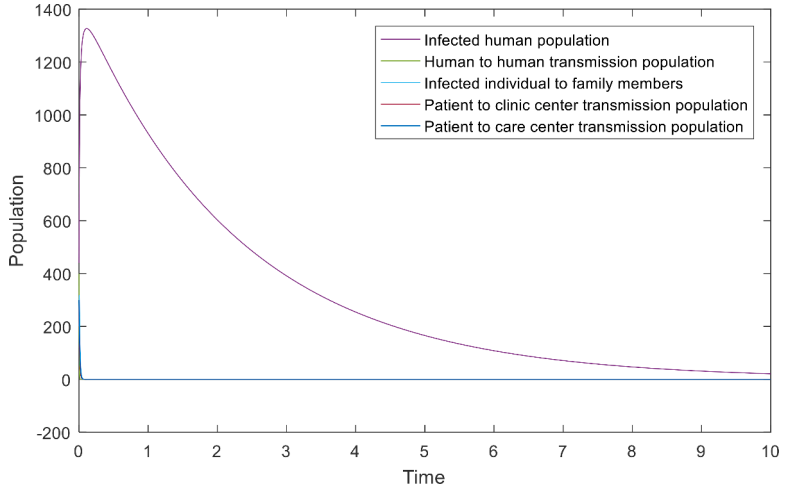

Figure 5. Dynamics of the populations when optimal control is applied to all the populations except the population of the infected individuals (no quarantine/isolation).

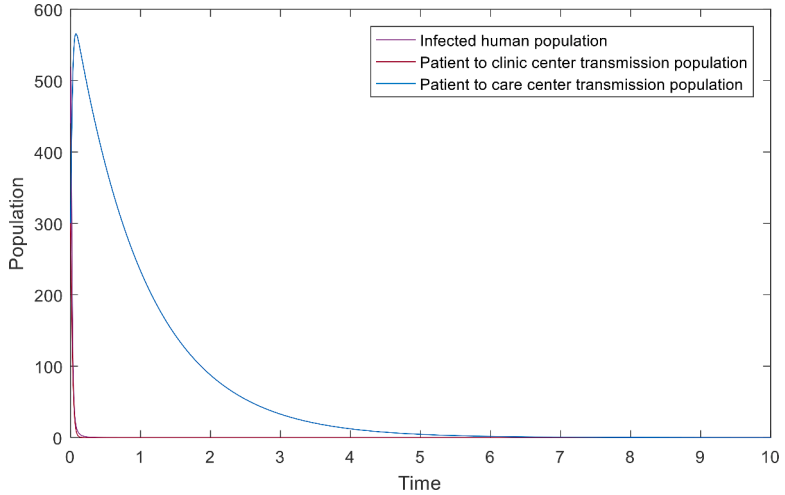

Figure 6. Dynamics of the populations when quarantine is the only control measure adopted.

eliminated, even if all the remaining control measures were not applied.

6. Conclusion

In conclusion, this paper studies a model that consists of a system of eight non – linear ordinary differential equations. The model studies the dynamics of COVID-19. An optimal control problem was constructed with the aim to study the most effective control strategies to prevent the proliferation of the disease. The existence of an optimal control function was established and the Pontryagin maximum principle was applied for

the characterization of the controller. Two equilibrium solutions; disease free equilibrium (DFE) and endemic equilibrium (EE) were found. Local stability analysis was carried out, and it was established that both DFE and EE depends on the magnitude of a threshold quantity, basic reproduction ratio (R_0). DFE is locally asymptotically stable when $R_0 < 1$, whereas EE is locally asymptotically stable when $R_0 > 1$.

Numerical simulations were carried out to support the analytic results and to explicitly show the significance of the control measures. It was obtained that $R_0 = 2.73$ when no control measures taken. But when infected individuals enter quarantine, people applied social distancing, family of infected individuals undergo self-isolation, sanitation adopted for medical center individuals and personal protective equipment for care center individuals were used, then the pandemic will automatically be controlled. The significance of quarantine over the remaining control measures was also shown.

Author statement

This research is very important as per as the current pandemic is concerned. People are dying due to the pandemic. Policy makers need to design policies that will protect the citizens. We think this paper will give an insight into the various available measures that can be used to control the spread of the disease, and the significance of suchmeasures.

Availability of data and material

We have no available data.

Funding

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Authors Contribution

All authors of this research paper have directly participated in the planning, execution, or analysis of this study.

Declaration of Competing Interest

We have no complicit of interest.

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