Article

MMS

Mathematics and Mechanics of Solids
2024, Vol. 29(3) 531–538

DOI: 10.1177/10812865231202445
journals.sagepub.com/home/mms
Sage

Interaction between an edge dislocation and a circular incompressible liquid inclusion

Xu Wang
School of Mechanical and Power Engineering, East China University of Science and Technology, Shanghai, China

Peter Schiavone
Department of Mechanical Engineering, University of Alberta, Edmonton, AB, Canada

Received 30 August 2023; accepted 3 September 2023

Abstract
We use Muskhelishvili's complex variable formulation to study the interaction problem associated with a circular incompressible liquid inclusion embedded in an infinite isotropic elastic matrix subjected to the action of an edge dislocation at an arbitrary position. A closed-form solution to the problem is derived largely with the aid of analytic continuation. We obtain, in explicit form, expressions for the internal uniform hydrostatic stresses, nonuniform strains and nonuniform rigid body rotation within the liquid inclusion; the hoop stress along the liquid-solid interface on the matrix side and the image force acting on the edge dislocation. We observe that (1) the internal strains and rigid body rotation within the liquid inclusion are independent of the elastic property of the matrix; (2) the internal hydrostatic stress field within the liquid inclusion is unaffected by Poisson's ratio of the matrix and is proportional to the shear modulus of the matrix; and (3) an unstable equilibrium position always exists for a climbing dislocation.

Keywords
Incompressible liquid inclusion, edge dislocation, closed-form solution, analytic continuation, image force on dislocation, Peach–Koehler formula

1. Introduction

A composite consisting of a solid matrix and liquid inclusions (e.g., ionic liquids, liquid metals and ferrofluids) exhibits unique mechanical and physical properties such as enhancement of overall deformability [1] and a stiffening phenomenon when the liquid inclusions are "small" allowing for a significant surface

Corresponding authors:
Xu Wang, School of Mechanical and Power Engineering, East China University of Science and Technology, 130 Meilong Road, Shanghai 200237, China.
Email: xuwang@ecust.edu.cn

Peter Schiavone, Department of Mechanical Engineering, University of Alberta, 10-203 Donadeo Innovation Centre for Engineering, Edmonton, AB T6G 1H9, Canada.
Email: p.schiavone@ualberta.ca

effect [2–4]. The micromechanics analysis of liquid inclusions has attracted considerable attention from researchers recently. The objective is always to predict the local microscopic and overall macroscopic responses of composites containing liquid inclusions [2–11]. Although the study of dislocations interacting with elastic inclusions (or elastic inhomogeneities) has a long history [12–15], the problem of dislocations interacting with liquid inclusions has seldom been considered.

In this paper, using Muskhelishvili's [16] complex variable formulation for plane elasticity, we solve the interaction problem associated with a circular incompressible liquid inclusion in an infinite isotropic elastic matrix under the action of an edge dislocation located at an arbitrary position. We derive a closed-form solution to the problem with the aid of analytic continuation [17]. We obtain two pairs of analytic functions: one pair defined in the incompressible liquid inclusion and the other in the matrix. Elementary and explicit expressions for the internal uniform hydrostatic tension, nonuniform strains and nonuniform rigid body rotation within the liquid inclusion and the hoop stress along the liquid-solid interface on the matrix side are presented. We find that the internal strains and rigid body rotation within the liquid inclusion are independent of the elastic property of the matrix. In addition, we show that the internal uniform hydrostatic tension is proportional to the shear modulus of the matrix and is unaffected by Poisson's ratio of the matrix. The image force acting on the edge dislocation is obtained using the Peach–Koehler formula [12]. A gliding dislocation is always attracted to the circular interface, whereas an unstable equilibrium position emerges for a climbing dislocation. The existence of the equilibrium position for the climbing dislocation is attributed to the contribution from the internal uniform hydrostatic tension within the liquid inclusion. The acquired analytical solution for an edge dislocation can be further employed to study the interaction of a mode I or mode II finite crack interacting with a circular incompressible liquid inclusion under uniform remote in-plane stresses.

2. Complex variable formulation

We first establish a fixed rectangular coordinate system $\{x_i\}$ $(i=1,2,3)$. For plane strain deformations of an isotropic elastic material, the three in-plane stresses (σ_{11}, σ_{22} and σ_{12}), two in-plane displacements (u_1 and u_2), and two stress functions (φ_1 and φ_2) are given in terms of two analytic functions $\phi(z)$ and $\psi(z)$ of the complex variable $z=x_1+\mathrm{i}x_2=re^{\mathrm{i}\theta}$ (in which (r,θ) are polar coordinates) as [16]

$$\begin{aligned}\sigma_{11}+\sigma_{22}&=2\mu\left[\phi'(z)+\overline{\phi'(z)}\right],\\ \sigma_{22}-\sigma_{11}+2\mathrm{i}\sigma_{12}&=2\mu[\bar{z}\phi''(z)+\psi'(z)],\end{aligned}\tag{1}$$

and

$$\begin{aligned}2(u_1+\mathrm{i}u_2)&=\kappa\phi(z)-z\overline{\phi'(z)}-\overline{\psi(z)},\\ \varphi_1+\mathrm{i}\varphi_2&=\mathrm{i}\mu\left[\phi(z)+z\overline{\phi'(z)}+\overline{\psi(z)}\right],\end{aligned}\tag{2}$$

where $\kappa=3-4\nu$, μ and $\nu(0\leqslant\nu\leqslant 1/2)$ are the shear modulus and Poisson's ratio, respectively. In addition, the stresses are related to the two stress functions through [18]

$$\begin{aligned}\sigma_{11}&=-\varphi_{1,2},\quad \sigma_{12}=\varphi_{1,1},\\ \sigma_{21}&=-\varphi_{2,2},\quad \sigma_{22}=\varphi_{2,1}.\end{aligned}\tag{3}$$

Furthermore, the in-plane strains $(\varepsilon_{11},\varepsilon_{22},\varepsilon_{12})$ and the rigid body rotation $\varpi_{21}=\frac{1}{2}(u_{2,1}-u_{1,2})$ can also be expressed in terms of the two analytic functions $\phi(z)$ and $\psi(z)$ as

$$\begin{aligned}\varepsilon_{11}+\varepsilon_{22}+2\mathrm{i}\varpi_{21}&=\kappa\phi'(z)-\overline{\phi'(z)},\\ \varepsilon_{22}-\varepsilon_{11}+2\mathrm{i}\varepsilon_{12}&=\bar{z}\phi''(z)+\psi'(z).\end{aligned}\tag{4}$$

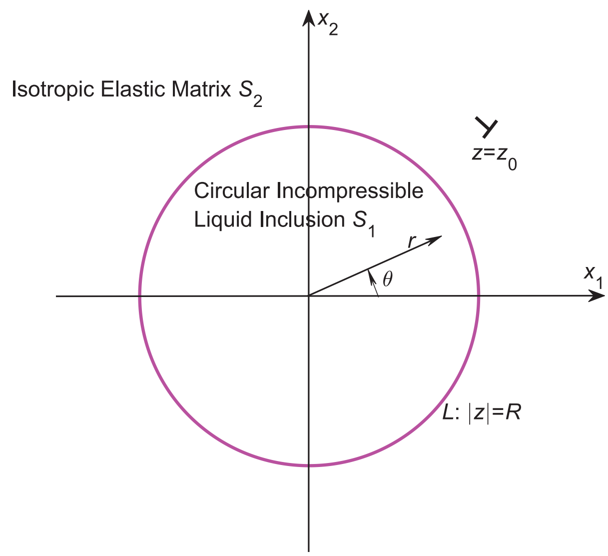

Figure 1. A circular incompressible liquid inclusion embedded in an infinite isotropic elastic matrix under the action of an edge dislocation located at $z = z_0 = \xi + i\eta$.

3. An edge dislocation interacting with a circular incompressible liquid inclusion

As shown in Figure 1, we consider a circular incompressible liquid inclusion of radius R, centered at the origin of the coordinate system, embedded in an infinite isotropic elastic matrix subjected to the action of an edge dislocation with Burgers vector $(b_1,\ b_2)$ located at an arbitrary position $z = z_0 = \xi + i\eta$. Let S_1 and S_2 denote the liquid inclusion and the matrix, respectively, which are perfectly bonded together across the circular interface $L : \{|z| = R\}$. In what follows, the subscripts 1 and 2 are used to identify the respective quantities in S_1 and S_2.

The most general form of the displacements and stress functions within the circular incompressible liquid inclusion is given by

$$\begin{aligned} &u_1 + iu_2 = \tfrac{1}{2}\left[\phi_1(z) - z\overline{\phi'_1(z)} - \overline{\psi_1(z)}\right], \\ &\varphi_1 + i\varphi_2 = i\sigma_0 z, \quad z \in S_1, \end{aligned} \tag{5}$$

where σ_0 is the internal uniform hydrostatic tension (to be determined). The condition of incompressibility of the liquid inclusion has been satisfied using the displacement expression in equation $(5)_1$ in view of equation $(4)_1$ with $\kappa_1 = 1$.

In the ensuing analysis, for convenience, we introduce the following analytic continuations [17]:

$$\phi_i(z) = -z\bar{\phi}'_i\left(\frac{R^2}{z}\right) - \bar{\psi}_i\left(\frac{R^2}{z}\right), \ i = 1,\ 2. \tag{6}$$

Thus, the continuity conditions of displacements and tractions across the perfect liquid–solid interface $|z| = R$ can be expressed concisely as

$$\begin{aligned} &\phi_1^+(z) + \phi_1^-(z) = \kappa_2\phi_2^-(z) + \phi_2^+(z), \\ &\phi_2^-(z) - \phi_2^+(z) = \frac{\sigma_0}{\mu_2}z, \quad |z| = R, \end{aligned} \tag{7}$$

where the superscripts " + " and "−" indicate the values when approaching the circle $|z| = R$ from inside and outside, respectively.

Equation $(7)_1$ can be rewritten as

$$\begin{aligned}&\phi_1^+(z)-\phi_2^+(z)-\overline{\phi'_1(0)}z-\kappa_2 A\ln(z-z_0)-A\ln\frac{z-R^2/\bar{z}_0}{z}+\frac{\bar{A}R^2\left(R^2-|z_0|^2\right)}{\bar{z}_0^3(z-R^2/\bar{z}_0)}\\&=\kappa_2\phi_2^-(z)-\phi_1^-(z)-\overline{\phi'_1(0)}z-\kappa_2 A\ln(z-z_0)-A\ln\frac{z-R^2/\bar{z}_0}{z}+\frac{\bar{A}R^2\left(R^2-|z_0|^2\right)}{\bar{z}_0^3(z-R^2/\bar{z}_0)},\quad |z|=R,\end{aligned}\tag{8}$$

where

$$A=\frac{b_2-\mathrm{i}b_1}{\pi(\kappa_2+1)}.\tag{9}$$

We can see that the left-hand side of equation (8) is analytic and single valued everywhere within the circle, and that the right-hand side of equation (8) is analytic and single valued everywhere outside the circle including the point at infinity. Using Liouville's theorem, we arrive at the following relationships

$$\begin{aligned}&\phi_2(z)=\phi_1(z)-\overline{\phi'_1(0)}z-\kappa_2 A\ln(z-z_0)-A\ln\frac{z-R^2/\bar{z}_0}{z}+\frac{\bar{A}R^2\left(R^2-|z_0|^2\right)}{\bar{z}_0^3(z-R^2/\bar{z}_0)},\quad |z|\leqslant R;\\&\phi_2(z)=\frac{1}{\kappa_2}\phi_1(z)+\frac{\overline{\phi'_1(0)}}{\kappa_2}z+A\ln(z-z_0)+\frac{A}{\kappa_2}\ln\frac{z-R^2/\bar{z}_0}{z}-\frac{\bar{A}R^2\left(R^2-|z_0|^2\right)}{\kappa_2\bar{z}_0^3(z-R^2/\bar{z}_0)},\quad |z|\geqslant R.\end{aligned}\tag{10}$$

Substituting equation (10) into equation $(7)_2$ yields

$$\begin{aligned}&\phi_1^+(z)-\overline{\phi'_1(0)}z+\frac{\sigma_0}{\mu_2}z-A(\kappa_2+1)\ln(z-z_0)\\&=\frac{1}{\kappa_2}\phi_1^-(z)+\frac{\overline{\phi'_1(0)}}{\kappa_2}z+\frac{A(\kappa_2+1)}{\kappa_2}\ln\frac{z-R^2/\bar{z}_0}{z}-\frac{\bar{A}R^2\left(R^2-|z_0|^2\right)(\kappa_2+1)}{\kappa_2\bar{z}_0^3(z-R^2/\bar{z}_0)},\quad |z|=R.\end{aligned}\tag{11}$$

The left-hand side of equation (11) is analytic and single valued everywhere inside the circle, and the right-hand side of equation (11) is analytic and single valued everywhere outside the circle including the point at infinity. Again using Liouville's theorem, we arrive at the following expressions for $\phi_1(z)$ for $|z|\leqslant R$ and its analytic continuation

$$\begin{aligned}&\phi_1(z)=\overline{\phi'_1(0)}z-\frac{\sigma_0}{\mu_2}z+A(\kappa_2+1)\ln(z-z_0),\quad |z|\leqslant R;\\&\phi_1(z)=-\overline{\phi'_1(0)}z-A(\kappa_2+1)\ln\frac{z-R^2/\bar{z}_0}{z}+\frac{\bar{A}R^2\left(R^2-|z_0|^2\right)(\kappa_2+1)}{\bar{z}_0^3(z-R^2/\bar{z}_0)},\quad |z|\geqslant R.\end{aligned}\tag{12}$$

Inserting equation (12) into equation (10), we deduce that the function $\phi_2(z)$ for $|z|\geqslant R$ and its analytic continuation are given as follows

$$\begin{aligned}&\phi_2(z)=-A\ln\frac{z-R^2/\bar{z}_0}{z}+\frac{\bar{A}R^2\left(R^2-|z_0|^2\right)}{\bar{z}_0^3(z-R^2/\bar{z}_0)}+A\ln(z-z_0)-\frac{\sigma_0}{\mu_2}z,\quad |z|\leqslant R;\\&\phi_2(z)=A\ln(z-z_0)-A\ln\frac{z-R^2/\bar{z}_0}{z}+\frac{\bar{A}R^2\left(R^2-|z_0|^2\right)}{\bar{z}_0^3(z-R^2/\bar{z}_0)},\quad |z|\geqslant R.\end{aligned}\tag{13}$$

From equation $(12)_1$, we find that

$$\phi_1'(0) = \overline{\phi'_1(0)} - \frac{\sigma_0}{\mu_2} - \frac{A(\kappa_2+1)}{z_0}. \tag{14}$$

Consequently, σ_0 and $\mathrm{Im}\{\phi_1'(0)\}$ can be determined from equation (14) as

$$\begin{aligned}\frac{\sigma_0}{\mu_2} &= -(\kappa_2+1)\mathrm{Re}\left\{\frac{A}{z_0}\right\} = \frac{b_1\eta - b_2\xi}{\pi(\xi^2+\eta^2)},\\ \mathrm{Im}\{\phi'_1(0)\} &= -\frac{\kappa_2+1}{2}\mathrm{Im}\left\{\frac{A}{z_0}\right\} = \frac{b_1\xi + b_2\eta}{2\pi(\xi^2+\eta^2)}.\end{aligned} \tag{15}$$

From equation $(15)_1$, we see that the internal hydrostatic tension σ_0 is independent of Poisson's ratio of the matrix, and is proportional to the shear modulus of the matrix. Using the analytic continuations in equation (6) and making using of equation $(15)_1$, the original two pairs of analytic functions $\phi_1(z)$, $\psi_1(z)$ defined in the liquid inclusion and $\phi_2(z)$, $\psi_2(z)$ defined in the matrix can be written as

$$\begin{aligned}\phi_1(z) &= \left(\overline{\phi'_1(0)} - \frac{\sigma_0}{\mu_2}\right)z + A(\kappa_2+1)\ln(z-z_0),\\ \psi_1(z) &= \bar{A}(\kappa_2+1)\ln(z-z_0) - \frac{A\bar{z}_0(\kappa_2+1)}{z-z_0}, \quad |z| \leqslant R;\end{aligned} \tag{16}$$

and

$$\begin{aligned}\phi_2(z) &= A\ln(z-z_0) - A\ln\frac{z-R^2/\bar{z}_0}{z} + \frac{\bar{A}R^2\left(R^2-|z_0|^2\right)}{\bar{z}_0^3(z-R^2/\bar{z}_0)},\\ \psi_2(z) &= \bar{A}\ln(z-z_0) - \frac{A\bar{z}_0}{z-z_0} - \bar{A}\ln\frac{z-R^2/\bar{z}_0}{z} + \left[A\bar{z}_0 - \frac{\bar{A}\left(R^2-|z_0|^2\right)}{\bar{z}_0}\right]\frac{1}{z-R^2/\bar{z}_0}\\ &+ \frac{\bar{A}R^2\left(R^2-|z_0|^2\right)}{\bar{z}_0^2(z-R^2/\bar{z}_0)^2} + \mathrm{Re}\left\{\frac{A}{z_0}\right\}\left[2\left(R^2-|z_0|^2\right) - R^2(\kappa_2+1)\right]\frac{1}{z} - \frac{AR^2}{z^2}, \quad |z| \geqslant R.\end{aligned} \tag{17}$$

The internal nonuniform strains and nonuniform rigid body rotation within the liquid inclusion can be obtained from equation (4) with $\kappa_1 = 1$ and equation (16) as follows

$$\begin{aligned}\varpi_{21} &= -\frac{b_1\xi + b_2\eta}{2\pi(\xi^2+\eta^2)} - \frac{b_1(x_1-\xi) + b_2(x_2-\eta)}{\pi\left[(x_1-\xi)^2 + (x_2-\eta)^2\right]},\\ \varepsilon_{22} &= -\varepsilon_{11} = \frac{b_2(x_1-\xi) + b_1(x_2-\eta)}{2\pi\left[(x_1-\xi)^2+(x_2-\eta)^2\right]}\\ &+ \frac{b_1(x_2-\eta)\left[3(x_1-\xi)^2 - (x_2-\eta)^2\right] + b_2(x_1-\xi)\left[3(x_2-\eta)^2 - (x_1-\xi)^2\right]}{2\pi\left[(x_1-\xi)^2+(x_2-\eta)^2\right]^2},\\ \varepsilon_{12} &= \frac{b_1(x_1-\xi) - b_2(x_2-\eta)}{2\pi\left[(x_1-\xi)^2+(x_2-\eta)^2\right]}\\ &+ \frac{b_1(x_1-\xi)\left[(x_1-\xi)^2 - 3(x_2-\eta)^2\right] + b_2(x_2-\eta)\left[3(x_1-\xi)^2 - (x_2-\eta)^2\right]}{2\pi\left[(x_1-\xi)^2+(x_2-\eta)^2\right]^2}, \; z \in S_1,\end{aligned} \tag{18}$$

which is independent of the elastic property of the matrix

The hoop stress along the liquid-solid interface on the matrix side can be derived as

$$\frac{\sigma_{\theta\theta}}{\mu_2} = 4\left(|z_0|^2 - R^2\right)\mathrm{Re}\left\{\frac{A}{R(Re^{i\theta} - z_0)(e^{i\theta}\bar{z}_0 - R)} + \frac{\bar{A}}{\bar{z}_0(e^{i\theta}\bar{z}_0 - R)^2}\right\}$$
$$+ 4\mathrm{Re}\left\{\frac{A}{Re^{i\theta}}\right\} + (\kappa_2 + 1)\mathrm{Re}\left\{\frac{A}{z_0}\right\}. \tag{19}$$

When the edge dislocation lies simply on the circular interface with $z_0 = Re^{i\theta_0}$, equation (19) reduces to

$$\frac{\sigma_{\theta\theta}}{\mu_2} = \frac{4(b_2\cos\theta - b_1\sin\theta)}{R\pi(\kappa_2 + 1)} + \frac{b_2\cos\theta_0 - b_1\sin\theta_0}{R\pi}(\theta \neq \theta_0). \tag{20}$$

Using the Peach–Koehler formula [12], the image force acting on an edge dislocation lying on the x_1-axis with $\eta = 0$ is

$$F_1 - iF_2 = 2\mu_2(b_2 + ib_1)\mathrm{Re}\{\phi'_R(\xi)\} + \mu_2(b_2 - ib_1)[\xi\phi''_R(\xi) + \psi'_R(\xi)], \tag{21}$$

where F_1 and F_2 are, respectively, the image force components along the x_1- and x_2-directions, and the two terms $\phi'_R(\xi)$ and $\xi\phi''_R(\xi) + \psi'_R(\xi)$ are given by

$$\phi'_R(\xi) = \frac{(\bar{A} - A)R^2}{\xi\left(\xi^2 - R^2\right)},$$
$$\xi\phi''_R(\xi) + \psi'_R(\xi) = -\frac{2\bar{A}\xi}{\xi^2 - R^2} - \frac{2i\mathrm{Im}\{A\}}{\xi} - \mathrm{Re}\{A\}\left[2\left(R^2 - \xi^2\right) - R^2(\kappa_2 + 1)\right]\frac{1}{\xi^3} + \frac{2AR^2}{\xi^3}. \tag{22}$$

Consequently, we obtain from equations (21) and (22) the horizontal and vertical components of the image force as

$$F_1 = \frac{\mu_2}{\pi(\kappa_2 + 1)}\left\{-\frac{2R^2\left(b_1^2 + b_2^2\right)}{\xi\left(\xi^2 - R^2\right)} + \left[b_2^2(\kappa_2 + 1) - 2b_1^2\right]\frac{R^2}{\xi^3}\right\},$$
$$F_2 = \frac{\mu_2 b_1 b_2 R^2(\kappa_2 + 3)}{\pi(\kappa_2 + 1)\xi^3}. \tag{23}$$

It is seen from equation $(23)_2$ that the vertical component of the image force F_2 is nonzero when both components of the Burgers vector are nonzero (i.e., $b_1 b_2 \neq 0$). When the edge dislocation lying on the x_1-axis contains only the gliding component with $b_2 = 0$, equation (23) becomes

$$F_1 = -\frac{2\mu_2 R^2 b_1^2}{\pi(\kappa_2 + 1)}\left[\frac{1}{\xi\left(\xi^2 - R^2\right)} + \frac{1}{\xi^3}\right], F_2 = 0. \tag{24}$$

In this case, the gliding dislocation is always attracted to the circular interface. Equation (24) becomes equation (7.8) with $\alpha = -1$ by Dundurs [12] for a traction-free circular hole in view of the fact that $\sigma_0 = 0$ for a gliding dislocation.

When the edge dislocation lying on the x_1-axis contains only the climbing component with $b_1 = 0$, equation (23) becomes

$$F_1 = \frac{\mu_2 R^2 b_2^2}{\pi(\kappa_2 + 1)} \cdot \frac{\xi^2(\kappa_2 - 1) - R^2(\kappa_2 + 1)}{\xi^3\left(\xi^2 - R^2\right)}, F_2 = 0. \tag{25}$$

Equation (25) cannot reduce to equation (7.9) with $\alpha = -1$ by Dundurs [12] for a traction-free circular hole in view of the fact that $\sigma_0 \neq 0$ for a climbing dislocation. The difference between the two is simply

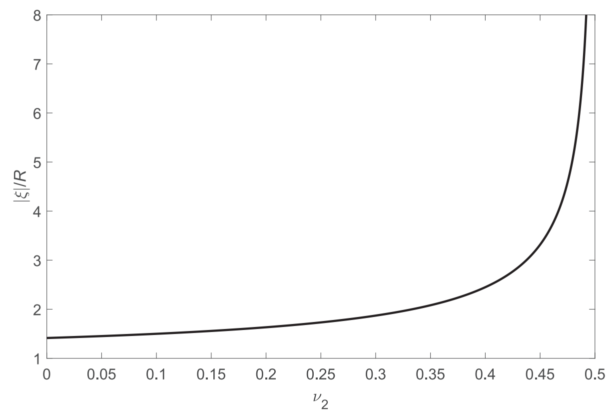

Figure 2. Variation of the unstable equilibrium position for a climbing dislocation as an increasing function of ν_2 determined from equation (28).

the contribution of the hydrostatic tension σ_0 to the image force. It is seen from equation (25) that the climbing dislocation will be attracted to the circular interface when

$$R<|\xi|<R\sqrt{\frac{\kappa_2+1}{\kappa_2-1}}, \tag{26}$$

it is repelled from the interface when

$$|\xi|>R\sqrt{\frac{\kappa_2+1}{\kappa_2-1}}, \tag{27}$$

and it has an unstable equilibrium position at

$$|\xi|=R\sqrt{\frac{\kappa_2+1}{\kappa_2-1}}>R. \tag{28}$$

The climbing dislocation is always attracted to the circular interface when the matrix becomes incompressible with $\kappa_2=1$. The unstable equilibrium position for a climbing dislocation as an increasing function of Poisson's ratio of the matrix ν_2 determined from equation (28) is illustrated in Figure 2. It is seen there that $|\xi|/R$ lies in the range from $|\xi|/R=1.414$ for $\nu_2=0$ to $|\xi|/R\to\infty$ for $\nu_2\to 0.5$.

4. Conclusion

A closed-form solution has been derived to the plane strain problem associated with an edge dislocation located at an arbitrary position in an infinite isotropic elastic matrix interacting with a nearby circular incompressible liquid inclusion. The two pairs of analytic functions $\phi_1(z)$, $\psi_1(z)$ defined in the inclusion and $\phi_2(z)$, $\psi_2(z)$ defined in the matrix are given by equations (16) and (17), respectively. The internal uniform hydrostatic tension, nonuniform strains and nonuniform rigid body rotation within the liquid inclusion are obtained in elementary form in equations $(15)_1$ and (18), and the hoop stress along the circular interface on the matrix side is determined explicitly in equation (19). Using the Peach–Koehler formula, the image force acting on the edge dislocation is presented concisely in equation (23). Our analysis indicates that a gliding dislocation is always attracted to the circular interface, and that there exists an unstable equilibrium position for a climbing dislocation. The existence of the unstable equilibrium position cannot be deduced from the classical result for a traction-free circular hole by Dundurs [12].

Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This work was supported by a Discovery Grant from the Natural Sciences and Engineering Research Council of Canada (grant no. RGPIN-2023-03227 Schiavo).

ORCID iD

Peter Schiavone https://orcid.org/0000-0003-4741-0165

References

[1] Lefevre, V, Danas, K, and Lopez-Pamies, O. A general result for the magnetoelastic response of isotropic suspensions of iron and ferrofluid particles in rubber, with applications to spherical and cylindrical specimens. *J Mech Phys Solids* 2017; 107: 343–364.

[2] Style, RW, Boltyanskiy, R, Allen, B, et al. Stiffening solids with liquid inclusions. *Nat Phys* 2015; 11(1): 82–87.

[3] Ghosh, K, and Lopez-Pamies, O. Elastomers filled with liquid inclusions: theory, numerical implementation, and some basic results. *J Mech Phys Solids* 2022; 166: 104930.

[4] Ghosh, K, Lefevre, V, and Lopez-Pamies, O. The effective shear modulus of a random isotropic suspension of monodisperse liquid *n*-spheres: from the dilute limit to the percolation threshold. *Soft Matter* 2023; 19: 208–224.

[5] Style, RW, Wettlaufer, JS, and Dufresne, ER. Surface tension and the mechanics of liquid inclusions in compliant solids. *Soft Matter* 2015; 11(4): 672–679.

[6] Wu, J, Ru, CQ, and Zhang, L. An elliptical liquid inclusion in an infinite elastic plane. *P Roy Soc A: Math Phy* 2018; 474(2215): 20170813.

[7] Chen, X, Li, MX, Yang, M, et al. The elastic fields of a compressible liquid inclusion. *Extreme Mech Lett* 2018; 22: 122–130.

[8] Dai, M, Hua, J, and Schiavone, P. Compressible liquid/gas inclusion with high initial pressure in plane deformation: modified boundary conditions and related analytical solutions. *Eur J Mech A: Solid* 2020; 82: 104000.

[9] Dai, M, Huang, C, and Schiavone, P. Modified closed-form solutions for three-dimensional elastic deformations of a composite structure containing macro-scale spherical gas/liquid inclusions. *Appl Math Model* 2021; 97: 57–68.

[10] Ti, F, Chen, X, Li, MX, et al. Cylindrical compressible liquid inclusion with surface effects. *J Mech Phys Solids* 2022; 161: 104813.

[11] Ghosh, K, Lefevre, V, and Lopez-Pamies, O. Homogenization of elastomers filled with liquid inclusions: the small-deformation limit. *J Elasticity*. Epub ahead of print 7 February 2023. DOI: 10.1007/s10659-023-09992-x.

[12] Dundurs, J. Elastic interaction of dislocations with inhomogeneities. In: Mura, T (ed.) *Mathematical theory of dislocations*. New York: American Society of Mechanical Engineers (ASME), 1969, pp. 70–115.

[13] Mura, T. *Micromechanics of defects in solids*. 2nd rev. ed. Dordrecht: Martinus Nijhoff, 1987.

[14] Mura, T, Shodja, HM, and Hirose, Y. Inclusion problems. *Appl Mech Rev* 1996; 49: S118–S127.

[15] Zhou, K, Hoh, HJ, Wang, X, et al. A review of recent works on inclusions. *Mech Mater* 2013; 60: 144–158.

[16] Muskhelishvili, NI. *Some basic problems of the mathematical theory of elasticity*. Groningen: P. Noordhoff Ltd, 1953.

[17] England, AH. *Complex variable methods in elasticity*. New York: John Wiley & Sons, 1971.

[18] Ting, TCT. *Anisotropic elasticity: theory and applications*. New York: Oxford University Press, 1996.