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ac.cn)Entanglement Guarantees Emergence of
Cooperation in Quantum Prisoner's
Dilemma Games on Networks

Angsheng Li & Xi Yong

State Key Laboratory of Computer Science, Institute of Software, Chinese Academy of Sciences.

It was known that cooperation of evolutionary prisoner's dilemma games fails to emerge in homogenous networks such as random graphs. Here we proposed a quantum prisoner's dilemma game. The game consists of two players, in which each player has three choices of strategy: cooperator (C), defector (D) and super cooperator (denoted by Q). We found that quantum entanglement guarantees emergence of a new cooperation, the super cooperation of the quantum prisoner's dilemma games, and that entanglement is the mechanism of guaranteed emergence of cooperation of evolutionary prisoner's dilemma games on networks. We showed that for a game with temptation b , there exists a threshold $\arccos \frac{\sqrt{b}}{b}$ for a measurement of entanglement, beyond which, (super) cooperation of evolutionary quantum prisoner's dilemma games is guaranteed to quickly emerge, giving rise to stochastic convergence of the cooperations, that if the entanglement degree γ is less than the threshold $\arccos \frac{\sqrt{b}}{b}$, then the equilibrium frequency of cooperations of the games is positively correlated to the entanglement degree γ , and that if γ is less than $\arccos \frac{\sqrt{b}}{b}$ and b is beyond some boundary, then the equilibrium frequency of cooperations of the games on random graphs decreases as the average degree of the graphs increases.

It has been a longstanding challenge to understand the emergence and convergence (or guaranteed emergence) of cooperations of games on networks, and the principles behind them. Our quantum prisoner's dilemma game extends the classical prisoner's dilemma game naturally by introducing a *super cooperator*, and by using a measurement of entanglement, which we called *entanglement degree* as a metaphor for the relationship between players. This gives rise to a natural generation of the prisoner's dilemma game. Our game better reflects real games in nature and society. Our results explore a threshold theory for the role of quantum entanglement in emergence and convergence of cooperations in evolutionary quantum prisoner's dilemma games on networks, providing a foundation for exploring the emergence and convergence, and the principles behind, of cooperations of prisoner's dilemma games on complex networks. Equally important, we notice that our quantum game can be played by classic devices. This provides an example that quantum entanglement plays an essential role in games, and that the advantage of quantum games can be fully achieved by classic devices.

The prisoner's dilemma (PD, for short) game is one of the well-known games, having implications in a wide range of disciplines. In a PD game, two players simultaneously decide their strategy C (cooperator) or D (defector). For mutual cooperation, both players receive a payoff R and receive P upon mutual defection. If one cooperates and the other defects, then the cooperator gains payoff S and the traitor gains temptation T . The payoff rank for the PD game is given by $T > R > P \geq S$. In a PD game, the best strategy for both players is to defect regardless of the other's decision, in which case, the payoffs of the players are minimized.

Real games are played in a system characterized by a graph in which the nodes are players, and the edges represent the games played by the two endpoints of each of the edges. The games in a graph are evolving by rounds. In this case, emergence of cooperation of the evolutionary games on a graph implies a maximal global payoff evolved from minimal local payoffs of the graphs. It plays an essential role in the organizations of the graphs such as systems of biosphere and human society.

The reasons why does cooperation emerge in evolutionary games on a system could be many, one of which seems the special structures of the system. This is indeed the case. We know that social structures do play essential roles in emergence of cooperation, referred to^{1–3}. Nowak and May found that cooperation emerges in a very simple spatial structure⁴. Santos and Pacheco showed that the scale-free networks promote emergence of cooperation, with higher cooperation ratios than that on random graphs for both PD games and the snowdrift games (SG)⁵. All these results showed that heterogeneity does promote emergence of cooperation in different games.



This property has also been found in other games, for instance, in the public goods games (PGG)⁶, which further demonstrates that heterogeneity of networks, a metaphor for social diversity, favors the emergence of cooperative behavior. On the other hand, cooperation is unlikely to emerge in the evolutionary prisoner's dilemma games on homogeneous networks, referred to⁷⁻⁹.

This means that network structure is essential to emergence of cooperation of the PD games. However, in real world, we are usually unable to change the network structures. A fundamental question is thus: How can we guarantee emergence of cooperations of the prisoner's dilemma games on general (probably random) graphs, including both heterogeneous and homogeneous networks?

In the present paper, we will answer this question. For simplicity, we choose the parameters of the PD game such that $R = 1, P = S = 0$, and $T = b$ for b with $1 < b < 2$.

Before answering the question, we reexamine the prisoner's dilemma game to open the window for a new form of cooperations. The new possibility is that in classic prisoner's dilemma game, both players are independent of each other, and each of them is absolutely selfish and perfectly rational. However, in real world games, the two players in a game may always have some entangled relationship, which is the way that quantum comes into the game naturally.

Quantum Prisoner's Dilemma Game

Eisert *et al* in 1999¹⁰ proposed a quantum game in which each player chooses a strategy from a unitary operator:

$$\hat{U}(\phi, \theta) = \begin{pmatrix} e^{i\phi} \cos \theta/2 & \sin \theta/2 \\ -\sin \theta/2 & e^{-i\phi} \cos \theta/2 \end{pmatrix},$$

for ϕ and θ ranging from 0 to $\frac{\pi}{2}$ and π respectively.

Li *et al*¹¹⁻¹³ implemented some experiments on evolutionary quantum games on some networks, in which each player randomly chooses a strategy from the whole unitary space. The experiments showed that quantum strategies promote cooperations in the games for some choices of the strategies. In particular, for two special quantum strategies, the Hadamard operation and the Q strategy (the same as our super cooperator), the experiments in¹² showed that the grid network is most easily invaded by quantum strategies and that a scale-free network can be invaded by agents adopting quantum strategies only if a hub of the network is occupied by an agent with a quantum strategy or if the fraction of agents with quantum strategies in the population is significantly large. Pawela and Sladkowski¹⁴ investigated the evolution of strategies on hyper-graph networks with three quantum strategies. The experiments in¹⁴ showed that in the case of a three player game on a hyper-graph networks, quantum strategies are not necessarily stable strategies, and that in some cases, the defection strategy is as good as a quantum strategy. Chappell, Iqbal and Abbott¹⁵ proposed an analysis of three-player quantum games in an EPR type setup by using the Clifford geometric algebra.

Here we propose a *quantum prisoner's dilemma* (QPD, for short) game. Our idea is to introduce a special quantum strategy, written by Q , which joins the classical prisoner's dilemma game as a *super cooperator*. Therefore, our game consists of three strategies: C , D and Q , given by $C = \hat{U}(0, 0)$, $D = \hat{U}(\pi, 0)$, and $Q = \hat{U}(0, \pi)$.

Our game consists of two players, Alice and Bob. We assume that Alice and Bob have an *entanglement degree* $\gamma \in [0, \frac{\pi}{2}]$.

Then the payoff matrix of our quantum prisoner's dilemma game is thus given by Table 1.

In our game, we choose just one quantum strategy, $Q = \hat{U}(0, \pi)$, and allow quantum entanglement degree γ to change from 0 to $\frac{\pi}{2}$. We notice that the payoff matrix is deduced from quantum mechanics, but the game itself can be simulated classically. By this reason, our

quantum prisoner's dilemma is a game based on quantum mechanics that can be played by both quantum and classic devices. More importantly, as we will see, that this generalization solves many problems of the prisoner's dilemma games. All the games in the experiments of our paper are played classically.

From Table 1, we have the following results: 1) If $\gamma = 0$, then the Q strategy collapses at C ; 2) If both players choose the strategy Q , then they will get payoff 1, the same as that they both choose the strategy C , and Q never takes advantage from C ; 3) If $\gamma = \frac{\pi}{2}$, then a (Q, C) game gets payoffs $(0, 0)$ respectively, and if $\gamma = \frac{\pi}{2}$, then a (Q, D) game gets payoffs b and 0 respectively, meaning that Q takes advantage from D , and if $\gamma > 0$, then Q always reduces the gain of a D strategy; and 4) when γ increases, (Q, Q) becomes more and more a stable game than the (C, C) game due to the entanglement of the two players, and if $\gamma = \frac{\pi}{2}$, then (Q, Q) is a Nash equilibrium that maximizes the payoffs.

By 1), the classical prisoner's dilemma game is a special case of our game. By 2), we can interpret Q as a cooperator. By 4), if $\gamma = \frac{\pi}{2}$, then Q is completely different from C , meaning that Q is indeed a new strategy. By 3), Q is against D . By 4), the game (Q, Q) would be a Nash Equilibrium, and simultaneously be Pareto optimal. Therefore, our game is a natural generalization of the prisoner's dilemma with a new strategy, the super cooperator Q .

We explain that our Q -strategy and quantum prisoner's dilemma game capture new phenomena in nature and society. In biological species evolution, initially we have a population C , later on, a species of invaders D appears, the invaders take advantage from the population C . In this case, there will be a new type mutated from population C , which is the Q -strategy. The new type Q is consistent with C , but against the invaders D . Eventually, the new type Q will evolve as a new dominating species. In human society, we interpret the honest crowds as C , and dishonest crowds as D . To prevent the invasions of the C -strategy crowds from the D -strategy invaders, a party Q naturally and gradually grows up from C to restrain D so that D becomes a minority in the society. Our quantum prisoner's dilemma game may explain some new phenomena in economics. For instance, it explains the reason why a win-win strategy in economics is possible and provides a mechanism to guarantee such a win-win strategy in economical activities. A remarkable characteristic of our quantum prisoner's dilemma game is that the quantum game is completely different from the classic prisoner's dilemma game, and that our quantum prisoner's dilemma game can be played by classic devices. This provides a remarkable example to show that classic devices fully realize our quantum prisoner's dilemma game, which is essentially different from the classic prisoner's dilemma game. This means that some theoretical advantage of quantum mechanics can be fully achieved by classic devices, leading to potential new physics.

Evolutionary Quantum Prisoner's Dilemma Games

Evolutions of our quantum prisoner's dilemma games on networks are defined similarly to that of the classical prisoner's dilemma games.

Let $G = (V, E)$ be a graph. The evolutions of our games proceed by stages: At stage 0, for every node $v \in V$, define the strategy of v ,

Table 1 | The expectation payoff matrix of our quantum prisoner's dilemma game

	C	D	Q
C	(1, 1)	(0, b)	($\cos^2 \gamma$, $\cos^2 \gamma$)
D	(b, 0)	(0, 0)	($b \cdot \cos^2 \gamma$, $b \cdot \sin^2 \gamma$)
Q	($\cos^2 \gamma$, $\cos^2 \gamma$)	($b \cdot \sin^2 \gamma$, $b \cdot \cos^2 \gamma$)	(1, 1)



written by $s(v)[0]$, to be one of the C , D and Q with probabilities p_1, p_2 and p_3 respectively. (In our experiments, we set $p_i = \frac{1}{3}$ for each $i = 1, 2, 3$.) Suppose that the strategy of v at stage t , written $s(v)[t]$, are defined for all $v \in V$. Then for each $X = C, D$, or Q , let $\rho_X[t]$ be the fraction of nodes in V that share strategy X during stage t , and let $P(v)[t]$ be the total payoff of the games between v and all its neighbors by the end of stage t . At stage $t + 1$, for every node $v \in V$, pick randomly and uniformly a neighbor, u say, of v as its reference. If $P(u)[t] > P(v)[t]$, then with probability $\frac{P(u)[t] - P(v)[t]}{\max\{d_u, d_v\} \cdot b}$, set $s(v)[t + 1] = s(u)[t]$, where d_x is the degree of node x in G , otherwise, then keep $s(v)$ unchanged.

Generally speaking, the evolutionary system may reach a dynamical equilibrium after sufficiently many stages¹⁶. However, the interesting thing is the emergence and stochastic convergence of cooperations. As mentioned above, for the classical prisoner's dilemma games, cooperation fails to emerge in homogeneous networks⁷⁻⁹. We will see that this is no longer true for our quantum prisoner's dilemma games.

Emergence of Cooperation of Games on Homogenous Networks

We consider two classical models both generating homogeneous graphs. The first is the Erdős - Rényi (ER, for short)¹⁷ model, and the second is the small world model¹⁸. In the ER model, for given n nodes, and a number p , for every pair of the nodes i and j , with probability p , create an edge between i and j . For the small world model, we give n nodes in a circle, create an edge between any two nodes of distance within a number r , and for each node i , with some probability p , change an edge from i to a randomly and uniformly picked node k .

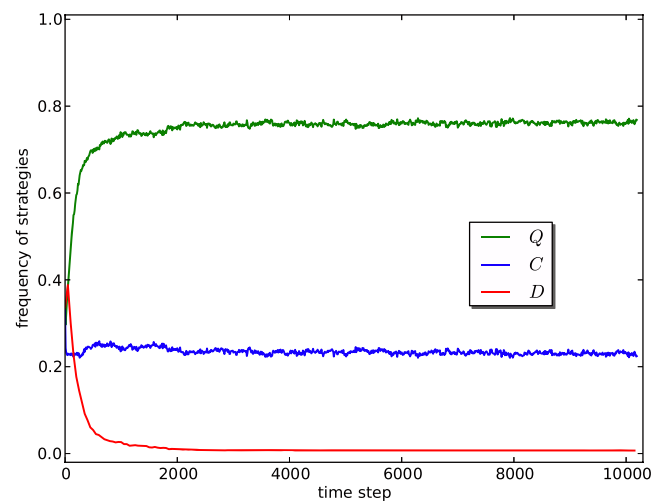
We depict the curves of frequencies of the C -, D - and Q -strategy nodes in the evolutionary quantum prisoner's dilemma games on a graph from the ER model in Figure 1 (a), and a graph from the small world model in Figure 1 (b).

From Figure 1, we observe that: 1) The frequencies of the three strategies evolve with time, and finally reach some stochastically stable states; 2) In the graph of the ER model, the D -strategy nodes almost vanishes quickly, after which the majority Q -strategy nodes and a fraction of C -strategy nodes share the graph; 3) In the graph of the small world model, the C -strategy nodes almost vanish quickly, after which the majority Q -strategy nodes and a fraction of D -strategy nodes share the graph; and 4) In both graphs, Q quickly becomes the dominating strategy of the evolutionary games, and keeps as the dominating strategy once it emerged. This experiment shows the emergence of cooperation of evolutionary quantum prisoner's dilemma games on homogeneous networks.

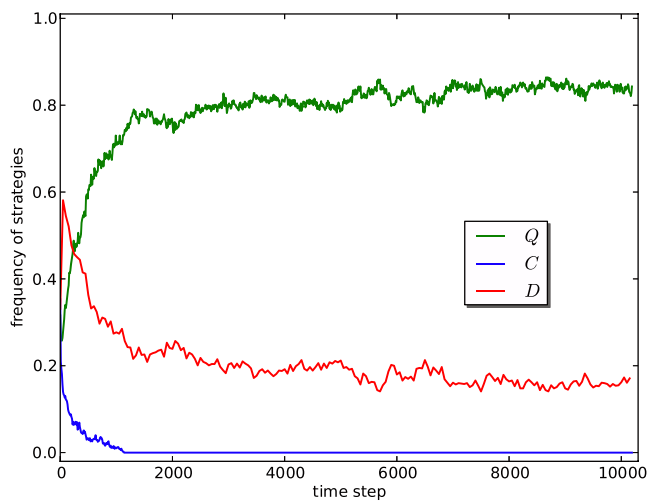
Classical and Quantum Prisoner's Dilemma Games

We investigate the stochastically convergent strategies of the evolutionary quantum prisoner's dilemma games on random graphs generated by the ER model with different entanglement degrees and different expected average degrees. Figures 2(a) and 2(b) give the curves of C -, D - and Q -strategies of the games for $\gamma = 0$ and expected average degree $d = 4$ and 8 respectively. In this case, Q collapses at C so that the games are classical ones. Figures 2 (c) and 2 (d) give the curves of C -, D - and Q -strategies of the games with $\gamma = \frac{\pi}{7}$, and expected average degree $d = 4$ and 8 respectively.

By observing Figure 2, we have the following results: (1) For $\gamma = 0$. Then the D -strategy will quickly become the dominating strategy for b greater than 1.2 and 1.1 for $d = 4$ and 8 respectively. (2) For $\gamma = \frac{\pi}{7}$, and $d = 4$. Then the super cooperator Q will become the dominating strategy for all b , including $b = 2$; and (3) For $\gamma = \frac{\pi}{7}$ and $d = 8$. Then



(a)



(b)

Figure 1 | The evolution of three strategies with steps on a graph of the ER model (the upper part) and one from the small world model (the lower part) respectively. The temptation factor $b = 1.18$, the expected average degree is $d = 4$ and the is $\gamma = \pi/12$ for both the graphs.

Q will be the dominating strategy for most b 's, for which only if $b > 1.8$, the D -strategy nodes may surpass the super cooperators.

By (1), if $\gamma = 0$, then the games are the classical ones, in which case, for non-trivial b 's (not too small), D is always the dominating strategy, so that cooperation fails to emerge. By (2) and (3), if γ is non-trivial (not too small), then Q will be the dominating strategy, unless b is too large.

This experiment shows that our quantum prisoner's dilemma game is remarkably different from the classical PD game, and that super cooperation emerges on random graphs, unless γ is too small, or b is too large. This poses the question: what is the relationship between γ and b which guarantees the emergence of cooperations of the games on random graphs?

Threshold of Entanglement for Emergence and Convergence

We investigate the relationship between γ and b that guarantees emergence of super cooperation of the quantum prisoner's dilemma games on random graphs of the ER model.

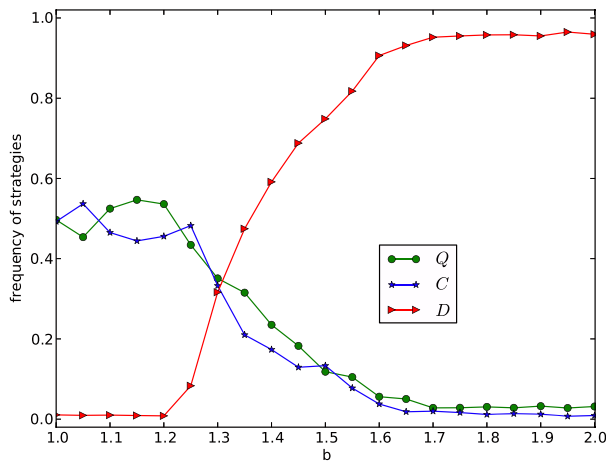
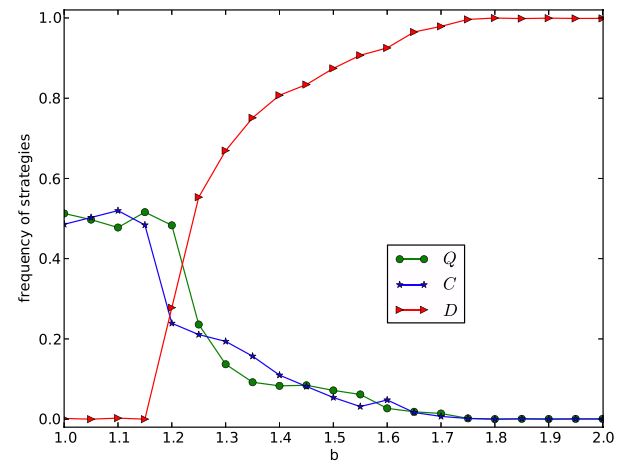
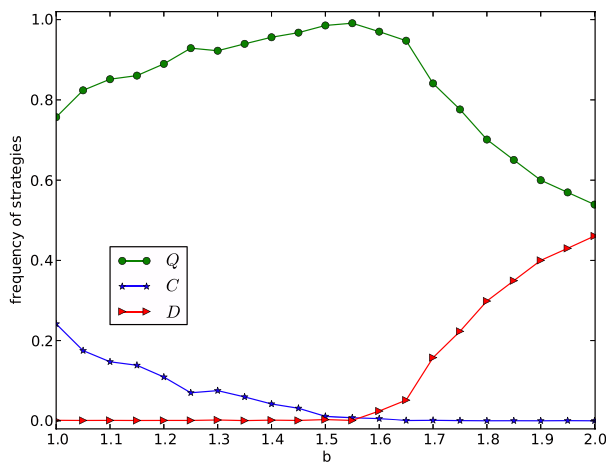
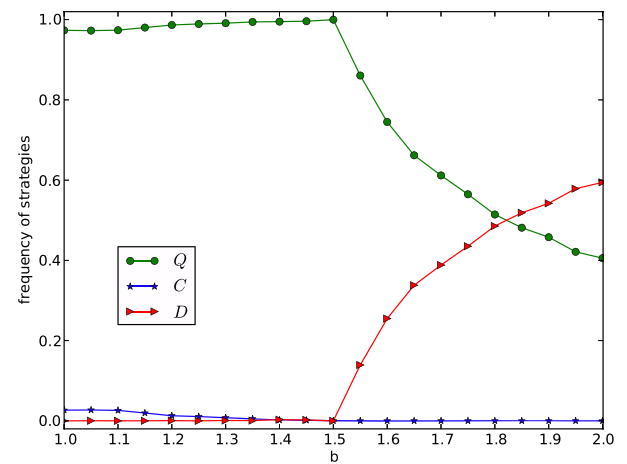
(a) $d = 4, \gamma = 0$ (b) $d = 8, \gamma = 0$ (c) $d = 4, \gamma = \frac{\pi}{7}$ (d) $d = 8, \gamma = \frac{\pi}{7}$

Figure 2 | The equilibrium fraction of three strategies is plotted as a function of b . In order to compare our results with the classical one, we did the experiment when the $\gamma = 0$ and $\gamma = \pi/7$ respectively.

Given $b \in (1, 2)$ and $\gamma \in [0, \frac{\pi}{2}]$, suppose that Alice and Bob play the game with payoff matrix given in Table 1.

Suppose that $\gamma > \arccos \frac{\sqrt{b}}{b}$. We consider the following scenario: If Alice chooses a Q-strategy, then the maximal payoff of Bob is $\max\{1, \cos^2 \gamma, b \cdot \cos^2 \gamma\}$. By the assumption of $\gamma > \arccos \frac{\sqrt{b}}{b}$, the best payoff of Bob $\max\{1, \cos^2 \gamma, b \cdot \cos^2 \gamma\}$ is minimized at 1, which occurs only if Bob chooses Q too. Therefore, if $\gamma > \arccos \frac{\sqrt{b}}{b}$, then by choosing Q, Alice forces Bob to choose Q if Bob wants to maximize his payoff. At the same time, when both Alice and Bob choose Q, the payoffs of both players and the total game are maximized. By the same reason, in this case, (Q, Q) is the unique Nash equilibrium of our quantum prisoner's dilemma game.

This result implies that in the case of $\gamma > \arccos \frac{\sqrt{b}}{b}$, the (super) cooperation is guaranteed to emerge in the evolutionary quantum prisoner's dilemma games and is guaranteed to dominate almost all

nodes of the network, for arbitrarily given network, including random graphs. This gives, for the first time, a theoretical threshold for emergence and stochastic convergence of quantum prisoner's dilemma games on an arbitrarily given graph.

The result above explores a remarkable role of the Q strategy that in the case of $\gamma > \arccos \frac{\sqrt{b}}{b}$, by choosing Q, a player may force its opponent to choose a particular strategy. This is a phenomenon frequently occurring in real world games, but never in classical prisoner's dilemma games.

The result demonstrates that the Q-strategy plays a fundamental role in emergence and stochastic convergence of cooperations, that our quantum prisoner's dilemma game is remarkably different from the classical prisoner's dilemma game, and that our quantum prisoner's dilemma game does better reflect the nature of real world games.

To understand the result, we verify it by experiments. In Figures 3 (a), (b), (c) and (d), we depict the curves of each of the strategies for a random graph with expected average degree $d = 8$, and with $b = 1.2, 1.5, 1.8$ and 2.1 respectively. In each of the figures, it does show that, if $\gamma > \arccos \frac{\sqrt{b}}{b}$, then the super cooperation strategy Q emerges, stoch-



astically converges and dominates the evolutionary games on the graph. For example, we choose $b = 1.5$, then $\arccos \frac{\sqrt{b}}{b} \approx 35^\circ$. From Figure 3 (b), we observe that, if $\gamma \geq 3\pi/16 = 33.75^\circ$, then the frequency of Q -strategy nodes is almost equal to 1. For $b = 2.1$, $\arccos \frac{\sqrt{b}}{b} \approx 46^\circ$, by observing Figure 3 (d), if $\gamma \geq \pi/4$, then Q emerges, stochastically converges and dominates the games on the graph.

On the other hand, by observing Figures 3 (a), (b), (c) and (d), we have that if $\gamma \leq \arccos \frac{\sqrt{b}}{b}$, then the equilibrium frequency of super cooperations of the evolutionary quantum prisoner's dilemma games increases as the entanglement degree γ increases.

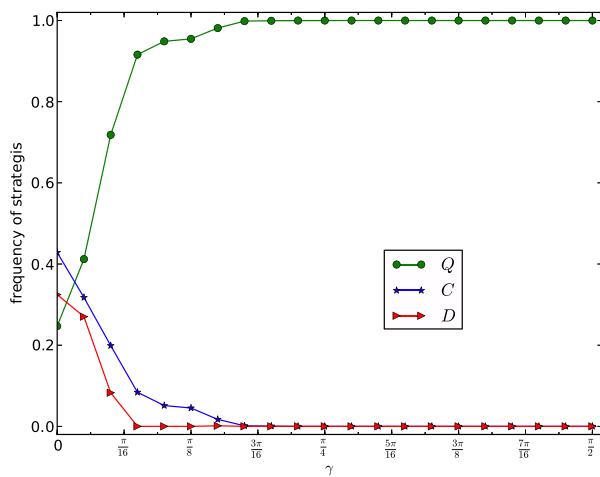
We thus have that, if $\gamma > \arccos \frac{\sqrt{b}}{b}$, then super cooperation emerges, stochastically converges (almost surely emerges) and dominates the evolutionary quantum prisoner's dilemma games, and that if $\gamma \leq \arccos \frac{\sqrt{b}}{b}$, then the equilibrium frequency of super coopera-

tions of the evolutionary quantum prisoner's dilemma games is positively correlated to the entanglement degree γ 's. We emphasize that the results here principally hold for games on, not only random, but also arbitrarily given networks, although there must be new phenomena for different structures of networks.

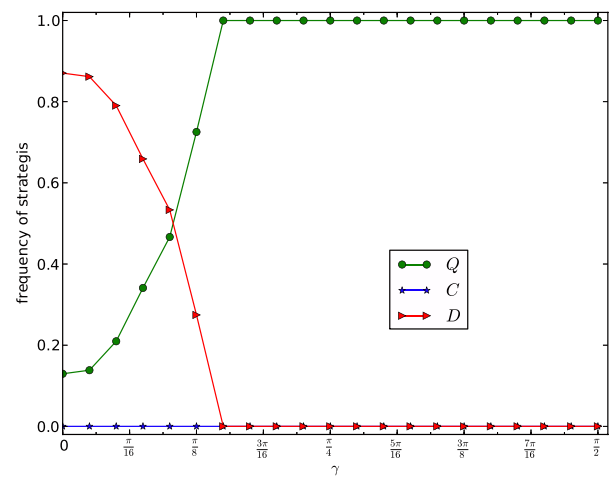
The Role of Structure of Graphs

We have seen that if $\gamma > \arccos \frac{\sqrt{b}}{b}$, then super cooperation emerges, stochastically converges and dominates the evolutionary quantum prisoner's dilemma games on random graphs, and that if $\gamma \leq \arccos \frac{\sqrt{b}}{b}$, then the equilibrium frequency of super cooperation is positively correlated to γ .

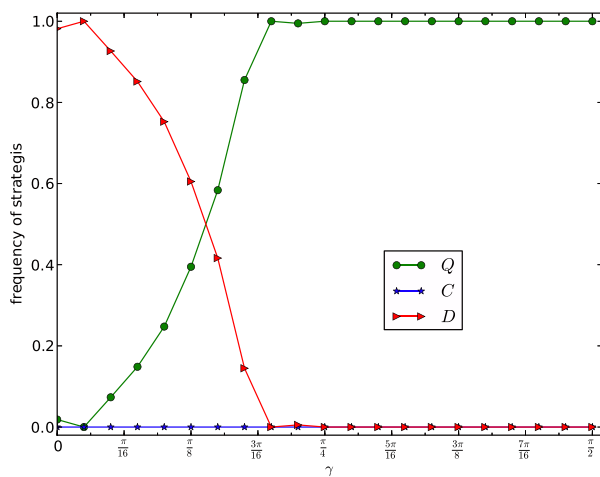
However, if $\gamma \leq \arccos \frac{\sqrt{b}}{b}$, then super cooperation may emerge, or may fail to emerge, depending on both the entanglement degree γ and the structure of the network. What roles do the structures of networks play in the emergence of cooperation of the games on the networks?



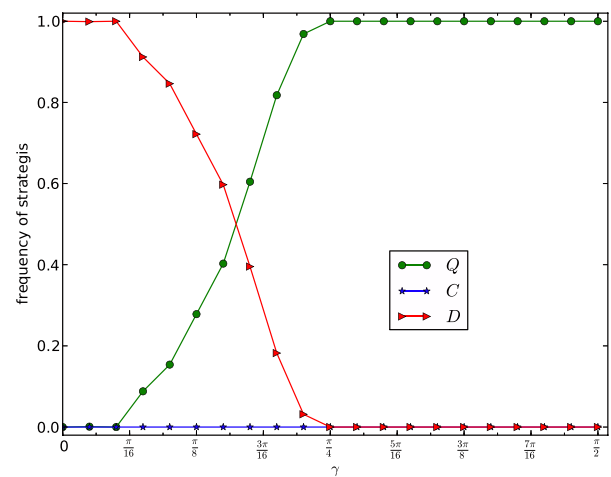
(a) $b = 1.2$



(b) $b = 1.5$



(c) $b = 1.8$



(d) $b = 2.1$

Figure 3 | The equilibrium fraction of three strategies is plotted as a function of γ when $b = 1.2$, $b = 1.5$, $b = 1.8$, $b = 2.1$ respectively. The size of the network is 10000 and the average degree is 8.

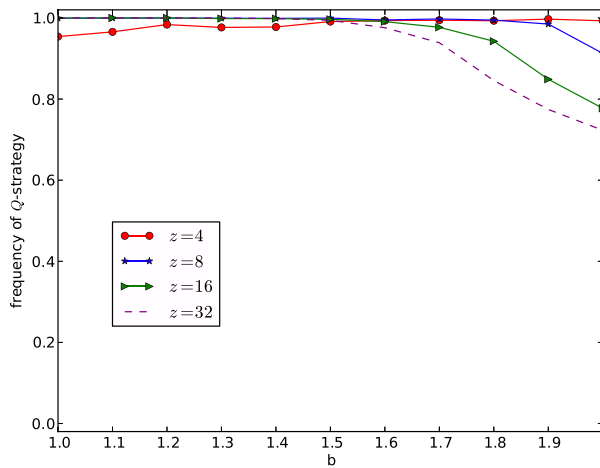
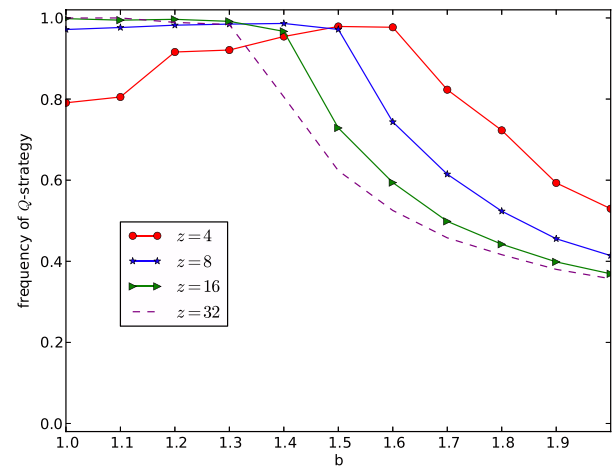
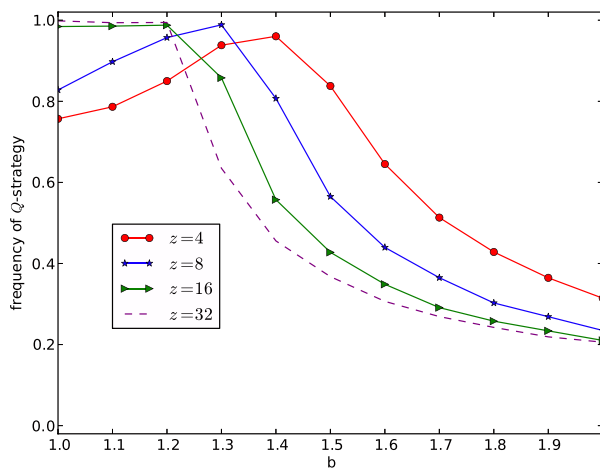
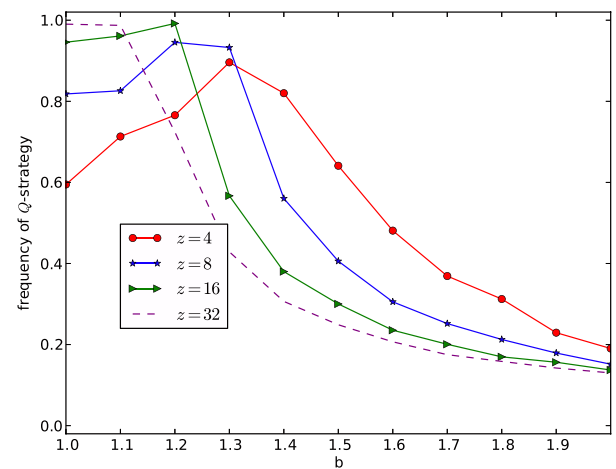
(a) $\gamma=5$ (b) $\gamma=7$ (c) $\gamma=9$ (d) $\gamma=11$

Figure 4 | The equilibrium fraction of super cooperators (the Q -strategy nodes) is plotted as a function of b with different values of expected average degree z .

To understand this, we study the curves of equilibrium frequencies of super cooperators of the games on random graphs of the ER model with different expected average degrees.

In Figures 4 (a), (b), (c) and (d), we depict the curves of super cooperators of four graphs with expected average degrees $z = 4, 8, 16$ and 32 , and with entanglement degrees $\gamma = \pi/5, \gamma = \pi/7, \gamma = \pi/9, \gamma = \pi/11$ respectively.

From the experiments, we observe the following results: (1) If the b 's are very small, then the equilibrium frequencies of super cooperations increase as the average degrees of the random graphs increase, and that if the b 's are appropriately large, then the equilibrium frequencies of cooperations decrease as the average degrees of the random graphs increase. This shows the role of structures and explores an interesting new phenomenon, calling for a new theory to explain.

Conclusions and Discussion

We proposed a quantum prisoner's dilemma game and investigated the emergence, and stochastic convergence (guaranteed emergence) of (super) cooperation of the evolutionary quantum prisoner's dilemma games on random graphs. Our results can be summarized

as follows: 1) if $\gamma > \arccos \frac{\sqrt{b}}{b}$, then (super) cooperation always emerges and stochastically converges on every network, 2) if $\gamma \leq \arccos \frac{\sqrt{b}}{b}$, then equilibrium frequencies of super cooperations are positively correlated to the entanglement degree γ on every network, 3) if $\gamma \leq \arccos \frac{\sqrt{b}}{b}$ and b is very small, then the equilibrium frequencies of super cooperations on random graphs increase as the average degrees of the graphs increase, and 4) if $\gamma \leq \arccos \frac{\sqrt{b}}{b}$ and b is appropriately large, then the equilibrium frequencies of super cooperations on random graphs decrease as the average degrees of the graphs increase. The first two results provide a theory for emergence and stochastic convergence of cooperations of prisoner's dilemma games on arbitrarily given networks. The later results explore a new phenomenon of emergence of cooperations, predicting a new theory for understanding the roles of structures of networks in emergence of cooperations. Our results pose a fundamental question for future investigation: Given a network G , find the least entangle-



ment degree γ_0 such that for every $\gamma \geq \gamma_0$ and for every $b \in (1, 2]$, super cooperation of the evolutionary quantum prisoner's dilemma games on G is guaranteed to emerge. This could be a general scientific problem, calling for solutions from mathematics, physics, informatics, economics, biology and social sciences etc. Our game is derived from quantum theory, but can be played classically. This suggests a new direction to characterize the quantum theory that can be realized by classic devices.

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Author contributions

A.L. designed the research and wrote the paper, X.Y. designed the research and performed the experiments. All authors reviewed the paper.

Additional information

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