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An efficiency-based interval type-2 fuzzy multi-criteria group decision making for makeshift hospital selection

Ze-hui Chen [a], Shu-ping Wan [a,*], Jiu-ying Dong [b]

[a] School of Information Management, Jiangxi University of Finance and Economics, Nanchang 330013, China
[b] School of Statistics, Jiangxi University of Finance and Economics, Nanchang 330013, China

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A B S T R A C T

Since makeshift hospitals have strong ability in blocking the spread of the virus, how to design some methods to select the reasonable sites of makeshift hospitals is vitally important for containing COVID-19. This paper investigates an efficiency-based multi-criteria group decision making (MCGDM) method by combining the best-worst method (BWM) and data envelopment analysis (DEA) in trapezoidal interval type-2 fuzzy (TrIT2F) environment. This MCGDM method is called *TrIT2F-BWM-DEA*, where the *TrIT2F-BWM* is used to determine the weights of criteria and decision-makers, and the *TrIT2F-DEA* is employed to rank alternatives by measuring their overall efficiencies. Based on cut set theory, the expectation and average expectation (AE) of TrIT2FSs are successively defined. To solve three key issues in the development of the *TrIT2F-BWM*, this paper proposes a flexible ranking relation of TrIT2FSs to transform the TrIT2F constraints, initiates an efficient theorem to normalize the TrIT2F weights, and designs an input-based consistency ratio to check the reliability of the determined weights. A fully *TrIT2F-DEA* model is originally built to measure the TrIT2F efficiencies of alternatives. The alternatives are finally ranked according to the AEs of alternatives' TrIT2F efficiencies. A site selection case of Fangcang hospitals and some comparative analyses are provided to confirm the validity and merits of the proposed *TrIT2F-BWM-DEA*.

1. Introduction

According to the historical data at the website of World Health Organization (WHO) [1], since the 21st century, human beings have undergone a series of public health emergencies, including six "public health emergencies of international concern" confirmed by WHO from 2005 to 2020. These public health emergencies caused numerous human deaths and economic losses. Two representative examples are the well-known H1N1 pandemic in 2009 and the ongoing global COVID-19. H1N1 pandemic 2009 broke out from April 2009 to August 2010 across the world, causing at least 24 million confirmed cases of H1N1 pandemic, including more than 15,000 deaths. The ongoing COVID-19 (which was identified in late 2019) has attacked a large number of countries and regions around the world. Globally, as of 14 October 2021, more than 239.9 million confirmed cases of COVID-19 have been reported, including over 4.88 million deaths [1]. Unfortunately, H1N1 pandemic 2009 and COVID-19 were not two exceptions but rather yet additions to a string of public health emergencies. Indisputably, despite the adverse impacts (e.g., high infectivity and fatality rate) caused by public health emergencies are extremely serious, they can be weakened by making some scientific preparations and response efforts.

As one of the most important response efforts, timely isolation and treatment of confirmed patients are crucial to contain the rapid spread of pandemics. Large-scale outbreak of pandemics will certainly trigger a huge number of confirmed cases, which will inevitably result in a serious imbalance between the tremendous confirmed patients and the limited available medical/hospital resources. In such a situation, large numbers of confirmed patients cannot receive timely isolation and treatment, which will further increase the risk of the population cross-infection and the rapid spread of pandemics. To improve the efficiency of pandemic prevention, one of the scientific countermeasures is to differently isolate and treat patients according to their conditions. That is, the patients with different symptoms should be cared with differential medical/hospital resources. For example, during the outbreak of COVID-19 in Wuhan, China, the severe-to-serious and mild-to-moderate patients were cared in the designated hospitals (e.g., Wuhan Jinyintan hospital) and makeshift hospitals (e.g., Jianghan Fangcang hospital), respectively. Generally, the designated hospitals can be selected from the existing available hospitals, while makeshift hospitals need

* Correspondence to: School of Information Management, Jiangxi University of Finance and Economics, No. 169, East Shuang-gang Road, Economic and Technological Development district, Nanchang 330013, China.
E-mail address: shupingwan@163.com (S.-p. Wan).

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to be temporarily designed and constructed based on the selected reasonable sites. Thus, the urgent concern for local government is how to select the reasonable sites for makeshift hospitals. Undoubtedly, reasonable makeshift hospital sites are beneficial to effective isolation and treatment of mild-to-moderate confirmed patients, which is extremely instrumental in repelling pandemics. Conversely, unreasonable makeshift hospital sites are inevitably followed by inefficient treatment performance, which primarily manifests in the fact that more confirmed patients cannot receive timely isolation and treatment. Therefore, to enhance the isolation and treatment performance of mild-to-moderate confirmed patients, it is vitally important to determine the reasonable sites for makeshift hospitals.

Makeshift (Fangcang) hospital, originally implemented in China to tackle pandemics and natural disasters (e.g., SARS in 2003, Yushu earthquake in 2010 and COVID-19 in 2020), is a special form of hospital and a novel public health concept [2]. Fangcang hospital is large-size temporary place by rapidly reconstructing existent building (e.g., stadium and exhibition center) into healthcare facility. Since Fangcang hospital possesses five key virtues (i.e., rapid construction, massive scale, low cost, good mobility and strong environmental adaptability) and six essential functions (i.e., isolation, triage, basic medical care, frequent monitoring, rapid referral, and essential living), it shows strong ability in undertaking emergency medical rescue missions. Currently, during the period of anti-COVID-19, Fangcang hospital has become a popular choice for numerous countries (e.g., Italy, Brazil, Russia, the United Kingdom, the United States, etc.) [3].

Generally, the site selection of makeshift hospitals mainly includes the following four successive stages:

Stage I. Identify candidate sites of makeshift hospitals (i.e., alternatives). Alternatives are usually identified by preliminary filtration of some large-size public buildings in the city where the pandemic is occurring.

Stage II. Select criteria. Criteria are obviously essential to measure the performance of candidate sites. The criteria for makeshift hospital selection can be extracted from the related construction standards and technical requirements. For example, according to the "Technical requirements for the design and conversion of makeshift (Fangcang) hospitals", Wan et al. [4] extracted eight criteria as follows: geographical position, infrastructure, regional communication convenience, capacity, traffic convenience, environmental protection, reconstruction difficulty and reconstruction cost. In this regard, the site selection of makeshift hospitals can surely be regarded as a multi-criteria decision making (MCDM) problem. In addition, due to the limited cognitive of decision-maker (DM), it is not easy for a single DM to thoroughly assess the alternatives on all criteria [5,6], which results in the appearance of multi-criteria group decision making (MCGDM). Hence, to acquire synthetic decision results, it is necessary to recruit a group of experts/DMs to assess each alternative under the selected criteria.

Stage III. Compare criteria and evaluate alternatives. In this stage, the project leader would invite a panel of experts from different expertise fields to express their reference comparisons (RCs) on criteria and evaluate alternatives under the selected criteria.

Stage IV. Determine the best alternative. In this stage, the following two issues need to be solved.

(i) Determine the weights of DMs and criteria. Since DMs usually have different expertise levels, it is more reasonable to assign different weights to different DMs. However, DMs' weights were often fully/partially predetermined in some achievements [7,8], which is obviously not in line with the actual decision making problems. The similar scenario also appears in the assignment of criteria weights [9]. Hence, to acquire more reasonable decision results, it is expected to use/develop some scientific techniques to determine the weights of DMs and criteria. Currently, analytic hierarchy process (AHP) [10], analytic network process (ANP) [11] and best-worst method (BWM) [12] are the three most popular tools to determine the weights of objects (e.g., DMs and criteria).

(ii) Rank alternatives. Based on the obtained weights of objects and the evaluation matrices of alternatives, alternatives can be synthetically evaluated by diverse MCDM methods, such as TOPSIS (Technique for order Preference by Similarity to an Ideal Solution) [13], MULTIMOORA (multi-objective optimization by ratio analysis plus the full multiplicative form) [14], TODIM (TOmada de Decisão Iterativa Multicritério) [8], VIKOR (VIsekriterijumska optimizacija i KOm-promisno Resenje) [15], DEA (data envelopment analysis) [16], etc. In particular, DEA is a frequently used technique that measures the performance of a group of decision making units (DMUs) with a number of outputs and inputs. Due to the strong theoretical foundation, DEA has been widely applied to multifarious real world problems, e.g., smart product service module selection [17], portfolio selection [18], mutual funds evaluation [19], healthcare [20], etc. However, the DEA models used in [18–20] are all crisp, namely, the input (output) parameters and the decision variables in these models are all taken as crisp numbers. This will surely limit the application of DEA to more complex and uncertain decision making problems.

In practice MCDM problems, it is convenient for DMs to express their judgments with linguistic terms. Nevertheless, there is an inescapable fact the same linguistic term usually means different things for different people. For example, several DMs express their judgments with the term "weak importance", but this term may have different semantic implications for these DMs. Regrettably, most of the existing studies on MCDMs ignored this difference. Currently, type-1 fuzzy sets (T1FSs) [21–23] are the mostly-used representations of linguistic terms,15. T1FSs make more sense than crisp numbers since they have crisp membership grades in the interval [0,1]. However, it is impossible for crisp membership grades to completely portray the uncertainty appearing in "the same word means different things for different people". To tie this issue, Zadeh [24] proposed type-2 fuzzy set (T2FS) to handle DMs' different opinions by introducing a secondary membership function. Besides, to broaden the applications of T2FSs, some achievements [15,25] developed interval approach-based type-2-fuzzistics methodology to encode linguistic terms. Presently, trapezoidal interval type-2 fuzzy set (TrIT2FS), a simplified format of T2FS, has been widely used to various fuzzy decision making problems [7,8,13,15] due to its low computational complexity and high efficiency in handling uncertainty. To illustrate the specific semantics of TrIT2FS in real applications, an example is presented as follows.

Example 1. A DM wishes to assess the importance of the selected criteria. Five terms are used to express DM's preference of one criterion over another: weak importance (WI), moderate importance (MI), strong importance (SI), very strong importance (VSI) and extreme importance (EI). Assume that a word survey was conducted by investigating 50 respondents. These respondents were preinformed that each of the five terms describes an interval falling somewhere between 1 and 9. For example, the range for the term "WI" might lie between 1 and 3. Firstly, each respondent was asked to provide his/her judgment on the possible range of each term. Then, based on all respondents' judgments and the interval approach [25], it is easy to construct a mathematical model to determine the footprint of uncertainty (FOU) for each term. Lastly, the FOU for each term can be determined in form of TrIT2FS, as graphically shown in Fig. 1. Hence, the specific semantics of a TrIT2FS can be exactly interpreted as an overall opinion of all respondents on the same linguistic term. From this perspective,

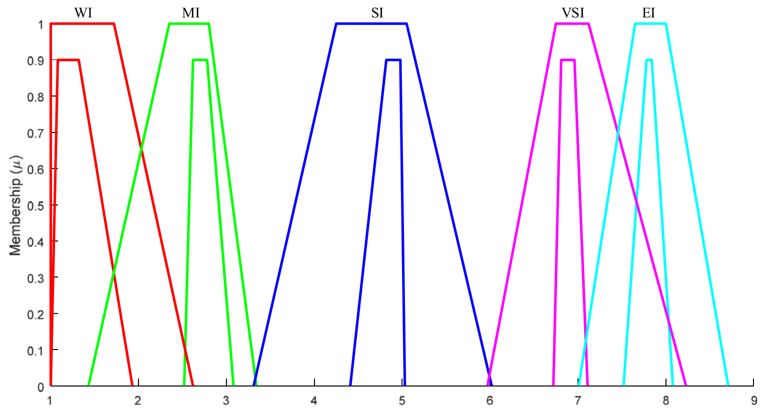

Fig. 1. FOUs for terms in form of TrIT2FSs.

TrIT2FS indeed has the power in dealing with the uncertainty steamed from the fact "the same word means different things for different people".

Due to the advantages of TrIT2FS and the inevitable uncertainty during Fangcang hospital location (FHL) decision making, it is impeccable to extend the application of TrIT2FS to the FHL problem. To this end, it is necessary to stipulate that the decision information is represented by TrIT2FSs during the FHL decision process, where TrIT2FSs are encoded from DMs' linguistic judgments by using the type-2-fuzzistics method in [15,25].

For real-life FHL decision problems, the following questions need to be investigated: (i) How to deal with the TrIT2F information during the FHL decision process. (ii) How to determine the weights of DMs and criteria. (iii) How to assess the performance of the candidate sites based on a set of criteria. To cover these questions, this paper intends to develop an integrated method (called *TrIT2F-BWM-DEA*) by combining BWM and DEA in TrIT2F environment. The reason for choosing the *TrIT2F-BWM-DEA* to the FHL problem is discussed as follows:

For the FHL problem with TrIT2F information, crisp weights might be inadequate to perfectly reflect the importance of the objects (e.g., DMs and criteria). As such, it is more reasonable to replace crisp weights with TrIT2F weights. Recently, BWM has become a popular choice to determine the weights of objects since it has several outstanding advantages (e.g., requirement of few RCs, generation of high reliable weights and easy integration with other methods). Generally, the selected criteria for measuring the performance of alternatives could be divided into two types: benefit criteria (e.g., capacity) and cost criteria (e.g., reconstruction difficulty), and the evaluations of alternatives under these criteria might be multi-scaled. Compared with other MCDM methods [8,13–15], DEA has greater power in handling the MCDMs with the multi-scale cost and benefit criteria, and quickly identifying the best alternatives distributing along the production frontiers [17]. Given the advantages of BWM and DEA, this paper intends to develop an integrated method by combining BWM and DEA (called *BWM-DEA*). However, due to the great differences of different decision environments, both the crisp BWM and DEA along with their existing fuzzy versions cannot be directly applied to TrIT2F environment. Thus, it is meaningful to develop a TrIT2F version of *BWM-DEA* (called *TrIT2F-BWM-DEA*) for the FHL problem, where *TrIT2F-BWM* and *TrIT2F-DEA* are used to determine the TrIT2F weights and rank alternatives, respectively.

Additionally, in order to better clarify the main framework of this paper, some research assumptions and objectives are stated as follows:

(1) Research assumptions: (i) The decision results would be affected by DM's risk attitudes. (ii) The proposed approaches (see Theorems 3.1 and 4.3) are expected to avoid information loss or distortion of TrIT2FSs. (iii) Type-2 fuzzy information is hopeful to generate more reasonable decision results than type-1 fuzzy information. These assumptions provide a clear orientation for implementing this paper.

(2) Research objectives: (i) Put forward some novel approaches to flexibly handling TrIT2F information. (ii) Develop a *TrIT2F-BWM* to determine the TrIT2F weights of DMs and criteria more accurately. (ii) Construct a fully *TrIT2F-DEA* model to measure the efficiency of each alternative reasonably. These objectives are the basic requirements to solve the above-mentioned three questions.

The layout of this paper is taken as follows. Section 2 briefly reviews the literature concerning emergency shelter location, interval type-2 fuzzy (IT2F) decision making, BWM and DEA. Section 3 introduces some preliminaries related to T2FSs and TrIT2FSs. Section 4 develops a *TrIT2F-BWM* to determine the weights of DMs and criteria. Section 5 originally proposes a fully *TrIT2F-DEA* to rank alternatives. Section 6 completes the integration of the *TrIT2F-BWM* and *TrIT2F-DEA*. A real FHL case and some comparative analyses are conducted in Section 7. Section 8 terminates this paper with some remarkable conclusions and research prospects.

2. Literature review and contributions of this paper

This section roughly reviews some essential achievements on interval type-2 fuzzy (IT2F) decision making, BWM, DEA and emergency shelter location. By carefully analyzing the reviewed achievements, several research gaps have surfaced. Then, the main contributions of this paper are summarized.

2.1. Review on IT2F decision making

Since interval type-2 fuzzy set (IT2FS) has powerful ability in capturing uncertain and complex information by applying the secondary membership [24,26], it has been extensively concerned and applied to various decision making problems, such as supplier selections [27,28], stock selection [29], etc. In addition, some classical decision methods have been extended into IT2F environment, e.g., TOPSIS [7,13], likelihood-based decision making method [27], BWM [4], VIKOR [30], analytic hierarchy process (AHP) [31], etc. Celik et al. [32] performed a systematic review concerning IT2F MCDMs, which provides an insightful orientation for researchers to further study IT2F MCDMs. It is clear that the defuzzification of fuzzy sets/numbers is an essential content in fuzzy MCDMs. Currently, the centroid-based and possibility mean values (PMV)-based approaches [15,28] are commonly used to defuzzify TrIT2FSs. However, approaches [15,28] have the following limitations: (i) in approach [15], the calculation of the centroid of TrIT2FS is complex, which might limit the wide application of TrIT2FSs; (ii) in approach [28], the calculation of the PMV of TrIT2FS is based on two pseudo level sets, where two different cut sets need to be separately considered. Hence, approaches [15,28] are not quite handy to defuzzify TrIT2FSs. To this end, the first research gap is to propose a more convenient approach to defuzzifying TrIT2FSs.

2.2. Review on BWM

Since the advent of BWM [12] in 2015, it has been applied to multifarious decision making problems, such as maintenance assessment in the hospitals [33], hybrid vehicle engine selection [34], Fangcang hospital selection [4], supplier selection [35], etc. Mi et al. [36] conducted a systematic review on the applications of BWM.

Due to the complexity and uncertainty of practical MCDM problems, it is not easy for DMs to express their judgments on objects with crisp values [37]. As such, BWM needs to be combined with uncertain information in form of fuzzy numbers/sets, e.g., triangular fuzzy numbers (TFNs) [38], intuitionistic fuzzy values (IFVs) [39], hesitant fuzzy sets (HFSs) [40], IT2FSs [4,15], etc. In fuzzy environments, the crisp weight-determining model (WDM) [12,41] will be extended into various fuzzy versions. The key issue to solve a fuzzy WDM is how to transform it into a crisp one, in which two sub-issues need to be solved: (i) how to transform the objective of the fuzzy WDM; (ii) how to normalize the fuzzy weights. To cover the first sub-issue, methods [15,22,38,39] used a crisp absolute deviation to convert the fuzzy minmax objective into a crisp minimization objective. To solve the second sub-issue, methods [15,22] normalized the fuzzy weights with direct defuzzification approaches, and method [4] proposed a weight-normalizing approach in which the upper and lower heights of TrIT2FSs are neglected. Although methods [4,15,22,38,39] are enforceable in handling the above-mentioned two key sub-issues, there might still exist the appearance of information loss or distortion of fuzzy sets due to their imprecise disposals. Hence, the second research gap is to develop some more reasonable approaches to transforming the TrIT2F WDM into a crisp one, which is also the difficulty of investigating the *TrIT2F-BWM*.

2.3. Review on DEA

DEA is a data-driven linear programming method, which is used to assess the relative efficiencies of numerous DMUs. Up to now, multitudinous investigations on DEA have been reported. Chen & Ming [17] proposed an integrated method by combining BWM and DEA for solving the selection of smart product service modules. Lim et al. [18] proposed a way of using DEA cross-efficiency evaluation in portfolio selection. Lin & Liu [19] extended the multiplier dynamic DEA by using directional distance function for evaluating the performance of mutual funds. Xiao et al. [42] considered the uncertainty of parameter and constructed three diversification consistent DEA models for the estimation of portfolio efficiency. Ebrahimi et al. [43] put forward a novel mixed binary linear DEA model for finding the most efficient DMU by considering DMs' preferences. Zhou et al. [44] established a novel dynamic network DEA model with desirable and undesirable indicators to compute the detailed efficiencies of sustainable supply chain. Otay et al. [20] developed a multi-expert fuzzy method to evaluate the performance of healthcare institutions by integrating intuitionistic fuzzy DEA and AHP. Emrouznejad & Yang [45] finished a survey on the extensions and applications of DEA from 1978 to 2016, which can offer researchers and practitioners with pragmatic guidelines.

Despite these investigations have strong power in solving MCDM problems, they still have some shortages. For example, two limitations appeared in [18,19]: (i) Refs. [18,19] overlooked the different influences of inputs and outputs on the performance of alternatives (DMUs); (ii) Refs. [18,19] assumed that the input and output criteria were tangible, i.e., the input and output data is objective. In fact, in some actual decision making problems, the input and output criteria are usually intangible. In such a case, the DEA models presented in [18,19] will be invalid. To solve these issues, Otay et al. [20] integrated DEA with AHP in intuitionistic fuzzy environment, where AHP and DEA are used to derive the weights of inputs/outputs and evaluate DMUs' efficiencies, respectively. Chen & Ming [17] combined BWM and DEA in rough-fuzzy environment. Although methods [17,20] can effectively solve the above two limitations, they built non-fully fuzzy DEA models to derive crisp efficiencies, which might result in some inaccurate decision results. Intuitively, it is more reasonable to derive fuzzy efficiencies with fully fuzzy inputs and outputs. Besides, it has been assumed that the decision information in this paper is represented by TrIT2FSs. Obviously, the processing of TrIT2F information is quite different from those of intuitionistic fuzzy and rough-fuzzy information. Based on the above analysis, it is imperative to construct a fully *TrIT2F-DEA* model to accurately measure the efficiencies of DMUs. This is the third research gap.

2.4. Review on emergency shelter location

Presently, only few progresses [46–48] related to makeshift (Fangcang) hospitals have been reported. A common feature of [46–48] is that they mainly discussed the introduction of Fangcang hospital, such as the functions of Fangcang hospital [46,48] and the significance of Fangcang hospital [47]. Although Wan et al. [4] investigated a multi-criteria group decision making (MCGDM) method and applied it to locate makeshift (Fangcang) hospitals, they overlooked the upper and lower heights of TrIT2FSs. Considering that Fangcang hospital is also a special form of emergency shelters, some studies [49–53] concerning the site selection of emergency shelters are also reviewed. These studies mainly focused on the location of emergency shelters for post-natural disaster rescue operations. It is evident that natural disaster emergencies are quite different from public health emergencies, which implies that methods [49–53] are not suitable for solving the FHL problem in this paper. In recent years, BWM [12,41] has become an effective and popular tool to determine the weights of criteria. However, only few studies [4,15] have completed the integration of classical MCDM methods and BWM in TrIT2F environment. Therefore, the fourth research gap is to develop a combined method to address the FHL problem in TrIT2F environment. This is also a challenge of integrating the *TrIT2F-BWM* and *TrIT2F-DEA*.

2.5. Contributions of this paper

To narrow and fill the aforementioned research gaps, this paper investigates a *TrIT2F-BWM-DEA* for the FHL problem. Firstly, a *TrIT2F-BWM* is proposed to determine the TrIT2F weights of DMs and criteria. Then, a *TrIT2F-DEA* is developed to measure the overall efficiencies of alternatives. Compared with previous achievements, this paper highlights the following contributions:

(1) This paper defines the expectation of TrIT2FS based on cut set theory, where only one cut set is considered. Then, the average expectation (AE) of TrIT2FS is defined to defuzzify TrIT2FSs. The

rationality of the defined AE of TrIT2FS is demonstrated with the mean absolute percentage error (MAPE) between the AEs and centroids of TrIT2FSs. This achieves the first research gap.

(2) Based on the defined expectation of TrIT2FS, this paper proposes a novel flexible ranking relation of TrIT2FSs and a TrIT2F weight-normalizing theorem. The ranking relation of TrIT2FSs used in [4,15] is defined by the rigorous ranks of several pairs of numbers related to TrIT2FSs, it is very simple but too strict. The ranking relation of TrIT2FSs proposed in this paper is more flexible and suitable for portraying the vagueness of TrIT2FSs than that used in [4,15]. To normalize TrIT2F weights, Wan et al. [4] designed a support-core-based weight-normalizing approach to replacing the centroid-based one used in [15]. Despite the former is more effectual in preserving the fuzzy information of TrIT2FSs than the latter, it ignored the lower and upper heights of TrIT2FSs which are two essential components of TrIT2FSs. To cover this flaw, the proposed TrIT2F weight-normalizing theorem considers the lower and upper heights of TrIT2FSs. Consequently, both the proposed flexible ranking relation of TrIT2FSs and TrIT2F weight-normalizing theorem are greatly helpful to preserve the inherent information of TrIT2FSs. This fills the second research gap.

(3) A fully *TrIT2F-DEA* is proposed to evaluate the efficiencies of alternatives. Considering that the decision information is finally represented by TrIT2FSs, this paper constructs a fully *TrIT2F-DEA* model to measure the TrIT2F efficiencies of alternatives. In this model, all the parameters and variables are represented by TrIT2FSs. This finishes the third research gap.

(4) To solve the FHL problem, this paper develops a *TrIT2F-BWM-DEA*, in which the *TrIT2F-BWM* is used to determine the TrIT2F weights of DMs and criteria, and the *TrIT2F-DEA* is employed to measure the TrIT2F efficiencies of alternatives. The interaction of the *TrIT2F-BWM-DEA* mainly relies on the proposed *TrIT2F-BWM* and *TrIT2F-DEA*, which is detailed as follows: (i) Based on the determined weights of DMs and criteria, the individual TrIT2F evaluation matrices can be aggregated into an overall TrIT2F evaluation matrix. (ii) From the overall TrIT2F evaluation matrix, it is easy to extract all alternatives' TrIT2F inputs and outputs which are further used to the constructed fully *TrIT2F-DEA* model. This completes the fourth research gap.

3. Preliminaries

In this section, some basic concepts concerning intervals and TrIT2FSs are briefly reviewed. Then, the expectation of TrIT2FS is defined and a novel flexible ranking relation of TrIT2FSs is proposed.

3.1. Basic concepts of intervals

Let $\bar{h}_i = [h_i^-, h_i^+]$ $(i = 1, 2)$ be two intervals, $m(\bar{h}_i) = (h_i^+ + h_i^-)/2$ and $r(\bar{h}_i) = (h_i^+ - h_i^-)/2$ be the midpoint and radius of $\bar{h}_i$, respectively. The terminology "$\bar{h}_1$ is not greater than $\bar{h}_2$" is denoted by $\bar{h}_1 \preccurlyeq \bar{h}_2$.

Definition 3.1 (*[23]*). Let $\bar{h}_1 = [h_1^-, h_1^+]$ and $\bar{h}_2 = [h_2^-, h_2^+]$ be two intervals. The order relation $\bar{h}_1 \preccurlyeq \bar{h}_2$ is considered to be a fuzzy set, whose membership degree is defined as follows:

$$\mu(\bar{h}_1 \preccurlyeq \bar{h}_2) = \begin{cases} 1, & \text{if } h_1^+ \le h_2^- \\ 1^+, & \text{if } h_1^- \le h_2^- \le h_1^+ \le h_2^+ \wedge r(\bar{h}_1) > 0 \\ \dfrac{(h_2^+ - h_1^+)}{2[r(\bar{h}_2) - r(\bar{h}_1)]}, & \text{if } h_2^- \le h_1^- \le h_1^+ \le h_2^+ \wedge r(\bar{h}_2) > r(\bar{h}_1) \\ 0.5, & \text{if } r(\bar{h}_2) = r(\bar{h}_1) \wedge h_2^- = h_1^- \end{cases} \tag{3.1}$$

where 1^- denotes a fuzzy number being less than 1, which represents that $\bar{h}_1$ is weakly not greater than $\bar{h}_2$. $\mu(\bar{h}_1 \preccurlyeq \bar{h}_2)$ is interpreted as DM's acceptability of the ranking relation $\bar{h}_1 \preccurlyeq \bar{h}_2$.

It is obvious that $0 \le \mu(\bar{h}_1 \preccurlyeq \bar{h}_2) \le 1$. If $\mu(\bar{h}_1 \preccurlyeq \bar{h}_2) = 0$, then DM totally declines $\bar{h}_1 \preccurlyeq \bar{h}_2$. If $\mu(\bar{h}_1 \preccurlyeq \bar{h}_2) = 1$, then DM completely accepts $\bar{h}_1 \preccurlyeq \bar{h}_2$. Otherwise, DM accepts $\bar{h}_1 \preccurlyeq \bar{h}_2$ with different satisfactory degrees between 0 and 1. Similarly, the terminology "$\bar{h}_1$ is not less than $\bar{h}_2$", i.e., $\bar{h}_1 \succcurlyeq \bar{h}_2$, is defined below.

Definition 3.2 (*[23]*). Let $\bar{h}_1 = [h_1^-, h_1^+]$ and $\bar{h}_2 = [h_2^-, h_2^+]$ be two intervals. The order relation $\bar{h}_1 \succcurlyeq \bar{h}_2$ is considered to be a fuzzy set, whose membership degree is defined as follows:

$$\mu(\bar{h}_1 \succcurlyeq \bar{h}_2) = \begin{cases} 0, & \text{if } h_1^+ \le h_2^- \\ 0^+, & \text{if } h_1^- \le h_2^- \le h_1^+ \le h_2^+ \wedge r(\bar{h}_1) > 0 \\ \dfrac{(h_1^- - h_2^-)}{2[r(\bar{h}_2) - r(\bar{h}_1)]}, & \text{if } h_2^- \le h_1^- \le h_1^+ \le h_2^+ \wedge r(\bar{h}_2) > r(\bar{h}_1) \\ 0.5, & \text{if } r(\bar{h}_2) = r(\bar{h}_1) \wedge h_2^- = h_1^- \end{cases} \tag{3.2}$$

where 0^+ means a fuzzy number being greater than 0, which indicates that $\bar{h}_1$ is weakly not less than $\bar{h}_2$.

If $\bar{h}_1 \succcurlyeq \bar{h}_2$ and $\bar{h}_1 \preccurlyeq \bar{h}_2$, then $\bar{h}_1 \approx \bar{h}_2$, where the symbol "$\succcurlyeq$" is an interval version of the ranking relation "$\ge$" in the real value set and has the linguistic term of "essentially not less than". Other symbols "$\preccurlyeq$" and "$\approx$" are interpreted similarly.

Definition 3.3 (*[23]*). Let $\bar{h}_1 = [h_1^-, h_1^+]$ and $\bar{h}_2 = [h_2^-, h_2^+]$ be two intervals. The satisfactory crisp equivalent form of the interval ranking relation $\bar{h}_1 \preccurlyeq \bar{h}_2$ is defined as:

$$h_1^+ \le h_2^+ \text{ and } \mu(\bar{h}_1 \succcurlyeq \bar{h}_2) \le \vartheta \tag{3.3}$$

where $\vartheta \in [0, 1]$ signifies DM's acceptance degree of $\bar{h}_1 \preccurlyeq \bar{h}_2$ to be violated. The value of ϑ is pre-given according to DM's risk attitudes. In particular, $\vartheta \in [0, 0.5)$ means that DM is conservative, $\vartheta = 0.5$ indicates that DM is neutral, and $\vartheta \in (0.5, 1]$ signifies that DM is adventurous.

The acceptance degree $\vartheta \in [0, 1]$ implies that DM may allow the relation $\bar{h}_1 \preccurlyeq \bar{h}_2$ to be violated with the acceptance degree between 0 and 1. In particular, $\vartheta = 0$ stands for that DM fully rejects the relation $\bar{h}_1 \preccurlyeq \bar{h}_2$ to be violated, and $\vartheta = 1$ indicates that DM totally accepts the relation $\bar{h}_1 \preccurlyeq \bar{h}_2$ to be violated.

Similarly, the satisfactory crisp equivalent form of $\bar{h}_1 \succcurlyeq \bar{h}_2$ is defined as:

$$h_1^- \ge h_2^- \text{ and } \mu(\bar{h}_1 \preccurlyeq \bar{h}_2) \le \vartheta \tag{3.4}$$

3.2. Basic concepts of TrIT2FS

Let X be the universe of discourse.

Definition 3.4 (*[17]*). A type-2 fuzzy set (T2FS) $\widetilde{B}$ in X is defined as:

$$\widetilde{B} = \{(x, \upsilon), \mu_{\widetilde{B}}(x, \upsilon) | \forall x \in X, \forall \upsilon \in J_x\} \tag{3.5}$$

where $J_x \in [0, 1]$ represents the main membership at x, $\mu_{\widetilde{B}}(x, \upsilon) \in [0, 1]$ indicates the secondary grade of (x, υ), x and υ are the primary and the secondary variables, respectively.

Definition 3.5 (*[54]*). Let $\widetilde{B}$ be a T2FS defined in X. If $\mu_{\widetilde{B}}(x, \upsilon) = 1$ $(x \in X, \upsilon \in J_x)$, then $\widetilde{B}$ is called an interval type-2 fuzzy set (IT2FS).

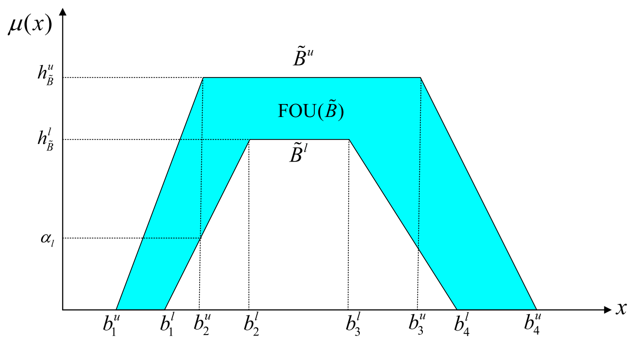

Fig. 2. Trapezoidal interval type-2 fuzzy set.

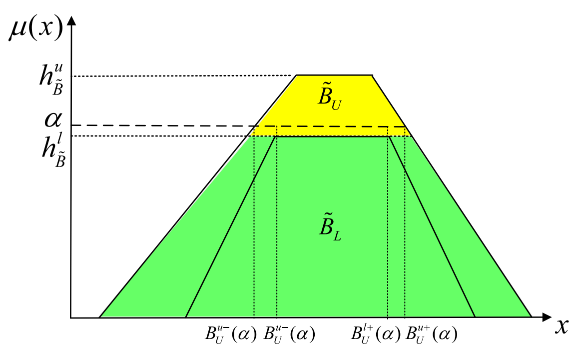

Fig. 3. Geometrical interpretation of a TrIT2FS $\widetilde{B}$.

Definition 3.6 ([4]). An IT2FS $\widetilde{B} = [\widetilde{B}^l, \widetilde{B}^u] = [(b^l_1, b^l_2, b^l_3, b^l_4; h^l_{\widetilde{B}}), (b^u_1, b^u_2, b^u_3, b^u_4; h^u_{\widetilde{B}})]$ is called TrIT2FS, its LMF and UMF are formulated as follows:

$$\mu_{\widetilde{B}^l}(x) = \begin{cases} \dfrac{x - b^l_1}{b^l_2 - b^l_1} h^l_{\widetilde{B}}, & \text{if } b^l_1 \le x < b^l_2 \\ h^l_{\widetilde{B}}, & \text{if } b^l_2 \le x \le b^l_3 \\ \dfrac{b^l_4 - x}{b^l_4 - b^l_3} h^l_{\widetilde{B}}, & \text{if } b^l_3 < x \le b^l_4 \\ 0, & \text{otherwise} \end{cases} \quad (3.6)$$

and

$$\mu_{\widetilde{B}^u}(x) = \begin{cases} \dfrac{x - b^u_1}{b^u_2 - b^u_1} h^u_{\widetilde{B}}, & \text{if } b^u_1 \le x < b^u_2 \\ h^u_{\widetilde{B}}, & \text{if } b^u_2 \le x \le b^u_3 \\ \dfrac{b^u_4 - x}{b^u_4 - b^u_3} h^u_{\widetilde{B}}, & \text{if } b^u_3 < x \le b^u_4 \\ 0, & \text{otherwise} \end{cases} \quad (3.7)$$

where $h^l_{\widetilde{B}}$ and $h^u_{\widetilde{B}}$ are the lower and upper heights of $\widetilde{B}$ respectively, $h^l_{\widetilde{B}}$, $h^l_{\widetilde{B}}$, b^l_i and b^u_i $(i = 1, 2, 3, 4)$ are all real numbers and satisfy $b^l_1 \le b^l_2 \le b^l_3 \le b^l_4$, $b^u_1 \le b^u_2 \le b^u_3 \le b^u_4$, $b^u_1 \le b^l_1$, $b^l_4 \le b^u_4$ and $0 \le h^l_{\widetilde{B}} \le h^u_{\widetilde{B}} \le 1$. If $b^u_1 \ge 0$, then $\widetilde{B}$ is called a non-negative TrIT2FS.

Obviously, if $b^l_i = b^u_i$ $(i = 1, 2, 3, 4)$ and $h^u_{\widetilde{B}} = h^l_{\widetilde{B}} = 1$, then $\widetilde{B}$ reduces to a trapezoidal fuzzy number (TrFN); if $b^l_i = b^u_i = b$ $(i = 1, 2, 3, 4)$ and $h^u_{\widetilde{B}} = h^l_{\widetilde{B}} = 1$, then $\widetilde{B}$ degenerates to a real number which is denoted by $b^* = [(b, b, b, b; 1), (b, b, b, b; 1)]$.

According to Definition 3.6, it is easy to plot the image of TrIT2FS, as shown in Fig. 2, where the shaded region is the footprint of uncertainty (FOU) of $\widetilde{B}$, denoted by FOU($\widetilde{B}$). The upper membership function (UMF) is associated with the upper bound of FOU($\widetilde{M}$) and denoted by $\widetilde{B}^u$. The lower membership function (LMF) is associated with the lower bound of FOU($\widetilde{B}$) and denoted by $\widetilde{B}^l$.

Definition 3.7 ([55]). Let $\widetilde{B} = [(b^l_1, b^l_2, b^l_3, b^l_4; h^l_{\widetilde{B}}), (b^u_1, b^u_2, b^u_3, b^u_4; h^u_{\widetilde{B}})]$ and $\widetilde{B}_i = [(b^l_{i1}, b^l_{i2}, b^l_{i3}, b^l_{i4}; h^l_{\widetilde{B}}), (b^u_{i1}, b^u_{i2}, b^u_{i3}, b^u_{i4}; h^u_{\widetilde{B}})]$ $(i = 1, 2)$ be three arbitrary TrIT2FSs. Some basic arithmetic operations of TrIT2FSs are defined as follows:

(i) Addition:

$$\widetilde{B}_1 + \widetilde{B}_2 = \begin{bmatrix} (b^l_{11} + b^l_{21}, b^l_{12} + b^l_{22}, b^l_{13} + b^l_{23}, b^l_{14} + b^l_{24}; \min\{h^l_{\widetilde{B}_1}, h^l_{\widetilde{B}_2}\}), \\ (b^u_{11} + b^u_{21}, b^u_{12} + b^u_{22}, b^u_{13} + b^u_{23}, b^u_{14} + b^u_{24}; \min\{h^u_{\widetilde{B}_1}, h^u_{\widetilde{B}_2}\}) \end{bmatrix} \quad (3.8)$$

(ii) Subtraction:

$$\widetilde{B}_1 - \widetilde{B}_2 = \begin{bmatrix} (b^l_{11} - b^l_{24}, b^l_{12} - b^l_{23}, b^l_{13} - b^l_{22}, b^l_{14} - b^l_{21}; \min\{h^l_{\widetilde{B}_1}, h^l_{\widetilde{B}_2}\}), \\ (b^u_{11} - b^u_{24}, b^u_{12} - b^u_{23}, b^u_{13} - b^u_{22}, b^u_{14} - b^u_{21}; \min\{h^u_{\widetilde{B}_1}, h^u_{\widetilde{B}_2}\}) \end{bmatrix} \quad (3.9)$$

(iii) Multiplication:

$$\widetilde{B}_1\widetilde{B}_2 = \begin{bmatrix} (b^l_{11}b^l_{21}, b^l_{12}b^l_{22}, b^l_{13}b^l_{23}, b^l_{14}b^l_{24}; \min\{h^l_{\widetilde{B}_1}, h^l_{\widetilde{B}_2}\}), \\ (b^u_{11}b^u_{21}, b^u_{12}b^u_{22}, b^u_{13}b^u_{23}, b^u_{14}b^u_{24}; \min\{h^u_{\widetilde{B}_1}, h^u_{\widetilde{B}_2}\}) \end{bmatrix} \quad (3.10)$$

where b^l_{ik} and b^u_{ik} $(i = 1, 2; k = 1, 2, 3, 4)$ are all positive real numbers.

(iv) Division:

$$\widetilde{B}_1/\widetilde{B}_2 = \begin{bmatrix} (b^l_{11}/b^l_{24}, b^l_{12}/b^l_{23}, b^l_{13}/b^l_{22}, b^l_{14}/b^l_{21}; \min\{h^l_{\widetilde{B}_1}, h^l_{\widetilde{B}_2}\}), \\ (b^u_{11}/b^u_{24}, b^u_{12}/b^u_{23}, b^u_{13}/b^u_{22}, b^u_{14}/b^u_{21}; \min\{h^u_{\widetilde{B}_1}, h^u_{\widetilde{B}_2}\}) \end{bmatrix} \quad (3.11)$$

where b^l_{ik} and b^u_{ik} $(i = 1, 2; k = 1, 2, 3, 4)$ are non-zero positive real numbers.

(v) Scalar multiplication:

$$k\widetilde{B} = \begin{cases} [(kb^l_1, kb^l_2, kb^l_3, kb^l_4; h^l_{\widetilde{B}}), (kb^u_1, kb^u_2, kb^u_3, kb^u_4; h^u_{\widetilde{B}})], & k \ge 0 \\ [(kb^l_4, kb^l_3, kb^l_2, kb^l_1; h^l_{\widetilde{B}}), (kb^u_4, kb^u_3, kb^u_2, kb^u_1; h^u_{\widetilde{B}})], & k < 0 \end{cases} \quad (3.12)$$

3.3. A novel flexible ranking method for TrIT2FSs

As shown in Fig. 3, a TrIT2FS $\widetilde{B}$ can surely be regarded to be composed of an upper TrIT2FS $\widetilde{B}_U$ (see the yellow area) and a lower TrIT2FS $\widetilde{B}_L$ (see the green area). In this regard, $\widetilde{B}$ is mathematically expressed by $\widetilde{B} = \widetilde{B}_U \cup \widetilde{B}_L$.

For $\widetilde{B}^l = (b^l_1, b^l_2, b^l_3, b^l_4; h^l_{\widetilde{B}})$, if $\alpha = h^l_{\widetilde{B}}$, then $x \in [b^l_2, b^l_3]$, which means that x belongs to the core $[b^l_2, b^l_3]$ of $\widetilde{B}^l$ with membership (possibility) $h^l_{\widetilde{B}}$. Intuitively, when $\alpha \ge h^l_{\widetilde{B}}$, $x \in [b^l_2, b^l_3]$ still holds. Based on this, to make the heights of $\widetilde{B}^l$ and $\widetilde{B}^u$ equal, it is reasonable to assume that if $\alpha \in [h^l_{\widetilde{B}}, h^u_{\widetilde{B}}]$, then $x \in [b^l_2, b^l_3]$ (see Fig. 3).

As such, according to Fig. 3, it is easy to determine the α-cut set of $\widetilde{B}$ as follows:

$$\widetilde{B}(\alpha)=\begin{cases}\widetilde{B}_L(\alpha)=\begin{bmatrix}[B_L^{l-}(\alpha),B_L^{l+}(\alpha)],\\ [B_L^{u-}(\alpha),B_L^{u+}(\alpha)]\end{bmatrix}\\ \quad=\begin{bmatrix}[b_1^l+\alpha(b_2^l-b_1^l)/h_{\widetilde{B}}^l,b_4^l-\alpha(b_4^l-b_3^l)/h_{\widetilde{B}}^l],\\ [b_1^u+\alpha(b_2^u-b_1^u)/h_{\widetilde{B}}^u,b_4^u-\alpha(b_4^u-b_3^u)/h_{\widetilde{B}}^u]\end{bmatrix},\\ \quad \text{if } \alpha\in[0,h_{\widetilde{B}}^l]\\ \widetilde{B}_U(\alpha)=\begin{bmatrix}[B_U^{l-}(\alpha),B_U^{l+}(\alpha)],\\ [B_U^{u-}(\alpha),B_U^{u+}(\alpha)]\end{bmatrix}\\ \quad=\begin{bmatrix}[b_2^l,b_3^l],\\ [b_1^u+\alpha(b_2^u-b_1^u)/h_{\widetilde{B}}^u,b_4^u-\alpha(b_4^u-b_3^u)/h_{\widetilde{B}}^u]\end{bmatrix},\\ \quad \text{if } \alpha\in[h_{\widetilde{B}}^l,h_{\widetilde{B}}^u]\end{cases} \tag{3.13}$$

Definition 3.8. The expectation $E(\widetilde{B})$ of a TrIT2FS $\widetilde{B}$ is defined as

$$E(\widetilde{B})=\int_0^{h_{\widetilde{B}}^l}\widetilde{B}_L(\alpha)d\alpha+\int_{h_{\widetilde{B}}^l}^{h_{\widetilde{B}}^u}\widetilde{B}_U(\alpha)d\alpha \tag{3.14}$$

where

$$\int_0^{h_{\widetilde{B}}^l}\widetilde{B}_L(\alpha)d\alpha=\left[[E(B_L)^{l-},E(B_L)^{l+}],[E(B_L)^{u-},E(B_L)^{u+}]\right]$$
$$=\left[\begin{array}{l}[\frac{b_1^l+b_2^l}{2}h_{\widetilde{B}}^l,\frac{b_3^l+b_4^l}{2}h_{\widetilde{B}}^l],\\ [b_1^uh_{\widetilde{B}}^l+\frac{b_2^u-b_1^u}{2h_{\widetilde{B}}^u}(h_{\widetilde{B}}^l)^2,b_4^uh_{\widetilde{B}}^l-\frac{b_4^u-b_3^u}{2h_{\widetilde{B}}^u}(h_{\widetilde{B}}^l)^2]\end{array}\right] \tag{3.15}$$

and

$$\int_{h_{\widetilde{B}}^l}^{h_{\widetilde{B}}^u}\widetilde{B}_U(\alpha)d\alpha$$
$$=\left[[E(B_U)^{l-},E(B_U)^{l+}],[E(B_U)^{u-},E(B_U)^{u+}]\right]$$
$$=(h_{\widetilde{B}}^u-h_{\widetilde{B}}^l)\left[\begin{array}{l}[b_2^l,b_3^l],[\frac{b_2^uh_{\widetilde{B}}^u+m_2^uh_{\widetilde{B}}^l+m_1^uh_{\widetilde{B}}^u-m_1^uh_{\widetilde{B}}^l}{2h_{\widetilde{B}}^u},\\ \frac{m_3^uh_{\widetilde{B}}^l+m_3^uh_{\widetilde{B}}^u+m_4^uh_{\widetilde{B}}^u-m_4^uh_{\widetilde{B}}^l}{2h_{\widetilde{B}}^u}]\end{array}\right] \tag{3.16}$$

Similar to the addition of TrIT2FSs, it follows that

$$E(\widetilde{B})=\left[\begin{array}{l}[\frac{2b_2^lh_{\widetilde{B}}^u+(b_1^l-b_2^l)h_{\widetilde{B}}^l}{2},\frac{2b_3^lh_{\widetilde{B}}^u+(b_4^l-b_3^l)h_{\widetilde{B}}^l}{2}],\\ [\frac{b_1^u+b_2^u}{2}h_{\widetilde{B}}^u,\frac{b_3^u+b_4^u}{2}h_{\widetilde{B}}^u]\end{array}\right] \tag{3.17}$$

For convenience, denote

$$E(\widetilde{B})_L=[E(\widetilde{B})_L^-,E(\widetilde{B})_L^+]=[\frac{2b_2^lh_{\widetilde{B}}^u+(b_1^l-b_2^l)h_{\widetilde{B}}^l}{2},\frac{2b_3^lh_{\widetilde{B}}^u+(b_4^l-b_3^l)h_{\widetilde{B}}^l}{2}] \tag{3.18}$$

and

$$E(\widetilde{B})_U=[E(\widetilde{B})_U^-,E(\widetilde{B})_U^+]=[\frac{b_1^u+b_2^u}{2}h_{\widetilde{B}}^u,\frac{b_3^u+b_4^u}{2}h_{\widetilde{B}}^u] \tag{3.19}$$

Then, Eq. (3.17) is rewritten as $E(\widetilde{B})=[E(\widetilde{B})_L,E(\widetilde{B})_U]$.

Motivated by the definition of centroid for TrIT2FS [15,56], the average expectation (AE) of TrIT2FS $\widetilde{B}$ is given below.

Definition 3.9. Let $\widetilde{B}=[(b_1^l,b_2^l,b_3^l,b_4^l;h_{\widetilde{B}}^l),(b_1^u,b_2^u,b_3^u,b_4^u;h_{\widetilde{B}}^u)]$ be a TrIT2FS. Its AE is defined as

$$AE(\widetilde{B})=\frac{1}{2}\left(\frac{E(\widetilde{B})_L^-+E(\widetilde{B})_L^+}{2}+\frac{E(\widetilde{B})_U^-+E(\widetilde{B})_U^+}{2}\right)$$
$$=\frac{1}{8}[(b_1^l-b_2^l+b_4^l-b_3^l)h_{\widetilde{B}}^l+(2b_2^l+2b_3^l+b_1^u+b_2^u+b_3^u+b_4^u)h_{\widetilde{B}}^u] \tag{3.20}$$

In order to verify the validity of the defined AE of TrIT2FS, the AEs and centroids of different TrIT2FSs are calculated and shown in Table 1. It is easy to observe from Table 1 that for the same TrIT2FS, the difference between its AE and centroid is tiny.

Besides, the mean absolute percentage error (MAPE) is employed to calculate the error between AEs and centroids of different TrIT2FSs. The MAPE measure is calculated as

$$MAPE=\frac{1}{n}\sum_{i=1}^{n}\left|\frac{AE_i-Centroid_i}{Centroid_i}\right|\times 100\%=0.69\%$$

where n ($n=9$, herein) is the total number of observations, AE_i and $Centroid_i$ are the values of the AE and centroid of the ith observation ($i=1,2,\ldots,n$), respectively. Obviously, the smaller the value of MAPE, the more accurate and trustworthy the AE of TrIT2FS defined by this paper. Particularly, if $MAPE=0$, then the defined AE of TrIT2FSs is perfectly trustworthy according to the centroid of TrIT2FSs; if $MAPE<5\%$, then the defined AE of TrIT2FSs is considered to be very accurate and trustworthy according to the centroid of TrIT2FS. It is calculated that the MAPE between AEs and centroids of different TrIT2FSs is equal to **0.6%**. Thus, the defined AE of TrIT2FS is very reliable, which can further verify the reliability and validity of Definition 3.8.

Remark 1. According to the above analysis, it is easy to conclude that there is only little difference between the defuzzified values of TrIT2FSs derived by Eq. (3.20) and those derived by centroid-based approach [15,56]. However, the calculation of the former is remarkably simpler than that of the latter. Thus, the proposed AE of TrIT2FS is more practical than the centroid of TrIT2FS [15,56].

Definition 3.10. Let $\widetilde{B}_i=[(b_{i1}^l,b_{i2}^l,b_{i3}^l,b_{i4}^l;h_{\widetilde{B}_i}^l),(b_{i1}^u,b_{i2}^u,b_{i3}^u,b_{i4}^u;h_{\widetilde{B}_i}^u)]$ ($i=1,2$) be any two TrIT2FSs. The AE-based order relations between them are defined as follows:

(i) If $AE(\widetilde{B}_1)>AE(\widetilde{B}_2)$, then $\widetilde{B}_1\succ\widetilde{B}_2$ (i.e., $\widetilde{B}_1$ is superior to $\widetilde{B}_2$);
(ii) If $AE(\widetilde{B}_1)<AE(\widetilde{B}_2)$, then $\widetilde{B}_1\prec\widetilde{B}_2$ (i.e., $\widetilde{B}_1$ is inferior to $\widetilde{B}_2$);
(iii) If $AE(\widetilde{B}_1)=AE(\widetilde{B}_2)$, then $\widetilde{B}_1\approx\widetilde{B}_2$ (i.e., $\widetilde{B}_1$ is indifferent to $\widetilde{B}_2$).

In this paper, Eq. (3.20) is used to defuzzify the TrIT2F efficiencies of alternatives into the corresponding AE efficiencies. Then, the AE-based order relations of TrIT2FSs are used to rank alternatives based on the AE efficiencies of alternatives.

To effectively transform the TrIT2F objectives of the proposed BWM and DEA models, a flexible ranking relation of TrIT2FSs is proposed below.

Definition 3.11. Let $\widetilde{B}_i=[(b_{i1}^l,b_{i2}^l,b_{i3}^l,b_{i4}^l;h_{\widetilde{B}_i}^l),(b_{i1}^u,b_{i2}^u,b_{i3}^u,b_{i4}^u;h_{\widetilde{B}_i}^u)]$ ($i=1,2$) be two TrIT2FSs. The ranking relations between them are prescribed as follows:

(i) $\widetilde{B}_1\preccurlyeq\widetilde{B}_2$ iff $E(\widetilde{B}_1)\preccurlyeq E(\widetilde{B}_2)$;
(ii) $\widetilde{B}_1\succcurlyeq\widetilde{B}_2$ iff $E(\widetilde{B}_1)\succcurlyeq E(\widetilde{B}_2)$;
(iii) $\widetilde{B}_1\approx\widetilde{B}_2$ iff $E(\widetilde{B}_1)\succcurlyeq E(\widetilde{B}_2)$ and $E(\widetilde{B}_1)\preccurlyeq E(\widetilde{B}_2)$.

Table 1
TrIT2F reference comparisons for different linguistic terms.

Pairwise linguistic term	Crisp RC	TrIT2F RC	Centroid	AE	Error
Completely equal importance (CEI)	1	[(1.00, 1.00, 1.00, 1.00; 1), (1.00, 1.00, 1.00, 1.00; 1)]	1	1	0
Weak importance (WI)	2	[(1.00, 1.08, 1.32, 1.93; 0.9), (1.00, 1.00, 1.72, 2.62; 1)]	1.494	1.452	0.042
Moderate importance (MI)	3	[(2.52, 2.62, 2.78, 3.08; 0.9), (1.43, 2.35, 2.80, 3.34; 1)]	2.596	2.612	0.016
Moderate plus importance (MPI)	4	[(3.36, 3.48, 3.60, 3.83; 0.9), (2.15, 3.00, 3.85, 4.82; 1)]	3.507	3.510	0.003
Strong importance (SI)	5	[(4.41, 4.82, 4.98, 5.03; 0.9), (3.31, 4.25, 5.05, 6.02; 1)]	4.717	4.738	0.021
Strong plus importance (SPI)	6	[(5.64, 5.78, 5.97, 6.06; 0.9), (4.69, 5.50, 6.20, 6.95; 1)]	5.843	5.849	0.006
Very strong importance (VSI)	7	[(6.72, 6.81, 6.96, 7.11; 0.9), (5.97, 6.75, 7.12, 8.23; 1)]	6.989	6.958	0.031
Extreme importance (EI)	8	[(7.52, 7.78, 7.84, 8.08; 0.9), (7.01, 7.65, 8.00, 8.71; 1)]	7.827	7.824	0.003
Extreme more importance (EMI)	9	[(8.87, 8.92, 8.98, 9.00; 0.9), (8.03, 8.86, 9.00, 9.00; 1)]	8.758	8.833	0.075

Definition 3.12. Let $\widetilde{B}_i = [(b^l_{i1}, b^l_{i2}, b^l_{i3}, b^l_{i4}; h^l_{\widetilde{B}_i}), (b^u_{i1}, b^u_{i2}, b^u_{i3}, b^u_{i4}; h^u_{\widetilde{B}_i})]$ $(i = 1, 2)$ be two TrIT2FSs. The ranking relations between $E(\widetilde{B}_i)$ $(i = 1, 2)$ are specified as follows:

(i) $E(\widetilde{B}_1) \preccurlyeq E(\widetilde{B}_2)$ iff $E(\widetilde{B}_1)_L \preccurlyeq E(\widetilde{B}_2)_L$ and $E(\widetilde{B}_1)_U \preccurlyeq E(\widetilde{B}_2)_U$;
(ii) $E(\widetilde{B}_1) \succcurlyeq E(\widetilde{B}_2)$ iff $E(\widetilde{B}_1)_L \succcurlyeq E(\widetilde{B}_2)_L$ and $E(\widetilde{B}_1)_U \succcurlyeq E(\widetilde{B}_2)_U$;
(iii) $E(\widetilde{B}_1) \approx E(\widetilde{B}_2)$ iff $E(\widetilde{B}_1)_L \approx E(\widetilde{B}_2)_L$ and $E(\widetilde{B}_1)_U \approx E(\widetilde{B}_2)_U$.

Theorem 3.1. *Let* $\widetilde{B}_i = [(b^l_{i1}, b^l_{i2}, b^l_{i3}, b^l_{i4}; h^l_{\widetilde{B}_i}), (b^u_{i1}, b^u_{i2}, b^u_{i3}, b^u_{i4}; h^u_{\widetilde{B}_i})]$ $(i = 1, 2)$ *be two TrIT2FSs. The ranking relations of them can be converted as follows:*

(i) $\widetilde{B}_1 \preccurlyeq \widetilde{B}_2$ *iff*

$$\begin{cases} 2b^l_{13}h^u_{\widetilde{B}_1} + (b^l_{14} - b^l_{13})h^l_{\widetilde{B}_1} \\ \quad \leq 2b^l_{23}h^u_{\widetilde{B}_2} + (b^l_{24} - b^l_{23})h^l_{\widetilde{B}_2}, (b^u_{13} + b^u_{14})h^u_{\widetilde{B}_1} \leq (b^u_{23} + b^u_{24})h^u_{\widetilde{B}_2} \\ (1-2\vartheta)[2b^l_{12}h^u_{\widetilde{B}_1} + (b^l_{11} - b^l_{12})h^l_{\widetilde{B}_1} - 2b^l_{22}h^u_{\widetilde{B}_2} - (b^l_{21} - b^l_{22})h^l_{\widetilde{B}_2}] \\ \quad \leq 2\vartheta[2b^l_{23}h^u_{\widetilde{B}_2} + (b^l_{24} - b^l_{23})h^l_{\widetilde{B}_2} - 2b^l_{13}h^u_{\widetilde{B}_1} - (b^l_{14} - b^l_{13})h^l_{\widetilde{B}_1}] \\ (1-2\vartheta)[(b^u_{11} + b^u_{12})h^u_{\widetilde{B}_1} - (b^u_{21} + b^u_{22})h^u_{\widetilde{B}_2}] \\ \quad \leq 2\vartheta[(b^u_{23} + b^u_{24})h^u_{\widetilde{B}_2} - (b^u_{13} + b^u_{14})h^u_{\widetilde{B}_1}] \end{cases} \tag{3.21}$$

(ii) $\widetilde{B}_1 \succcurlyeq \widetilde{B}_2$ *iff*

$$\begin{cases} (b^u_{11} + b^u_{12})h^u_{\widetilde{B}_1} \\ \quad \geq (b^u_{21} - b^u_{22})h^u_{\widetilde{B}_2}, 2h^u_{\widetilde{B}_1}b^l_{12} + (b^l_{11} - b^l_{12})h^l_{\widetilde{B}_1} \\ \quad \geq 2h^u_{\widetilde{B}_2}b^l_{22} + (b^l_{21} - b^l_{22})h^l_{\widetilde{B}_2} \\ (1-2\vartheta)[2h^u_{\widetilde{B}_1}b^l_{13} + (b^l_{14} - b^l_{13})h^l_{\widetilde{B}_1} - 2h^u_{\widetilde{B}_2}b^l_{23} - (b^l_{24} - b^l_{23})h^l_{\widetilde{B}_2}] \\ \quad \leq 2\vartheta[2h^u_{\widetilde{B}_2}b^l_{22} + (b^l_{21} - b^l_{22})h^l_{\widetilde{B}_2} - 2h^u_{\widetilde{B}_1}b^l_{12} - (b^l_{11} - b^l_{12})h^l_{\widetilde{B}_1}] \\ (1-2\vartheta)[(b^u_{13} + b^u_{14})h^u_{\widetilde{B}_1} - (b^u_{23} + b^u_{24})h^u_{\widetilde{B}_2}] \\ \quad \leq 2\vartheta[(b^u_{21} + b^u_{22})h^u_{\widetilde{B}_2} - (b^u_{11} + b^u_{12})h^u_{\widetilde{B}_1}] \end{cases} \tag{3.22}$$

(iii) $\widetilde{B}_1 \approx \widetilde{B}_2$ *iff Eqs.* (3.21) *and* (3.22) *hold, simultaneously.*

Proof. (i) From Definition 3.3, $E(\widetilde{B}_1)_L \preccurlyeq E(\widetilde{B}_2)_L$ is equivalent to

$$\begin{cases} 2b^l_{13}h^u_{\widetilde{B}_1} + (b^l_{14} - b^l_{13})h^l_{\widetilde{B}_1} \leq 2b^l_{23}h^u_{\widetilde{B}_2} + (b^l_{24} - b^l_{23})h^l_{\widetilde{B}_2} \\ \mu(IE(\widetilde{B}_1)^l \succcurlyeq IE(\widetilde{B}_2)^l) \leq \vartheta \end{cases}$$

By Definition 3.2, $\mu\left(E(\widetilde{B}_1)_L \succcurlyeq E(\widetilde{B}_2)_L\right) \leq \vartheta$ is rewritten as given in Box I, which is further equivalent to

$$(1-2\vartheta)[2b^l_{12}h^u_{\widetilde{B}_1} + (b^l_{11} - b^l_{12})h^l_{\widetilde{B}_1} - 2b^l_{22}h^u_{\widetilde{B}_2} - (b^l_{21} - b^l_{22})h^l_{\widetilde{B}_2}]$$
$$\leq 2\vartheta[2b^l_{23}h^u_{\widetilde{B}_2} + (b^l_{24} - b^l_{23})h^l_{\widetilde{B}_2} - 2b^l_{13}h^u_{\widetilde{B}_1} - (b^l_{14} - b^l_{13})h^l_{\widetilde{B}_1}]$$

Hence, $E(\widetilde{B}_1)_L \preccurlyeq E(\widetilde{B}_2)_L$ is equivalent to

$$\begin{cases} 2b^l_{13}h^u_{\widetilde{B}_1} + (b^l_{14} - b^l_{13})h^l_{\widetilde{B}_1} \leq 2b^l_{23}h^u_{\widetilde{B}_2} + (b^l_{24} - b^l_{23})h^l_{\widetilde{B}_2} \\ (1-2\vartheta)[2b^l_{12}h^u_{\widetilde{B}_1} + (b^l_{11} - b^l_{12})h^l_{\widetilde{B}_1} - 2b^l_{22}h^u_{\widetilde{B}_2} - (b^l_{21} - b^l_{22})h^l_{\widetilde{B}_2}] \\ \quad \leq 2\vartheta[2b^l_{23}h^u_{\widetilde{B}_2} + (b^l_{24} - b^l_{23})h^l_{\widetilde{B}_2} - 2b^l_{13}h^u_{\widetilde{B}_1} - (b^l_{14} - b^l_{13})h^l_{\widetilde{B}_1}] \end{cases}$$

Similarly, $E(\widetilde{B}_1)_U \preccurlyeq E(\widetilde{B}_2)_U$ is equivalent to

$$\begin{cases} (b^u_{13} + b^u_{14})h^u_{\widetilde{B}_1} \leq (b^u_{23} + b^u_{24})h^u_{\widetilde{B}_2} \\ (1-2\vartheta)[(b^u_{11} + b^u_{12})h^u_{\widetilde{B}_1} - (b^u_{21} + b^u_{22})h^u_{\widetilde{B}_2}] \\ \quad \leq 2\vartheta[(b^u_{23} + b^u_{24})h^l_{\widetilde{B}_2} - (b^u_{13} + b^u_{14})h^u_{\widetilde{B}_1}] \end{cases}$$

By Definitions 3.11 and 3.12, it follows that $\widetilde{B}_1 \preccurlyeq \widetilde{B}_2$ iff Eq. (3.21) holds.

Similarly, the proof of Theorem 3.1(ii) can also be proven. □

Remark 2. Obviously, if $b^l_{i1} = b^u_{i1}, b^l_{i2} = b^u_{i2}, b^l_{i3} = b^u_{i3}, b^l_{i4} = b^u_{i4}$ and $h^l_{\widetilde{B}_i} = h^u_{\widetilde{B}_i} = 1$, then TrIT2FSs $\widetilde{B}_i$ $(i = 1, 2)$ degenerates into two TrFNs. In such a case, it holds that $\widetilde{B}_1 \preccurlyeq \widetilde{B}_2$ iff

$$\begin{cases} b_{13} + b_{14} \leq b_{23} + b_{24} \\ (1-2\vartheta)(b_{11} + b_{12} - b_{21} - b_{22}) \leq 2\vartheta(b_{23} + b_{24} - b_{13} - b_{14}) \end{cases}$$

which is exactly the crisp equivalent constraints of the relation $\widetilde{B}_1 \preccurlyeq \widetilde{B}_2$ proposed by Dong & Wan [23]. Similarly, the crisp equivalent constraints of the relation $\widetilde{B}_1 \succcurlyeq \widetilde{B}_2$ proposed by Dong & Wan [23] are also the special form of Theorem 3.1(ii). Hence, the ranking order of TrIT2FSs proposed in this paper generalizes the ranking order of TrFNs proposed in [23], which justifies Definition 3.11.

4. Extend traditional BWM into TrIT2F environment

This section develops a *TrIT2F-BWM* by extending the classical BWM [12,41] into TrIT2F environment. To achieve this extension, the following efforts need to be made.

(i) Transform the TrIT2F objective of the WDM (see Section 4.2);

(ii) Normalize the TrIT2F weights in the WDM (see Section 4.3);

(iii) Determine the consistency ratio (CR) for checking the reliability of the determined weights (see Section 4.5).

For convenience, some notations are prescribed as follows:

(i) $K = \{k|k = 1, 2, \ldots, t\}$, $J = \{j|j = 1, 2, \ldots, n\}$ and $I = \{i|i = 1, 2, \ldots, m\}$ are three index sets of DMs, criteria and alternatives, respectively;

(ii) $C = C_{\text{I}} \cup C_{\text{II}} = \{c_j | j \in J\}$ is the set of n criteria, where C_{I} and C_{II} respectively represent the sets of cost and benefit criteria, satisfying $C_{\text{I}} \cap C_{\text{II}} = \phi$ (ϕ indicates an empty set);

$$\frac{2b_{12}^{l}h_{\widetilde{B}_1}^{u}+(b_{11}^{l}-b_{12}^{l})h_{\widetilde{B}_1}^{l}-2b_{22}^{l}h_{\widetilde{B}_2}^{u}-(b_{21}^{l}-b_{22}^{l})h_{\widetilde{B}_2}^{l}}{2b_{23}^{l}h_{\widetilde{B}_2}^{u}+(b_{24}^{l}-b_{23}^{l})h_{\widetilde{B}_2}^{l}-2b_{22}^{l}h_{\widetilde{B}_2}^{u}-(b_{21}^{l}-b_{22}^{l})h_{\widetilde{B}_2}^{l}-2b_{13}^{l}h_{\widetilde{B}_1}^{u}-(b_{14}^{l}-b_{13}^{l})h_{\widetilde{B}_1}^{l}+2b_{12}^{l}h_{\widetilde{B}_1}^{u}+(b_{11}^{l}-b_{12}^{l})h_{\widetilde{B}_1}^{l}}\leq 2\vartheta$$

Box I.

(iii) d_k stands for the kth DM/expert;
(iv) $\widetilde{w}_k = [(w_{k1}^l, \lambda_{k2}^l, \lambda_{k3}^l, \lambda_{k4}^l; h_{\widetilde{w}_k}^l), (w_{k1}^u, w_{k2}^u, w_{k3}^u, w_{k4}^u; h_{\widetilde{w}_k}^u)]$ refers to the weight of DM d_k ($k \in K$);
(v) $\widetilde{w}_j^k = [(w_{j1}^{kl}, w_{j2}^{kl}, w_{j3}^{kl}, w_{j4}^{kl}; h_{\widetilde{w}_j^k}^l), (w_{j1}^{ku}, w_{j2}^{ku}, w_{j3}^{ku}, w_{j4}^{ku}; h_{\widetilde{w}_j^k}^u)]$ is the weight of c_j with respect to d_k ($j \in J$; $k \in K$).

4.1. Construct weight-determining model for TrIT2F-BWM

Taking the determination of DMs' weights as example, the construction steps of the WDM for the *TrIT2F-BWM* are described in detail as follows:

Step 1. Obtain TrIT2F best-to-others vector (BOV) and TrIT2F others-to-worst vector (OWV) on DMs

Firstly, according to the background information about DMs (e.g., education background, professional experience, years of experience, etc.), the project leader selects the best DM d_B and the worst DM d_W, where $B, W \in K$.

Secondly, the project leader uses linguistic RCs to express his/her preferences of d_B over other DMs d_k ($k \in K$). These linguistic RCs are further encoded into TrIT2FSs by using the interval approach-based type-2-fuzzistics methodology [15,25]. The linguistic RCs and the corresponding encoded TrIT2FSs are all listed in Table 1.

Then, the TrIT2F-BOV on DMs is determined as follows:

$$\widetilde{\boldsymbol{A}}_B = (\widetilde{a}_{B1}, \widetilde{a}_{B2}, \ldots, \widetilde{a}_{Bt}) \tag{4.1}$$

where $\widetilde{a}_{Bk} = [(a_{Bk1}^l, a_{Bk2}^l, a_{Bk3}^l, a_{Bk4}^l; h_{\widetilde{a}_{Bk}}^l), (a_{Bk1}^u, a_{Bk2}^u, a_{Bk3}^u, a_{Bk4}^u; h_{\widetilde{a}_{Bk}}^u)]$ denotes the TrIT2F RC of d_B over d_k.

Analogically, the TrIT2F-OWV on DMs can also be obtained:

$$\widetilde{\boldsymbol{A}}_W = (\widetilde{a}_{1W}, \widetilde{a}_{2W}, \ldots, \widetilde{a}_{tW})^T \tag{4.2}$$

where $\widetilde{a}_{kW} = [(a_{kW1}^l, a_{kW2}^l, a_{kW3}^l, a_{kW4}^l; h_{\widetilde{a}_{kW}}^l), (a_{kW1}^u, a_{kW2}^u, a_{kW3}^u, a_{kW4}^u; h_{\widetilde{a}_{kW}}^u)]$ refers to the TrIT2F RC of d_k over d_W.

Step 2. Construct TrIT2F WDM

The TrIT2F WDM is constructed by extending the crisp one into TrIT2F environment, as follows:

$$\begin{aligned}&\min\ \max_{k\in K}\{|\widetilde{w}_B-\widetilde{a}_{Bk}\widetilde{w}_k|, |\widetilde{w}_k-\widetilde{a}_{kW}\widetilde{w}_W|\}\\ &\text{s.t.}\begin{cases}\sum_{k=1}^{t}\widetilde{w}_k\approx\widetilde{1}\\ \widetilde{w}_k\succcurlyeq\widetilde{0}(k\in K)\end{cases}\end{aligned} \tag{M1}$$

4.2. Transform the objective of model (M1)

Let $\widetilde{\delta} = \max_{k\in K}\{|\widetilde{w}_B-\widetilde{a}_{Bk}\widetilde{w}_k|, |\widetilde{w}_k-\widetilde{a}_{kW}\widetilde{w}_W|\}$, where $\widetilde{\delta} = [(\delta_1^l, \delta_2^l, \delta_3^l, \delta_4^l; h_{\widetilde{\delta}}^l), (\delta_1^u, \delta_2^u, \delta_3^u, \delta_4^u; h_{\widetilde{\delta}}^u)]$ denotes a maximum TrIT2F deviation. Then, model (M1) is reformulated as

$$\begin{aligned}&\min\ \widetilde{\delta}\\ &\text{s.t.}\begin{cases}|\widetilde{w}_B-\widetilde{a}_{Bk}\widetilde{w}_k|\preccurlyeq\widetilde{\delta}(k\in K)\\ |\widetilde{w}_k-\widetilde{a}_{kW}\widetilde{w}_W|\preccurlyeq\widetilde{\delta}(k\in K)\\ \sum_{k=1}^{t}\widetilde{w}_k\approx\widetilde{1}\\ \widetilde{w}_k\succcurlyeq\widetilde{0}(k\in K)\end{cases}\end{aligned} \tag{M2}$$

Theorem 4.1. *Model (M2) is equivalent to model (M3) in the sense of Definition 3.11.*

$$\begin{aligned}&\min z = E(\widetilde{\delta})\\ &\text{s.t.Constraints are the } \textit{same} \text{ as } \textit{those} \text{ of } \textit{model} \text{ (M2)}\end{aligned} \tag{M3}$$

Proof. Let Θ be the set of feasible solutions of model (M2). It is obvious that Θ is also the set of feasible solutions of model (M3). Suppose that $\widetilde{\delta}^*$ is the optimal solution of model (M2). Then, for any $\widetilde{\delta} \in \Theta$, it holds that $\widetilde{\delta}^* \preccurlyeq \widetilde{\delta}$. According to Definition 3.11, $\widetilde{\delta}^* \preccurlyeq \widetilde{\delta}$ is equivalent to $E(\widetilde{\delta}^*) \preccurlyeq E(\widetilde{\delta})$. Thus, $\widetilde{\delta}^*$ is the optimal solution of model (M3).

Analogously, it can be proven that $\widetilde{\delta}^*$ is an optimal solution of model (M2) if it is an optimal solution of model (M3). This finishes the proof of Theorem 4.1. □

For convenience, we denote

$$\widetilde{w}_{Bk}=\widetilde{w}_B-\widetilde{a}_{Bk}\widetilde{w}_k=\begin{bmatrix}(w_{B1}^l-a_{Bk4}^lw_{k4}^l, w_{B2}^l-a_{Bk3}^lw_{k3}^l, w_{B3}^l-a_{Bk2}^lw_{k2}^l,\\ w_{B4}^l-a_{Bk1}^lw_{k1}^l; \min\{h_{\widetilde{w}_B}^l, h_{\widetilde{w}_k}^l, h_{\widetilde{a}_{Bk}}^l\}),\\ (w_{B1}^u-a_{Bk4}^uw_{k4}^u, w_{B2}^u-a_{Bk3}^uw_{k3}^u, w_{B3}^u-a_{Bk2}^uw_{k2}^u,\\ w_{B4}^u-a_{Bk1}^uw_{k1}^u; \min\{h_{\widetilde{w}_B}^u, h_{\widetilde{w}_k}^u, h_{\widetilde{a}_{Bk}}^u\})\end{bmatrix} \tag{4.3}$$

and

$$\widetilde{w}_{kW}=\widetilde{w}_k-\widetilde{a}_{kW}\widetilde{w}_W=\begin{bmatrix}(w_{k1}^l-a_{kW4}^lw_{W4}^l, w_{k2}^l-a_{kW3}^lw_{W3}^l, w_{k3}^l-a_{kW2}^lw_{W2}^l,\\ w_{k4}^l-a_{kW1}^lw_{W1}^l; \min\{h_{\widetilde{w}_W}^l, h_{\widetilde{w}_k}^l, h_{\widetilde{a}_{kW}}^l\}),\\ (w_{k1}^u-a_{kW4}^uw_{W4}^u, w_{k2}^u-a_{kW3}^uw_{W3}^u, w_{k3}^u-a_{kW2}^uw_{W2}^u,\\ w_{k4}^u-a_{kW1}^uw_{W1}^u; \min\{h_{\widetilde{w}_W}^u, h_{\widetilde{w}_k}^u, h_{\widetilde{a}_{kW}}^u\})\end{bmatrix} \tag{4.4}$$

Then, it holds that

$$|\widetilde{w}_B-\widetilde{a}_{Bk}\widetilde{w}_k|\preccurlyeq\widetilde{\delta}\Leftrightarrow\widetilde{w}_{Bk}\preccurlyeq\widetilde{\delta}\wedge\widetilde{w}_{Bk}\succcurlyeq-\widetilde{\delta}$$

and

$$|\widetilde{w}_k-\widetilde{a}_{kW}\widetilde{w}_W|\preccurlyeq\widetilde{\delta}\Leftrightarrow\widetilde{w}_{kW}\preccurlyeq\widetilde{\delta}\wedge\widetilde{w}_{kW}\succcurlyeq-\widetilde{\delta}.$$

According to Eqs. (3.21)–(3.22), $\widetilde{w}_{Bk} \preccurlyeq \widetilde{\delta}$, $\widetilde{w}_{kW} \preccurlyeq \widetilde{\delta}$, $\widetilde{w}_{Bk} \succcurlyeq -\widetilde{\delta}$ and $\widetilde{w}_{kW} \succcurlyeq -\widetilde{\delta}$ are equivalent to Eqs. (4.5)–(4.8), respectively.

$$\begin{cases}2h_{\widetilde{w}_{Bk}}^uw_{Bk3}^l+(w_{Bk4}^l-w_{Bk3}^l)h_{\widetilde{w}_{Bk}}^l\\ \quad\leq 2h_{\widetilde{\delta}}^u\delta_3^l+(\delta_4^l-\delta_3^l)h_{\widetilde{\delta}}^l, (w_{Bk3}^u+w_{Bk4}^u)h_{\widetilde{w}_{Bk}}^u\leq(\delta_3^u+\delta_4^u)h_{\widetilde{\delta}}^u\\ (1-2\vartheta)[2w_{Bk2}^lh_{\widetilde{w}_{Bk}}^u+(w_{Bk1}^l-w_{Bk2}^l)h_{\widetilde{w}_{Bk}}^l-2\delta_2^lh_{\widetilde{\delta}}^u\\ \quad-(\delta_1^l-\delta_2^l)h_{\widetilde{\delta}}^l]\leq 2\vartheta[2h_{\widetilde{\delta}}^u\delta_3^l+(\delta_4^l-\delta_3^l)h_{\widetilde{\delta}}^l\\ \quad-2h_{\widetilde{w}_{Bk}}^uw_{Bk3}^l-(w_{Bk4}^l-w_{Bk3}^l)h_{\widetilde{w}_{Bk}}^l]\\ (1-2\vartheta)[(w_{Bk1}^u+w_{Bk2}^u)h_{\widetilde{w}_{Bk}}^u-(\delta_1^u+\delta_2^u)h_{\widetilde{\delta}}^u]\\ \quad\leq 2\vartheta[(\delta_3^u+\delta_4^u)h_{\widetilde{\delta}}^u-(w_{Bk3}^u+w_{Bk4}^u)h_{\widetilde{w}_{Bk}}^u]\end{cases} \tag{4.5}$$

$$\begin{cases} 2h^u_{\widetilde{w}_{kW}} w^l_{kW3} + (w^l_{kW4} - w^l_{kW3})h^l_{\widetilde{w}_{kW}} \\ \quad \leq 2h^u_{\widetilde{\delta}}\delta^l_3 + (\delta^l_4 - \delta^l_3)h^l_{\widetilde{\delta}}, (w^u_{kW3} + w^u_{kW4})h^u_{\widetilde{w}_{kW}} \leq (\delta^u_3 + \delta^u_4)h^u_{\widetilde{\delta}} \\ (1-2\vartheta)[2w^l_{Bk2}h^u_{\widetilde{w}_{kW}} + (w^l_{kW1} - w^l_{kW2})h^l_{\widetilde{w}_{kW}} - 2\delta^l_2 h^u_{\widetilde{\delta}} \\ \quad -(\delta^l_1 - \delta^l_2)h^l_{\widetilde{\delta}}] \leq 2\vartheta[2h^u_{\widetilde{\delta}}\delta^l_3 + (\delta^l_4 - \delta^l_3)h^l_{\widetilde{\delta}} \\ \quad - 2h^u_{\widetilde{w}_{kW}} w^l_{kW3} - (w^l_{kW4} - w^l_{kW3})h^l_{\widetilde{w}_{kW}}] \\ (1-2\vartheta)[(w^u_{kW1} + w^u_{kW2})h^u_{\widetilde{w}_{kW}} - (\delta^u_1 + \delta^u_2)h^u_{\widetilde{\delta}}] \\ \quad \leq 2\vartheta[(\delta^u_3 + \delta^u_4)h^u_{\widetilde{\delta}} - (w^u_{kW3} + w^u_{kW4})h^u_{\widetilde{w}_{kW}}] \end{cases} \tag{4.6}$$

$$\begin{cases} 2h^u_{\widetilde{w}_{Bk}} w^l_{Bk2} + (w^l_{Bk1} - w^l_{Bk2})h^l_{\widetilde{w}_{Bk}} \\ \quad \geq -2\delta^l_3 h^u_{\widetilde{\delta}} + (\delta^l_3 - \delta^l_4)h^l_{\widetilde{\delta}}, (w^u_{Bk1} + w^u_{Bk2})h^u_{\widetilde{w}_{Bk}} \geq -(\delta^u_3 + \delta^u_4)h^u_{\widetilde{\delta}} \\ (1-2\vartheta)[2h^u_{\widetilde{w}_{Bk}} w^l_{Bk3} + (w^l_{Bk4} - w^l_{Bk3})h^l_{\widetilde{w}_{Bk}} + 2\delta^l_2 h^u_{\widetilde{\delta}} - (\delta^l_2 \\ \quad - \delta^l_1)h^l_{\widetilde{\delta}}] \geq 2\vartheta[-2\delta^l_3 h^u_{\widetilde{\delta}} + (\delta^l_3 - \delta^l_4)h^l_{\widetilde{\delta}} \\ \quad - 2h^u_{\widetilde{w}_{Bk}} w^l_{Bk2} - (w^l_{Bk1} - w^l_{Bk2})h^l_{\widetilde{w}_{Bk}}] \\ (1-2\vartheta)[(w^u_{Bk3} + w^u_{Bk4})h^u_{\widetilde{w}_{Bk}} + (\delta^u_1 + \delta^u_2)h^u_{\widetilde{\delta}}] \\ \quad \geq 2\vartheta[-(\delta^u_3 - \delta^u_4)h^u_{\widetilde{\delta}} - (w^u_{Bk1} + w^u_{Bk2})h^u_{\widetilde{w}_{Bk}}] \end{cases} \tag{4.7}$$

$$\begin{cases} 2h^u_{\widetilde{w}_{kW}} w^l_{kW2} + (w^l_{kW1} - w^l_{kW2})h^l_{\widetilde{w}_{kW}} \\ \quad \geq -2\delta^l_3 h^u_{\widetilde{\delta}} + (\delta^l_3 - \delta^l_4)h^l_{\widetilde{\delta}}, (w^u_{kW1} + w^u_{kW2})h^u_{\widetilde{w}_{kW}} \\ \quad \geq -(\delta^u_3 + \delta^u_4)h^u_{\widetilde{\delta}} \\ (1-2\vartheta)[2h^u_{\widetilde{w}_{kW}} w^l_{kW3} + (w^l_{kW4} - w^l_{kW3})h^l_{\widetilde{w}_{kW}} + 2\delta^l_2 h^u_{\widetilde{\delta}} - (\delta^l_2 \\ \quad - \delta^l_1)h^l_{\widetilde{\delta}}] \geq 2\vartheta[-2\delta^l_3 h^u_{\widetilde{\delta}} + (\delta^l_3 - \delta^l_4)h^l_{\widetilde{\delta}} \\ \quad - 2h^u_{\widetilde{w}_{kW}} w^l_{kW2} - (w^l_{kW1} - w^l_{kW2})h^l_{\widetilde{w}_{kW}}] \\ (1-2\vartheta)[(w^u_{kW3} + w^u_{kW4})h^u_{\widetilde{w}_{kW}} + (\delta^u_1 + \delta^u_2)h^u_{\widetilde{\delta}}] \\ \quad \geq 2\vartheta[-(\delta^u_3 + \delta^u_4)h^u_{\widetilde{\delta}} - (w^u_{kW1} + w^u_{kW2})h^u_{\widetilde{w}_{kW}}] \end{cases} \tag{4.8}$$

Motivated by Definition 5 presented in [23], model (M3) is further transformed into a multi-objective programming model as follows:

$$\begin{aligned} &\min\ z_1 = h^u_{\widetilde{\delta}}\delta^l_3 + (\delta^l_4 - \delta^l_3)h^l_{\widetilde{\delta}}/2 \\ &\min\ z_2 = h^u_{\widetilde{\delta}}(\delta^l_3 + \delta^l_2)/2 + (\delta^l_1 - \delta^l_2 + \delta^l_4 - \delta^l_3)h^l_{\widetilde{\delta}}/4 \\ &\min\ z_3 = (\delta^u_3 + \delta^u_4)h^u_{\widetilde{\delta}}/2 \\ &\min\ z_4 = (\delta^u_1 + \delta^u_2 + \delta^u_3 + \delta^u_4)h^u_{\widetilde{\delta}}/4 \\ &\text{s.t.} \begin{cases} \text{Eqs. (4.5)–(4.8)}\ (k \in K) \\ 0 \leq \delta^u_1 \leq \delta^l_1;\ \delta^l_4 \leq \delta^u_4;\ 0 \leq h^l_{\widetilde{\delta}} \leq h^u_{\widetilde{\delta}} \\ \delta^l_1 \leq \delta^l_2 \leq \delta^l_3 \leq \delta^l_4;\ \delta^u_1 \leq \delta^u_2 \leq \delta^u_3 \leq \delta^u_4 \\ \sum_{k=1}^{t} \widetilde{w}_k \approx \widetilde{1} \\ \widetilde{w}_k \succcurlyeq \widetilde{0}(k \in K) \end{cases} \end{aligned} \tag{M4}$$

In the following, the fuzzy constraint $\sum_{k=1}^{t} \widetilde{w}_k \approx \widetilde{1}$ of model (M4) needs to be equivalently converted into a crisp form. To complete this conversion, a TrIT2F weight normalization approach is designed.

4.3. Normalize TrIT2F weights

To normalize interval weights, Wang & Elhag [57] developed a weight-normalization approach as follows:

Theorem 4.2 ([57]). *Let* $\boldsymbol{W} = (w_1, w_2, \ldots, w_t)$ *be an interval weight vector.* $\boldsymbol{W}$ *is normalized iff the following inequalities hold:*

$$\begin{cases} w^+_k - w^-_k + \sum_{j=1}^{t} w^-_j \leq 1 (k \in K) \\ w^-_k - w^+_k + \sum_{j=1}^{t} w^+_j \geq 1 (k \in K) \end{cases} \tag{4.9}$$

where $w_k = [w^-_k, w^+_k]$ *and* $0 \leq w^-_k \leq w^+_k$ $(k \in K)$.

Definition 4.1. Let $\widetilde{\boldsymbol{W}} = (\widetilde{w}_1, \widetilde{w}_2, \ldots, \widetilde{w}_t)$ be a TrIT2F weight vector of DMs and $E(\widetilde{\boldsymbol{W}}) = (E(\widetilde{w}_1), E(\widetilde{w}_2), \ldots, E(\widetilde{w}_t))$ be the expectation of $\widetilde{\boldsymbol{W}}$. If $E(\widetilde{\boldsymbol{W}})$ is normalized, then $\widetilde{\boldsymbol{W}}$ is normalized.

Definition 4.2. $E(\widetilde{\boldsymbol{W}})$ is normalized iff all its elements $E(\widetilde{w}_k)$ $(k \in K)$ are normalized.

Definition 4.3. $E(\widetilde{w}_k)$ $(k \in K)$ are normalized iff $E(\widetilde{w}_k)_L$ $(k \in K)$ and $E(\widetilde{w}_k)_U$ $(k \in K)$ are normalized, simultaneously.

According to Definitions 4.1–4.3 and Theorem 4.2, an effective weight-normalizing theorem is initiated to normalized TrIT2F weights.

Theorem 4.3. *Let* $\widetilde{\boldsymbol{W}}$ *be a TrIT2F weight vector of DMs.* $\widetilde{\boldsymbol{W}}$ *is normalized if the following constraints hold:*

$$\begin{cases} 2w^l_{k3}h^u_{\widetilde{w}_k} + (w^l_{k4} - w^l_{k3})h^l_{\widetilde{w}_k} - 2w^l_{k2}h^u_{\widetilde{w}_k} - (w^l_{k1} - w^l_{k2})h^l_{\widetilde{w}_k} \\ \quad + \sum_{\gamma \in K}[2w^l_{\gamma 2}h^u_{\widetilde{w}_\gamma} + (w^l_{\gamma 1} - w^l_{\gamma 2})h^l_{\widetilde{w}_\gamma}] \leq 2(k \in K) \\ 2w^l_{k2}h^u_{\widetilde{w}_k} + (w^l_{k1} - w^l_{k2})h^l_{\widetilde{w}_k} - 2w^l_{k3}h^u_{\widetilde{w}_k} - (w^l_{k4} - w^l_{k3})h^l_{\widetilde{w}_k} \\ \quad + \sum_{\gamma \in K}[2w^l_{\gamma 3}h^u_{\widetilde{w}_\gamma} + (w^l_{\gamma 4} - w^l_{\gamma 3})h^l_{\widetilde{w}_\gamma}] \geq 2(k \in K) \\ (w^u_{k3} + w^u_{k4})h^u_{\widetilde{w}_k} - (w^u_{i1} + w^u_{i2})h^u_{\widetilde{w}_k} \\ \quad + \sum_{\gamma \in K}[(w^u_{\gamma 1} + w^u_{\gamma 2})h^u_{\widetilde{w}_\gamma}] \leq 2(k \in K) \\ (w^u_{k1} + w^u_{k2})h^u_{\widetilde{w}_k} - (w^u_{k3} + w^u_{k4})h^u_{\widetilde{w}_k} \\ \quad + \sum_{\gamma \in K}[(w^u_{\gamma 3} + w^u_{\gamma 4})h^u_{\widetilde{w}_\gamma}] \geq 2(k \in K) \\ 0 \leq w^u_{k1} \leq w^l_{k1};\ w^l_{k4} \leq w^u_{k4};\ 0 \leq h^l_{\widetilde{w}_k} \leq h^u_{\widetilde{w}_k} \leq 1(k \in K) \\ w^l_{k1} \leq w^l_{k2} \leq w^l_{k3} \leq w^l_{k4};\ w^u_{k1} \leq w^u_{k2} \leq w^u_{k3} \leq w^u_{k4}(k \in K) \end{cases} \tag{4.10}$$

Proof. According to Definition 4.1, $\widetilde{\boldsymbol{W}}$ is normalized if $E(\widetilde{\boldsymbol{W}})$ is normalized. By Definition 4.2, $E(\widetilde{\boldsymbol{W}})$ is normalized iff $E(\widetilde{w}_k)$ $(k \in K)$ are normalized. In terms of Definition 4.3, $E(\widetilde{w}_k)$ $(k \in K)$ are normalized iff $E(\widetilde{w}_k)_L$ $(k \in K)$ and $E(\widetilde{w}_k)_U$ $(k \in K)$ are normalized, simultaneously. Thus, according to Theorem 4.2 and the definition of TrIT2FSs, $\widetilde{\boldsymbol{W}}$ is normalized if Eq. (4.10) holds. This fulfills the proof of Theorem 4.3. □

Remark 3. It is obvious that Eq. (4.10) has higher power in preserving the information of TrIT2FSs than the centroid-based defuzzification approaches [15]. In addition, the lower and upper heights of TrIT2FSs are contained in Eq. (4.10), while they are overlooked in approach [4]. Since the lower and upper heights are the essential components of TrIT2FSs, the proposed TrIT2F weight-normalizing approach can retain more inherent information of TrIT2FSs.

4.4. Solve the transformed crisp model

For convenience, let $h^l_{\widetilde{\delta}} = h^l_{\widetilde{w}_k} = \min_k\{h^l_{\widetilde{a}_{Bk}}, h^l_{\widetilde{a}_{kW}}\}$ and $h^u_{\widetilde{\delta}} = h^u_{\widetilde{w}_k} = \min_k\{h^u_{\widetilde{a}_{Bk}}, h^u_{\widetilde{a}_{kW}}\}$. Based on model (M4) and Eq. (4.10), the TrIT2F model (M1) of the proposed *TrIT2F-BWM* is finally converted into the following crisp multi-objective linear programming model.

$$\begin{aligned}
&\min\ z_1 = h^u_{\widetilde{\delta}}\delta_3^l + (\delta_4^l - \delta_3^l)h^l_{\widetilde{\delta}}/2\\
&\min\ z_2 = h^u_{\widetilde{\delta}}(\delta_3^l + \delta_2^l)/2 + (\delta_1^l - \delta_2^l + \delta_4^l - \delta_3^l)h^l_{\widetilde{\delta}}/4\\
&\min\ z_3 = (\delta_3^u + \delta_4^u)h^u_{\widetilde{\delta}}/2\\
&\min\ z_4 = (\delta_1^u + \delta_2^u + \delta_3^u + \delta_4^u)h^u_{\widetilde{\delta}}/4\\
&\text{s.t.}\begin{cases}\text{Eqs. (4.5)–(4.8) and (4.10) } (k \in K)\\ 0 \le \delta_1^u \le \delta_1^l;\ \delta_4^l \le \delta_4^u;\ 0 \le h^l_{\widetilde{\delta}} \le h^u_{\widetilde{\delta}}\\ \delta_1^l \le \delta_2^l \le \delta_3^l \le \delta_4^l;\ \delta_1^u \le \delta_2^u \le \delta_3^u \le \delta_4^u\end{cases}
\end{aligned}\tag{M5}$$

In the following, the efficient goal programming approach [23] is employed to solve model (M5).

Step 1. Obtain the goal of each objective z_κ by solving the following single-objective model:

$$\begin{aligned}&\min\ z_\kappa\\ &\text{s.t. constraints are the same as those of model (M5)}\end{aligned}$$

It is evident that the objectives z_κ ($\kappa = 1, 2, 3, 4$) can be regarded as the functions of the auxiliary variable vector $\boldsymbol{\delta} = (\delta_1^l, \delta_2^l, \delta_3^l, \delta_4^l, \delta_1^u, \delta_2^u, \delta_3^u, \delta_4^u)$. Then, the relation between z_κ and $\boldsymbol{\delta}$ can be denoted by $z_\kappa = z_\kappa(\boldsymbol{\delta})$ ($\kappa = 1, 2, 3, 4$). Solving the above single-objective model with Lingo software, the optimal objective value $z_\kappa^{\min}$ and its corresponding solution $\boldsymbol{\delta}_\kappa^*$ can be easily acquired.

Generally, the goal of objective z_κ can be written as $\sigma z_\kappa^{\min}$ ($\kappa = 1, 2, 3, 4$), where σ is the proportion parameter, such as $\sigma = 0.85$ and $\sigma = 0.9$.

Step 2. Obtain the priority factor φ_k allocated to each objective z_κ ($\kappa = 1, 2, 3, 4$). The priority factor φ_κ can be predetermined by DMs.

Step 3. The TrIT2F weights of DMs can be determined by solving the following linear goal programming model.

$$\begin{aligned}&\min\ \sum_{\kappa=1}^{4}\varphi_\kappa(q_\kappa^- + q_\kappa^+)\\ &\text{s.t.}\begin{cases} z_\kappa + q_\kappa^- - q_\kappa^+ = \sigma z_\kappa^{\min}(\kappa = 1, 2, 3, 4)\\ q_\kappa^-, q_\kappa^+ \ge 0(\kappa = 1, 2, 3, 4)\\ \text{constraints are the same as those of model (M5)}\end{cases}\end{aligned}\tag{M6}$$

4.5. Determine CR for the TrIT2F-BWM

Liang et al. [58] proposed an input-based CR in view of the cardinal consistency $a_{bi} \times a_{iw} - a_{bw}$ ($a_{bw} \in [1, 9]$) for checking the consistency of the sort of the results against the sort of the RCs provided by DM.

$$CR^{In} = \max\{CR_i^{In} | i = 1, 2, \ldots, n\}\tag{4.11}$$

where

$$CR_i^{In} = \begin{cases}\dfrac{|a_{bi} \times a_{iw} - a_{bw}|}{a_{bw} \times a_{bw} - a_{bw}}, & \text{if } a_{bw} > 1\\ 0, & \text{if } a_{bw} = 1\end{cases}\tag{4.12}$$

CR^{In} is the global input-based CR for all criteria, CR_i^{In} denotes the local consistency level associated with criterion C_i. Liang et al. [58] also presented the approximated thresholds for the input-based CR^{In} (see Table 3 in [58]). Inspired by the idea of [58], a global input-based CR for the proposed *TrIT2F-BWM* is defined as follows:

$$CR_{TrIT2FS}^{In} = \max_{i=1}^{n}\{|AE(\widetilde{CR}_i^{In})|\}\tag{4.13}$$

where

$$\widetilde{CR}_i^{In} = \begin{cases}\dfrac{\widetilde{a}_{Bi} \times \widetilde{a}_{iW} - \widetilde{a}_{BW}}{\widetilde{a}_{BW} \times \widetilde{a}_{BW} - \widetilde{a}_{BW}}, & \text{if } \widetilde{a}_{BW} \succ \widetilde{1}\\ 0, & \text{if } \widetilde{a}_{BW} \approx \widetilde{1}\end{cases}\tag{4.14}$$

Remark 4. Before determining the weights of DMs and criteria, it requires to use $CR_{TrIT2FS}^{In}$ to check the consistency of RCs. In this paper, the thresholds of $CR_{TrIT2FS}^{In}$ can refer to those of CR^{In} presented in [58]. For the same scale and number of criteria, if $CR_{TrIT2FS}^{In} \le \xi$ (ξ is the threshold of CR^{In}), then the determined TrIT2F weights are regarded to be acceptable. Otherwise, the TrIT2F RCs of c_B over c_k and c_k over c_W must be adjusted until $CR_{TrIT2FS}^{In} \le \xi$.

5. Extend DEA into TrIT2F environment

Traditional DEA is an efficient tool to measure DMUs' performance, in which all inputs and outputs are quantitative and identified with crisp values. Thus, traditional DEA is inapplicable to the decision problems involving complexity, imprecision and uncertainty. Although various fuzzy versions have been developed, such as triangular fuzzy DEA [59] and intuitionistic fuzzy DEA [20], they are inapplicable for solving the FHL problem in TrIT2F environment. Therefore, it is necessary to extend DEA into TrIT2F environment. In what follows, a fully *TrIT2F-DEA* is presented, in which all the parameters and variables are represented by TrIT2FSs.

For the convenience of the following text, some symbols and variables are interpreted as follows:

(i) $i \in \{1, 2, \ldots, m\}$ implies the subscript of DMU (alternative).

(ii) $\tau \in \{1, 2, \ldots, p\}$ means the subscript of input (cost) criterion, and $C_I = \{c_\tau^I | \tau = 1, 2, \ldots, p\}$.

(iii) $r \in \{1, 2, \ldots, s\}$ indicates the subscript of output (benefit) criterion, and $C_{II} = \{c_r^O | r = 1, 2, \ldots, s\}$.

(iv) $\widetilde{u}_r$ ($\widetilde{v}_\tau$) represents the TrIT2F weight of the rth (ith) output (input) criterion for DMUs.

(v) $\widetilde{y}_{ri}$ and $\widetilde{x}_{\tau i}$ refer to the TrIT2F amounts of c_r^O and c_τ^I for DMU_i, respectively.

(vi) $\widetilde{y}_{ro}$ and $\widetilde{x}_{\tau o}$ are the TrIT2F amounts of c_r^O and c_τ^I for the observed DMU_o, respectively.

Based on the above description, a fully *TrIT2F-DEA* model is constructed as follows:

$$\begin{aligned}&\max \widetilde{z}_o = \sum_{r=1}^{s}\widetilde{u}_r\widetilde{y}_{ro}\\ &\text{s.t.}\begin{cases}\sum_{\tau=1}^{p}\widetilde{v}_\tau\widetilde{x}_{\tau o} \approx \widetilde{1}\\ \sum_{r=1}^{s}\widetilde{u}_r\widetilde{y}_{ri} \preccurlyeq \sum_{\tau=1}^{p}\widetilde{v}_\tau\widetilde{x}_{\tau i}(i = 1, 2, \ldots, m)\\ \widetilde{u}_r, \widetilde{v}_\tau \succcurlyeq \widetilde{0}(r = 1, 2, \ldots, s;\ \tau = 1, 2, \ldots, p)\end{cases}\end{aligned}\tag{M7}$$

Let $\widetilde{Y}_o = \sum_{r=1}^{s}\widetilde{u}_r\widetilde{y}_{ro}$, $\widetilde{X}'_{\tau o} = \widetilde{v}_\tau\widetilde{x}_{\tau o}$, $\widetilde{Y}_i = \sum_{r=1}^{s}\widetilde{u}_r\widetilde{y}_{ri}$, $\widetilde{X}_i = \sum_{\tau=1}^{p}\widetilde{v}_\tau\widetilde{x}_{\tau i}$, $h^l_{\widetilde{u}_r} = \min_i\{h^l_{\widetilde{y}_{ri}}\}$, $h^u_{\widetilde{u}_r} = \min_i\{h^u_{\widetilde{y}_{ri}}\}$, $h^l_{\widetilde{v}_\tau} = \min_i\{h^l_{\widetilde{x}_{\tau i}}\}$ and $h^u_{\widetilde{v}_\tau} = \min_i\{h^u_{\widetilde{x}_{\tau i}}\}$. Then, similar to the transformation from model (M2) into model (M5), model (M7) can be transformed into a crisp multi-objective linear programming model.

$$
\begin{aligned}
&\max z_o^1 = h^u_{\widetilde{Y}_o} Y^l_{o2} + (Y^l_{o1} - Y^l_{o2}) h^l_{\widetilde{Y}_o}/2\\
&\max z_o^2 = h^u_{\widetilde{Y}_o} (Y^l_{o2} + Y^l_{o3})/2 + (Y^l_{o1} - Y^l_{o2} + Y^l_{o4} - Y^l_{o3}) h^l_{\widetilde{Y}_o}/4\\
&\max z_o^3 = (Y^u_{o1} + Y^u_{o2}) h^u_{\widetilde{Y}_o}/2\\
&\max z_o^4 = (Y^u_{o1} + Y^u_{o2} + Y^u_{o3} + Y^u_{o4}) h^u_{\widetilde{Y}_o}/2\\
&\text{s.t.}\begin{cases}
2x'^l_{\tau o3} h_{\widetilde{x}'^u_{\tau o}} + (x'^l_{\tau o4} - x'^l_{\tau o3}) h_{\widetilde{x}'^l_{\tau o}} - 2x'^l_{\tau o2} h_{\widetilde{x}'^u_{\tau o}}\\
\quad - (x'^l_{\tau o1} - x'^l_{\tau o2}) h_{\widetilde{x}'^l_{\tau o}}\\
\quad + \sum_{\gamma=1}^{p} [2x'^l_{\gamma o2} h_{\widetilde{x}'^u_{\gamma o}} + (x'^l_{\gamma o1} - x'^l_{\gamma o2}) h_{\widetilde{x}'^l_{\gamma o}}]\\
\ \leq 2 (\tau = 1, 2, \ldots, p)\\
2x'^l_{\tau o2} h_{\widetilde{x}'^u_{\tau o}} + (x'^l_{\tau o1} - x'^l_{\tau o2}) h_{\widetilde{x}'^l_{\tau o}} - 2w^l_{k3} h_{\widetilde{x}'^u_{\tau o}}\\
\quad - (x'^l_{\tau o4} - x'^l_{\tau o3}) h_{\widetilde{x}'^l_{\tau o}} + \sum_{\gamma=1}^{p} [2x'^l_{\gamma o3} h_{\widetilde{x}'^u_{\gamma o}}\\
\quad + (x'^l_{\gamma o4} - x'^l_{\gamma o3}) h_{\widetilde{x}'^l_{\gamma o}}]\\
\ \geq 2 (\tau = 1, 2, \ldots, p)\\
(x'^u_{\tau o3} + x'^u_{\tau o4}) h_{\widetilde{x}'^u_{\tau o}} - (x'^u_{\tau o1} + x'^u_{\tau o2}) h_{\widetilde{x}'^u_{\tau o}}\\
\quad + \sum_{\gamma=1}^{t} [(x'^u_{\gamma o1} + x'^u_{\gamma o2}) h_{\widetilde{x}'^u_{\gamma o}}] \leq 2 (\tau = 1, 2, \ldots, p)\\
(x'^u_{\tau o1} + x'^u_{\tau o2}) h_{\widetilde{x}'^u_{\tau o}} - (x'^u_{\tau o3} + x'^u_{\tau o4}) h_{\widetilde{x}'^u_{\tau o}}\\
\quad + \sum_{\gamma=1}^{t} [(x'^u_{\gamma o3} + x'^u_{\gamma o4}) h_{\widetilde{x}'^u_{\gamma o}}] \geq 2 (\tau = 1, 2, \ldots, p)\\
2h^u_{\widetilde{Y}_i} Y^l_{i3} + (Y^l_{i4} - Y^l_{i3}) h^l_{\widetilde{Y}_i} \leq 2h^u_{\widetilde{X}_i} X^l_{i3}\\
\quad + (X^l_{i4} - X^l_{i3}) h^l_{\widetilde{X}_i}, (Y^u_{i3} + Y^u_{i4}) h^u_{\widetilde{Y}_i}\\
\ \leq (X^u_{i3} + X^u_{i4}) h^u_{\widetilde{X}_i} (i = 1, 2, \ldots, m)\\
(1 - 2\upsilon)[2h^u_{\widetilde{Y}_i} Y^l_{i2} + (Y^l_{i1} - Y^l_{i2}) h^l_{\widetilde{Y}_i} - 2h^u_{\widetilde{X}_i} X^l_{i2}\\
\quad - (X^l_{i1} - X^l_{i2}) h^l_{\widetilde{X}_i}] \leq 2\upsilon[2h^u_{\widetilde{X}_i} X^l_{i3} + (X^l_{i4} - X^l_{i3}) h^l_{\widetilde{X}_i}\\
\quad - 2h^u_{\widetilde{Y}_i} Y^l_{i3} - (Y^l_{i4} - Y^l_{i3}) h^l_{\widetilde{Y}_i}] (i = 1, 2, \ldots, m)\\
(1 - 2\upsilon)[(Y^u_{i1} + Y^u_{i2}) h^u_{\widetilde{Y}_i} - (X^u_{i1} + X^u_{i2}) h^u_{\widetilde{X}_i}]\\
\ \leq 2\upsilon[(X^u_{i3} + X^u_{i4}) h^u_{\widetilde{X}_i} - (Y^u_{i3} + Y^u_{i4}) h^u_{\widetilde{Y}_i}] (i = 1, 2, \ldots, m)\\
0 \leq u^l_{r1} \leq \upsilon^l_{\tau 1}; \upsilon^u_{\tau 4} \leq u^u_{r1} (r = 1, 2, \ldots, s; \tau = 1, 2, \ldots, p)\\
u^l_{r1} \leq u^l_{r2} \leq u^l_{r3} \leq u^l_{r4}; u^u_{r1} \leq u^u_{r2} \leq u^u_{r3} \leq u^u_{r4};\\
\upsilon^l_{\tau 1} \leq \upsilon^l_{\tau 2} \leq \upsilon^l_{\tau 3} \leq \upsilon^l_{\tau 4}; \upsilon^u_{\tau 1} \leq \upsilon^u_{\tau 2} \leq \upsilon^u_{\tau 3} \leq \upsilon^u_{\tau 4}\\
(r = 1, 2, \ldots, s; \tau = 1, 2, \ldots, p)
\end{cases}
\end{aligned}
\tag{M8}
$$

Remark 5. Undoubtedly, it is inadmissible that the DMU's total output is larger than its total input, which indicates that the fuzzy constraint $\sum_{r=1}^{s} \widetilde{u}_r \widetilde{y}_{ri} \precsim \sum_{\tau=1}^{p} \widetilde{\upsilon}_\tau \widetilde{x}_{\tau i}$ is completely not permitted to be violated. Hence, the value of acceptance degree υ in model (M8) is taken as 0.

By using the same method presented in Section 4.4, model (M8) is finally converted into a crisp single objective model as follows:

$$
\begin{aligned}
&\min \sum_{\kappa=1}^{4} \varphi_\kappa (q_\kappa^- + q_\kappa^+)\\
&\text{s.t.}\begin{cases}
z_o^\kappa + q_\kappa^- - q_\kappa^+ = \rho z_o^{\kappa-\max} (\kappa = 1, 2, 3, 4)\\
q_\kappa^-, q_\kappa^+ \geq 0 (\kappa = 1, 2, 3, 4)\\
\text{constraints are the same as those of model (M8)}
\end{cases}
\end{aligned}
\tag{M9}
$$

where ρ is the proportion parameter, such as $\rho = 1.15$ and $\rho = 1.1$, $z_o^{\kappa-\max}$ means the optimal objective value of model (M8) when only the objective z_o^κ is considered ($\kappa = 1, 2, 3, 4$).

By solving model (M9), the optimal values of $\widetilde{u}_r^* = [(u^{l*}_{r1}, u^{l*}_{r2}, u^{l*}_{r3}, u^{l*}_{r4}; h^l_{\widetilde{u}_r}), (u^{u*}_{r1}, u^{u*}_{r2}, u^{u*}_{r3}, u^{u*}_{r4}; h^u_{\widetilde{u}_r})]$ $(r = 1, 2, \ldots, s)$ can be acquired. Then, the optimal TrIT2F efficiency of the observed DMU_o is calculated as $\widetilde{z}_o^* = \sum_{r=1}^{s} \widetilde{u}_r^* \widetilde{y}_{ro}$. The optimal TrIT2F efficiency of DMU_i is denoted by $\widetilde{z}_i^*$ $(i \in I)$. Lastly, in terms of Definition 3.10, the priority of alternatives can be ranked in descending order according to the values of $AE(\widetilde{z}_i^*)$ $(i \in I)$.

6. An integrated TrIT2F BWM & DEA methodology

This section aims to investigate an integrated methodology (i.e., *TrIT2F-BWM-DEA*) by combining the proposed *TrIT2F-BWM* and *TrIT2F-DEA*. In this decision making system, there exist m alternatives (DMUs) χ_i $(i \in I)$, t DMs d_k $(k \in K)$ and n criteria c_j $(j \in J)$. Without loss of generality, let $C_I = \{c_1, c, \ldots, c_p\}$ and $C_{II} = \{c_{p+1}, c_{p+2}, \ldots, c_{p+s}\}$, where $p + s = n$. DMs use the linguistic terms presented in Table 2 of [7] to express their evaluations of alternatives under each criterion.

The steps of the proposed *TrIT2F-BWM-DEA* are stated as follows:

Step 1. Determine the TrIT2F weights of DMs and criteria

The TrIT2F weights of DMs $\widetilde{w}_k$ $(k \in K)$ and the TrIT2F weights of criteria $\widetilde{w}_j^k$ $(k \in K, j \in J)$ can be determined by the proposed *TrIT2F-BWM* presented in Section 4.

Step 2. Acquire the TrIT2F evaluation matrices $\widetilde{Q}^k = (\widetilde{q}_{ji}^k)_{n \times m}$

$$
\widetilde{Q}^k = \begin{array}{c} \\ c_1 \\ \vdots \\ c_p \\ c_{p+1} \\ \vdots \\ c_{p+s} \end{array}
\begin{array}{c} \begin{array}{cccc} \chi_1 & \chi_2 & \cdots & \chi_m \end{array} \\
\begin{pmatrix}
\widetilde{q}_{11}^k & \widetilde{q}_{12}^k & \cdots & \widetilde{q}_{1m}^k\\
\vdots & \vdots & \ddots & \vdots\\
\widetilde{q}_{p1}^k & \widetilde{q}_{p2}^k & \cdots & \widetilde{q}_{pm}^k\\
\widetilde{q}_{p+1,1}^k & \widetilde{q}_{p+1,2}^k & \cdots & \widetilde{q}_{p+1,m}^k\\
\vdots & \vdots & \ddots & \vdots\\
\widetilde{q}_{p+s,1}^k & \widetilde{q}_{p+s,2}^k & \cdots & \widetilde{q}_{p+s,m}^k
\end{pmatrix}\end{array}
\tag{6.1}
$$

where $\widetilde{q}_{ji}^k$ refers to the TrIT2F evaluation of χ_i on c_j, which is encoded from the linguistic evaluation provided by d_k $(i \in I, j \in J, k \in K)$ with the type-2-fuzzistics methodology in [15,25].

Step 3. Calculate the weighted evaluation matrix (WEM) $\widetilde{Q}_w^k = (\widetilde{\widetilde{q}}_{ji}^k)_{n \times m}$ as follows:

$$
\begin{aligned}
\widetilde{\widetilde{q}}_{ji}^k = \widetilde{w}_j^k \widetilde{q}_{ji}^k = [&(w_{j1}^{kl} q_{ji1}^{kl}, w_{j2}^{kl} q_{ji2}^{kl}, w_{j3}^{kl} q_{ji3}^{kl}, w_{j4}^{kl} q_{ji4}^{kl}; \min\{h^l_{\widetilde{q}_{ji}^k}, h^l_{\widetilde{w}_j^k}\})\\
&, (w_{j1}^{ku} q_{ji1}^{ku}, w_{j2}^{ku} q_{ji2}^{ku}, w_{j3}^{ku} q_{ji3}^{ku}, w_{j4}^{ku} q_{ji4}^{ku}; \min\{h^u_{\widetilde{q}_{ji}^k}, h^u_{\widetilde{w}_j^k}\})]
\end{aligned}
\tag{6.2}
$$

Step 4. Obtain the aggregated evaluation matrix (AEM) $\widetilde{Q}_g$ as follows:

$$\widetilde{Q}_g = \begin{matrix} & \chi_1 & \chi_2 & \cdots & \chi_m \\ c_1 & \sum_{k\in K}\widetilde{w}_k\widetilde{w}_1^k\widetilde{q}_{11} & \sum_{k\in K}\widetilde{w}_k\widetilde{w}_1^k\widetilde{q}_{12} & \cdots & \sum_{k\in K}\widetilde{w}_k\widetilde{w}_1^k\widetilde{q}_{1m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_p & \sum_{k\in K}\widetilde{w}_k\widetilde{w}_p^k\widetilde{q}_{p1} & \sum_{k\in K}\widetilde{w}_k\widetilde{w}_p^k\widetilde{q}_{p2} & \cdots & \sum_{k\in K}\widetilde{w}_k\widetilde{w}_p^k\widetilde{q}_{pm} \\ c_{p+1} & \sum_{k\in K}\widetilde{w}_k\widetilde{w}_{p+1}^k\widetilde{q}_{p+1,1} & \sum_{k\in K}\widetilde{w}_k\widetilde{w}_{p+1}^k\widetilde{q}_{p+1,2} & \cdots & \sum_{k\in K}\widetilde{w}_k\widetilde{w}_{p+1}^k\widetilde{q}_{p+1,m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{p+s} & \sum_{k\in K}\widetilde{w}_k\widetilde{w}_{p+s}^k\widetilde{q}_{p+s,1} & \sum_{k\in K}\widetilde{w}_k\widetilde{w}_{p+s}^k\widetilde{q}_{p+s,2} & \cdots & \sum_{k\in K}\widetilde{w}_k\widetilde{w}_{p+s}^k\widetilde{q}_{p+s,m} \end{matrix}$$

$$= \begin{matrix} & \chi_1 & \chi_2 & \cdots & \chi_m \\ c_1 & \widetilde{x}_{11} & \widetilde{x}_{12} & \cdots & \widetilde{x}_{1m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_p & \widetilde{x}_{p1} & \widetilde{x}_{p2} & \cdots & \widetilde{x}_{pm} \\ c_{p+1} & \widetilde{y}_{11} & \widetilde{y}_{12} & \cdots & \widetilde{y}_{1m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{p+s} & \widetilde{y}_{s1} & \widetilde{y}_{s2} & \cdots & \widetilde{y}_{sm} \end{matrix} \quad (6.3)$$

Step 5. Rank alternatives

From Eq. (6.3), it is easy to obtain each alternative's TrIT2F inputs and outputs (i.e., $\widetilde{y}_{ri}$ and $\widetilde{x}_{\tau i}$). Solving model (M7) with these outputs and inputs, the TrIT2F efficiencies of all alternatives can be determined. Then, according to Definition 3.10, the alternatives can be ranked.

The whole flowchart of the proposed *TrIT2F-BWM-DEA* is shown in Fig. 4.

7. Real application to Fangcang hospital location amid COVID-19

In this section, the validity of the proposed *TrIT2F-BWM-DEA* is demonstrated with the real location problem of Fangcang hospitals. The detailed information about this real case can be consulted in [4]. Some comparative analyses are conducted to illustrate the stability, and merits of the *TrIT2F-BWM-DEA*.

7.1. Evaluation process

Most countries around the world have been attacked by COVID-19 since it was identified at the end of 2019. Unfortunately, Wuhan was also seriously attacked by COVID-19 from December 2019 to April 2020. To jointly treat the COVID-19 patients with mild symptom and realize the mission of "leave no one unattended", Hubei provincial government launched the design and conversion of Fangcang hospitals in Wuhan. The proposed *TrIT2F-BWM-DEA* is used to solve the FHL problem. Without loss of generality, this real case only considers the site selection for the first Fangcang hospital. Four selection stages are implemented as follows:

Stage I. Identify alternatives. Five alternatives are initially qualified to be converted into Fangcang hospitals, i.e., Wuhan Sports Center (χ_1), Wuhan International Conference & Exhibition Center (χ_2), Hongshan Gymnasium (χ_3), Wuhan Gymnasium (χ_4) and Wuhan Mass Fitness Center (χ_5).

Stage II. Select criteria. Eight criteria can be identified based on the requirements on the design and reconstruction of makeshift (Fangcang) hospitals (see http://zjt.hubei.gov.cn/) as follows: reconstruction difficulty (c_1), reconstruction cost (c_2), geographical position (c_3), infrastructure (c_4), regional communication convenience (c_5), capacity (c_6), traffic convenience (c_7) and environmental protection (c_8). It is obvious that $c_1, c_2 \in C_{\text{I}}$ and $c_\gamma \in C_{\text{II}}$ ($\gamma = 3, 4, \ldots, 8$).

Stage III. Compare criteria and evaluate alternatives. Five DMs d_k ($k = 1, 2, \ldots, 5$) from different professional fields (e.g., architectural design institutes, universities and hospitals) are invited to jointly participate in this FHL decision making.

Note The linguistic BOV and OWV on DMs' expertise given by the project leader are the same as those in Table 6 of [4]. The linguistic BOV and OWV on the importance of criteria given by each DM are the same as those in Table 7 of [4]. The linguistic evaluations of alternatives χ_i ($i = 1, 2, \ldots, 5$) on criteria c_j ($j = 1, 2, \ldots, 8$) given by each DM are the same as those in Table 8 of [4].

Stage IV. Determine the best alternative. The proposed *TrIT2F-BWM-DEA* is employed to determine the weights of DMs (criteria) and measure the efficiencies of alternatives χ_i ($i = 1, 2, \ldots, 5$).

Taking the case of $\vartheta = 0$ (i.e., DM is completely conservative) as a representative example, the assessment process is presented as follows:

Step 1. Determine the TrIT2F weights of DMs and criteria by using the proposed *TrIT2F-BWM.*

Based on Table 1, the linguistic RCs of DMs given by project leader can be encoded into TrIT2F ones. By using Eq. (4.13), the input-based CR of these TrIT2F RCs are calculated as $CR_{TrIT2FS}^{In} = 0.289$. Since 0.289 is smaller than the threshold 0.3062 than presented in [58], the TrIT2F RCs are acceptable and can be used to determine the weights of DMs. Without loss of generality, let $\varphi_1 = \varphi_3 = 100$, $\varphi_2 = \varphi_4 = 1$ and $\sigma = 0.9$. Then, the TrIT2F weights of DMs are derived by solving model (M6).

$$\widetilde{w}_1 = [(0.054,\ 0.054,\ 0.054,\ 0.054; 0.9),\ (0.049,\ 0.049,\ 0.049,\ 0.054; 1)];$$

$$\widetilde{w}_2 = [(0.109,\ 0.109,\ 0.109,\ 0.109; 0.9),\ (0.109,\ 0.11,\ 0.12,\ 0.12; 1)];$$

$$\widetilde{w}_3 = [(0.548,\ 0.551,\ 0.565,\ 0.569; 0.9),\ (0.548,\ 0.548,\ 0.548,\ 0.569; 1)];$$

$$\widetilde{w}_4 = [(0.084,\ 0.084,\ 0.101,\ 0.101; 0.9),\ (0.084,\ 0.097,\ 0.1,\ 0.101; 1)];$$

$$\widetilde{w}_5 = [(0.17,\ 0.17,\ 0.187,\ 0.187; 0.9),\ (0.17,\ 0.171,\ 0.174,\ 0.187; 1)].$$

Similarly, it is easy to verify that all the TrIT2F RCs on criteria given by DMs are also acceptable. Then, the criteria weights $\widetilde{w}_j^k$ ($k = 1, 2 \cdots, 5; j = 1, 2, \ldots, 8$) can also be determined by the proposed *TrIT2F-BWM.*

Step 2. Identify the TrIT2F evaluation matrix $\widetilde{Q}^k = (\widetilde{q}_{ji}^k)_{n\times m}$ given by d_k. According to Table 2 of [7], the linguistic evaluations of alternatives can be encoded into TrIT2F evaluations. Then, the TrIT2F evaluation matrices $\widetilde{Q}^k = (\widetilde{q}_{ji}^k)_{n\times m}$ ($k = 1, 2, \ldots, 5$) are easily obtained.

Step 3. Construct the weighted evaluation matrix $\widetilde{Q}_w^k = (\overline{\widetilde{q}}_{ji}^k)_{n\times m}$ ($k = 1, 2, \ldots, 5$). Based on the identified $\widetilde{Q}^k$ and the obtained criteria weights $\widetilde{w}_j^k$ ($j = 1, 2, \ldots, 8$), the weighted evaluation matrices $\widetilde{Q}_w^k$ ($k = 1, 2, \ldots, 5$) can be constructed by using Eq. (6.2).

Step 4. Obtain the aggregated evaluation matrix $\widetilde{Q}_g$. According to Eq. (6.3), $\widetilde{Q}_g$ is obtained. Here, only the values of $\widetilde{x}_{11}$ and $\widetilde{y}_{11}$

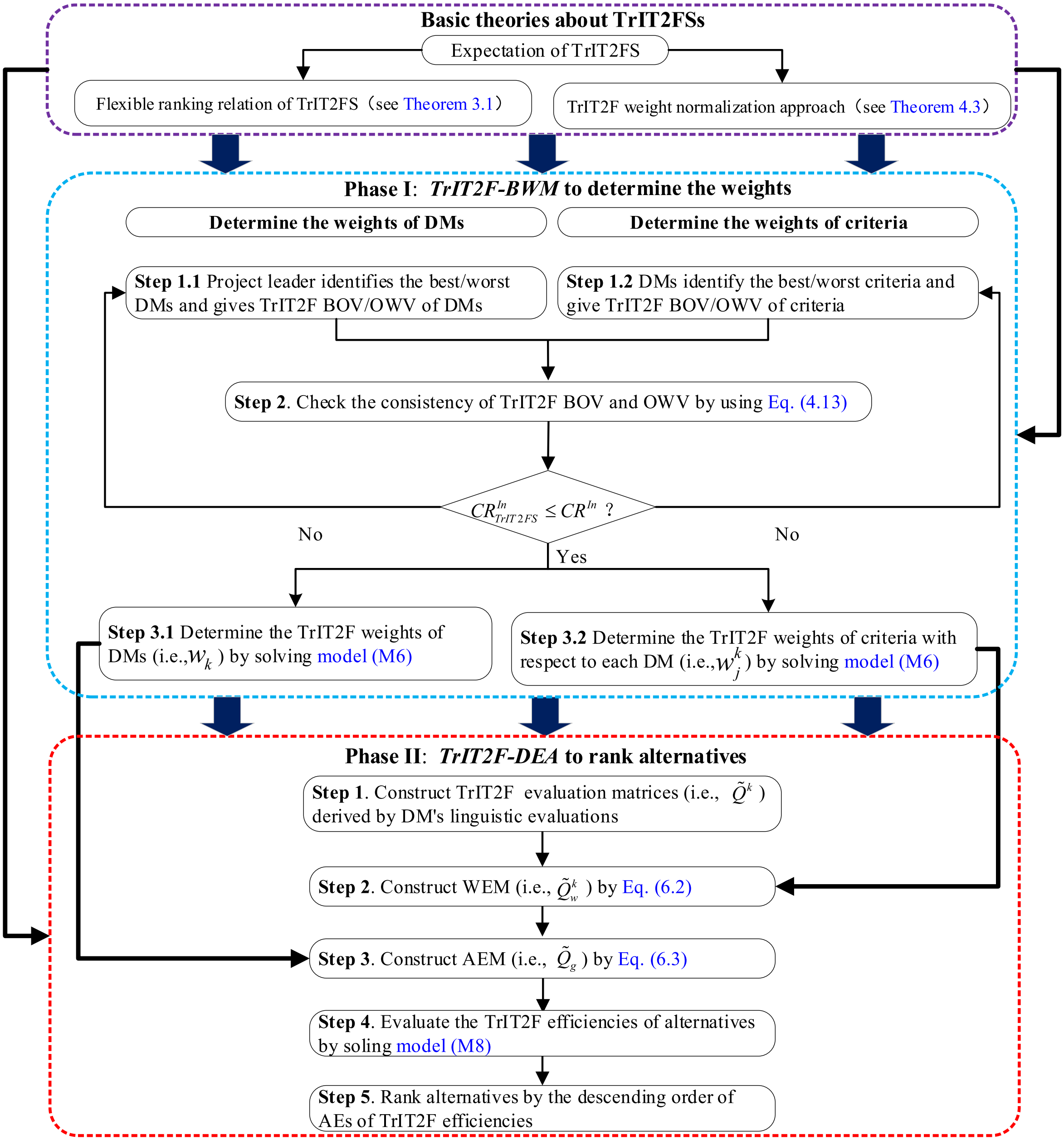

Fig. 4. Flowchart of the proposed *TrIT2F-BWM-DEA*.

are exhibited as follows:

$$\widetilde{x}_{11} = [(0.0058, 0.0124, 0.0192, 0.0345; 0.9), (0.0025, 0.0088, 0.0113, 0.0515; 1)]$$

and

$$\widetilde{y}_{11} = [(0.0121, 0.0302, 0.0370, 0.0536; 0.9), (0.0074, 0.0262, 0.0266, 0.075; 1)].$$

Step 5. Rank alternatives

Without loss of generality, let $\varphi_1 = \varphi_3 = 100$, $\varphi_2 = \varphi_4 = 1$ and $\rho = 1.1$. Then, the TrIT2F efficiencies of all alternatives χ_i $(i = 1, 2, \ldots, 5)$ are determined by solving model (M7).

$$\widetilde{e}_1 = [(0.395, 0.471, 0.95, 1.076; 0.9), (0.153, 0.287, 0.948, 1.406; 1)];$$

$$\widetilde{e}_2 = [(0.527, 0.603, 1.374, 1.501; 0.9), (0.273, 0.429, 1.111, 1.896; 1)];$$

$$\widetilde{e}_3 = [(0.507, 0.574, 1.225, 1.738; 0.9), (0.009, 0.026, 1.666, 2.299; 1)];$$

$$\widetilde{e}_4 = [(0.210, 0.295, 0.581, 0.803; 0.9), (0.099, 0.176, 0.178, 1.33; 1)];$$

$$\widetilde{e}_5 = [(0.424, 0.470, 1.081, 1.174; 0.9), (0.214, 0.356, 0.917, 1.378; 1)].$$

Then, by using Eq. (3.20), the AEs of $\widetilde{e}_i$ $(i = 1, 2, \ldots, 5)$ are calculated as follows: $AE(\widetilde{e}_1) = 0.71$, $AE(\widetilde{e}_2) = 0.96$, $AE(\widetilde{e}_3) = 1$, $AE(\widetilde{e}_4) = 0.457$ and $AE(\widetilde{e}_5) = 0.751$.

Therefore, according to Definition 3.10, the ranking order of alternatives is determined as $\chi_3 \succ \chi_2 \succ \chi_5 \succ \chi_1 \succ \chi_4$, and the best alternative is χ_3.

Table 2
Decision results for different types of fuzzy information.

Fuzzy information	Case	$AE(\widetilde{e}_1)$	$AE(\widetilde{e}_2)$	$AE(\widetilde{e}_3)$	$AE(\widetilde{e}_4)$	$AE(\widetilde{e}_5)$	Ranking order of alternatives
TrIT2FSs (Type-2 fuzzy sets)	$\vartheta = 0$	0.710	0.960	1	0.457	0.751	$\chi_3 \succ \chi_2 \succ \chi_5 \succ \chi_1 \succ \chi_4$
	$\vartheta = 0.5$	0.699	0.977	1	0.572	0.745	$\chi_3 \succ \chi_2 \succ \chi_5 \succ \chi_1 \succ \chi_4$
	$\vartheta = 1$	0.733	0.962	1	0.597	0.855	$\chi_3 \succ \chi_2 \succ \chi_5 \succ \chi_1 \succ \chi_4$
TrFNs (Type-1 fuzzy sets)	$\vartheta = 0$	0.727	0.968	0.852	0.570	0.761	$\chi_2 \succ \chi_3 \succ \chi_5 \succ \chi_1 \succ \chi_4$
	$\vartheta = 0.5$	0.691	1.000	0.852	0.493	0.762	$\chi_2 \succ \chi_3 \succ \chi_5 \succ \chi_1 \succ \chi_4$
	$\vartheta = 1$	0.707	0.971	0.852	0.574	0.874	$\chi_2 \succ \chi_3 \succ \chi_5 \succ \chi_1 \succ \chi_4$

Table 3
Ranking orders of alternatives with different methods.

Method	Ranking order of alternatives
Chen & Lee's method [7]	$\chi_2 \succ \chi_3 \succ \chi_5 \succ \chi_1 \succ \chi_4$
Wang's et al. method [48]	$\chi_2 \succ \chi_3 \succ \chi_1 \succ \chi_4 \succ \chi_5$
Wu's et al. method [15]	$\chi_3 \succ \chi_2 \succ \chi_5 \succ \chi_1 \succ \chi_4$
Wan's et al. method [4]	$\chi_3 \succ \chi_2 \succ \chi_1 \succ \chi_5 \succ \chi_4$
The proposed method	$\chi_3 \succ \chi_2 \succ \chi_5 \succ \chi_1 \succ \chi_4$

Similarly, the decision results in the cases of $\vartheta = 0.5$ (i.e., DM is risk-neutral) and $\vartheta = 1$ (i.e., DM is adventurous) are obtained and shown in Table 2.

It can be observed from Table 2 that the best alternative derived by proposed *TrIT2F-BWM-DEA* with different values of ϑ is always χ_3, which is perfectly consistent with the fact that Hongshan Gymnasium (χ_3) is the one of the first reconverted Fangcang hospitals in Wuhan [60]. This observation can surely illustrate the high robustness of proposed *TrIT2F-BWM-DEA* along with the objectivity and credibility of the obtained decision results.

In addition, the sums of all alternatives' efficiencies in the cases of $\vartheta = 0$, $\vartheta = 0.5$ and $\vartheta = 1$ are 3.878, 3.993 and 4.147, respectively. Thus, the sum of all alternatives' efficiencies has an increasing tendency as the value of ϑ increases. The explication of this tendency is that the proposed flexible ranking order of TrIT2FSs considers DM's acceptance degree of fuzzy constraints to be violated, which is more suitable for complex decision-making problems.

7.2. Comparative analyses

7.2.1. Comparison with existing methods

In order to illustrate the validity and superiorities of the proposed method, the decision results derived by the proposed *TrIT2F-BWM-DEA* are compared with those derived by Wan's et al. method [4], Chen & Lee's method [7], Wu's et al. method [15], Wang's et al. method [48]. The obtained decision results with these methods are listed in Table 3. Meanwhile, the ranking orders of alternatives derived by different methods are graphically depicted in Fig. 5.

It is easy to see from Table 3 and Fig. 5 that the ranking order of alternatives determined by the proposed method is only slightly different from those determined by methods [4,7,15,48], which indicates the proposed method is validity. Moreover, compared with methods [4,15], the proposed method has some superiorities which are emphasized as follows:

(i) Superiority in transforming the minimax objective of model (M1). By using method [15], the maximum TrIT2F deviation $\widetilde{\delta} = [(\delta_1^l, \delta_2^l, \delta_3^l, \delta_4^l; h_{\widetilde{\delta}}^l), (\delta_1^u, \delta_2^u, \delta_3^u, \delta_4^u; h_{\widetilde{\delta}}^u)]$ is replaced by a crisp deviation $\delta = [(\delta, \delta, \delta, \delta; 1), (\delta, \delta, \delta, \delta; 1)]$ to transform the minimax objective of model (M1). This transformation will cause information distortion of TrIT2FSs. For example, there are two constraints that contain the crisp and TrIT2F maximum absolute deviations, respectively:

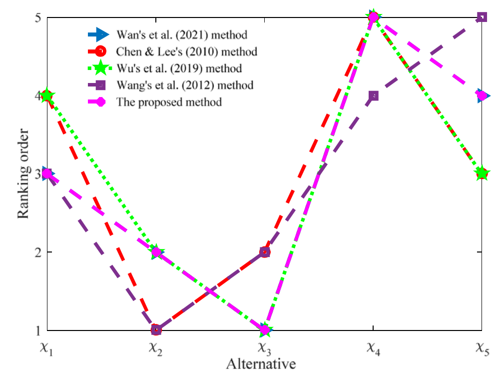

Fig. 5. Ranking orders of alternatives with different methods.

$$[(3, 4, 5, 6; 0.9), (1, 2, 7, 8; 1)] \preccurlyeq [(\delta, \delta, \delta, \delta; 1), (\delta, \delta, \delta, \delta; 1)] = \delta$$

and

$$[(3, 4, 5, 6; 0.9), (1, 2, 7, 8; 1)] \preccurlyeq [(\delta_1^l, \delta_2^l, \delta_3^l, \delta_4^l; h_{\widetilde{\delta}}^l), (\delta_1^u, \delta_2^u, \delta_3^u, \delta_4^u; h_{\widetilde{\delta}}^u)] = \widetilde{\delta}.$$

By method [15], it obviously holds that:

$\delta = [(8, 8, 8, 8; 1), (8, 8, 8, 8; 1)]$ and
$\widetilde{\delta} = [(3, 4, 5, 6; 0.9), (1, 2, 7, 8; 1)]$.

Undoubtedly, $\widetilde{\delta}$ can perfectly retain the inherent information of TrIT2FS, while δ only retains the maximum component of TrIT2FS, which inevitably results in the information distortion and loss of TrIT2FS.

To facilitate comparison, we define two types of crisp converted WDMs (denoted by WDM-I and WDM-II). WDM-I (or WDM-II) is converted from the TrIT2F WDM whose objective is transformed by using the crisp deviation approach [15] (or the proposed TrIT2F deviation approach). By solving WDM-I and WDM-II with six different sets of input parameters, the optimal crisp deviations and TrIT2F deviations (in terms of their AEs) can be acquired, as shown in Fig. 6, where "*DW*" represents the determination of DMs' weights and "CW_k" denotes the determination of criteria weights with respect to d_k $(k = 1, 2, 3, 4, 5)$.

It is easy to observe from Fig. 6 that all the AEs of TrIT2F deviations are remarkably smaller than the crisp deviations. Thus,

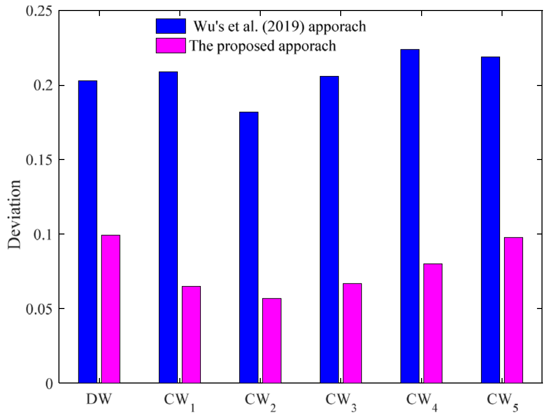

Fig. 6. Crisp and TrIT2F deviations.

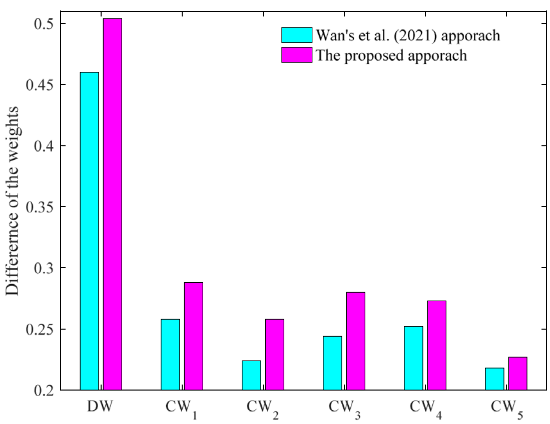

Fig. 7. Difference between the best and worst weights.

the transformation approach of TrIT2F objective proposed in this paper has more superiority in retaining the inherent information of TrIT2FSs than that proposed in [15].

(ii) Superiority of the proposed TrIT2F weight-normalizing approach. The lower and upper heights of TrIT2FSs are considered in the proposed TrIT2F weight-normalizing approach, while they are ignored in approach [4]. Since the lower and upper heights are the essential information of TrIT2FSs, the proposed TrIT2F weight-normalizing approach would be more rigorous and credible, which can be reflected in the differences between the weights of the best and worst DMs (or criteria). It can be seen from Fig. 7 that the differences of the best and worst weights derived by the proposed approach are all larger than those derived by approach [4], which shows that the proposed approach has more advantage in distinguishing the best and worst DMs (or criteria).

7.2.2. Comparison with trapezoidal fuzzy BWM-DEA

To further illustrate the practicability and superiority of using TrIT2FSs, the proposed *TrIT2F-BWM-DEA* (denoted by **method I**) is reduced to trapezoidal fuzzy *BWM-DEA* (denoted by **method II**), in which the trapezoidal fuzzy RCs and evaluations are represented by the UMFs of the TrIT2F RCs and evaluations that have been used in **method I**, respectively.

It has mentioned in Definition 3.6 that a TrFN can be rewritten as the format of a TrIT2FS. Therefore, the proposed *TrIT2F-BWM-DEA* can certainly be utilized to solve the FHL problem in trapezoidal (type-1) fuzzy environment, which implies its high flexibility. The decision results in trapezoidal (type-1) fuzzy environment are also listed in Table 2. To clearly compare **methods I** and **II**, the decision results derived by them are graphically displayed in Fig. 8.

It can be seen from Table 2 and Fig. 8 that the ranking first alternatives derived by **method II** are always χ_2, which exactly tallies with the fact that Wuhan International Conference & Exhibition Center (χ_2) is the one of the first reconverted Fangcang hospitals in Wuhan [60]. Therefore, it is practicable to reduce **method I** into trapezoidal (type-1) fuzzy environment. Although both **methods I** and **II** can generate objective and reasonable decision results, the former has two outstanding advantages as follows:

(i) Advantage in identifying DEA-efficient alternatives. The efficiencies of the optimal alternatives derived by **method I** are all equal to 1, which indicates that these optimal alternatives are DEA-efficient. However, only when $\vartheta = 0.5$, the optimal alternative derived by **method II** is DEA-efficient. Hence, compared with **method II**, **method I** has the advantage in identifying more numbers of DEA-efficient alternatives.

(ii) Advantage in obtaining more realistic decision results. Since χ_3 and χ_2 are the first batch of the reconverted Fangcang hospitals in Wuhan [60], it is realistic to conclude that the difference between the efficiencies of them should be little. It is easy to see from Table 2 that difference between the efficiencies of χ_3 and χ_2 measured by **method I** is remarkably smaller than the one measured by **method II**. Hence, the decision results derived by **method I** are more realistic than those derived by **method II**.

Based on these advantages, it is easy to conclude that compared with T1FSs, T2FSs are more applicable to complex decision-makings since they can capture the vague assessment information by incorporating the FOU into T1FSs.

7.2.3. Comparison with crisp BWM-DEA

In this subsection, the proposed *TrIT2F-BWM-DEA* (i.e., **method I**) is compared with a crisp *BWM-DEA* (denoted by **method III**). In **method III**, the RCs and evaluations are all crisp. Note that the crisp RCs are listed in the second column of Table 1 and the crisp evaluations are represented by the AEs of TrIT2FS evaluations that have been used in method I. The decision results derived by **methods I** and **III** are graphically displayed in Figs. 9–10.

The ranking order of alternatives derived by **method III** is $\chi_2 \sim \chi_3 \sim \chi_5 \succ \chi_1 \succ \chi_4$, which means that alternatives χ_2, χ_3 and χ_5 have the same priority and any of them can be selected as the optimal alternative. However, it has known from [60] that χ_5 was not selected since its capacity (c_6) is not large enough. Thus, the ranking order of alternatives derived by **method III** is not credible. It has mentioned in Section 7.2.1 that the ranking orders of alternatives derived by **method I** are accurate and reasonable. The main reason for the difference between the decision results derived by **methods I** and **III** is that **method I** has adequate ability in perfectly handling TrIT2F information by using the developed flexible ranking order of TrIT2FSs and normalization approach of TrIT2FSs. Hence, **method I** can produce more objective and credible decision results according to DMs' actual needs for complex decision making problems.

7.3. Managerial implications

Based on the proposed method and the results of the studied real case, this subsection summarizes the following specific suggestions (managerial insights) for health care system (HCS) managers and scholars.

(i) It is easy to see from Section 7.1 that DM's acceptance degree has an impact on the efficiencies of alternatives. Consequently, the introduction of DM's acceptance degree (i.e., parameter ϑ) is meaningful. When it comes to practical decision-making

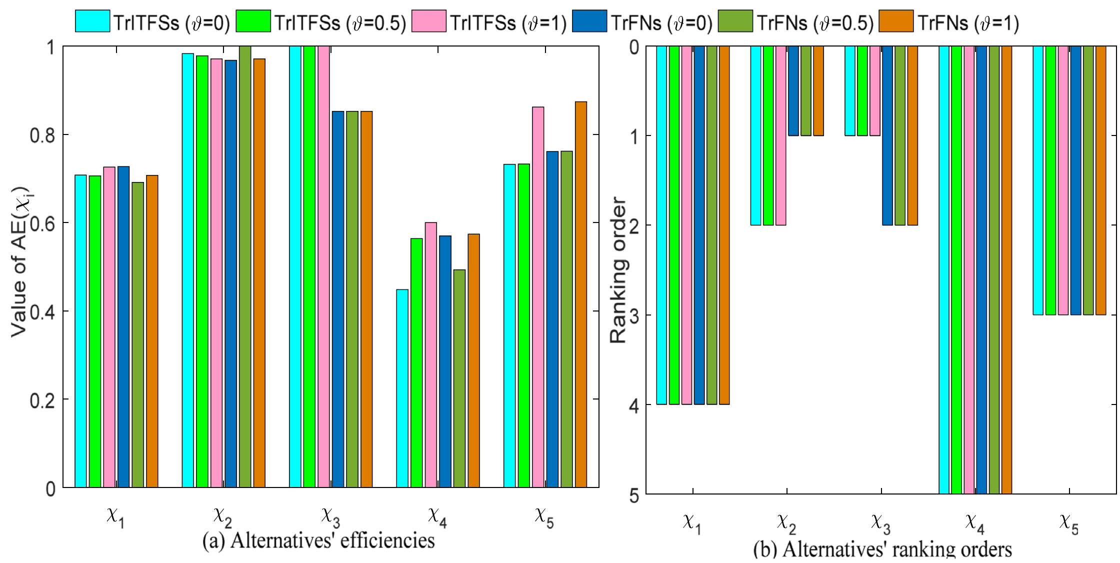

Fig. 8. Alternatives' efficiencies and ranking orders in type-1 and type-2 fuzzy environments.

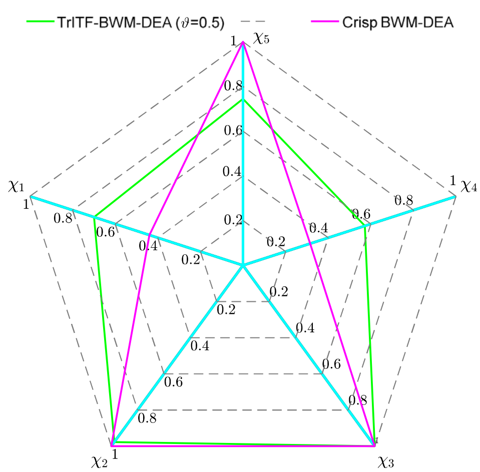

Fig. 9. Efficiencies of alternatives.

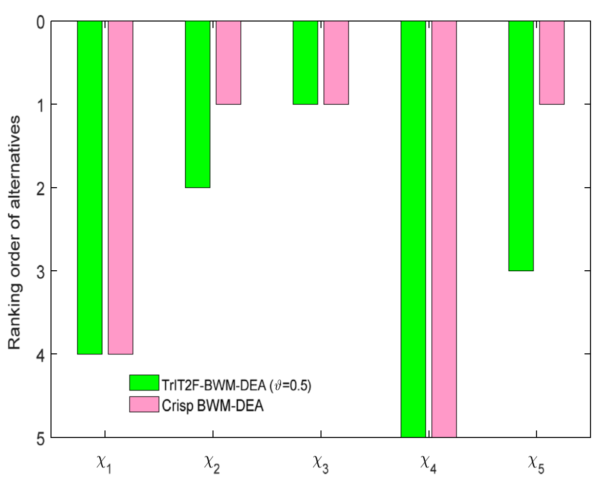

Fig. 10. Ranking order of alternatives.

process, HCS managers can select different values of ϑ according the urgency of the ongoing epidemic. Generally, the more urgent the epidemic, the more pessimistic HCS managers (i.e., the smaller the value of ϑ) should be, and vice versa. This implies that when a very urgent epidemic occurs, the value of ϑ should be as close to 0 as much as possible.

(ii) In the proposed method, there are two places where TrIT2F information needs to be processed cautiously. One is the transformation of the objective of the TrIT2F model (e.g., model (M1)), another is the normalization of the TrIT2F weights. From Section 7.2.1, it is natural to suggest HCS scholars to select the proposed transformation and normalization approaches (see Theorems 3.1 and 4.3) to process TrIT2F data since they have high ability in avoiding information distortion and loss of TrIT2FSs.

(iii) From the comparison result in Section 7.2.3, it is easy to conclude that TrIT2FSs are more suitable than TrFNs in representing uncertain information. Thereby, HCS managers are recommended to apply TrIT2FSs to represent DM's linguistic judgments in the complex practical FHL problems.

(iv) Since FHL problem usually involves multiple criteria, it is hard for a single DM to thoroughly evaluate every alternative under all criteria. Thus, HCS managers should consider the group opinions from multi-experts. Accordingly, HCS managers are recommended to apply the proposed MCGDM technique *TrIT2F-BWM-DEA* to solve the FHL problem, since it has strong power in quantitatively expressing all experts' opinions in an uncertain environment.

8. Conclusion

This paper proposes a *TrIT2F-BWM-DEA* for solving the FHL problem. According to the decision results of the real application and comparative analyses, some conclusions can be drawn as follows:

(1) The expectation of TrIT2FS is defined based on cut set theory. Then, a flexible ranking relation of TrIT2FSs and a TrIT2F weight-normalizing approach are proposed and used to transform the TrIT2F models of the *TrIT2F-BWM* and *TrIT2F-DEA* into crisp ones.

(2) A *TrIT2F-BWM* is proposed to determine the weights of DMs and criteria, in which an input-based consistency ratio is designed to check the consistency of TrIT2F RCs.

(3) A fully *TrIT2F-DEA* model is constructed to measure the TrIT2F efficiencies of alternatives.

(4) The validity and superiorities of the proposed *TrIT2F-BWM-DEA* are confirmed with a real FHL case and some comparative analyses.

The decision results show that the proposed *TrIT2F-BWM-DEA* can effectively solve the FHL problem. To enrich the application of the proposed *TrIT2F-BWM-DEA*, it is expected to be applied to solve other real-world decision making problems, e.g., evaluation of hospital performance [61], selection of smart product service module [17], etc. Besides, this paper has some limitations that need to be consummated.

(i) Remark 4 points out that if the CR exceeds a given threshold, then DMs' RCs are regarded to be unacceptably inconsistent and decision results are usually inaccurate. In such a case, DMs' RCs must be adjusted. However, this paper lacks a deep discussion on how to effectively adjust DMs' RCs. To cover this defect, some adjustment algorithms are required to be developed in the further research.

(ii) This paper uses TrIT2FSs to represent the decision information. However, it is demonstrated that the preference information provided by people tends to obey Gaussian distribution [62], which implies that Gaussian IT2FSs have more advantage in capturing uncertain information than TrIT2FSs. Therefore, the development of an extended *BWM-DEA* with Gaussian IT2F information may be a deserving research direction.

CRediT authorship contribution statement

Ze-hui Chen: Conceptualization, Software, Writing – original draft, Writing – review & editing. **Shu-ping Wan:** Supervision, Data curation, Writing – original draft, Writing – review & editing, Validation, Funding acquisition. **Jiu-ying Dong:** Resources, Investigation, Methodology, Formal analysis, Project administration.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

[1] World Health Organization, https://www.who.int/. Accessed: 2021.
[2] W.Z. Wang, X.W. Liu, Y. Qin, Multi-attribute group decision making models under interval type-2 fuzzy environment, Knowl.-Based Syst. 30 (2012) 121–128.
[3] China Daily, http://cn.chinadaily.com.cn/a/202003/26/WS5e7c8493a3107bb6b57a8f9c.html. Accessed: 2021.
[4] S.P. Wan, Z.H. Chen, J.Y. Dong, An integrated interval type-2 fuzzy technique for democratic-autocratic multi-criteria decision making, Knowl.-Based Syst. 214 (2021) 106735.
[5] M. Kadziński, K. Martyn, M. cinelli, et al., Preference disaggregation method for value-based multi-decision sorting problems with a real-world application in nanotechnology, Knowl.-Based Syst. 218 (2021) 106879.
[6] S.H. Kim, B.S. Ahn, Interactive group decision making procedure under incomplete information, European J. Oper. Res. 116 (1999) 498–507.
[7] S.M. Chen, L.W. Lee, Fuzzy multiple attributes group decision-making based on the interval type-2 TOPSIS method, Expert Syst. Appl. 37 (2010) 2790–2798.
[8] Q. Wu, X. Liu, J. Qin, W. Wang, L. Zhou, A linguistic distribution behavioral multi-criteria group decision making model integrating extended generalized TODIM and quantum decision theory, Appl. Soft Comput. 98 (2021) 106757.
[9] D.F. Li, S.P. Wan, A fuzzy inhomogenous multiattribute group decision making approach to solve outsourcing provider selection problems, Knowl.-Based Syst. 67 (2014) 71–89.
[10] C. Lin, G. Kou, A heuristic method to rank the alternatives in the AHP synthesis, Appl. Soft Comput. 100 (2021) 106916.
[11] T. Supeekit, T. Somboonwiwat, D. Kritchanchai, DEMATEL-modified ANP to evaluate internal hospital supply chain performance, Comput. Ind. Eng. 102 (2016) 318–330.
[12] J. Rezaei, Best-worst multi-criteria decision-making method, Omega 53 (2015) 49–57.
[13] T. Wu, X.W. Liu, F. Liu, An interval type-2 fuzzy TOPSIS model for large scale group decision making problems with social network information, Inform. Sci. 432 (2018) 392–410.
[14] P.D. Liu, H. Gao, H. Fujita, The new extension of the MULTIMOORA method for sustainable supplier selection with intuitionistic linguistic rough numbers, Appl. Soft Comput. 99 (2021) 106893.
[15] Q. Wu, L.G. Zhou, Y. Chen, H.Y. Chen, An integrated approach to green supplier selection based on the interval type-2 fuzzy best-worst and extended VIKOR methods, Inform. Sci. 502 (2019) 394–417.
[16] A. Charnes, W.W. Cooper, E. Rhodes, Measuring the efficiency of decision making units, European J. Oper. Res. 6 (1978) 429–444.
[17] Z.H. Chen, X.G. Ming, A rough-fuzzy approach integrating best-worst method and data envelopment analysis to multi-criteria selection of smart product service module, Appl. Soft Comput. 94 (2020) 106479.
[18] S. Lim, K.W. Oh, J. Zhu, Use of DEA cross-efficiency evaluation in portfolio selection: An application to Korean stock market, European J. Oper. Res. 236 (2014) 361–368.
[19] R.Y. Lin, Q. Liu, Multiplier dynamic data envelopment analysis based on directional distance function: An application to mutual funds, European J. Oper. Res. 293 (2021) 1043–1057.
[20] I. Otay, B. Oztaysi, S.C. Onar, C. Kahraman, Multi-expert performance evaluation of healthcare institutions using an integrated intuitionistic fuzzy AHP & DEA methodology, Knowl.-Based Syst. 133 (2017) 90–106.
[21] J.Y. Dong, S.P. Wan, S.M. Chen, Fuzzy best-worst method based on triangular fuzzy numbers for multi-criteria decision-making, Inform. Sci. 547 (2021) 1080–1104.
[22] S. Guo, H. Zhao, Fuzzy best-worst multi-criteria decision-making method and its applications, Knowl.-Based Syst. 121 (2017) 23–31.
[23] J.Y. Dong, S.P. Wan, A new trapezoidal fuzzy linear programming method considering the acceptance degree of fuzzy constraints violated, Knowl.-Based Syst. 148 (2018) 100–114.
[24] L.A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning-I, Inform. Sci. 8 (1975) 199–249.
[25] D. Wu, J.M. Mendel, S. Coupland, Enhanced interval approach for encoding words into interval type-2 fuzzy sets and its convergence analysis, IEEE Trans. Fuzzy Syst. 20 (2012) 499–513.
[26] S.M. Chen, D. Barman, Adaptive weighted fuzzy interpolative reasoning based on representative values and similarity measures of interval type-2 fuzzy sets, Inform. Sci. 478 (2019) 167–185.
[27] S. Hendiani, H. Liao, R. Ren, B. Lev, A likelihood-based multi-criteria sustainable supplier selection approach with complex preference information, Inform. Sci. 536 (2020) 135–155.
[28] Y. Gong, N. Hu, J. Zhang, G. Liu, J. Deng, Multi-attribute group decision making method based on geometric Bonferroni mean operator of trapezoidal interval type-2 fuzzy numbers, Comput. Ind. Eng. 81 (2015) 167–176.
[29] X.Z. Sang, Y.H. Zhou, X.Y. Yu, An uncertain possibility-probability information fusion method under interval type-2 fuzzy environment and its application in stock selection, Inform. Sci. 504 (2019) 546–560.
[30] J.D. Qin, X.W. Liu, W. Pedrycz, An extended VIKOR method based on prospect theory for multiple attribute decision making under interval type-2 fuzzy environment, Knowl.-Based Syst. 86 (2015) 116–130.
[31] C. Kahraman, B. Öztayşi, İ.U. Sarı, E. Turanoğlu, Fuzzy analytic hierarchy process with interval type-2 fuzzy sets, Knowl.-Based Syst. 59 (2014) 48–57.
[32] E. Celik, M. Gul, N. Aydin, A.T. Gumus, A.F. Guneri, A comprehensive review of multi criteria decision making approaches based on interval type-2 fuzzy sets, Knowl.-Based Syst. 85 (2015) 329–341.
[33] H. Karimi, M. Sadeghi-Dastaki, M. Javan, A fully fuzzy best-worst multi attribute decision making method with triangular fuzzy number: A case study of maintenance assessment in the hospitals, Appl. Soft Comput. 86 (2020) 105882.
[34] A. Hafezalkotob, A. Hafezalkotob, H.C. Liao, F. Herrera, Interval MULTIMOORA method integrating interval Bordar rule and interval best-worst-method-based weighting model: case study on hybrid vehicle engine selection, IEEE Trans. Cybern. 50 (2020) 1157–1169.
[35] H. Aboutorab, M. Saberi, M.R. Asadabadi, O. Hussain, E. Chang, ZBWM: The Z-number extension of best worst method and its application for supplier development, Expert Syst. Appl. 107 (2018) 115–125.

[36] X.M. Mi, M. Tang, H.C. Liao, W.J. Shen, B. Lev, The state-of-the-art survey on integrations and applications of the best worst method in decision making: Why, what, what for and what's next? Omega 87 (2019) 205–225.
[37] S.P. Wan, G.L. Xu, F. Wang, J.Y. Dong, A new method for atanassov's interval-valued intuitionistic fuzzy MAGDM with incomplete attribute weight information, Inform. Sci. 316 (2015) 329–347.
[38] A. Hafezalkotob, A. Hafezalkotob, A novel approach for combination of individual and group decisions based on fuzzy best-worst method, Appl. Soft Comput. 59 (2017) 316–325.
[39] Q. Mou, Z.S. Xu, H.C. Liao, A graph based group decision making approach with intuitionistic fuzzy preference relations, Comput. Ind. Eng. 110 (2017) 138–150.
[40] X.M. Mi, H.C. Liao, An integrated approach to multiple criteria decision making based on the average solution and normalized weights of criteria deduced by the hesitant fuzzy best worst method, Comput. Ind. Eng. 133 (2019) 83–94.
[41] J. Rezaei, Best-worst multi-criteria decision-making method: Some properties and a linear model, Omega 64 (2016) 126–130.
[42] H.L. Xiao, T.T. Ren, Z.B. Zhou, W.B. Liu, Parameter uncertainty in estimation of portfolio efficiency: Evidence from an interval diversification-consistent DEA approach, Omega 103 (2021) 102357.
[43] B. Ebrahimi, M. Tavana, M. Toloo, V. Charles, A novel mixed binary linear DEA model for ranking decision-making units with preference information, Comput. Ind. Eng. 149 (2020) 106720.
[44] X.Y. Zhou, Y. Wang, J. Chai, L.Q. Wang, S.Y. Wang, B. Lev, Sustainable supply chain evaluation: A dynamic double frontier network DEA model with interval type-2 fuzzy data, Inform. Sci. 504 (2019) 394–421.
[45] A. Emrouznejad, G.L. Yang, A survey and analysis of the first 40 years of scholarly literature in DEA: 1978–2016, Socio-Econ. Plan. Sci. 61 (2017) 4–8.
[46] S.M. Chen, Z.J. Zhang, J.T. Yang, et al., Fangcang shelter hospitals: a novel concept for responding to public health emergencies, Lancet 395 (2020) 1305–1314.
[47] L. Shang, J. Xu, B. Cao, Fangcang shelter hospitals in COVID-19 pandemic: the practice and its significance, Clin. Microbiol. Infect. 26 (2020) 976–978.
[48] K.W. Wang, J. Gao, X.X. Song, et al., Fangcang shelter hospitals are a one health approach for responding to the COVID-19 outbreak in Wuhan, China, One Health 10 (2020) 100167.
[49] A.E. Amideo, M. Scaparra, P.K. Kotiadis, Optimizing shelter location and evacuation routing operations: The critical issues, European J. Oper. Res. 279 (2019) 279–295.
[50] Ö.B. Kınay, B.Y. Kara, F. Saldanha-da Gamac, I. Correia, Modeling the shelter site location problem using chance constraints: A case study for Istanbul, European J. Oper. Res. 270 (2018) 132–145.
[51] J. Gu, Y. Zhou, A. Das, I. Moon, G.M. Lee, Medical relief shelter location problem with patient severity under a limited relief budget, Comput. Ind. Eng. 125 (2018) 720–728.
[52] E. Ozbay, Ö. Çavuş, B.Y. Kara, Shelter site location under multi-hazard scenarios, Comput. Oper. Res. 106 (2019) 102–118.
[53] H.Y. Wu, P.J. Ren, Z.S. Xu, Addressing site selection for earthquake shelters with hesitant multiplicative linguistic preference relation, Inform. Sci. 516 (2020) 370–387.
[54] J.M. Mendel, R.I. John, F. Liu, Interval type-2 fuzzy logic systems made simple, IEEE Trans. Fuzzy Syst. 14 (2006) 808–821.
[55] T.Y. Chen, A linear assignment method for multiple-criteria decision analysis with interval type-2 fuzzy sets, Appl. Soft Comput. 13 (2013) 2735–2748.
[56] J.T. Starczewski, Centroid of triangular and Gaussian type-2 fuzzy sets, Inform. Sci. 280 (2014) 289–306.
[57] Y.M. Wang, T.M.S. Elhag, On the normalization of interval and fuzzy weights, Fuzzy Sets and Systems 157 (2006) 2456–2471.
[58] F. Liang, M. Brunelli, J. Rezaei, Consistency issues in the best worst method: Measurements and thresholds, Omega 96 (2020) 102175.
[59] K. Rashidi, K. Cullinane, A comparison of fuzzy DEA and fuzzy TOPSIS in sustainable supplier selection: Implications for sourcing strategy, Expert Syst. Appl. 121 (2019) 266–281.
[60] People's Daily, http://society.people.com.cn/n1/2020/0206/c1008-31573109.html Accessed: 2021.
[61] M. Amiri, M. Hashemi-Tabatabaei, M. Ghahremanloo, et al., A new fuzzy approach based on BWM and fuzzy preference programming for hospital performance evaluation: A case study, Appl. Soft Comput. 92 (2020) 106279.
[62] X. Pan, Y. Wang, K.-S. Chin, A large-scale group decision-making method for site selection of waste to energy project under interval type-2 fuzzy environment, Sustainable Cities Soc. 71 (2021) 103003.