OPEN ACCESS Freely available online

PLOS ONE

Lifespan and Aggregate Size Variables in Specifications of Mortality or Survivorship

Michael Epelbaum*

Independent Multidisciplinary Scientist, Nashville, Tennessee, United States of America

Abstract

A specification of mortality or survivorship provides respective explicit details about mortality's or survivorship's relationships with one or more other variables (e.g., age, sex, etc.). Previous studies have discovered and analyzed diverse specifications of mortality or survivorship; these discoveries and analyses suggest that additional specifications of mortality or survivorship have yet to be discovered and analyzed. In consistency with previous research, multivariable limited powered polynomials regression analyses of mortality and survivorship of selected humans (Swedes, 1760–2008) and selected insects (caged medflies) show age-specific, historical-time-specific, environmental-context-specific, and sex-specific mortality and survivorship. These analyses also present discoveries of hitherto unknown lifespan-specific, contemporary-aggregate-size-specific, and lifespan-aggregate-size-specific mortality and survivorship. The results of this investigation and results of previous research help identify variables for inclusion in regression models of mortality or survivorship. Moreover, these results and results of previous research strengthen the suggestion that additional specifications of mortality or survivorship have yet to be discovered and analyzed, and they also suggest that specifications of mortality and survivorship indicate corresponding specifications of frailty and vitality. Furthermore, the present analyses reveal the usefulness of a multivariable limited powered polynomials regression model-building approach. This article shows that much has yet to be learned about specifications of mortality or survivorship of diverse kinds of individuals in diverse times and places.

Citation: Epelbaum M (2014) Lifespan and Aggregate Size Variables in Specifications of Mortality or Survivorship. PLoS ONE 9(1): e84156. doi:10.1371/journal.pone.0084156

Editor: Cédric Sueur, Institut Pluridisciplinaire Hubert Curien, France

Received April 10, 2013; **Accepted** November 21, 2013; **Published** January 15, 2014

Funding: No current external funding sources for this study.

Competing Interests: Author's patent US 8219600 B2 and other potential patents. The author hereby confirms that these competing interests do not alter adherence to all the PLOS ONE policies on sharing data and materials, as detailed online in the guide for authors.

* E-mail: mepelbaum3@gmail.com

Introduction

A specification of mortality or survivorship provides explicit details about mortality's or survivorship's relationships with one or more other variables. For example, *X*-specific mortality or survivorship provides explicit details about mortality's or survivorship's relationship with variable *X*. Previous investigations present discoveries and analyses of diverse specifications of mortality or survivorship, as illustrated by discoveries and analyses of age-specific [1–40], environmental-context-specific [10,13,17,35,41,42], historical-time-specific [17,43–47], physical-size-specific [2,10,12,20,34,35,48–53], sex-specific [10,35,54], birth-cohort-specific [23,36–40,44,47,55,56], exposure-specific [12,13,44,47,57], density-specific [10,17,41,58–60], and disease-specific [61] mortality or survivorship. These considerations suggest that additional specifications of mortality or survivorship have yet to be discovered and analyzed. This article presents discoveries and analyses of hitherto unknown lifespan-specific, contemporary-aggregate-size-specific, and lifespan-aggregate-size-specific mortality and survivorship.

Lifespan is the total time span of an individual's existence [21,62], such that $L_{iq} = L_i = t_{iz} - t_{i0}$, where L_{iq} refers to the lifespan of a natural or artificial individual i at time t_q, $z \geq q$, t_{iz} is the time of the individual's cessation of existence, t_{i0} is the time of the individual's initiation of existence, and L_i is constant for all t_q in t_{i0}:t_{iz}. The time of birth typically indicates time t_{i0}, and the time of death typically indicates time t_{iz}, but these typical notions of time of birth and time of death as limits of lifespan do not apply to all kinds of individuals [10,20,63]. The lifespan aggregate includes all the individuals that are identically characterized with respect to lifespan and every other condition in a data set. The individuals that are included in a lifespan aggregate begin their existence in coexistence at the beginning of the lifespan, they coexist through said lifespan, and they cease to exist and cease to coexist at the conclusion of this lifespan. Therefore, a lifespan aggregate's composition and size are constant from the time of the initiation of existence of this aggregate to the time of its cessation of existence. In some cases, the lifespan aggregate consists only of a respective single natural or artificial individual, but in many cases the lifespan aggregate consists of more than one individual. An individual's lifespan aggregate is included in every contemporary aggregate of this individual. The contemporary aggregate includes all the individuals that are identically characterized with respect to every condition in a data set at a point of cessation or continuation of existence, except that these individuals share or do not share an identical lifespan. These considerations indicate that the contemporary aggregate's composition and size are time-specific and changeable through time. Additionally, the size of an individual's contemporary aggregate is equal to – or greater than – the size of this individual's corresponding lifespan aggregate.

Every natural or artificial individual is characterized by a lifespan, a contemporary aggregate, and a lifespan aggregate at every point of continuation of existence (i.e., survivorship) and at the point of cessation of existence (i.e., mortality). Previous

investigations posit that age-specific aggregates are characterized by a "longevity factor" [5–7]; this longevity factor has been implemented in logistic models of mortality or survivorship [3,5–7] and in frailty models of survival time [15,64]. However, lifespan-specific, contemporary-aggregate-size-specific, and lifespan-aggregate-size-specific mortality or survivorship have not been discovered or analyzed in previous empirical research. Therefore, it is useful to search for lifespan-specific, contemporary-aggregate-size-specific, and lifespan-aggregate-size-specific mortality or survivorship. This search is conducted here in empirical analyses of mortality and survivorship of selected humans and selected insects.

Materials and Methods

Data

Deaths 1×1 and exposures 1×1 tables (last modified on 14 July, 2010) from the Human Mortalities Database are employed here in the compilation of data on aggregate age-sex-year-specific deaths and age-sex-year-specific exposures of males and females in ages 0 to 110+ in Sweden 1751–2008 [65]. Computer intensive analyses impose restrictions on the size of the data file for the present analyses. Therefore, the analytic data file is restricted here to 188,087 weighted cases with 79,164,608 events of death or survival of all individuals born in Sweden in decennial years 1760–1930, with deaths occurring between 1760 and 2008. The selected aggregate data are converted here to yearly events of each individual's death or survival, where each individual-level case is weighted by its corresponding number of age-lifespan-sex-specific identical individuals (i.e., the number of sex-specific individuals who are born in the year of birth of the criterion individual and who die in the year of death of the criterion individual). Each case includes data on an age-sex-year-specific event of death or survival of one individual, year of the event, the individual's sex, the individual's age at the time of the event, the individual's lifespan, number of age-lifespan-sex-specific identical individuals (i.e., this is the weight variable in the analyses, and it is also the lifespan aggregate size variable in the respective models of mortality and survivorship), and the number of age-sex-specific individuals that are exposed to the risk of death and prospect of survival during the year of the event (i.e., this is the contemporary aggregate size variable in the respective models of the selected humans' mortality and survivorship).

The data on mortality and survivorship of the selected insects – Mediterranean fruit flies, *Ceratitis capitata*, commonly known as medflies – were collected in 1991 at the Moscamed medflies mass-rearing facility in Metapa, a small village located about 20 kilometers from the city of Tapachula in the state of Mexico [9,10]. These data have been previously analyzed – using diverse compilations and methods – in studies that have been reported in diverse publications [9,10,27–29]. The original data file contains information on numbers of age-cage-and-sex-specific deaths of 1,203,646 male and female medflies, where insects are distributed in 167 cages, and where the numbers of age-cage-sex-specific dead individuals are counted daily [66]. Computer intensive analyses impose restrictions on the size of the data file that is analyzed here. Therefore, the analytic data file is restricted here to cases of physical size #5 and birth aggregate batch #2. In these selected cases, individuals lived and died in one of thirteen cages, where the cages averaged 3,646.3 sex-specific insects per cage at age 0 to 1 days. These aggregate data are converted here to daily events of each individual's death or survival, where each case is weighted by the number of sex-cage-specific individuals that were born in the day of birth of the criterion individual and that died in the day of death of the criterion individual. The resultant analytic data file includes 50,716 cases with 2,211,782 events of individual insects' deaths or survivals. Each case includes data on an age-cage-and-sex-specific event of death or survival of one individual, the individual's sex, the individual's age at the time of the event, the individual's lifespan, cage specifier, number of corresponding age-lifespan-cage-sex-specific identical individuals (i.e., number of cage-sex-specific individuals with identical birth day and identical death day to the criterion individual, which is the weight variable in the analyses, and which is also the lifespan aggregate size variable in the respective models of mortality and survivorship), and the number of age-cage-sex-specific individuals that are exposed to the risk of death and prospect of survival during the day of the event (i.e., this variable is also the contemporary aggregate size variable in the respective models of the selected insects' mortality and survivorship).

Model-building approach

Mortality refers here to cessation of existence of an individual, and survivorship refers here to continuation of existence of an individual. Therefore, an explanatory model of mortality or survivorship – i.e., a model that is dedicated to the explanation of an individual's cessation or continuation of existence – requires a binary response model. Additionally, multiple specifications of mortality or survivorship – and avoidance of the omitted variables bias in models of mortality or survivorship [67–71] – require multivariable models. Furthermore, previous research shows that trajectories of specific mortality or survivorship tend to be nonlinear [31,32]; therefore, the explanatory multivariable binary response model of mortality or survivorship should allow for nonlinearity. Previous research also shows that mortality and survivorship correspond to power laws and scaling laws [22,26,30,35,48–53,60,72–81]; therefore, the explanatory multivariable nonlinear binary response model of mortality or survivorship should enable investigation of power laws and scaling laws. The multivariable fractional polynomials regression model-building approach [31,32,34] enables investigation of explanatory multivariable nonlinear binary response models of mortality or survivorship. However, by allowing more than one power coefficient for each relevant right-hand side variable and by not searching for precise power coefficients, this model-building approach may disable the investigation of power laws and scaling laws of mortality or survivorship. Related to the multivariable fractional polynomials regression model-building approach, a multivariable limited powered polynomials regression model-building approach enables investigation of explanatory multivariable nonlinear binary response models, power laws, and scaling laws.

A multivariable limited powered polynomials regression model is specified here with

$$Y = \beta_0 + \sum_{q=1}^{n} \sum_{k=1}^{r_q} [\beta_{qk} \{(X_q)^{p_q}\}^k] + \sum_{v=1}^{u} (\beta_v W_v) \qquad (1)$$

where – in the present context – the left-hand side variable Y denotes mortality M or survivorship S, β denotes a regression coefficient, X denotes an ordinal or higher-level variable, and W denotes a categoric variable. In this regression model, a distinct precise power coefficient p_q of a distinct variable X_q is common to all k in each limited power series $\sum_{k=1}^{r_q} [\beta_{qk} \{(X_q)^{p_q}\}^k]$, and length r_q of each of these limited power series is distinct to each variable X_q. These characteristics of the multivariable limited powered polynomials regression model enable investigation of power laws,

scaling laws, and post-estimation marginal probabilities and derivatives for each $(X_q)^{pq}$ variable. In the following example of a multivariable limited powered polynomials regression model

$$Y = \beta_0 + \beta_1\left\{(X_1)^{1.4}\right\}^1 + \beta_2\left\{(X_2)^{0.5}\right\}^1 + \beta_3\left\{(X_2)^{0.5}\right\}^2 + \beta_4\left\{(X_2)^{0.5}\right\}^3 + \beta_5\left\{(X_3)^{0.7}\right\}^1 + \beta_6\left\{(X_3)^{0.7}\right\}^2 + \beta_7\left\{(X_3)^{0.7}\right\}^3 + \beta_8\left\{(X_3)^{0.7}\right\}^4 + \beta_9 W_1 + B_{10} W_2 \quad (2)$$

variable X_1 has a $q = 1$ index, a power coefficient $p_1 = 1.4$, a limited power series of length $r_1 = 1$, and one respective regression coefficient β; variable X_2 has a $q = 2$ index, a power coefficient $p_2 = 0.5$, a limited power series of length $r_2 = 3$, and three respective regression coefficients β; variable X_3 has a $q = 3$ index, a power coefficient $p_3 = 0.7$, a limited power series of length $r_3 = 4$, and four respective regression coefficients β; categoric variable W_1 has a $v = 1$ index and one respective regression coefficient β; and categoric variable W_2 has a $v = 2$ index and one respective regression coefficient β. The multivariable limited powered polynomials regression model in this example includes eleven regression coefficients β that are distributed as follows: one coefficient β for the intercept, eight coefficients β for the three X_q variables and their respective limited power series, and two regression coefficients β for the two respective W variables. The model in this example enables investigation of power laws, scaling laws, and respective post-estimation marginal probabilities and derivatives for $(X_1)^{1.4}$,$(X_2)^{0.5}$, and$(X_3)^{0.7}$.

Statistical analyses

Analyses of mortality and survivorship of the selected humans analyze the following multivariable limited powered polynomials binary random effects weighted model:

$$Y_{ij} = \beta_0 + \sum_{k=1}^{r_A}[\beta_{Ak}\{(A_{ij})^{p_A}\}^k] + \sum_{k=1}^{r_L}[\beta_{Lk}\{(L_{ij})^{p_L}\}^k] + \sum_{k=1}^{r_C}[\beta_{Ck}\{(C_{ij})^{p_C}\}^k] + \sum_{k=1}^{r_\Lambda}[\beta_{\Lambda k}\{(\Lambda_{ij})^{p_\Lambda}\}^k] + \sum_{k=1}^{r_H}[\beta_{Hk}\{(H_{ij})^{p_H}\}^k] + \beta_F F_{ij} + \xi_{ij} + \varepsilon_{ij} \quad (3)$$

where Y_{ij} refers to mortality M_{ij} or survivorship S_{ij} of an individual human i that continues to exist (i.e., $M_{ij} = 0$ and $S_{ij} = 1$) or ceases to exist (i.e., $M_{ij} = 1$ and $S_{ij} = 0$) at observation j; A_{ij}, L_{ij}, C_{ij}, Λ_{ij}, F_{ij}, and H_{ij} are respective right-hand side variables corresponding to individual i at observation j; A denotes age, L denotes lifespan, C denotes contemporary aggregate size, Λ (the Greek capital letter *Lambda*) denotes lifespan aggregate size, H denotes historical time, and F (in reference to being or not being female) denotes sex; ξ_{ij} denotes a random effects component corresponding to individual i at observation j; and ε_{ij} denotes an error corresponding to individual i at observation j. Every $(X_q)^{pq}$ in Model (3) is a power transformation of a corresponding variable X_q using a corresponding specific power coefficient p_q (e.g., $(A_{ij})^{p_A}$ is a power transformation of A_{ij}). Previous research provides evidence of unobserved heterogeneity in models of mortality or survivorship [9,13–16,28]; by denoting a random effects component of individual i at observation j, coefficient ξ_{ij} in Model (3) accommodates and implements unobserved heterogeneity [82]. Additionally, previous research shows that regression models of mortality, survivorship, and other phenomena are often encumbered by the age-period-cohort problem (also known as the "APC conundrum") of separating the effects of age-groups, periods, and cohorts in regression models [23,36–40]. Inclusion of the variables age, lifespan, contemporary aggregate size, lifespan aggregate size, and historical time variables as separate and distinct variables in Model (3) shows that this model is not encumbered by the age-period-cohort problem.

Corresponding analyses of mortality and survivorship of the selected insects analyze the following multivariable limited powered polynomials binary random effects weighted model:

$$Y_{ij} = \beta_0 + \sum_{k=1}^{r_A}[\beta_{Ak}\{(A_{ij})^{p_A}\}^k] + \sum_{k=1}^{r_L}[\beta_{Lk}\{(L_{ij})^{p_L}\}^k] + \sum_{k=1}^{r_C}[\beta_{Ck}\{(C_{ij})^{p_C}\}^k] + \sum_{k=1}^{r_\Lambda}[\beta_{\Lambda k}\{(\Lambda_{ij})^{p_\Lambda}\}^k] + \beta_F F_{ij} + \{\beta_{Ec} E_{ic}\} + \xi_{ij} + \varepsilon_{ij} \quad (4)$$

where Y_{ij}, A_{ij}, L_{ij}, C_{ij}, Λ_{ij}, F_{ij}, p_q, k, β_{qk}, ξ_{ij}, and ε_{ij} denote as in Model (3); E_{ic} denotes the environmental context E of individual i, such that c in β_{Ec} and E_{ic} denotes a specific cage c, such that $c = 1{:}13$ cages, such that Model (4) includes one of 13 respective terms $\{\beta_{Ec}E_{ic}\}$, such that one of these 13 respective terms applies to a respective individual i.

Statistical analyses of limited powered polynomials binary random effects weighted regression Models (3) and (4) are conducted here using the Stata software [83]. Stata restricts the statistical analyses of random effects binary response models to respective analyses of logit, probit, and complementary log-log models with a Gaussian distribution of unobserved heterogeneity. Goodness-of-fit (GOF) of a model is indicated here by minimization of the Akaike information criterion, AIC, and minimization of the Bayesian information criterion, BIC [83–85]. Statistical analyses of Models (3) and (4) consist here of data-driven stepwise tests of improvements in GOF in respective weighted random effects logit, probit, or complementary log-log regression analyses of these models.

Initial steps in the stepwise analyses employ $k = 1$ of all n right-hand side variables $(X_q)^{p_q}$ of Model (3) or (4), testing diverse power coefficients p_q (using $ln(X_q)$ for $p_q = 0$), searching for the power coefficient p_q for each specific $(X_q)^{p_q}$ variable that most improves the model's GOF, stopping respective testing of a specific $(X_q)^{p_q}$ when a specific change in p_q for this specific $(X_q)^{p_q}$ ceases to improve the model's GOF, dropping variables $(X_q)^{p_q}$ that fail to improve the model's GOF, and retaining variables $(X_q)^{p_q}$ that most improve the model's GOF. The distinct power coefficients p_q of respective distinct variables $(X_q)^{p_q}$ that are retained when $k = 1$ are kept constant in all the subsequent GOF tests of $(X_q)^{p_q}$ variables with $k > 1$. If GOF tests of $(X_q)^{p_q}$ variables with $k > 1$ improve the model's GOF, then increasing k and continuing stepwise reiterations of these tests, until no further improvements in the model's GOF are achieved. The best-fitting model is also required to enable calculations of post-estimation marginal probabilities and marginal derivatives; if these calculations are not achieved then calculations are attempted with the most preceding improved model until success in such calculations is achieved. Thus, a best-fitting model here is the model whose right-hand side variables X_q and W, power coefficients p_q, and respective limited power series coefficients k and r_q minimize AIC and BIC and enable successful calculations of post-estimation marginal

probabilities and marginal derivatives. Statistical analyses culminate here in selections of a best-fitting model of the selected humans' mortality, a best-fitting model of the selected humans' survivorship, a best-fitting model of the selected insects' mortality, and a best-fitting model of the selected insects' survivorship. The best-fitting models yield z-ratios and respective probabilities $P(|z|)$ for these ratios, where $z = \beta / SE(\beta)$. Coefficients $P(|z|)$ serve here as respective indicators of respective specifications of mortality or survivorship.

Results

Best-fitting models and specifications of mortality and survivorship

The analyses yield a best-fitting multivariable limited powered polynomials random effects logit weighted model of the selected humans' mortality. Table 1 presents respective β, p_q, k, $SE(\beta)$, z, and $P(|z|)$ coefficients of this best-fitting model of the selected humans' mortality. This model is computed on the basis of Model (3) and – employing β coefficients from Table 1 – it is specified with

$$\begin{aligned}\eta_{ij} = {} & 511.78 - 1074.55\left(A_{ij}^{0.16}\right) + 546.12\left(A_{ij}^{0.16}\right)^2 - 17.12\left(L_{ij}^{0.88}\right) \\ & + 0.101\left(L_{ij}^{0.88}\right)^2 + 0.006\left(C_{ij}^{0.75}\right) - (4.39e-7)\left(C_{ij}^{0.75}\right)^2 \\ & + 6.19\left(\Lambda_{ij}^{0.30}\right) - 0.35\left(\Lambda_{ij}^{0.30}\right)^2 - 0.008\left(H_{ij}^{1.41}\right) \\ & + (1.92e-6)\left(H_{ij}^{1.41}\right)^2 - (7.97e-10)\left(H_{ij}^{1.41}\right)^3 \\ & - 1.13\left(F_{ij}\right) + \xi_{ij}\end{aligned} \quad (5)$$

such that $M_{ij} = exp(\eta_{ij}) / \{1 + exp(\eta_{ij})\}$, where i denotes an individual, j is the consecutive number for the respective consecutive observation of this individual's cessation or continuation of existence, M_{ij} denotes the logit fitted probability of mortality of individual i at observation j, A_{ij} denotes the individual's age (in years) at observation j, L_{ij} denotes the individual's lifespan (in years) at observation j, C_{ij} denotes the individual's contemporary aggregate size at observation j, Λ_{ij} denotes the individual's lifespan aggregate size at observation j, H_{ij} denotes the individual's historical time at observation j where j denotes a calendar year transformed to a sequential number, $F_{ij} = 1$ when the individual is female and $F_{ij} = 0$ otherwise, and ξ_{ij} denotes the random effects component corresponding to individual i at observation j.

The analyses also yield a corresponding best-fitting multivariable limited powered polynomials random effects logit weighted model of the selected humans' survivorship. Table 2 presents respective β, p_q, k, $SE(\beta)$, z, and $P(|z|)$ coefficients of this best-fitting model of the selected humans' survivorship. This model is computed on the basis of Model (3) and – employing β coefficients from Table 2 – it is specified with

$$\begin{aligned}\eta_{ij} = {} & -511.78 + 1074.55\left(A_{ij}^{0.16}\right) - 546.12\left(A_{ij}^{0.16}\right)^2 \\ & + 17.12\left(L_{ij}^{0.88}\right) - 0.101\left(L_{ij}^{0.88}\right)^2 - 0.006\left(C_{ij}^{0.75}\right) \\ & + (4.39e-7)\left(C_{ij}^{0.75}\right)^2 - 6.19\left(\Lambda_{ij}^{0.30}\right) + 0.35\left(\Lambda_{ij}^{0.30}\right)^2 \\ & + 0.008\left(H_{ij}^{1.41}\right) - (1.92e-6)\left(H_{ij}^{1.41}\right)^2 \\ & + (7.97e-10)\left(H_{ij}^{1.41}\right)^3 + 1.13\left(F_{ij}\right) + \xi_{ij}\end{aligned} \quad (6)$$

such that $S_{ij} = exp(\eta_{ij}) / \{1 + exp(\eta_{ij})\}$, where S_{ij} denotes the logit fitted probability of survival of individual i at observation j, and all other denotations are as in Model (5).

The analyses yield a best-fitting multivariable limited powered polynomials random effects logit weighted model of the selected insects' mortality. Table 3 presents respective β, p_q, k, $SE(\beta)$, z, and $P(|z|)$ coefficients of this best-fitting model of the selected insects' mortality. This model is computed on the basis of Model (4) and – employing β coefficients from Table 3 – it is specified with

Table 1. Coefficients of the best-fitting multivariable limited powered polynomials random effects weighted logit model of the selected humans' mortality.[1]

Variable	β Index	β	$SE(\beta)$	z-ratio	$P(\|z\|)$	β's 95% Confidence Interval	
Constant	β_0	511.7836	0.577069	886.87	0.00	510.6526	512.9146
$A^{0.16}$	β_{A1}	−1074.55	1.208563	−889.12	0.00	−1076.92	−1072.19
$(A^{0.16})^2$	β_{A2}	546.1184	0.613552	890.09	0.00	544.9158	547.3209
$L^{0.88}$	β_{L1}	−17.1193	0.019416	−881.73	0.00	−17.1574	−17.0813
$(L^{0.88})^2$	β_{L2}	0.100631	0.00012	839.59	0.00	0.100396	0.100866
$C^{0.75}$	β_{C1}	0.006233	2.08e-05	299.18	0.00	0.006192	0.006273
$(C^{0.75})^2$	β_{C2}	−4.39e-07	3.42e-09	−128.68	0.00	−4.46e-07	−4.33e-07
$\Lambda^{0.3}$	$\beta_{\Lambda 1}$	6.186891	0.00919	673.25	0.00	6.16888	6.204902
$(\Lambda^{0.3})^2$	$\beta_{\Lambda 2}$	−0.34869	0.000512	−681.26	0.00	−0.34969	−0.34768
F	β_F	−1.12889	0.004455	−253.37	0.00	−1.13762	−1.12016
$H^{1.41}$	β_{H1}	−0.00784	3.58e-05	−219.24	0.00	−0.00791	−0.00777
$(H^{1.41})^2$	β_{H2}	1.92e-06	2.90e-08	66.06	0.00	1.86e-06	1.97e-06
$(H^{1.41})^3$	β_{H3}	−7.97e-10	7.70e-12	−103.52	0.00	−8.12e-10	−7.82e-10

[1]Variables are right-hand side (rhs) variables of the best-fitting model. Variables include: A denoting age (in years), L denoting lifespan (in years), C denoting contemporary aggregate size, Λ denoting lifespan aggregate size, F denoting sex, and H denoting historical time (i.e., indicated by a specific year). Coefficient β denotes a regression coefficient of the respective best-fitting model, $SE(\beta)$ denotes the standard error of β, z denotes a specific z-ratio calculated with $z = \beta / SE(\beta)$, and $P(|z|)$ denotes a respective probability of $|z|$.

doi:10.1371/journal.pone.0084156.t001

Table 2. Coefficients of the best-fitting multivariable limited powered polynomials random effects weighted logit model of the selected humans' survivorship.[1]

Variable	β Index	β	*SE*(β)	*z-ratio*	*P*(|*z*|)	β's 95% Confidence Interval	
Constant	β_0	−511.784	0.577069	−886.87	0.00	−512.915	−510.653
$A^{0.16}$	β_{A1}	1074.553	1.208563	889.12	0.00	1072.185	1076.922
$(A^{0.16})^2$	β_{A2}	−546.118	0.613552	−890.09	0.00	−547.321	−544.916
$L^{0.88}$	β_{L1}	17.11934	0.019416	881.73	0.00	17.08128	17.15739
$(L^{0.88})^2$	β_{L2}	−0.10063	0.00012	−839.59	0.00	−0.10087	−0.1004
$C^{0.75}$	β_{C1}	−0.00623	2.08e-05	−299.18	0.00	−0.00627	−0.00619
$(C^{0.75})^2$	β_{C2}	4.39e-07	3.42e-09	128.68	0.00	4.33e-07	4.46e-07
$\Lambda^{0.3}$	$\beta_{\Lambda 1}$	−6.18689	0.00919	−673.25	0.00	−6.2049	−6.16888
$(\Lambda^{0.3})^2$	$\beta_{\Lambda 2}$	0.348686	0.000512	681.26	0.00	0.347683	0.349689
F	β_F	1.128888	0.004455	253.37	0.00	1.120156	1.137621
$H^{1.41}$	β_{H1}	0.007839	3.58e-05	219.24	0.00	0.007769	0.007909
$(H^{1.41})^2$	β_{H2}	−1.92e-06	2.90e-08	−66.06	0.00	−1.97e-06	−1.86e-06
$(H^{1.41})^3$	β_{H3}	7.97e-10	7.70e-12	103.52	0.00	7.82e-10	8.12e-10

[1]As in the footnote of Table 1.

doi:10.1371/journal.pone.0084156.t002

Table 3. Coefficients of the best-fitting multivariable limited powered polynomials random effects weighted logit model of the selected insects' mortality.[1]

Variable	β Index	β	*SE*(β)	*z-ratio*	*P*(|*z*|)	β's 95% Confidence Interval	
Constant	β_0	1391.92	8.754245	159.00	0.00	1374.76	1409.08
$A^{0.13}$	β_{A1}	−2648.52	16.6719	−158.86	0.00	−2681.2	−2615.85
$(A^{0.13})^2$	β_{A2}	1295.76	8.161646	158.76	0.00	1279.76	1311.76
$L^{0.98}$	β_{L1}	−16.67	0.106237	−156.94	0.00	−16.88	−16.46
$(L^{0.98})^2$	β_{L2}	0.095159	0.00062	153.51	0.00	0.093944	0.096374
$C^{1.02}$	β_{C1}	−0.00632	0.000103	−61.38	0.00	−0.00652	−0.00612
$(C^{1.02})^2$	β_{C2}	6.85e-07	1.55e-08	44.10	0.00	6.54e-07	7.15e-07
$\Lambda^{0.95}$	$\beta_{\Lambda 1}$	−0.09025	0.001541	−58.58	0.00	−0.09327	−0.08723
$(\Lambda^{0.95})^2$	$\beta_{\Lambda 2}$	0.000263	4.91e-06	53.64	0.00	0.000254	0.000273
F	β_F	−1.82694	0.040958	−44.61	0.00	−1.90722	−1.74667
E_1	β_{E1}	−1.41705	0.096002	−14.76	0.00	−1.60521	−1.22889
E_2	β_{E2}	0.631117	0.102214	6.17	0.00	0.430781	0.831452
E_3	β_{E3}	0.756026	0.099505	7.60	0.00	0.561	0.951052
E_4	β_{E4}	1.597167	0.09196	17.37	0.00	1.416928	1.777405
E_5	β_{E5}	2.7985	0.096029	29.14	0.00	2.610285	2.986714
E_6	β_{E6}	0.996794	0.088613	11.25	0.00	0.823116	1.170472
E_7	β_{E7}	−4.41742	0.104782	−42.16	0.00	−4.62279	−4.21205
E_8	β_{E8}	2.042812	0.100085	20.41	0.00	1.846648	2.238976
E_9	β_{E9}	3.962415	0.093918	42.19	0.00	3.77834	4.146491
E_{10}	β_{E10}	−0.86276	0.10499	−8.22	0.00	−1.06854	−0.65699
E_{11}	β_{E11}	2.069779	0.097441	21.24	0.00	1.878799	2.26076
E_{12}	β_{E12}	1.655298	0.08756	18.9	0.00	1.483683	1.826912

[1]Variables are right-hand side variables of the best-fitting model. Variables' respective designators include: *A* designating age (in days), *L* designating lifespan (in days), and *E* designating environmental context (i.e., indicated by a specific cage index). All else is as in the footnote of Table 1.

doi:10.1371/journal.pone.0084156.t003

$$\begin{aligned}\eta_{ij} = {} & 1391.92 - 2{,}648.52\left(A_{ij}^{\ 0.13}\right) + 1295.76\left(A_{ij}^{\ 0.13}\right)^2 \\ & - 16.67\left(L_{ij}^{\ 0.98}\right) + 0.095\left(L_{ij}^{\ 0.98}\right)^2 - 0.006\left(C_{ij}^{\ 1.02}\right) \\ & + (6.85e-07)\left(C_{ij}^{\ 1.02}\right)^2 - 0.09\left(\Lambda_{ij}^{\ 0.95}\right) \\ & + 0.00026\left(\Lambda_{ij}^{\ 0.95}\right)^2 - 1.83\left(F_{ij}\right) \\ & + \left\{ \begin{array}{l} -1.42(E_{i1}) + 0.63(E_{i2}) + 0.76(E_{i3}) + 1.59(E_{i4}) + 2.80(E_{i5}) \\ +0.997(E_{i6}) - 4.42(E_{i7}) + 2.043(E_{i8}) + 3.96(E_{i9}) \\ -0.863(E_{i10}) + 2.069(E_{i11}) + 1.65(E_{i12}) + 1.00(E_{i13}) \end{array} \right\} \\ & + \xi_{ij} \end{aligned} \tag{7}$$

$$\begin{aligned}\eta_{ij} = {} & -732.74 + 1402.49\left(A_{ij}^{\ 0.16}\right) - 706.62\left(A_{ij}^{\ 0.16}\right)^2 + 19.24\left(L_{ij}^{\ 0.94}\right) \\ & - 0.12\left(L_{ij}^{\ 0.94}\right)^2 + 0.0041\left(C_{ij}^{\ 1.02}\right) - (4.03E-07)\left(C_{ij}^{\ 1.02}\right)^2 \\ & + 0.11\left(\Lambda_{ij}^{\ 0.88}\right) - 0.000491\left(\Lambda_{ij}^{\ 0.88}\right)^2 + 1.28\left(F_{ij}\right) \\ & + \left\{ \begin{array}{l} +0.96(E_{i1}) - 0.669(E_{i2}) - 1.03(E_{i3}) - 0.75(E_{i4}) \\ -1.53(E_{i5}) - 0.0.52(E_{i6}) + 3.25(E_{i7}) - 1.15(E_{i8}) \\ -2.63(E_{i9}) + 0.83(E_{i10}) - 1.12(E_{i11}) - 0.53(E_{i12}) \\ +1.00(E_{i13}) \end{array} \right\} \\ & + \xi_{ij} \end{aligned} \tag{8}$$

where M_{ij}, L_{ij}, C_{ij}, Λ_{ij}, F_{ij}, and ξ_{ij} denote as in Model (5), A_{ij} denotes the age (in days) of individual i at observation j, and coefficients E_{ic} respectively denote the environmental context of an individual i in one of $c = 1{:}13$ cages, such that only one of the 13 terms of coefficients E_{ic} applies to individual i within parentheses {} of Model (7).

The analyses also yield a best-fitting multivariable limited powered polynomials random effects complementary log-log weighted model of the selected insects' survivorship. Table 4 presents respective β, p_q, k, $SE(\beta)$, z, and $P(|z|)$ coefficients of this best-fitting model of the selected insects' survivorship. This model is computed on the basis of Model (4) and – employing β coefficients from Table 4 – it is specified with

such that $S_{ij} = 1 - exp\{-exp(\eta_{ij})\}$, where S_{ij} denotes the complementary log-log fitted probability of survival of individual i at observation j, and all other denotations are as in Model (7).

Coefficients $P(|z|)$ in Tables 1 and 3 provide evidence of respective age-specific, lifespan-specific, contemporary-aggregate-size-specific, historical-time-specific, environmental-context-specific, and sex-specific mortality. Similarly, coefficients $P(|z|)$ in Tables 2 and 4 provide evidence of respective age-specific, lifespan-specific, contemporary-aggregate-size-specific, historical-time-specific, environmental-context-specific, and sex-specific survivorship. Thus, as noted, the best-fitting models yield respective evidence of respective age-specific, lifespan-specific, contemporary-aggregate-size-specific, lifespan-aggregate-size-specific, historical-time-specific, environmental-context-specific, and sex-specific

Table 4. Coefficients of the best-fitting multivariable limited powered polynomials random effects weighted complementary log-log model of the selected insects' survivorship.[1]

Variable	β Index	β	$SE(\beta)$	z-ratio	$P(\|z\|)$	β's 95% Confidence Interval	
Constant	β_0	−732.741	7.57595	−96.72	0.00	−747.59	−717.893
$A^{0.16}$	β_{A1}	1402.486	14.67483	95.57	0.00	1373.72	1431.25
$(A^{0.16})^2$	β_{A2}	−706.621	7.484498	−94.41	0.00	−721.29	−691.951
$L^{0.94}$	β_{L1}	19.23612	0.212374	90.58	0.00	18.81988	19.65237
$(L^{0.94})^2$	β_{L2}	−0.11759	0.001355	−86.8	0.00	−0.12025	−0.11493
$C^{1.02}$	β_{C1}	0.004073	0.000101	40.4	0.00	0.003875	0.004271
$(C^{1.02})^2$	β_{C2}	−4.03e-07	1.49e-08	−26.99	0.00	−4.32e-07	−3.74e-07
$\Lambda^{0.88}$	$\beta_{\Lambda 1}$	0.112845	0.002865	39.38	0.00	0.10723	0.118461
$(\Lambda^{0.88})^2$	$\beta_{\Lambda 2}$	−0.00049	1.38e-05	−35.57	0.00	−0.00052	−0.00046
F	β_F	1.282721	0.041707	30.76	0.00	1.200977	1.364465
E_1	β_{E1}	0.964724	0.10298	9.37	0.00	0.762886	1.166561
E_2	β_{E2}	−0.66944	0.121445	−5.51	0.00	−0.90747	−0.43141
E_3	β_{E3}	−1.03296	0.102814	−10.05	0.00	−1.23448	−0.83145
E_4	β_{E4}	−0.74903	0.115518	−6.48	0.00	−0.97544	−0.52262
E_5	β_{E5}	−1.53755	0.118571	−12.97	0.00	−1.76995	−1.30516
E_6	β_{E6}	−0.52188	0.106935	−4.88	0.00	−0.73147	−0.31229
E_7	β_{E7}	3.253259	0.103586	31.41	0.00	3.050235	3.456284
E_8	β_{E8}	−1.1545	0.141288	−8.17	0.00	−1.43142	−0.87758
E_9	β_{E9}	−2.6341	0.115133	−22.88	0.00	−2.85975	−2.40844
E_{10}	β_{E10}	0.831803	0.116184	7.16	0.00	0.604087	1.059518
E_{11}	β_{E11}	−1.1248	0.131534	−8.55	0.00	−1.3826	−0.867
E_{12}	β_{E12}	−0.53327	0.109138	−4.89	0.00	−0.74718	−0.31936

[1]As in the footnote of Table 3.

doi:10.1371/journal.pone.0084156.t004

mortality and survivorship of the selected humans and the selected insects in this investigation.

Discussion

Making reference to existence of any natural or artificial individual (i.e., individuals from apples to zithers; for example, particles, plants, planets, viruses, insects, humans, bicycles, books, and poems), survivorship refers here to continuation of existence, and mortality refers here to cessation of existence. Every natural or artificial individual is characterized by age, lifespan, contemporary aggregate size, lifespan aggregate size, historical time, and environmental context at every point of continuation of existence and at the point of cessation of existence. Similarly, every sexual individual is characterized by sex at every point of continuation of existence and at the point of cessation of existence. These considerations – and the specifications in the present investigation and in past research – provide guidance to the inclusion of the variables age, lifespan, contemporary aggregate size, lifespan aggregate size, historical time, and environmental context in regression models of mortality or survivorship of diverse kinds of natural or artificial individuals in diverse times and places. Similarly, these considerations – and the specifications in the present investigation and in past research – provide guidance to the inclusion of the variable sex in regression models of mortality or survivorship of diverse kinds of sexual individuals in diverse times and places.

As noted, respective age-specific, historical-time-specific, environmental-context-specific, and sex-specific mortality or survivorship have already been discovered and analyzed in previous empirical research. However, respective lifespan-specific, contemporary-aggregate-size-specific, and lifespan-aggregate-size-specific mortality or survivorship have not been discovered or analyzed in previous empirical research. The present discoveries and analyses of hitherto unknown lifespan-specific, contemporary-aggregate-size-specific, and lifespan-aggregate-size-specific mortality and survivorship reveal that much has yet to be learned about these specifications among diverse kinds of individuals in diverse times and places. Additionally, the new discoveries and analyses strengthen the suggestion that additional specifications of mortality or survivorship have yet to be discovered and analyzed.

As noted, previous research shows that regression models of mortality or survivorship are encumbered by the omitted variables bias [67–71] and by unobserved heterogeneity bias [9,13–16,28]. The present investigation suggests that addition of lifespan, contemporary aggregate size, or lifespan aggregate size variables to the right-hand side of regression models of mortality or survivorship – and employment of multivariable limited powered polynomials regression models – reduce or eliminate these biases. Furthermore, as noted, previous research shows that trajectories of mortality or survivorship tend to be nonlinear [31,32], that mortality and survivorship correspond to power laws and scaling laws [22,26,30,35,48–53,60,72–81], and that previous models of mortality or survivorship are encumbered by the age-period-cohort problem of separating the effects of age-groups, periods, and cohorts in regression models [23,36–40]. The present investigation suggests that multivariable limited powered polynomials binary regression models of mortality or survivorship help capture nonlinearity, contribute to analyses of power laws and scaling laws, and provide a useful solution to the age-period-cohort problem. These suggestions can now be investigated further.

This investigation and previous research elucidate – and are elucidated by – considerations of frailty and vitality. Frailty is typically conceptualized in negative terms conveying vulnerability, susceptibility, weakness, debility, defenselessness, helplessness, exposure, liability, lack, absence, decay, decline, exhaustion, or depletion. Vitality is typically conceptualized in positive terms conveying liveliness, vigor, strength, resistance, robustness, animation, verve, dynamism, vim, resistance, success, accomplishment, achievement, or expansion. Notions of frailty and vitality have been prevalent in many cultures throughout human evolution and history, as exemplified by expressions of these notions in vitalism, *yinyang*, *élan vital*, or *conatus*. There is ample research on frailty in mortality or survivorship [14–16,19,64,86–91], and there is also ample research on vitality in mortality or survivorship [1,2,11,12,19,22,28,92–115]. Previous research posits the existence of an "inherent vitality" that is defined as "the total potential capacity of an [individual] to perform vital actions, in the complete absence of matter or energy of exogenous derivation" ([2], pp. 108, 147). This conception of inherent vitality – together with general notions of frailty and vitality as well as previous research on frailty and vitality in mortality and survivorship – suggest that each natural or artificial individual is characterized by lifespan-specific vitality and a corresponding lifespan-specific frailty. Additionally, previous research focuses on attritions of individuals from respective frailty-based aggregates – and retentions of individuals in respective vitality-based aggregates – through the life course [9,10,13–16,28,64,89,90,100,116–127], suggesting an affinity between frailty and aggregate-size-specific mortality and suggesting an affinity between vitality and aggregate-size-specific survivorship. These suggestions are generalized here by positing that specifications of mortality indicate corresponding specifications of frailty and by positing that specifications of survivorship indicate corresponding specifications of vitality. These suggested generalizations reveal that much remains to be learned about specifications of mortality, survivorship, frailty, and vitality of diverse kinds of individuals in diverse times and places.

Author Contributions

Conceived and designed the experiments: ME. Performed the experiments: ME. Analyzed the data: ME. Contributed reagents/materials/analysis tools: ME. Wrote the paper: ME.

References

1. Gompertz B (1825) On the nature of the function expressive of the law of human mortality, and on a new mode of determining the value of life contingencies. Philosophical Transactions of the Royal Society of London 115: 513–585.
2. Pearl R (1928) The Rate of Living. New York: Alfred A. Knopf.
3. Perks W (1932) On some experiments in the graduation of mortality statistics. Journal of the Institute of Actuaries 63: 12–57.
4. Greenwood M, Irwin JO (1939) The biostatistics of senility. Human Biology 11: 1–23.
5. Beard RE (1959) Note on some mathematical mortality models. In: Wolstenholme GEW, Cameron MP, editors. Ciba Foundation Colloquia on Aging: The Lifespan of Animals. Boston: Little, Brown. pp. 302–311.
6. Beard RE (1964) Some observations on stochastic processes with particular reference to mortality studies. International Congress of Actuaries 3: 463–477.
7. Beard RE (1971) Some aspects of theories of mortality, cause of death analysis, forecasting and stochastic processes. In: Brass W, editor. Biological Aspects of Demography. London: Taylor & Francis. pp. 57–68.
8. Bebbington M, Lai C-D, Zitikis RA (2011) Modelling deceleration in senescent mortality. Mathematical Population Studies 18: 18–37.
9. Carey JR, Liedo P, Orozco D, Vaupel JW (1992) Slowing of mortality rates at older ages in large medfly cohorts. Science 258: 457–461.
10. Carey JR (2003) Longevity: The Biology and Demography of Life Span. Princeton: Princeton University Press. 278 p.
11. Strehler BL, Mildvan AS (1960) General theory of mortality and aging. Science 132: 14:21.

12. Strehler BL (1977) Time, Cells, and Aging. New York: Academic Press.
13. Manton GK, Stallard E (1984) Recent Trends in Mortality Analysis. Orlando: Academic Press. 342 p.
14. Vaupel JW, Carey JR, Christensen K, Johnson TE (1998) Biodemographic trajectories of longevity. Science 280: 855–860.
15. Vaupel JW, Manton KG, Stallard E (1979) The impact of heterogeneity in individual frailty on the dynamics of mortality. Demography 16: 439–454.
16. Vaupel JW, Yashin AI (1985) Heterogeneity's ruses: Some surprising effects of selection on population dynamic. The American Statistician 39: 176–185.
17. Wrigley EA, Oeppen JE, Schofield RS (1997) English Population History from Family Reconstitution 1580–1937. Cambridge: Cambridge University Press.
18. Medawar PB ([1946] 1957) Old age and natural death. In: Medawar PB, editor. The Uniqueness of the Individual. London: Methuen. pp. 17–43.
19. Medawar PB ([1952] 1957) An unsolved problem in biology. In: Medawar PB, editor. The Uniqueness of the Individual. London: Methuen. pp. 44–70.
20. Finch CE (1990) Longevity, Senescence, and the Genome. Chicago: University of Chicago Press.
21. Kirkwood TBL (1996) Human senescence. BioEssays 18: 1009–1016.
22. Atlan H, Miquel J, Helmle LC, Dolkas CB (1976) Thermodynamics of aging in Drosophila melanogaster. Mechanisms of Ageing and Development 5: 371–387.
23. O'Brien RM, Hudson K, Stockard J (2008) A mixed model estimation of age, period, and cohort effects. Sociological Methods Research 36: 402–428.
24. Carnes BA, Olshansky SJ, Grahn D (1996) Continuing the search for a law of mortality. Population and Development Review 22: 231–264.
25. Hamilton WD (1966) The moulding of senescence by natural selection. Journal of Theoretical Biology 12: 12–45
26. Juckett DA, Rosenberg B (1982) The kinetics and thermodynamics of lysis of young and old sheep red blood cells. Mechanisms of Ageing and Development 18: 33–45.
27. Koenker R, Geling O (2001) Reappraising medfly longevity. Journal of the American Statistical Association 96: 458–468.
28. Li T, Anderson JJ (2009) The vitality model: A way to understand population survival and demographic heterogeneity. Theoretical Population Biology 76: 118–131.
29. Milne EMG (2008) The natural distribution of survival. Journal of Theoretical Biology 255: 223–236.
30. Rosenberg B, Kemeny G, Smith LG, Skurnick ID, Bandurski MJ (1973) The kinetics and thermodynamics of death in multicellular organisms. Mechanisms of Ageing and Development 2: 275–293.
31. Royston P, Altman DG (1994) Regression using fractional polynomials of continuous covariates: parsimonious parametric modelling (with discussion). Journal of the Royal Statistical Society Series C (Applied Statistics) 43: 429–467.
32. Royston P, Sauerbrei W (2008) Multivariable Model-Building: A Pragmatic Approach to Regression Analysis Based on Fractional Polynomials for Modelling Continuous Variables. Chichester, England: John Wiley.
33. Williams GC (1957) Pleiotropy, natural selection, and the evolution of senescence. Evolution 11: 398–411.
34. Wong E, Wang B, Garrison L, Alfonso-Cristancho R, Flum D, et al. (2011) Examining the BMI-mortality relationship using fractional polynomials. BMC Medical Research Methodology 11: 175.
35. Charnov EL (1993) Life History Invariants; Some Explorations of Symmetry in Evolutionary Ecology. Oxford: Oxford University Press. 167 p.
36. Greenberg BG, Wright JJ, Sheps CG (1950) A technique for analyzing some factors affecting the incidence of syphilis. Journal of the American Statistical Association 25: 373–399.
37. Hobcraft J, Menken J, Preston S (1982) Age, period, and cohort as sources of variation in demography. Population Index 48: 4–43.
38. Mason KO, Mason WM, Winsborough HH, Poole K (1973) Some methodological issues in cohort analysis of archival data. American Sociological Review 38: 242–258.
39. Mason WM, Feinberg SE (1985) Cohort Analysis in Social Research: Beyond the Identification Problem. New York: Springer-Verlag.
40. O'Brien RM (2011) The age period cohort conundrum as two fundamental problems. Quality & Quantity: 1–16.
41. Sinclair ARE (1988) Population regulation in animals. In: Cherrett JM, editor. Ecological Concepts: The Contribution of Ecology to an Understanding of the Natural World. Oxford: Blackwell. pp. 197–241.
42. Lindheim R, Syme SL (1983) Environments, people, and health. Annual Review of Public Health 4: 335–359.
43. Cohen MN (1989) Health and the Rise of Civilization. New Haven: Yale University Press.
44. Vasi F, Travisano M, Lenski RE (1994) Long-term experimental evolution in Escherichia coli. II. Changes in life-history traits during adaptation to a seasonal environment. The American Naturalist 144: 432–456.
45. Riley JC (2001) Rising Life Expectancy: A Global History. Cambridge: Cambridge University Press.
46. Riley JC (2008) Low Income, Social Growth, and Good Health: A History of Twelve Countries. Berkeley: University of California Press.
47. Lenski RE (2011) Evolution in action: A 50,000-generation salute to Charles Darwin. Microbe 6: 30–33.
48. Brown JH, West GB (2000) Scaling in Biology. Oxford: Oxford University Press. 352 p.
49. Calder WAI (1983) Body size, mortality, and longevity. Journal of Theoretical Biology 102: 135–144.
50. McGurk MD (1986) Natural mortality of marine pelagic fish eggs and larvae: Role of spatial patchiness. Marine Ecology Progress Series 34: 227–242.
51. Rossetto M, De Leo GA, Bevacqua D, Micheli F (2012) Allometric scaling of mortality rates with body mass in abalones. Oecologia 168: 989–996.
52. Savage VM, Gillooly JF, Woodruff WH, West GB, Allen AP, et al. (2004) The predominance of quarter-power scaling in biology. Functional Ecology 18: 257–282.
53. West GB, Brown JH, Enquist BJ (1997) A general model for the origin of allometric scaling laws in biology. Science 276: 122–126.
54. Owens PF (2002) Sex differences in mortality rates. Science 297: 2008–2009.
55. Macunovich DJ (1999) Relative cohort size: Source of a unifying theory of global fertility transition. Center for Policy Research 139.
56. Kermack WO, McKendrick AG, McKinlay PL (1934) Death-rates in Great Britain and Sweden and some general regularities and their significance. The Lancet 223: 698–703.
57. MacMahon B, Trichopoulos D (1996) Epidemiology: Principles and Methods. Boston: Little, Brown and Company.
58. Lee RD (1987) Population dynamics of humans and other animals. Demography 24: 443–465.
59. Malthus TR ([1798] 1992) An Essay on the Principle of Population; Winch D, editor. Cambridge: Cambridge University Press. 392 p.
60. Gillis DM, Kramer DL, Bell G (1986) Taylor's power law as a consequence of Fretwell's ideal free distribution. Journal of Theoretical Biology 123: 281–287.
61. World Health Organization, International Classification of Diseases website. Available: http://www.who.int/classifications/icd/en/. Accessed 2013 March 12.
62. Carnes B (2011) What is lifespan regulation and why does it exist? Biogerontology 12: 367–374.
63. Rasmussen S, Bedau MA, Chen L, Deamer D, Krakauer DC, et al, editors(2009) Protocells: Bridging Non-living and Living Matter. Cambridge: MIT Press.
64. Duchateau L, Janssen P (2008) The Frailty Model. New York: Springer.
65. Human Mortality Database website. Available: www.mortality.org. Accessed 2010 May 2.
66. Author received a copy of the 1991 Moscamed data from Professor James R. Carey, Department of Entomology, University of California, Davis, California, USA, on June 2, 1997.
67. Theil H (1957) Specification errors and the estimation of economic relationships. Revue de l'Institut International de Statistique/Review of the International Statistical Institute 25: 41–51.
68. Heckman JJ (1979) Sample selection bias as a specification error. Econometrica 47: 153–161.
69. Heckman JJ (1981) Heterogeneity and state dependence. In: Rosen S, editor. Studies in Labor Markets. Chicago: University of Chicago Press. pp. 91–140.
70. Heckman JJ, Singer B (1986) Econometric analysis of longitudinal data. In: Grilliches Z, lntriligator MD, editors. Handbook of Econometrics. New York: Elsevier. pp. 1690–1763.
71. Greene WH (2008) Econometric Analysis. New York: Prentice-Hall.
72. Epelbaum M (1990) Sociomonetary patterns and specifications. Social Science Research 19: 322–347.
73. Becerra Ó, Johnson N, Meier P, Restrepo J, Spagat M (2006) Natural disasters, casualties and power laws: A comparative analysis with armed conflict. Presented at the Annual Conference of American Political Science Association. Available on website http://irevolution.files.wordpress.com/2011/07/apsa-paper-2006-harvard.pdf. Accessed 2012 March 5.
74. Bigger JT, Steinman RC, Rolnitzky LM, Fleiss JL, Albrecht P, et al. (1996) Power law behavior of RR-interval variability in healthy middle-aged persons, patients with recent acute myocardial infarction, and patients with heart transplants. Circulation 93: 2142–2151.
75. Clauset A, Shalizi CR, Newman MEJ (2009) Power-law distributions in empirical data. SIAM Review 51: 661–703.
76. Clauset A, Wiegel FW (2010) A generalized aggregation-disintegration model for the frequency of severe terrorist attacks. Journal of Conflict Resolution 54: 179–197.
77. Clauset A, Young M, Gleditsch KS (2007) On the frequency of severe terrorist events. Journal of Conflict Resolution 51: 58–87.
78. Richardson LF (1960) Statistics of Deadly Quarrels. Pittsburgh: The Boxwood Press.
79. Novozhilov AS, Karev GP, Koonin EV (2006) Biological applications of the theory of birth-and-death processes. Briefings in Bioinformatics 7: 70–85.
80. Olivola CY, Sagara N (2009) Distributions of observed death tolls govern sensitivity to human fatalities. Proceedings of the National Academy of Sciences 106: 22151–22156.
81. Bevacqua D, Melià P, De Leo GA, Gatto M (2011) Intra-specific scaling of natural mortality in fish: the paradigmatic case of the European eel. Oecologia 165: 333–339.
82. Jenkins SP (2008) Survival Analysis with Stata: Module EC968. Available on website: http://www.iser.essex.ac.uk/resources/survival-analysis-with-stata-module-ec968. Accessed 2011 May 15.
83. StataCorp (2009) Stata Statistical Software Release 11. College Station, TX: Stata Corporation.

84. Akaike H (1973) Information theory and an extension of the maximum likelihood principle. In: Petrov B, Csaki F, editors. Second International Symposium on Information Theory. Budapest: Akademiai Kiado. pp. 267–281.
85. Raftery AE (1996) Bayesian model selection in social research. In: Marsden PC, editor. Sociological Methodology. Oxford: Basil Blackwell. pp. 111–163.
86. Swinne C, Cornette P, Schoeverdts D, Latteur V, Melon C (1998) Frailty in the medical literature. Age and Ageing 27: 411–413
87. Kannisto V (1991) Frailty and survival. Genus 47: 101–118.
88. Bergman H, Ferrucci L, Guralnik J, Hogan DB, Hummel S, et al. (2007) Frailty: An emerging research and clinical paradigm: Issues and controversies. The Journals of Gerontology Series A: Biological Sciences and Medical Sciences 62: 731–737.
89. Steinsaltz DR, Wachter KW (2006) Understanding mortality rate deceleration and heterogeneity. Mathematical Population Studies 13: 19–37.
90. Yashin AI, Iachine IA, Begun AS (2000) Mortality modeling: A review. Mathematical Population Studies 8: 305–332.
91. Golubev A (2009) How could the Gompertz-Makeham law evolve. Journal of Theoretical Biology 258: 1–17.
92. Atlan H (1968) Strehler's theory of mortality and the second principle of thermodynamics. Journal of Gerontology A 23: 196–200.
93. Miquel J, Lundgren PR, Bensch KG, Atlan H (1976) Effects of temperature on the life span, vitality and fine structure of Drosophila melanogaster. Mechanisms of Ageing and Development 5: 347–370.
94. Corbaux F (1833) On the Natural and Mathematical Laws Concerning Population, Vitality, and Mortality. London: W. Wilson.
95. Gompertz B (1862) A Supplement to two papers published in the Transactions of the Royal Society, 'on the science connected with human mortality;' the one published in 1820, and the other in 1825. Philosophical Transactions of the Royal Society of London 152: 511–559.
96. Hooker PF (1965) Benjamin Gompertz: March 5, 1779–July 14, 1865. Journal of the Institute of Actuaries 91: 203–212.
97. Makeham WM (1867) On the law of mortality and the construction of annuity tables. Journal of the Institute of Actuaries and Assurance 13: 325–367.
98. Makeham WM (1889) On the further development of Gompertz's law. Journal of the Institute of Actuaries 28: 152–160, 185–192.
99. Olshansky SJ, Carnes BA (1997) Ever since Gompertz. Demography 34: 1–15.
100. Anderson JJ, Gildea MC, Williams DW, Li T (2008) Linking growth, survival, and heterogeneity through vitality. The American Naturalist 171: E20–E43.
101. Pearl R, Miner JR (1935) Experimental studies on the duration of life. Quarterly Review of Biology 10: 60–79.
102. Brody S (1923) The kinetics of senescence. Journal of General Physiology 6: 245–257.
103. Failla G (1958) The aging process and cancerogenesis. Annals of the New York Academy of Sciences 71: 1124–1140.
104. Jones HB (1956) A special consideration of the aging process, disease and life expectancy. Advances in Biological and Medical Physics 4: 281–337.
105. Simms HS (1946) Logarithmic increase in mortality as a manifestation of aging. Journal of Gerontology 1: 13–26.
106. Zheng H, Yang Y, Land K (2011) Heterogeneity in the Strehler-Mildvan general theory of mortality and aging. Demography 48: 267–290.
107. Anderson JJ (1992) A vitality-based stochastic model for organism survival. In: DeAngelis DL, Gross LJ, editors. Individual-based models and approaches in ecology. New York: Chapman & Hall. pp. 256–277.
108. Anderson JJ (2000) Vitality-based model relating stressors and environmental properties to organism survival. Ecological Monographs 70: 445–470.
109. Zuev SM, Yashin AI, Manton KG, Dowd E, Pogojev IB, et al. (2000) Vitality index in survival modeling: how physiological aging influences mortality. Journal of Gerontology A 55: B10–19.
110. Ruggiero C, Metter EJ, Melenovsky V, Cherubini A, Najjar SS, et al. (2008) High basal metabolic rate is a risk factor for mortality: The Baltimore longitudinal study of aging. Journal of Gerontology A: Biological Social Medical Sciences 63: 698–706.
111. Economos A (1981) Beyond rate of living. Gerontology 27: 258–265.
112. Economos AC (1982) Rate of aging, rate of dying and the mechanism of mortality. Archives of Gerontology and Geriatrics 1: 3–27.
113. Lints FA (1989) The rate of living theory revisited. Gerontology 35: 36–57.
114. Johnson SE, Abrams MD (2009) Age class, longevity and growth rate relationships: protracted growth increases in old trees in the eastern United States. Tree Physiology 29: 1317–1328.
115. Comfort A (1957) The biological approach in the comparative study of ageing. In: Wolstenholme GEW, O'Connor CM, editors. CIBA Foundation Colloquia on Ageing. Boston: Little, Brown and Company. pp. 2–19.
116. Curtsinger JW, Fukui HH, Townsend DR, Vaupel JW (1992) Demography of genotypes: failure of the limited life-span paradigm in Drosophila melanogaster. Science 258: 461–463.
117. Carnes BA, Olshansky SJ (2001) Heterogeneity and its biodemographic implications for longevity and mortality. Experimental Gerontology 36: 419–430.
118. Rossolini G, Piantanelli L (2001) Mathematical modeling of the aging processes and the mechanisms of mortality: paramount role of heterogeneity. Experimental Gerontology 36: 1277–1288.
119. Service PM (2000) Heterogeneity in individual mortality risk and its importance for evolutionary studies of senescence. The American Naturalist 156: 1–13.
120. Service PM (2004) Demographic heterogeneity explains age-specific patterns of genetic variance in mortality rates. Experimental Gerontology 39: 25–30.
121. Steinsaltz D (2005) Re-evaluating a test of the heterogeneity explanation for mortality plateaus. Experimental Gerontology 40: 101–113.
122. Wu D, Cypser JR, Yashin AI, Johnson TE (2009) Multiple mild heat-shocks decrease the Gompertz component of mortality in Caenorhabditis elegans. Experimental Gerontology 44: 607–612.
123. Wu D, Rea SL, Yashin AI, Johnson TE (2006) Visualizing hidden heterogeneity in isogenic populations of C. elegans. Experimental Gerontology 41: 261–270.
124. Hougaard P (1984) Life table methods for heterogeneous populations: Distributions describing the heterogeneity. Biometrika 71: 75–83.
125. Manton GK, Stallard E (1984) Heterogeneity and its effect on mortality measurement. In: Vallin J, Pollard JH, Heligman L, editors. Methodologies for the Collection and Analysis of Mortality Data. Liège, Belgium: Ordina Editions. pp. 265–301.
126. Manton GK, Stallard E, Vaupel JW (1986) Alternative models for the heterogeneity of mortality risk among the aged. Journal of the American Statistical Association 81: 635–644.
127. Clayton DG (1978) A model for association in bivariate life tables and its application in epidemiological studies of familial tendency in chronic disease incidence. Biometrika 65: 141–151.